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Vector Meson Dominance Model for Radiative Decays Involving Light Scalar Mesons

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We study a vector dominance model which predicts a fairly large number of currently interesting decay amplitudes of the types $S \to \gamma\gamma$, $V \to S\gamma$ and $S \to V\gamma$, where S and V denote scalar and vector mesons, in terms of three parameters. As an application, the model makes it easy to study in detail a recent proposal to boost the ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0^0\gamma)$ by including the isospin violating a_0^0-f_0 mixing. However we find that this effect is actually small in our model.

There is increasing interest in a possible nonet of light scalar mesons (all of mass $< 1\,\mathrm{GeV}$). In addition to the well established $f_0(980)$ and $a_0(980)$ evidence of both experimental and theoretical nature for a very broad σ ($\simeq 560$) and a very broad κ ($\simeq 900$) has been presented [1]. The latter two resonances are difficult to identify cleanly because they appear to be of non Breit-Wigner type, signaling strong interference with the non-resonant background.

Such a nonet would most likely represent meson states more complicated than quark-anti quark type and hence would be of great importance for a full understanding of QCD in its non-perturbative low energy regime.

Clearly it is important to study the properties of the $f_0(980)$ and $a_0(980)$ from the point of view of how they fit into a putative nonet family. In particular, the reactions $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ have recently been observed [2] with good accuracy and are considered as useful probes of scalar properties. The theoretical analysis was initiated by Achasov and Ivanchenko [3] and followed up by many others [4]. The models employed are essentially variants of the single K meson loop diagram to which a ϕ-type vector meson, a photon and two pseudoscalars or a scalar are attached.

In the present note we introduce a complementary approach which emphasizes the "family" or symmetry aspects of the analysis. This enables us to study the correlations among a fairly large number of related radiative amplitudes in terms of a few parameters, without making a commitment to a particular quark structure for the scalars.

Our framework is that of a standard non-linear chiral Lagrangian containing, in addition to the pseudoscalar nonet matrix field ϕ, the vector meson nonet matrix ρ_μ and a scalar nonet matrix field denoted N. Under chiral unitary transformations of the three light quarks; $q_{\mathrm{L,R}} \to U_{\mathrm{L,R}} \cdot q_{\mathrm{L,R}}$, the chiral matrix $U = \exp(2i\phi/F_\pi)$, where $F_\pi \simeq 0.131\,\mathrm{GeV}$, transforms as $U \to U_{\mathrm{L}} \cdot U \cdot U_{\mathrm{R}}^\dagger$. The convenient matrix $K(U_{\mathrm{L}}, U_{\mathrm{R}}, \phi)$ [5] is defined by the following transformation property of ξ ($U = \xi^2$): $\xi \to U_{\mathrm{L}} \cdot \xi \cdot K^\dagger = K \cdot \xi \cdot U_{\mathrm{R}}^\dagger$, and specifies the transformations of "constituent-type" objects. The fields we need transform as

$$
\begin{aligned}
& N \to K \cdot N \cdot K^\dagger \ , \\
& \rho_\mu \to K \cdot \rho_\mu \cdot K^\dagger + \frac{i}{\tilde{g}} K \cdot \partial_\mu K^\dagger \ , \\
& F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}\,[\rho_\mu\,,\,\rho_\nu] \to K \cdot F_{\mu\nu} \cdot K^\dagger ,
\end{aligned} \tag{1}
$$

where the coupling constant $\tilde{g}$ is about 4.04. One may refer to Ref. [6] for our treatment of the pseudoscalar-vector Lagrangian and to Ref. [7] for the scalar addition. The entire Lagrangian is chiral invariant (modulo the quark mass term induced symmetry breaking pieces) and, when electromagnetism is added, gauge invariant.

It should be remarked that the effect of adding vectors to the chiral Lagrangian of pseudoscalars only is to replace the photon coupling to the charged pseudoscalars as,

$$
\begin{aligned}
& ie\mathcal{A}_\mu \,\mathrm{Tr}\left(Q\phi \overleftrightarrow{\partial_\mu} \phi \right) \to \\
& e\mathcal{A}_\mu \left[k\tilde{g}F_\pi^2 \mathrm{Tr}\,(Q\rho_\mu) \right. \\
& \left. \quad + i\left(1 - \frac{k}{2}\right) \mathrm{Tr}\left(Q\phi \overleftrightarrow{\partial_\mu} \phi \right) \right] + \cdots \ ,
\end{aligned} \tag{2}
$$

