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Are Three Flavors Special?

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Are three flavors special?

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It has become clearer recently that the regular pattern of three flavor nonets describing the low spin meson multiplets seems to require some modification for the case of the spin 0 scalar mesons. One picture which has had some success, treats the scalars in a chiral Lagrangian framework and considers them to populate two nonets. These are, in turn, taken to result from the mixing of two "bare" nonets, one of which is of quark- antiquark type and the other of two quark- two antiquark type. Here we show that such a mixing is, before chiral symmetry breaking terms are included, only possible for three flavors. In other cases, the two types of structure can not have the same chiral symmetry transformation property. Specifically, our criterion would lead one to believe that scalar and pseudoscalar states containing charm would not have "four quark" admixtures. This work is of potential interest for constructing chiral Lagrangians based on exact chiral symmetry which is then broken by well known specific terms. It may also be of interest in studying some kinds of technicolor theories.

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I. INTRODUCTION

Historically, the nonet structure of elementary particle multiplets has suggested the spin 1/2 quark substructure and, with the help of the "slightly" broken flavor symmetry SU(3), has provided an enormous amount of information about the properties of the observed low lying hadronic states. For example, the lightest meson multiplets appear to be those of the pseudoscalars and vectors, consistent with s-wave quark- antiquark bound states. The next heaviest set of meson multiplets seems to be generally consistent with p-wave bound states, yielding a scalar nonet, a tensor nonet and two axial vector nonets.

Available evidence indicates that the predicted states arising from the addition of the charm and beauty quarks would fit in with corresponding SU(4) and SU(5) extensions (having respectively 16 and 25 members) of the SU(3) nonets. Of course a possible extension to states made with top quarks is of less interest, owing to the rapid weak decay of the top quark. Naturally the much heavier masses of the c and b quarks make the SU(4) and SU(5) symmetries not as good as SU(3). Nevertheless the observed particles still fit into the extended multiplets.

However, in the last few years there has been a growing recognition [1] - [26] that the lightest nine scalar states do not seem to fit well into the above classification. In terms of increasing mass these comprise the isosinglet $\sigma(600)$, the two isodoublets $\kappa(800)$ and the roughly degenerate isosinglet $f_0(980)$ and isotriplet $a_0(980)$. There are two unexpected features. First the masses of these states are significantly lower than the other "constituent quark model "p-wave states" (i.e. tensors and two axial vectors with different C properties). Secondly, the order, with increasing mass - isosinglet, isodoublet and roughly degenerate isosinglet with isotriplet - seems to be reversed compared to that of the "standard" vector meson nonet.

Clearly such a light and reversed order nonet requires some rethinking of the standard picture of the scalar mesons. Actually, a long time ago, it was observed [25] that the reversed order could be explained if the light scalar nonet were actually composed of two quarks and two antiquarks. In that case the number of strange quarks (which determines the direction of increasing masses) rises with the reversed order given. For example the lowest mass "isolated" isosinglet scalar $\sigma(600)$ would look like $(u\bar{u} + d\bar{d})^2$ while, for comparison, the highest mass isolated vector isosinglet $\phi(1020)$ looks like $s\bar{s}$. At that time the existence of a light sigma and a light kappa was considered dubious. More recent work by a great many people has now pretty much confirmed the existence of such states as well as the plausibility that they fit into a three flavor nonet.

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The above does not necessarily explain the unusual lightness of such a scalar nonet. A possible explanation was advanced [20] which proposed that the mixing between a $qq\bar{q}\bar{q}$ scalar nonet together with a usual p-wave $q\bar{q}$ nonet could produce this effect due to the "level repulsion" expected in quantum mechanics perturbation theory. This was done using a non-linear realization of chiral symmetry. Further work, to be discussed below, makes use of linear realizations. Related models for thermodynamic properties of QCD have been presented in ref. [27].

II. TWO CHIRAL NONETS

Of course, it has been realized for a long time that the nonet structure of mesons with respect to SU(3) flavor transformations should, at a more fundamental level, be expanded to SU(3) chiral symmetry transformations; this amounts to an SU(3) for massless left-handed quarks and another SU(3) for massless right-handed quarks. This chiral symmetry is that of the fundamental QCD Lagrangian itself, with neglect of quark mass terms. It is accepted, for example, that the SU(3) baryon properties do not depend much on the u, d and s quark masses. The spontaneous breaking of this symmetry, which gives zero mass pseudoscalars, is a basic part of the present understanding of low energy QCD. The light quark mass terms play a relatively small role and are treated as perturbations. It thus appears that chiral (rather than just the vector) symmetry should be considered the first approximation for an understanding of the structure of hadrons.

