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Heavy Meson Radiative Decays and Light Vector Meson Dominance

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(May, 1994)

Abstract

Electromagnetic interactions are introduced in the effective chiral Lagrangian for heavy mesons which includes light vector particles. A suitable notion of vector meson dominance is formulated. The constraints on the heavy meson-light vector and heavy meson-light pseudoscalar coupling constants are obtained using experimental $D^* \to D \gamma$ branching ratios. These constraints are compared with values estimated from semi-leptonic transition amplitudes as well as from extension of the light meson coupling pattern. Application to the heavy baryon spectrum in the “bound state” model is made.
I. INTRODUCTION

Effective Lagrangians combining heavy quark symmetry and chiral invariance [1] provide promising tools for understanding the “soft” interactions of the heavy mesons. The apparent dominance of the decays \( D \rightarrow \bar{K}^* l\nu \) over \( D \rightarrow \bar{K}\pi l\nu \) [2], as well as general considerations, suggest the inclusion of the light vector mesons in addition to the light pseudoscalars. The total Lagrangian is the sum of a “light” part describing the three flavors \( u, d, s \) and a “heavy” part describing the “heavy” meson multiplet \( H \) and its interaction with the light sector:

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}}. \tag{1.1}
\]

The relevant light fields belong to the 3 \( \times \) 3 matrix of pseudoscalars, \( \phi \), and to the 3 \( \times \) 3 matrix of vectors, \( \rho_\mu \). It is convenient to define objects which transform simply under the action of the chiral group,

\[
\xi = \exp(i\phi/F_\pi), \quad U = \xi^2, \\
A_\mu^L = \xi\rho_\mu\xi^\dagger + \frac{i}{\tilde{g}}\xi\partial_\mu\xi^\dagger, \\
A_\mu^R = \xi^\dagger\rho_\mu\xi + \frac{i}{\tilde{g}}\xi^\dagger\partial_\mu\xi, \\
F_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu], \tag{1.2}
\]

where \( F_\pi \approx 0.132 \text{ GeV} \) and \( \tilde{g} \approx 3.93 \) for a typical fit.

The heavy multiplet field combining the heavy pseudoscalar \( P' \) and the heavy vector \( Q'_\mu \), both moving with a fixed 4-velocity \( V_\mu \) is given by:

\[
H = \frac{1}{2} - i\gamma_\mu V_\mu(i\gamma_5 P' + i\gamma_\nu Q'\nu), \quad \bar{H} \equiv \gamma_4 H^\dagger \gamma_4. \tag{1.3}
\]

In our convention \( H \) has the canonical dimension one.

The light part of the action under consideration has been most recently discussed in [3]. Apart from SU(3) and chiral symmetry breaking terms and terms proportional to the Levi-Civita symbol, it may be written as

\[
\mathcal{L}_{\text{light}} = -\frac{1}{4} Tr(F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)) - \frac{m_v^2(1 + k)}{8k} Tr(A_\mu^L A_\mu^L + A_\mu^R A_\mu^R) + \frac{m_v^2(1 - k)}{4k} Tr(A_\mu^L U A_\mu^R U^\dagger), \tag{1.4}
\]

where \( m_v \approx 0.77 \text{ GeV} \) is the light vector mass and \( k = \frac{m_v^2}{(F_\pi \tilde{g})^2} \). An alternate “hidden symmetry” approach [4] leads to the identical Lagrangian.

