An Improved Limit on the Rate of the Decay $K^+ \rightarrow \pi^+ \mu^+ e^-$

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An Improved Limit on the Rate of the Decay $K^+ \rightarrow \pi^+\mu^+e^-$


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We report results of a search for the lepton-family number violating decay $K^+ \rightarrow \pi^+\mu^+e^-$ from data collected by experiment E865 in 1996 at the Alternating Gradient Synchrotron of Brookhaven National Laboratory. We place an upper limit on the branching ratio at $3 \times 10^{-10}$ (90% C.L.). Together with results based on data collected in 1995 and an earlier experiment, E777, this result establishes a combined 90% confidence level upper limit on the branching ratio at $2.8 \times 10^{-11}$. We also report a new upper limit on the branching ratio for $\pi^0 \rightarrow \mu^+e^-$ of $3.8 \times 10^{-10}$ (90% C.L.).

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Despite the success of the Standard Model in describing elementary particle physics, several key issues remain unresolved. One is the gauge-hierarchy problem, i.e., that the scalar Higgs field, introduced to give mass to the W and Z vector bosons, suggests a particle of mass $\approx 100$ GeV/c$^2$ while its renormalization implies a mass scale at least ten orders of magnitude larger. Theoretical extensions to the Standard Model, such as Horizontal Gauge models, Technicolor, and Supersymmetry, were developed primarily to address the gauge hierarchy problem. These extensions permit lepton-family number (LFN) non-conserving decays. Observation of such phenomena would indicate new physics: physics beyond the Standard Model.

To test these theories in the kaon sector several experiments have recently been performed: $K^0_L \rightarrow \mu^+e^-$ [11], $K^0_L \rightarrow \pi^0\mu^+e^-$ [12] and $K^+ \rightarrow \pi^+\mu^+e^-$ ($K_{\pi\mu e}$). In the mid and late 1980s an experiment at the Brookhaven AGS, BNL E777, searched for the $K_{\pi\mu e}$ decay. Prior to that experiment the upper limit on the branching ratio for $K_{\pi\mu e}$ was measured at CERN to be $4 \times 10^{-9}$ [13]. E777 reduced that limit to $2.1 \times 10^{-10}$ (90% C.L.) [14]. In 1992 a more sensitive experiment, BNL E865, continued the search for the $K_{\pi\mu e}$ mode. In the 1995 running period E865 achieved the same limit as E777 [14]. In this paper, we report new results, from data collected in 1996.

The E865 detector system and trajectories from a simulated $K_{\pi\mu e}$ decay are shown in Fig. 1. Some of the details of the detector have been previously discussed in the context of an associated measurement of the branching ratio $K^+ \rightarrow \pi^+e^+e^-$ [1].

The apparatus resided in an unseparated beam directly downstream from a 5 m long vacuum decay volume. With $10^{13}$ protons impinging on a 10 cm long Cu production target, a secondary unseparated 6 GeV/c beam was produced containing about $10^{8}$ $K^+$ and $2 \times 10^{9}$ $\pi^+$ and protons per 1.6 second AGS pulse. All detector elements were either desensitized or removed from the beam region. A dipole magnet at the exit of the decay region approximately separated $K^+$ decay products by charge (negative to beam left, positive to beam right) and re-

FIG. 1. Plan view of the E865 apparatus.
duced the charged particle background originating upstream of the detector. Proportional chamber packages (P1-P4), each containing four planes of chambers, were arrayed on either side of a second dipole magnet to form a momentum analyzing spectrometer system. The momentum resolution of this configuration was \( \sigma_p \approx 0.003 P^2 \) GeV/c, where the momentum of the decay products, \( P \), ranged from 0.6 to 4 GeV/c.

Correct particle identification (PID) was of critical importance in reducing backgrounds. The first elements of the PID system were atmospheric pressure Čerenkov counters upstream and downstream of the spectrometer magnet. The left sides of each, C1L and C2L, were filled with hydrogen gas to detect \( e^- \) and not \( \pi^- \), and had a light yield of \( \approx 1.7 \) photoelectrons (p.e.) for \( e^- \). To reject \( e^+ \), the right sides (C1R and C2R) were filled with methane with a light yield of about 4.5 p.e. for \( e^+ \). The two sides were separated by a thin membrane. In order to reduce beam gas interactions, closed tubes of hydrogen gas were placed in the beam region of C1R and C2R.

