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An Examination of Elementary Teachers Perspectives on the Learning and Mastery of Basic Math Facts

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Abstract

A popular goal in every mathematics classroom is conceptual and computational fluency. Fluency in mathematics includes efficient use of strategies, accuracy when applying those strategies to provide an answer, and flexibility in problem solving with known and unknown problems. This paper explains my interest in looking into the relationship between what teachers believe about teaching and teaching techniques and how those same teachers execute teaching techniques in their classroom. The following sections include literature review that provides definitions of the key terms and support for my interest in researching this relationship. It also includes evidence of beliefs from interviews with real teachers compared to observations of those teachers' classrooms. The final section of this paper explains the connection between the literature review and my findings to help convince the reader why this topic is important to consider in the field of education.

Executive Summary

This paper is an analysis of beliefs about teaching and implementation of different teaching techniques in regards to fostering fluency in mathematics. This paper provides literature review, summaries of interviews with current teachers, and summaries of observations of those teachers implementing techniques in their classrooms. First, fluency in math includes three main ideas: efficiency, accuracy, and flexibility. A student who is fluent with a specific topic will be efficient in using strategies or memorization to get an answer, will provide accurate answers, and will be flexible with strategies when problem solving with unknown problems. In order to provide accurate answers, a student must either retrieve, from memory, the answer itself, or retrieve a strategy that helps them get to the answer. Retrieval from memory is the re-accessing of information that has been encoded in the brain. Examining how quickly students can retrieve an answer, either by retrieving the actual answer or retrieving the strategy that helps them retrieve the answer, can allow a researcher to determine how strong the relationship of the fact or strategy is in the students' brain. The strategies talked about in regards to retrieval would fall under the category of number sense. A child who has number sense understands numbers and can break them apart and understand their different parts and uses. In regards to basic fact learning, a student who has number sense has ways to answer unknown questions because they have a variety of strategies to use in order to estimate or get to the correct answer. In comparison to this, rote memorization can be described as the learning of isolated facts, where no fact has influence on any other fact. In reality, rote memorization will lead to some sort of number sense, even if that is not the original goal; however, this paper looks into these two topics and their subtopics in regards to teachers' beliefs and practices.

This paper starts with a review of related literature in order to gain a basis for the start of my research project. I look into topics such as fluency, retrieval, number sense, and memorization in a variety of different scholarly sources. These sources are used to help gain an understanding of all of the different beliefs and topics that teachers are exposed to, in order to gain a better understanding of where teachers develop ideas and practices. I also wanted to be well versed in the ideas behind education so that my interviews could be rooted in research based terminology and ideas.

I continue in the literature review with a section on the already existing research on the disconnect between teachers' beliefs and practices. These articles were used to validate my research questions showing that there have been other researchers who looked into very similar topics. My research topic wanted to expand on this topic by investigating how teachers think about students' understanding of arithmetic and how that influences their future math understanding. I also wanted to look at, if there is a gap between the two, what external forces account for these differences, which was a topic I did not see very well covered in the research articles that I read. I believed that researching teachers' beliefs on students' understanding about basic math facts was important because arithmetic is the building block of most other forms of mathematics that students will encounter in their futures.

After the literature review, I describe my methods of gathering data. I conducted hour-long interviews with 6 teachers of varying grade levels from 2nd to 4th grades. I got a chance to observe the classrooms of three out of those 6 teachers and compare those classroom observations to the overall beliefs of each individual teacher as well as the beliefs of the teachers in general. I used inductive and qualitative analysis of the transcribed interviews and my field notes from observations in order to analyze the data and come to a conclusion about teacher beliefs and practices, and even learned a little about what external forces might have accounted for any differences. Many of the teachers gave me insight into what factors limit their idealistic classroom from becoming a reality. I also used my analysis to draw other conclusions from these transcripts and field notes.

Across my participants, teachers defined fluency as being able to provide an answer to a problem efficiently and accurately, but had different opinions about how fluency should be promoted in their classrooms. Most of the teachers believed that number sense is an important concept but knowing facts from memory is imperative. This belief influences teaching techniques to be more memorization based. Many of these teachers believe that time is a big reason why they cannot focus more on number sense based activates and instead rely on quick, memorization based ones. I saw many examples of misalignments between teacher beliefs such as believing that number sense is an important concept, but using timed tests, during the classroom observation, to assess student learning. The separations explained in the findings section provide some evidence that it is important to be aware of possible misalignments in order to teach students to the best of one's ability.

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Advice to Future Honors Students

My advice to anyone completing an Honors Capstone Project would be picking to research something that you love. I love teaching, so my project did not feel too much like a burden. I was interested in everything that I was doing, which made this very long process much more enjoyable. I would also suggest getting an early start. The more headway you get done in the early stages of the project, the less stressed you will be when the submission date comes around. I started very early and am still editing up until the day that it is due! My last piece of advice is to try your best to get a great advisor. My advisor is one of the few reasons that I pursued through this entire process. I would not have been able to do it without his encouragement and his guidance.

Chapter 1: Rationale

When I was in elementary school, the emphasis in mathematics lessons, from my perspective, was placed on the memorization of basic math facts. Flash cards and timed math tests were used to reinforce these lessons. It seemed that this method prepared us for future math classes as well as life in general. In Algebra II, a student does not have time to think about what 6 times 7 is, he or she needs to know it is 42. Currently in some school districts, the emphasis has moved away from rote memorization and toward a deeper understanding of the meaning of math facts. The prevailing opinion in those districts is rote memorization is not the best form of learning. It is important to understand relative advantages and disadvantages of rote memorization versus number sense because the knowledge of these concepts will help teachers be more aware of what they believe they should teach and how they should teach it. Ultimately, I would like to analyze what teachers believe are the best ways to teach students and how those beliefs either align or do not align with their practices in their classroom. My end goal was examine different possible beliefs that teachers could have, interview different teachers about their beliefs, and examine whether or not those teachers' beliefs are fully apparent in their classrooms to make a generalization. I wanted to understand the ways that teachers think about sense-making compared to memorization as learning strategies for basic number facts and the implications for students future success and how their opinion related to their classroom teaching. This generalization would be used to help teachers think more about how to connect their beliefs with their practices to optimize education for all students.

This question fits into the existing field of education because teachers' beliefs and practices are at the core of what makes education work. This question is important because learning basic facts through rote memorization can affect students' abilities to perform more extensive mathematics problems in the future as well as help them as adults. According to Mason (1996) the heart of teaching mathematics is the development of mathematical generalization. He believes that foundation of algebraic thinking should begin its development in the early stages of education, and flourish throughout childhood. (Mason, 1996, p.65) Students' successful thinking and number sense will directly affect their ability to think algebraically in their future mathematical careers. Teachers' beliefs will also affect how they orient their classrooms to either promote a rote memorization style of learning or more of an investigative style of learning. I want to investigate how well teachers' beliefs about different styles of learning line up with what they can actually do in practice. A study done by Ortiz (2014) investigated how much of the brain is active while adults do simple math problems. This study showed that faster and more accurate students used fewer resources than slower or less accurate students. The researchers suggest the subjects relied on recall of answers from memory and quick strategies. This document reports the findings of my investigation of my investigation into how teachers view and practice different methods of teaching, and how this could potentially affect implementation of new or altered teaching techniques.

My research combines scholarly articles and interviews with practicing teachers in order to get a full view of the possible beliefs that teachers could have, the actual beliefs that practicing teachers current have, and how those relate to classroom instruction. This information could provide key evidence in the constantly changing field of education. Also, these findings could provide insights into how to more accurately align your beliefs as a teacher to your practices. My research combines many different sources including scholarly journals, articles written by teachers, interviews with teachers, and observations of classroom instruction. The eclectic group of sources should provide insights into relationships between teachers' beliefs and their practices. If nothing more, I hope that my research can be read by current and future teachers and offer them views other than their own to help them teach their future students to the best of their ability.

This is an important topic to research because teaching can only be improved through reflection. Teachers must understand what they believe about education and compare that to the lessons they are producing in the classroom in order to bring those two ideals closer together. This paper consists of evidence from literature review as well as from my personal research that shows the apparent disconnect between teacher beliefs and practices caused by a variety of factors but still apparent in every day classrooms.

The research questions that guided my project were:

1. What are elementary teachers' beliefs about how students develop fluency in arithmetic?
2. What is the relationship between teacher beliefs and their classroom practices?
3. What external forces might account for these differences?

Chapter 2

Literature Review

Teacher Beliefs and Their Relationship to Teaching and Student Learning

What teachers believe does not necessarily transition to what they produce in a classroom environment. Tait-McCutcheon, Drake, and Sherley (2011) report the changes in practice of one teacher that were brought about by her reflection and effectiveness of her teaching and learning program and discusses how this teacher created a process which helped make learning basic facts more relevant and meaningful to her students. The authors suggest that teachers should build from existing knowledge and strategies their students may have by using materials, using images, and then using number properties, to actively have the students construct new knowledge and strategies on their own, instead of providing them with the rules directly and having them memorized. In a section of this article, proponents of student-lead learning described noting a shift in thinking from "expectations of knowing" to understanding basic facts and students working collaboratively instead of independently in order to socially construct knowledge (Tait-McCutcheon, Drake, Sherley, 2011). However, there was a discrepancy in how teacher participants described their teaching and the way students were practicing and learning their basic facts. The discrepancy arose when the assessment portion of the class came into play. The teachers assessed their students through timed tests, which was the way that the students had always been assessed, but did not build upon any of the ideas that the teachers believed they should. The timed tests were a summative assessment of student memorization, when the teachers were trying to foster collaborative construction of knowledge and understanding. One teacher's reflections upon this discrepancy helped her understand that by not communicating the intent of her basic-facts program to students and by using generic assessment practices (such as timed tests) that relied on speed and accuracy, she was inadvertently reinforcing the traditional memorization instead of what she meant to be reinforcing which was purposeful construction of unknown facts (Tait-McCutcheon, Drake, Sherley, 2011). She noticed that her beliefs were not aligned with her teaching, and made a change to help bring them closer together. She started by analyzing her student's achievements and identifying areas of lower success, placing an emphasis on shared understanding of the purpose behind each aspect of her basic-facts learning. In this way, she began to try to align her beliefs with her teaching practices, and tried to help her fellow teachers do so as well. However, none of this would have happened without her reflection on her teaching in the first place.

