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0. Introduction

There is no need to argue in favor of concise, clear, complete, consistent, descriptions of programming languages, nor to recite the cost in time, energy, money and effectiveness which is incurred when a description falls short of these standards. Reliable, high-quality computer programming is impossible without a clear and precise understanding of the language in which the programs are written -- this being true quite independently of the merits of the language as a language.

In this study we tried to discover the current state of the methodology of definition of programming languages. We sought to separate the question (as far as it can be done) of definition from that of design. Our goal was to get at ways of specifying programming languages rather than ways of inventing them or of implementing them on machines. We realise however that these questions are not completely separable.

Many programming languages are poorly defined. Nowadays -- indeed ever since the influential and pioneering example of the formal definition of ALGOL-60 -- one finds (usually) that the syntax of a programming language is defined very clearly and compactly, but that the semantics is (usually) explained badly.

This discrepancy seems to be due to the fact that the BNF notation (with its accompanying framework of concepts for dealing with sets of
strings over a given alphabet of characters) has become a powerful, standard tool for specifying a syntax; whereas nothing comparable has yet received universal acceptance for specifying a semantics. Until relatively recently, indeed, nothing comparable existed. However, since about 1970 there has been a fruitful research effort under way in formal semantics which has produced a sound mathematical theory of considerable power and elegance. The product of this research has been a system of notation (with an accompanying framework of concepts and results) which, in the opinion of the present writer, more than redresses the imbalance between the methodology of syntax and that of semantics.

There has also been, over the past decade or so, a trend towards improving the technique of syntax-specification, summed up in the distinction between concrete and abstract syntax. The earlier BNF-influenced notion was that a syntax was a system of sets of strings of characters. The structure of a given string had to be computed (as a labelled tree figure of some kind) by means of the operation of parsing the string. The details tended to be fussy, minute, many, and yet inessential to the overall mission of the language.

The best current practice is to present the syntax of a language in as abstract a form as is consistent with the objective of creating suitable vehicles for the intended semantic roles of the various syntactic constructs. Thus, one might specify, for example, that a conditional expression is simply a "thing" that has three immediate constituents: a premiss, a conclusion, and an alternative; and that the premiss is a sentence while the conclusion and the alternative are both expressions. Only two basic requirements on one's abstract specification need be imposed explicitly:
the requirement of unique decomposition and the requirement of finite composition.

The first requirement, for example, means that two conditional expressions are the same conditional expression if and only if they have the same premiss, the same conclusion, and the same alternative.

The second requirement says that there must be no infinite sequences

\[ x_1, x_2, \ldots \]

of expressions in which each next expression \( x_{j+1} \) is an immediate constituent of the preceding expression, \( x_j \).

With such an abstract specification, the particular details of how a conditional expression shall be written, e.g. whether as one of the following

\[
\begin{align*}
A & \rightarrow B, C \\
\text{If } A \text{ then } B \text{ else } C \\
si A \ alore \ B \ ou \ C \\
A? & \rightarrow \begin{cases}
\text{Yes} & B \\
\text{No} & C
\end{cases}
\end{align*}
\]

or some other "syntactically sugared" bit of text, does not matter: so long as the premiss \( A \), the conclusion \( B \) and the alternative \( C \) are computable from the text by the selection functions provided as part of the concrete, sugared representation.

The benefits of using abstract syntax specification are obvious and will not be argued here. Suffice it to remark that, in cases where the language being specified is intended (as, e.g., with JOVIAL) to be implemented
on a wide variety of machines and for a large community of users, one gains flexibility and portability at the expense merely of the writing of "concrete-to-abstract" translators, and "abstract-to-concrete" representors. The abstract syntax is a suitable interlingua between the different implementations of the same language.

In the opinion of the present writer there is a widespread understanding of and adherence to the principle of specifying syntax abstractly. At any rate, the techniques appropriate for doing so are not new or difficult. In the sequel a relatively unorthodox viewpoint (that syntactic objects are functions, namely from a finite set of selectors to a set of other syntactic objects as their immediate constituents) is presented as part of a general approach to programming language definition in which every entity is construed as a function of some kind, or else as unanalyzed, primitive. However, there is nothing particularly novel in this point of view. It is the method of handling semantic definitions which is new and important.

The main content of this study will therefore be a summary of the best current principles and practice of formal semantic specification.
0.1 Abstract vs. Concrete Semantics

In preparing the critical survey we studied several contrasting methods of programming language definition: (a) the method used by the VIENNA group to define PL/I and which has subsequently been applied to other languages, such as ALCOL-60; (b) the SEMANOL method, developed by a group at TRW for RADC, as a method for specifying the language JOVIAL; (c) the SECD method devised by Landin [7] for defining the semantics of programming languages based on the lambda-calculus; and subsequently [8] elaborated to a method based on the SHARING machine; (d) the AXIOMATIC method introduced by Hoare [9] for specifying the semantics of isolated programming language constructs such as assignment, while-do, and if-then-else, principally for the purpose of providing a formal basis for proving properties of programs built out of these constructs; (e) the OXFORD method devised by Strachey and Scott [6] as an extension of the well-established TARSKI set-theoretic semantics [10] used in mathematical logic for several decades.

The literature on these various methods is extensive, and is only just beginning to include expositions designed for the general computing public (as opposed to the specialist research workers in the field of programming language design and implementation).

There is, in the opinion of the present writer, one outstanding issue which forces itself on the attention of the reader of the programming language definition research literature: the issue of whether to specify semantics in a concrete, or an abstract, way.

Concrete specifications are those which, in one way or another, involve the description of a MACHINE or INTERPRETER, and take the form, in one way or another, of stating how the interpreter behaves when given
a program text, together with some data-describing text, as input.

The interpreter may be described at various levels of conceptual abstraction -- at one end, details of behavior being insisted upon; at the other, merely broad essentials being laid down -- but the general principle at work is the same one: to specify an OPERATIONAL semantics by saying what will happen when the defining machine is driven by the defined program in the presence of given data objects.

Abstract specifications of semantics avoid the introduction of the definitional interpreter [11]. They proceed instead by describing the entity denoted by the program text -- the "meaning" of the text -- in a way that is independent of any particular concrete realization or representation of that entity.

The advantages of abstract over concrete semantics are similar to those of abstract over concrete syntax: essential features are isolated from the irrelevant detail which accompanies particular realizations of them; a wider range of implementations -- possibly involving completely different conventions, codings, and software technologies -- is admitted; correctness proofs for compilers and interpreters are immeasurably simplified and shortened; documentation of the language is made concise and transparent.

It is in the OXFORD methodology of definition that the principle of abstract semantics reaches its highest development to date. The Oxford definition method will in the opinion of the present writer evolve, with use, into the standard tool of the professional programming language designer for specifying, developing, analyzing, and explaining not only constructs but entire languages.

We therefore turn to a closer look at the ideas involved.
1. The Oxford Definition Method


The ODM approach takes as its main mathematical point of departure a beautiful and important new mathematical theory, due to Scott, which deals with the foundations of our ideas about computation, especially as these bear upon approximating (describing) infinite objects by means of finite ones. The details of Scott's theory are quite difficult and will not be given here. In any case, Scott's theory is rather like Dedekind's "Schnitt" theory of the real numbers, in two respects: not only technically, but also in that one is assured, by its existence, that certain intuitive conceptions are "consistent" and "safe", and may be rigorously justified by precise constructions, without having to perform the constructions when using the conceptions routinely and intuitively.