The second PID element was a Bashlyk-style electromagnetic calorimeter (EM cal) \([2]\). This device consisted of 600 modules, each 11.4 by 11.4 cm\(^2\) in cross section and 30 cm (15 radiation lengths) along the beam, arrayed 30 horizontally by 20 vertically with 18 modules removed in the beam region. The approximate resolution of the array for electrons was \( \sigma_E/E = 0.09/\sqrt{E} \), where \( E \) is the electron energy measured in GeV. Typical energy deposition of a minimum ionizing particle was 250 MeV.

The third element was a muon range stack with 24 planes of proportional tubes, with tubes alternately oriented horizontally and vertically, and plates of steel placed between each pair of planes. The steel thickness was 5 cm between the first eight pairs of planes and 10 cm between the last four.

Trigger hodoscopes were located directly downstream of P1 (D hod), immediately upstream of the calorimeter (A hod), and between the eighth and ninth pairs of proportional tubes in the range stack (B hod).

The first-level trigger (T0) selected three charged particle tracks based on hodoscope and calorimeter hit patterns consistent with kaon three body decays, and with at least one particle on each side of the apparatus. The \( \pi\mu\) trigger added a signal from B hod, and at least 0.25 p.e. from C1L and C2L. The T0 trigger, downsampled by \( 10^4 \), also served as the trigger for our normalizing process, \( K^+ \rightarrow \pi^+\pi^+\pi^- (K_\tau) \). Satisfaction of any requested trigger, roughly 700 times per machine pulse, resulted in all information in the various data buffers being read out to a Fastbus based data acquisition system in about 100 \( \mu s \).

This paragraph describes the most important offline selection criteria \([3]\). Common requirements for the \( K_{\pi\mu\tau} \) and the \( K_\tau \) normalizing mode included a vertex formed from three reconstructed tracks, one negatively and two positively charged, with total vector momentum consistent with the beam phase space. Events with an extra (accidental) track on the left, not on the vertex, capable of making a C1L or C2L signal were removed. For \( K_{\pi\mu\tau} \) candidates, additional cuts were required. Background from \( \pi^0 \rightarrow \gamma e^+e^- \) (Dalitz) was suppressed by removing events with \( M_{ee} \) (effective mass of the \( e^- \) with either positive particle interpreted as \( e^+ \) ) < 50 MeV/c\(^2\). Electron PID required a signal in both C1L and C2L with corrected timing within \( \pm 4 \) ns, and energy deposited in the calorimeter divided by the particle measured momentum \((E/P)\) to be \( \pm 0.2 \). Pions were required to have C1R and C2R < 1.2 p.e., \( E/P < 0.85 \), and to not have penetrated the range stack to a depth of their expected ionization loss range. Muons were required to have C1R and C2R < 1.2 p.e., less than 450 MeV deposited in the calorimeter, and range stack penetration depth consistent with their range. High rates on the right, mostly at the one p.e. level, from high momentum muons and charged particle scintillation, prevented use of lower \( \pi \) and \( \mu \) thresholds in C1R and C2R. Table I summarizes the final PID efficiencies and probabilities of misidentification \([13]\). These measurements were made with particles of known identities from \( K_\tau \) and \( K_{\pi\mu\tau} \) with a subsequent Dalitz decay of the \( \pi^0 \).

<table>
<thead>
<tr>
<th>( \rightarrow \pi )</th>
<th>( \rightarrow \mu )</th>
<th>( \rightarrow e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ )</td>
<td>0.780 ± 0.004</td>
<td>0.049 ± 0.017</td>
</tr>
<tr>
<td>( \pi^- )</td>
<td>0.969 ± 0.002</td>
<td>-</td>
</tr>
<tr>
<td>( \mu^+ )</td>
<td>-</td>
<td>0.743 ± 0.014</td>
</tr>
<tr>
<td>( e^+ )</td>
<td>(1.7 ± .7) ( \times 10^{-6} )</td>
<td>&lt; 1.7 ( \times 10^{-6} )</td>
</tr>
<tr>
<td>( e^- )</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE I.** PID efficiencies and probabilities of misidentification. The symbol “\( \rightarrow \)” denotes “identified as”.