Cohen (1990), reporting the case of Mrs. Outblier, describes a teacher who sees herself as a success for the new policy of replacing mechanical memorization with mathematical understanding. She believes that she has revolutionized her mathematics teaching, but as Cohen explains, through observations of her classroom, the innovations of her teaching have been filtered through a very traditional approach to instruction. The result is a large discontinuity from her beliefs about her own teaching to her practice in a classroom. In this study, Cohen describes several clips of interviews with this teacher, where she is describing the changes she has made because of this new policy. Then, Cohen goes on to describe different aspects of her classroom that go directly against what Mrs. Outblier believed she was achieving. Without this observer coming into her classroom, Mrs. Outblier never would have known that her teaching practices were not aligning with what she believed she was achieving. Many of the specific examples showed that sometimes what she was doing in her classroom went directly against what she believed she was getting across. One example of this is how her teaching reflected the new framework by her adoption of innovative instructional materials and activities, designed to help students make sense of mathematics. However, during observations she would use the new materials as though mathematics contained only right and wrong answers. She revised the curriculum to help students understand math but was limiting their ability to explore different ideas by reinforcing the wrong idea that math only has right and wrong answers.

This literature provides definitions of key ideas and vocabulary so that this paper can be grounded in the vocabulary of education. I used these ideas when constructing my question in the next sections for the teacher interviews. In order to investigate teacher beliefs and practices, I needed to be well versed in the literature surrounding teachers and influencing them daily. I used this opportunity to do as much research as I could to be able to discuss a variety of different ideas that must be considered when a teacher decides which techniques he or she will or will not implement in his or her classroom. As you can see, there is some research into the disconnect between what teachers believe and what they do in practice, however my research question was more specifically about teachers' beliefs regarding students' understanding of arithmetic. I focused on basic math facts because arithmetic is the building block of students' future math understanding. I wanted to investigate specifically how teachers believed that these building blocks should be taught and if those teachers were teaching the way they believed they should be, or if not, what factors were influencing their teaching practices. The reason that I wanted to investigate beliefs versus practices is because teacher beliefs influence what they do. As you can see in some of the literature, there is an apparent disconnect, because every classroom has obstacles and no one teacher is perfect. By investigating teacher beliefs and comparing them to practices, I can further influence the field of education in a positive way by attempting to help teachers bring their beliefs closer to their practices.

How Students Respond to Mathematical Tasks

To begin my research, I started by drawing on existing literature to define the notion of fluency. By investigating the current definition of fluency, I could more accurately research how to encourage fluency in a mathematics classroom. Also, by starting with a definition of fluency, I would be able to explain to participants what exactly I was talking about when I was using this term. Researching the notion of fluency led me to research other terms that had bearing on my project to provide background knowledge of the concepts I was investigating.

One goal for every mathematics teacher is to encourage and promote efficiency and fluency in children's' learning. Before I started this research, my first goal was to define fluency because a topic cannot be researched without a definition. Through my readings, I have run into many different definitions of the term fluency. Gojak (2013) claims, "Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently" (p.1)

Wallace and Gurganus (2005) discuss how teachers should teach multiplication for fluency. The authors also cite *Principles and Standards for School Mathematics* (2000), which states, "Developing fluency requires a balance and connection between conceptual understanding and computational proficiency" (p. 35). Wallace and Gurganus (2005) also explicitly state that rote memorization is not considered fluency because "fluency with multiplication includes a deeper understanding of the concepts and flexible, ready to use of computation skills across a variety of applications" (p. 26).

Wallace and Gurganus (2005) raise the point that students who are just taught straight multiplication facts may not realize that multiplication can have a variety of meanings, and in that case, would not be able to see its connection to the real world. These students need to be made aware of the fact that multiplication is not just a sequence of isolated facts. Fluency in this case would be the ability to apply the knowledge of the concept of multiplication to solve different kinds of problems such as repeated addition (there are eight crayons in a box, how many are there in three boxes?), scalar (Marcus has eight marbles. His brother has three times as many marbles. How many marbles does Marcus's brother have?), area, etc. Students who are taught straight multiplication facts may have trouble relating those facts to word problems such as these. According to Wallace and Gurganus, fluency is making the connection between conceptual understanding and computational fluency, instead of solely rote memorization.

Kling and Williams (2014) cite two definitions of fluency, one from the Common Core State Standards for Mathematics (CCSSM) and the other from Baroody (2006). The CCSSM document describes procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately (CCSSI, 2010, p.6)." Baroody describes basic fact fluency as "the efficient, appropriate, and flexible application of single-digit calculation skills and… an essential aspect of mathematical proficiency" (p. 22). These two definitions share similarities even in the exact words that they use. Flexibility, efficiency, and appropriate application of mathematical skills are what both of these definitions require for fluency. These are very general definitions so they can be applied to all different sections of mathematics but they imply a common idea, namely, that a student who is fluent in a specific topic should be able to answer questions efficiently, correctly, and should be able to use their knowledge to answer questions they may not have seen before.

According to Russell (2000), fluency includes three ideas: efficiency, accuracy, and flexibility. Efficiency implies that the student does not get bogged down in many steps or lose track of the logic of strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making sure of intermediate results to solve the problem. Accuracy depends on several aspects of the problem-solving process, among them, careful recording, the knowledge of basic number combinations and other important number relationships, and concerns for double-checking results. Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Children must have an understanding of the meaning of the operations and their relationships to each other (inverse relationship between multiplication and division), the knowledge of a large repertoire or number relationships (how 4x5 is related to 4x50), and a thorough understanding of the number system and how the relationships vary (24+10=34 vs. 24x10=240). “The reason that mathematics can be solves in multiple ways is that mathematics does not consist of isolated rules, but connected ideas” (Russell, 2000, p. 155).

Russell (2000) also describes the cases of two students, Eleana and David, who both have some good ideas about number relationships and operations, but who have not learned to apply these ideas sensibly in problem solving on their own. With teacher guidance, they were able to identify their errors and correct them (aligning and carrying wrong). Good mathematicians are able to identify their mistakes quickly and correct them, and teachers should teach their students how to do that. Our jobs as teachers is to help students connect procedures, properties of operations, and understanding of place value rather than have them learn these concepts as separate, compartmentalized pieces of knowledge.

The ways that we define fluency may impact the ways that we think about how to help lead children to fluency. Two main instructional strategies that are often contrasted are rote memorization versus teaching children by developing their number sense.

One topic that relates fluency, number sense, and memorization is retrieval. Retrieval of memory refers to the "subsequent re-accessing of events or information from the past, which have been previously encoded and stored in the brain" (Martin 2010). One of the aspects of fluency is the ability to accurately produce an answer. Children produce an accurate answer either through retrieval of the answer itself, or retrieval of the strategy that will help them come up with the answer. Knowing facts from memory is the goal of teaching math facts though number sense, but the different arises in how you get there. Rote memorization of a fact such as 6x8 could be described as retrieval of a fact. However, children who use repeated addition, or another thinking strategy, multiple times to solve the same problem will eventually have that fact stored in their memory and will have easy retrieval of that fact without having to reconstruct it each time. In both cases, fact retrieval is the goal; the difference arises in how a student gets to that goal. Memory retrieval requires "re-visiting the nerve pathways the brain formed when encoding the memory, and the strength of those pathways determines how quickly the memory can be recalled" (Martin 2010). Examining how quickly students can retrieve an answer, either by retrieving the actual answer or retrieving the strategy that helps them retrieve the answer, can allow a researcher to determine how strong the relationship of the fact or strategy is in the students' brain.

Mahler (2011) discusses retrieval in a way that is more oriented toward number sense. Mahler presents a Language/Story approach to teaching mathematics both in general and, more specifically, to children who have disabilities. According to Mahler, sometimes children understand a concept but cannot memorize related facts. They will learn it one day after you drill them, and then the next day the information will be completely gone. *Semantic memory* involves knowledge of facts, rules, symbols, meanings and ideas that are not necessarily connected to specific incidents (in contrast to episodic memory) (Mahler, p. 5). Retrieval is the ability, to, upon demand, access and give expression to a specific fact, procedure, or piece of knowledge, within the generally accepted timeframe for that item.

Mahler's language/story approach involves assigning a rhyming mnemonic (3 = tree, 4 = door) and then weaving them into a funny and visually memorable story. The words that represent the numbers are supposed to rhyme with those numbers so that when you say "tree x door = elf" it sounds like "three x four = twelve." The stories were then used in hands-on activities such as drawing with felt pens or shaving cream, modeling with pipe cleaners or clay, acting or dancing, etc. This would help the students remember the stories, which in turn would help them remember the math facts. Although the stories might make the students take a little longer to recite the math fact, it is better than them not being able to recite it at all.

Baroody (1985) discusses how certain families or groups of numbers may be stored within a child's mind. One of the models raises the possibility that a certain fact could be stored as a specific fact as well as be represented by a general rule. For example, 1+0 (in the zero addition family) may be stored as the fact 1+0=1, in which case, when asked what 1+0 is, the child would respond with 1 without having to think about it. In addition, 1+0 could also be stored and represented by the rule (N+0=N and 0+N=N). The student would not necessarily need to retrieve this rule in order to solve the problem 1+0, but would understand that 1+0 would be part of that family. However, less familiar members of the zero addition family, such as 0+9989 or 78+0 would probably be represented by the rule alone, because it would be a waste for a child to memorize 0 plus every number as individual facts when the student could just memorize the zero addition rule. However, there is a limit to how many families of rules can be stored as both a fact and a rule, so there must be other ways to create fluency (Baroody 1985).

Knowledge of rules, procedures, or principles could interact in different families of facts in order to interact and generate number combinations. For example, many subtraction combinations may be generated by knowledge of addition combinations. If a child knows that 3+2=5, then they would be able to figure out that 5-3=2, given sufficient number sense and interconnected associations with numbers.

Some rules and families are just more common and easier to access because of the availability and repetition in the classrooms. Svenson (1975) measured reaction times of third graders in response to different examples of families of numbers involving addition. Svenson (1975) found that, by third grade, reaction times increased with addend size but dropped off with the (10+N and N+10) family" (Baroody, 92). The relatively fast reactions times with this family can be attributed to the emphasis on 10 factors in elementary school. There is also evidence cited by Baroody that said that there is a lower error rate for the (Nx5 and 5xN) family because children are taught the rule that says if a problem has a 5, the product must end in a 5 or a 0, which gets rid of a lot of the possible errors that children could make. This accessibility may be related to the availability of redundant information that is given out throughout elementary school. There are many rules that are taught, such as the product of two odd numbers will also be odd, that are reinforced all the time. This information is stored in semantic memory, which goes on to create a web of information that will assist with retrieval of facts as well as families and rules. Students can then build off of the information that they have to solve unfamiliar problems that may not be stored as a fact or family in their semantic memory yet.