The intuitions addressed by Scott's theory concern the collections of functions which appear to be the subject-matter of such rich programming languages as the full ("typeless") lambda-calculus of Church and Curry, which underlies so many of the modern high-level programming languages in use today. For the purposes of this study we need only say that Scott's theory rigorously justifies the use, as one's defining metalanguage, of the typeless, classical lambda calculus enriched with a suitable collection of primitives such as the conditional expression and the traditional set-theoretical notation for denoting the set of all functions with a given
domain and range. We shall rely on this justification in the simplest and most direct way, namely by using such a metalanguage freely and un­critically in an intuitive, informal fashion. We proceed by going through some examples of the use of ODM.

1.1 Environments

The ODM approach is to define every significant structured entity of a programming language as a mathematical *function*. This is a deliberate attempt to avoid unnecessary specification and to employ *abstract* objects, rather than representations of them, as elements of the specified system.

An easy introductory example of this is the analysis of an *environment* of a computation -- namely the association between the *identifiers* in a program and their *values* or *denotations* -- as a function from the set ID of identifiers to the set D of denotable values. Thus, the environments form a set ENV, and the equation defining it is

\[ \text{ENV} = (\text{ID} \rightarrow \text{D}) \]

where the notation on the right is the traditional one for the set of functions from ID to D. In the ODM framework of ideas, the set \((\text{ID} \rightarrow \text{D})\) does not in fact contain *all* functions whatsoever from ID to D, when ID is an *infinite* set, but only those which, in Scott's theory, qualify as "continuous". In the present study we can safely neglect this foundational aspect of ODM and proceed on an informal intuitive basis.

A particular environment \(\rho\):

\[ \rho \in \text{ENV} \]

behaves as one would expect: it gives the denotation

\[ \rho \xi \]

of every identifier \(\xi\) in the set ID. By \(\rho \xi\) we mean *the value of the function*
\(\rho \) at the argument \(\xi\), and we indicate the application of a function \(f\) to its argument \(a\) in general by writing down \(f\) followed by \(a\):

\[fa\]

When \(fa\) is again a function \(g\) (as we shall see, in ODM applications this is very often the case), and when \(a\) is itself a function (same comment) we may write

\[fa \ b\]

to indicate

\[(fa) \ b\]

i.e.

\[g \ b\]

but if \(ab\) is a function \(h\) then we would write

\[f(ab)\]

to signify that the intended result is \(fh\), not \(gb\). This convention is called association to the left. It is convenient to have an explicit notation for application of functions to arguments; and we shall sometimes use the solid wedge \(\triangleright\) for this. Thus

\[f \triangleright x\]

means the same as

\[fx\]

and we can even reverse the direction of the wedge if we like, writing

\[x \triangleleft f\]

The wedge points from the function being applied, to the argument.

1.2 Stores

It is easy to give, using the ODM approach, a clear and satisfactory
analysis of a computer memory setup, and of the mechanism of assignment which is present in virtually all programming languages.

A store is taken to be a function \( \sigma \) in the set

\[
S = (L \rightarrow V)
\]

of functions from locations \( a \) in \( L \) to storable values \( \beta \) in \( V \).

A particular \( \sigma \) is thus a possible state of the memory.

Two particular functions may then be defined.

\[\text{(1) define: contents} \in L \rightarrow (S \rightarrow V) \]
\[\text{by:} \ (\text{contents} \triangleright a) \triangleright \sigma = \sigma \triangleright a \]

and

\[\text{(2) define: update} \in (L \times V) \rightarrow (S \rightarrow S) \]
\[\text{by:} \ (\text{update} \triangleright (a, \beta)) \triangleright \sigma = \sigma' \]
\[\text{where:} \ \sigma' = \lambda a'. \text{ if } a = a' \text{ then } \beta \text{ else } (\sigma \triangleright a') \]

The use of the lambda notation

\[\lambda \, x. \, a\]

to denote the function whose value at \( t \) is the result of evaluating the expression \( a \) in an environment in which \( x \) is bound to \( t \), is assumed to be familiar.

1.3 Expressions

The expressions of a programming language comprise a set \( \text{EXP} \) of things which, as we shall see in the example of the next section, can also be construed, in the spirit of ODM, as functions. For the present discussion we assume nothing about the structure of expressions, but proceed to discuss the
processes involved in assignment under the assumption that each expression $\varepsilon$ in EXP, when evaluated, yields some value which depends on the particular structure of $\varepsilon$ and on the environment in which the evaluation takes place.

Assignment involves an expression $\varepsilon_0$ which denotes a location, and an expression $\varepsilon_1$ which denotes a value; and the assignment command:

$$\varepsilon_0 := \varepsilon_1$$

causes the value of $\varepsilon_1$ to be stored in the location which is the value of $\varepsilon_0$. ($\varepsilon_0$ might be arbitrarily complex; e.g. it might be $A[i+j, B[k+i]]$

where $A$ and $B$ are arrays).

Two functions, $L$ and $R$ (for "left-value" and "right-value") are introduced to analyse this operation.

(1) \[
\text{define: } L \in (\text{EXP} \to (\text{ENV} \to (S \to (L \times S)))) \\
\text{by: } (\text{detailed specification of } L)
\]

(2) \[
\text{define: } R \in (\text{EXP} \to (\text{ENV} \to (S \to (V \times S)))) \\
\text{by: } (\text{detailed specification of } R)
\]

The detailed specifications of $L$ and $R$ will depend on the syntactic structures of the expressions $\varepsilon$ in EXP. But the general idea is straightforward.

$(L \triangleright \varepsilon_0) \triangleright \rho$ is, as the type specification in (1) shows, a function in

$$(S \to (L \times S))$$

which, when applied to a store $\sigma_0$, yields a location $\alpha$ and a (possibly altered) store $\sigma_1$

(3) \[
((L \triangleright \varepsilon_0) \triangleright \rho) \triangleright \sigma_0 = (\alpha, \sigma_1)
\]

The possibly altered store $\sigma_1$ is to allow, in the analysis, for possible
side-effects which may occur during the evaluation of $\epsilon_0$ in $\rho$. Intuitively, $\alpha$ is the location in the memory which $\epsilon_0$ denotes in $\rho$.

In carrying out the operation $\epsilon_0 := \epsilon_1$, then, the first step is to do equation (3). The second step is to get the value of $\epsilon_1$ by using the function $R$. By referring to the type of $R$ given in (2), we see that

$$ (R \epsilon_1)\rho $$

is a function in

$$(S \rightarrow (V \times S))$$

which, when applied to a store $\sigma_1$, yields a value $\beta$ and a (possibly altered, again because of side effects of the evaluation process) store $\sigma_2$:

$$ ((R \epsilon_1)\rho)\sigma_1 = (\beta, \sigma_2) $$

Intuitively, $\beta$ is the value denoted by $\epsilon_1$ in the environment $\rho$ of identifier-bindings when the memory is in state $\sigma_1$. In this model of storage and assignment, the environment is not changed by assignments, but rather the state of the memory, (the store), is what changes.

