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GROWTH AND CONVERGENCE: A PROFILE OF DISTRIBUTION DYNAMICS AND MOBILITY

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Abstract. In this paper we focus primarily on the dynamic evolution of the world distribution of growth rates in per capita GDP. We propose new concepts and measures of "convergence," or "divergence" that are based on entropy distances and dominance relations between groups of countries over time.

We update the sample period to include the most recent decade of data available, and we offer traditional parametric and *new* nonparametric estimates of the most widely used growth regressions for two important subgroups of countries, OECD and non-OECD. Traditional parametric models are rejected by the data, however, using robust nonparametric methods we find strong evidence in favor of "polarization" and "within group" mobility.

Key Words: Growth, convergence, distribution dynamics, entropy, stochastic dominance, nonparametric, international cross-section.

JEL Classification: C13, C21, C22, C23, C33, D30, E13, F43, Q30, Q41.

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1. Introduction

Recent research on growth and convergence has provided a fertile interface between economic theorists, empirical economists and, increasingly, modern econometricians. It is now more widely accepted that the research effort in this area should be directed less toward questions of whether realizations from, or moments of, the distribution of growth rates converge, and more to questions concerning the "laws" that generate the distribution of growth rates, or incomes, and their evolution over time. This focus on whole distributions would hide less of the pertinent facts, and is more conducive to learning the nature and degree of what appears to be an "unconditional" divergence in growth rates and incomes. There is a well established tradition for our approach in the "income distribution" literature where ranking of distributions by, for example, Lorenz and Stochastic Dominance criteria, and the study of mobility, are well developed. Quah's work is rightly associated with the introduction of the distribution approach in the "growth convergence" literature; see Quah (1993, 1997).

In this paper we focus on significant features of the probability laws that generate *growth rates* that go beyond both the "β-convergence" and "σ-convergence." It is perhaps necessary to emphasize how narrow these two concepts are. The former concept refers to the possible equality of a single coefficient of a variable in the conditional mean of a distribution of growth rates! The latter, while being derivative of a commonplace notion of "goodness of fit," also is in reference to the mere fit of a conditional mean regression, and is additionally rather defunct when facing nonlinear, nonguassian, or multimodal distributions commonly observed for growth and income distributions. We will examine the entire distribution of growth rates, as well as the distributions of parametrically and nonparametrically fitted and *residual* growth rates relative to a space of popular conditioning variables in this literature. New concepts of convergence and "conditional convergence" emerge as we introduce new *entropy* measures of distance between distributions to statistically examine a deeper question of convergence or divergence.

Some of our findings may be viewed as alternative quantifications *and characterizations* of the distributional dynamics discussed in Quah (1993, 1997). Quah focuses on the distribution of *per capita* incomes (and relative incomes) for the same panel of countries in the world. He examines diffusion processes for the probability law generating these incomes, and a measure of "transition

probabilities," the stochastic kernel, to examine the evolution of the relative per capita incomes. On the other hand, we examine the distribution of the *growth rates* themselves, and use entropy distance metrics that reveal divergences, reflect the nature of divergences, and is closely related to welfare-theoretic notions of income mobility embodied in the inequality reducing measures of Shorrocks-Maasoumi-Zandvakili; see Maasoumi (1998).

Our findings are largely based on distributional dynamics and conform more closely with the theoretical models which take cross-country interactions into account (such as in Lucas (1993), and Quah (1997)) or which allow for elements of multiple regimes and certain types of non-convexities (as in Durlauf and Johnson (1995)). Employing recent techniques for handling mixed discrete/continuous variables, we also present *new* nonparametric estimates of both the growth rate distributions (see Li and Racine (2003, 2004), Racine and Li (2004), and Hall, Racine, and Li (forthcoming)). While we strongly agree with Quah on the limitations of the traditional panel regression (conditional mean) analysis in this area, we do connect to, and accommodate the current literature by applying our nonparametric techniques to the estimation of the most widely analyzed extended form of the original Solow-Swan regression model (as in Mankiw, Romer and Weil (1992)). But here too we offer a different (entropy) measure of "fit" for these regressions which may be viewed as an enhancement of the concept of σ-convergence since it involves many more moments than just the variance. Making summary statements with conditional means (averages) is not without value, but our modest message is that one can make better statements *and* one must caution that some distributions are poorly summarized by their means and/or variances.

The availability of data on a number of important dimensions that describe domestic economic activity in a given country and the collection of these individual country data into an international data source, such as in Summers and Heston (1988) and King and Levine (1993), has allowed a systematic examination of cross-country growth regressions. Focusing on the conditional means, the vast majority of the contributions to the empirics of economic growth have assumed that the main attributes that characterize growth such as physical and human capital exert the same effect on economic growth both across countries (intratemporally) and across time (intertemporally) and have assumed a (log) linear relationship (see Barro (1991) and Barro and Sala-i-Martin (1995)). There have been some recent studies that question the assumption of linearity and propose nonlinear

alternatives that allow for multiple regimes of growth patterns among different countries. These models are consistent with the presence of multiple steady-state equilibria that classify countries into different groups with different convergence characteristics (see Quah (1996) for a discussion of the evidence against the convergence hypothesis that underlies the standard approach). In this context, Bernard and Durlauf (1993) offer an explanation for the apparent strong evidence in favor of the convergence hypothesis (see Mankiw *et al.* (1992)). They argue that the convergence properties for all countries in the misspecified linear model are inherited from the convergence of a group of countries associated with a common steady state in the correctly specified *multiple regime* growth model.

Motivated by recent theories emphasizing threshold externalities (Azariadis and Drazen, (1990)), Durlauf and Johnson (1995) postulate that countries obey different laws of motion to the steady state. They employ regression tree methodology and divide countries into four subgroups according to their initial level of per capita income and literacy rate. They infer distinct linear laws of motion for the four subgroups. Thus, their work rejects the presumption on which the majority of the cross-country empirical growth literature is based. In particular, they find substantial differences in their estimate of the coefficient for the secondary enrollment ratio: it is insignificant for two of the subsamples and is positive for the other two (it is a third larger in magnitude for the middle income economies as compared to the high income ones). Hansen (2000) uses a threshold regression framework to test for sample splitting between different groups of countries and he finds evidence of such groupings. In a related study using some of the same methods as ours, Liu and Stengos (1999) allow for two nonlinear components, one for the initial level of GDP and the other for the secondary enrollment rate. They find that the presence of nonlinearities were mainly due to groupings of countries according to their level of initial income, whereas the effect of human capital (as measured by the secondary enrollment rate) was in essence linear.

As has been pointed out by Durlauf and Quah (1999), the dominant focus in these studies is on certain aspects of estimated conditional means, such as the sign or significance of the coefficient of initial incomes, how it might change if other conditioning variables are included, or with other functional forms for the production function or regressions. Many of these empirical models, including panel data regressions, fail to serve as vehicles to identify and distinguish underlying

economic theories with sometimes radically different implications and predictions. Many also run counter to observed income distributional dynamics, or are unable to explain them. In addition, all of the above studies rely on "correlation" criteria to assess goodness of fit and to evaluate "convergence." Our first step is to rectify this shortcoming, especially when considering nonlinear and/or nonparametric regressions. This we achieve with two entropy measures of fit. The resulting analysis produces "fitted values" of growth rates, as well as "residual growth rates" which will be used for fresh looks at the question of "conditional" convergence. Our nonparametric kernel estimates of conditional growth are free from some of the functional form misspecifications that have been pointed out by various authors in this area. We shed some light on potential nonlinearities in growth relations.

Turning to the main objective of this paper, we examine the relation between growth rate *distributions* for different country groups, as well as the evolution of the generating law over time, both within and between country groups. The nonparametric density method of Hall *et al.* (forthcoming) is utilized to analyze these questions. We quantify these distribution distances and movements by entropy measures, and use the latter to examine convergence (conditional and unconditional) as a new statistical hypothesis. Our data are extended beyond previous studies and span the last 35 years of available data.

The plan of the paper is as follows. In Section 2 we present the elements of the traditional "work horse" model of this literature. In Section 3 we propose to fit parametric and nonparametric regression models on the data panel for two different groups of countries, the OECD and the "rest of the world" consisting of the lesser developed countries. We also offer a conditional moment test of the traditional parametric specification. In Section 4 we present the unconditional distribution of the growth rates, and the distribution of their fitted values. Next, we obtain k-class entropies of each distribution, especially for two values of k, the Shannon entropy, and for k=1/2 (see Granger, Maasoumi and Racine (forthcoming)). Our approach is appealing because the distribution of growth rates across countries and time cannot be successfully summarized by their variances alone (unless they are normal). Additionally, inferences regarding the fit of these models is assessed by a metric entropy measure of distance between the actual and fitted distributions for each country group. We report the entropy distance between the two groups of countries (both for fitted and actual growth

rates). The distance based on "raw" growth rates is a new measure of unconditional convergence. The one based on the fitted values is a new measure of "conditional convergence." These entropies and entropy distances reveal how far apart (dispersed) are the economies within each group and between the two groups. If indeed there is statistically significant convergence to a common steady state then one expects that these distance measures "shrink" in size as one moves from the 1960's to the 1970's through the 1990's. We find that the empirical evidence is compatible with bipolar development and "clubs."