One of the main ideas that also relates to the idea of memorization, number sense, and fluency is the fact that children who have not yet learned a specific problem are able to use estimation strategies to provide an answer. Baroody (1993) reports the results of a study of 42 third-graders before they had been introduced to multiplication in school. Nearly one third of these students apparently used estimation strategies when presented with unknown multiplication knowledge depending on prior knowledge. Baroody found that the associative strength between a problem and its answers was tested using an overt-strategies prohibited task, which is an arithmetic task that encourages children to respond quickly without giving them the ability to count. According to the distribution-of-associations model, "the probability of stating an incorrect or correct answer to a problem is proportional to the associative strength between the answer and the problem" (Baroody, p. 215). However, a schema-based view "suggests that children who have not yet memorized the single-digit addition combinations may respond to such a task by using estimation strategies. So if children are solely focused on memorizing problems, then when presented with a problem they do not have memorized, they should have a random guess. However, if the children have number sense, they should be able to use some kind of estimation strategy that can be identified.

The task given to kindergarteners was to respond to the questions provided while keeping their hands folded, as to prevent the children from counting to get the answer. The training for this process only helped the children understand what addition was and what the written representation of addition was, no practice with it. The groups of response patters that were expected to be seen were: correct answer, state a number several more than the larger addend, state a teen, transformation of an addend to a teen, state the number after an addend, state an addend, and other (Baroody 224). These strategies encompassed many of the common estimation strategies that children used when encountering unknown addition problems.

According to Baroody (1992), identifying different error patters and examining them can help identify the level of a child's number sense. Depending on which identifiable estimation strategy they use, you can see their level of number sense. For example, a child with little or no number sense would use a strategy such as stating the number after the addend such as 4+6=7. This shows a lack of number sense because if you add a number bigger than 1 to 6, you should get a number bigger than 7. However, if a child does not understand the n+1 rule, they would not be able to understand that 4+6 should be bigger than 7. The same logic would apply to the stating an addend rule. For example, if the problem was 4+6 and a child answered with 6, they would clearly not understand the concept of addition at all. A more sophisticated estimation strategy would be stating a number several more than the larger addend. This is more sophisticated because the child understands that adding a number more than one to another number should make that second number larger. An example of this would be 4+6=13 because the student knows that 4+6 equals a number much greater than both 4 and 6, even if it is the wrong answer. By examining the children's' estimation strategies, researchers can see the progression of children's number sense.

This topic relates to number sense and memorization because the more advanced estimation strategies the child had the more advanced number sense that a child must have. If a child has learned addition or multiplication facts through memorization of specific isolated facts, and then they were presented with a unpracticed problem, that child would more likely resort to a strategy such as stating a teen or N+teen, just because they have no sense of what answer might be the right one. However, if a child has been taught the definition of addition or multiplication, they should be able to use strategies to get the answer or a close-miss error, which would be using a strategy such as the state a number larger than the largest addend. They realize that addition involves adding two numbers together to get a larger number.

Baroody (1993) also presents other strategies that could be used to get the answer if a student did not know it right away with a study of 42-third graders who had not been introduced to multiplication in school yet. When encountering multiplication problems, some of these third graders were taught that multiplication is basically just repeated addition. A strategy that helps children figure out an answer relatively quickly is called skip counting. For example, if you are trying to solve 8x3, you count "3,6,9,12,15,18,21,24". However, this strategy can create computational errors: "factor related errors such as 8x3=21 (adding the multiplicand too many or too few times) and close-miss errors such as 8x3=23 (due to minor counting errors)" (p. 95). These are more advanced estimation strategies than the ones talked about in the previous article because they require that a student have the number sense to know that multiplication is repeated addition and they also require that the student know all the factors of specific numbers. However, if a child is starting multiplication, they should be aware of both of these facts, which would make examining these strategies and errors much more meaningful. Baroody also looked at estimation strategies, but in multiplication. There were specific strategies that the researchers charted and were looking for. The mechanical strategies included stating a factor (3x5 = 5 or 3), sum of the factors (5x3 = 8), difference of the factors (5x3 = 2), factor plus or minus 1 (5x3 = 4 or 6), or combining the factors (5x3 = 53). The semi mechanical strategies included stating a teen (an answer 11-19) or stating a decade (5x3 = 60). On the preliminary test, "almost one-third of the children chose at least half their answers from a single class of numbers (e.g., the sum or different of the factors, the teens, the decades)" (Baroody 106). These systematic error patters are consistent with the view that the children partaking in these tests used some sort of mechanical or semi mechanical estimation strategy when they were not sure of the answer. Consistent with the schema-based view, after the multiplication training, no experimental subjects relied on mechanical strategies and only one used a semi-mechanical strategy. This suggests that those children began to understand the concept of multiplication and knew the answers correctly, so did not have to use estimation strategies (Baroody 1993).

Baroody (1985) presents further investigation of children's estimation. Baroody discusses how "Winkelman and Schmidt (1974) propose that addition and multiplication facts share a parallel organization . . . they argue that there are associations between two digits (e.g., 3+3) and both their sum (6) and product (9)" (Baroody, p. 86). Baroody states that a result of this associative interference is that there is a greater tendency to associate 9 with 3+3, compared to 7. Another model proposes that addition facts are represented in memory as a printed table. The time needed to figure out an answer is measured by the distance you have to travel in the table in your mind to find the intersection of the two numbers. This accounts for why it may take a child longer to answer a problem with larger addends. Groen and Parkman suggest a direct-access model in which all memorized facts are equally accessible and those facts that are not memorized are figured out by an immature counting-on strategy.

All of these processes could account for different estimation errors. In the case of addition and multiplication being paralleled in memory, a child could respond to an addition problem with the product (e.g., 3+3=9) or respond to a multiplication problem with the sum (3x3 = 6). In the case of addition and multiplication facts being represented as a table in a person's mind, the estimation errors could be accounted to factor errors or close-miss errors. For example, in multiplication, if a problem was given as 3x4, and the child went into the table in their memory, there is a chance they could mistake the answer for either 3x3 or 3x5, just because of factor errors. In addition, if the child were given 3+4, there are many answers that could be off by one or two, just because they may read that memory table wrong. In the direct-access view, the children could mess up with the counting-on strategy when they do not know the answer. Counting-on is an immature strategy because it takes the longest when you do not know the answer, but could also prove the least able to create an estimation error.

Based on the ways that researchers and others have examined fluency, I define fluency as the efficient and accurate recall of answers to known problems and the flexible use of strategies to answer unknown problems. With this definition in place, I now turn to examining how students develop the knowledge and understandings needed for fluency.

How Students Learn and Develop Knowledge and Understanding

Wallace and Gurganus (2005) believe that multiplication should be taught through four steps: introducing the concepts through problem solving situations and linking new concepts to prior knowledge, providing concrete experiences and semi-concrete representation to purely symbolic notions, teaching rules explicitly, and providing mixed practice. These steps represent the goal of number sense. Through these steps, the students will have to discover new information and patters for themselves and then relate that information together before they are even taught anything. Number sense relies a lot on allowing students to discover patterns for themselves so they may discover strategies that work best for them. Another aspect of number sense discussed by Wallace and Gurganus is the fact that students should be taught through physical demonstration before they move on, so that they will be able to visualize those physical demonstrations in their head if they need to in the future. These authors believe that teaching for mastery of multiplication is more than rote memorization of the basic facts, instead they believe that students must develop conceptual understanding first and then move on to computational fluency.

Baroody (1985) cites Brownell (1935) on the definition of *meaning theory* and the true test of learning: "the true test is an intelligent grasp upon number relations and the ability to deal with arithmetical situations with proper comprehension of their mathematical as well as their practical significance (p.19)". Baroody compares meaning theory to drill theory, and cites the advantages and disadvantages of both of these learning techniques. Meaning theory falls under the category of number sense for a variety of reasons. First, learning mathematics, including mastery of number combinations, is viewed as a slow process. In other words, "the child's psychological readiness is considered" (Baroody, 1985, p. 85). This theory indicates that teaching children thinking strategies is more effective than the contradicting drill theory in facilitating learning, retention and transfer of basic combinations. Many educators and curriculums believe that learning the basic facts of mathematics should be a straightforward process that should be accomplished quickly. Many guidelines in schools overestimate how quickly children should be able to master all of the basic combinations. However, many contradicting sources from those who have researched education as well as those that have partaken in education have discovered that many things worth knowing take a long time to learn (Baroody 1985). Meaning theory relies on "discovering, labeling, and internalizing relationships" (Baroody, 1985, p. 95).

Another important aspect of meaning theory that relates directly to the field of teaching is the idea of teaching thinking strategies so that children will be able to learn numerical relationships. Children with disabilities have a harder time making these connections between families of numbers, which is why one of the most significant deficits displayed by children having difficulty with mathematics is weakness in their knowledge of basic number combinations. Without the connections, children are memorizing combinations of numbers as isolated facts (so 5+7 could be different than 7+5). Although this article discussed this topic for children with disabilities, all children could benefit from learning mathematics this way, in order to avoid doing twice the amount of work when memorizing isolated facts. Teachers should focus on teaching thinking strategies because is has been advocated to help children learn numerical relationships and foster the automatic recall of number facts.

Baroody (1992) presents another view that relates to number sense called the *schema-based view*.. A schema-based view focuses on number sense and claims that with practice and instruction, error patterns will evolve creating more reasonable responses to unpracticed mathematics problems. This view also focuses on allowing children to discover patterns that allow them to invent rules to help them respond to unpracticed problems. Baroody's (1992) research study examined the responses of kindergarten children to the overt-strategies prohibited task before and after they had received training. The overt strategies prohibited task asks the students to respond to addition problems without counting on their fingers. In this way, this test was designed to attempt to provide a reasonable estimate of whether children use identifiable estimation strategies on problems they have never encountered. Baroody found that most children in this study used some kind of identifiable error pattern suggesting the use of estimation strategies in both the pre and post tests, however, the use of mechanical or semi-mechanical strategies in the experimental groups dropped dramatically. This suggests that they began to connect the addition doubles rule to multiplication combinations involving two.

Baroody's study illustrates a way that number sense can be assessed through observation of developing error patterns. Early mental-arithmetic error patters lean more towards educated guessing. Children who do not know the answer to a problem and who are not allowed to compute will probably revert to estimating. This means, "when time or social expectations do not permit figuring out the exact answer, children may strategically employ general knowledge associated with a problem or related problems to make a judgment about the unknown answer" (p. 96). However, children's earliest estimation strategies will be ineffective and mechanical. For example, if they are presented with a problem such as 5x3, they may create answers by stating the sum of the factors or using the factors to create a two-digit product. As the children begin to develop their number sense, they will change the errors that they make. The schema-based view takes into account that children can discover their own techniques to help them compute answers. Two, more sophisticated, short cuts for computing answers they may not know from memory are skip counting and counting-on. Skip counting is used to compute products by counting up by factors. If the student were presented with 5x3 again, they would skip count by 5's, which they should be able to do. If you were to use the counting-on strategy to solve 5x3, a student may already know that 5x2=10 and then they would just have to count-on five more times (11,12,13,14,15). The development of these strategies would allow teachers and professionals to see the development of a child's number sense. As children start to understand numbers, they will be able to understand the errors they make because they can see which answers make sense and which do not.