where $\mathcal{A}_\mu$ is the photon field, $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ and $k = \left(\frac{m_v}{\tilde{g}F_\pi}\right)^2$ with $m_v \simeq 0.76\,\mathrm{GeV}$. The ellipses stand for symmetry breaking corrections. We see that in this model, Sakurai's vector meson dominance [8] simply amounts to the statement that $k = 2$ (the KSRF relation [9]). This is a reasonable numerical approximation which is essentially stable to the addition of symmetry breakers [6, 10] and we employ it here by neglecting the last term in Eq. (2). Although vector meson dominance must be somewhat modified in cases where the axial anomaly plays a role [11], it generally works quite well for processes such as those we study here.

The new feature of the present work is the inclusion of strong trilinear scalar-vector-vector terms in the effective Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{SVV} &= \beta_A\, \epsilon_{abc}\epsilon^{a'b'c'} \left[F_{\mu\nu}(\rho)\right]^a_{a'} \left[F_{\mu\nu}(\rho)\right]^b_{b'} N^c_{c'} \\
&\quad + \beta_B\, \mathrm{Tr}\left[N\right] \mathrm{Tr}\left[F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)\right] \\
&\quad + \beta_C\, \mathrm{Tr}\left[N F_{\mu\nu}(\rho)\right] \mathrm{Tr}\left[F_{\mu\nu}(\rho)\right] \\
&\quad + \beta_D\, \mathrm{Tr}\left[N\right] \mathrm{Tr}\left[F_{\mu\nu}(\rho)\right] \mathrm{Tr}\left[F_{\mu\nu}(\rho)\right] \ . \qquad (3)
\end{aligned}
$$

Chiral invariance is evident from (1) and the four flavor-invariants are needed for generality. (A term $\sim \mathrm{Tr}(FFN)$ is linearly dependent on the four shown). Actually the β_D term will not contribute in our model so there are only three relevant parameters β_A, β_B and β_C. Equation (3) is analogous to the PVV interaction which was originally introduced as a $\pi\rho\omega$ coupling a long time ago [12]. It is intended to be a leading point-like [13] description of the production mechanism. With (2) one can now compute the amplitudes for $S \to \gamma\gamma$ and $V \to S\gamma$ according to the diagrams of Fig. 1.

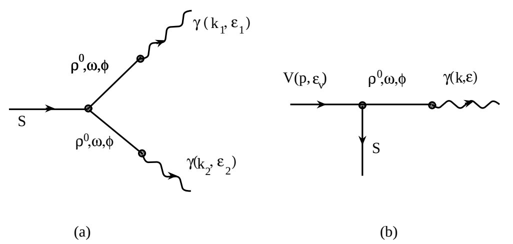

FIG. 1: Feynman diagrams for (a) $S \to \gamma\gamma$ and (b) $V \to S\gamma$.

The decay matrix element for $S \to \gamma\gamma$ is written as $(e^2/\tilde{g}^2) X_S \times (k_1 \cdot k_2\, \epsilon_1 \cdot \epsilon_2 - k_1 \cdot \epsilon_2\, k_2 \cdot \epsilon_1)$ where ϵ_μ stands for the photon polarization vector. It is related to the width by

$$
\Gamma\left(S \to \gamma\gamma\right) = \alpha^2 \frac{\pi}{4} m_S^3 \left|\frac{X_S}{\tilde{g}^2}\right|^2 \ , \qquad (4)
$$

and X_S takes on the specific forms:

$$
\begin{aligned}
X_\sigma &= \frac{4}{9}\beta_A\left(\sqrt{2}s - 4c\right) + \frac{8}{3}\beta_B\left(c - \sqrt{2}s\right) \ , \\
X_{f_0} &= -\frac{4}{9}\beta_A\left(\sqrt{2}c + 4s\right) + \frac{8}{3}\beta_B\left(\sqrt{2}c + s\right) \ , \\
X_{a_0} &= \frac{4\sqrt{2}}{3}\beta_A \ . \qquad (5)
\end{aligned}
$$

Here $\alpha = e^2/(4\pi)$, $s = \sin\theta_S$ and $c = \cos\theta_S$ where the scalar mixing angle, θ_S is defined from

$$
\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} N_3^3 \\ (N_1^1 + N_2^2)/\sqrt{2} \end{pmatrix} \ . \qquad (6)
$$

Furthermore ideal mixing for the vectors, with $\rho^0 = (\rho_1^1 - \rho_2^2)/\sqrt{2}$, $\omega = (\rho_1^1 + \rho_2^2)/\sqrt{2}$, $\phi = \rho_3^3$, was assumed for simplicity.