Contrary to commonly assumed models, the evolution of these distances or laws may not be "linear." For example it may be that the distance first decreases and then increases. Within each group, even if one finds β-convergence (the coefficient of initial income may be negative, signifying that a country with a lower GDP will have higher growth rates thereby catching up with the rest of the countries in the same group), entropy within each group will reveal any unequal pattern of growth rates (conditional and unconditional). If the growth rates are roughly equal, entropy will take its maximum value (log N, in the case of Shannon's, where N is the number of countries in the group). Thus we are able to reveal more of the growth mobility dynamics even within groups. This offers an examination of mobility dynamics which tells us how distributions change and by how much, in the sense of Shorrocks-Maasoumi. In other words we are able to capture nonlinearities in the growth dynamics of different income classes (heterogeneity in the growth paths). Quah (1997), looking at the per capita incomes, examines the probabilities of related transitions. This approach captures the cross-sectional heterogeneity and the tendency towards polarization of the cross-country distribution.[1] The two approaches are clearly interconnected and complementary but different. Maasoumi (1998) sheds light on the relation between these two notions of mobility.

Our reported entropies in the distributions of growth rates and model *residuals* for all countries and both groups reveal why it has been false to assert convergence, in any sense, without grouping of countries.

What the proposed approach does that has not been done before is to define, measure, and test for convergence in the probability laws that generate cross-country growth rates, explicitly allow for

[1]Fiaschi and Lavezzi (2003) have tried to combine the two approaches in a Markov transition matrix framework. However, their approach suffers from the complexity of the state space in terms of both income levels and growth rates, since there is no natural way to obtain its partition ex-ante.

heterogeneity between different country groups, and base inferences on more robust nonparametric estimators.[2]

2. The Traditional Parametric Setting.

It is helpful to first present the mechanics of the traditional regression models of the conditional mean of the distribution which will be the primary focus of our work. This regression has been the main focus in the literature. Our recollection in this section helps to identify some popular conditioning variables. But we also offer some advances in the analysis of this conditional mean which would be helpful when one wishes to make statements that are useful "on average" for sufficiently homogeneous country groups. Mankiw *et al.* (1992) assume a production function of the form $Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta}$, where Y, K, H, and L represent total output, physical capital stock, human capital stock and labor, respectively, and A is a technological parameter. Technology is assumed to grow exponentially at the rate ϕ, or $A_t = A_0 e^{\phi t}$. By linearizing the transition path around the steady state, they derive the path of output per effective worker $\widetilde{y}$ ($\widetilde{y} = Y/AL$) between time period T and $T + r$ as follows:

$$\ln \widetilde{y}_{T+r} = \theta \ln \widetilde{y}^* + (1 - \theta) \ln \widetilde{y}_T, \tag{1}$$

where $\theta = (1 - e^{-\lambda r})$, λ is the rate of convergence and $\widetilde{y}^*$ is the steady state level of output per effective worker. In order to derive the growth of output per worker (Y/L), they substitute for the steady state level of output per worker ($\ln \widetilde{y}^* = \alpha \ln k^* + \beta \ln h^*$), noting that the steady state levels of capital per effective worker (k^*) and human capital per effective worker (h^*) depend on the share of output devoted to physical capital accumulation (s^k), the share of output devoted to human capital accumulation (s^h), the growth of the labor force (n), and the depreciation rate for (human and physical) capital (δ). Finally, the growth of output per worker between period T and $T + r$ of country i is obtained by noting that $\ln \widetilde{y}_T = \ln(Y/L)_T - \ln A_0 - \phi T$ and subtracting initial

[2]Quah (1996, 1997) looks at the distributions of per capita incomes and its various transformations, and their evolution into a bipolar set. Quah's work is similar in spirit to ours but does not offer measures of "distance" between distributions, as we do.

income from both sides of (1) to arrive at:

$$\begin{aligned}\ln\left(\frac{Y}{L}\right)_{i,T+r} - \ln\left(\frac{Y}{L}\right)_{i,T} &= \phi r + \theta(\ln A_0 + \phi T) + \theta\frac{\alpha}{1-\alpha-\beta}\ln s_i^k \\ &- \theta\frac{\alpha+\beta}{1-\alpha-\beta}\ln(n_i+\phi+\delta) \\ &+ \theta\left(\frac{\beta}{1-\alpha-\beta}\right)\ln s_i^h - \theta\ln\left(\frac{Y}{L}\right)_{i,T}.\end{aligned} \tag{2a}$$

Mankiw *et al.* (1992, p. 418) point out that the steady state level of output per worker can also be expressed in terms of the (steady state) level of human capital (h^*), rather than s^h. In this case, the growth of output per worker becomes:

$$\begin{aligned}\ln\left(\frac{Y}{L}\right)_{i,T+r} - \ln\left(\frac{Y}{L}\right)_{i,T} &= \phi r + \theta(\ln A_0 + \phi T)\left(\frac{1-\alpha-\beta}{1-\alpha}\right) \\ &+ \theta\frac{\alpha}{1-\alpha}\ln s_i^k - \theta\frac{\alpha}{1-\alpha}\ln(n_i+\phi+\delta) \\ &+ \theta\left(\frac{\beta}{1-\alpha}\right)\ln h_i^* - \theta\ln\left(\frac{Y}{L}\right)_{i,T}.\end{aligned} \tag{2b}$$

As they point out, testing depends on "...whether the available data on human capital correspond more closely to the rate of accumulation (s_h) or the level of human capital (h)." The early literature used data on rates of enrollment corresponding to the model in (2a). More recent contributions have used estimates of the number of years of schooling of the working age population corresponding more closely to the formulation in (2b).

Mankiw *et al.* (1992) estimated the model in (2a) with cross-section data and used the ratio of investment to GDP to measure s^k and the secondary enrollment rate (adjusted for the proportion of the population that is of secondary school age) to measure human capital (s^h). Others have used primary as well as secondary enrollment rates to measure human capital (see Barro and Sala-i Martin (1995)).

As it is common with most recent contributions we employ panel data over seven 5-year periods: 1960-1964, 1965-1969, 1970-1974, 1975-1979, 1980-1984, 1985-1989 and 1990-1994. We estimate

the unrestricted versions of the models in (2b) as follows:

$$y_{it} = a_0 + a_1 D_t + a_2 D_j + a_3 \ln s_{it}^k + a_4 \ln(n_{it} + \phi + \delta)$$
$$+ a_5 \ln x_{it} + a_6 \ln h_{it} + \varepsilon_{it}, \tag{3}$$

where y_{it} refers to the growth rate of income per capita during each period, x_{it} is per capita income at the beginning of each period, h_{it} is human capital measured either as a stock or as a flow. D_t and D_j are dummy variables for each period and for certain regions such as Latin America or Sub Saharan Africa, respectively. The need for dummies to identify the time period over which the model is estimated is clear from equation (2b). Regional dummies have been included by many previous researchers to account for idiosyncratic economic conditions in these two regions. Initial income estimates are from the Summers-Heston data base, as are the estimates of the the average investment/GDP ratio for 5-year period. The average growth rate of the per capita GDP and the average annual population growth for each period are from the World Bank. Finally, the average years of schooling in the population above 15 years of age are obtained from Barro and Lee (2000).

Durlauf and Quah (1999) have provided an insightful summary of the empirical results from these regressions, their extensions, and their ability or inability to address the validity and predictions of both the exogenous and endogenous growth theories, with different treatments of human capital and technical assumptions. It is clear that negativity or significance of the impact of initial income in these regressions is insufficient evidence to distinguish between the underlying models/theories. It is the distributional dynamics, or "mobility" characteristics of these economies that are more interesting, less fragile as evidence, and more relevant especially in explaining within group interactions of economies that are either geographically close, or within trade groups, or similar in stage of social and economic development. Nevertheless, we include in the next section our more robust findings regarding the above regression models.

3. Growth Regressions and Their Fit.

3.1. **Parametric Results.** We first consider a linear parametric model which has been used to model this relationship. Note that this model is linear and additive in nature, while there is no interaction between the categorical variables (year, OECD status) and the continuous variables.

Table 1 summarizes the estimated model (`R` code and data to reproduce these results are available upon request from the authors).

Table 1. Parametric Model Summary

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	6.5101	3.8180	1.71	0.0887
OECD	−0.0043	0.0044	−0.97	0.3311
d1970	0.0001	0.0039	0.02	0.9816
d1975	0.0028	0.0040	0.70	0.4853
d1980	−0.0073	0.0040	−1.81	0.0712
d1985	−0.0238	0.0041	−5.78	0.0000
d1990	−0.0136	0.0042	−3.25	0.0012
d1995	−0.0187	0.0043	−4.35	0.0000
initgdp	−3.3940	2.0025	−1.69	0.0906
initgdp^2	0.6572	0.3908	1.68	0.0931
initgdp^3	−0.0558	0.0336	−1.66	0.0975
initgdp^4	0.0018	0.0011	1.63	0.1043
popgro	−0.0172	0.0105	−1.63	0.1035
inv	0.0185	0.0023	7.93	0.0000
humancap	0.0007	0.0032	0.21	0.8366
humancap^2	0.0011	0.0021	0.51	0.6084
humancap^3	0.0005	0.0011	0.45	0.6512
Residual standard error: 0.026 on 599 degrees of freedom				
Multiple R-Squared: 0.2856, Adjusted R-squared: 0.2665				
F-statistic: 14.97 on 16 and 599 DF, p-value: $< 2.2 \times 10^{16}$				

Before proceeding further, we offer a new test for correct parametric specification of the model summarized in Table 1 since inference based upon incorrectly specified parametric models will be unreliable. We proceed with a robust consistent nonparametric test developed in Hsiao, Li, and Racine (under revision) which we now briefly outline.

If we denote the parametric model by $m(x_i, \gamma)$ and the true but unknown conditional mean by $E(y_i|x_i)$, then a test for correct parametric specification is a test of the hypothesis $H_0 : E(y_i|x_i) = m(x_i, \gamma)$ almost everywhere versus the alternative $H_1 : E(y_i|x_i) \neq m(x_i, \gamma)$ on a set having positive measure. Equivalently, letting $u_i = y_i - m(x_i, \gamma)$, correct specification requires that $E(u_i|x_i) = 0$ almost everywhere, with a consequence of incorrect functional specification being that $E(u_i|x_i) \neq 0$ on a set with positive measure, or, equivalently, that $[E(u_i|x_i)]^2 = 0$ and $[E(u_i|x_i)]^2 \geq 0$. To avoid problems arising from the presence of a random denominator in the nonparametric estimator of $E(u_i|x_i)$, a density weighted version given by $[E(u_i|x_i)]^2 f(x_i)$ is deployed.