Part of Baroody's (1992) goal was to illustrate the differences between the distribution-of-associations model and the schema-based view, and to discover which of these two topics were complete. The distribution-of-associations model will be discussed in the next section regarding teaching children through rote memorization. However, Baroody claims that the schema-based view, in this study, "predicts that children who are taught or who discover the relationship between multiplication combinations involving two and their knowledge of the addition doubles should be able to respond efficiently to unpracticed combination, at least for those for which they have internalized the related addition double" (Baroody, 1992, p. 99). Basically, Baroody concludes that children who realize that multiplication problems that involve two can be related to addition (5x2 is just 5+5) will be able to be successful with unpracticed problems that have the same format, or a similar format. Baroody's conclusion was that number sense and a discovery-learning approach was equally as effective as a direct-instruction approach in fostering mastery of multiplication combinations, and that the discovery-learning approach that develops number sense is more likely than direct instruction to foster autonomy and mathematical power in children.

In a separate study, Baroody, Bajwa, and Eiland (2009) describe the *active construction* view, which involves "constructing a well-structured or –connected body of knowledge that involves patterns, relations, algebraic rules, and automatic reasoning processes, as well as facts" (p. 69). In this view, there are three phases involved in children memorizing basic combinations:

- Phase 1 – counting strategies (using object or verbal counting to determine the answers)
- Phase 2 – reasoning strategies (using known facts and relations to deduce the answer of an unknown combination)
- Phase 3 – mastery (efficiently producing answers from a memory network). This article discusses these phases in terms of both rote memorization and meaningful memorization.

Most importantly, Baroody explicitly connected his research to the concept of number sense. Baroody (2009) defines *number sense* as mastery with fluency growing out of the development of meaningful and well-interconnected knowledge about numbers. Most schools now are aiming to develop their students' number sense, instead of just relying on rote memorization. Fluency with basic mathematics facts in children "grows out of the development of meaningful and well-interconnected knowledge about numbers", or number sense (Baroody, p. 70). For this reason, the Active Construction View is often called the Number Sense View. Phases 1 and 2, as presented by Baroody, are critical to developing a network of facts, relationships, and strategies that become the basis for fluency, which is Phase 3. One example presented by Baroody is that once children recognize that adding zero to a number does not change that number, this concept can be stored as a rule: any number plus zero is that number.

Dr. Charlotte Sharpe, an assistant professor in mathematics education at Syracuse

University combined with these previous views in a short interview (C. Sharpe, personal communication, 2017). While discussing the Common Core, Dr. Sharpe suggested that it is very important to make a distinction between *memorizing* and *knowing from memory*. She said the Common Core still requires that students know facts from memory but rely on other methods besides straight memorization to develop their knowledge of those facts. According to Dr. Sharpe, the Common Core has even adjusted its expectations so students focus on developing other skills before memorizing. Dr. Sharpe then described a bingo game that can be used to help develop number sense. Each player creates his or her own bingo board containing positive whole numbers. A player rolls two dice and if the product of the pips shown on the dice is on the board, then it is crossed off.

Playing the game involves thinking about and understanding multiplication combinations. The players need to know that the numbers that are placed on the bingo board have to be multiples of the numbers on the dice. After playing a couple of times, the players could learn the numbers that seem to pop up often could be put in the same row or in the corners to help you win. Dr. Sharpe explained that this game could be used as a memorization game if players are told to write down the multiplication sentences they are using to get the product. After playing a couple of times, the players can look back and see what numbers they have used before and which math sentences provide the same answers. As learners play the bingo game, they record their dice rolls as math sentences on a recording sheet. After a couple weeks, the students or players can be given a new recording sheet to complete. This puts them in a position either to solve all the multiplication equations again or to recall answers from memory. Dr. Sharpe closed by saying that this game develops memorization as well as number sense about factors and strategy about what numbers to use to increase the chances of winning the game.

One topic that I wanted to discuss regarding fluency as well as retrieval is memorization. Memorization is how I learned mathematics when I was younger. I believe that there were some aspects of number sense in my elementary school, but what I remember most clearly was timed tests and flashcards. In this sense, I want to define rote memorization as focusing not on understanding but on learning by means of repetition (i.e. flashcards). Baroody (2009) describes the process of memorizing a basic number fact by rote as "a simple nonconceptual process of forming a bond or association between an expression such as 7+6 and its answer 13" (p. 70). Baroody characterizes this mode of learning and knowing as the *Passive Storage View*. Recall that Baroody describes three phases (Counting strategies, reasoning strategies, and mastery) that children typically progress through when learning basic facts and families of basic facts. According to the Passive Storage View, which I associate with memorization, Phases 1 and 2 can facilitate, but are not necessary for, memorizing facts. According to Baroody, early proponents of the Passive Storage View looked at counting and reasoning strategies as unnecessary and a bad habit to avoid doing the real work of memorizing the facts. Although Phases 1 and 2 allow for opportunities to understand the meanings behind basic facts, they are unnecessary because all the facts could theoretically be learned by extensive practice.

According to proponents of the Passive Storage View, practice is the key factor in rote memorization of basic facts forming and strengthening of specific associations. This begins to relate to the topic of retrieval when the idea of a constant-response-time (CRT) is introduced. In this approach, beginning in grade 2, after children have been given some time to use informal methods, children should be given only a few seconds to answer and be provided the correct answer if their either respond incorrectly or do not respond in the set amount of time. This method is supposed to minimize the children's ability to use informal strategies such as counting on their hands or reasoning strategies. After discussing this Passive Storage View, Baroody (2009) does go on to explain that some research indicates that this fact drill process is less effective in fostering fluency in children than instruction that focuses on patterns, relations, and reasoning strategies. This may be because children who do not know the answer to an unpracticed problem, will have no strategies to help them figure out the answer if all they have been taught is fast retrieval of an answer. However, there are definitely pros of drill practice, because eventually, whether the children understand the concepts behind basic facts or not, they will be required to know the answers for their future.

A separate perspective is the *Distribution-of-Associations* model (Baroody, 1993). Baroody (1993) studied the responses to multiplication questions by 42 third-graders who had not been introduced to multiplication in school yet. According to the distribution-of-associations model, "the representation of a basic multiplication combination consists of a set of possible answers, which very in their associative strength with the combination. In this view, there are three factors that influence the correctness of answers: related knowledge, difficulty of executing back-up strategies, and the frequency of a problem exposure. The current distribution-of-associations model suggests that mental-multiplication errors are due to faulty retrieval. In this article, Baroody (1993) cites Siegler (1988) who has stated that children with little opportunity to practice a multiplication combination should have a flat distribution of associations, which means that they should have a wide range of answers to an unpracticed problem. Also, according to the distribution-of-associations model, previous associations can affect the answer that a child may give to a posed problem. An example of this would be because addition and multiplication are related there is a high possibility that children will incorrectly state the sum of the digits when asked for the product (4x3=7).

A basic assumption of this model is that “people associate whatever answer they state, whether correct or incorrect, with the problem on which they state it” (Baroody, 1993, p. 95). So, if a student is studying a problem such as 4x3, and they answer 7, even though that answer is wrong, they have now created an association between that problem and that incorrect answer. However, with sufficient practice, the correct answer will become so highly associated with the problem that the child will almost always respond with it (p. 95). This concept is called memorization. According to this model, however, since the facts are memorized as individual problems instead of as a family of problems, knowing one fact should not increase the probability of responding correctly to other combinations (this model did not take into account nx0=0 or nx1=n relationships). It follows that practicing 7x2 will not increase the likelihood of responding correctly to 8x2. In this case, memorization would not help develop children’s understanding of numbers and the way they interact. Also in this case, children would have to memorize each individual fact and would have no reasoning strategies to figure out unpracticed problems.

The goal of this section was to explain why fluency with number combinations is important and that fluency includes the ability to efficiently and correctly produce responses to number-combination tasks. This literature shows that students might develop such fluency through either rote memorization or through developing number sense, or a combination of the two. However, students who develop some number sense and schemas for operations are better prepared to solve new problems compared to students who have only develop memory for specific number combinations.

Fluency is defined as the efficient, accurate, and flexible use of strategies and recall to provide answers to known and unknown problems. Developing fluency requires a balance between conceptual understanding and procedural fluency. Fluency can be fostered in students through a combination of the development of number sense as well as practice with number combinations and emphasis on some aspects of memorization. Teachers can help develop fluency in their students by helping them to see the concepts that make up different aspects of arithmetic by either showing them strategies or allowing students to develop strategies on their own, or a combination of these two. Teachers' beliefs about fluency and learning are important because whatever teachers believe influences what they produce in a classroom. Teachers use their beliefs in order to create different learning opportunities for their students to help foster fluency in all aspects of mathematics. Focusing on elementary teachers' beliefs is imperative because the foundation of future mathematics understanding starts with learning basic arithmetic. Arithmetic is used in almost all other aspects of mathematics, so understanding how elementary teachers think about teaching is important to help foster the fluency in every student, currently and in their futures.

Chapter 3

Methods

I interviewed six teachers from several middle-class suburban schools regarding their ideas about basic mathematics facts and the best ways they have seen in their careers to teach children those facts. By analyzing these interviews, I will attempt to compare the way the participants described their beliefs and how students learn arithmetic. I then compared the teacher interviews with observations of their classrooms to examine the alignment between their beliefs and practices. Some teachers are aware of a misalignment for a variety of reasons and explain those reasons in their interviews.