Similarly, the decay matrix element for $V \to S\gamma$ is written as $(e/\tilde{g}) C_V^S \times [p \cdot k \epsilon_V \cdot \epsilon - p \cdot \epsilon k \cdot \epsilon_V]$. It is related to the width by

$$
\Gamma(V \to S\gamma) = \frac{\alpha}{3}\left|k_V^S\right|^3 \left|\frac{C_V^S}{\tilde{g}}\right|^2 \ , \qquad (7)
$$

where $k_V^S = (m_V^2 - m_S^2)/(2m_V)$ is the photon momentum in the V rest frame. For the energetically allowed $V \to S\gamma$ processes we have

$$
\begin{aligned}
C_\phi^{f_0} &= \frac{2\sqrt{2}}{3}\beta_A c - \frac{4}{3}\beta_B\left(\sqrt{2}c + s\right) \\
&\quad + \frac{\sqrt{2}}{3}\beta_C\left(c - \sqrt{2}s\right) \ , \\
C_\phi^\sigma &= -\frac{2\sqrt{2}}{3}\beta_A s - \frac{4}{3}\beta_B\left(c - \sqrt{2}s\right) \\
&\quad - \frac{2}{3}\beta_C\left(c + \frac{1}{\sqrt{2}}s\right) \ , \\
C_\phi^{a_0} &= \sqrt{2}\left(\beta_C - 2\beta_A\right) \ , \\
C_\omega^\sigma &= \frac{2\sqrt{2}}{3}\beta_A\left(c + \sqrt{2}s\right) + \frac{2\sqrt{2}}{3}\beta_B\left(c - \sqrt{2}s\right) \\
&\quad - \frac{2}{3}\beta_C\left(\sqrt{2}c + s\right), \\
C_{\rho^0}^\sigma &= -2\sqrt{2}\beta_A c + 2\sqrt{2}\beta_B\left(c - \sqrt{2}s\right). \qquad (8)
\end{aligned}
$$

In addition, the same model predicts amplitudes for the energetically allowed $S \to V\gamma$ processes: $f_0 \to \omega\gamma$, $f_0 \to \rho^0\gamma$, $a_0^0 \to \omega\gamma$, $a_0^0 \to \rho^0\gamma$ and, if κ^0 is sufficiently heavy $\kappa^0 \to K^{*0}\gamma$. The corresponding width is

$$
\Gamma(S \to V\gamma) = \alpha\left|k_S^V\right|^3 \left|\frac{D_S^V}{\tilde{g}}\right|^2 \ , \qquad (9)
$$

where $k_S^V = (m_S^2 - m_V^2)/(2m_S)$ and

$$
\begin{aligned}
D_{f_0}^\omega &= \frac{2}{3}\beta_A\left(-2c + \sqrt{2}s\right) + \frac{2}{3}\beta_B\left(2c + \sqrt{2}s\right) \\
&\quad + \frac{2}{3}\beta_C\left(c - \sqrt{2}s\right) \ , \\
D_{f_0}^{\rho^0} &= -2\sqrt{2}\beta_A s + 2\beta_B\left(2c + \sqrt{2}s\right) \ , \\
D_{a_0}^\omega &= 2\beta_C \ , \\
D_{a_0}^{\rho^0} &= \frac{4}{3}\beta_A \ , \\
D_{\kappa^0}^{K^{*0}} &= -\frac{8}{3}\beta_A \ . \qquad (10)
\end{aligned}
$$

All the different decay amplitudes are described by the parameters β_A, β_B and β_C. The reason β_D does not appear at all and β_C does not appear for $S \to \gamma\gamma$ is that, noting Eq. (2), the $\mathrm{Tr}(F_{\mu\nu})$ factor is seen to give zero when coupled to an external photon line. Because the σ and κ are so broad, the simple two body final state approximation in decays like ω, $\phi \to \sigma\gamma \to \pi^0\pi^0\gamma$ is not accurate. It is better to consider these decays as having three body final states with the terms in Eq. (3) giving the vertices and to take into account large width corrections in the scalar propagators as well as non resonant background.