With $\alpha$ and $\beta$ as given by (3) and (4), the remaining work involved in the meaning of $\epsilon_0 := \epsilon_1$ is given by:

$$ (\text{update } (\alpha, \beta)) \sigma_2 = \sigma_3 $$

That is, the overall effect of executing $\epsilon_0 := \epsilon_1$ when the environment is $\rho$ and the store is $\sigma_0$ is to change the store to $\sigma_3$.

If the meaning of commands $\alpha$ in the set CMD of the language is given by a function:

$$ C \in (\text{CMD} \rightarrow (\text{ENV} \rightarrow (S \rightarrow S))) $$

i.e. $(C \gamma)\rho$ is a state transition function $\theta$ in $(S \rightarrow S)$ for each command $\gamma$ and environment $\rho$, then part of the detailed specification of $C$ would be:
or equivalently

\[
\begin{align*}
C(\varepsilon_0 := \varepsilon_1)\rho = \lambda \sigma_0. \text{update} (a, \beta) \sigma_2 \\
\text{where } (\beta, \sigma_2) = ((R \varepsilon_1)\rho)\sigma_1 \\
\text{where } (a, \sigma_1) = ((L \varepsilon_0)\rho)\sigma_0.
\end{align*}
\]

using the lambda notation to define the state transformation \( \theta \) explicitly.

1.4 Commands

In general, commands are program elements \( \gamma \) in a set \( \text{CMD} \) thereof whose syntax will be part of the specification of the programming language. There will be a semantic function \( C \) which gives the meaning of a command \( \gamma \), just as the semantic functions \( L \) and \( R \) gave the "left" and "right" meanings of expressions \( \varepsilon \) in EXP.

The specification of the language will largely consist of the detailed definition of these semantic functions and of the syntactic structure of their (textual) domains.

A command \( \gamma \) is construed as denoting, in each environment \( \rho \), a state transition function \( \theta = C\gamma\rho \)

\[ \theta \in (S \rightarrow S). \]

The execution of a sequence \( \gamma_0 \gamma_1 \ldots \gamma_n \) of commands thus corresponds to the transition of the initial state \( \sigma_0 \) successively to \( \sigma_1 = C\gamma_0 \sigma_0 \), to \( \sigma_2 = C\gamma_1 \sigma_1 \), etc. The open wedge \( \triangleright \) notation represents composition of functions: \( f \triangleright g \) being the function whose application to \( x \) yields the result
given by:

\[(f \circ g) \circ x = f \circ (g \circ x)\].

As with \(\circ\), the direction of \(\circ\) may be reversed without changing the meaning, provided we also reverse its two operands:

\[f \circ g = g \circ f\]

Then the composition of state transitions \(\theta_0 \ldots \theta_n\) (in the order of their application) is the state transition:

\[\theta = \theta_0 \circ \ldots \circ \theta_n\]

and we may write the entire transition as:

\[\sigma_0 \circ \theta = \sigma_0 \circ (\theta_0 \circ \ldots \circ \theta_n)\]

\[= \sigma_n\]

With this notation, the sequence of commands \(\gamma_0 \ldots \gamma_n\) corresponds to the state transition

\[(C\gamma_0 \rho) \circ \ldots \circ (C\gamma_n \rho)\].

1.5 Sequencing

One of the more troublesome aspects of programming language semantics is the specification of "flow of control".

Suppose that \(\gamma = \gamma_0 ; \gamma_1\) is the command consisting of "\(\gamma_0\) followed by \(\gamma_1\)" -- to use the notation standard in many programming languages. Now in order to specify the state-transition for \(\gamma\):

\[C(\gamma_0 ; \gamma_1)\rho = \theta\]

we shall have to know whether the execution of \(\gamma_0\) will involve a jump or not -- which in general we cannot know until \(\gamma_0\) is actually being executed, since, in general, whether \(\gamma_1\) is the (dynamically) next command after \(\gamma_0\), or whether some other command somewhere else in the program will be the
dynamically next one, is not a fixed feature of the command $\gamma_0 ; \gamma_1$, but of the entire program and its detailed potential interactions with possible environments and stores.

The ODM approach deals with this complication via an ingenious device, known as the device of continuation functions, which was hit upon independently around 1970 by several authors, including F. L. Morris [3] of Syracuse University.

The essential idea of the continuation function method is to define the semantics of a program element (such as a sequential command $\gamma_0 ; \gamma_1$) not in isolation but rather in context as part of a complete program. (There is a flavor here of passing from context-free semantics to context-sensitive semantics.) To specify the transition function denoted by $\gamma_0 ; \gamma_1$, in an environment $\rho$ we must therefore, according to the continuation method, supply a context $\theta$, the nature of which will be derived as the discussion develops, and define the transition as depending upon $\theta$.

$$(1) \quad (P(\gamma_0 ; \gamma_1)\rho)\theta$$

as well as upon $\rho$. The analysis, by F. L. Morris and the others, of the nature of $\theta$ in this application, leads to the construing of its as a transition. Since (1) must also denote a transition, i.e. must be in the set

$$C = S \rightarrow S$$

we have that the type of $P$ is

$$(\text{CMD} \rightarrow (\text{ENV} \rightarrow (C \rightarrow C))).$$

The intuitive interpretation of the "continuation function" $\theta$ in (1) is that it is the function corresponding to the remainder of the operation specified by the entire program, factored out (literally: in the sense of functional composition) and to be executed in the event that $\gamma_0 ; \gamma_1$ does not involve a jump to some other part of the program. That is, $\theta$ represents the "normal continuation" of the flow of control.
As F. L. Morris remarks: [3]

"the function compiled for the subexpression should be passed as an argument something which says what more is waiting to be done, so that the subexpression can decide whether to do it or not."

Thus, if the execution of $\gamma$ does not involve a jump, we would specify that

$$((P \gamma)p)\theta)\sigma_0 = \theta(((C \gamma)p)\sigma_0)$$

i.e. the next state after $\sigma_0$ would be reached by first going to the state

$$\sigma_1 = (C \gamma p)\sigma_0.$$  
which is reached by executing the command $\gamma$, and then applying to that state, the normal continuation $\theta$, to get:

$$\sigma_2 = \theta\sigma_1.$$  

In the event of a jump, some other ("abnormal") transition, or continuation, will be specified as having to be applied to $\sigma_1$ to get $\sigma_2$. That is, some commands will "ignore their normal continuations".

In terms of this idea, the definition of $P$ for the case of a sequencing command $\gamma_0 \cdot \gamma_1$ is then:

$$P (\gamma_0 \cdot \gamma_1)p\theta\sigma = (P\gamma_0p)(P\gamma_1p\theta)\sigma$$

which says, intuitively, that if $\theta$ is the normal continuation, then the transition:

(1)  
$$P (\gamma_0 : \gamma_1)p\theta$$

is explained as giving to the routine:

(2)  
$$P \gamma_0p$$

the normal continuation:

(3)  
$$P \gamma_1p\theta$$
so that, if in the execution of (2) there are no jumps, the remainder of
the computation will be the application of the transition (3) to the state
produced by executing (2).