\( \mathcal{L}_{\text{heavy}} \) has been discussed by several authors [3] [8]. Following the notations of ref. [3], it is, to leading order in \( M \):

\[
\frac{\mathcal{L}_{\text{heavy}}}{M} = iV_\mu Tr \left[ H (\partial_\mu - i\alpha \tilde{g} \rho_\mu - i(1 - \alpha)v_\mu) \bar{H} \right] + id Tr \left[ H \gamma_\mu \gamma_5 p_\mu \bar{H} \right] + \frac{ic}{m_v} Tr \left[ H \gamma_\mu \gamma_5 F_{\mu\nu}(\rho) \bar{H} \right], \tag{1.5}
\]
where $M$ is the mass of the heavy meson and
\[ v_\mu, p_\mu \equiv \frac{i}{2} (\xi \partial_\mu \xi^\dagger \pm \xi^\dagger \partial_\mu \xi). \] (1.6)

$\alpha, c, d$ are dimensionless coupling constants for the heavy-light interactions; they are crucial for discussing the soft dynamics of the heavy mesons as well as other applications. In particular, we are interested in the dynamics of the heavy baryons in the bound state model, wherein all three parameter enter in an important way. The object of this note is to obtain useful restrictions on these three parameters for the purpose of studying the heavy baryons.

A key ingredient in the analysis will be estimates of the $D^* \to D \gamma$ rates based on a suitable notion of vector meson dominance for the electromagnetic interactions of heavy mesons. Analogous restrictions have been discussed [9–11] for the model in which light vectors are not present. The model where vectors are included was used to calculate the rates $D^* \to D \gamma$ [12] with $c$ and $d$ obtained from assumed pole fits to semi-leptonic form factors. Especially for $c$, there is no detailed experimental confirmation that the form factors have exactly the pole dependence. Hence, it is desirable to proceed in a more general way. Here we shall use a similar model to get estimated bounds on $c$ and $d$. In addition, we will formulate the model in such a way that vector meson dominance can be relaxed and deviations calculated.

At the present stage it seems most reasonable to work in the leading order in $M$ as well as tree order in the light fields. Before proceeding we make some preliminary remarks about the coupling constants.

The coupling constant $d$ in (1.3) is related to the $D^* \to D \pi$ decay widths as follows,
\[
\begin{align*}
\Gamma(D^{*0} \to D^0 \pi^0) &= \frac{d^2 p_{00}^3}{12\pi F_\pi^2}, \\
\Gamma(D^{*+} \to D^0 \pi^+) &= \frac{d^2 p_{0+}^3}{6\pi F_\pi^2}, \\
\Gamma(D^{*+} \to D^+ \pi^0) &= \frac{d^2 p_{+0}^3}{12\pi F_\pi^2},
\end{align*}
\] (1.7)

where $p_{ab}$ is the decay 3-momentum in the parent rest frame for the $D^a \pi^b$ final state. Note that $D^{*0} \to D^+ \pi^-$ in addition to all of the $B^* \to B \pi$ decays are energetically forbidden. $D_{s^{*+}} \to D_{s^+} \pi^0$ is energetically allowed but is suppressed [13] due to isospin conservation and has not yet been observed. An experimental bound, $\Gamma_{total}(D^{*+}) < 131$ KeV [23], gives using (1.7), the restriction
\[ |d| < 0.70 \] (1.8)

If we were to go to higher orders in a $\frac{1}{M}$ expansion, the $d$ appearing in (1.7) and (1.8) should be interpreted as an effective one [24] which deviates by a small amount from the $d$ parameter defined in (1.3).

The coupling constant $\alpha$ in (1.3) was introduced in [6] as a measure of vector meson dominance in the light-heavy direct interaction. We will also verify here that $\alpha = 1$ corresponds to vector meson dominance for the diagonal matrix elements of the light electromagnetic current between heavy meson states, $\langle A | J_{\mu}^{\text{light}} | A \rangle$, where $A = P$ or $Q_\mu$. 

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In section II we briefly discuss some general features of heavy meson radiative decays in a preliminary way using the constituent quark model. While the vector meson dominance approach yields amplitudes with the same general structure, and so cannot be considered essentially superior just for the purpose of calculating these decays, it does express the amplitudes in terms of the coupling constant $c$ which is of great interest. The point is made that, even though it is perhaps unmerited by the “state of the art” (including experiments) to go beyond a leading order in $M$ calculation, there is an unavoidable “accidentally” important subleading contribution due to the fact that two different pieces of the electromagnetic current are being probed.