This chiral point of view may be especially relevant for studying the light scalars since they are the "chiral partners" of the zero mass pseudoscalars. To implement this picture systematically one may introduce a $q\bar{q}$ chiral nonet containing 9 scalar and 9 pseudoscalar fields as well as a $qq\bar{q}\bar{q}$ nonet also containing 9 scalars and 9 pseudoscalars. Furthermore, the light quark mass terms should be added as well as suitable terms to mock up the $U(1)_A$ anomaly of QCD.

Even though one can not write down the exact QCD wave functions of the low lying mesons it is easy to write down schematic descriptions of how quark fields may combine to give particles with specified transformation properties. The usual chiral nonet $M(x)$ realizing the $q\bar{q}$ structure is then written as:

$$M_a^{\dot{b}} = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}, \tag{1}$$

where a and A are respectively flavor and color indices. For clarity, on the left hand side the undotted index transforms under the left SU(3) while the dotted index transforms under the right SU(3).

One possibility for the $qq\bar{q}\bar{q}$ states is to make them as "molecules" from two quark-antiquark nonets. This leads to the following schematic form:

$$M_a^{(2)\dot{b}} = \epsilon_{acd}\epsilon^{\dot{b}\dot{e}\dot{f}} \left(M^\dagger\right)_{\dot{e}}^{c} \left(M^\dagger\right)_{\dot{f}}^{d}. \tag{2}$$

Note that the fields M and $M^{(2)}$ transform in the same way under chiral SU(3) as well as under the discrete P and C symmetries, as required if they are to mix with each other according to the scheme shown above. As noted in the Appendix, the axial U(1) transformation properties of M and $M^{(2)}$ differ from each other and provide a measure of whether the state is of one quark-antiquark type, two quark - antiquark type etc. In the chiral Lagrangian there are terms which break the axial U(1) in a manner dictated by the QCD axial anomaly. In the Appendix it is also pointed out that schematic fields $M^{(3)}$ and $M^{(4)}$ which have "diquark-antidiquark" forms instead of the "molecular" form can also be constructed. There has been some discussion in the literature about which type is favored [28]. In the present approach either is allowable. In fact it was shown in the first of [29] that the molecular form can be transformed using Fierz identities to a linear combination of the "diquark-antidiquark" forms. We thus assume that some unspecified linear combination of $M^{(2)}$, $M^{(3)}$ and $M^{(4)}$, denoted by M', represents the $qq\bar{q}\bar{q}$ chiral nonet which mixes with M. The decomposition into pseudoscalar and scalar fields is given by,

$$M = S + i\phi, \quad M' = S' + i\phi'. \tag{3}$$

The initial discussion of the chiral Lagrangian using these fields was presented in [30]. A more detailed picture with a particular choice of interaction terms was given in the first of [23]. In a series of papers, the model was explored for an arbitrary choice of interactions [31], a choice of interactions based on including terms containing less than a fixed number of underlying quark or antiquark fields [32] and the zero quark mass limit [33]. In addition the modeling of the axial anomaly was discussed [29] as well as the details of pion pion scattering [34]. In the most recent of these papers [35], the possible identification with all observed states was studied in further detail; after mixing there are two physical scalar nonets and two physical pseudoscalar nonets. Since each nonet has one isovector, two conjugate isospinors, and two isosinglets, there are altogether sixteen different masses involved. The model has eight inputs so the other eight masses are predictions. There are in fact experimental states which are candidates for identification with all the particles of the model and the agreement is reasonable. Additional predictions are given for the 4 $\times$ 4 orthogonal matrices which mix each of the four isosinglet scalars and each of the four isosinglet pseudoscalars. Perhaps, most interestingly, the lighter scalar mesons are predicted to be mainly of two quark - two antiquark type while the heavier scalar mesons are mainly of quark - antiquark type. The situation is opposite, as expected, for the pseudoscalar mesons, where the lighter ones are mainly of quark - antiquark type.