Participants

Participants were identified from among personal contacts and by soliciting suggestions from administrators of schools. I emailed the prospective teachers and explained what my research project was investigating. If they believed that they would be a helpful resource to me, I set up a one-hour interview in a public or semi-private area with them, depending on their preference. The participants are recorded in the Table 1:

Table 1: Participants and Grade Levels

Participants Psuedonyms	Grade Level Taught	School Psuedonym	
Ms. Lewis	2nd	Brookdale Elementary School	-Since the interview, this teacher has transitioned to teaching 1st grade at a different school -Has been teaching for 13 years -Has taught 1st, 2nd,3rd, and 4th grades
Mrs. Peterson	3rd	Brookdale Elementary School	-Has been teaching for 27 years -Before she taught 3rd grade, she taught 6th grade Language Arts -Has been teaching 3rd grade for about 12 years -Her love for language arts is aparent when looking around her toom
Orangebrook School 3rd	3rd	Orangebrook School	These teachers did not have time to meet in person for an interview so instead they submitted

Grade Cohort			a google doc with general consensus answers for each of the interview questions
Mrs. Smith	4th	Brookdale Elementary School	-Has taught 4th grade for only about 5 years -Since the interview, Mrs. Smith has moved down to 2nd grade because she felt too restricted by the intense standards required in fourth grade.

Data Collection

Data were collected through interviews with teachers and through observations of some teachers' classrooms. I conducted the interviews in person, public or semi-private areas, depending on the participants' preferences. Three of the interviews took place in the respective teacher's classroom and the other three were conducted asynchronously through a shared document in Google Docs. Participants were asked to answer a series of questions about their profession and teaching methods. I inquired about their ideas and opinions regarding my topic. Each interview took no more than one-hour.

I was also able to observe the classrooms of teachers who had participated in one-on-one interviews to see how their interview relates to their actual practices. Observing the classroom helped me to see how students react to the different ways that teachers may introduce methods to learn basic math facts. These interviews and observations took place in May and June of 2017.

I recorded the interviews on an iPad. The recordings were voice only and were later transcribed. I transcribed the interviews on my own so I would have time to go through and analyze the teachers' responses, while also creating a tangible source of the interviews.

Questions posed throughout the interview:

-What are your strategies for helping children learn basic math facts?

-What are some activities that you use to help students learn basic math facts?

-What are some of the ways that you assess whether students are learning basic math facts?

-How do you identify fluency with basic math facts in your students?

-In your opinion, how important is it for students to know basic math facts from memory?

-Have changes in policy, such as the NCTM Standards or the Common Core Standards, affected your ways of helping students to learn basic math facts?

-What has been the effect on the students because of this change in instruction?

-Are your methods for teaching basic math facts the same now as when you started your career?

-If not, how are they different? Why do you think your methods changed? How much of the change was self-motivated? How much was motivated by outside influences?

-Are you familiar with the term "number sense" and if so, what does it mean to you?

-How is the implementation of number sense activities in classrooms helping students know from memory in comparison to just memorizing? (Give an example from the current curriculum)

-If you were to list the strategies that you think help students to learn basic math facts, where would memorization appear in that list

The classroom observations all took place after the interviews with the teachers. I wanted to use the information that I acquired from the interview to form the ideas that I was going to be on the look for when I went into the classroom. My goal for observing the classrooms of these teachers was to be able to compare the teachers' beliefs to their practices. In order to compare these two, I needed to observe the teachers in their classroom environments. I was hoping that these classroom observations would help answer the question regarding the extent of alignment between teachers' beliefs and their classroom practices. In general, I was looking for the following things: memorization activities (flash cards, timed tests, etc.), student explanations of their solutions, identifiable strategy errors, use of accurate rules, overt strategies, success or failure with unpracticed problems, any student-invented rules, and fluency. Some of these ideas were more difficult to observe in just one classroom observation; however, having a general idea of what I was looking for definitely helped me close in on more specific ideas to take notes on. I went into the classroom observations with a chart that contained rows with each of the previous ideas written in them, and then columns that followed in order to write what I observed in regards to those ideas as well as other notes. Throughout the classroom observations, I checked off the ideas that I saw being implemented and also took extensive notes to later compare to the interviews I conducted. After the classroom observations, I was able to go back through the interviews in order to highlight key ideas that I saw implemented.

Data Analysis

In order to analyze this data, I listened to and transcribed each interview myself in order to gather information as well as create a document without interruptions or unnecessary words such as "um" and "like" where the teachers were pausing to think about their answers. I used qualitative methods and inductive analysis to compare teachers' statements from the interviews to the events, statements, and actions that were captured in my field notes during the observation. By comparing the transcripts of the interviews to field notes, I identified ways in which classroom events were aligned with teachers' beliefs, and ways that classroom events contrasted against stated beliefs.

Chapter 4

Findings

In this section, I present findings describing relationships between participants' beliefs about education and their enacted practices. The grade level of the teachers' students, starting with a Second Grade teacher at Brookdale Elementary School named Ms. Lewis and continuing through to findings from a Fourth Grade Teacher, also at Brookdale Elementary School, Mrs. Smith orders these findings. The following sections describe the beliefs of these teachers regarding number sense and rote memorization as well as their feelings about timed tests, fluency, and how to best foster an understanding of numbers in their students. These teachers also talk about the "Strategy Continuum" compared to memorization and how students should be practicing math facts every day. These beliefs are also compared to classroom observations in most cases in order to see if the beliefs that the teachers are describing line up with their actions in the classroom, and if not, then why.

Across my participants, these teachers defined fluency as being able to provide an answer to a problem efficiently and accurately, but had different opinions on how that efficiency and accuracy should be fostered. Most of my teachers believed that number sense is an important aspect of fluency in mathematics; however, knowing basic facts from memory is a more important aspect of students' future success in mathematics and their educational careers. This influences their teaching to rely more on fostering memorization of basic facts when it comes down to it, rather than spending a lot of time on fostering number sense. Many of my teachers shared the belief that they do not have enough time to achieve all of their academic goals for each of their students, impeding their ability to directly line up their beliefs with their practices.

Throughout my observations, the primary strategies that I saw being used were mostly memorization based. I saw parts of different lessons that hinted at an emphasis on number sense, but instead became mostly based on helping the students memorize the basic information needed to understand the concept. This compares with different aspects of the interviews by aligning with the fact that they do not have enough time to dedicate to number sense during their lessons but going directly against what most teachers believed were important aspects of teaching basic math facts. These separations are examples of exactly what I was researching and provide some evidence that it is important to be aware of the possible misalignment in order to teach to the best of one's ability.

Mrs. Lewis: Misgivings About Timed Tests, Believes Fluency is More Than Just Accuracy and Speed

Mrs. Lewis is a 2nd Grade Teacher at Brookdale Elementary School. Mrs. Lewis has been a teacher for 13 years and has taught 1st, 2nd, 3rd, and 4th grades. She believes that number sense has to do with the students understanding of the general concepts of addition and subtraction either making the answer larger or smaller than either of the addends. She also believes that fluency involves being fast and accurate but that those are not the only concepts that can make students fluent in mathematics.

Number sense means knowing that addition makes bigger and subtraction makes smaller. Mrs. Lewis believes that in second grade, "number sense . . . means that they understand that when you add, you get a higher number, and when you subtract, you get a lower number" (H. Lewis, personal communication, June 21, 2017). She cited some of the strategies, which were doubles, skip counting, and doubles near doubles. Doubles is teaching 2+2=4 and 4+4=8 and then as soon as the students know those facts, they will be able to figure out 2+3=5 because 5 is one more than 4 and 3 is one more than 2. Mrs. Lewis also explained the idea of counting on, for addition, which means that you would be given a problem like 5+3, and you would start at 5, and then count on 6,7,8. The first way that children would be able to learn this process is with a number line. The number line is key because it helps the children with visual learning that also helps them to create a sense of how numbers work in their mind. A process that Mrs. Lewis explained that seemed unique was called touch counting. Mrs. Lewis said, "Basically, you draw the dots on the different numbers and then they can use them to know how many the number represents, as well as using this to learn how to count on, and eventually add…while they are overtly counting the dots on all the numbers, they also see the number sentence" (H. Lewis, personal communication, June 21, 2017). So while they are overtly counting dots, they are also subconsciously seeing that 5 dots is what the number 5 represents.

Mrs. Lewis stated that *add* and *subtract* are not words that the students hear every day, so it is not something that they will easily understand. Making sense of these terms and being able to use them to do problems in their heads is key for number sense with addition and subtraction facts. When asked how she was implementing activities that would develop these words (add and subtract) in her classroom, Mrs. Lewis responded with using manipulatives and coloring so the students can visualize and learn to see the difference of two quantities being added or subtracted. Eventually, the students should understand the meaning of add and subtract and be able to visualize past activities in their heads to help them figure out unfamiliar problems, instead of giving up because they do not have specific fact memorized.

Mrs. Lewis gave insight about her beliefs regarding the effects of the Common Core on her classroom. She said "the standards are emphasizing harder problems before the students are ready and trying to force them to think outside the box" (H. Lewis, personal communication, June 21, 2017). Mrs. Lewis expressed a belief that "thinking outside the box" should be a skill that the students develop when they are ready, not something that is taught to them. In her opinion, the standards expect teachers to teach the students complicated word problems before the students have mastered their basic facts. Complicated word problems could definitely be confused with number sense but those two are not the same. Mrs. Lewis expressed that number sense is students understanding numbers, not being able to decipher what she perceives as unnecessarily hard word problems that do not add to their knowledge of math.

Fluency is about speed, accuracy, and method of computation. Mrs. Lewis believes that fluency in computation requires that the student be both quick and accurate when producing the answer. However, later on in the interview she also described thinking about fluency as including how students compute as well as their speed and accuracy. Mrs. Lewis described a process of carrying around a checklist when she gives her students independent work so she can look and see who is struggling and who is completing the tasks quickly and accurately. These are the ideas that she looks for in her students because this is how she sees fluency. Mrs. Lewis said, "[her] definition of fluency is if the students can complete addition and subtraction problems quickly and accurately" (H. Lewis, personal communication, June 21, 2017). In order to assess these qualities, Mrs. Lewis said that she needs to watch the students complete the problems. She can see which students are using their fingers, or which students can do all the work in their heads just by looking over their shoulders. Mrs. Lewis also believes that students knowing their basic math facts from memory is extremely important, claiming, "addition is the basis of everything to come in math, so being able to do it quickly and accurately is extremely important" (H. Lewis, personal communication, June 21, 2017). She also believes doing simple procedures quickly and accurately will help students succeed, as the problems get harder. One example that Mrs. Lewis discussed was the example of 3+4. If a student does not know what 3+4 is, there is no chance that they will know 30+40 or 300+400. Knowing 3+4, opens the door to knowing 30+40, and 300+400, as well as any facts that can be related to 3+4.