The analysis of the meaning of a jump command

goto ε

is now clear in terms of continuation functions. We shall have:

\[(P(\text{goto } ε)\rho)θ = θ'\]

where θ' is the transition function which is obtained by evaluating the
"label expression" ε in the environment ρ:

\[θ' = (E ε)ρ\]

Thus the ODM approach yields a simple and straightforward analysis of the
this is used very effectively to explain the semantics of the control aspects
of that extremely powerful language, in which one can construct such un-
orthodox control regimes as co-routines and quasiparallel processes.
2. **ASCL**

The programming language ASCL ("A Small Continuation Language") is a language devised by C. Strachey and C. Wadsworth in [5] to illustrate the ODM concept of continuation functions. It also is well suited to illustrate the other features of ODM definitions, and we shall now go through its definition, as a more extended example of the ODM approach.

### 2.1 Syntactic Categories of ASCL

Four sets of syntactic entities:

\[ \text{ID} \quad \text{CMD} \quad \text{EXP} \quad \text{FN} \]

are declared and given the names exhibited in the line above. Simultaneously, four sets of (metalinguistic) variables are declared, namely the lower-case Greek letters

\[ \xi \quad \gamma \quad \epsilon \quad \phi \]

These variables, with or without numeric subscripts, will be used in the definitions with the ranges:

\[ \xi \in \text{ID} \quad \gamma \in \text{CMD} \quad \epsilon \in \text{EXP} \quad \phi \in \text{FN} \]

Informally, we are told that

- ID is the set of identifiers of ASCL. No particular assumptions need be made (for the purposes of the definition) about the members of ID.

- CMD is the set of commands (this is the same as what is commonly called a statement; but the word "command" has the more appropriate "imperative" connotation).

- EXP is the set of expressions of ASCL.

- FN is the set of primitive commands of ASCL.

### 2.2 Syntax Equations for ASCL

The following two equations assert that CMD and EXP are the unions of several disjoint sets of syntactic subcategories, or cases:
CMD = FN + DUM + SEQ + IFC + WDO + GTO + BLK + ESC

EXP = ID + YES + NO + IFX + CMP

These "cases equations" give part of the information contained in a BNF equation, namely the breakdown into cases of the syntactic category denoted by the name on the left hand side. The remaining information, namely the structural descriptions of the syntactic subcategories, is given by a structural equation for each such subcategory. In the present example, the subcategories are:

DUM  SEQ  IFC  WDO  GTO  BLK  ESC
YES  NO  IFX  CMP

Informally, we remark that:

- DUM is the set whose only element is the dummy command "skip".
- YES is the set whose only element is the Boolean constant "true".
- NO is the set whose only element is the Boolean constant "false".
- SEQ is the set of compound commands often written as the (semi-colon-separated) sequence $\gamma_0 ; \gamma_1$.
- IFC is the set of conditional commands often written if $\varepsilon$ then $\gamma_0$ else $\gamma_1$, or as: $\varepsilon \rightarrow \gamma_0, \gamma_1$.
- WDO is the set of repetitive constructs often written as while $\varepsilon$ do $\gamma$.
- GTO is the set of jumps often written as goto $\varepsilon$.
- BLK is the set of blocks or programs often written as:
  \begin{verbatim}
  begin $\gamma_0; \xi_1: \gamma_1; \ldots; \xi_n: \gamma_n end
  \end{verbatim}
  with each $\xi_i$ being a label (or "tag") in the body of the block.
- ESC is the set of escape commands which return a value from a subroutine; these are often written as: result is $\varepsilon$, or return $\varepsilon$, or escape with $\varepsilon$.
- IFX is the set of conditional expressions, often written as
  \begin{verbatim}
  if $\xi_0$ then $\varepsilon_1$ else $\varepsilon_2$, or as $\varepsilon_0 \rightarrow \varepsilon_1, \varepsilon_2$.
  \end{verbatim}
- CMP is the set of value-returning computations, sometimes written as: value of $\gamma$. 
These informal remarks are not part of the formal definition; nor are the particular concrete representations, such as: \( \text{if } \varepsilon \text{ then } \gamma_0 \text{ else } \gamma_1 \), for the various syntactic constructions. The syntactic structure is abstract. The equations which follow are descriptions of each syntactic construct as a function of selectors. To decompose each construct into its immediate constituents one merely applies the construct (as a function) to the selector(s) in its domain.

Note that FN and ID are not cited as subcategories since they have already been cited as categories.

Each subcategory is defined by a characteristic construction. The general form of the construction is given by an equation whose left-hand side is the name of the subcategory, and whose right-hand side is a structural description showing the immediate constituents and their categories, or an explicit listing of the members of the category (if the category is a finite set). In the ASCL subcategories there are three of the second kind, namely:

- \( \text{DUM} = \{ \text{skip} \} \)
- \( \text{YES} = \{ \text{true} \} \)
- \( \text{NO} = \{ \text{false} \} \)

and eight of the first kind, namely:

- \( \text{SEQ} = \{ \langle \text{first } \gamma_0 \rangle \hspace{1em} \langle \text{next } \gamma_1 \rangle \} \)
- \( \text{IFC} = \{ \langle \text{test } \varepsilon \rangle \hspace{1em} \langle \text{positive } \gamma_0 \rangle \hspace{1em} \langle \text{negative } \gamma_1 \rangle \} \)
- \( \text{WDO} = \{ \langle \text{condition } \varepsilon \rangle \hspace{1em} \langle \text{repeat } \gamma \rangle \} \)
- \( \text{GTO} = \{ \langle \text{label } \varepsilon \rangle \} \)
- \( \text{BLK} = \{ \langle \text{entry } \gamma_0 \rangle \hspace{1em} \langle \text{tag}_1 \varepsilon_1 \rangle \hspace{1em} \langle \text{command}_1 \gamma_1 \rangle \hspace{1em} \ldots \hspace{1em} \langle \text{tag}_n \varepsilon_n \rangle \hspace{1em} \langle \text{command}_n \gamma_n \rangle \} \)
- \( \text{ESC} = \{ \langle \text{value } \varepsilon \rangle \} \)
- \( \text{IFX} = \{ \langle \text{premiss } \varepsilon_0 \rangle \hspace{1em} \langle \text{conclusion } \varepsilon_1 \rangle \hspace{1em} \langle \text{alternative } \varepsilon_2 \rangle \} \)
- \( \text{CMP} = \{ \langle \text{code } \gamma \rangle \} \)
These structural equations declare that DUM is a finite set, whose only member is the entity skip. This is intended (as the semantic specifications which follow will formally show) to be a "dummy" command whose execution has no effect on the state of the computation. The sets YES and NO are, likewise, singletons, whose members are intended respectively to be the ASCL constants denoting truth (tt) and falsehood (ff).