In section III we discuss how to add the electromagnetic interactions to the effective chiral Lagrangian. The parameter choices which yield vector meson dominance are noted. In this limit and in the leading order of the chiral expansion we compute the $D^* \to D \gamma$ amplitudes. Comparing with the existing experimental data yields somewhat restrictive bounds on the coupling constants $d$ and $c$ ($d$ enters the problem because present experiments give values for the branching ratios but not the total rates).

In section IV, the allowed region for $c$ and $d$ is compared with two attempts at determining these parameters. It is found that if they are obtained by considering the strange quark to be the heavy one, we get a point slightly outside the allowed region. Estimates of $d$ and $c$ based on fitting the $D \to K^*$ and $D \to K$ semi-leptonic decays to pole-form are noted to lie within the allowed region. The constraints are also used to limit the prediction of the heavy baryon mass spectrum in the soliton “bound state” model.

II. HEAVY VECTOR MESON RADIATIVE DECAYS

The decays of the type $D^* \to D \gamma$ are governed by the fundamental electromagnetic interaction:

$$\mathcal{L}_{EM} = eJ^EM_\mu A_\mu$$

where $e$ is the proton charge and $A_\mu$ is the photon field. It is important to note that we need both the light and the heavy pieces in the decomposition:

$$J^EM_\mu = J^{light}_\mu + J^{heavy}_\mu,$$

$$J^{light}_\mu = i \left[ \frac{2}{3} \bar{u}_\mu u - \frac{1}{3} \left( \bar{d}_\mu d + \bar{s}_\mu s \right) \right],$$

$$J^{heavy}_\mu = iCQ_\mu Q + \cdots ,$$

where $C$ is the electric charge of the particular heavy quark under consideration (e.g. $\frac{2}{3}$ for the $c$ quark).

For orientation purposes it is useful to consider how the $D^* \to D \gamma$ decays are computed in the simplest non-relativistic “constituent” quark model [14]. The interaction Hamiltonian is $-\mu \cdot B$ where the magnetic moment operator is

$$\mu = e \left( \frac{C}{M} S^{heavy} + \frac{\bar{q}}{m} S^{light} \right),$$
while $M$ and $m$ are the heavy and light constituent quark masses respectively. $q$ is the charge of the light anti-quark in the heavy meson. The two terms in (2.3) illustrate the decomposition of $J_{\mu}^{EM}$ in (2.2). After sandwiching (2.3) between vector and pseudoscalar spin wavefunctions we find that the amplitude for $D^* \to D \gamma$ is proportional to $(\frac{C}{M} - \frac{m}{M})$. This means that the amplitudes for $D^0 \to D^0 \gamma$, $D^+ \to D^+ \gamma$, and $D_s^+ \to D_s^+ \gamma$ stand in the ratios:

$$(1 + \frac{m}{M}) : (-\frac{1}{2} + \frac{m}{M}) : (-\frac{m}{2m_s} + \frac{m}{M}) \quad (2.4)$$

where $m_s$ is the constituent strange quark mass. (For comparison the corresponding radiative amplitudes for $\bar{B}^* \to \bar{B} \gamma$, $\bar{B}^0 \to \bar{B}^0 \gamma$ and $\bar{B}^{*0} \to \bar{B}_s^0 \gamma$ stand in the ratios \((-2 + \frac{m}{M}) : (1 + \frac{m}{M}) : (\frac{m}{m_s} + \frac{m}{M})\)). Now we would like to start working in the leading $M \to \infty$ limit. However, (2.4) indicates that this is a rather questionable approximation for $D^* \to D \gamma$ since $m \approx 0.35$ GeV while $M$ is in the 1.5 - 1.8 GeV range. In the case of $D^* \to D^+ \gamma$ the piece proportional to $\frac{1}{M}$ is expected to be almost half of the leading term and opposite in sign.