Classroom observation: What do numbers mean? In the classroom observation, I saw evidence of Mrs. Lewis teaching students with the question "what do numbers mean?" in mind. This lesson regarding partial sums had more to do with Mrs. Lewis trying to help the students understand numbers instead of encouraging them to complete the problems quickly. One example that gave Mrs. Lewis a hard time was 70 + 40. In this case, the students were supposed to know that 7+4=11 and if you get a number bigger than 9, you need to carry. However, since none of the previous examples required carrying, the students did not relate the same concept to 70 + 40. Mrs. Lewis tried to stress that knowing 7+4 would help the students figure out the answer, but this problem requires a great amount of number sense. Even though Mrs. Lewis probably would have wanted the answer to be quick and accurate, the main point of this lesson was to have the students understand that 7+4 is 11 ones whereas 70+40 is 11 tens, or 110. Mrs. Lewis stressed the understanding of numbers in this lesson instead of speed and accuracy. Mrs. Lewis ended up having to count on her fingers to overtly show her students how to compute the answer. Her hope seemed to be that the students would eventually be able to do this counting and thinking in their heads.

During my classroom observation of Mrs. Lewis, I saw an additional idea about number sense that was not expressed in her initial interview. She said she believed that number sense was the basic idea of knowing that addition means you make the number bigger and subtraction means you make the number smaller; however, during her lesson, she discussed the number 562, and what each of those digits represented. Knowing how to break apart numbers is a key aspect of students' number sense that Mrs. Lewis may not even realize she is discussing. However, even though Mrs. Lewis may not realize that this is a key part of her students' learning, she is fostering the idea regardless. The main point of the lesson was to teach her students partial sums and different ways to add up bigger numbers. For example, she wanted her students to add 52 + 74, and in order to do that, she was teaching them to break 52 into 50 and 2 and 74 into 70 and 4, and then just add up the ones and the tens, respectively. In order to be able to recognize 52 and 50+2 as the same thing, students must understand the ideas of numbers and what they represent. Through observation of her lesson, I can see the emphasis on understanding numbers rather than just knowing that addition is making something larger.

Misgivings about timed tests as an assessment of student learning. Though not visible through observation of her classroom, this dislike for timed tests may be a concern about the fact that she believes that fluency is not just about speed and accuracy mixed with a concert about how timed tests may not be a correct representation of student knowledge. The way her students are tested on these basic math facts is through timed tests. When asked about how she tested her students, Mrs. Lewis responded with "Unfortunately, through timed tests" (H. Lewis, personal communication, June 21, 2017). So clearly Mrs. Lewis is unhappy with the way that she is testing her students. In a follow up interview I would ask Mrs. Lewis how she would prefer to assess her students if not through timed tests.

Mrs. Lewis believes that number sense involves understanding the concepts behind number operations, instead of relying solely on the memorization of individual facts. However, in her classroom observation, she demonstrated an idea that is very important to point out, not included in her interview. During Mrs. Lewis' classroom observation, I watched her walk students through breaking numbers apart, which is another key aspect of number sense. Through her interview, Mrs. Lewis explained that she believes fluency is about speed, accuracy, and method of computation, but in the classroom, she seemed to rely more on student understanding that quickness of answers. Mrs. Lewis also provides important insight with her feelings about timed tests. She explains that she uses timed tests to test her students which aligns with her idea of fluency being about speed, but seemed to directly contrast how she runs her classroom. Mrs. Lewis' case illustrates one example of a disconnect between beliefs and practices, for reasons such as lack of time and promoting understanding instead of relying solely on speed.

Mrs. Peterson: Memorization is Most Efficient, But Teaches with a "Strategy Continuum"

Mrs. Peterson is a 3rd grade teacher also at Brookdale Elementary School. She has been teaching for 27 years. Before she taught 3rd grade, Mrs. Peterson was a 6th grade Language Arts teacher. Her love for language arts is apparent when looking around her room. There are no posters about math, and most of the posters that are up revolve around English. She has been teaching 3rd grade for about 12 years. Mrs. Peterson believes that multiplication can be taught with a "strategy continuum" but most of her beliefs lie with memorization being the most efficient way to learn basic facts. She believes that practice with facts is the best way to become efficient with the facts.

Students can learn multiplication with a "strategy continuum", but memorization is more efficient. Describing how students learn multiplication, Mrs. Peterson referred to a *strategy continuum* (C. Peterson, personal communication, June 14, 2017). Mrs. Peterson says that in 3rd grade her students are first learning the concept of multiplication. First, multiplication is described as repeated addition, so if you have 3 groups of 2 items, you have 2+2+2. The continuum starts with using actual counters to show the problem 3x2. Students use manipulatives to create 3 groups with 2 counters in each. Then after that, the class shifts to drawing their own pictures to represent the problem. After that, they move on to writing a repeated addition equation, so 3 groups of 2 is 2+2+2. As soon as the students can understand multiplication as repeated addition, Mrs. Peterson says she teaches them skip counting. She says she does not want them to use counters, or their fingers, or pictures anymore, she wants them to skip count in their heads. As the students start to progress in their mathematical abilities, the problems will begin to become harder, and skip counting will also become harder. So then they can use a strategy called the distributive property. For example, a student might not know 9 times 7, but they know 5 time 7 and 4 times 7 because both of those are "friendly" multiplication facts. However, after explaining all of these processes, Mrs. Peterson concluded by saying "Ultimately, we say you can do all of this work, the skip counting, the distributive property, where you are breaking up your facts . . . but the easiest thing to do and the most efficient is to memorize them" (C. Peterson, personal communication, June 14, 2017). In order to accomplish this memorization, Mrs. Peterson says that her students do a lot of repeated drills starting with the easiest numbers to memorize and moving to the most difficult. These ideas show the progression from forming the students' number sense to having them ultimately memorize the problems anyway. Being able to understand multiplication is key in 3rd grade mathematics, but in order to move onto higher mathematics, the students should be able to recall their facts quickly, which is where the rote memorization comes in.

Number sense is like spelling and playing with numbers**.** Mrs. Peterson explained that she believes there is a connection between being a good speller and proficient at basic math facts because both of those skills relate to being a visual learner. This relates to the idea of number sense because number sense has been described as being able to picture the solution, or the strategies to get to a solution, in your mind. Mrs. Peterson believes that if you are a visual learner, you can look at either the spelling of a word or the solution to a mathematics problem and know immediately that it does not look right. Later in the interview, Mrs. Peterson described number sense as the ability to "decompose numbers" and "play around with numbers" in their brain. This belief is consistent with Mrs. Peterson's notion that visual learners have an easier time memorizing basic math facts, because they have better number sense and an easier time visualizing the answer to the problems. Another aspect of Mrs. Peterson's beliefs about number sense, which lines up with the earlier discussed strategy continuum, is the idea that students can play around with numbers and create strategies that may have never been taught to them. This is number sense in the most pure way because students can come up with their own strategies for solving problems because they completely understand how numbers work.

Practice and memorization are the most efficient ways to know basic math facts. After thoroughly discussing number sense, Mrs. Peterson admitted that she believes that the most efficient way to know your basic math facts is to practice and memorize them. One way that she claims to help her students memorize the basic math facts is though speed drills. She will start with the easier problems, like starting with just the 2 times tables, and then move on to harder times tables as the students build their confidence. She also explained that sometimes, randomly, she would have the students play multiplication bingo or war to practice their facts and begin to memorize them. There is also a math website where the students can go on to practice their math facts. All of these activities rely mostly on practice to lead to memorization. This belief is mirrored in Mrs. Peterson's description of using timed tests to assess students' knowledge. She says "the goal is to get 60 problems in 5 minutes, in third grade" (C. Peterson, personal communication, June 14, 2017). Mrs. Peterson said the goal was to get 60 problems in 5 minutes, not to understand how they got their answer, just to get them right. There is research that shows that some students, who rely solely on memorization to learn basic math facts, do not completely understand what they are doing, even if they have strategies to get the right answer.

Mrs. Peterson also discussed that on the students' report cards, the teachers must assess them on two different Common Core third grade standards. The two standards are "multiplies within 100" and "applies multiplication strategies." Mrs. Peterson said how she believes the first standards is asking, "do the students know their facts?" The second standard relies more on the multiplication strategy continuum, which explains more why Mrs. Peterson might rely on teaching those strategies early on. She describes how she assesses her students on these two standards. For "multiplies within 100," she says "if you can consistently recall multiplication facts within 100, you have achieved this standard" (C. Peterson, personal communication, June 14, 2017). Then there are lower numbers for how often students are able, if they are able at all, to recall their facts within 100. For "applies multiplication strategies," she says "this is what we are shooting for, consistently demonstrat[ing] understanding of multiplication strategies to solve a variety of equations through the use of tools" (C. Peterson, personal communication, June 14, 2017). This may be what the standards are striving for, but Mrs. Peterson seems to want the students to be able to multiply numbers in their heads without using strategies. However, this standard may be just that they are able to use strategies, even if they do not necessarily need to.

In the later part of the interview, Mrs. Peterson shared her ideas about how she values timed tests as assessments, how she believes the most efficient way to know your basic math facts is to practice and memorize, and how she believes fluency is being able to come up with the answer without using strategies. So there was a little bit of conflict between these ideas and the ideas that you should teach multiplication with a continuum of strategies**.** In the interview, Mrs. Peterson told me "to be fluent, is to not have to rely on skip counting on your fingers, not having to do this break apart…fluent to me is that you can say [the answer] within three seconds" (C. Peterson, personal communication, June 14, 2017). This statement shows that Mrs. Peterson consciously values accuracy and efficiency when solving basic math problems. She likes to teach the students strategies for multiplication first, but then she focuses mostly on practice and memorization.

Classroom observation: Using visualization to support estimation. During the observation of Mrs. Peterson's class, she was teaching the students about estimating. Estimating can be considered number sense because the students must be able to visualize and compare items in their head. One of the questions asked in the class was what is the closest estimate for a soda can – 350 mL or 35 L? This question requires the students to be able to picture what a soda can would look like next to items that could represent 350 mL as well as 35 L. Estimating liquids requires visualizing and can vary based on how the student may be visualizing it. To cope with that fact, Mrs. Peterson showed her water bottle as a physical example of 500 mL. This gives the students a reference point for what a soda can could be compared to and allows access to all students. Although this observation did not show any aspects of the multiplication strategy continuum, it showed a good idea of how number sense can be used in the real world, aside from basic facts. If the observation had taken place earlier in the year, I believe I would have seen these strategies form over time, but for now I just have to take Mrs. Peterson's word for it that that is the progression of her class.

One other disconnect I noticed was between what Mrs. Peterson was saying in her interview, and the class observation. In the observation, Mrs. Peterson was relying a lot on number sense and sense making activities to help the students estimate and picture amounts in their heads. This could result from the fact that this was the opening day of a unit, so she started off with the concept and would then have the students memorize the procedure. This unit is also not based around basic math facts, and is number sense based, which also could have been the reason for this disconnect.