These formulas can be used for different choices of the quark structure of the scalar nonet N_a^b (e.g. the usual $q_a\bar{q}^b$ scenario or the "dual" scenario $Q_a\bar{Q}^b$ where $Q_a \sim \epsilon_{abc}\bar{q}^b\bar{q}^c$). The characteristic mixing angle θ_S is expected to differ, depending on the scheme. In the literature, besides conventional $q\bar{q}$ models, $qq\bar{q}\bar{q}$ models [14], meson-meson "molecule" models [15] and unitarized meson-meson [16] models have been investigated. Recently models featuring mixing between a $qq\bar{q}\bar{q}$ nonet and a heavier $q\bar{q}$ nonet have been proposed [17]; in this case two sets of interactions like Eq. (3) should be included.

Now we shall illustrate the procedure for the model of a single putative scalar nonet [7] with a mixing angle, $\theta_S \simeq -20^\circ$ (characteristic of $qq\bar{q}\bar{q}$ type scalars).

The parameters β_A and β_B may be estimated from the $S \to \gamma\gamma$ processes. Substituting $\Gamma_{\rm exp}(a_0 \to \gamma\gamma) = (0.28 \pm 0.09)\,\mathrm{keV}$ (obtained using [19] $B(a_0 \to K\bar{K})/B(a_0 \to \eta\pi) = 0.177 \pm 0.024$) into Eqs. (4) and (5) yields $\beta_A = (0.72 \pm 0.12)\,\mathrm{GeV}$ (assumed positive in sign). Of course, this value is independent of the value of θ_S. Then, $\Gamma_{\rm exp}(f_0 \to \gamma\gamma) = 0.39 \pm 0.13\,\mathrm{keV}$ yields either $\beta_B = (0.61 \pm 0.10)\,\mathrm{GeV}^{-1}$ or $\beta_B = (-0.62 \pm 0.10)\,\mathrm{GeV}^{-1}$. In turn we formally predict $\Gamma(\sigma \to \gamma\gamma)$ to be either $(0.024 \pm 0.023)\,\mathrm{keV}$ or $(0.38 \pm 0.09)\,\mathrm{keV}$ respectively.

Next consider the ϕ radiative decays. Assuming $\phi \to \eta\pi^0\gamma$ is dominated by $\phi \to a_0\gamma$, $\Gamma_{\rm exp}(\phi \to a_0\gamma) = (0.47 \pm 0.07)\,\mathrm{keV}$ and Eq. (8) determines β_C as either $(7.7 \pm 0.5)\,\mathrm{GeV}^{-1}$ or $(-4.8 \pm 0.5)\,\mathrm{GeV}^{-1}$. Note that $|\beta_A|$ and $|\beta_B|$ are almost an order of magnitude smaller than $|\beta_C|$. Thus, the ϕ radiative decay rates are mainly determined by $|\beta_C|$. Knowing β_A, β_B and β_C we can predict $\Gamma(\phi \to f_0\gamma)$ using Eq. (8). There are four possibilities due to the two possibilities each for β_B and β_C. The largest number, $\Gamma(\phi \to f_0\gamma) = (0.21 \pm 0.03)\,\mathrm{keV}$ corresponds [18] to the choice $\beta_B = (-0.62 \pm 0.10)\,\mathrm{GeV}^{-1}$ and $\beta_C = (7.7 \pm 0.5)\,\mathrm{GeV}^{-1}$.

Unfortunately this is still considerably smaller than the listed value [19]: $\Gamma_{\rm exp}(\phi \to f_0\gamma) = (1.51 \pm 0.41)\,\mathrm{keV}$ [20]. Recently Close and Kirk [21] proposed that the ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ could be boosted by considering the effects of the isospin violating $a_0^0(980)$-$f_0(980)$ mixing. We will now see that these effects are small in our model. One may simply introduce the mixing by a term in the effective Lagrangian: $\mathcal{L}_{af} = A_{af}a_0^0 f_0$. A recent calculation [22] for the purpose of finding the effect of the scalar mesons in the $\eta \to 3\pi$ process obtained the value $A_{af} = -4.66 \times 10^{-3}\,\mathrm{GeV}^2$. It is convenient to treat this term as a perturbation. Then the amplitude for $\phi \to f_0\gamma$ includes a correction term consisting of the $\phi \to a_0^0\gamma$ amplitude given in Eq. (8) multiplied by A_{af} and by the a_0 propagator. The $\phi \to a_0^0\gamma$ amplitude has a similar correction. In terms of the amplitudes in Eq.(8) the desired ratio is then,