We note that the structures of $J_{\mu}^{\text{light}}$ and $J_{\mu}^{\text{heavy}}$ are essentially different so there is no reason to expect that $J_{\mu}^{\text{light}}$ has large $\frac{1}{M}$ corrections. Thus we consider the present calculation to be correct to order 1, but wherein the “accidentally” important $\frac{1}{M}$ contamination due to the leading $J_{\mu}^{\text{heavy}}$ has been included. That $\frac{1}{M}$ piece is fixed \cite{10} from its relation to the heavy quark number current and corresponds to a term in the effective Lagrangian:

$$-\frac{1}{2}eCA_{\mu}Tr[\partial_{\nu}(H\bar{H}\sigma_{\mu\nu})]. \quad (2.5)$$

III. VECTOR MESON DOMINANCE

First, let us review how to add electromagnetic interactions to the light particle Lagrangian (1.4). More details are given in \cite{15} and in section III of \cite{16}. The fields $A_{\mu}^{L}$ and $A_{\mu}^{R}$ introduced in (1.2) are taken to transform under a local chiral transformation, $U_{L,R} = 1 + E_{L,R}$, as

$$\delta A_{\mu}^{L,R} = -[A_{\mu}^{L,R}, E_{L,R}] - \frac{i}{\hat{g}} \partial_{\mu}E_{L,R}. \quad (3.1)$$

External fields, $B_{\mu}^{L,R}$ transform as

$$\delta B_{\mu}^{L,R} = -[B_{\mu}^{L,R}, E_{L,R}] - \frac{i}{\hat{h}} \partial_{\mu}E_{L,R}, \quad (3.2)$$

where $h$ is the external field coupling constant. Then it is clear that (1.4) will become locally gauge invariant if we make the substitutions $A_{\mu}^{L,R} \to A_{\mu}^{L,R} - \frac{\hat{h}}{\hat{g}} B_{\mu}^{L,R}$. Here we are interested in the case of electromagnetism, which corresponds to the choice

\footnote{\hat{g} is called $g$ in these references}
\[ hB_{\mu}^{L,R} = eQA_\mu, \quad Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}). \]  

The resulting electromagnetic interaction piece from (1.4) may be expanded out (see (14) of [15]) to yield the leading relevant terms

\[ eA_\mu[k\tilde{g}F_\pi^2 Tr(Q\rho_\mu) + i(1 - \frac{k}{2}) Tr[Q(\phi\partial_\mu\phi - \partial_\mu\phi\phi)] + \cdots]. \]  

It is possible to eliminate the photon-vector meson cross term by rediagonalization but it is more conventional to keep it in this form. Notice that the special choice \( k = 2 \) is denoted the Kawarabayashi-Suzuki-Riazuddin-Fayazuddin (KSRF) relation [17]. When this holds the second term in (3.4) vanishes and the photon couples to the charged light pseudoscalars via its mixing with the light neutral vector mesons in the first term. Actually \( k \) seems to be 10% higher than the value required by the KSRF relation so that this picture is reasonably good but not perfect.

A leading \( O(M^0) \) contribution to the \( D^* \to D \gamma \) decays arises via the \( c \) term in (1.5) (which is locally chiral invariant) giving \( D^* \rho_0^0 \) and \( D^* \omega_0^0 \) vertices followed by the \( \rho^0 - \gamma \) and \( \omega^0 - \gamma \) mixings in (3.4). In addition, it is possible to construct a direct \( H\bar{H}\gamma \) locally chiral invariant interaction term as:

\[ \frac{ieM\delta}{2m_v} Tr(H\gamma_\mu\gamma_\nu[\xi^\dagger F_{\mu\nu}(B_L)\xi + \xi F_{\mu\nu}(B_R)\xi^\dagger]\bar{H}), \]  

whose strength is measured by the parameter \( \delta \).