To test whether $E(u_i|x_i) = 0$ holds over the entire support of the regression function, the statistic $I \stackrel{def}{=} E\{[E(u_i|x_i)]^2 f(x_i)\} \geq 0$ is chosen. Note that $I = 0$ if and only if H_0 is true, and I therefore serves as a valid candidate for testing H_0. The sample analogue of I is obtained by replacing u_i with the residuals obtained from the parametric null model, $\hat{u}_i = y_i - m(x_i, \hat{\gamma})$, and by replacing $E(u_i|x_i)$ and $f(x_i)$ with their consistent kernel estimators, while the null distribution of the statistic is obtained via resampling methods ('wild-bootstrapping'). This test is directly applicable to the problem at hand involving a mix of discrete and continuous data. The test has been shown to have correct size and, being consistent, to possess good power properties against a wide class of alternative models (see Hsiao, Li, Racine (under revision) for further details).

Applying this test to the parametric model summarized in Table 1 yields a p-value of 4.07087×10^{-06}. Unsurprisingly, this is extremely strong evidence against the null and indicates parametric misspecification; See Durlauf and Johnson (1995) for similar findings based on other methods. Given that we *reject* the null of (this) parametric specification, and given the presence of both discrete and continuous data, we choose to proceed with a rather new nonparametric approach.

3.2. **Nonparametric Results.** For what follows, we consider a fully nonparametric local linear specification using the estimator of Li and Racine (2004) that permits us to model the mix of discrete and continuous data types found in the present context. We summarize the nonparametric results using partial regression plots. These plots simply present the estimated multivariate regression function via a series of bivariate plots in which the regressors not appearing on the horizontal axis of a given plot have been held constant at their respective medians. That is, if we wish to present the nonparametric regression of y on x_1, x_2, and x_3, we plot y versus $E(x_1, \bar{x}_2, \bar{x}_3)$, y versus $E(\bar{x}_1, x_2, \bar{x}_3)$, and y versus $E(\bar{x}_1, \bar{x}_2, x_3)$ where the bar denotes a median which allows one to visualize the multivariate regression surface via a series of two-dimensional plots. One of the appealing features of this approach is that it permits direct comparison of the parametric and nonparametric results.

The profiles presented in figures 1 and 2 are constructed using our panel of 616 observations in the following manner. First, least-squares cross-validation is used to obtain the appropriate bandwidths for the discrete and continuous regressors (see Li and Racine (2004) for details). Next we

generate and plot the partial regression relationships between GDP Growth (Y) and each continuous explanatory variable holding the remaining continuous variables constant at their respective medians (year = 1980, initial GDP = 7.8, population growth = -2.6, investment = -1.8, human capital = 1.4 respectively). We also plot the partial parametric regression surfaces, and we consider separate plots for OECD versus Non-OECD members.

Our nonparametric approach allows for interactions among all variables and also allows for nonlinearities in and among all variables. Furthermore, the method has two defining features; i) if the underlying relationship is linear in a variable(s) then the cross-validated smoothing parameter is capable of automatically detecting this; ii) the method has better finite-sample properties than the popular local constant kernel estimator, in particular, it is minimax efficient and is known to possess one of the best boundary correction methods available. A summary of the particulars of the nonparametric method for this panel (bandwidths and so forth) are available upon request from the authors.

In the literature on growth convergence a great deal of attention has been paid to the relationship between GDP Growth and Initial GDP. This relationship is given in the first plot in Figure 1. It is clear that as Initial GDP rises, ceteris paribus, GDP growth falls. This would seem to offer evidence in favor of "β-convergence" As Durlauf and Quah (1999), point out, however, this is not evidence necessarily in favor of the traditional exogenous technical change, Solow-Swan model and its extended forms. A negative "coefficient" of initial GDP is not empirically incompatible with sometimes radically different theories.

An interesting feature arises when considering the conditional relationship between GDP growth and population growth for OECD versus Non-OECD countries. Note that for OECD countries, population growth "hurts" GDP growth. However, for Non-OECD countries, low levels of population growth are *beneficial* while only high levels hurt growth. This is a reflection of an apparent threshold level for population size which may support economic advancement. Many smaller and economically less developed countries consider their population size to be a handicap in supporting major industrial developments and investment.

These graphs make clear the importance of decomposition by country groups. Aggregating these countries hides the very different impact that each group has experienced from investment,

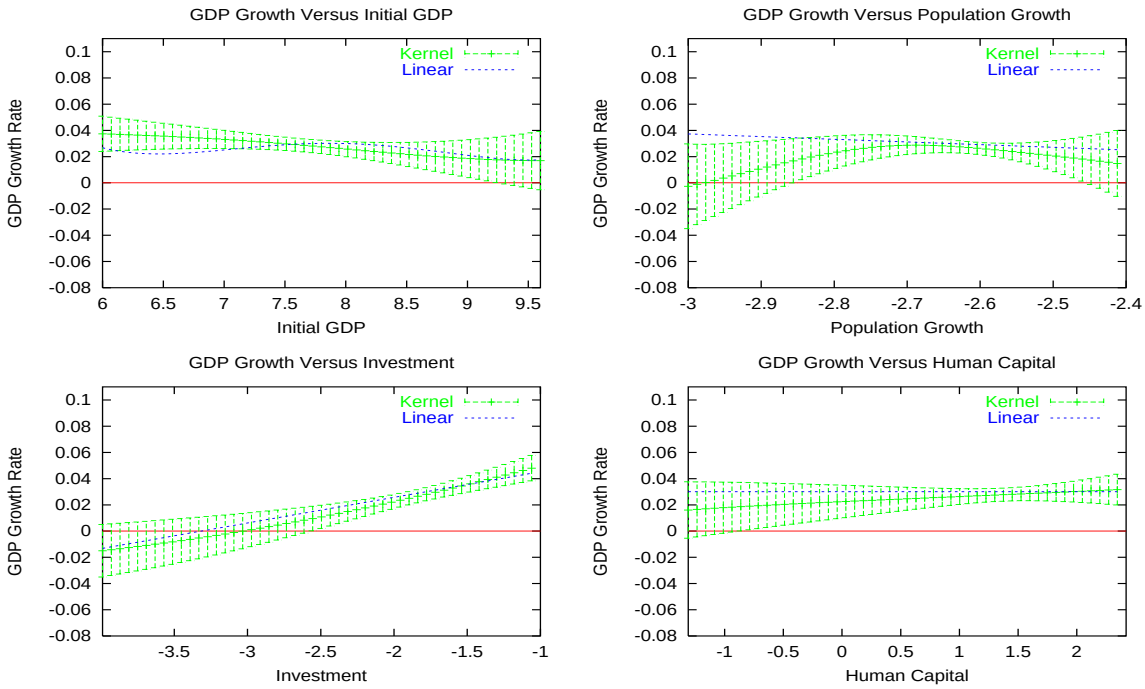


FIGURE 1. Nonparametric Partial Regression Plots for Non-OECD Countries.

population growth, and especially "human capital" upon its growth rates. While human capital has an increasing and positive relation with growth for OECDs, it has a tenuous impact for Non-OECDs. But a general association of low human capital and low growth rates is common to both groups.

Given these observations, it is rather interesting that, for the parametric regression on all of the countries, the OECD status (dummy) variable is insignificant. This underscores the dangers inherent to the unquestioning use of linear models!

4. Evolution of Cross-Section Distributions

In view of the evident limitations of conditional means (or even variances) as vehicles for analysing diversity (convergence!) within distributions, certainly of incomes, we now turn to the central analysis of this paper based on the whole distribution of growth rates. The stylized facts concerning the cross-section *distributions* of growth rates and their evolution are well laid out in Durlauf and Quah (1999). The most important of these are a "polarization" effect being largely an evolution

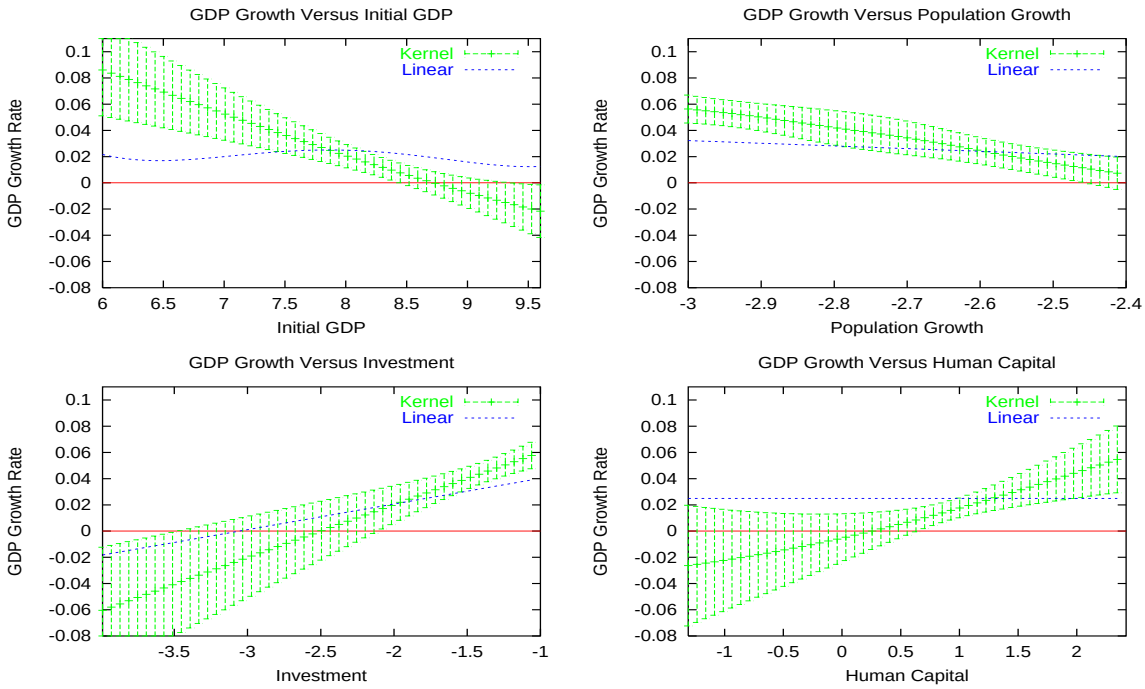


FIGURE 2. Nonparametric Partial Regression Plots for OECD Countries.