III. OTHER THAN THREE FLAVORS

Our initial motivation for this work was the recent experimental discovery [36] of the semileptonic decay mode,

$$D_s^+(1968) \rightarrow f_0(980) e^+ \nu_e, \tag{4}$$

in which the $f_0(980)$ was identified from its two pion decay mode. This provides some motivation for formulating a four flavor version of the model so that the charmed meson D_s would be conveniently contained.

There is no problem finding a chiral formulation for a $q\bar{q}$ 16-plet, $M_a^{\dot{b}}$. However we can not find a suitable schematic meson wave function with the same chiral transformation property constructed, for example, as a "molecule" out of two such states. The closest we can come for a two-part "molecule" is:

$$M_{ag}^{(2)\dot{b}\dot{h}} = \epsilon_{agcd} \epsilon^{\dot{b}\dot{h}\dot{e}\dot{f}} \left(M^\dagger\right)_{\dot{e}}^{c} \left(M^\dagger\right)_{\dot{f}}^{d}. \tag{5}$$

However, instead of transforming under SU(4)$_L$ $\times$ SU(4)$_R$ as $(L,R) = (4,\bar{4})$ as desired, this object transforms as $(L,R) = (6,\bar{6})$, owing to the two sets of antisymmetric indices (ag and $\dot{b}\dot{h}$) which appear. Hence, it should not mix in the chiral symmetry limit with the initial four flavor $q\bar{q}$ state. (See Eq.(1)) Of course it would be possible to multiply the right hand side of Eq.(5) by a third field $\left(M^\dagger\right)_{\dot{h}}^{g}$. That does give the correct transformation property to mix with the four flavor version of Eq.(1). However it corresponds to a three quark- three antiquark molecule. We assume that, especially after quark mass terms are added, an "elementary particle" state of such a form is unlikely to be bound.

The same problem emerges in the four flavor case when we alternatively construct composites of the diquark - antidiquark states given in Eqs.(A5) and (A7) of the Appendix. As above, this yields a composite state transforming like $(6,\bar{6})$ (rather than the desired $(4,\bar{4})$):

$$M_{gp}^{(3)\dot{f}\dot{q}} = \left(L^{gpE}\right)^\dagger R^{\dot{f}\dot{q}E}, \tag{6}$$

where,

$$\begin{aligned} L^{gpE} &= \epsilon^{gpab} \epsilon^{EAB} q_{aA}^T C^{-1} \frac{1+\gamma_5}{2} q_{bB}, \\ R^{\dot{f}\dot{q}E} &= \epsilon^{\dot{f}\dot{q}\dot{c}\dot{d}} \epsilon^{EAB} q_{\dot{c}A}^T C^{-1} \frac{1-\gamma_5}{2} q_{\dot{d}B}. \end{aligned} \tag{7}$$

We could contract L^{gpE} with a left handed quark field and $R^{\dot{f}\dot{q}E}$ with a right handed quark field to obtain the desired overall transformation property at the expense of having a three quark- three antiquark state (which we are assuming to be unbound).

It is clear that essentially the same argument would hold for five or more quark flavors.

Going in the direction of fewer flavors, we now note that there is also no suitable schematic "molecular" wavefunction available in the 2-flavor case for mixing with the quark- antiquark state. The closest we can come here for a "molecule" has the form:

$$M^{(2)} = \epsilon_{cd} \epsilon^{\dot{e}\dot{f}} \left(M^\dagger\right)_{\dot{e}}^{c} \left(M^\dagger\right)_{\dot{f}}^{d}. \tag{8}$$

This is clearly unsatisfactory since it transforms like (1,1) under SU(2)$_L$ $\times$ SU(2)$_R$ rather than the (2,2) required for mixing according to our assumed model. Actually one must be a little more careful because it is well known that the object $M_a^{\dot{b}}$ is not irreducible under chiral transformations in the 2-flavor case. It may be interesting to show that the same result is obtained when this fact is taken into account. The irreducible representations are formed by making use of the fact that $\tau_2 M^* \tau_2$ transforms in the same way as M. Then we may consider the irreducible linear combinations:

$$\frac{1}{\sqrt{2}}(M + \tau_2 M^* \tau_2) \equiv \sigma I + i\pi \cdot \tau$$
$$\frac{1}{\sqrt{2}}(M - \tau_2 M^* \tau_2) \equiv i\eta I + \mathbf{a} \cdot \tau, \tag{9}$$

where the usual SU(2) chiral multiplet containing π and σ is recognized as well as the parity reversed one containing η and the isovector scalar particle **a**. Since $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is equivalent to the group SO(4)we may consider the fields π and σ as making up an isotopic four vector, p_μ and the fields **a** and η as comprising another four vector q_μ. A "molecule" state which could mix with, say p_μ would have to be another four vector made as a product of p_μ and q_μ. The combination $p_\mu q_\mu$ is a singlet, the combination $\epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta$ has six components and the symmetric traceless combination has nine components. This confirms that there is no allowed mixing with a possible molecule at the chiral level in the two flavor case.

One might wonder why, if mixing is possible in the three flavor case, it is not possible in the two flavor case, which is just a subset of the former. The answer is already contained in Eq.(2). If we want to find something that mixes with the quark- antiquark π^+ particle we should look at the 12 matrix element. On the right hand side, one sees that the "molecule" field which mixes contains an extra $s\bar{s}$ pair, which is simply not present in the two flavor model.

Thus we see that flavor SU(3) has some interesting special features for schematically constructing bound states with well defined chiral transformation properties.

A possibility for the mixing of a quark antiquark state with a different state not of "molecular" (or more generally, two quark - two antiquark) type, would be to consider a so called radial excitation. For mixing with $M_a^{\dot{b}}$, such a state could be schematically written as $f(\Box)M_a^{\dot{b}}$, where f is a function of the d'Alembertian. In this case, one would not expect the inverted multiplets which appear in the "molecular" picture.

IV. TESTABLE CONSEQUENCES

At the three flavor level the existence of two quark - two antiquark states, suggested by our kinematical criterion, seems to have some experimental support. This gave rise to mixtures with the original quark - antiquark states and a doubling of the scalar and pseudoscalar meson spectra, as discussed in detail in ref. [35]. As an example, there are two established low lying isovector scalars - $a_0(980)$ and the $a_0(1450)$- rather than the single one predicted by the non-relativistic quark model.

What does it mean to say that the extra states, and hence the mixing, is not allowed at the four flavor level? Clearly the scalar and pseudoscalar 16-plets can not be completely absent since they contain also the three flavor nonets. Thus we conclude that the kinematical criterion should imply that 16 -9 = 7 members of the possible 16-plets for scalars and for pseudoscalars should not be doubled. The states which should not be doubled can be conveniently described using the notation of Eq.(3). The states which should not appear are:

$$\begin{aligned} scalars: &\quad {S'}_a^4, \quad {S'}_4^a, \quad {S'}_4^4, \\ pseudoscalars: &\quad {\phi'}_a^4, \quad {\phi'}_4^a, \quad {\phi'}_4^4. \end{aligned} \tag{10}$$

Here a = 1,2,3 and the quark corresponence is 1=u, 2=d, 3=s, 4=c. Furthermore subscripts denote the quark transformation property and the superscripts denote the antiquark transformation property.

Clearly the excluded states are those having non-zero charm. It will be interesting to see whether this holds using the large amount of new data expected from LHC.

V. SUMMARY AND DISCUSSION

A three flavor chiral model of scalar and pseudoscalar mesons as mixtures of "quark-antiquark" with "two quark-two antiquark" fields has previously been seen to be able to explain the unusual pattern of light scalar meson masses. That approach used a chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ linear sigma model which was supplemented by invariant terms which model the axial U(1) anomaly as well as the usual terms which model the quark masses. Before it was broken, the $U(1)_A$ quantum number distinguished the "two quark- two antiquark" mesons from the "quark-antiquark" mesons. The starting point for the mixing was that a schematic two quark - two antiquark product state could be constructed with the same $SU(3)_L \times SU(3)_R$ transformation property as the original "quark-antiquark" state. Of course this is just a "kinematic" statement and does not presume to say that the dynamical binding has been established or that large quark masses do not change this picture.