Putting the contributions to \( D^* \to D \gamma \) from [(1.5) and (3.4)] and from (3.6) together with the subleading one from (2.5) finally yields the width expressions:

\[ \Gamma(D^{*0} \to D^0 \gamma) = \frac{e^2 p_{0\gamma}^3}{12\pi} \left( \frac{2}{3M} + \frac{8}{3m_v}(\delta + \frac{c}{g}) \right)^2, \]

\[ \Gamma(D^{*+} \to D^+ \gamma) = \frac{e^2 p_{+\gamma}^3}{12\pi} \left( \frac{2}{3M} - \frac{4}{3m_v}(\delta + \frac{c}{g}) \right)^2, \]

\[ \Gamma(D^{s*+} \to D^{s+} \gamma) \approx \Gamma(D^{*+} \to D^+ \gamma), \]  

wherein \( p_{0\gamma} \) and \( p_{+\gamma} \) are the 3-momenta in the \( D^{*0} \) and \( D^{*+} \) rest frames respectively. The third approximate equality utilizes the coincidence that the phase space factors are approximately equal for the two reactions. We shall not make use of the \( D^{s*+} \to D^{s+} \gamma \) reaction here. To compute it more accurately in the present model (even at the tree level) involves taking into account several SU(3) symmetry breaking terms discussed in [3]. Notice that the structure of the amplitudes in (3.7) is essentially the same as that in the naive quark
model, (2.4). The parameter $M$ in (3.7), however, is more reasonably considered to be the heavy meson mass. A natural notion of light vector meson dominance for $D^* \rightarrow D \gamma$ is to set

$$\delta = 0$$

(3.8)

which corresponds to the photon interacting with the light electromagnetic transition moment only through its mixing with the light vectors in (3.4). Of course, there is no a priori reason for the assumption (3.8) to be perfect, but usual low energy phenomenology suggests that it is very sensible.

Now let us compare with experiment. The latest data from CLEO II [18] yields

$$\frac{\Gamma(D^*\rightarrow D^0 \gamma)}{\Gamma(D^*\rightarrow D^0 \pi^0)} = 0.57 \pm 0.14,$$

$$\frac{\Gamma(D^{*+}\rightarrow D^{+} \gamma)}{\Gamma(D^{*+}\rightarrow D^{0} \pi^{+})} < 0.059.$$  (3.9)

These numbers should be equated to the predictions from (3.7) and (1.7). Defining, for temporary convenience,

$$A = \frac{2}{3Md}, \quad B = \frac{4}{3m_v d}(\delta + \frac{c}{g}),$$

(3.10)

we then get $|A + 2B| = 3.40 \pm 0.42 \text{ GeV}^{-1}$ as well as $|A - B| < 1.37 \text{ GeV}^{-1}$. Taking $B > 0$, the allowed region is shown in Fig. 1. Notice that $B$ is leading in $M$ while $A$ represents the subleading contribution. Hence it is natural to consider $B > A$ as the region of interest. This corresponds to the allowed region OPQR. (We shall not consider further here the region PTSQ for which $A > B$.) Furthermore the bound (1.8) would require $A$ to lie to the right of the dashed line (obtained by setting $M = 1.8 \text{ GeV}$ in (3.10)). We see that $B$ is restricted to lie between 1 and 1.65. With the vector meson dominance assumption that $\delta = 0$, this leads to the bounds on the interesting $c/d$ ratio

$$2.27 < \frac{c}{d} < 3.75.$$  (3.11)

Choosing $M = 1.8 \text{ GeV}$ and comparing with (1.8) shows that $0.29 < d < 0.70$.

It should be stressed that the constraints on $c$ and $d$ just discussed have been based on the assumptions

- i) light vector meson dominance,

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2We would like to thank S. Stone for pointing out to us that the value quoted in [18] for $\frac{\Gamma(D^{+}\rightarrow D^{0} \gamma)}{\Gamma_{\text{tot.}}}$ is best interpreted as an upper bound of 4%.