into a "bipolar" world, and "churning" or what we prefer to call "within group mobility" which, when examined in greater detail, points to possible "multimodality" and "clubs."[3]

As noted earlier, several authors, including Binachi (1997), Jones (1997), and Quah (1993, 1997), have examined the more interesting aspects of the dynamics in the distribution of growth rates in light of the predictions of various growth models. This section's analysis, and our main interest, is in the same spirit. In particular, Binachi too obtains (different) nonparametric density estimates for growth rate distributions at each point in time, whereas Quah (1997) examined the (relative) per capita income distributions and their "transition" laws by analyzing transition probabilities and their continuous counterpart, stochastic kernels. Examination of mobility (in any attribute) has traditionally been conducted in two ways. Transition matrices (kernels) and indices defined

[3]Classification of countries by proximity or trade are given in Quah (1997) and others. We believe this question deserves greater attention and is perhaps best left to studies that consider multidimensional clustering which combine two different techniques. The multidimensionality aspects may be addressed in the manner of Maasoumi and J. H. Jeong (1985) who considered composite measures of well being for the world, including per capita incomes. The clustering techniques of Hirschberg, Maasoumi and Slottje (2001) may then be applied to these multidimensional indices.

over them, or inequality reducing measures based on "distances" between distributions and how they evolve toward the "equal" distribution over time. Ideal indices of mobility based on the latter approach are connected to those in the former, but a full understanding of the relations is not yet at hand. See Maasoumi (1998) for an extended discussion. In addition, per capita income and growth rates of incomes are at least statistically distinct (but surely related) variables. In comparing our results with the complementary findings of Quah (1997), these distinctions must be born in mind. Our findings reinforce the notion of divergence and polarization in both incomes and their growth rates. We also find that some groupings of countries identify somewhat more uniform sets, but neither identifies the causes of divergence in incomes or growth rates. Perhaps there is substance in the view that "conditional convergence" is a rather vacuous concept. Of course there are causes for the observed divergence.

4.1. **Distribution Dynamics: Actual Growth Rates.** For what follows, we focus attention on the probability density function (PDF) and cumulative distribution function (CDF) of growth rates, focusing on how the distribution of growth rates evolves over time and behaves with respect to OECD status. Rather than presume that growth rates are generated from a known parametric family of distributions, we use robust nonparametric methods capable of providing consistent estimates of the unknown PDF and CDF. We elect to use kernel methods, and we estimate Rosenblatt-Parzen type density estimates. Data-driven methods of bandwidth selection are employed, and bandwidths are selected via likelihood cross-validation, which results in estimates that are close to the true density in terms of the Kullback-Leibler information distance $\int f(y|x) \log\{f(y|x)/\hat{f}(y|x)\}\, dy$ where $f(y|x)$ represents the conditional density function (see Silverman (1986, page 53) and Hall (1987)).

We begin by modeling the PDF and CDF of the actual growth rates conditional on OECD status (0/1) and year (1965, 1970, ...). Note that, by modeling the joint distribution of growth rates, year, and OECD status and then conditioning on OECD status and year, we obtain a kernel density estimate having improved finite-sample properties relative to the traditional univariate kernel density estimate for growth rates for a particular year and OECD status (the latter using only a subset of the data used to construct the former). The conditional density estimator found in Hall *et al.* (forthcoming) is used due to the mix of continuous and discrete data present.

Figure 3 presents plots of the conditional PDF and CDF for all combinations of OECD status and year, while Figure 4 presents a plot of all OECD distributions for all years and all Non-OECD ones again for all years.

Several important features of these results may be noted:

(1) The growth distributions for the OECD and Non-OECD countries are very different, and have remained very different from 1965 to 1995.

(2) The distribution for OECDs is less dispersed and is symmetrical, becoming more so over time.

(3) The distribution for Non-OECDs is less symmetrical, and not converging to any particular form, and becoming less concentrated. It appears to be forming a bimodality of its own suggesting multimodality that, while not incompatible with parametric/traditional regression models, may be difficult for "regression techniques" to identify and examine. Within group mobility in the Non-OECDs is made evident by these graphs. It is possible to derive "mobility profiles" in the manner of Maasoumi and Zandvakili (1990), but we leave this to future work.

(4) When combined, the previous two observations agree and further explain the often observed and expanding multimodality in the world distribution of growth rates; For example see Durlauf and Johnson (1995) who arrive at compatible inferences based on multiple regressions and regression trees.

(5) Linton, Maasoumi and Whang (2002) consider welfare-theoretic bases for assessing the relations between distributions. They propose subsampling based tests of First, Second (and higher) Order Stochastic Dominance, FSD and SSD, respectively. Although one of us has applied these tests to some of the cases in this paper, we partially agree with Quah (1997) who suggests one has a *census* of all countries in the population here, not a sample. Given this point of view, the following observations may be viewed as free from sampling variation:

   (a) In 1965-1970 OECD First Order Stochastically Dominates (FSD) the non-OECDs since its CDF lies everywhere to the right of the latter. From 1975 there is no FSD ranking

between these two groups, but there is Second Order SD (SSD) of OECDs over non-OECDs through 1990 (with the possible exception of 1980). The order rankings are inconclusive and almost identical for the 1980 and 1995 pair! It should be noted that FSD is a very strong rank order and implies SSD. SSD obtains on the basis of welfare functions that are increasing and concave (equality preferring). Thus, one might conclude that the evolution of the non-OECD distribution has been positive, and it is a higher degree of "convergence" of growth rates amongst the OECDs that contributes to its SSD over the non-OECDs in later years. Some of these observations are explained by the movement of China from a large population, low growth economy, to a large population, high growth economy status. There is much "churning," or "exchange mobility," and no "convergence" within the non-OECDs, and a tangible convergence and "growth mobility" within the OECDs. There are clearly a minimum of "two clubs" on the basis of growth rates alone. Similarly, Quah (1997) finds credible evidence of relative per capita income "clubs" on the basis of geographical proximity, as well as trading practices.

(b) Regarding the evolution of each group over time, again we find "a tale of two cities." OECDs have clearly "*deteriorated*" over time since the 1965, whereas the situation for non-OECDs is far less clearcut. The OECD growth distribution in 1965 *First order dominates* all other years. There is a clear break in the 1980s, resulting in a gradual strengthening of this rank order as they evolve toward 1995. Note that this period contained two recessions in the 1980s and early 90s. It would be interesting to re-examine this hierarchy when more recent data become available. It is interesting to note that, since FSD implies SSD, whatever small convergence in growth rates of OECD, if any, it is not enough to topple the SSD ranking (greater "equality") of earlier years over the latter years.

(c) Regarding the non-OECDs, the only clearcut ranking is that 1985 is First order dominated by every other year except 1995. Clearly this was not a "good" year for growth globally. But, the differential development within this group is well reflected by a lack

of FSD amongst other years. It is possible that 1965 weakly Second order dominates 1995, yet another reflection of a lack of "convergence" in these distributions.

We are in fact able to quantify the magnitudes of these movements between entire distributions! Thus we will report entropy distances and related tests in a subsequent section which shed light on the "magnitude" of these distances and focus on convergence.

4.2. **Distribution Dynamics: Nonparametric Fitted Growth Rates.** Our nonparametric regressions have produced what might be considered *robust fitted values of the growth rates* in the plane of the most popular conditioning variables.

We present the PDF and CDF of these "fitted" or estimated growth rates. Plots of these conditional PDFs and CDFs for all combinations of OECD status and year are followed, finally, by plots of all OECD distributions for all years as well as all Non-OECD countries for all years. The FSD and SSD rankings are similar to the "raw" growth distributions. The evolution of growth rates, as predicted by popular explanatory variables and free of "residual sources," tends to conform to the "unconditional" evolution analysis provided in the previous section. Several caveats are in order, however:

(1) The FSD rankings between the OECD and others is even stronger than for raw growth rates, becoming less strong toward 1980, whereby it is only a SSD ranking with decreasing strength toward 1995 where there may be at most a Third Order SD ranking between them. "Bipolarity" is surely not questioned.

(2) All of our previous statements regarding the "time path" of these distributions for each group are intact, but somewhat stronger rankings are possible for the non-OECD distributions over time (compare this with generally consistent results of Quah (1997) for per capita incomes, and Durlauf and Quah (1999)).