A likelihood analysis was used to evaluate the probability that selected events fit particular hypotheses, e.g., \( K_{\pi\mu\tau}, K_\tau \), accidental. In this method distributions derived from data were used as probability density functions (PDFs). These distributions included vertex and track quality, reconstructed beam parameters, PID detector responses and timing, and the invariant mass of the three decay products, to mention the most important. The extremes of these distributions were cut to allow the survival of about 95% of the events for the respective hypotheses. The resulting PDFs were then the templates to determine the probabilities that the variables in a given event originated from the hypothesized mode, say the \( i \)th mode. The logarithms of these probabilities were added to form the joint log-likelihood \((L_i)\) for the \( i \)th decay hypothesis. In the case of \( K_{\pi\mu\tau} \) the PDF for the invariant mass of the decay products was generated by Monte Carlo simulation. All other PDFs were generated from data.

An example of the final \( L_i \) distribution is seen in Fig. 2 where we display data and Monte Carlo simulated \( K_\tau \)
events. The 10% and 20% points on these plots represent likelihood values for which the probabilities of finding smaller likelihood are 10% and 20%, respectively.

![Graph](image)

**FIG. 2.** $K_\tau$ log-likelihood comparison between data and Monte-Carlo events. The vertical lines (solid for data and dash for MC) show the 10% and 20% likelihood points.

Candidate $K_{\pi\mu e}$ events were first subjected to the cuts described above, and for those that survived the $L_{\pi\mu e}$ was determined. The results of this analysis are presented in Fig. 3 as a scatter plot of $L_{\pi\mu e}$ vs. invariant mass of the decay products. The top plot is data where the invariant mass cut was increased from its nominal value of 3σ to 6σ for display purposes. The bottom plot shows simulated $K_{\pi\mu e}$ events, where a vector interaction for the decay was assumed in the simulation. Only seven data events with correct PID, but for which the three particles did not originate from a single decay and occurred accidentally in time.

The number and $L_{\pi\mu e}$ distribution of accidental events was estimated using events for which the variable describing the rms deviation from the mean time of all participating counters ($T_{rms}$) was more than 3 standard deviations, but which otherwise satisfied all $K_{\pi\mu e}$ cuts. Estimation of the relative probability that such events would have an acceptable $T_{rms}$ was made by evaluating the $T_{rms}$ distribution for events with total momentum greater than 6.5 GeV/c, i.e., events which are primarily accidental. The $T_{rms}$ distribution for high momentum events also gave the $T_{rms}$ PDF for accidental events with acceptable values of $T_{rms}$. The latter was used to randomly replace the $T_{rms}$ value for accidental events described above in forming the full $L_{\pi\mu e}$ distribution for accidentals.

![Graph](image)

**FIG. 3.** Scatter plot of ES65 1996 $K_{\pi\mu e}$ data (top) and Monte Carlo (bottom). The abscissa is the log-likelihood of the reconstructed events under the $K_{\pi\mu e}$ hypothesis, the ordinate is the invariant mass of the detected particles. The horizontal lines demark the 3σ mass region.

The number and $L_{\pi\mu e}$ distribution of accidentals was estimated using events for which the variable describing the rms deviation from the mean time of all participating counters ($T_{rms}$) was more than 3 standard deviations, but which otherwise satisfied all $K_{\pi\mu e}$ cuts. Estimation of the relative probability that such events would have an acceptable $T_{rms}$ was made by evaluating the $T_{rms}$ distribution for events with total momentum greater than 6.5 GeV/c, i.e., events which are primarily accidental. The $T_{rms}$ distribution for high momentum events also gave the $T_{rms}$ PDF for accidental events with acceptable values of $T_{rms}$. The latter was used to randomly replace the $T_{rms}$ value for accidental events described above in forming the full $L_{\pi\mu e}$ distribution for accidentals.

The $L_{\pi\mu e}$ distributions for $K_{\pi\mu e}$, Dalitz, and accidental modes are shown in Fig. 3. The estimated number of such events that would pass all $K_{\pi\mu e}$ cuts and appear in Fig. 3 within the accepted $M_{\pi\mu e}$ region is 2.6 ± 1.0: 0.06 ± 0.03 $K_\tau$ events, 0.1 ± 0.1 Dalitz events, and 2.4 ± 1.0 accidental events, in good agreement with the three events observed.