Mrs. Peterson believes that practice and memorization are the most efficient ways to learn and remember basic math facts. In the classroom observation, Mrs. Peterson relied heavily on visuals to help the students learn about estimation. Mrs. Peterson illustrates an example of a teacher who is aware of multiple strategies, or a strategy continuum, but expresses beliefs that promote memorization and practice as the most efficient way to learn basic facts. Mrs. Peterson demonstrates number sense related activities in her classroom observation, which seems to go against some of the statements made in her interview; however, since this was only one classroom observation, no overarching generalizations can be made. From this case study, Mrs. Peterson illustrates a teacher who believes strongly in the ideas of practice and multiplication but shows some aspects of number sense in her classroom, whether or not she is aware of the fact that she is doing so.

Orangebrook School 3rd Grade Cohort: Number Sense is About How Students Know and Understand Numbers, But Knowing Basic Facts From Memory Helps Students Complete Harder Problems

Three 3rd grade teachers from Orangebrook Elementary School chose to participate in this survey together, and completed it via Google Docs. These teachers are relatively new, with anywhere between 1-5 years of experience. Although this data is different from Ms. Lewis or Mrs. Peterson, and though I did not get to conduct observations, I was still able to learn about different strategies these teachers used to teach their students and beliefs they had about timed tests and number sense. These teachers believe that knowing basic facts from memory, through memorization, will help ease students frustration with math in the future. They seemed to focus on activities such as timed tests to assess their students' understanding and help them prepare for their future mathematics endeavors. However, these teachers also share their beliefs about the importance of students creating their own strategies to solve problems and being able to understand numbers.

I was unable to complete any observations of their classrooms because of time restraints, but their thoughts are still important to this study. These teachers had unlimited time to consider their answers and respond to the questions as a group. Since these teachers had time to think, their answers included ideas that other teachers might believe but could not necessarily think of during the interview. Some of the Orangebrook teachers' beliefs can be seen throughout the classroom observations that were conducted in other schools.

The Orangebrook Elementary School third grade teachers believe that multiplication should be taught in a strategic order to help students learn strategies to solve facts quickly. These teachers teach the facts in this order: 0s, 1s, 2s, 4s, 8s, 3s, 6s, 7s, 5s, 10s, 9s. They start with the 0 multiplication facts because the rule is the easiest to teach students: "any factor times zero is 0." Then the 1 times tables are easy as well because the students can use the Identity Property to show that any factor times 1 is itself. In order to teach 2s, these teachers say that the students can either skip count by twos, which most students are highly capable of, or double the other factor. After mastering the twos, the teachers move on to showing that with your twos you can learn your fours and then with your fours you can learn your eights. This is a logical way to go about showing the students how to multiply by four and eight because it involves building off what they already know. After mastering those, these teachers move on to teaching threes because that can also build off twos. For multiplying by three, just use your two facts plus one more group. After the students know their three facts, they can figure out the sixes, and so on. These teachers orient their lessons to teach the students these facts in a specific order to help develop the student's strategy skills. Also, by teaching multiplication facts in this way, the teachers are fostering their students' sense of numbers, by showing that multiplying by four is just the same thing as multiplying by 2 twice.

Some of the ways that these teachers help their students practice multiplication is with timed tests and activities. For the timed tests, once a student "passes" a level, they move on to the next factor. This was not explicitly stated, but one would assume that these students take the multiplication tests in the order in which they are taught, starting with zero and ending with 9. Once all of the levels have been completed, these students move on to a text with a mixture of all the multiplication problems. When asked how they assess whether students are learning basic facts and how they identify fluency with basic facts in their students, the only answers were "through timed fact tests and through observations" (Orangebrook Elementary School teachers, personal communication, June 2017). These teachers seem to rely heavily on timed fact tests which would seem to show that they also believe that fluency means being able to complete problems quickly and accurately. These teachers also talked a little about the games they play to help children practice multiplication. One of the games they described playing was called "Around the World," where the students compete against each other. One student stands up, next to another student's desk, and the teacher shows a flash card. Whoever gets the right answer faster, wins that round and moved on to stand next to another desk and compete with another student. This game also seems to rely on speed and accuracy. These teachers encourage students and parents to practice flash cards each night and to make piles of "I know it by heart" and “I need to keep practicing.” Encouraging students to use flash cards is yet another example of these teachers encouraging quick retrieval of facts.

These teachers believe that knowing basic facts from memory helps students complete harder problems more easily, while also reducing frustration level. According to their interview, these teachers believe that with addition and subtraction, most students can get by without having the basic facts all memorized. They believe that many students can count on their fingers for these skills and it ends up being manageable. However, these teachers also believe that “memorizing multiplication facts is crucial for success in upper grades” (Orangebrook Elementary School teachers, personal communication, June 2017). They believe that multiplication facts help you to quickly identify least common multiples and greatest common factors – both of which are needed for fractions. These facts also help students find common denominators when adding, subtracting or reducing fractions. These are all complex-concepts and can be made even more challenging when students must do extra work just to find products, common multiples, and common factors. One teacher described her work with middle schools students – some who have their facts memorized and others who have not and she explains that the difference of success is significant. She also said that she notices a significant difference in overall attitude and frustration level because of the extra time and effort they need to put into each problem if their facts are not memorized.

These teachers believe that number sense is all about how well students know and understand numbers, and how well they can break those numbers apart. These teachers also believe that number sense is how well they understand numbers mean and where they are relative to each other. These teachers found that games are great ways to help improve number sense. They said, “The more students come up with their own strategies and have more exposure and practice with these skills, the more they [the skills] stick and the more comfortable they become with these skills" (Orangebrook Elementary School teachers, personal communication, June 2017). The teachers also said that the more students discuss and explain their strategies to and with each other, the more they learn what others are thinking, and the more they develop this number sense. These teachers believe that memorization is important but that number sense goes being just memorizing facts. They claim that a student who has only memorized their facts wills struggle when the problems and concepts go beyond just multiplying numbers together (e.g. word problems). For example, "a student may be able to tell you what 4x6 is and what 8x3 is independent of each other, but may be unable to name 2 factors of 24 from memory" (Orangebrook Elementary School teachers, personal communication, June 2017).

The Orangebrook Elementary School third grade teachers' believe that knowing basic facts from memory can help students' complete harder problems more easily, while reducing frustration level. This idea was a very important finding in my research study. These teachers explained that they relied more heavily on timed tests to assess student learning but expressed that they taught multiplication in a strategic way to promote the best learning environment and believed that students should come up with their own strategies to optimize learning opportunities. Even without a classroom observation, there is a misalignment with these beliefs that should reflected upon to improve the alignment of their beliefs with their practices as well as every teachers' ability to teach their students effectively.

Mrs. Smith – Number Sense is Useful but Quick Retrieval is Necessary

Mrs. Smith is a fourth grade teacher working at Brookdale Elementary School. Mrs. Smith has taught in fourth grade for only about five years. Since this interview took place, Mrs. Smith has moved down to second grade because she felt too restricted by the intense standards required in fourth grade. I have not been in contact with her since the move down to second grade.

Games can help students develop number sense and fact retrieval. Mrs. Smith gives fact tests two days a week, usually Tuesday and Thursday. Before the individual tests are given out, Mrs. Smith partners her students up by ability and has them play one of two different games. One of the games that her students are allowed to play is called math wars. During this game, the students have a set of cards and each of the students turn over two cards and multiply, and whoever has the larger answer wins. If the students end up getting the same answer, they actually have a math war. For example, if one student turns over a King (12) and an Ace (1) their answer would be 12x1 = 12 and if the student they are playing with turns over a 7 and a 4, their answer would be 7x4 = 28 and the second student would win that game of war. This game involves memorization and number sense. The memorization part of this game comes in when the students have to quickly solve basic facts in their heads. The number sense aspect is the fact that the students are able to see which numbers multiply to the same number. For example, if two students got a 4 times a 3 and a 6 times a 2, they would be able to see that both of those multiplication facts multiply to twelve. Then, they would have a math war. Mrs. Smith explains that the students do math wars until about January, and then they switch to the factor game. For this game, the students hold a card to their head and their partner must say all of the factors of that number until the person with the card guesses what number it is. This game is a little more challenging, but develops number sense because factors are not what the students are used to seeing. The idea of factors is much less concrete to students so it is important to have them practicing factors while also reviewing for their fact test.

Mrs. Smith keeps track of the fact tests on a chart on the back wall. Each student gets a sticker in the section they complete once they have gotten a 100% on that section. The fact tests start off with Mrs. Smith giving the students 30 problems in 5 minutes. If they get a 100% on this test, they will get 30 again next time in 5 minutes. Once they have two 100s, they move up to 40 problems in 5 minutes, and after that they have 50 problems in 5 minutes all the way up to 100 problems in 5 minutes. After they have mastered 100 problems in 5 minutes, the students can move on to division and follow the same pattern. Since the students are always on different tests, the chart on the back wall is to keep track and also to see how each students is doing individually. The organization of these tests is scaffolded so that the students continually challenge themselves.

During the classroom observation, I was able to see the students play the factor game. It was interesting to see the students participate in this activity. Mrs. Smith walked around during this activity to make sure that all the students were on task and to help out any students who were struggling coming up with factors to tell their partner. With a number like 5, there are only 2 factors to say, and if someone's partner does not guess it with those two factors there is not much more to say. So, Mrs. Smith has to walk around and monitor the progress of each student. Mrs. Smith also had to explain to one student why the factor game was helping the students review for their factor test. She explained that factors are all the numbers that can be multiplied together in order to get the number you are looking for. For example, the factors of 12 are 1 and 12, 3 and 4, and 2 and 6 because those numbers multiplied together all get 12.

Daily practice of quick recall is important. Mrs. Smith believes that her students should be practicing math facts every day so at the end of the day, the students have to line up to leave the classroom and she will fire a math fact at them. If they get it wrong, they have to go to the back of the line and try again. Mrs. Smith explained that she used to go through their fact tests and pick out the problems they were struggling with and fire those at them. However, she explained that she does not really have time to do that for every student anymore so she just picks the ones that a lot of students struggle with. This activity is very much memorization based. The students are supposed to respond quickly and accurately before leaving the classroom. This is a similar definition to what some of the other teachers have said for fluency with basic math facts. This may indicate that Mrs. Smith also believes that fluency is responding quickly and accurately.