4.3. **Residual Growth Rates by OECD Status for all Years.** Appendix A presents results based on the distribution of our nonparametric regression "residuals," organized in the same manner as the last two sections. These residuals may be regarded as "conditional" growth rates in the usual meaning of conditioning in econometrics. The residuals are growth rates after controlling for the influence of conditioning variables. Of course this control is only achieved on the mean of the

growth rates, and the variables may continue to impact other distributional characteristics. This residual analysis is valuable since our residuals are robust to functional forms and any evidence of "convergence" of their distributions may be interpreted as evidence of "conditional convergence."[4] We summarize as follows:

(1) There is no FSD between the OECD and non-OECD groups. There is generally no SSD either, with the possibility of week SSD or higher orders for some later years. Once the mean differences due to conditional variables are removed, uniform ranking of these groups by dominance criteria vanishes. Interestingly, even the dispersion aspects of these two distributions are generally not sufficiently different to produce higher order (SSD) rankings. This is evidence in favor of "conditional convergence" in the sense developed in this paper.[5]

(2) The "fit" is generally good for the regressions, but less good for non-OECD data because of their heterogeneity.

(3) There is not much to separate these distributions over the successive five year intervals. The fit is equally good (bad) for each cross-section.

Also, note that the residuals are effectively 'smoothed' over time so that differences in the *residual* series are negligible for different time periods.

5. Entropy Measures of Distributional Distance

In this section we provide a formal quantification of the distributional distances and evolutions observed in the last section. This is done by using a *metric entropy* measure suggested in Granger *et al.* (forthcoming).[6] Any entropy measure is useful as an indicator of divergence from the uniform distribution, and is thus a measure of "equality," or concentration in the corresponding distribution. The characterization of a density afforded by entropies is only a little short of that provided by characteristic functions. Thus entropies are generally superior to other moment-based

[4]We are sympathetic, however, to the view that considers "conditional convergence" as rather lacking in meaning or consequence, especially relative to substantive theories and hypotheses which initially motivated this area of research.

[5]It is worth noting that "strong" non-uniform rankings are not ruled out. There do exist cardinal (welfare) criteria according to which these distributions may be ranked. Variance is one such criterion, however unlikely in this situation.

[6]For comparison purposes, we also computed the Kullback-Leibler divergence measure. These were removed to save space but give consistent results. The KL measure is the most popular index of divergence between distributions, but it is not a metric and unsuited for precisely the type of comparisons of "distances" we need to make in this application.

criteria. Unfortunately, Shannon's popular entropy is not a metric and thus fails to be useful for multiple comparisons, exemplified by our application here where several years and/or groups of distributions are being compared. Granger *et al.* (forthcoming) developed a normalized entropy measure of "dependence" that has several desirable properties as well as being a proper distance metric. Some of these properties are briefly enumerated here for convenience. A measure of similarity/distance/dependence for a pair of random variables X and Y may be required to satisfy the following six "ideal" properties:

(i) It is well defined for both continuous and discrete variables. (ii) It is normalized to zero if X and Y are identical, and is conveniently normalized to lie between 0 and +1. (iii) The modulus of the measure is equal to unity if there is a *measurable* exact (nonlinear) relationship, $Y = g(X)$ say, between the random variables. This is useful in our use of this measure for assessing the fit of regressions. (iv) It is equal to or has a simple relationship with the (linear) correlation coefficient in the case of a bivariate normal distribution. Again, this is useful in our use of this measure for assessing the fit of regressions. (v) It is metric, that is, it is a true measure of "distance" and not just of divergence. (vi) The measure is invariant under continuous and strictly increasing transformations $h(\cdot)$. This is useful since X and Y are independent if and only if $h(X)$ and $h(Y)$ are independent. Invariance is important since otherwise clever or inadvertent transformations would produce different levels of dependence. This leads to a normalization of the Bhattacharya-Matusita-Hellinger measure of dependence/distance given by

$$S_\rho = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(f_1^{\frac{1}{2}} - f_2^{\frac{1}{2}}\right)^2 dx\, dy, \tag{1}$$

where $f_1 = f(x)$ and $f_2 = f(y)$ are the marginal densities of the random variables X and Y. If f_1 and f_2 are equal this metric will yield the value zero, and is otherwise positive and less than one. Granger *et al.* (forthcoming) demonstrate the relation of this normalized measure to k-class entropy *divergence* measures, as well as copulae. We use it as our primary means of assessing the distances between distributions. Testing for convergence is based on the null hypothesis that $S_\rho = 0$.

Below, two types of use are made of these entropy measures that reflect their universal role as both measures of "divergence" and measures of "fit" or "dependence." Tables that report entropies for the fit of the growth regressions allow an assessment of the "goodness of fit" of these models,

and represent new results in their own right. Since our regressions are not linear, the traditional measures of correlation and *linear dependence*, such as R^2, are clearly inadequate. Thus in these tables we offer the first robust dependence results for the fit of the traditional growth regression variables.[7]

TABLE 2. Shannon's Entropy (-$\int_{-\infty}^{\infty} f(x)\ln(f(x))\,dx$).

OECD	1965	1970	1975	1980	1985	1990	1995
Actual Growth Rate	-2.432	-2.324	-2.479	-2.584	-2.774	-2.686	-2.532
Parametric Fit	-3.533	-3.561	-3.582	-3.766	-3.733	-3.784	-3.768
Nonparametric Fit	-2.692	-2.689	-2.745	-3.027	-3.113	-3.092	-3.091
Parametric Residual	-2.668	-2.607	-2.683	-2.706	-2.806	-2.725	-2.669
Nonparametric Residual	-3.012	-2.956	-3.031	-3.012	-3.039	-3.054	-2.948
Non-OECD	1965	1970	1975	1980	1985	1990	1995
Actual Growth Rate	-2.077	-2.158	-2.029	-1.966	-1.942	-1.970	-1.800
Parametric Fit	-2.839	-2.855	-2.804	-2.881	-2.867	-2.868	-2.814
Nonparametric Fit	-2.871	-2.738	-2.525	-2.812	-2.729	-2.641	-2.226
Parametric Residual	-2.104	-2.195	-2.147	-2.054	-2.055	-2.087	-2.048
Nonparametric Residual	-2.262	-2.314	-2.281	-2.213	-2.229	-2.262	-2.267

In terms of Shannon's entropy (reported in Table 2), the *actual* growth rate distributions for OECD were becoming somewhat more concentrated until 1985, whereafter increasing in dispersion levels of 1965. For non-OECDs the increase in dispersion/inequality of growth rates is a steady pattern. Neither of these changes are "large" in absolute value (but see below for statistical evaluation).

TABLE 3. KL Entropy ($\int_{-\infty}^{\infty} f(x)\ln(f(x)/g(x))\,dx$) ($f(x)$=Non-OECD, $g(x)$=OECD). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference between OECD and Non-OECD countries. Kernel Evaluation of KL Entropy: OECD versus Non-OECD

	1965	1970	1975	1980	1985	1990	1995
Actual Growth Rate	0.803	0.160	0.383	0.630	1.378	1.237	0.580
	[0.182, 0.212]	[0.157, 0.187]	[0.184, 0.211]	[0.190, 0.228]	[0.194, 0.235]	[0.174, 0.216]	[0.323, 0.352]
Parametric Fit	1.623	1.628	1.064	1.098	1.270	1.669	1.572
	[0.294, 0.340]	[0.255, 0.304]	[0.261, 0.304]	[0.270, 0.339]	[0.284, 0.360]	[0.290, 0.346]	[0.301, 0.349]
Nonparametric Fit	1.154	0.504	0.476	0.085	0.512	0.950	1.047
	[0.240, 0.291]	[0.147, 0.182]	[0.226, 0.284]	[0.100, 0.128]	[0.128, 0.154]	[0.224, 0.272]	[0.349, 0.424]

[7]We compute all entropy measures in the following manner: (i) Compute the conditional Rosenblatt-Parzen density estimates with covariates OECD status and Year via cross-validation. (ii) Generate a grid in $[-0.25, 0.25]$ having grain 0.001 (there are 501 points on this grid). (iii) Evaluate the Rosenblatt-Parzen kernel estimator on the grid of 501 points. Note that at the edges of the grid $\hat{f}(x|\text{OECD}, \text{Year}) = 0.0$. (iv) Evaluate each respective entropy via numerical quadrature.

TABLE 4. S_ρ Entropy $(\frac{1}{2}\int_{-\infty}^{\infty}[\sqrt{f(x)} - \sqrt{g(x)}]^2\,dx)$ ($f(x)$=Non-OECD, $g(x)$=OECD). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference between OECD and Non-OECD countries.

	1965	1970	1975	1980	1985	1990	1995
Actual Growth Rate	0.127	0.035	0.069	0.089	0.182	0.180	0.112
	[0.040, 0.048]	[0.032, 0.040]	[0.043, 0.049]	[0.039, 0.044]	[0.040, 0.047]	[0.038, 0.045]	[0.054, 0.065]
Parametric Fit	0.259	0.232	0.156	0.147	0.174	0.175	0.198
	[0.061, 0.070]	[0.053, 0.063]	[0.056, 0.068]	[0.059, 0.069]	[0.057, 0.067]	[0.059, 0.069]	[0.060, 0.069]
Nonparametric Fit	0.252	0.111	0.067	0.015	0.077	0.141	0.143
	[0.043, 0.051]	[0.031, 0.038]	[0.042, 0.051]	[0.022, 0.029]	[0.027, 0.032]	[0.038, 0.044]	[0.069, 0.077]

In table 2 we note that the distances S_ρ (also KL divergences not reported here) between OECD and others is significant at the 95% level for every date except 1970. Over time, we see that these distances declined in the 1960s, thereafter growing steadily until 1990, but seem to have declined in 1990-1995.

TABLE 5. S_ρ Entropy $(\frac{1}{2}\int_{-\infty}^{\infty}[\sqrt{f(x)} - \sqrt{g(x)}]^2\,dx)$ ($f(x)$=Actual, $g(x)$=Predicted).