With the $L_{\pi\mu e}$ distributions for $K_{\pi\mu e}$, and those of the most dominant background modes weighted in relative proportion as determined above, a χ² function for Poisson-distributed data was minimized to determine the most probable number of $K_{\pi\mu e}$ and total background events in the data distribution of Fig. 3. Those numbers were 0.0 and 3.0, respectively, consistent with our
estimation of 2.6 background events. To determine the 90% confidence interval for this result, the Frequentist approach was used, with the number of background events assigned to be 2.6. Including the uncertainty in the assigned background level, the result of that analysis was that the expected number of 

$$K_{\pi\mu\nu e}$$

events in our data sample is less than 2.5 at the 90% confidence level.

![Histograms](image)

FIG. 4. $K_{\pi\mu\nu e}$ likelihood distributions for signal, Dalitz, and accidentals. The histograms are smoothed to compensate for the lack of statistics. The vertical scales for Dalitz and accidental distributions are normalized to their respective number of expected events.

The upper limit on the $K_{\pi\mu\nu e}$ branching ratio, normalized to the $K_\tau$ branching ratio, is calculated according to the formula:

$$BR(\pi\mu e) < \frac{BR(\pi\pi\pi)}{BR(\pi\mu e)} \cdot \frac{N(\pi\mu e)}{N(\pi\pi\pi)} \cdot \frac{A_{\pi\pi\pi}}{A_{\pi\mu e}} \cdot C,$$

where $BR$ denotes the branching ratio, $BR(\pi\pi\pi) = 0.0559 \pm 0.0005$ [16]; $N(\pi\mu e) = 2.5$, the 90% C.L. number of signal events; $N(\pi\pi\pi) = 2.19 \times 10^{10}$, the number of $K_\tau$ events adjusted for downscale factors; $A$ represents the geometrical acceptance of the detector system for the specific decay mode, $A_{\pi\pi\pi}/A_{\pi\mu e} = 1.64 \pm 0.02$, with 0.01 contribution from systematic uncertainty; and $C = 3.78 \pm 0.08$, the product of correction factors accounting for efficiency differences between the two modes. The bulk of $C$, 3.15, is the reciprocal of the product of the $\pi^+, \mu^+, e^-$ PID efficiencies shown in Table I, while the remaining $1.20$ results from acceptance differences between $K_{\pi\mu e}$ and $K_\tau$ due to cuts [13].

Employing these factors, we set a limit on the branching ratio $BR(\pi\mu e) < 3.9 \times 10^{-11}$ (90% C.L.). Combining this result with data collected in 1995, $BR < 2.1 \times 10^{-10}$ [14], and the E777 experiment, $BR < 2.0 \times 10^{-10}$ [15], yields a new upper limit of the branching ratio for $K_{\pi\mu e}$ of $2.8 \times 10^{-11}$ (90% C.L.).

This branching ratio implies that an intermediate boson in models described by a horizontal gauge interaction, e.g., Ref. [8], with purely vector coupling and strength equal to that of the weak interaction, would have a mass greater than 60 TeV.

Since the process $\pi^0 \to \mu^+\mu^-$ would be observed in our data through $K^+ \to \pi^+\pi^0$, $\pi^0 \to \mu^+\mu^-$, we also set an upper limit on its branching ratio. The only candidate events are the three discussed above, but their $M_{\mu\mu}$ values of 0.226, 0.282, and 0.332 GeV/$c^2$ are too far from $M_{\mu\mu}$ for the events to have originated from $\pi^0$ decays. We thus place an upper limit on the expected number of $\pi^0 \to \mu^+\mu^-$ events at 2.44 (90% C.L.). We again normalize to the $K_\tau$ mode with the ratio of acceptances being $A_{\pi\pi\pi}/A_{\pi\pi\pi(\mu\mu)} = 4.07 \pm 0.02$, the factor $C = 3.32 \pm 0.07$, and the branching ratio for $K_\tau$ is 21.16% [16]. Combining these factors gives an upper limit on the decay branching ratio for $\pi^0 \to \mu^+\mu^-$ of $3.8 \times 10^{-10}$ (90% C.L.), compared with the Review of Particle Physics limit of $1.72 \times 10^{-8}$ for the combined $\pi^0 \to (\mu^+\mu^- + \mu^-e^+)$ decays [16].

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[14] There were insufficient $K_\tau$ events with acceptable $K_{\pi\mu\nu e}$ kinematics to derive a $L$ distribution for that mode.