Development of fluency requires constant reinforcement of basic skills. Mrs. Smith works to identify fluency within her students by including multiplication and division problems on every test given to the students. She said that she includes 3 by 1 digit or 3 by 2 digit problems on every test so that her students are continually working on the skill. She also explained an activity that she developed which she described as a fun multiplication game. The game is called Zapp. These cards are not solely basic facts, they are either 2 digit by 2 digit, or 2 digit by 3 digit. The students play in groups of four and they have a white board and markers. They are also provided with a calculator. What they are supposed to do is, the first person would turn over a card and that is the problem. Then, all of the students solve the problem. Mrs. Smith explained that she wants every student to solve the problem because she does not like anyone sitting there doing nothing. If they all have the same answer, then the student who chose the card was right and he or she gets a point. If a student does not have the answer correct, all of the students have to work to find the mistake and whoever finds the mistake and explains it to the student who made it gets a point. There are Zapp cards in the deck, and if a student pulls a Zapp card they lose all their points. This fun game helps the students practice basic facts within harder problems and also helps the students be able to find their mistake or other's mistakes. Being able to identify where a mistake is helps students develop their number sense because it helps them deconstruct numbers and figure out what exactly makes an answer correct.

Basic facts are important for all other concepts. Mrs. Smith believes that basic facts lead to everything else and not knowing them impedes learning all other concepts. She explained that there was a girl in her class that was struggling with conversions of gallons, pints, quarts, and ounces because she knew the steps but could not remember her basic facts. She ended up letting the little girl use a calculator because she really needed the little girl to learn how to convert and the math facts would not just suddenly come to her. Mrs. Smith also brought up the fact that in your life, outside of school, when you are cooking and you are doubling or tripling the recipe you need to know how to multiply. Mrs. Smith is hinting at the fact that basic facts influence and lead to everything else in a student's math career and life. Mrs. Smith also explains that she can see how frustrated this little girl from her class is, but there is no time to stop the lesson and help her learn the facts. She says, "I think that will also cause a fear of math in her future, people hate math because they don't understand it, so she might be like 'I'm not good at math, so I'm not even going to try'" (G. Smith, personal communication, June 9, 2017). However, Mrs. Smith also explained that this little girl understood the concepts and what she had to do, her basic facts just were not there. This is an interesting point that could raise a debate about memorization vs. number sense, because even though Mrs. Smith is explaining that this little girl knew the concepts, she could not complete the math because she had no strategies and no memorized facts to recall.

Mrs. Smith believes that number sense is when students can look at numbers and break them apart. Mrs. Smith describes how every once in a while she will give her students something like 12 times 4 and they will respond with "We are not supposed to know how to do that!" This is a great example of students believing that they should know "have to" know something from memory, when in reality, Mrs. Smith was asking them to figure it out – something they can do. She explained that when this comment arises she asks the students to think about the question before jumping to not being able to do it. She explains that she mostly only has her higher kids work on these types of questions because she likes to challenge them. Mrs. Smith explains how she wants to encourage the students to find their own strategies to break the numbers apart and use their number sense to figure out the solution.

One strategy she explains to teach her students subtraction for a problem like 8000-6422 when you have to regroup is to just take one away from each. So for this example, you would take one away from 8000 and have 7999 and then you do not have to regroup. However, she also explains that some of her students cannot visualize taking one away from 8000. Some of her students say that 8000 take away one is 7000, and she explains that she asked them to take away one not 1000. She explains that she lays a lot out for them so that they are able to see what they are supposed to know on paper and eventually be able to picture that in their heads. This comment hinted at the fact that she also believes number sense is the ability to picture numbers and strategies in your head. She also explains how she always incorporates visualizing into her classroom because drawing pictures and visualizing helps the students to remember and understand.

In the classroom observation, I saw an example of Mrs. Smith helping her students visualize. There was a problem presented on the board that said "7 $\frac{1}{2}$ pounds = ______ ounces?" She decided to teach them a trick because a lot of students get discouraged when they see fractions involved in anything. She said, first, why don't you try to figure out how many ounces are in 7 pounds. The students have a conversion chart and they are able to see that there are 16 ounces in one pound, so to find out how many are in 7 pounds they should be multiplying. Mrs. Smith wrote this problem out on the board. She first wrote the original problem, and then followed it up with the first step that she explained. One could infer that Mrs. Smith wanted the students to hear the directions but also be able to see the problems step by step. After asking them to find out what 16 times 7 was, she asked the students to find out what half of 16 was. She explained that she wanted them to find half of 16 because that was the other part of the number that they were trying to convert to ounces. She expected the students to know what half of 16 was, which they did. After these two steps, she explained that if you add those two numbers together you can get what 7 and a half pounds equals in ounces. The students seemed to understand that because you are looking for 7 and a half pounds into ounces, you are allowed to find out how many ounces are in 7 pounds and then also find out how many ounces are in half of a pound and just add them together. After this example, Mrs. Smith provided the students with a couple more examples to have them practice this skill and hopefully be able to picture it in their head instead of having to do it in three lengthy steps.

It is great if the students can see the concept of multiplication, but they need to "get it done" (G. Smith, personal communication, June 9, 2017). Mrs. Smith believes that students must know their basic facts to succeed. She also believes that it is important for students to get the number sense so that they can eventually see that, for example, 6 times 8, is 8 six times. She said, "I think it is good that they see that, but if they don't, quite frankly they have just got to get it done (G. Smith, personal communication, June 9, 2017)." She explains her thinking behind this statement a little bit. She says that information is moving so quickly so they really need the foundation of memorizing basic facts in order to succeed with all of the other material. Mrs.

Smith teaches number sense, or tries her best to teach number sense, but will not slow down her lessons to help students grasp the concepts of multiplication and division because if they do not get it the first couple of times, it is just important that they are able to access the facts quickly for the future.

Mrs. Smith believes that number sense is a useful tool for students to have but quick retrieval is necessary. This case study shows evidence of a teacher believing that number sense is important, but that quick retrieval, with or without number sense, is imperative. Her beliefs are shown in her classroom when she shows the students why a certain topic is true, but also asks them to just make sure they know how to complete the problem. Mrs. Smith represents a teacher that is aware of a misalignment but expects one because sometimes students need to just “get it done” (G. Smith, personal communication, June 9, 2017). Teachers could benefit from having ideal goals as well as realistic ones in order to strive for the best possible environment for students while also making sure that they are prepared for their futures.

Many of these teachers believe that number sense is an important concept if there is time, but knowing basic facts from memory is imperative in students’ future mathematical success. These teachers explain that it would be “nice” if students understood what multiplication meant, but in the end, the focus needs to be on having the students memorize basic math facts and know them from memory quickly. Many teachers expressed their beliefs on the importance of students’ conceptual understanding and number sense, but admitted, and also demonstrated in the classroom observations, a focus on memorization related activities and practices. The misalignment between beliefs and practices varied throughout these case studies, but overall showed the importance of self-reflection as a teacher. It also showed that there should be more emphasis put on aligning beliefs and practices in order to optimize teaching for every student.

From these interviews and classroom observations, time and the curriculum seem to be the biggest factors that influenced the misalignment between beliefs and practices. Teachers expressed annoyance with how much they must fit into a year of school, because it impacts their ability to help foster number sense in their students. In classroom observations, I saw first hand that math lessons went by very quickly, and it was hard to fit in all of the information, let alone number sense activities. By examining teachers' beliefs, comparing them to practices, and examining the different reasons for this misalignment, more information can be provided and used to try to attempt to close the gap and optimize education for all students.

Chapter 5: Conclusion

This research project adds to the existing evidence that teachers' beliefs do not necessarily line up with their practices in the classroom (Cohen, 1990). This was an important research topic because it brought to light something that most teachers may not realize. A first step for teachers to improve their teaching is to recognize differences between how they believe they are teaching and how they are actually teaching, which may not be as aligned as they may think. By understanding this, teachers can be more open to reflecting on their teaching to help improve their practices.

For example, Mrs. Lewis believes fluency is more than just accuracy and speed, but assesses her students with timed tests that rely mostly on those two factors with no information on how the students got their answers. Mrs. Lewis expresses misgivings about timed tests as an assessment of student learning but continually uses them for any number of different reasons. This misalignment between beliefs and practices can be attributed to lack of time, routine, and many other factors that might influence it. Mrs. Lewis seems slightly aware of the fact that there is some misalignment but did not seem as if she was doing much to try to alter her assessments. Another example is Mrs. Peterson. She believes that memorization is the most efficient way to learn basic facts. She is aware of the fact that teachers can teach with a strategy continuum, but believes that memorization is the best way. Her classroom seemed to be focused mostly on English and Language Arts studies, so I do not know how much time is usually dedicated to math on an every day basis. However, if she focuses mostly on English, time could be a concern and promote the use of memorization strategies over number sense ones.

The third grade teachers from Orangebrook Elementary School believe that timed tests and practice are the best ways to promote the learning of basic facts in their students. However, they also explained that they teach multiplication in a strategic order that even if they are not fully aware of it, promotes number sense as well as memorization. These teachers also believe that helping the students learn basic math facts will help them in their future math careers and will lower their frustrations level, which might be a factor that influences a focus on memorization. Lastly, Mrs. Smith shares her belief that number sense is useful, but quick retrieval is necessary. Since she believes that quick retrieval is necessary, she focuses on memorization related activities such as timed tests. Mrs. Smith focuses on daily practice of recall and games that promote memorization, even though she also believes that number sense is important. With reflection, her number sense beliefs would be able to come out more in her classroom and close the gap between her beliefs and practices.

In order to investigate this topic, I read literary sources, conducted interviews with elementary school teachers, and observed those teachers' classrooms to understand the connection between what they believed they were doing in their classroom and what they were actually doing. The purpose of this research objective was to investigate if there was a disparity between beliefs and practices and to understand how and where the disparity between beliefs and practices came from. Through interviews and classroom observations, I have identified that there is a disparity between beliefs and practices and that most teachers, even very good ones, can benefit from self-assessment of their teaching.

This research yields important findings about the ways that elementary teachers think about student learning and about the influences that lead them to teach in ways that are not entirely aligned with their beliefs. My research combined many different sources from scholarly journals, to articles written by teachers, and the eclectic group of sources should provide a relatively unbiased view of the different teaching techniques as well as teachers' own views on their teaching. I hope that this research can help teachers improve their own teaching by understanding that self-assessment is key to improvement.

This is an important topic to research because teaching can only be improved through reflection. Teachers must understand what they believe about education and compare that to the lessons they are producing in the classroom in order to bring those two ideals closer together. In this paper, you saw evidence from literature review as well as from my personal research that show that teachers sometimes overlook whether or not their ideas about education are showing up in their classroom. This project was created to help raise awareness to current and future teachers about the significance of teacher self-reflection and of understanding that sometimes our beliefs do not transition well into our practice. By understanding this, teachers will be one step closer to improving their instruction to better help their students.

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