OECD	1965	1970	1975	1980	1985	1990	1995
Parametric	0.196	0.232	0.228	0.240	0.173	0.215	0.243
Nonparametric	0.020	0.041	0.019	0.047	0.037	0.038	0.069
Non-OECD	1965	1970	1975	1980	1985	1990	1995
Parametric	0.140	0.117	0.143	0.188	0.183	0.174	0.209
Nonparametric	0.125	0.080	0.062	0.155	0.154	0.103	0.043

Table 3 reports the "goodness of fit" values for the parametric as well as our own nonparametric models. For OECDs, the parametric fit is much better than the nonparametric one. This is predictable from the relative homogeneity in this group. The nonparametric fit is much better for the non-OECDs, but deteriorates in the later parts of the sample.

TABLE 6. S_ρ Entropy $(\frac{1}{2}\int_{-\infty}^{\infty}[\sqrt{f(x)} - \sqrt{g(x)}]^2\,dx)$ ($f(x)$=Year$_t$, $g(x)$=Year$_{t+5}$). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference over time.

	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95
Pooled	0.003	0.006	0.014	0.032	0.008	0.008
	[0.013, 0.016]	[0.014, 0.017]	[0.015, 0.017]	[0.014, 0.017]	[0.015, 0.017]	[0.015, 0.016]
OECD	0.017	0.009	0.071	0.037	0.074	0.093
	[0.037, 0.045]	[0.038, 0.045]	[0.038, 0.048]	[0.036, 0.044]	[0.039, 0.045]	[0.036, 0.045]
Non-OECD	0.007	0.011	0.022	0.047	0.010	0.023
	[0.026, 0.030]	[0.025, 0.029]	[0.027, 0.030]	[0.026, 0.029]	[0.026, 0.030]	[0.026, 0.030]

Table 4 corresponds to our earlier graphical analysis of evolution through time. For the *pooled sample*, only the distances between 1980-85 are significant at 95% level. These distances first increase to 1985, but thereafter become small again. By these indices, one would infer "convergence" except in 1980-85, demonstrating the difficulty of analyzing distributional dynamics by *strong/complete* but non-uniform criteria. In the absence of uniform SD rankings, there will exist some "criterion function" which may reverse the conclusion of "convergence," by another criterion. This may explain some of the quandary in the current literature with different conclusions being reached by different authors on the question of convergence.

By either measure of divergence, the OECD countries moved forward by small amounts in the late 1960s and early 70s, but changing significantly in later periods (except for 1980-85). For the non-OECD growth distributions, on the other hand, the two measures suggest that their distributions have been changing slowly, indeed only significantly so in 1980-85.

The magnitude of changes over time are generally much larger for OECDs than others ('the rich get richer and the poor get poorer'). These observations add credence to those in Durlauf and Quah (1999) and elsewhere, that the most interesting aspects of the growth phenomenon appear to be in different distributional dynamics and mobility profiles of different country groups, rather than in the growth regressions.

Appendix C reports similar analysis for "conditional growth rates," i.e., the residuals of both the parametric and nonparametric growth regressions. Our earlier observations are confirmed by these entropy tests.

(1) The "fit" is generally good for the regressions, but less good for non-OECD data because of their less homogeneous membership.

(2) There is not much to separate these distributions over the successive five year intervals. The fit is equally good (bad) for each cross-section. Also, note that the residuals are effectively 'smoothed' over time so that differences in the *residual* series are negligible for different time periods.

(3) There is no significant change in these residual growth distributions at the 95% level, and almost always, even at the 90% level (the exception is, again, 1980-85 for OECDs which are significant at the 90% level).

(4) There is a further interpretation of these entropy measures of dynamic residual movements. Following Granger *et al.* (forthcoming), the entropies in this context may be regarded as robust measures of possibly nonlinear *serial dependence.* Accordingly, our results indicate that there is no evidence of significant serial dependence of residuals between these five year periods.

A summary of the conditional and unconditional ("actual") growth rates and their distributional characteristics is given in tables 11 and 12 of Appendix C.

6. Conclusions

Employing nonparametric kernel density and regression techniques, we have examined the otherwise traditional growth relationship and given new entropy measures of fit, as well as residual correlation for them. We have identified distinct effects of the major conditioning variables on the growth rates of different groups of countries. This leaves very little doubt that separate models are required to examine different groups of countries.

We have further examined the dynamics of cross-section distributions of actual growth rates, as well as "conditional" and "fitted" growth rates. Our study of these dynamics was based on Stochastic Dominance rankings, as well as tests based on entropy distances which shed further light on the mobility between and within groups of countries. Our robust findings tend to confirm the hypotheses of "convergence clubs" and polarization.

We agree with the conclusions of Durlauf and Quah (1999) that future work needs to address more successfully the need for modeling of cross-country interactions and remain consistent with the rich distributional dynamics observed here, and studied in the mobility literature as, for example, in Maasoumi and Zandvakili (1990). There is also a need to extend the scope of this field by considering other attributes of well-being than per capita incomes, and connecting to the literature which deals with its related issues; see, for example, Hirschberg *et al.* (2001), and Maasoumi and Jeong (1985).