Each of the remaining equations has a right-hand side which is a finite set of ordered pairs, i.e., a function with finite domain. The elements of the domains of these functions are called selectors. Thus, the domains for each of these eight functions are:

- SEQ: \{first, next\}
- IFC: \{test, positive, negative\}
- WDO: \{condition, repeat\}
- GTO: \{label\}
- BLK: \{entry, tag, body\}
- ESC: \{value\}
- IFX: \{premiss, conclusion, alternative\}
- CMP: \{code\}

For example, a conditional command \( \gamma \) (i.e. a member of IFC) has three immediate constituents: they are found by applying \( \gamma \) to each of the three selectors test, positive, negative:

\[
\begin{align*}
\epsilon &= \text{test} \ \gamma \\
\gamma_0 &= \text{positive} \ \gamma \\
\gamma_1 &= \text{negative} \ \gamma
\end{align*}
\]

where we are writing the function \( \gamma \) to the right of its argument.
In general, application of a function to its argument may be indicated by writing the function on the left or on the right, whichever is convenient. To help, avoid confusion as to which direction is intended, we use the solid wedge notation \( \bullet \) for application. The thick end is where the function goes; the pointed end is the "point of application", where the argument goes. Thus

\[ f \bullet x \]

is: \( f \) applied to \( x \). This is exactly the same as

\[ x \bullet f \]

which is read: \( f \) applied to \( x \), i.e. it is read from right to left. The application symbol \( \bullet \) can be used in any direction, including both vertical ones. Thus, \( f \) applied to \( x \) may also be written

\[
\begin{array}{c}
  f \\
  \downarrow \\
  x
\end{array}
\]

and

\[
\begin{array}{c}
  x \\
  \uparrow \\
  f
\end{array}
\]

It is of course the usual convention to omit an explicit symbol for application; we shall do so only when it is clear "which direction application goes" in the given context.

In particular, the application of a syntactic construct to a selector goes from right to left, e.g.

\[ \text{first} \, \gamma = \text{first} \, \bullet \, \gamma \]

with the intended "pun" that the meaning

\[ \text{first} \, \gamma = \text{first} \, \bullet \, \gamma \]
could also be thought of. It is quite usual to think of selectors as functions which "pick out" immediate constituents from a syntactic construct. For example, in LISP a nonempty list L has a head (car) and a tail (cdr). We write (car L), (cdr L) thinking of car, cdr as functions and L as argument. But in our system the same notation is used with the opposite wedge direction intended. Each structural equation has the reading: the set on the left is the set of all functions having the form exhibited on the right. For example the equation

\[ \text{SEQ} = \{ <\text{first} \gamma_0, \text{next} \gamma_1> \} \]

if written out rigorously would be:

\[ \text{SEQ} = \{ \text{first, next} \} \rightarrow \text{CMD} \]

while the equation

\[ \text{IFC} = \{ <\text{test} \epsilon_0, \text{positive} \gamma_0, \text{negative} \gamma_1> \} \]

would be

\[ \text{IFC} = \{ \text{positive, negative} \} \rightarrow \text{CMD, } \{ \text{test} \} \rightarrow \text{EXP.} \]

The second translation illustrates the "extended functionality notation". The usual notation for the set of functions from the set A to the set B is

\[ A \rightarrow B \]

so that the assertion:

\[ f \in (A \rightarrow B) \]

means:

\[ f \text{ is defined for each element } x \text{ of } A, \text{ and when } x \text{ is in } A \text{ the value of } f \text{ at } x \text{ is an element of } B \]
we generalise this notation to a list of such domain \rightarrow range elements:

\[ A_1 \rightarrow B_1, \ldots, A_n \rightarrow B_n \]

so that the assertion:

\[ f \in (A_1 \rightarrow B_1, \ldots, A_n \rightarrow B_n) \]

means:

\[ f \text{ is defined for each element of the set } A_1 \cup \ldots \cup A_n, \text{ and when } x \text{ is in } A_i, \text{ the value of } f \text{ at } x \text{ is in the set } B_i, \ i = 1, \ldots, n \]

For this extended notation to make sense it is of course necessary that the sets \( A_1, \ldots, A_n \) be disjoint.

The syntactic equations for ASCL are intended as a characterization of a family of sets -- the syntactic classes (categories and subcategories) of ASCL -- whose union is the set of ASCL texts or program elements. The information essentially contained in the equations is that each program element is either simple (such as an identifier or a primitive command or a primitive expression) or composite; and that in the second case it has a unique analysis into immediate constituents of given syntactic classes which can be obtained formally by applying the program element (as a function) to its selectors (as argument). Those who are wedded to traditional mores are permitted to view this immediate constituent analysis, by a suitably contrived symbolic pun, as the application of the various selectors (as functions) to the text (as argument).

So much, for the time being, for the syntax of ASCL and for the several definitional techniques for specifying syntactic structures. We proceed next to its semantics, the heart of the matter.
2.4 Value Domains of ASCL

There are six sets which are specified to be semantic domains for ASCL. They are:

\[ T \quad S \quad C \quad D \quad E \quad K \]

The first two are primitive domains.

- \( T \) is the domain of truth values
- \( S \) is the domain of machine states.

The other five are compound domains:

- \( C = S \rightarrow S \) is the set of command continuations, or state transition functions:
- \( D = T + C \) is the set of denotations, i.e., the set of entities which can be denoted by identifiers;
- \( E = T + C \) is the set of expression results, i.e., the set of entities which can be the results of computing the value of an expression;
- \( K = D \rightarrow C \) is the set of expression continuations, i.e., the set of entities which correspond to the possible contexts which can arise in a computation when an expression is evaluated.

In the subsequent parts of the definition of ASCL the following variables (with or without subscripts) will be used to range over the sets shown:

\[
\begin{array}{c|c}
\theta & C \\
\delta & D \\
\delta & E \\
\kappa & K \\
\end{array}
\]

The sets of continuations will be explained shortly. For the present, we simply accept the declaration of six sets, two outright, and four as simple constructs of them. \( D \) and \( E \) are identical as sets in ASCL; they are declared separately because in general the entities which can be
denoted by identifiers are not always all of those which can be computed, i.e. which can be values of expressions.

In a richer language than ASCL there would be further value domains such as the set of integers, the set of characters, and the set of locations.

2.5 Semantic Functions of ASCL

The set ENV of environments is a composite domain whose elements have two constituents: a binding and a resumption. The binding is a function

\[ \beta \in (\text{ID} \to \text{D}) \]

which gives, for each identifier \( \xi \) in ID, the entity \( \beta \triangleright \xi \) to which \( \xi \) is bound, or which \( \xi \) denotes. The resumption is a function

\[ \kappa \in \text{K} \]

which is an expression continuation whose significance will be made clear shortly.

One could therefore take (as is done, e.g. in C. Strachey and C. Wadsworth [5]) an element \( \rho \) of ENV to be the ordered pair \( <\beta, \kappa> \) of its two immediate constituents. We prefer to use the same technique here as in the case of syntactical constituents: to construe each \( \rho \) in ENV as a function with domain

\[ \text{ID} \cup \{ \text{res} \} \]

so that we can write the equation

\[ \text{ENV} = (\text{ID} \to \text{D}), \{ \text{res} \} \to \text{K}. \]
Two particular functions, \( P \) and \( E \), are declared to have the functionalities:

\[
P \in \text{CMD} \to (\text{ENV} \to (C \to C))
\]

\[
E \in \text{EXP} \to (\text{ENV} \to (K \to C))
\]

and their detailed definition and explication is the central part of the specification of ASCL. We shall now discuss the ideas underlying the formal definition of \( \text{ENV}, P \) and \( E \).