$$\frac{amp(\phi \to f_0\gamma)}{amp(\phi \to a_0^0\gamma)} = \frac{C_\phi^f + A_{af}C_\phi^a/D_a(m_f^2)}{C_\phi^a + A_{af}C_\phi^f/D_f(m_a^2)}, \tag{11}$$

where $D_a(m_f^2) = -m_f^2 + m_a^2 - im_a\Gamma_a$ and $D_f(m_a^2) = -m_a^2 + m_f^2 - im_f\Gamma_f$. In this approach the propagators are diagonal in the isospin basis. The numerical values of these resonance widths and masses are, according to the Review of Particle Physics [19] $m_{a_0} = (984.7 \pm 1.3)\,\mathrm{MeV}$, $\Gamma_{a_0} = 50$–$100\,\mathrm{MeV}$, $m_{f_0} = 980 \pm 10\,\mathrm{MeV}$ and $\Gamma_{f_0} = 40$–$100\,\mathrm{MeV}$. For definiteness, from column 1 of Table II in Ref. [23] we take $m_{f_0} = 987\,\mathrm{MeV}$ and $\Gamma_{f_0} = 65\,\mathrm{MeV}$ while in Eq. (4.2) of Ref. [24] we take $\Gamma_{a_0} = 70\,\mathrm{MeV}$. In fact the main conclusion does not depend on these precise values. It is easy to see that the mixing factors are approximately given by

$$\frac{A_{af}}{D_a(m_f^2)} \approx \frac{A_{af}}{D_f(m_a^2)} \approx \frac{iA_{af}}{m_a\Gamma_a} \approx -0.07i. \tag{12}$$

Noting that $C_\phi^f/C_\phi^a \approx 0.75$ in the present model, the ratio in Eq.(11) is roughly $(0.75 - 0.07i)/(1 - 0.05i)$. Clearly, the correction to $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ due to a_0-f_0^0 mixing only amounts to a few per cent, nowhere near the huge effect suggested in [21]. It may be remarked that Eq.(11) is practically accurate to all orders in A_{af}, corresponding to iterating any number of a_0-f_0 transitions. Then, after summing a geometric series, the numerator picks up a correction factor $[1 - A_{af}^2/(D_a(m_f^2)D_f(m_f^2))]^{-1}$ and the denominator, the similar factor $[1 - A_{af}^2/(D_a(m_a^2)D_f(m_a^2))]^{-1}$.

Vector meson dominance, together with the assumptions of $SU(3)$ flavor symmetry and a single nonet of scalar mesons makes many more predictions. These are listed in Table I for two of the allowed $(\beta_A, \beta_B, \beta_C)$ parameter sets, neglecting a_0-f_0 mixing. It will be interesting to see if future experiments confirm the pattern of predicted widths.

We have given a leading order correlation of many radiative decays involving scalars, based on flavor symmetry and vector meson dominance. Clearly further improvements can be made. Elsewhere, we will study flavor symmetry breaking effects, higher drivative interaction terms, treatment of the $S\gamma$ final states as $PP\gamma$, and the case of mixed $q\bar{q}$ and $qq\bar{q}\bar{q}$ scalar nonets.

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β_A	0.72 ± 0.12	0.72 ± 0.12
β_B	0.61 ± 0.10	-0.62 ± 0.10
β_C	7.7 ± 0.52	7.7 ± 0.52
f_0/a_0 ratio	0.26 ± 0.06	0.46 ± 0.09
$\Gamma(\sigma \to \gamma\gamma)$	0.024 ± 0.023	0.38 ± 0.09
$\Gamma(\phi \to \sigma\gamma)$	137 ± 19	33 ± 9
$\Gamma(\omega \to \sigma\gamma)$	16 ± 3	33 ± 4
$\Gamma(\rho \to \sigma\gamma)$	0.23 ± 0.47	17 ± 4
$\Gamma(f_0 \to \omega\gamma)$	126 ± 20	88 ± 17
$\Gamma(f_0 \to \rho\gamma)$	19 ± 5	3.3 ± 2.0
$\Gamma(a_0 \to \omega\gamma)$	641 ± 87	641 ± 87
$\Gamma(a_0 \to \rho\gamma)$	3.0 ± 1.0	3.0 ± 1.0

TABLE I: Fitted values of β_A, β_B and β_C together with the predicted values of the ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ and the decay widths of $V \to S + \gamma$ and $S \to V + \gamma$. Units of β_A, β_B and β_C are GeV^{-1} and those of the decay widths are keV.

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