3Another region is obtained by reversing the signs of both $A$ and $B$. However, this choice would disagree with the sign of (4.3).
• ii) leading order in \( \frac{1}{M} \) for \( J_{\mu}^{light} \),

• iii) SU(3) invariance,

• iv) leading order in chiral perturbation theory.

(To some extent ii) can be handled by considering \( c \) and \( d \) to be effective values rather than the ones defined in (1.5).) We thus expect that the constraints can be systematically improved. Nevertheless, it seems to us very worthwhile to give an idealized analysis which is suited both to the present experimental accuracy and the accuracy of many present applications.

To end this section we comment on the first, kinetic term in (1.5), which contains the "chiral derivative" \( D_\mu \bar{H} = \left[ \partial_\mu - i\alpha\tilde{g}\rho_\mu - (1 - \alpha)v_\mu \right]\bar{H} \). The presence of \( D_\mu \) guarantees invariance under global transformations belonging to the chiral group. When including electromagnetism we should naturally add the terms \(-ieA_\mu(Q - C)\) to \( D_\mu \). This is not sufficient since both \( v_\mu \) and \( \rho_\mu \) pick up inhomogeneous pieces under local electromagnetic \( U(1) \) transformations. Thus, we should replace them by the properly covariant quantities

\[
\tilde{v}_\mu = v_\mu + eA_\mu \left[ \frac{1}{2}(\xi Q\xi^\dagger + \xi^\dagger Q\xi) - Q \right],
\]

\[
\tilde{\rho}_\mu = \rho_\mu - \frac{e}{g}QA_\mu.
\]

The net result is an "electrified chiral derivative" to be used in (1.5):

\[
D_{\mu}^{ECD} \bar{H} = \left[ \partial_\mu - ieA_\mu(Q - C) - i\alpha\tilde{g}\tilde{\rho}_\mu - i(1 - \alpha)\tilde{v}_\mu \right]\bar{H}
= D_\mu \bar{H} + ieA_\mu[Q - C - \frac{1}{2}(1 - \alpha)(\xi Q\xi^\dagger + \xi^\dagger Q\xi)]\bar{H}.
\]  

(3.12)

From this expression, it is evident that the choice \( \alpha = 1 \) corresponds to no direct photon coupling to the light part of the heavy meson field \( \bar{H} \). The indirect coupling via photon mixing with the light vectors ensures proper normalization of the electromagnetic form factors of the heavy meson at zero momentum transfer. Since the first term in (1.5) is diagonal, of course, it does not contribute to the off-diagonal transition matrix elements of interest. Even though \( \alpha = 1 \) has been seen to be the choice for vector meson dominance of the diagonal matrix elements of light electromagnetic currents between heavy meson states, one still does not know experimentally just how good that assumption really is.

IV. OTHER ESTIMATES AND AN APPLICATION

We first consider two other estimates for the light pseudoscalar-heavy meson coupling constant \( d \) and the light vector-heavy meson coupling constant \( c \).

One way to get a handle on \( c \) and \( d \) is to imagine that the flavor SU(3) invariant expression for the vector-vector-pseudoscalar interaction [see (2.18) of [14], for example]:

\[
L_{VV\phi} = -ig_{VV\phi}\epsilon_{\mu\nu\alpha\beta}Tr(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\phi),
\]

(4.1)
continues to hold when the strange quark is formally considered to be “heavy”. Then the $K^+$ field, normally denoted as $\phi_1^+$, would be called $\bar{P}_1$, while the $K^{*+}$ field would be called $\bar{Q}_1\mu$ (in the notation of [4]). If we consider both of the vectors in (4.1) to be heavy (with the pseudoscalar light) the resulting $Q\phi\bar{Q}$ interaction is actually part of the $d$-term in (1.5) (see (3.20) of [6]). Similarly the $P\rho\bar{Q}$ piece from (4.1) is part of the $c$-term in (1.5) (see (3.22) of [6]). These identifications give the estimates:

\[ -g_{V\phi} = \frac{2d}{F_\pi} = \frac{4c}{m_\psi}. \] (4.2)

We immediately see that the ratio:

\[ \frac{c}{d} = \frac{m_\psi}{2F_\pi} = 2.92 \] (4.3)

is compatible with (3.11). A typical estimate [16], $|g_{V\phi}F_\pi| \approx 1.8$ yields the additional expectation that $|d| \approx 0.9$. The resulting point is denoted $x$ in Fig. 1; it is slightly outside the allowed region.