In the present note we have shown that this kinematical feature in the chiral limit does not hold for $SU(n)_L \times SU(n)_R$ when n is different from three. In the case of n = 4, it was seen that three quark- three antiquark states could have the same transformation property but we assumed that the 6-object bound state and other higher ones (needed for still larger n) would be unlikely to be bound as an "elementary particle".

As for our initial motivation, mentioned in section III, to construct a 4 flavor model for studying semi-leptonic decays of charmed mesons into scalar plus leptons, a kind of hybrid chiral model will be discussed elsewhere.

We have also noted a possible experimental test of the kinematical criterion for the doubling of scalar and pseudoscalar states in the charm sector.

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Appendix A: Notation and further details

Here we briefly discuss some notational and technical details. The γ matrices and the charge conjugation matrix have the form:

$$\gamma_i = \begin{bmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{bmatrix}, \qquad \gamma_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{bmatrix}. \tag{A1}$$

Our convention for matrix notation is $M_a^b \to M_{ab}$. Then M transforms under chiral $SU(3)_L \times SU(3)_R$, charge conjugation C and parity P as

$$\begin{aligned} &M \to U_L M U_R^\dagger \\ &C: \quad M \to M^T, \qquad P: \quad M(\mathbf{x}) \to M^\dagger(-\mathbf{x}). \end{aligned} \tag{A2}$$

Here U_L and U_R are unitary, unimodular matrices associated with the transformations on the left handed ($q_L = \frac{1}{2}(1+\gamma_5)\,q$) and right handed ($q_R = \frac{1}{2}(1-\gamma_5)\,q$) quark projections. For the $U(1)_A$ transformation one has:

$$M \to e^{2i\nu} M. \tag{A3}$$

Next consider nonets with " four quark", $qq\bar{q}\bar{q}$ structures. An alternate possibility to the one given in Eq.(2)of section II is that such states may be bound states of a diquark and an anti-diquark. There are two choices if the diquark is required to belong to a $\bar{3}$ representation of flavor SU(3). In the first case it belongs to a $\bar{3}$ of color and is a spin singlet with the structure,

$$\begin{aligned} L^{gE} &= \epsilon^{gab}\epsilon^{EAB} q_{aA}^T C^{-1} \frac{1+\gamma_5}{2} q_{bB}, \\ R^{\dot{g}E} &= \epsilon^{\dot{g}\dot{a}\dot{b}}\epsilon^{EAB} q_{\dot{a}A}^T C^{-1} \frac{1-\gamma_5}{2} q_{\dot{b}B}. \end{aligned} \tag{A4}$$

Then the matrix M has the form:

$$M_g^{(3)\dot{f}} = \left(L^{gA}\right)^\dagger R^{fA}. \tag{A5}$$

In a second alternate possibility, the diquark belongs to a 6 representation of color and has spin 1. It has the schematic chiral realization:

$$\begin{aligned} L^g_{\mu\nu,AB} = L^g_{\mu\nu,BA} &= \epsilon^{gab} q_{aA}^T C^{-1} \sigma_{\mu\nu} \frac{1+\gamma_5}{2} q_{bB}, \\ R^{\dot{g}}_{\mu\nu,AB} = R^{\dot{g}}_{\mu\nu,BA} &= \epsilon^{\dot{g}\dot{a}\dot{b}} q_{\dot{a}A}^T C^{-1} \sigma_{\mu\nu} \frac{1-\gamma_5}{2} q_{\dot{b}B}, \end{aligned} \tag{A6}$$

where $\sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu]$. The corresponding M matrix has the form

$$M_g^{(4)\dot{f}} = \left(L^g_{\mu\nu,AB}\right)^\dagger R^f_{\mu\nu,AB}, \tag{A7}$$

where the dagger operation includes a factor $(-1)^{\delta_{\mu 4}+\delta_{\nu 4}}$.The nonets $M^{(2)}$, $M^{(3)}$ and $M^{(4)}$ transform like M under all of $SU(3)_L \times SU(3)_R$, C, P. Under $U(1)_A$ all three transform with the phase $e^{-4i\nu}$, e.g.:

$$M^{(2)} \to e^{-4i\nu} M^{(2)}. \tag{A8}$$

It is seen that the $U(1)_A$ transformation distinguishes the "four quark" from the "two quark" states. In the full chiral Lagrangian treatment of the model under discussion there are explicit terms which model the breaking of this symmetry and hence cause the mixing.

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