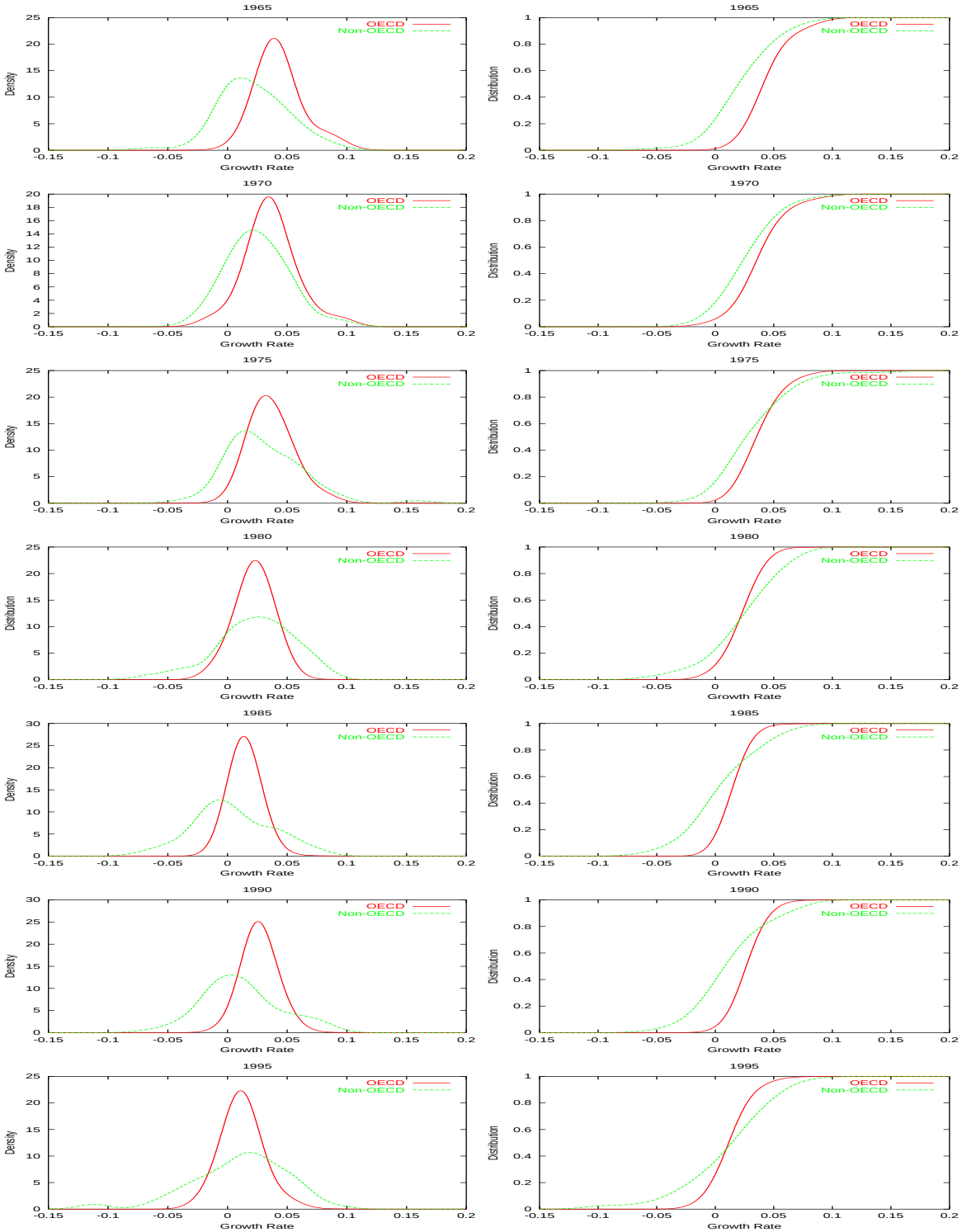


FIGURE 3. Growth Rate Distributions by Year and OECD Status

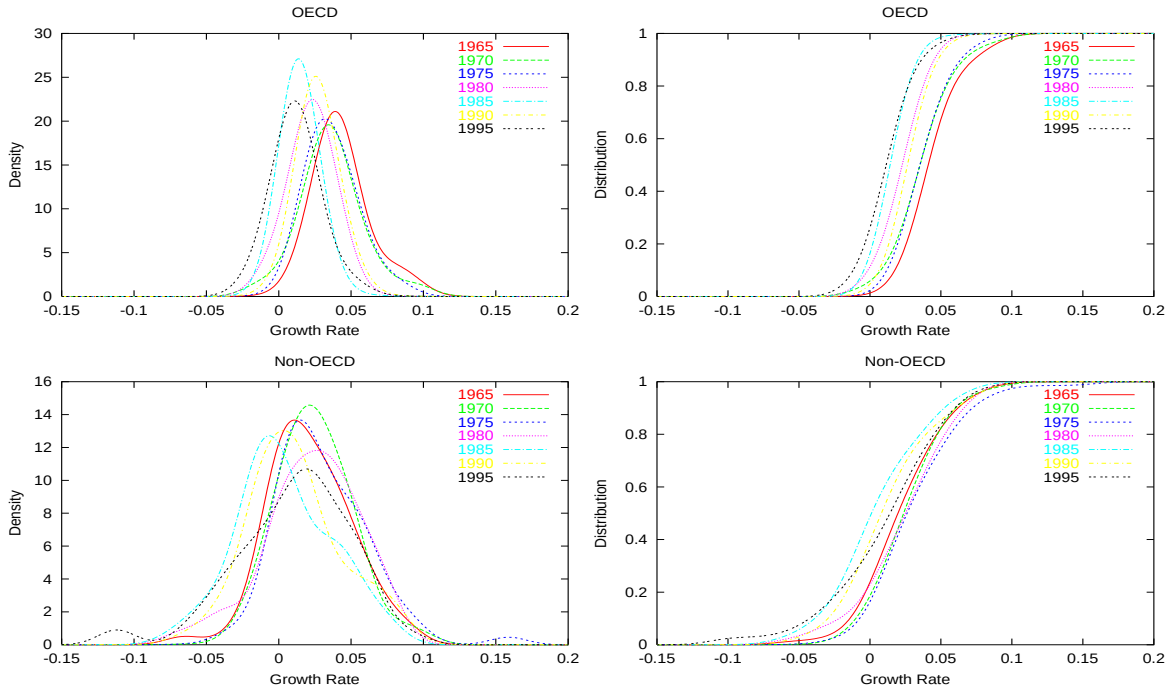


FIGURE 4. Growth Rate Distributions By OECD Status for All Years

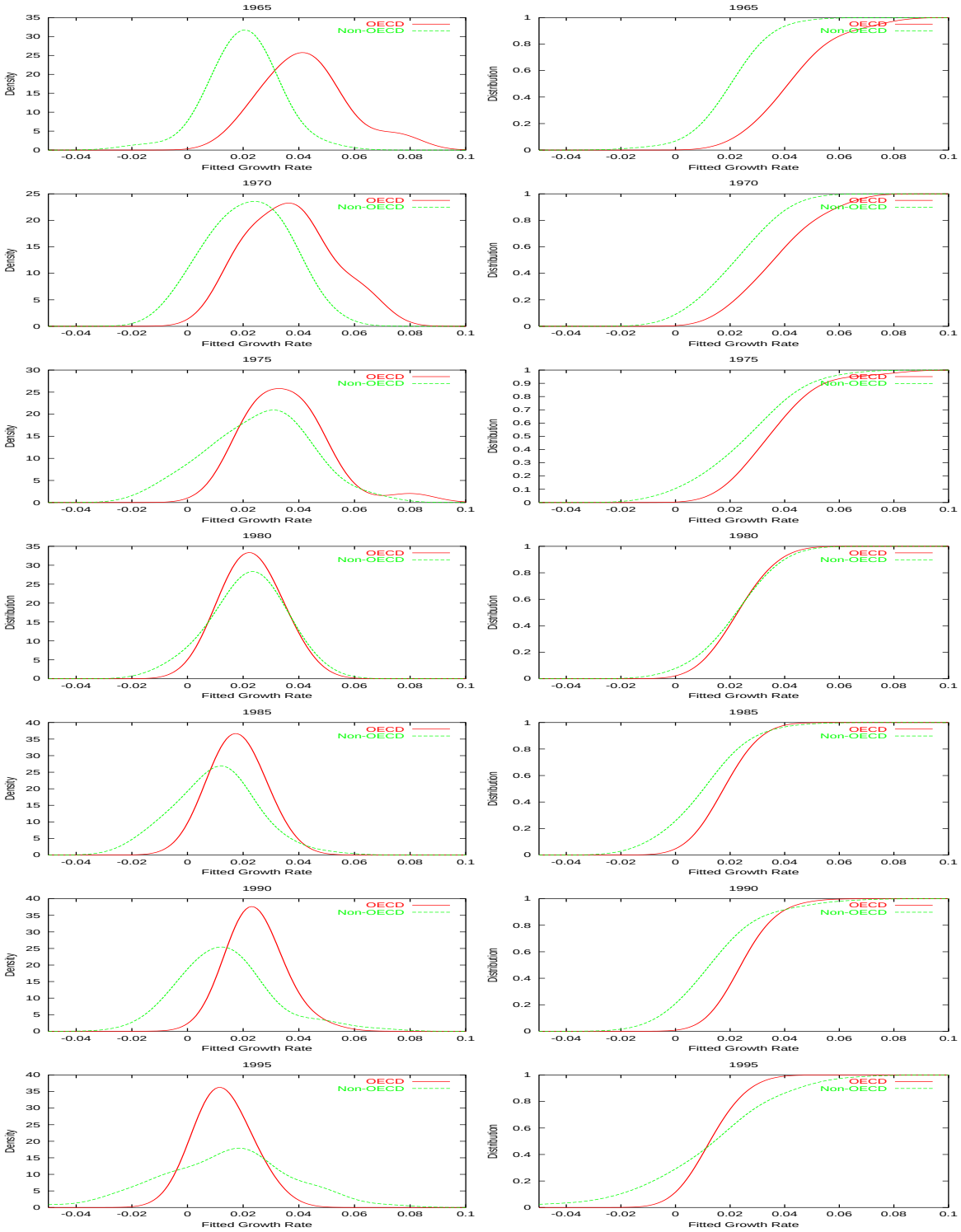


FIGURE 5. Nonparametric Fitted Growth Rates

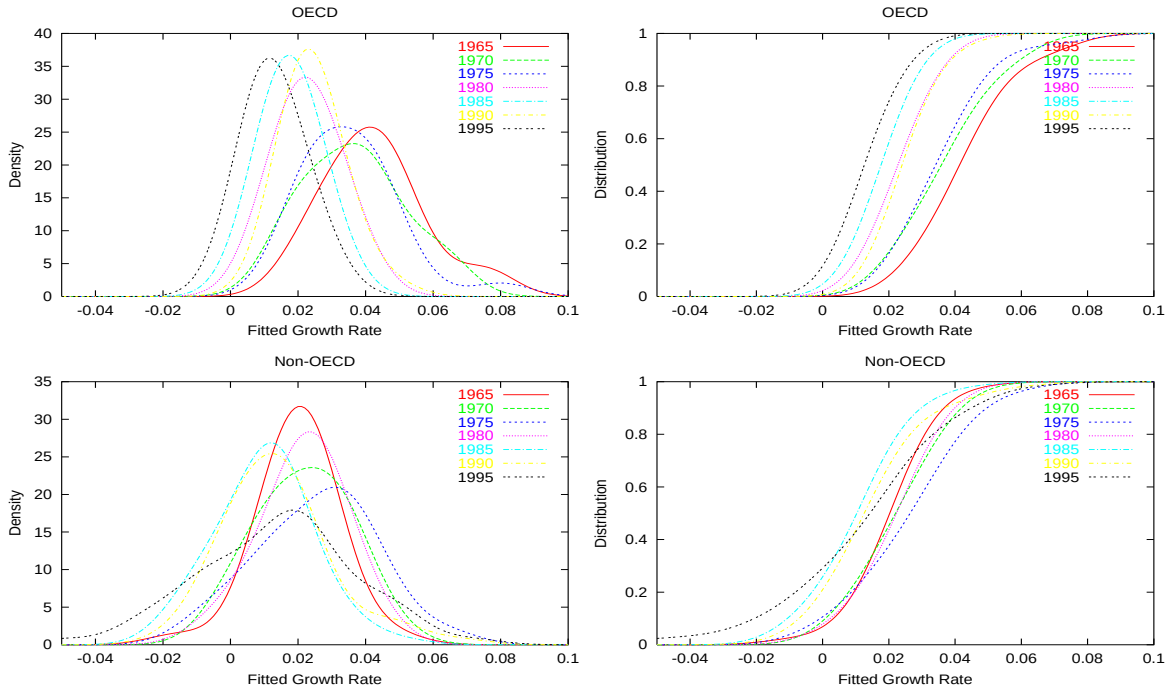
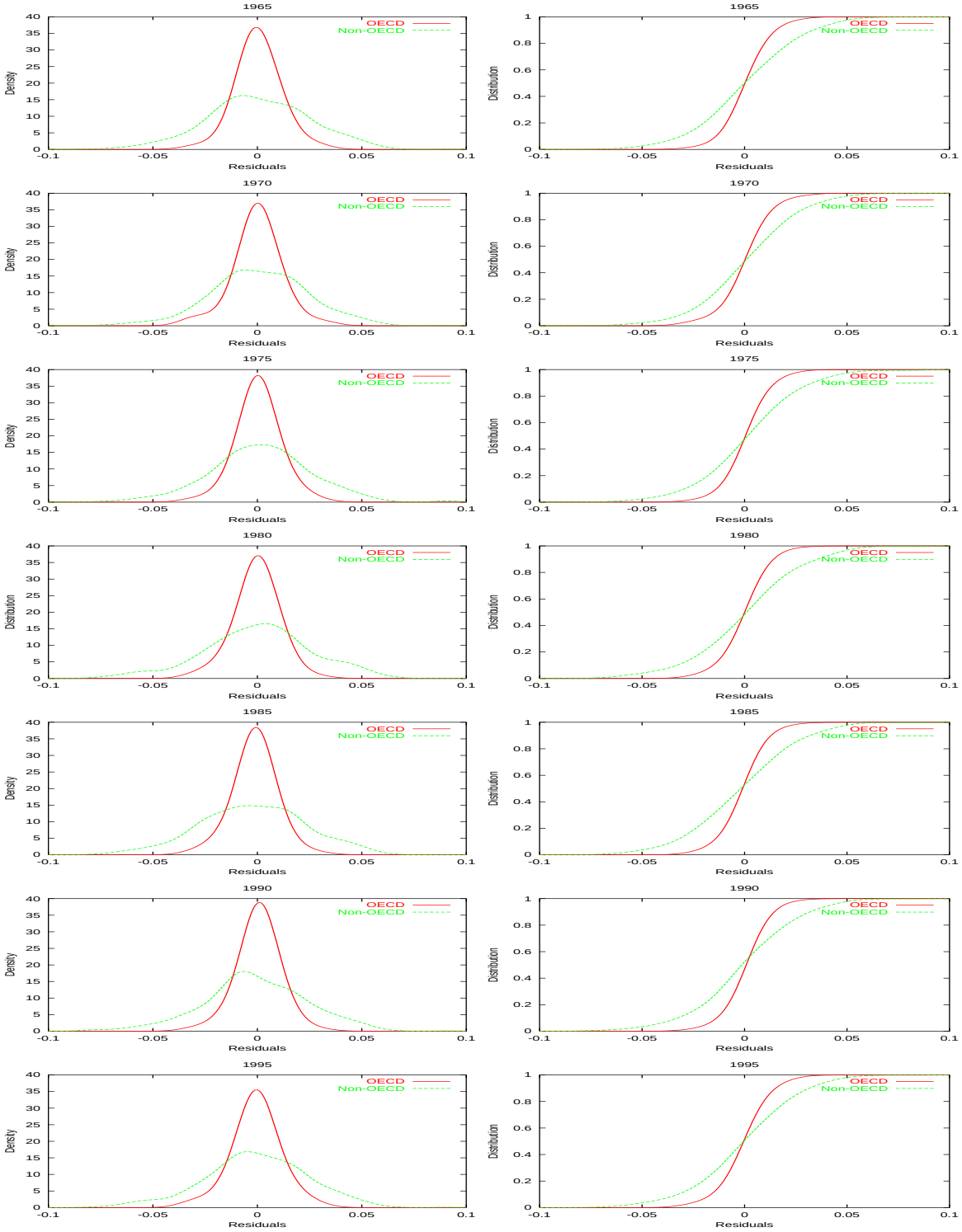
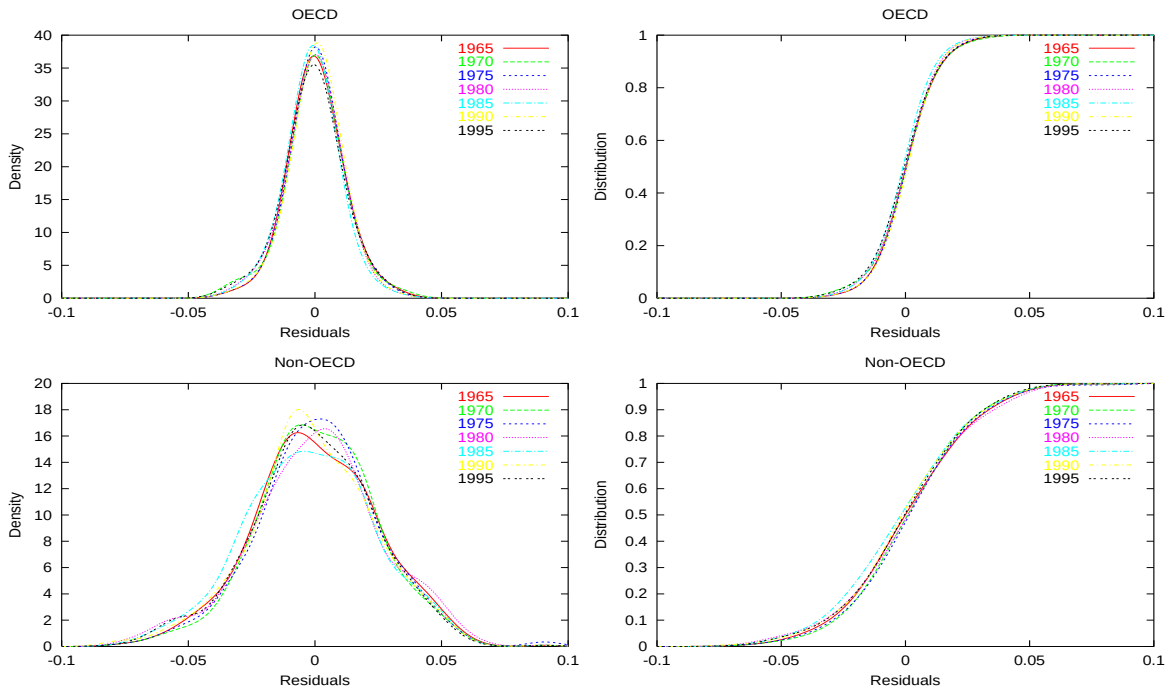


FIGURE 6. Predicted Growth Rates By OECD Status for All Years

Appendix A. Residuals By Year and OECD Status



Appendix B. Residuals By OECD Status for All Years



Appendix C. Growth Rate Dynamics

Table 7. KL Entropy for Parametric Residuals ($\int_{-\infty}^{\infty} f(x)\ln(f(x)/g(x))\,dx$) ($f(x)$=Year$_t$, $g(x)$=Year$_{t+5}$). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference over time.

	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95
OECD	0.020	0.019	0.012	0.053	0.010	0.037
	[0.032, 0.039]	[0.032, 0.040]	[0.033, 0.039]	[0.032, 0.038]	[0.032, 0.039]	[0.033, 0.040]
Non-OECD	0.014	0.006	0.030	0.012	0.005	0.015
	[0.025, 0.030]	[0.025, 0.029]	[0.025, 0.029]	[0.026, 0.030]	[0.026, 0.030]	[0.026, 0.030]

Table 8. S_ρ Entropy Dynamic for Parametric Residuals ($\frac{1}{2}\int_{-\infty}^{\infty}[\sqrt{f(x)} - \sqrt{g(x)}]^2\,dx$) ($f(x)$=Year$_t$, $g(x)$=Year$_{t+5}$). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference over time.

	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95
OECD	0.006	0.004	0.003	0.013	0.003	0.009
	[0.008, 0.009]	[0.008, 0.010]	[0.008, 0.010]	[0.008, 0.009]	[0.008, 0.010]	[0.008, 0.010]
Non-OECD	0.003	0.002	0.007	0.003	0.001	0.004
	[0.006, 0.007]	[0.006, 0.007]	[0.006, 0.007]	[0.006, 0.007]	[0.006, 0.007]	[0.006, 0.007]

Table 9. KL Entropy for Kernel Residuals ($\int_{-\infty}^{\infty} f(x)\ln(f(x)/g(x))\,dx$) ($f(x)$=Year$_t$, $g(x)$=Year$_{t+5}$). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference over time.