A more direct experimental approach to finding $d$ is based on fitting [19] the $D \to K$ semi-leptonic transition form factor to $R_q^2 + m_\rho^2 (D^*)$, where $R = \text{constant}$. In the present model, $R = \frac{dM^2D}{F_\pi}$, where $M$ is the $D$ mass. This leads to (see (5.2) of [20])

\[ |d| \approx 0.53, \] (4.4)

where $F_D \approx 0.25$ GeV was taken [21]. A similar approach to finding $c$ can be based on the study of the $D \to K^*$ semi-leptonic transition vector type form factor. There is more theoretical as well as experimental uncertainty in this case but one gets [22, 5, 2] :

\[ |c| \approx 1.6. \] (4.5)

The point $y$ in Fig. 1 corresponds to choosing both (4.4) and (4.3); it clearly lies within the allowed region.

It may be helpful to give some predictions resulting from the typical parameter choices $d = 0.53, c = 1.6, \delta = 0$ and $M = 1.8$ GeV. Then the branching ratios in (3.9) turn out to be $\Gamma(D^{*0} \to D^0\gamma)/\Gamma(D^{*0} \to D^0\pi^0) = 0.55$ and $\Gamma(D^{*+} \to D^{+}\gamma)/\Gamma(D^{*+} \to D^0\pi^+) = 0.01$. The total widths are estimated as $\Gamma_{tot}(D^{*0}) = 0.056$ MeV and $\Gamma_{tot}(D^{*+}) = 0.081$ MeV. With a heavy mass choice of 5.28 GeV the radiative B widths are estimated as $\Gamma(\bar{B}^{*-} \to \bar{B}^{-}\gamma) = 0.43$ KeV and $\Gamma(B^{*0} \to B^{0}\gamma) = 0.14$ KeV.

Now let us discuss the application of this analysis to the computation of the heavy baryon spectrum in the bound state picture [24], which was our original motivation. In this picture the heavy baryon is treated as a bound state of the heavy meson and an ordinary light baryon (considered as a Skyrme soliton). To the first rough approximation the problem reduces to that of relative motion in a spherical harmonic oscillator potential [26, 20, 27]

\[ V(r) = V_0 + \frac{1}{2} \kappa r^2, \] (4.6)

where $V_0$ and $\kappa$ are numbers which are computable in terms of the coupling constants $d, c$ and $\alpha$ as well as the light baryon Skyrme “profiles”. A recent analysis [28] has shown that
the above quadratic approximation is not as good for the charmed baryons as it is for the bottom baryons. Unfortunately, most of the data pertains to the charmed baryons. The energy levels which follow from (4.6) are:

\[ E_n = V_0 + \sqrt{\frac{\kappa}{\mu}}(n + \frac{3}{2}) \]  

(4.7)

where \( \mu \) is the reduced mass (which we take as 0.633 GeV for the charmed baryon case). The ground state is labeled by \( n = 0 \). We will compare (4.7) with the two pieces of experimental data:

\[ E_0 = m(\Lambda_c) - m(N) - m(D) = -0.63 \text{ GeV}, \]
\[ E_1 - E_0 = m(\Lambda'_c) - m(\Lambda_c) = 0.31 \text{ GeV}, \]  

(4.8)

where \( m(\Lambda'_c) \) is taken from the experimental evidence of [29]. Now the predictions of the bound state model are given in (5.1) of [20]:

\[ V_0 = -1.19d - 0.39c - 0.26\alpha \text{ GeV}, \]
\[ \kappa = 0.27d + 0.12c + 0.04\alpha \text{ GeV}^3, \]  

(4.9)

wherein we have reversed the sign of the \( \alpha \) term as discussed in [28]. The \( d, c \) and \( \alpha \) terms correspond to the contributions of \( \pi \) mesons, \( \rho \) mesons and \( \omega \) mesons, respectively.