2.6 Oxford Semantics

The ideas underlying the formal definitions of \( \text{ENV}, P \) and \( E \) are the heart of ODM. Certain aspects of these ideas will be unfamiliar at first, and require faith and practice for full acceptance. Persistence is strongly recommended: these are ideas whose time has come.

The ODM plan is to explain the meaning of programs and program elements by supplying semantic functions (here, for ASCL, they are the functions \( P \) and \( E \); in general they are a set of functions, one for each main syntactic category of the language being defined). The meaning of a text is then the entity yielded by applying the appropriate semantic function to that text.

Thus, in the case of ASCL, every text \( \tau \) is either a command \( \gamma \) or an expression \( \epsilon \). In the first case we apply \( P \) to \( \gamma \); in the second case, we apply \( E \) to \( \epsilon \). In either case, the entity yielded is the meaning. By referring to the functionality declarations for \( P \) and \( E \):

\[
P \in \text{CMD} \to (\text{ENV} \to (C \to C))
\]

\[
E \in \text{EXP} \to (\text{ENV} \to (K \to C))
\]
we see that we have:

\[ P \uparrow \gamma \in \text{ENV} \rightarrow (C \rightarrow C) \]

\[ E \uparrow \varepsilon \in \text{ENV} \rightarrow (K \rightarrow C) \]

that is, the meaning of a command \( \gamma \) or of an expression \( \varepsilon \) is a function which can be applied to any environment \( \rho \). It is this function which is the meaning of the ASCL text \( \tau \).

Our present task is to grasp the intention behind this. Let us first confine our attention to commands. We note that the function \( P \uparrow \gamma \) provides, for each environment \( \rho \) in \( \text{ENV} \), a function:

\[ (P \uparrow \gamma) \uparrow \rho \in (C \rightarrow C) \]

which, when applied to a state-transition-function \( \theta_0 \) in \( C \), yields another state-transition-function \( \theta_1 \) in \( C \).

Let us recall the distinction between an open subroutine and a closed subroutine.

An open subroutine is a piece of machine code \( \theta \) which, when executed, causes the machine to make a transition from the state \( \sigma_0 \) before the execution of \( \theta \), to the state \( \sigma_1 \) after the execution of \( \theta \). Thus \( \theta \) is essentially a state-transition-function \( \theta : S \rightarrow S \) from machine states to machine states.

A closed subroutine \( \mu \) is a piece of machine code containing a "return address" parameter \( \theta \) which must be specialized to (the address of) an open subroutine before \( \mu \uparrow \theta \) becomes an open subroutine. In other words, \( \mu \uparrow \theta \) is an open subroutine, for each open subroutine \( \theta \); which is to say that the functionality of \( \mu \) is "open subroutines to open subroutines";
\[ \mu \in (S \to S) \to (S \to S) \]

or

\[ \mu \in (C \to C). \]

In terms of this distinction, \(((P \triangleright \gamma) \triangleright \rho) = \mu\) is a closed subroutine; in order to run it as part of a program we must provide it with a \(\theta\) in \(C = S \to S\) as a \textit{continuation}, by e.g. giving it as an argument the address of \(\theta\) as \textit{return address}.

This account has to be slightly complicated for the case that the text is an expression \(\varepsilon\) instead of a command \(\gamma\).

In order to run the "closed subroutine" \(\pi = (E \triangleright \varepsilon) \triangleright \rho\) as part of a program we must provide it with (the return address of) a "\textit{special open subroutine}\(\kappa\) which will yield a simple open subroutine \(\pi \triangleright \kappa\) corresponding to the whole program.

However, since in general the meaning of \(\pi\) will yield both a value \(\delta\) (that of the expression \(\varepsilon\) in the environment \(\rho\)) and a new machine state \(\sigma\) (that produced as a side effect of the process of computing \(\delta\)) the special open subroutine \(\kappa\) must be one which takes both \(\delta\) and \(\sigma\) as its inputs in order to produce the final state of the whole program. It is convenient to construe \(\kappa\) as a function in the set:

\[ E \to (S \to S) \]

that is, we regard the whole program execution as producing the state

\[ (\kappa \triangleright \xi) \triangleright \sigma_1 \]

from the state \(\sigma_0\), where \(\delta\) is the value of \(\varepsilon\) in \(\rho\), and where \(\sigma_1\) is the state produced from \(\sigma_0\) by evaluating \(\varepsilon\) in \(\rho\) with the machine initially in \(\sigma_0\).
So the functionality of $\pi$ is "special open subroutines to open subroutines":

$$\pi \in (E \to (S \to S)) \to (S \to S)$$

or

$$\pi \in (K \to C).$$

The function $K$ is also called a continuation, since it corresponds to the open subroutine which continues the program to completion; but it is called an expression continuation in order to distinguish it from the simpler kind of continuations in $(S \to S)$, which are called command continuations.

Let us summarise this discussion of continuations.

A program text, which may be a command $\gamma$ or an expression $\varepsilon$, is "compiled" into a routine by applying to it the appropriate semantic function, $P$ or $E$. The routines thus compiled are closed. They require, at "run time", to be supplied with open routines as continuations, which will take over from them and continue the computation to completion.

Thus the closed routines become open routines when supplied with appropriate open subroutines:

- $\mu \uparrow \theta$ is open: i.e. is in $S \to S$
- $\theta$ is open: i.e. is in $S \to S$
- $\therefore \mu$ is closed: i.e. is in $(S \to S) \to (S \to S)$

where $\mu = (P \uparrow \gamma) \uparrow \rho$

- $P$ is in $(CMD \to (ENV \to ((S \to S) \to (S \to S))))$
- $\pi \uparrow K$ is open: i.e. is in $(S \to S)$
- $K$ is open: i.e. is in $(E \to (S \to S))$
- $\therefore \pi$ is closed: i.e. is in $(E \to (S \to S)) \to (S \to S)$

where $\pi = (E \uparrow \varepsilon) \uparrow \rho$

- $E$ is in $(EXP \to (ENV \to ((E \to (S \to S)) \to (S \to S))))$
2.7 Semantic Equations

The detailed specification of $P$ and $E$ is given by writing equations. We shall have eight equations, $C_1$ to $C_8$ for $P$ and five equations $E_1$ to $E_5$ for $E$.

**FN**

| $C_1.\quad P \triangleright \phi \triangleright \rho = \text{some given function } \mu \text{ in } C \rightarrow C, \text{ to be associated with } \phi. \) |

A primitive command $\phi$ in the set $FN$ yields some constant routine $\mu$ in each environment $\rho$.

**DUM**

| $C_2.\quad \rho \triangleright \text{skip} \triangleright \rho \triangleright \theta = \theta$ |

The command $\text{skip}$ compiles to the identity function in $(C \rightarrow C)$.