	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95
OECD	0.008	0.009	0.004	0.005	0.014	0.013
	[0.013, 0.016]	[0.014, 0.017]	[0.013, 0.017]	[0.013, 0.016]	[0.013, 0.016]	[0.014, 0.016]
Non-OECD	0.005	0.004	0.013	0.012	0.010	0.004
	[0.012, 0.013]	[0.011, 0.013]	[0.011, 0.012]	[0.011, 0.013]	[0.012, 0.013]	[0.011, 0.013]

Table 10. S_ρ Entropy for Kernel Residuals ($\frac{1}{2}\int_{-\infty}^{\infty}[\sqrt{f(x)} - \sqrt{g(x)}]^2\,dx$) ($f(x)$=Year$_t$, $g(x)$=Year$_{t+5}$). The values in brackets are the 90th and 95th percentiles obtained under the null of no difference over time.

	1965-70	1970-75	1975-80	1980-85	1985-90	1990-95
OECD	0.002	0.002	0.001	0.001	0.003	0.003
	[0.003, 0.004]	[0.003, 0.004]	[0.003, 0.004]	[0.003, 0.004]	[0.003, 0.004]	[0.003, 0.004]
Non-OECD	0.001	0.001	0.003	0.003	0.003	0.001
	[0.003, 0.003]	[0.003, 0.003]	[0.003, 0.003]	[0.003, 0.003]	[0.003, 0.003]	[0.003, 0.003]

Table 11. Actual Growth Rates Summary

	Mean		Median		σ		IQR	
Year	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD
1965	0.044	0.022	0.039	0.020	0.018	0.028	0.015	0.037
1970	0.037	0.025	0.035	0.025	0.021	0.025	0.017	0.033
1975	0.037	0.031	0.033	0.025	0.016	0.031	0.022	0.042
1980	0.022	0.023	0.022	0.027	0.013	0.032	0.012	0.042
1985	0.014	0.003	0.014	-0.002	0.007	0.033	0.008	0.041
1990	0.027	0.011	0.025	0.009	0.010	0.032	0.012	0.034
1995	0.011	0.010	0.011	0.014	0.014	0.041	0.012	0.052

Table 12. Kernel Predicted Growth Rates Summary

	Mean		Median		σ		IQR	
Year	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD
1965	0.042	0.020	0.041	0.020	0.015	0.011	0.017	0.013
1970	0.037	0.022	0.038	0.021	0.015	0.013	0.018	0.022
1975	0.035	0.026	0.032	0.028	0.014	0.018	0.018	0.022
1980	0.023	0.022	0.022	0.023	0.008	0.012	0.011	0.013
1985	0.018	0.010	0.018	0.012	0.007	0.013	0.008	0.016
1990	0.025	0.014	0.021	0.014	0.007	0.016	0.009	0.017
1995	0.013	0.012	0.011	0.015	0.007	0.028	0.009	0.031

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