First let us consider the situation when only the \( d \) term is present. (This corresponds to initial treatments in which the light vector mesons are neglected [23]). For the typical choice \( d = 0.53 \) as in (4.4) we have \( E_0 = 0.09 \text{ GeV} \) and \( E_1 - E_0 = 0.48 \text{ GeV} \). Thus the ground state baryon is not even bound in this case. To improve this situation we want \( d \) to be as large as possible (going to the left in Fig. 1 according to (3.10)). The experimental bound (1.8) sets this value as \( d = 0.7 \). In this case \( E_0 = -0.02 \text{ GeV} \) and \( E_1 - E_0 = 0.54 \text{ GeV} \). Here the ground state is just barely bound. One might expect [28] the binding strength to be increased by perhaps 0.1 GeV when going beyond the quadratic approximation but even so the prediction is very unsatisfactory when compared to (4.8).

A very large improvement is obtained by including the light vector mesons. First it is necessary to specify the parameter \( \alpha \), on which we have no experimental information. However the fact that light vector meson dominance has been seen to be reasonable for the \( D^* \to D\gamma \) decays suggests that the vector meson dominance choice \( \alpha = 1 \) is also reasonable. Let us fix \( \alpha = 1 \), \( d = 0.53 \) and allow \( c \) to vary near its allowed range \( 2.56 < \frac{c}{d} < 3.61 \) (as read off from Fig. 1). Then we find from (4.7) and (4.9) that \( (E_0, E_1 - E_0) = (-0.31, 0.74) \text{ GeV},(-0.36, 0.76) \text{ GeV} \) and \( (-0.41, 0.80) \text{ GeV} \) for \( \frac{c}{d} = 2.5, 3.0 \) and \( 3.5 \) respectively. It is clear that the binding energy is now in the right ballpark, but somewhat too small. The excitation energy \( E_1 - E_0 \) is too large, but it has been observed [28] that this is expected to get significantly improved when we go beyond the quadratic approximation (4.7).

We have discussed the introduction of the electromagnetic interaction in the framework of the effective chiral Lagrangian for heavy mesons which includes light vector mesons. A suitable notion of light vector meson dominance was formulated. Application was made to the radiative decays of the \( D^* \) mesons with the goal of determining the light heavy coupling constants \( c \) and \( d \). It was found that the acceptable range of values, on the assumption
of vector meson dominance, was compatible with information extracted from semi-leptonic $D$ decays. The structure of the radiative amplitudes had the same form as in the simple quark model. Apart from the necessity to include the $\frac{1}{M}$ piece describing the heavy part of the electromagnetic current (which is “accidentally” enhanced for $D^*$ radiative decays) we worked to leading order $M^0$. Furthermore, higher derivatives, loops and SU(3) symmetry breaking were neglected. Together with more accurate measurements of $\Gamma(D^{*+} \rightarrow D^+ \gamma)$, these additional corrections may, in the future, greatly clarify the situation. Finally the constraints on $c$ and $d$ were used for discussing the predicted heavy baryon spectrum in the “bound state model”.

**V. ACKNOWLEDGEMENT**

We would like to thank G.C. Moneti, N. Horwitz, S. Stone and A. Subbaraman for helpful discussions. This work was supported in part by the U.S. DOE contract nos. DE-FG-02-85ER40231 and DE-FG05-91ER-40636.
REFERENCES

[16] P. Jain et. al. in Ref. 3 above.  

FIGURES

FIG. 1. Physically allowed region for $A$ and $B$. 