**SEQ**

| $C_3.\quad P \triangleright \{<\text{first } \gamma_0> <\text{second } \gamma_1>\} \triangleright \rho \triangleright \theta = P \triangleright \gamma_0 \triangleright \rho \ (P \triangleright \gamma_1 \triangleright \rho \triangleright \theta)$ |

The sequence of two commands $\gamma_0; \gamma_1$ compiles to code which, when given the continuation routine $\theta$, runs the code for $\gamma_0$ to which it gives the continuation routine $P \triangleright \gamma_1 \triangleright \rho \triangleright \theta$. This latter routine is the code for $\gamma_1$ furnished with the continuation $\theta$. Thus (running $[\gamma_0; \gamma_1]$ - followed-by-$\theta$) is the same as (running $[\gamma_0]$.followed-by-(running $\gamma_1$.followed-by-$\theta$)).

**IFC**

| $C_4.\quad P \triangleright \{<\text{test } \varepsilon> <\text{positive } \gamma_0> <\text{negative } \gamma_1>\} \triangleright \rho \triangleright \theta$

$= (E \triangleright \varepsilon \triangleright \rho) \ (\text{cond } (P \triangleright \gamma_0 \triangleright \rho \triangleright \theta, \ P \triangleright \gamma_1 \triangleright \rho \triangleright \theta))$ |

The conditional command $if \ varepsilon \ then \ \gamma_0 \ else \ \gamma_1$ compiles to code which, when given the continuation routine $\theta$, carries out either $\gamma_0$-followed-by-$\theta$ or $\gamma_1$-followed-by-$\theta$, according as the expression $\varepsilon$ evaluates to $tt$ or $ff$.

The function cond thus must be of the type $(C \times C) \rightarrow (T \rightarrow C)$
so that the code

\[ \text{cond} \ (P \triangleright \gamma_0 \triangleright \rho \triangleright \theta, \ P \triangleright \gamma_1 \triangleright \rho \triangleright \theta) \]

has the type

\[ (T \to C) \]

of an expression continuation which can be used by the closed routine

\[ (E \triangleright \varepsilon \triangleright \rho) \]

as the continuation to which it transfers control after obtaining the value of \( \varepsilon \) in \( \rho \). The function \( \text{cond} \) is of course defined by:

\[
\text{cond} (\theta_1, \theta_2) > tt = \theta_1 \\
\text{cond} (\theta_1, \theta_2) > ff = \theta_2.
\]

This equation gives the semantics of the command:

\[
\textit{while } \varepsilon \textit{ do } \gamma
\]

which is the normal while-loop which repeatedly evaluates \( \varepsilon \), and each time \( \varepsilon \) is found to be true executes \( \gamma \); the first time \( \varepsilon \) is found to be false marks the completion of the event asked for by the command.

The equation says that \( P \) produces code for this command which, when run with the continuation routine \( \theta \) as its follow-on, has the effect of the code displayed on the right-hand side.

Let us examine the right-hand side:

\[ Y \triangleright (\lambda \theta'. (E \triangleright \varepsilon \triangleright \rho) \triangleright (\text{cond} \triangleright (P \triangleright \gamma \triangleright \rho \triangleright \theta', \theta))) \]

and figure out what it does.

The function \( Y \) is the minimal fixed point operator which obeys the characteristic law:

\[ f \triangleright (Y \triangleright f) = Y \triangleright f \]
for every function \( f \) in its domain.

Consider the function \( H \), where:

\[
H \cdot \theta' = (E \cdot \epsilon \cdot \rho) \cdot (\text{cond} \cdot (P \cdot \gamma \cdot \rho \cdot \theta', \theta))
\]

For any particular \( \theta' \), \( H \cdot \theta' \) is a state-transformation whose effect on a state \( \sigma \) is this: evaluate \( \epsilon \) in \( \rho \) with state \( \sigma \); let the result be \( \delta \) and the new state \( \sigma' \). If \( \delta \) is \( tt \), carry out \( \gamma \) in \( \rho \) with initial state \( \sigma' \) and continuation \( \theta' \); but if \( \delta \) is \( ff \), simply carry out the continuation \( \theta \) on the state \( \sigma' \). In this account, if we identify \( \theta' \) with \( H \cdot \theta' \), we get a spelling out of the repetitive nature of the while-loop. But the argument of \( Y \) in the right-hand side of equation C5 is the function \( H \):

\[
H = \lambda \theta'. (E \cdot \epsilon \cdot \rho) \cdot (\text{cond} \cdot (P \cdot \gamma \cdot \rho \cdot \theta', \theta))
\]

and so the right-hand side of C5 describes the minimal solution of the equation

\[
\theta' = H \cdot \theta',
\]

which is just what is required.

\[
P \cdot \langle \text{label } \epsilon \rangle \cdot \rho \cdot \theta = (E \cdot \epsilon \cdot \rho) \cdot \text{Jump}
\]

This is the equation which gives the meaning of the \( \text{goto } \epsilon \) command. It says that if the evaluation of \( \epsilon \) in \( \rho \) with state \( \sigma \) is a continuation \( \theta' \), and if the evaluation alters the state to \( \sigma' \), the effect of \( \text{goto } \epsilon \) will be

\[
P \cdot \text{goto } \epsilon \cdot \rho \cdot \theta \cdot \sigma = (\text{Jump} \cdot \theta') \cdot \sigma'
\]

which in turn we want to be simply

\[
\theta' \cdot \sigma'
\]

no matter what the continuation \( \theta \) is. For this to work out, \( \text{Jump} \) must be the expression continuation

\[
\text{Jump } \epsilon \cdot E \rightarrow (S \rightarrow S)
\]
which satisfies the equation

\[(\text{Jump} \triangleright \theta') = \theta'\]

for all \(
\theta'
\) in \(\mathbb{E}\). We recall that

\[\mathbb{E} = \mathbb{T} + \mathbb{C} = \mathbb{T} + (S \rightarrow S).\]

Application of \text{Jump} to a truth value is a run-time type error; we therefore complete the definition of \text{Jump} by

\[\text{Jump} \triangleright \varepsilon = \text{Jump} \triangleright \mathfrak{f}f = \lambda \sigma. \sigma \text{error}\]

a continuation which will abort any computation in a distinguished error state.

This is the most elaborate of the semantic equations, but its import is straightforward. It says that the code for a block

\[\text{begin } \gamma_0 ; \xi_1 : \gamma_1 ; \ldots ; \xi_n : \gamma_n \text{ end}\]

must first set up an extended environment

\[\rho' = \rho [\theta_1 \ldots \theta_n / \xi_1 \ldots \xi_n]\]

in which the identifiers \(\xi_1 \ldots \xi_n\) are bound to the continuations \(\theta_1 \ldots \theta_n\) described by the equations (*)
The state-transformation for the whole block (with continuation $\theta$) is then $\theta_0$, where (back-substituting the above equations)

$$\theta_0 = (P \gamma_0 \rho') (P \gamma_1 \rho') \ldots (P \gamma_{n-1} \rho') (P \gamma_n \rho') \theta.$$ 

Within the block, occurrences of the labels $\xi_1 \ldots \xi_n$ will be evaluated in the environment $\rho'$ set up at block-entry, and hence will be found to denote the continuations shown, which is the intended meaning.

This equation describes the meaning of the command

$$\text{result is } \varepsilon$$

which causes control to jump out to the smallest textually surrounding block, with the value of the expression $\varepsilon$. Let us therefore consider this equation together with that for the expression:

$$\text{value of } \gamma$$

This equation says that, to carry out the value of $\gamma$ in $\rho$ block with continuation $\kappa$ we set up a modified environment $\rho_{[\kappa/res]}$ in which the continuation $\kappa$ is the resumption component, and run the code for $\gamma$ with continuation Fail. The reason for the failure continuation is that we do not expect the code for $\gamma$ to continue in this way, but rather to terminate by executing a result is $\varepsilon$ command and hence continuing with the resumption code denoted by $res$, namely the continuation $\kappa$.

Thus C.8 can now be seen to describe the intended behavior of the result is $\varepsilon$ command: evaluation of $\varepsilon$ followed by the continuation denoted by the identifier $res$.

Again, it is suitable to define

$$\text{Fail} = \lambda \sigma.\sigma_{\text{error}}$$
The remaining expression equations are straightforward:

$$E \mapsto \xi \mapsto \rho \mapsto \kappa = \kappa \mapsto (\rho \mapsto \xi) = \xi \mapsto (\rho \mapsto \kappa)$$

This equation says that we evaluate an identifier by first looking its value up in the environment and then applying the given continuation to this value.

YES

$$E \mapsto \text{true} \mapsto \rho \mapsto \kappa = \kappa \mapsto tt$$

NO

$$E \mapsto \text{false} \mapsto \rho \mapsto \kappa = \kappa \mapsto ff$$

E1, E2, E3 say that identifiers, and the constants \text{true} and \text{false}, can be evaluated without side-effects: the machine state is the same after their evaluation as before. E.g.

$$(E \mapsto \text{true} \mapsto \rho \mapsto \kappa) \mapsto \sigma = (\kappa \mapsto tt) \mapsto \sigma$$

says that the routine consisting of the work (evaluating \text{true})-followed-by-\kappa has the same effect on \sigma as that of the routine (\kappa \mapsto tt).

Finally, equation E4 describes the semantics of conditional expressions analogously to equation C4:

$$E \mapsto \{\text{premiss } \varepsilon_0 \overset{\text{conclusion } \varepsilon_1}{\text{alternative } \varepsilon_2}\} \mapsto \rho \mapsto \kappa$$

$$= (E \mapsto \varepsilon_0 \mapsto \rho) \mapsto (\text{cond} \mapsto (E \mapsto \varepsilon_1 \mapsto \rho \mapsto \kappa, E \mapsto \varepsilon_2 \mapsto \rho \mapsto \kappa))$$
3. The General Case for ODM

There is an important distinction which must be made between the semantics (to say of a given program what function it computes) and the implementation of a language (to say how a machine is to be organised so as to carry out the actions which are specified by its programs).

The role of mathematical semantics is to give a precise, unambiguous definition of what programs mean, sufficient to determine their outcome, while remaining uncommitted as to the details of how this outcome is to be achieved on a (real or abstract) computing machine.

[2], p. 21.

The technique of describing the meanings of programs as functions over certain domains is to give the same sort of "uncommitted" account of semantics as an abstract syntax is able to give of syntax: only the essential specifications are explicitly formulated, thereby leaving open all questions which do not need to be decided.

By contrast, other definition methods are to a greater or lesser extent committed to some form of implementation: even if this is only to the SECD-machine type of stack-oriented interpreter, or to the reduction-rule rewriting systems of a Markov or Church-Curry kind.
4. SEMANOL vs. ODM

The SEMANOL system for programming language definition is subject to several of the limitations which ODM is intended to avoid. As Strachey remarks:

Most of the work on syntax and some on semantics has been at the level of symbol manipulation—that is to say it has been concerned with the representations (generally on paper) rather than with the mathematical objects represented.

[2], pp. 2, 3.

4.1 Concrete vs Abstract Syntax

SEMANOL is wedded to concrete syntax. Specifically it assumes that all programming languages will have as their syntactic domains sets of strings of ASCII characters; that a context-free grammar will be supplied to characterize these sets.

As a consequence of this assumption a large part of a SEMANOL definition is concerned with a mass of detail which has to be abstracted away again by the user of the definition, in order that the definition be structured intelligibly and its complexity controlled sufficiently for the mind to grasp its essentials.

4.2 Interpreter-based Semantics

SEMANOL is itself a programming language; and SEMANOL definitions are programs written in it. The meaning of a program P in a language L is to be found in the process which ensues when a pair (P, D) consisting of P together with an input datum D is given as input to the definition DEF. The output

DEF (P, D)
of DEF for this pair as input, is declared to be the output of P for the input D. Thus DEF is a "universal machine" in the sense of Turing: we must program it in SEMANOL, and our understanding of L is then one and the same with our understanding of the program DEF.

For example DEF for the language JOVIAL (73) is neither concise nor clear. It is a 321-page book containing over 2,000 program elements. Its complexity is very large and one is left, having struggled through the details, with less than a firm grasp on the essentials.

In general in order that a SEMANOL program DEF shall serve as a specification of a programming language L one must have a clear comprehension of the language SEMANOL itself. In the nature of the interpreter-oriented approach this means that we have to study the SEMANOL interpreter in order to find out what SEMANOL programs, i.e. programming language definitions, really are saying. There are two sources available for this: a Reference Manual for SEMANOL, and a booklet documenting the Interpreter Program (which is written in FORTRAN).

The Reference Manual is

... not a tutorial; it is terse, and presupposes considerable familiarity with the underlying ideas of SEMANOL (73). The presentation is in a "top-down" sequence -- many structures are defined in terms of structures not yet defined; thus it is intended for use by experienced (or at least well versed) SEMANOL (73) programmers.


Now it must be admitted that ODM requires the user to be "well-versed" in its own intellectual and notational apparatus: but in the case of ODM this apparatus is no more than the universal, traditional mathematical framework of set-theoretic conventions supplemented by the formalism of the lambda-calculus and the theory of continuous
functions devised by Scott.

ODM definitions are concise and precise. Those of SEMANOL are prolix and are exact only by arbitrary fiat (in that a SEMANOL program DEF does what the FORTRAN interpreter program makes it do: one cannot deny that this kind of explanation of semantics does resolve ambiguities). In Tennant [1] an ODM definition is given of Reynolds' enormously powerful and general language GEDANKEN. The definition occupies just over two pages of the Communications of the A.C.M. and involves 24 semantic equations most of which are very short.

The ODM approach is a rigorous, mathematical technique for the specification of programming languages. It offers at last a general notation for semantic specification which is based on a language-independent framework of semantical concepts. By comparison, previous methods (such as that embodied in the SEMANOL system) involve a relatively superficial semantic, and in some cases even syntactic, explication of the language under definition. We close with a quotation summing up the spirit of ODM:

The point of our approach is to allow a proper balance between rigorous formulation, generality of application and conceptual simplicity. One essential achievement of the method we shall wish to claim is that by insisting on a suitable level of abstraction and by emphasizing the right details we are going to hit squarely what can be called the mathematical meaning of a language.

References:


