EXTRACTION AND LOCALIZATION OF NOISE-RELATED FLOW STRUCTURES IN HIGH SPEED JETS

Pinqing Kan
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ABSTRACT

The jet noise problem, or the noise radiation from high-speed flow interactions, still remains unresolved after over sixty years of research. With the growing aviation industry, more communities are exposed to noise pollution from aircrafts; and better control strategies are needed to meet the more stringent noise regulations.

This thesis analyzes two sets of high-speed jet flow data and aims at better understanding the flow dynamics and noise radiation mechanism. The first set of data consists of subsonic / transonic axisymmetric jets and the second dataset is of a more advanced configuration: a two-stream supersonic rectangular jet with a flat plate extending from the nozzle exit. For both flow configurations, the analysis consists of two parts. Firstly, the flow and acoustic features will be analyzed using statistical and continuous wavelet techniques. With the subsonic case, several diagnostic signals are constructed and the flow regions related to noise radiation are identified. With the supersonic case, the near-field flow structures are categorized and their frequency-specific propagation pathways and interaction patterns are depicted. The second part of the analysis involves devising algorithms to extract noise-related events, analyzing the event features and looking for event-related flow structures. The algorithms combine multi-correlations, continuous wavelet and pattern recognitions and are able to identify noise-related events with fewer than 10\% false matches. The algorithms are tested rigorously using several approaches and the extracted events exhibit features consistent with existing theories on noise sources.
EXTRACTION AND LOCALIZATION OF NOISE-RELATED FLOW STRUCTURES IN HIGH SPEED JETS

by

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4.49 The cross-correlations between the velocity signals along the Inner SL (their locations are indicated by the cyan markers in the left plot) and the pressure signal along the acoustic propagation path (the red marker).
Nomenclature

\( X, x \) = the streamwise direction
\( \Delta x \) = the grid size in streamwise direction
\( r \) = the radial direction
\( \theta \) = the azimuthal direction
\( Y, y \) = the transverse direction
\( Z, z \) = the spanwise direction
\( V \) = volume
\( t \) = the time axis
\( \Delta t \) = the time resolution
\( t' \) = translation in time
\( s \) = the scale / dilation
\( \lambda \) = the wavelength
\( f \) = the frequency axis
\( \Delta f \) = the frequency resolution
\( St_{1}, St_{3} \) = non-dimensionalized Strouhal number using main jet / wall jet parameters
\( \tau \) = the time lag between two signals
\( \tau_0 \) = the best mean lag of peak correlation
\( \Delta \tau_0 \) = the time extent between correlation peak and adjacent zero-crossing
\( \Delta \tau \) = the difference between time lags
\( S_0 \) = the sign of the peak correlation
\( T_0 \) = the local period
\( D \) = diameter of nozzle (circular) exit
\( D_h \) = hydraulic diameter of rectangular nozzle
\( D_p \) = thickness of the splitter plate (supersonic jet)
\( H \) = height of the nozzle exit
\( W \) = width of the nozzle exit
\( \epsilon \) = angle of shear layer expansion in subsonic jet
\( \beta \) = the observation angle measured from the jet centerline
\( \phi \) = the Mach angle
\( \rho \) = density (of air)
\( T \) = temperature
\( S \) = entropy
\( Ma \) = Mach number
\( Ma_1 \) = Mach number of the core jet
\( Ma_3 \) = Mach number of the wall jet
\( M_c \) = Mach number of the convected eddies
\( c_0 \) = the speed of sound at the nozzle exit
\( c_a \) = the speed of sound at the ambient for supersonic jet
\( U_c \) = convection velocity of turbulence eddies
\( U_j \) = speed of the jet stream
\( U_x \) = the characteristic velocity derivative
\( U, u \) = streamwise velocity
\( V, v \) = radial / transverse velocity
\( W, w \) = spanwise velocity
\( \omega \) = vorticity
\( \Omega \) = vorticity tensor
\( div \) = velocity divergence
\( s_{ij} \) = rate of strain of velocity
\( S \) = rate of strain tensor
\( det \) = 2D determinant of velocity strain rate
\( Q \) = Q criterion
\( p \) = pressure; also used as a generic signal
\( P \) = the acoustic power per mass
\( T_{ij} \) = the Lighthill stress tensor
\( \alpha \) = the Proudman constant
\( \bar{L} \) = the Powell-Howe Lamb vector
\( \bar{p} \) = vector p
\( \bar{p} \) = non-dimensionalized quantity p
\( p' \) = fluctuating quantity p (mean removed)
\( p^* \) = the complex conjugate of \( p \)
\(|p|\) = the absolute value of quantity \( p \)
\( B(p) \) = the probability density function of signal \( p \)
\( \bar{p} \) = the mean value of quantity \( p \)
\( \sigma_p \) = the RMS value of the signal \( p \)
\( \hat{p} \) = normalized quantity \( p \) (mean removed; divided by \( L^N \) norm)
\( \|p\| \) = Euclidean norm of matrix \( p \)
\( \|p\|_N \) = \( L^N \)-norm of signal \( p \) (\( N = 2 \) is the Euclidean norm)
\( \tilde{p} \) = the wavelet transform (wavelet coefficients) of \( p \)
\( \tilde{p}_2 \) = the Mexican hat wavelet transform of \( p \)
\( \tilde{p}_M \) = the Morlet wavelet transform of \( p \)
\( \psi \) = wavelet basis function or mother wavelet
\( C_\psi \) = the admissible constant for wavelet transform
\( \psi_2 \) = Mexican hat wavelet basis function
\( \psi_{M,z_0} \) = Morlet wavelet basis function
\( z_0 \) = the envelope factor for Morlet wavelet
\( \mathcal{R} \) = the real part of a complex value
\( \mathcal{F} \) = the Fourier transform of a signal
\( \mathcal{F}^{-1} \) = the inverse Fourier transform
\( \mathcal{F}_s \) = the short-time Fourier transform of a signal
\( \mathcal{W} \) = window function
\( \mathcal{G} \) = Gaussian filter of a signal
\( E_p \) = the energy of signal \( p \)
\( e_f \) = the Fourier spectrum of a signal
\( e_c \) = the compensated Fourier spectrum of a signal
\( E_\psi \) = wavelet spectrum
\( E_2 \) = compensated Mexican hat spectrum of a signal
\( E_M \) = compensated Morlet spectrum of a signal
\( \bar{E}_M \) = normalized compensated Morlet spectrum
\( \mathcal{X} \) = the cross-correlation between two signals
\( \Xi \) = the frequency-resolved cross-correlation

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\( \xi \) = the cross-correlation coefficient in time-frequency domain
\( C_\xi \) = criterion on cross-correlation coefficient
\( C_M \) = criterion on Morlet wavelet coefficient
\( C_c \) = criterion on cross-correlation between local excerpts
\( C_x \) = criterion on multi-correlation between local excerpts
\( D, M \) = the local excerpt of D / M signal
\( N_T \) = number of periods
\( rand \) = a randomly generated time series
Abreviations

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<thead>
<tr>
<th>Abbr</th>
<th>Description</th>
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<tbody>
<tr>
<td>BSAN</td>
<td>Broadband shock associated noise</td>
</tr>
<tr>
<td>D signal</td>
<td>the Diagnostic signal</td>
</tr>
<tr>
<td>FF</td>
<td>Far-Field</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>Filtered D signal</td>
<td>the Space-Filtered Diagnostic signal</td>
</tr>
<tr>
<td>FPER</td>
<td>the false-positive-to-event ratio</td>
</tr>
<tr>
<td>K signal</td>
<td>the down-sampled Kulite (pressure) signal</td>
</tr>
<tr>
<td>LES</td>
<td>Large Eddy Simulation</td>
</tr>
<tr>
<td>LSE</td>
<td>Linear stochastic estimation</td>
</tr>
<tr>
<td>M signal</td>
<td>the down-sampled Microphone (pressure) signal</td>
</tr>
<tr>
<td>NF</td>
<td>Near-Field</td>
</tr>
<tr>
<td>NPR</td>
<td>Nozzle pressure ratio (between total and ambient pressure)</td>
</tr>
<tr>
<td>OASPL</td>
<td>Overall Sound Pressure Level</td>
</tr>
<tr>
<td>POD</td>
<td>Proper orthogonal decomposition</td>
</tr>
<tr>
<td>Raw K signal</td>
<td>the original Kulite (pressure) signal</td>
</tr>
<tr>
<td>Raw M signal</td>
<td>the original Microphone (pressure) signal</td>
</tr>
<tr>
<td>SL</td>
<td>Shear Layer</td>
</tr>
<tr>
<td>SNR</td>
<td>the signal-to-noise ratio</td>
</tr>
<tr>
<td>STFT</td>
<td>short-time Fourier transform</td>
</tr>
<tr>
<td>UV signal</td>
<td>the original streamwise (U) and radial velocity (V)</td>
</tr>
<tr>
<td>WGN</td>
<td>White Gaussian noise</td>
</tr>
<tr>
<td>XY0</td>
<td>the plane of symmetry at $Z = 0$</td>
</tr>
<tr>
<td>XZ0</td>
<td>the horizontal plane at $Y = 0$</td>
</tr>
<tr>
<td>YZ1, 2, 4, 8</td>
<td>the transverse planes at $X/D = 1, 2, 4, 8$</td>
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Chapter 1

Jet Noise Research

1.1 Background and Motivation

Aeroacoustics, of which jet noise research is a part, started as a branch of fluid dynamics that studies the noise generated aerodynamically. The first research relating the acoustics with jet flows can be traced back to the work of Sondhauss, who described the sound produced by the interaction between jet flow and a solid edge in 1854, and the analysis by Rayleigh, who discussed how sound waves excited the jet flows and enhanced turbulent mixing [25, 108]. After World War I, aircraft noise became of interest due to the need for locating airplanes by their noise emission. Around the end of the World War II, turbojet engines became operational and the first commercial jet-powered aircraft came into service in 1952. The prospect of large scale aircraft for both military and civilian transportation and their entailed noise problem led to publication of the renowned Lighthill’s paper [78], which laid the theoretical foundation
of aeroacoustics as an independent discipline.

Aircraft noise is a concern for both military and civilian transportation. For military aircraft, noise reduction is desired for stealth operation and surveillance. Exposure to high noise level in a sustained period of time can lead to permanent hearing loss and thus jet noise is especially adverse for aircrews and aircraft carrier personnel [6].

Aircraft noise is also problematic for airport neighboring communities. In the 1970s, supersonic commercial airliners were developed but later terminated as they exceeded the noise regulations [22]. Progress was made in the 1980s, when engines with higher bypass ratio led to considerable noise reduction [121]. However, further increase in bypass ratio may decrease fuel efficiency and increase fan noise [98]. Moreover, with the pursuit towards higher speed for tactical and commercial jetliners, the sonic fatigue on aircraft structures, i.e., the material crack or failure caused by sound wave fluctuations, is becoming a bigger threat. Nowadays, more stringent noise regulations are put forward with the growing aviation industry and the expansion of communities [82]. The need is increasingly pressing for better understanding the aircraft noise sources and devising better control strategies.

Among the various sources of aircraft noise, the engine exhaust is the most dominant during take-off and fly-over and it is as important a source as the airframe at landing condition [21]. Thus extensive research has been conducted focusing on jet noise and advanced nozzle configurations. Jet noise is also of interest in research on combustion and rocket engines. The various approaches towards resolving the jet noise problem can be roughly sorted into three categories. The first category, the acoustic analogy, was
founded on Lighthill’s paper in 1952 [78], and it involves rearranging the Navier-Stokes equations and separating the wave propagator from the source term. The second category or the experimental approach was mostly to collect datasets and to investigate the influencing parameters of the radiated noise in the 1950s. The 1970s and the 1980s saw various sound measurements conducted in anechoic chambers and the technology development enabled the high resolution flow visualization in the 1990s, which was further facilitated with time resolution in the twentieth century. The resolved acoustic field through numerical simulations (the third category) was largely hindered by the large difference in the flow and acoustic field energy. The computational development enabled the direct acoustic computation or hybrid simulation approach in the 1990s. During these processes, the discovery of large coherent structures changed dramatically our view on jet noise [93] and the derivative wave packet modeling has shown great promise of linking the turbulence and the acoustics.

1.2 Major Theories on Jet Noise Mechanisms

1.2.1 Subsonic Jets

The noise mechanism theory on subsonic jet flows was founded by Lighthill’s acoustic analogy in 1952 [78]. Assuming that the non-steady fluctuating flow is constrained in a finite region, we can regard the acoustic field (far from the fluctuating flow) as an oscillation of very small amplitude about the uniform fluid at rest. The sound speed in this uniform medium is \( c_0 \). Also assuming that there is no external source of mass or
force, the mass and momentum conservation equations in the fluid can be expressed in
exact terms as

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} \rho u_i = 0, \quad \frac{\partial}{\partial t} \rho u_i + \frac{\partial}{\partial x_j} (\rho u_i u_j + p_{ij}) = 0 \]  

(1.1)

Here \( p_{ij} \) is the compressive stress tensor, which is in the \( x_i \) direction and applied across
a unit area. The inward normal of the surface area is in the \( x_j \) direction. Rearranging
the above equations, we have

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} \rho u_i = 0, \quad \frac{\partial}{\partial t} \rho u_i + c_0^2 \frac{\partial \rho}{\partial x_i} = - \frac{\partial T_{ij}}{\partial x_j}. \]  

(1.2)

Here \( T_{ij} \) is the Lighthill stress tensor defined as

\[ T_{ij} = \rho u_i u_j + p_{ij} - c_0^2 \rho \delta_{ij}. \]  

(1.3)

We can further simplify the equations by eliminating \( \rho u_i \) term and combine the two
equations:

\[ \left[ \frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2 \right] \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}. \]  

(1.4)

This is called the Lighthill’s inhomogeneous wave equation.

On the left hand side of Lighthill’s wave equation, we have the wave operator as in the
classic acoustics, while the term on the right hand side is interpreted as the spatial
distribution of acoustic sources. Suppose the source term is known or can be modeled,
we can use Green’s function to obtain the acoustic field. The integration of \( T_{ij} \) in space
is simplified by Proudman for low Mach high Reynolds number turbulent flow:

$$\rho(\vec{x}, t) = \frac{\rho_0}{4\pi c_0^4x} \int_V \frac{\partial^2 u_x^2}{\partial t^2} dV,$$

(1.5)

where $u_x$ is the velocity component in the $x$ direction. From a less quantitative analysis, or the dimensional analysis of the source term (integrated through space), Lighthill showed that the acoustic intensity is proportional to the eighth power of the flow velocity. This is confirmed by Proudman: $P = \alpha U^8 c_0^{-5} L^{-1}$, where $P$ stands for the acoustic power per mass and $\alpha$ is the Proudman constant [110]. It provides the accuracy check on experiments and simulations.

The Lighthill-Proudman theory has a relatively simple form, however, it does not distinguish the flow-acoustic interaction from the acoustic source. Thus various theories have been developed to account for this. The more widely accepted among these is the Lilley’s approach [79], which includes the influence of the mean flow (convection and mean-flow shear) as part of the wave propagator. However, its apparent potential for more complete representation is lost since simplification is required to interpret the now much more intricate equation. Another acoustic analogy approach started from Powell’s theory of vortex sound [107] and is extended by Howe [59]

$$\left[ \frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2 \right] \rho = \rho_0 \nabla \cdot \vec{L}.$$

(1.6)

Here $\vec{L}$ stands for the Powell-Howe Lamb vector: $\vec{L} = \vec{\omega} \times \vec{u} - T \nabla S$. It relates the sound production to vorticity ($\vec{\omega}$) and entropy inhomogeneity ($\nabla S$), with the second
term more significant for hot jets.

The majority of the established acoustic analogy theories were developed when the turbulent flow was regarded as isotropic and void of ordered structures (before 1970s). The acoustic source of the analogies is regarded as distributed point (compact) multipoles (quadrupole source for Lighthill-Proudman theory and dipole for Powell-Howe theory), which radiate to all quadrants. The convection by the mean flow and the refraction by turbulence help explain the directive sound radiation pattern.

However, the effect of refraction, scattering and absorption by turbulence is omitted in the analytical solutions. These effects are evaluated through experiments [3] or ray tracing [115, 39] and empirical models are developed to make up for their influence and produce the correct radiation pattern.

As the understanding of jet noise intertwines with that of turbulence, the discovery of large coherent structures changed the view on jet noise sources dramatically. The earliest research questioning the validity of regarding isotropic turbulence as the sole source dates back to the mid 1960s. From the acoustic spectra at specific frequency bands, Mollo-Christensen speculated that there might be two types of noise, one dominating at low frequencies and one at high frequencies [94]. Apart from the active frequency and directivity [94, 85], the existence of two distinctive noise mechanisms is also supported by various observations from the correlation patterns [40], the azimuthal components of velocity and pressure signals [92], the distributions of source locations [11], etc. The current consensus is that the noise sources of subsonic jets consist of both large coherent structures and isotropic turbulence.
The coherent structures are responsible for the peak of the acoustic spectra at aft-angles [94, 85] and it has been shown that the source coherence level is an important factor of noise radiation [91]. This noise component has been analyzed by various researchers and it is also the focus of this thesis. The general features of the coherent structures and their radiated noise have been confirmed by various research findings. The radiation from coherent structures has been found as highly intermittent [64] and highly directive [85]. The peak frequency of radiation has been narrowed down to low Strouhal number, especially around \( St = 0.2 \) to \( 0.3 \) [25, 12]. The radiation to aft-angles and low Strouhal is related to the centerline regions of the jet exit around the end of potential core [64, 12]. However, no universally accepted theory has yet been established as to the exact noise generation mechanism. Some researchers connected the noise radiation to the vortex movement: some investigated the noise generation from vortex pairing [34] while others stated that the vortex breakdown may be the contributing component [66]. Some researchers investigated the radiations from coherent structures through instability wave analysis or wave packet modeling. The nonlinear interaction between instability waves traveling at subsonic speed has been demonstrated to radiate low frequency noise with directivity [125], while the simulated results from wave packet models have shown good agreement with the radiation patterns [20]. More details about the research on coherent structures and wave-packets will be shown in later sections.
1.2.2 Supersonic Jets

To increase the speed of the jet stream, both convergent and convergent-divergent nozzles can be used. The flow condition outside the nozzle depends on the ratio between the total pressure of the fluid and that of the ambient atmosphere (NPR). For the convergent nozzles, the highest achievable velocity inside the nozzle occurs when the flow is choked at the nozzle exit. Further increasing the NPR means that the exit pressure is greater than the ambient pressure and under-expanded jets will form outside the nozzle. For a convergent-divergent nozzle, the choked state is achieved at the nozzle throat and the flow becomes supersonic downstream. Away from the design operating conditions, the exit flow will be over- or under-expanded and shock structures will be present in the jet stream.

Comparing with subsonic jet flows, new noise sources are introduced into the jet due to the presence of shock structures. These sources are categorized as shock-associated and there are two components, the broadband shock associated noise (BSAN) and the screech. The noise generation mechanism discussed in the subsonic section, namely, the turbulent mixing noise, also exists for supersonic jets. The turbulent mixing noise also has two components, the large coherent structures and isotropic turbulence. For subsonic jets, the convection speed of the coherent structures along the shear layers is subsonic: \( U_c \approx 70\% U_j \) [140, 97]. Here \( U_c \) stands for the convection velocity and \( U_j \) stands for the speed of jet stream. For supersonic jets, another noise component, the Mach wave radiation, takes place if the flow speed further increases such that the convection speed of the coherent structures becomes supersonic. In this section, a brief
overview will be provided on the existing theories on Mach wave radiation and the shock-associated noise.

Mach waves are radiated when the large-scale structures along the turbulent shear layer is convected at supersonic speeds. It has been found to be the most dominant noise component in the downstream direction, especially around the Mach angle

\[ \phi = \cos^{-1}(c_0/U_c). \]

Here \( \phi \) is the angle from the downstream axis and \( c_0 \) stands for the speed of sound at the ambient. This noise mechanism was first formulated by Phillips in 1960 [103]. Later Ffowcs Williams extended the Lighthill’s analogy to eddies with supersonic convection speed and derived that the radiation intensity scales as the cube of velocity [33]. The linear stability theory and wave packet modeling have been employed to understand this noise generation mechanism. Tam and Hu investigated the radiation of three different instability waves [131] and the Kelvin-Helmholtz (KH) instability is found as the governing mode for the wave packet dynamics and the radiation in the peak aft direction [120].

An intermittent and loud noise component known as the ‘crackle’ is sometimes present along with the Mach wave radiation and it is typically observed with full-scale heated jets. It was first identified by Ffowcs Williams et al. in 1975 and it is characterized by a shock-like wave front: a quick and strong pressure compression followed by a slow and weak expansion [35]. Due to the sawtooth shape of pressure, the skewness is often used as the identification criterion for crackle and it was suggested that the crackle is distinct for skewness larger than 0.4 and the jet is considered crackle free for skewness smaller than 0.3 [35].
The noise generation due to the presence of shock structures was first investigated by Powell in 1953 [105]. He investigated the noise now known as the ‘screech’ and traced this noise generation to the small disturbance at the nozzle exit and related it to the interaction between turbulence and shock waves. The screech is observed as very high amplitude peaks at discrete frequencies in the acoustic spectra. It may cause drastic fatigue failure of aircraft structures if its frequency matches the natural frequency of the structures. Several shock cell models have been developed and the prediction formula has been derived for the screech frequency [133], while the prediction for screech amplitude still remains elusive. The current theory regards the screech generation due to a feedback loop: a portion of the waves that are generated by the interaction between turbulence and shocks travels upstream; it gives rise to large scale disturbance at the nozzle exit; the reinforced turbulence structures interact with shocks and produce sound at discrete frequencies [105].

The waves moving upstream in a screeching jet are found to travel at the speed of sound and the primary peak frequency of the screech matches that of the broadband shock-associated noise (BSAN) in the upstream direction. Thus researchers have speculated that BSAN is the upstream portion of the feedback mechanism producing screech [133]. The interest in BSAN also results from the difficulty in its elimination: although various devices have been designed to eliminate screech, they often result in the enhancement of BSAN [124].

The BSAN, or the shock cell noise, is typically observed as multiple lobes (broad peaks) in the far-field acoustic spectra. Its generation is related to the interaction between the
large coherent structures along the shear layers and the shock structures. The BSAN radiates omni-directional, although it is more dominant for upstream observers since other noise components (e.g., Mach wave radiation) dominate in the downstream direction [128]. As the observation angle (measured from downstream direction) increases, the peak frequency decreases and the intensity increases [53]. The location of the BSAN source is found as the downstream regions of the shock cells [116]. Various models have been developed to predict the BSAN and most of them involve empirical formulas. One of the earliest models was developed by Harper-Bourne and Fisher in 1973 [53]. It was later extended and validated by Tam and Tanna [134] and it also shows good correspondence with the wave packet model developed by Suzuki [127]. The source model of BSAN consists of a line of correlated sources (waves radiate as the large structures pass each shock cell) and their interference produces the multi-lobe spectral pattern. The acoustic intensity is found to vary with the fully-expanded operating Mach number and the nozzle design Mach number; while the peak frequency formula is derived as

\[ f_{\text{peak}} = \frac{U_c}{L(1 - M_c \cos \beta)}. \] (1.7)

Here \( \beta \) stands for the observation angle and \( L \) is the separation between the sources in the model.
1.3 Coherent Structures and Wave Packets

The speculation that the turbulent flow is not completely chaotic, and that certain structures possibly exist can be found in the works of early to mid twentieth century. In the book by Townsend [141], which was published in 1956, he discussed the interaction and coalescence of some large energy-containing eddies that are repetitive and ordered. In the field of jet noise research, the coherent structures are often observed as a certain spatiotemporal structure, i.e., the wave packets. The existence of large structures in jet flows was first speculated by Mollo-Christensen in 1967 and he proposed a simple wave model for the noise source distribution [93]. In 1971, Crow and Champagne observed ordered wave forms in jet flows that were forced at low frequencies [25]. Since then, there have been numerous evidence confirming the existence of coherent structures in turbulent flows. The observation of the wave-packet form of near-field structures in jet flows and the recent development in wave packet modeling for jet noise sources have lead to a growing interest in wave packet analysis.

In an early paper by Yule et al [147], the characteristics of the coherent structures were listed and they included repetitive structures, coherence over distance much greater than the eddies’ length scale, birth-life-death cycle and being quasi deterministic. Currently, it is widely accepted that the large coherent structures have sizes comparable to the transverse length scale of the shear flow and negligible viscous effects. At the structure regions, there are concentrated regions of vorticity. Over the structures’ spatial extent, large-scale vorticity is instantaneously coherent. The criteria used for vortex detection are also used for coherent structure detection [144], such as Q criterion.
Coherent structures in the form of wave packets appear as intermittent ‘puffs’ that are convected with nearly constant velocity [25]. They are coherent in time and also over distances much larger than the integral scales of turbulent jets [63].

Arguably, coherent structures are regarded by many as the most dominant structures of turbulent shear flows. The interaction and amalgamation of coherent structures help provide better understanding to the mixing and entrainment processes [145, 17]. The control of the structures will lead to better control of the fluid mixing and is thus of great interest in the research of combustion, chemical processes, ejectors, etc. For jet flows, the coherent structures are found as the dominant structures in the noise-producing regions [14]. In subsonic jets, Michalke found that sources with larger coherence produce more noise at small observation angles [91], and a wave-packet ansatz has been used to predict this aft-angle noise by several researchers [20, 100]. In supersonic jets, Mach wave radiation is generated by coherent structures convected at supersonic speed [103] while the shock-associated noise is related to the interaction between coherent structures and shocks [134]. Wave packet modeling has been applied to simulate both Mach wave radiation and BSAN and the resulting acoustic features are consistent with experimental observations [120, 127].

To better capture the coherent structures and to yield more efficient control strategies, researchers have applied different kinds of analysis techniques. Here we give a brief overview of the techniques, emphasizing on those related to jet noise research and wave-packet analysis:

- Analytical approach:
- Some earlier research use double or triple decomposition to analyze the coherent component separately [62].

- Large coherent structures could be mathematically represented by instability models and stability analysis has been applied extensively to axisymmetric round jets [131, 96].

- Some earlier research on source modeling used the noncompact wavy line-source and it is able to capture the directivity pattern of jet noise radiation [25, 90]. For coherent structures convecting at subsonic speeds, a homogeneous amplitude can only produce a evanescent wave field, whereas a varied amplitude that grows and decays is able to radiates to the far-field [34, 112]. The radiation intensity is influenced by the modulation of the wave packets [24] which are temporally localized and intermittent [20, 69]. For wave packets with supersonic phase velocity, some researchers used the wavy wall analogy and strong propagating waves are emitted [129].

- Large coherent structures are implicit in the conventional statistical approach of turbulence research. Two similarity spectra were collapsed using far-field pressure of subsonic jets, which supported the two-source mechanism of jet noise, i.e. large structure and fine-scale turbulence [1, 130]. Some researchers inferred the wave-packet structure based on the correlation patterns of near-field pressure [93, 40]. The observations based on the directivity and intermittency also support the existence of large structures.
Some techniques share the general idea of decomposing the signals into various modes. Michalke and Fuchs applied azimuthal decomposition to subsonic round jets and they found sources with high azimuthal coherence are the most efficient [92]. The dominance of lower-order Fourier modes is confirmed by other researchers [4, 19]. Other researchers use modal decomposition (such as proper orthogonal decomposition or POD) for low-order representation of the flow field and look for the flow patterns related to noise production [56]. The vortex breakdown and the substructure interaction of a subsonic jet were investigated by Violato and Scarano by inspecting the POD modes of flow quantities [143].

Since coherent structures are characterized by a birth-life-death cycle and are quasi deterministic, researchers have used different techniques to extract and enhance the structures of interest:

- Conditional averaging has been used to extract the wave patterns related to noise production by several researchers and most of them assign a certain threshold on pressure magnitude [64, 47, 56]. This will be discussed in more details in the Event Detection section.
- The preferred mode or active frequency can be found through experiments with controlled excitation or through numerical simulations with initially organized modes [88]. The wave packet form and the vortex development have been visualized by several researchers in the near-field jets [25, 34].
- Linear stochastic estimation (LSE) is relatively more objective than the
previous techniques and has been used to educe the wave forms associated to pressure signals [104, 68].

Looking at the signals in time-frequency domain (e.g. wavelet analysis) is a relatively new technique (see wavelet section for more details). Srinivas et al. used two-dimensional wavelet to depict the coherent structures of jet flow within a life cycle [122]. Cavalieri et al. investigated the noise production parameters of the wave-packet ansatz using wavelet and other techniques [20].

1.4 Advanced Configurations for Noise Control

The ultimate goal of jet noise research is to devise better control strategies so as to modify the flow structures and reduce flow-induced noise. Flow control strategies are typically categorized as passive and active control. Passive control involves modification of the geometry or configuration of the devices, such as complex nozzle geometries, chevrons or serrations, deflectors for bypass flows, etc., while active control modifies the flow dynamics through injection or suction, synthetic jets, plasma-actuators, etc. The second half of this thesis (Chapter 4) is concerned with a multi-stream supersonic jets with a complex nozzle configuration. In this section, the research related to these configurations will be reviewed.

A three-stream engine technology has shown high benefit for aircraft performance and may be pursued by the United States Air Force. Its key features involve complex nozzle geometry, multi-stream supersonic flows and integrated propulsion system into the aircraft body. Recent research on complex nozzles and multi-stream jet flows have
shown potential benefit of noise reduction [15, 55]. Multi-stream engines also allows for high thrust while maintaining the thrust requirement [54]. The third stream allows the excess air at partial power to be ingested by the engine and used for cooling engine components or other purposes. The integration of the propulsion system has the benefit of reduced drag and the resulting asymmetric flow enables redirecting the noise radiation direction [27].

Non-circular jet flows have been studied extensively since the 1980s. Altering the nozzle geometry is a relatively low-cost passive control strategy which also can offer significant improvement such as enhanced mixing, improved combustion efficiency, noise suppression and thrust vector control [48]. Regarding its application in jet noise reduction, Ahuja et al. measured the noise radiation of several nozzle geometries and they found that rectangular jets produce lower noise comparing with circular jets in both subsonic and supersonic conditions [2]. Bridges investigated the influence of aspect ratios on noise production of rectangular jets and he found that the low-frequency noise at aft-angles is reduced with rectangular nozzles while the broadband high-frequency noise is actually enhanced [15].

Some research on the flow physics of jets from non-circular nozzles observed the general characteristics of flow transition and discussed the mixing enhancement by non-circular nozzles. Sforza et al. [118] and Trentacoste et al. [142] were among the first researchers to study the flow physics of non-circular jets and they defined the flow fields as three regions based on the axial velocity decay. Krothapalli et al. [70] and Hsia et al. [60] analyzed respectively the influence of aspect ratio and Mach number on the transition
to axisymmetric region of subsonic rectangular jet. The development of supersonic jet
from non-circular nozzles was observed by Gutmark et al. [50] and they found
non-circular nozzles produced more mixing. The effect of enhanced mixing was also
observed by Schadow et al. [113] for coaxial nozzles.

The shock structures and vortices are the focus of analysis for some earlier research on
three-dimensional nozzles. Gutmark et al. [49] related the near-field pressure
fluctuations of rectangular jets to the shock structures. Tam and Reddy [132] observed
that a shock was always present at the nozzle throat and at off-design conditions a
shock formed on the side walls. The pressure waves in the shear layers were found by
Shimizu et al. [119] as causing secondary flow and influencing the growth of vortices.

Tam and Thies [135] studied the instabilities in a rectangular nozzle and they concluded
that the corner vortices cause the saddle like profile [118, 142] and as the vortices decay
the flow becomes more circular. Bitting et al. [9] showed the corner flows and vortex
deformation may cause the axis switching phenomenon, which was observed by many
researchers for non-circular nozzles [48]. However some researchers [58] did not see this
phenomenon. This illustrates the complex nature of this type of flow field and a strong
dependence on initial conditions.

The multi-stream nozzles allow for additional variation of nozzle variation, such as
altering the offset of the third stream. Some multi-stream nozzles have eccentric
settings which offers the advantage of plume vectoring and noise redirecting. Some of
the first researchers to study three-stream nozzles were Dosanjh et al. [28]; they saw a
significant decrease in noise for supersonic concentric jets when additional streams were
added. The models for coaxial jets developed by Tanna and Morris [136] and Fisher et al. [37] found that as the inner shear layer is surrounded by the outer potential core, the convective Mach number and the turbulence level are reduced, leading to lower radiation efficiency. The addition of a third stream for noise reduction has only moderate benefit at subsonic conditions [55]. For supersonic jets, the non-concentric configuration of multi-stream nozzle is found to reduce noise more significantly, which could be due to the effect of jet plume vectoring [101].

With an integrated propulsion system, the exhaust will interact with the aircraft body, creating a complex flow field where part of the shear layer interacts with a surface. To model this, some researchers looked at a jet in the presence of a plate. Sforza et al. [117] showed that the rectangular wall jet can be broken into two parts, one acting like a free jet and one like a boundary layer. After the plate edge, the spreading occurs faster than a free jet and further downstream the jet looks axisymmetric. Hall and Ewing [51] performed LSE on a wall jet using pressure to acquire a three-dimensional representation of the velocity field. They detected large coherent structures that caused large fluctuations in the streamwise velocity. The effect of the plate on far-field acoustics was first analyzed by Powell as a dipole source, which is generated by the trailing edge of the plate [106]. The far-field acoustics are found as very dependent on Mach number, the plate aspect ratio and location and when the plate is attached to the nozzle edge, both screech and BSAN can be mitigated [148, 95].
Chapter 2

Signal Processing Techniques

2.1 Time-Frequency Analysis

In this section, the basic concepts and formulas of time-frequency analysis will be provided with the emphasis on continuous wavelet transform. A sample signal is constructed to compare the more conventional temporal analysis and frequency analysis with the time-frequency technique and a brief survey is provided on their application in jet noise research.

2.1.1 Temporal Analysis and Frequency Analysis

A typical signal acquired through experiments or simulations is a time series: \( p(t) \). Its instantaneous energy can be defined as \( |p(t)|^2 \). The time domain representation shows how the magnitude / energy / waveform varies with time. Juve et al. [64] compared the time traces of pressure, velocity and its second order derivative to relate large structures
to noise emission in subsonic jets. In Chapter 3, time series will be constructed from the
PIV snapshot of the jet near-field.

The signal can also be inspected in the frequency domain using Fourier transform:

$$\mathcal{F}(p) = \int_{-\infty}^{\infty} p(t)e^{-2\pi if^t}dt.$$  \hspace{1cm} (2.1)

The original signal can be reconstructed through inverse Fourier transform:

$$p(t) = \mathcal{F}^{-1}\{\mathcal{F}(p)\} = \int_{-\infty}^{\infty} \mathcal{F}(p)e^{2\pi if^t}df.$$  \hspace{1cm} (2.2)

If the upper and / or lower limit in the integration is set as a certain value, information
at some frequencies is removed, i.e. the signal is low- / high- / band-pass filtered.

Figure 2.1: Left plot: an example of a time series $p(t)$; middle plot: its energy spectrum $e_f(f)$; right plot: its compensated spectrum $e_c(f)$

The energy spectrum of the signal is related to the instantaneous power through the
Parseval’s theorem:

$$E_p = \frac{1}{2} \int_{-\infty}^{\infty} |p(t)|^2dt = \frac{1}{2} \int_{-\infty}^{\infty} |\mathcal{F}(p)|^2df.$$  \hspace{1cm} (2.3)

With a physical signal, both its magnitude and frequency vary greatly in order of
magnitude. The compensated spectrum shown in logarithmic scale provides clearer presentation [72], which is defined as:

\[ E_p = \int_{-\infty}^{\infty} e_c(f) \frac{df}{f}, \quad \text{where} \quad e_c(f) = f e_f(f), \quad e_f(f) = \frac{1}{2} |\mathcal{F}(p)|^2. \quad (2.4) \]

An example of the Fourier spectrum and the compensated spectrum is shown in Fig. 2.1. With the Fourier spectrum, the peak at low frequencies can be easily spotted, while the high frequency peaks stand out in the compensated spectrum without the need of visualizing a straight line of slope \(-1\) for power-law. The compensated spectrum will be used exclusively in this thesis.

Fourier analysis is a powerful tool, enabling the spectral analysis, inverse transform and signal filtering. The invention of Fast Fourier Transform (FFT) also facilitates the calculation of various other transforms [109], such as wavelet analysis, which will be discussed later. Some of the earliest jet noise research looking at the signals in the frequency domain includes the work by Mollo-Christensen et al [94, 93]. They analyzed the acoustic spectra in different frequency bands and observed the different features (e.g., directivity, location preference, etc.) at the low and high frequencies that are later attributed to two different noise mechanisms. The bandpass analysis was also combined with cross-correlations [40] and azimuthal decomposition [41] and provided additional evidence that the noise sources are not solely isotropic turbulence.

However, the frequency domain representation is obtained at the cost of the temporal information. The (discrete) Fourier transform decomposes the signals as sinusoidal waves, and while it is superior in treating periodic signals, it fails to reveal the transient
or intermittent features. Similarly, the time domain representation cannot localize the
signal with respect to frequency. The time-frequency techniques, such as wavelet
analysis, achieves a compromise between time and frequency resolution.

2.1.2 Time-Frequency Analysis and Continuous Wavelet

Transform

Putting wavelet analysis in historical context, the time-frequency analysis includes
various techniques to analyze the signals of transient nature (time-varying frequency
content), looking at the signals in time and frequency concurrently. They are widely
applied to various areas, such as communication system, radar / satellite, geoscience,
acoustics / speech, engineering, etc. With a time series, the time-frequency analysis
involves performing a time-frequency transform on the signal and looking for
information or conducting further analysis on the signal’s two-dimensional
time-frequency distribution. Various time-frequency distribution approaches have been
developed, some are better suited for certain types of signals, such as Gabor transform,
Wigner-Ville distribution, wavelet transform, S transform, etc [10]. This thesis focuses
on wavelet transforms, specifically the discretized implementation of continuous time
and frequency.

To obtain a spectrum of signal $p(t)$ that is localized in both time and frequency, we can
multiply the signal by a window $W(t')$ ($t'$ stands for the translation in time) and then
take the Fourier transform with respect to $t'$:

$$F_s\{p\}(t, f) = \mathcal{F}\{p(t')\mathcal{W}(t' - t)\}. \quad (2.5)$$

This is called the short-time Fourier transform (STFT). Gabor transform is one type of STFT developed in 1946 and it uses the Gaussian function as the window $\mathcal{W}(t')$ [42]. For implementation, the Gaussian filter has to be approximated to zero outside the window range, thus providing compact support around the time analyzed ($t'$). Gabor also derived the exact formula that governs the resolution of the transform window [42], which is known as the Heisenberg / Gabor uncertainty principle:

$$\Delta t \Delta f \leq \frac{1}{4\pi} \quad (2.6)$$

Note that the transform window $\mathcal{W}(t')$ does not have variable control of the frequency resolution $\Delta f$; thus with a given signal (fixed $\Delta t$), the STFT has fixed resolution across all frequencies. This is less than ideal for the analysis of turbulence and many other physical phenomena, since physical structures of larger scales have intrinsically larger wavelengths and are active at lower frequencies.

Wavelet transform was extended from the Gabor transform in the 1980s by Morlet et al. for the analysis of seismic data [44]. For a function $\psi$ to be considered a wavelet, it has to satisfy the admissibility condition:

$$C_\psi = \int_{-\infty}^{\infty} \frac{\vert \mathcal{F}\{\psi(f)\} \vert^2}{f} df < +\infty. \quad (2.7)$$
A sufficient condition for this is that the function has zero mean:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0.$$

(2.8)

When analyzing signals, a family of wavelet is used that are derived from the basis function or mother wavelet $\psi$:

$$\psi_{t',s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - t'}{s}\right)$$

(2.9)

Here $t'$ and $s$ stands for the time translation and the scale dilation of the wavelet. The wavelet transform of the signal $p(t)$ is formulated as:

$$\tilde{p}(t,s) = \int_{-\infty}^{\infty} p(t') \frac{1}{\sqrt{s}} \psi^*\left(\frac{t' - t}{s}\right)dt' = p \ast \Psi_s(t).$$

(2.10)

This is basically the convolution of the signal and the function $\Psi_s(t)$,

$$\Psi_s(t) = \frac{1}{s} \psi^*\left(\frac{-t}{s}\right).$$

(2.11)

Thus we can take advantage of FFT by obtaining its Fourier transform:

$$\mathcal{F}\{\Psi_s(f)\} = \sqrt{s}\mathcal{F}\{\psi^*(sf)\},$$

(2.12)
and the wavelet transform can be calculated as

\[ \tilde{p}(t, s) = p * \Psi_s(t) = \mathcal{F}^{-1}\{\sqrt{s}\mathcal{F}(p) \cdot \mathcal{F}(\psi^*(sf))\}. \] (2.13)

This also means that a wavelet transform is a band-pass filter: at a certain scale level, the Fourier coefficients of the signal are weighted by the wavelet and then inverse transformed. The signal \( p(t) \) can be reconstructed through inverse wavelet transform:

\[ p(t) = \frac{1}{C_\psi} \int_0^{\infty} \int_{-\infty}^{\infty} \tilde{p}(t', s) \frac{1}{\sqrt{s}} \psi\left(\frac{t - t'}{s}\right) dt' ds. \] (2.14)

The energy of the wavelet transformed signal also satisfies the Parseval’s relation:

\[ E_\psi = \frac{1}{2C_\psi} \int_0^{\infty} \int_{-\infty}^{\infty} |\tilde{p}(t', s)|^2 dt' ds. \] (2.15)

An example of wavelet analysis of the sample signal (Fig. 2.1) is shown in Fig. 2.2. As

![Figure 2.2: An example of the time-frequency analysis by wavelet transform: left plot - the compensated spectra of the sample signal; right plot - scalogram of the Mexican hat wavelet](image)

mentioned previously, the wavelet transform is a band-pass filter and as a result, the compensated wavelet spectra are a lot smoother than the compensated Fourier
spectrum (the left plot). The right plot shows the Mexican hat coefficients of the sample signal. The Mexican hat wavelet is able to identify the time and frequency of occurrence of the peaks and valleys of the signal, which shows up as red and blue patches in the scalogram.

Since the wavelet function is localized in time and frequency, it is good for analyzing transient signals. In addition, the scale factor $s$ indicates that the function is narrower or has smaller $\Delta t$ at higher frequencies. Thus wavelet analysis is especially suitable for analyzing turbulent structures. To better illustrate this, Fig. 2.3 shows the resolutions of STFT and discrete wavelet transform. The scale $s$ of the discrete wavelet transform is discretized on a logarithmic scale and a different sampling time is obtained with respect to each scale. The discrete wavelet transform, especially the orthogonal wavelet, is widely applied to signal / image compression and many other areas. In Fig. 2.3, each rectangle of both Gabor and wavelet transform has a fixed area, which is restricted by the uncertainty principle (Equation 2.6). When the wavelet distribution is shown in logarithmic scale, as is usually the practice for turbulence analysis, good time resolution is achieved without loss of resolution at higher frequencies.

Figure 2.3: Resolution illustration of different time-frequency distributions: left plot - STFT (e.g. Gabor transform); middle plot - wavelet transform in linear scale; right plot - wavelet transform in logarithmic scale
Wavelet analysis is a relatively new technique, especially in the field of turbulence research. The earliest analysis on turbulent flows can be found in the late 1980s to the early 1990s [31]. Farge et al. conducted two-dimensional Morlet wavelet transform on a plane turbulent flow [32] while Everson et al. looked at a moderate Reynolds number jet flow and a Brownian motion using two-dimensional Mexican hat wavelet transform [30]. Some later research combined wavelet with other analysis techniques, such as statistics and probability distribution [146], auto-correlation and cross-correlations [77] and ensemble averaging [114]. Some more sophisticated analysis include applying wavelet transform on the Navier-Stokes or other fluid mechanics equations [89, 73], using wavelet transform to decompose the signals and conduct further analysis on the structures of interest (e.g. coherent structures) [18, 74], etc. More specifically on jet noise research, Grizzi and Camussi combined wavelet transform with the cross-correlation between near-field pressure signals and separated the hydrodynamic and acoustic pressure components [46]. Three other separation techniques were proposed by Mancinelli et al. which made use of near-field-far-field cross-correlation, probability distributions and wavelet filtering [87]. Lewalle et al. analyzed the far-field acoustic signals at aft-angles using cross-correlation and continuous wavelet and they identified the acoustic events and estimated their near-field source locations [76]. Cavalieri et al. decomposed the acoustic field using azimuthal decomposition and wavelet analysis [19], while Koenig et al. combined POD and wavelet analysis for acoustic decomposition [69]. Their observation of the low-order representation of the flow field at the identified instants lead to better modeling of the acoustic source with the wave-packet ansatz.
2.1.3 Mexican Hat Wavelet and Morlet Wavelet

Since the invention of wavelet transform, numerous wavelet functions have been proposed by various researchers. Which wavelet function to use depends on the type of information that we are interested in extracting. As warned by Farge in her 1992 paper:

A very common pitfall when using any kind of transform is to forget the presence of the analyzing function in the transformed field, which may lead to severe misinterpretations, the structure of the analyzing function being interpreted as characteristic of the phenomena under study. To reduce this risk we should choose the analyzing function in accordance to the intrinsic structure of the field to be analyzed. [31]

For the analysis in the thesis, two wavelet functions have been used, which are the Mexican hat wavelet and the Morlet wavelet. The wave form of the Morlet wavelet makes it especially suitable for the event level analysis, since various researchers have used wave-packet ansatz for acoustic source modeling [20].

The Mexican hat wavelet is the second time derivative of the normalized Gaussian filter:

$$\psi_2(t, \sigma) = \frac{d^2 G}{dt^2}, \text{ where } G = \frac{1}{2\sqrt{\pi\sigma}} e^{-t^2/4\sigma} \text{ and } \sigma = s^2/2. \quad (2.16)$$

The Morlet wavelet consists of a Fourier wave inside a Gaussian envelope:

$$\psi_{M,z_0}(t) = e^{-2\pi^2 t^2/z_0^2}(e^{2i\pi t} - e^{-\frac{z_0^2}{2}}). \quad (2.17)$$
Here $z_0$ is the envelope factor and it controls the number of oscillations inside the envelope (the width is $z_0/\pi$). The term $e^{-z_0^2/2}$ is added to make the wavelet admissible.

The dilation in frequency ($f = 1/s$) is obtained through:

$$\psi_M(t, f) = \frac{f}{\sqrt{C_\psi}} \psi_{M,z_0}(tf) = \frac{f}{\sqrt{C_\psi}} e^{-2\pi^2 f^2 z_0^2 / z_0^2} \left( e^{2\pi i tf} - e^{-z_0^2 / 2} \right).$$

(2.18)

There is no closed-form expression for $C_\psi$ for Morlet transform and its value is obtained through numerical integration. With $z_0 = 5$, its value is 1.4406 [75].

The above notations for the Mexican hat and Morlet wavelets follow that of Lewalle et al. [75], which provide more convenient scalogram interpretation and simplified wavelet formulas (different from Equations 2.10):

$$\tilde{p}_M(t, f) = \int_{-\infty}^{\infty} p(t') \psi_M(t - t', f) dt', \quad \text{and} \quad \tilde{p}_M(t, f) = \int_{-\infty}^{\infty} p(t') \psi_M^*(t - t', f) dt'. \quad (2.19)$$

The inverse transform formulas are:

$$p(t) = -\int_0^\infty \sigma \tilde{p}_2(t, \sigma) \frac{d\sigma}{\sigma}, \quad \text{and} \quad p(t) = \int_0^\infty \int_{-\infty}^{\infty} \tilde{p}_M(t', f) \psi_M(t - t', f) dt' df. \quad (2.20)$$

And the compensated energy spectrum of the signals are:

$$E_2 = \int_0^\infty \int_{-\infty}^{\infty} 2\sigma^2 |\tilde{p}_2(t', \sigma)|^2 dt' \frac{d\sigma}{\sigma}, \quad \text{and} \quad E_M = \frac{1}{2} \int_0^\infty \int_{-\infty}^{\infty} |\tilde{p}_M(t', f)|^2 dt' df. \quad (2.21)$$

Fig. 2.2 and 2.4 show the Mexican hat and Morlet wavelet coefficients of the sample signal (Fig. 2.1) and the compensated spectra of these wavelets can be found in 30
Figure 2.4: Morlet wavelet transform: left plot - the absolute values of the complex coefficients; middle plot - the real part; right plot - phase.

Fig. 2.2. Unlike the Mexican hat wavelet, the Morlet wavelet is complex valued and from the coefficients, we can take the modulus (left plot in Fig. 2.4), the real part (middle plot) or the phase angle (right plot). The modulus focuses on the envelope while the individual oscillations are smoothed out; the real part shows the fluctuations within the envelope. Comparing the Mexican hat scalogram with that of the real part of Morlet, we can see that the Morlet wavelet provides better frequency resolution. This is also evident when comparing their compensated spectra (Fig. 2.2), as the Mexican hat spectrum is much smoother than that of Morlet. As discussed in the beginning of this section, the shapes of the wavelets determine the type of signals that they are more adept at analyzing: the Mexican hat wavelet (single peak) is better at isolating individual extrema while the Morlet wavelet (a packets of oscillations) is better at identifying local periodicity and short wave-trains. In the thesis, both wavelet functions have been tested and the Morlet wavelet is settled on for extracting the near-field structures related to noise production, which have been observed by many researchers as modulated wave packets [20].
2.2 Statistical Signal Analysis

2.2.1 Statistical Analysis and Multi-Correlations

In this section, some formulas of statistical analysis will be provided that are applied to jet noise research, which will also be used in this thesis. The techniques include probability density function $B(p)$, joint probability density $B(p,q)$, the moments, Fourier transform and spectral analysis, and auto- and cross-correlations. In particular, the correlation technique will be extended to multiple signals using the idea of Starke et al. [123]

The general formula for calculating the moments of a signal is shown in Equation 2.22. With the $n$ value varying from 1 to 4, we obtain the mean value, the variance (its square root is the standard deviation or RMS), the skewness and the kurtosis or flatness factor. To be noted here is the skewness factor, which can be used as the identification criterion for crackle noise in supersonic jets [35].

$$\bar{p}^n = \int_{-\infty}^{\infty} p^n B(p) dp. \quad (2.22)$$

The cross-correlation technique measures the similarity between two signals as one signal slides relative to the other. It is a function of the displacement between the two signals, or the time lag $\tau$ between two time series:

$$\mathcal{X}(\tau) = \int_{-\infty}^{\infty} \hat{p}^*(t)\hat{q}(t + \tau)dt. \quad (2.23)$$
The signals $p$ and $q$ are usually normalized: $\hat{p} = (p - \bar{p})/\sigma_p$, so that the correlation values range between $[-1, 1]$. If the two signals are the same, we obtain the auto-correlation. The integration of the auto-correlation across all $\tau$ values produces the integral length scale. It is used in turbulence research to estimate the time interval over which the flow is correlated with itself.

In the last section, the formulas of the Fourier transform and the energy spectrum have been provided. Notice that the correlation function is equivalent to the convolution between $\hat{p}^*(-t)$ and $\hat{q}$ and we can use the FFT to calculate the correlations:

$$X(\tau) = \hat{p}^*(-t) * \hat{q} = F^{-1}\{F^*\{\hat{p}\}F\{\hat{q}\}\}. \quad (2.24)$$

If the two signals are the same, the Fourier transform of the auto-correlation (the term inside the inverse transform of Equation 2.24) is actually the energy spectrum of the signal. Similarly, the cross-spectrum is used by some researchers, which is the spectral representation of the cross-correlations [41, 52].

In the thesis, multiple kinematic and acoustic signals will be compared and the multi-correlation can be used to simplify the comparison process [123]:

$$X(\tau_i) = \int_{-\infty}^{\infty} \prod_{i}^N \hat{p}_i(t + \tau_i) dt. \quad (2.25)$$

Here a total of $N$ signals are correlated and each signal is shifted relative to the first signal ($\tau_1 = 0$). This formula only applies to real-valued signals so as to remove the evaluation of the complex conjugate ($p^*$). To ensure that the correlation value still
varies between 0 and 1, with 1 being perfectly correlated, the signal $p$ is normalized by removing the mean and dividing by its $L^N$-norm:

$$\hat{p} = \frac{p'}{||p'||_N}, \text{ where } p' = p - \bar{p} \text{ and } ||p||_N = \left( \sum |p_i|^N \right)^{1/N}. \quad (2.26)$$

Here the $L^N$-norm is also referred to as the p-norm and with $N = 2$, we get the Euclidean norm. For two signals half a period out-of-phase, this normalization will bring them to the same phase. Thus the multi-correlation value is always positive.

### 2.2.2 Correlation and Time-Frequency Analysis

If we combine the correlation and continuous wavelet techniques, we can look at the contributions to cross-correlations in the time and frequency domain. From the Parseval’s theorem and following the wavelet normalization in the last section, the cross-correlation can be formulated as:

$$\mathcal{X}(\tau) = \int_0^\infty \int_{-\infty}^{\infty} \tilde{p}^*(t, f)\tilde{q}(t + \tau, f)dt\frac{df}{f} \quad (2.27)$$

Here $\tilde{p}$ and $\tilde{q}$ stand for the wavelet coefficients of these two signals. For a fixed time lag $\tau$, the integrand stands for the contributions to the corresponding correlation value at a specific time $(t)$ and frequency $(f)$:

$$\xi(\tau, t, f) = \tilde{p}^*(t, f)\tilde{q}(t + \tau, f). \quad (2.28)$$

34
\( \xi(\tau, t, f) \) is called the correlation coefficient and it is complex valued for Morlet wavelet. Some variations of Equation 2.28 are thus developed to obtain real valued Morlet correlation coefficients:

\[
\xi(\tau, t, f) = G\bigg( \mathcal{R}\{\check{p}(t, f)\}\mathcal{R}\{\check{q}(t + \tau, f)\} \bigg), \tag{2.29}
\]

\[
\xi(\tau, t, f) = \mathcal{R}\{\check{p}^*(t, f)\check{q}(t + \tau, f)\}. \tag{2.30}
\]

As an example, Fig. 2.5 compares the Morlet correlation coefficients obtained using these two methods. Both signals are set as the same sample signal used earlier (Fig. 2.1) and the correlation coefficients are basically the energy spectrum. In Equation 2.29, we first calculate the dot product of the real part of the Morlet coefficients. This produces the left plot in Fig. 2.5. This keeps the individual oscillations that include both peaks and valleys of the sample signal (Fig. 2.1). Since many researchers have modeled the acoustic source as modulated wave packets \cite{20}, we are more interested in identifying the time and frequency of occurrence for the envelope instead of the individual oscillations. Thus a Gaussian filter (\( G \)) is applied in Equation 2.29 (the middle plot in Fig. 2.5).

Another method to keep the envelope information is to take the dot product of the complex conjugate and the original coefficient and then take the real part (Equation 2.30). This produces the right plot in Fig. 2.5 and it has very similar patterns with that of the Gaussian filtered version. Both approaches will be of use in the thesis.

If we integrate \( \xi(\tau, t, f) \) across time at each frequency level, we see that it represents the
Figure 2.5: Left and middle plot is an example of Equation 2.29: the left shows the dot products between the real part of Morlet coefficients and the middle is the Gaussian filtered version of the left; right plot illustrates Equation 2.30: the real part of the product between complex Morlet coefficients spectral contribution to the cross-correlation:

\[
\Xi(\tau, f) = \int_{-\infty}^{\infty} \tilde{p}^*(t, f)\tilde{q}(t, f)dt = \int_{-\infty}^{\infty} \xi(\tau, t, f)dt. \tag{2.31}
\]

For a fixed \(\tau\), \(\Xi(\tau, f)\) shows the variation of the correlation across frequency and it can reveal the frequencies at which the signals are highly correlated. This is called the frequency-resolved cross-correlation.

In the above equations, we may choose to normalize the wavelet transformed signals (\(\tilde{p}\) and \(\tilde{q}\)) to zero mean and unit variance at each frequency level. This way, the frequency-resolved correlation \(\Xi(\tau, f)\) has values between \([-1, 1]\) and can be easily interpreted. However, this is avoided for some analysis and the original wavelet filtered signals are correlated. This is because a typical turbulence / acoustic signal is usually more active at certain frequencies. If the signals are normalized at each frequency, the wavelet coefficients are in the same range of values at all frequencies, unless they are very intermittent. Some physically-unimportant frequencies may turn out to be highly correlated and will be over-emphasized. Without the normalization, there is no longer a
fixed value for perfect correlation (1 or $-1$), thus the level of uncorrelated background noise will have to be inferred by inspecting $\Xi(\tau, f)$ across varying values of $\tau$. Please refer to the following chapters for more specific examples.

Lastly, the correlation contribution formulas will be extended to multi-correlations.

Equation 2.29 is used since it does not involve the dot products of complex numbers:

\begin{align*}
\xi(t, f) &= G \left( \prod_{i=1}^{N} |\mathcal{R}\{\tilde{p}_i(t + \tau_i, f)\}| \right), \\
\xi(t, f) &= G \left( \prod_{i=1}^{N} \hat{\mathcal{R}}\{\tilde{p}_i(t + \tau_i, f)\} \right).
\end{align*}

Here a fixed set of time lags will be assigned for the $N$ signals (thus the $\tau$ variable removed). Again, we may choose to use the original wavelet filtered signals or normalize the filtered signals following Equation 2.26. In Equation 2.32, the original wavelet filtered signals are used. Without the normalization, two signals that are the mirror image of each other (half-a-period out of phase) will contribute negatively to the multi-correlation coefficient (suppose all the other signals are in-phase), when they should be regarded as perfectly correlated. Thus the absolute values of the real part of the coefficients are used for the dot products. In Equation 2.33, the real part of the Morlet coefficients are normalized (thus the $\hat{\cdot}$ sign) to obtain multi-correlation coefficients ranging between 0 and 1.
2.3 Source Localization and Event Detection

In this section, we survey the existing research on source localization and event detection, which is an important component of this thesis. At the end of this section, the aspects to be improved of the existing research will be discussed and the approach and outline of the thesis will be provided.

The majority of the source localization techniques involve signal sampling at two or multiple locations outside the jet stream and obtaining an estimate of the source location through cross-correlation or cross-spectral analysis. In the 1970s, various techniques were developed such as cross-beam, acoustic mirror, acoustic telescope, polar correlation, etc. Damkevala et al. measured the density fluctuation outside a subsonic jet flow and used cross-correlation to obtain the apparent source distribution [26]. Parthasarathy measured the pressure signals in the FF of a supersonic flow and estimated the source location using time lag [102]. The acoustic mirror [71], telescope [8] and polar correlation [36] techniques involve more complicated experimental setting, such as placing a spherical reflector behind the microphone or using a microphone array. Despite the difference in their experimental settings, they are all based on the correlations (or cross-spectra) between sampled signals and they relate the acoustic radiation to flow regions around 2 to 12 nozzle diameters in the axial direction. Some more recent research on localizing the sources involve various far-field pressure sensors, e.g. Hileman et al. used the cross-correlation between eight FF microphones for three-dimensional source localization [57]. Some researchers use big microphone arrays and the beam-forming technique and they obtained apparent source distribution with
better resolutions [99, 126]. While these techniques can identify the flow regions associated with noise radiation, they cannot provide the answer to the nature of the sources or what kind of models will better represent the noise source. Another approach that can provide insights about the source location involves inspecting the field distribution of certain quantities. Bogey and Bailly looked at the NF distribution of cross-correlation, intermittency factor, frequency and convection velocity [13]. They related the jet centerline region at the end of the potential core to acoustic radiation at aft angles and these regions are highly intermittent and low Strouhal number dominant. As mentioned previously, the shock-like structure can be identified by high skewness factor. Some researchers looked at the near-field distribution of cross-correlation, skewness and kurtosis of velocity and identified the flow regions related to supersonic noise production [29, 83].

Event detection, or isolating patterns of interest from background noise, is of interest in many areas such as traffic management, biomedical research, telecommunications, etc. Many of the event detection algorithms involve thresholding on signal magnitude, persistence, correlation values or other quantities. More sophisticated machine learning algorithms can be developed if certain knowledge of the event is known beforehand and the models of the events can be improved iteratively based on the event information, such as statistics. In the field of turbulence and jet noise research, most of the event detection is performed by setting a threshold on pressure or energy magnitude. After the events are extracted, conditional averaging is usually performed to bring out the shared patterns; some researchers also performed event filtering and looked at the
statistics (e.g., spectrum) of the filtered signal, or inspected the time series or flow snapshots coupling with the identified events.

The event-level analysis in jet noise research is limited and here the research on separating hydrodynamic and acoustic pressure are also discussed. To identify the noise-related events, the magnitude of the FF pressure signal is the criterion used by most researchers. Guj et al. set the threshold as 30% of the maximum pressure magnitude [47] and Kearney et al. used 1.5 times the pressure RMS value [67]. Baars and Tinney added another criterion on pressure slope to identify the shock structures [7]. Some researchers resolved the pressure signal into time-frequency domain with wavelet analysis and set a threshold on the wavelet coefficient magnitude [19, 45]. Other event-selection criteria includes the amplitude of the second time derivative of velocity, which is used by Juve et al. [64], or the amplitude of the POD eigenvalue, which is used by Arndt et al. for NF pressure analysis [5]. Tinney et al. applied LSE in the spectral domain and set a threshold on the coherence spectra [137]. Some researchers investigated filtering algorithms to separate acoustic and hydrodynamic pressure components. Most of them filtered the signal by frequency since the hydrodynamic component tends to be related to low-Strouhal activity [139] and some researchers added a filtering criterion on the wavenumber [23].

To make the event-selection algorithm more rigorous, some researchers combined multiple criteria and some researchers developed more objective threshold selection methods. Lewalle et al. selected FF acoustic events based on the time lag and correlation level between three FF sensors and the FF pressure magnitude [76]. Freund
et al. introduced an estimate of the compactness of the noise source, and they filtered the acoustic source and compared the radiated field based on the compactness, the frequency and wavenumber [38]. Grassucci et al. optimized the threshold on pressure magnitude by defining a parameter that controls the event percentage and the kurtosis of the selected events [45]. Regarding the NF pressure filtering algorithms, Grizzi et al. optimized the threshold on wavelet coefficient magnitude such that the convection speed distribution and correlation values of the hydrodynamic and acoustic components are as expected [46]. Three wavelet-based filtering algorithms are proposed by Mancinelli et al. and they optimized the filtering results by improving the correlation values or the probability distribution of the filtered signals [87].

To look for the near-field structures responsible for noise production, inspecting the near-field flow snapshots seems straightforward but can only be meaningful if we know where to look. Some researchers use low-order presentation tools such as POD to reduce the amount of information; some narrow the search down by identifying the instants related to noise production, namely, the ‘events’. The existing few research on noise-related event detection is mostly performed in the temporal domain and involve subjective thresholding on acoustic pressure magnitude or energy. Although some researchers have developed more objective threshold selection and some have observed certain flow activities during event occurrence, no quantitative evaluations have been provided as to how many identified events are of significance and how many events are associated with such patterns.

In this thesis, two kinds of jet flows will be investigated and for each case, the analysis
consists of two parts. The first part looks at the general features of the kinematics and acoustics, i.e. the patterns of structures in the statistical sense. The near-field regions related to noise production (the apparent source locations) will be identified and will assist in sensor selection for event detection. The second part of the analysis involves devising algorithms to extract noise-related events and event-based analysis. The algorithms combine multi-correlation and continuous wavelet and look for events in the time-frequency domain that are the main contributors to NF-FF information propagation. The algorithm performance will be evaluated by synthetic signals and the events will be used to guide the search for noise-producing flow structures.
Chapter 3

Identification and Localization of Acoustic Events of Subsonic Axisymmetric Jets

In this chapter, an experimental dataset for subsonic axisymmetric jet is analyzed. The ultimate goal is to identify the flow structures that produce noise that radiate into the far-field at aft-angles. The chapter starts with a brief description of the experimental setup and the post-processing of the data. Then a statistical analysis is conducted that include spectral analysis, cross-correlations, multiple correlations and frequency-resolved cross-correlations. Through this process, the most promising sensors and parameters are identified. With these, the event-level analysis is able to isolate the dominant contributors to the information propagation between the near-field kinematics and far-field acoustics.
3.1 Description of the Experimental Datasets and Preliminary Processing

3.1.1 Experimental Setup and Data Acquisition

The experimental dataset was acquired by Low et al. in 2011 [84] at the large scale anechoic chamber and high speed flow lab at Syracuse University. The interior surfaces of the chamber were acoustically treated with high density fiberglass wedges and the chamber is anechoic down to 150 Hz. The free inner space dimensions of the chamber were $26 \times 20 \times 14\text{ft} (7,280\text{ft}^3)$.

The jet facility of the experiment is shown in Fig. 3.1. The jet rig was installed by Tinney et al. in 2004 [138] and the nozzle exit diameter was 2 inch. The discharged air from the jet rig was controlled with a pneumatically actuated valve and the static pressure just upstream the contraction region was monitored to achieve the desired Mach number. In the thesis, three different Mach number test cases are analyzed, including Mach 0.6, 0.85 and 1. The Reynolds number of the turbulent flow was approximately $7 \times 10^5$.

Figure 3.1: Syracuse University anechoic chamber and jet facility.
The experiments contained simultaneous measurements of Near-Field (NF) and Far-Field (FF) pressure at 40.96 kHz and NF velocity fields at 10 kHz. A National Instruments PXI-based data acquisition system was used to sample the pressure signals and the velocity field trigger signal. The pressure and velocity signals were aligned using the trigger signal. Low-pass filters were applied to half the sampling frequency to prevent aliasing.

The NF pressure was measured with high temperature miniature pressure transducers manufactured by Kulite. The Kulites have a 0.095 inch cross-section and have minimal intrusiveness into the flow field. The Kulite transducers were arranged into one azimuthal and one linear array. The azimuthal arrays included nine Kulites positioned in the \( r - \theta \) plane, at 6 diameters downstream of the jet exit (Fig. 3.2). The linear array included five Kulites placed at \( X/D = 4, 5, 6, 7 \) and 8. The Kulites were oriented radially and they followed the \( \epsilon = 5.5^\circ \) expansion line of the shear layer, 1 cm outside of the developing shear layer.

The FF acoustic signature was sampled by six 0.25 inch prepolarized free-field condenser microphones manufactured by G.R.A.S Sound and Vibration. The microphone array was oriented horizontally in the same plane with the jet centerline (Fig. 3.2). The microphones were positioned 75 jet diameters (\( D \)) away from the center of the nozzle exit and their diaphragms were directed towards the jet. The observation angles (\( \beta \)) of the microphones started from 15° all the way to 90° and they had 15° increment in between.

For flow measurements, the Spectral Energies Two-component Time-Resolved (TR)
Particle Image Velocimetry (PIV) system was utilized. The system consisted of a high energy diode pumped Neodym-YAG Laser and a Photron FASTCAM CCD camera. The laser head was oriented such that the beam shot parallel to the jet flow and deflected through the jet centerline. The laser sheet and camera system were mounted on a single traverse and were located on the left side of the jet. This way the reflection related noise was reduced for the FF microphones that were situated on the right side of the jet. The PIV snapshots were processed using single-image, dual frame cross-correlation technique and produced the two-component (axial and radial) velocity field. The PIV measurement windows were 3 inch × 3 inch and they were traversed downstream to cover the flow field from the lip of the jet to 7.5 diameters downstream.
3.1.2 Preliminary Processing of the Signals

The raw NF and FF pressure signals have longer record and higher sampling frequency than the velocity signals. After these signals are aligned using the trigger signal, the pressure signals have signal length of 49152 samples, which is 1.2 seconds of data. These are referred to as the Raw K (Kulite) and M (microphone) signals. The raw velocity signals consisted of 8623 samples, or 0.8623 seconds of data. They are referred to as the UV signals.

To calculate the cross-correlations, the Raw K and M signals have to be down-sampled and truncated to the same frequency and length with the UV signals. Resampling the signals at a lower frequency may result in aliasing, which is the contamination of the reconstructed signal by high-frequency information. A de-aliasing algorithm is first constructed using the Fourier filter following the two-thirds rule. An alternative is to use wavelet transform, which is basically a band-pass filter. The de-aliased signals thus obtained are very similar. Then these signals are resampled at 10kHz by taking the average of three adjacent data points. The resulted pressure signals also have 8623 data samples and they are referred to as K and M signals. Their energy spectra follow the same trend with the raw signals.

Despite the richness of the TRPIV data sets, there are some areas of deficiency, introduced by the particle seeding and some other limitation of the measurement technique. Each PIV frame has a 96 × 96 grid of UV signals. The edges of the PIV snapshots (5 columns / rows of data) are prone to error and they are not included for calculation. A filtering scheme is also applied to eliminate unreasonable big spikes. If
the absolute value of streamwise (U) or radial (V) velocity is too large compared with
its neighbors (a grid of 11 × 11), it is replaced by the average value. This process was
compared to the results from a more sophisticated scheme using GappyPOD. The two
methods produce basically the same results.
From the UV signals, several diagnostic signals (D signals) are calculated and they are
recorded frame by frame to form time series:

• instantaneous Mach number \( Ma = \sqrt{u^2 + v^2}/c_0 \),

• radial velocity \( \dot{v} = v/c_0 \),

• Reynolds stress \( \dot{u}v = (uv - \bar{u}\bar{v})/c_0^2 \),

• vorticity \( \dot{\omega} = \omega/U_x \),

• 2D divergence (trace of rate-of-strain) \( \dot{d}v = (\partial_x u + \partial_r v)/U_x = (s_{xx} + s_{rr})/U_x \),

• determinant of 2D rate-of-strain \( \dot{d}et = (s_{xx}s_{rr} - s_{xr}^2)/U_x^2 \),

• Q criterion \( \dot{Q} = (||\Omega||^2 - ||S||^2)/2U_x^2 = -\partial_j u_i \partial_i u_j /2U_x^2 \).

These quantities are non-dimensionalized (hence the dot \( \dot{\cdot} \) sign) by the characteristic
speed (\( c_0 \), speed of sound) and the characteristic velocity derivative
\( (U_x = Ma \cdot c_0/10\Delta x) \). Some variations of these D signals are also calculated, including
the fluctuating quantities (obtained from fluctuating velocities \( u' \) and \( v' \)), the absolute
values, the RMS values of the original and the absolute values. These variations are
compared with the originals and for a certain D signal, a certain variation is favored
over the others (e.g. the absolute value of the fluctuating vorticity \( |\dot{\omega}'| \) shows the most
promising statistics compared with all the other variations), which will be shown in the later sections. To be noted is the Q criterion, which is used to identify vortex cores [61] and it measures the relative contribution between rotation ($\Omega$ stands for the rotation rate) and deformation ($S$ stands for the strain rate). It is proportional to the Laplacian of pressure fluctuation of incompressible flow:

$$Q = \frac{1}{2} (||\Omega||^2 - ||S||^2) = -\frac{1}{2} \partial_i u_j \partial_j u_i = \frac{1}{2\rho} \nabla^2 p.$$  \hspace{1cm} (3.1)

### 3.2 General Description of Near-Field Flow and Far-Field Acoustics

#### 3.2.1 NF Kinematic Statistics and Information Propagation

Original UV and D signals

From the PIV snapshots, we obtain insights into the near-field flow physics. Firstly, focusing on the spectral information of the UV signals, each location in the PIV

![Figure 3.3: Peak frequency (normalized to Strouhal number) distributions of UV signals (shown in logarithmic scale); left plot is at Mach 0.6 and right plot is at Mach 0.85](image)
snapshots is regarded as a time series. The compensated spectra of the UV signals are calculated using the continuous Morlet wavelet. They show a smoother distribution compared with the Fourier spectrum and the peak frequency is easier to interpret. Fig. 3.3 shows the peak frequency distribution of the UV signals. The left two plots are at $Ma = 0.6$ and the right two plots are for the same snapshot locations at $Ma = 0.85$. The frequencies are normalized to Strouhal number using the jet main speed $Ma \cdot c_0$ and the nozzle diameter $D$. All the figures are plotted in logarithmic scale and all the plots use the same color scales. For the streamwise velocity $U$, the flow shows a clear trend of increasing frequency towards the centerline, while the spanwise velocity highlights the shear layers. As the Mach number increases, the flow peaks at higher frequencies. Along the shear layers, especially at $Ma = 0.85$, the flow seems to organize into large structures of size around $0.4D$.

Figure 3.4: Compensated Morlet spectra of D signals at selected NF locations

As mentioned in the previous section, various D signals are calculated from the UV signals. Fig. 3.4 shows the compensated Morlet spectra of the D signals at two different NF locations, one along the jet centerline (left plot) and one along the mixing shear
layer (right plot). The velocity-related quantities, including $Ma$, $\dot{v}$ and $\dot{uv}$, show similar spectral features that vary by location, which is consistent with Fig. 3.3. The features of $Ma$ are dominated by that of streamwise velocity $U$, while the spectrum of $\dot{uv}$ follows that of $\dot{v}$ and captures the smaller structures (higher frequency) along the shear layer.

The velocity-derivative-related D signals ($\dot{\omega}$, $\dot{\text{div}}$, $\dot{\text{det}}$, $\dot{Q}$) have very similar spectra and location seems not an influencing factor. The spectra at higher Mach numbers show consistent trends and thus omitted.

Figure 3.5: Cross-correlations between D signals at different NF locations ($Ma = 0.6$)

To make a wiser selection of the D signals and to assist in characterizing the flow structures, we look at the correlations between the D signals. Firstly, the D signals at the same locations are correlated and the pattern variations across locations and D signal pairs are investigated (Fig. 3.5). The left plot in Fig. 3.5 shows the correlations between D signals along the jet centerline. $\dot{\omega}$ is highly correlated with $\dot{v}$ and $\dot{uv}$, which also shows the high similarity between the radial velocity and Reynolds stress. The Q criterion is strongly correlated with strain-rate determinant, which is also a combination of squared velocity derivatives. When the location of the D signals shifts towards the
shear layer (the right plot in Fig. 3.5), the correlation pattern changes. The velocity-related D signals are highly correlated, while the velocity-derivatives show a different pattern. Note that in all these plots, the time lags of peak correlation is zero, except between $\dot{\omega}$ (and possibly $\dot{Q}$, low correlation levels) and $\dot{v}$ (and $\dot{uv}$) and it is possible that the radial velocity keeps better track of the edge of a vortex while $Q$ criterion tracks the center movement.

![Figure 3.6: Frequency-resolved cross-correlations between D signals at different NF locations (Ma = 0.6; all the plots use the same color scaling, i.e. $[-0.25, 0.25]$)](image)

The similarities between D signals can be further investigated through their frequency-resolved correlations. Using the Morlet continuous wavelet, the D signals are resolved into various frequency levels. Then at each frequency, the Morlet coefficients are normalized and correlated. Fig. 3.6 shows the frequency-resolved correlations between $\dot{\omega}$ and other D signals. The abscissa is the time lag and the ordinate is the frequency level (Strouhal). When located along the jet centerline (left plot), $\dot{\omega}$ is highly correlated with $\dot{v}$ (and $\dot{uv}$) over most of the frequencies. Meanwhile, it is not apparently correlated with $Ma$ or the velocity-derivatives. The strong purple or cyan patches at
very low frequencies (below Strouhal 0.05 approximately) are the artifacts of the normalized signals and do not correspond to physical structures (section 2.2.2). When located along the mixing shear layer (right plot), $\dot{\omega}$ is highly correlated with the squared velocity derivatives ($\dot{det}$, $\dot{Q}$) at both low and high frequencies. The dotted line shows the best time lag obtain from temporal correlation (Fig. 3.5). Although the correlation between $\dot{\omega}$ and $\dot{Q}$ has lower magnitudes comparing with that of $\dot{\omega}$ and $\dot{det}$, the correlation is consistently positive (purple patches) through all frequencies and this averages to the large temporal correlation magnitude (about 0.2 in Fig. 3.5). This is in contrast with the correlation between $\dot{\omega}$ and the velocity-signals. When correlated with $Ma$, which keeps track of the bulk flow evolution, only low frequencies around Strouhal 0.1 to 0.2 have common information. With the radial velocity $\dot{v}$ (and $\dot{uv}$), the correlation of the same sign as in the two-point correlation (Fig. 3.5), i.e. positive or purple patches, is limited at high frequencies around Strouhal 0.3 to 0.7.

![Image of space-time cross-correlations between D signals at different NF locations](image)

Figure 3.7: Space-time Cross-correlations between D signals at different NF locations ($Ma = 0.6$; all the plots use the same color scaling, i.e. $[-0.1, 0.1]$)

To obtain a more global view of the location variation, space-time correlation is applied to the D signals. The D signal at a selected location (e.g. the black cross in Fig. 3.7) is correlated with another D signal at each location of the whole PIV frame, which
produces the correlation level across the whole frame. Looking at the correlation
distribution at each time lag successively, we obtain an idea of the structure evolution.
The evolution pattern is summarized in Fig. 3.7, which is the space-time correlation at
\( \tau = 0 \). Two tracks of structures are observed: the first travel downstream along the jet
centerline (the bulk of jet stream) and the second travel downstream along the mixing
shear layers (rotating vortices). Combining this with the NF-FF correlations, which will
be presented later, the regions along the jet centerline \((-0.4 \leq Y/D \leq 0.2)\) are the focus
of analysis in this chapter.

**Filtered D signals**

As mentioned in the introduction, the NF coherent structures studied in this chapter
are thought to be the main source of FF acoustics [85]. To extract the large-scale
information, spatial filtering is applied to the D signals. Two sets of filtering windows
are used. The first set contains one large window that extends across the highly
correlated centerline region and is shown as the black box in the left plot in Fig. 3.8.
The second set contains five thinner windows at different radial locations and they are
shown as the dotted red boxes. Over the filtering window of each snapshot, the mean
values of D signals are calculated, which are recorded frame by frame to form time
series. The signals thus obtained are referred to as the Filtered D signals. In this
section, the first window setting (black box) is used. Its selection is based on the
location variation of D signals, the features of the Filtered D signals and the NF-FF
correlations. The second window setting hasn’t provided new information regarding the
general flow features. It will be used later in the event extraction section to compare the centerline and sideline regions.

Figure 3.8: Compensated Morlet spectra of the selected Filtered D signals (space-filtered at the window shown in the left plot)

The right plot in Fig. 3.8 shows the compensated Morlet spectra of selected Filtered D signals. $Ma$ and $\dot{v}$ are chosen as representatives of the velocity-related D signals, the former capturing the bulk stream and the latter the shear layers. $\dot{\omega}$ and $\dot{Q}$ complement each other and they together summarize the features of the velocity-derivatives. The spectra of these Filtered D signals show the same trend and peak frequency. If the size of the filtering window is reduced, the peak frequency increases, indicating smaller structures. Meanwhile, as the window size increases, the difference between the spectra gradually disappear. The spectra of several variations of the Filtered D signals (fluctuating quantities, absolute and RMS values, etc.) have been compared and no obvious difference has been observed. The absolute values of the fluctuating $\dot{v}$ and $\dot{\omega}$ are chosen, which will be further explained later.

The temporal and frequency resolved correlations between the Filtered D signals (Fig. 55)
Figure 3.9: Temporal (left plot) and frequency-resolved (right plot, color scaling of $[-0.25, 0.25]$) cross-correlations between the Filtered D signals at $Ma = 0.6$

3.9) are rather different from the correlations between the original D signals (Fig. 3.5, 3.6). The temporal correlation levels are greatly increased and the difference between D signal pairs is no longer obvious (the left plot in Fig. 3.9). This is even clearer looking at the frequency-contributions and the Filtered D signals are highly correlated over all frequencies (the right plot in Fig. 3.9). The windowing seems to smoothe over the difference between the D signals and the shared properties are enhanced.

**Near-Field K signals**

The K signals are sampled outside the shear layers in the NF and they contain both the hydrodynamic and acoustic information. Fig. 3.10 shows the compensated Fourier spectra of the Raw K signals and the spectra are block averaged to obtain a smoother curve. As the Raw K signal moves downstream, the peak frequency increases from around Strouhal 0.1 at $X/D = 4$ to Strouhal 0.3 at $X/D = 8$. This is possibly a result of the increased mixing and smaller structures in downstream direction. As the Mach
Figure 3.10: Compensated Fourier spectra of K signals at different streamwise locations. As the number increases (right plot), the spectrum magnitude increases and the peak frequency slightly increases. The shift towards higher frequency is smaller as the Raw K signals moves downstream and at $X/D = 8$, the peak frequency is the same in both cases. The spectra of the circular array of Raw K signals have also been examined and they show very good consistency, as would be expected with the axisymmetric jet.

Figure 3.11: Temporal (left plot) and frequency-resolved (right plot at $Ma = 0.6$, color scaling of $[-0.3, 0.3]$) cross-correlations between K signals at different streamwise locations. The correlations between K signals at different streamwise locations manifest the information propagation in the downstream direction (Fig. 3.11). The left plot in Fig.
3.11 shows the temporal correlations. At a given Mach number, the time lags of peak correlation between K signals are positive and as the distance between the K signal pairs increases, the time lag gradually increases. The difference in distance and the difference in best time lags indicate a constant information propagation speed, which can be calculated from

\[ U_c = \frac{D}{\cos \epsilon / \Delta \tau}. \]  \hspace{1cm} (3.2)

As the Mach number increases, the best time lag decreases, indicating larger propagation speed \( U_c \). However, regardless of the Mach number, the ratio between \( U_c \) and the jet stream speed \( U_j = Ma \cdot c_0 \) is approximately the same, i.e. 65%. This is consistent with the convection speed of turbulence eddies found in literature [140]. The right plot in Fig. 3.11 shows the frequency-resolved correlations at \( Ma = 0.6 \). When the K signals are placed just 1\( D \) apart, they are correlated at most of the frequencies. As the distance increases, the high frequency information is gradually lost while the low frequency information is preserved downstream.

Figure 3.12: Peak correlation level distributions (red-blue plots) and corresponding time lags (white-black plots) between K signals and D signals (\( Ma = 0.6 \))

The correlations between the D and K signals can further illustrate the propagation of NF structures. The peak values of cross-correlations between the K signal at

58
$X/D = 6D$ and $|\dot{v}'|$ are recorded at each NF location and the time lags corresponding to the peak correlations are also collected (the left plot in Fig. 3.12). The regions around the jet centerline is highly correlated and the best lag distribution is consistent with downstream propagation. Based on the difference in distance and the time lag, the propagation speed is evaluated and its value is very close to $U_c$. The right plot in Fig. 3.12 shows the peak correlation and best lag distributions between the K signal at $X/D = 6D$ and $|\dot{\omega}'|$. The same region is highlighted as active, but the time lag is considerably smaller than that of K and $|\dot{v}'|$. The propagation speed thus calculated is larger than the jet speed $U_j$. The correlation of K with $Ma$ is similar to that with $|\dot{v}'|$, while the correlation with $\dot{Q}$ is similar to that with $|\dot{\omega}'|$. It is possible that the velocity-related D signals and the velocity derivatives share different information with the K signals, the former combination capturing the convecting turbulent eddies and the latter revealing more acoustic information.

### 3.2.2 Statistical Features of FF Acoustics and NF-FF Propagation

**Far-Field M signals**

Turning our attention to the FF, the spectra and sound pressure level are first calculated of the M signals (Fig. 3.13). The Raw M signals are used and the spectra are block-averaged for smoother curves. The spectrum of M signal at an observation angle $\beta = 15^\circ$ peaks at Strouhal 0.2. As $\beta$ increases, the peak frequency increases and the spectrum become more broadband. The sound pressure level also varies with the
observation angle, first decreasing and then increasing as $\beta$ increases. These are consistent with the conclusions of many researchers that two kinds of sources exist in subsonic jets [1, 130]. The noise from large coherent structures are highly directional and the strongest at aft-angles while the noise from isotropic turbulence is more broadband [94, 85]. When the Mach number increases, the same trends are observed and the peak frequencies remain the same at all observation angles. This indicates the FF acoustics have the same nature and are not strongly dependent on subsonic jet speed.

![Compensated Spectrum](image1)

![Sound Pressure Level](image2)

Figure 3.13: Compensated Fourier spectra of M signals at different observation angles

Fig. 3.14 shows the temporal and frequency-resolved correlations between M signals. The M signals at $15^\circ$ and $30^\circ$ are highly correlated at most frequencies. As the difference in $\beta$ increases, only low frequency information is in common between the $15^\circ$ and $45^\circ$ M signals and the temporal correlation level is significantly lower. The shared information around Strouhal 0.2 no longer exists between the $15^\circ$ and $60^\circ$ M signals. The time lags between the $15^\circ$ M signals and the others are positive and they increase along with $\beta$, indicating the acoustic information reaches smaller observation angle first. Similarly, the $90^\circ$ M signal is correlated with the $75^\circ$ signal and it is only correlated
with the $60^\circ$ M signal at low frequencies. The correlations between M signals at higher Mach numbers are very similar to Fig. 3.14. The best time lags have the same values regardless of Mach number, which is as expected with the acoustic propagation in a fixed microphone array configuration.

![Correlation between M signals (Ma = 0.6)](image)

Figure 3.14: Temporal (left plot) and frequency-resolved (right plot, color scaling of $[-0.35, 0.35]$) cross-correlations between M signals at different observation angles

**Information Propagation between Near- and Far-Field**

When the FF signals are correlated with the NF signals, the NF regions related to FF acoustics can be extracted and we can obtain insights into the information propagation from the NF to the FF. Firstly, we look at the correlations between the original UV signals and the M signals at different observation angles (Fig. 3.15). The regions communicating to the FF are highlighted in the peak correlation distributions. When the observation angle is small, the jet centerline region is very active (the left plot); as $\beta$ increases, the correlation level gradually decreases and the correlated regions no longer come in obvious groups. This is consistent with some existing research that identified
the centerline region as the source location of aft-angle noise [11]. Again, the active region and variation trend do not change with Mach number.

Figure 3.15: Peak correlation level distributions between UV signals and M signals at different observation angles ($Ma = 0.6$)

From the UV signals, the D signals are calculated and they are correlated with the $15^\circ$ M signal, which contains the most noise from coherent structures (Fig. 3.16). The D signals are located along the jet centerline in the top plots of Fig. 3.16 and the bottom plots are generated using D signals along the shear layers. At $Ma = 0.6$, only $Ma$ along the centerline is obviously correlated with the $15^\circ$ M signal (the left plot). The time lag of peak correlation is 10.1 ms, which is the same with the propagation time calculated using the speed of sound. At $Ma = 1$, the correlation levels are increased and $\dot{Q}$ and $\dot{d}et$ at the centerline are also correlated with the M signal, but rather weakly. The correlation levels with D signals along shear layers are around the level of background oscillation regardless of the Mach number. This further illustrates the need to isolate the NF centerline region for further analysis.

Fig. 3.17 is produced using the Filtered D signals obtained from the filtering window in
Figure 3.16: Correlations between D signals at different sensor locations and 15° M signals

Fig. 3.8. The absolute values of the fluctuating $|\dot{v}'|$ and $|\dot{\omega}'|$ are used since they are the most highly correlated with the M signals among their variations. When correlated with the 15° M signal, all the selected Filtered D signals show peak correlation at the lag of acoustic propagation time. Their correlation levels are greatly increased when compared with their non-filtered version (Fig. 3.16). The current window setting (Fig. 3.8) is found to produce the largest NF-FF correlation and the selected D signals are among those most correlated. When the observation angle increases, the correlation level decreases while the time lag increases, consistent with the increase in distance. At $\beta = 75°$, the space-filtering of D signals is not effective in improving the correlation level. This is consistent with the proposition that noise to large $\beta$ is from isotropic turbulence [11, 80]. The correlation pattern at $\beta = 75°$ sustains for a few periods just above the background noise level and the best time lag is difficult to interpret.

The information propagation between the Filtered D signals and the M signals at different frequencies are exhibited in Fig. 3.18. Very similar patterns can be observed between different Filtered D signals and M signals at a given observation angle. At
Ma = 0.6, the peak correlated frequency is around Strouhal 0.15, which is around the peak frequencies of the energy spectra of Filtered D and M signals. When Mach number increases, the correlation extends to higher frequencies, which could be due to the higher frequency content of the original UV signals (Fig. 3.3). As the M signal moves to larger observation angles, the correlation level is weaker and the peak frequency decreases. This could indicate the presence of a different kind of source. For further investigation of the noise from large coherent structures, the M signals at aft-angles (15° and 30°) are the focus.

The correlations between the K and M signals reveal their shared acoustic components (Fig. 3.19). The temporal correlations between K and M signals show similar patterns with those between Filtered D and M signals. Strong correlation is found between K signals downstream (esp. those at 7D to 8D) and M signal at small β. The time lag of peak correlation between K signal at 8D and 15° M signal is 9.7ms. It is very close to the acoustic propagation time, which is estimated as 9.9ms (the exact position of the...
Figure 3.18: Frequency-resolved correlations between the Filtered D signals and M signals at different observation angles

Kulite sensors is not available). As the K signals move upstream, the best lag decreases slightly, leading to smaller propagation speed than the speed of sound. This could be due to the interference of the hydrodynamic information of K signals. But since the correlation level peak is less obvious starting from $X/D = 6$, the explanation remains uncertain. The frequency-resolved correlations are also the strongest using K signals at 7D and 8D, which are located downstream of the end of potential core. They are also correlated with the $15^\circ$ M signal over wider frequencies and the higher frequency information gradually disappears as the K signal moves upstream. In further analysis,
we would focus on the K signals at $X/D = 7$ and $X/D = 8$.

Figure 3.19: Temporal (left plot) and frequency-resolved (right plot) correlations between K signals and M signals ($Ma = 0.6$)

Lastly, the effectiveness of multi-correlation is investigated using the Filtered D signal, K signal at $X/D = 8$ and $15^\circ$ M signal (Fig. 3.20). In the left plot, the best time lags between the three signals are extracted using the temporal triple correlation (the left plot in Fig. 3.20). Except for $|\dot{v}'|$, all the other D signals are most correlated with M signal at 10.1 ms, consistent with acoustic propagation time. Similarly, the best lag between the Filtered D and K signal is around 0.3 to 0.4 ms. This is the time lag difference between the best lag of K-M correlation and that of D-M correlation. Having identified the best time lag, the peak frequency can be found using the frequency-resolved triple correlation (the right plot in Fig. 3.20). The correlation level is stronger than that of the D-M or K-M correlations and the triple correlation extracts the information common to all three signals. Two apparent peaks can be found for all the Filtered D signals. The first is around Strouhal 0.15, which is also around the peak frequency of energy spectra of the signals. The other is around Strouhal 0.02, which
could be an artifact of the signal normalization (see section 2.1.3). In the next section, the multi-correlation will be used to look at the common information in the various signals in more details.

![Graph showing triple correlations](image)

Figure 3.20: Triple temporal (left plot) and frequency-resolved (right plot) correlations between the Filtered D signals, K signal at $X/D = 8$ and $15^\circ$ M signal ($Ma = 0.6$)

### 3.3 Algorithm Rationale and Testing for Noise-Associated Near-Field Events

In this section, an algorithm will be constructed that isolates the dominant contributions to the cross-correlations between the NF and the FF. These dominant contributions will be identified at specific time and frequency and they are the ‘events’ that relate the NF structures to FF acoustics.
3.3.1 Dominant Contributions to NF-FF Correlations

As introduced previously, the signals / time series can be decomposed into time and frequency using wavelet transform. In this section, the Morlet continuous wavelet is used due to its good frequency resolution. By taking the dot products of the complex Morlet coefficients of D and M signals and then keeping the real part, the contributions to the D-M correlations are obtained in time and frequency domain (Equation 2.30). Here the Morlet transformed M signal is shifted by a certain time lag $\tau$. Its value is set as the best mean lag of two-point correlation when looking for the individual contributions to the peak correlations.

Fig. 3.21 shows the excerpts between 100$ms$ and 300$ms$ of the time-frequency contributions to D-M correlations. Here the D signals are spatial filtered using the second window setting (Fig. 3.8) and the window is located along the jet centerline (the window centered at $y/D = -0.08$). The contributions obtained using the first window setting do not show dramatic difference and are thus omitted. The M signal is located at 15$^\circ$ observation angle and its time lag is obtained from Fig. 3.17. For a given D signal, the time-frequency contribution contains numerous patches that have the same sign with the corresponding two-point correlation (Fig. 3.17). Comparing different D signals, various correlation contributors are in common, such as the patches around 110$ms$ and 165$ms$, etc. This corresponds to the very similar spectral and statistical features shown by these D signals (Fig. 3.9 and 3.18).

The D signals in Fig. 3.21 are highly correlated with the 15$^\circ$ M signal at the acoustic propagation time ($\tau \approx 10.1ms$). Large correlation coefficients with the same sign of the
two-point correlation peak show up for all these D signals. This is not the case when no apparent correlation peak can be found between the D-M pairs. In the first two plots in Fig. 3.22, the same 15° M signal is used while the filtering window locations of $|\omega'|$ are
Figure 3.22: Time-frequency contributions to D-M correlations (For the first two plots, D and M signals are from the same dataset and the D signals are spatial filtered at different window locations; in the last plot, the D signal is the same as in the second plot while the M signal is at from a different dataset)

different from that in the third plot in Fig. 3.21. The radial location of the center of the filtering window is indicated in each plot and the reader is referred to Fig. 3.8 for the exact window setting. When the window is centered at $y/D = -0.81$ (the first plot), more negative (blue) correlation coefficients are present. But their magnitudes (absolute value) are relatively small (mostly less than 0.4). As a result, when averaged in time and frequency, the two-point correlation level is less than 0.04 for all values of time lag. When the window is centered at $y/D = 0.65$ (the second plot), both strong positive
(t ≈ 228ms) and negative coefficients (t ≈ 170ms) can be found. They will cancel each other out and lead to low two-point correlation level. A further comparison is made in the last plot in Fig. 3.22. The same |\dot{\omega}'| signal is used as in the third plot of Fig. 3.21, whereas the 15° M signal is obtained from a different dataset. Their correlation level is less than 0.04 (in contrast to Fig. 3.17) and the patterns in their correlation contributions have also changed dramatically. Thus the strong correlation coefficients (combined with other criteria) can be extracted as individual events that contribute to the correlation peak. The D-M signal pair from different datasets will be used to estimate the amount of false-positive events.

![Figure 3.23: Time-frequency contributions for comparison of 15° and 30° M signals: top plot - correlation coefficients between |\dot{\omega}'| and 30° M signals; bottom plot - coefficients between 15° and 30° M signals](image)

The correlation without frequency resolution showed that the 15° and 30° M signals have similar features (Fig. 3.17 and 3.18). The patterns observed in correlation
contribution are consistent when the M signal that correlate with the D signal is changed from $15^\circ$ to $30^\circ$ (compare the third plot in Fig. 3.21 and the top plot in Fig. 3.23). All the strong red patches apparent in Fig. 3.21 also exist in Fig. 3.23, although some of them are a little weaker. The bottom plot in Fig. 3.21 shows the correlation coefficients between the $15^\circ$ and $30^\circ$ M signals. Note that the time axis is shifted such that it is synchronized with the top plot (the $|\dot{\omega}'|$ and $15^\circ$ M signals have a $10.1ms$ time lag). The coefficient magnitude is much larger as the two M signals are much closer to each other. Despite the difference in the two plots, some of the strong correlation contributors are in common, e.g., around $100ms$ in the top plot and $110ms$ in the bottom plot, around $260ms$ in the top plot and $270ms$ in the bottom plot, etc.

![Figure 3.24: Time-frequency contributions to D-M multi-correlations (the D signals are spatial-filtered along the jet sideline in the first plot and along the centerline in the second plot)](image)

The similarity between the correlation coefficients between the various D and M signals
indicates it may be beneficial to use multi-correlation and look for the patterns in common to multiple signals simultaneously (Equation 2.33). The real part of the Morlet coefficients of the D/M signals is used to simplify complex multiplication and a Gaussian filter is applied to the correlation coefficients to focus on the envelopes of individual oscillations. Here the Morlet coefficients are not normalized and the absolute values are taken of the real part coefficients. The readers are referred to the section 2.2.2 for more details. Fig. 3.24 shows the multi-correlation contributions between multiple D and M signals. Since more D signals than M signals are used, this will tend to place more weight on near-field kinematics and Equation 2.33 is thus adjusted:

\[
\xi(f, t) = G \left( \prod_{i}^{N_D} |\mathcal{R}\{\tilde{p}_{Di}(t, f)\}|^{\frac{1}{N_D}} \prod_{i}^{N_M} |\mathcal{R}\{\tilde{p}_{Mi}(t, f)\}|^{\frac{1}{N_M}} \right).
\]  

(3.3)

In the upper plot, the D signals are spatial-filtered along the jet sideline and the second plot is obtained with signals along the jet centerline. Much larger coefficient magnitudes are found with the centerline D signals and the large coefficients, which is consistent with the comparison by pair-wise correlation coefficients. The patterns by D signals at the centerline (dark grey patches in the bottom plot) are at the same time and frequency with the patterns in Fig. 3.21.

In the last section, the K signals have been used to track the information propagation between the NF and FF. Fig. 3.25 shows the correlation contributions between one pair of K-M signals. The variation in the K signal locations (downstream of the end of the potential core) and the M signal observation angles (aft angles) does not produce much difference in the scalograms. However, the patterns do not resemble greatly those with...
Figure 3.25: Time-frequency contributions to K-M correlations (top plot: K at X = 7D v.s. 15° M signals; second plot: K at X = 8D v.s. 15° M signals; last plot: K at X = 8D v.s. 30° M signals)

D-M correlations (Fig. 3.21). Fig. 3.26 shows the multi-correlation contributions between D-K-M signals (the normalized K signal coefficients are included in Equation 3.3). The difference between the sideline and the centerline is confirmed. The strong correlation contributors seem to appear at similar time-frequency instants with those of multiple D-M signals (Fig. 3.24). However, the shape of some strong contributors is changed (e.g. around 160ms) and the highlighted regions in the scalogram are shifted towards lower Strouhal number. This seems to indicate the hydrodynamic component is
more dominant in the K signals. The K signals may need to be decomposed before being included into the multi-correlation for event extraction.

![Figure 3.26: Time-frequency contributions to D-K-M multi-correlations (the D signals are spatial-filtered along the jet sideline in the first plot and along the centerline in the second plot)](image)

3.3.2 Event Extraction Algorithm

In this section, the algorithm is explained which extracts the individual events contributing to the NF-FF correlations. First presented will be the event extraction for one pair of D-M signals. This will be extended later to multiple signals. The events of interest are the time instants of the D and M signals that contribute to the peak of the D-M correlation. From the D-M correlation, the best time lag $\tau_0$ and the sign of the peak correlation $S_0$ (either positive or negative) can be obtained. From the correlation, we can also estimate the period of the most dominating structure by measuring the time
extent between the correlation peak and the zero-crossing next to the peak. This time extent is estimated as one quarter of the period and half the period is defined as $\Delta \tau_0$, the usage of which will be explained later.

As shown in the last section, an event candidate is a main correlation contributor in the time-frequency domain. Starting with the correlation coefficient scalogram (the M signal is shifted by $\tau_0$), e.g. Fig. 3.21, each point $(t_0, f_0)$ is scrutinized to see if they satisfy a set of criteria.

Firstly, an event should have relative large correlation coefficient (main correlation contributor), i.e., large absolute value in the scalogram:

$$|\xi(t_0, f_0)| \geq C_\xi; \quad (3.4)$$

Secondly, as the events are related to FF acoustics, they should be loud or have relatively large magnitude in the FF:

$$|\tilde{p}_M(t_0, f_0)| \geq C_M; \quad (3.5)$$

Here $\tilde{p}_M$ is the complex Morlet coefficient of the M signal. The third criterion is added for the sake of multi-correlation. The multi-correlation coefficient is always positive, whereas a positive correlation contribution should have the same sign with the two-point correlation, i.e., if $S_0$ is negative, the coefficient should be negative and vice versa. Thus the sign of the correlation between local D-M excerpts is used to enforce this. A local D / M excerpt is the real part of the Morlet coefficients centered at $(t_0, f_0)$
with $N_T$ periods on each side:

\[
D(t_0, f_0) = \mathcal{R}\{\tilde{p}_D(t_0 - N_T T_0 : t_0 + N_T T_0, f_0)\},
\]

\[
M(t_0, f_0) = \mathcal{R}\{\tilde{p}_M(t_0 - N_T T_0 : t_0 + N_T T_0, f_0)\}.
\]

The third criterion states that the correlation of D-M excerpts should have the right sign when shifted by the mean lag:

\[
\text{sgn}[\mathcal{X}(\tau_0)] = S_0, \quad \text{where } \mathcal{X}(\tau_0) = \int_{t_0 - N_T T_0}^{t_0 + N_T T_0} \mathcal{R}\{\tilde{p}_D(t, f_0)\} \mathcal{R}\{\tilde{p}_M(t + \tau_0, f_0)\} dt.
\]

Since the lags for individual events are likely to fluctuate around the best mean lag $\tau_0$, slight shift of the D-M excerpts around $\tau_0$ may improve their coherence. This range in which the optimal local lag is looked for should not deviate too much from $\tau_0$. It is set as half the local period $T_0/2 = 1/2f_0$ or $\Delta\tau_0$ away from $\tau_0$, whichever value is smaller.

As an event relating NF and FF, it is reasonable to assume that it should assume similar patterns in NF and FF. Thus the fourth criterion requires the local D-M correlation should be large enough when the excerpts are shifted by the best local lag $\tau_l$:

\[
\mathcal{X}(\tau_l) \geq C_c, \quad \text{where } \tau_l \in [\tau_0 - \Delta\tau_0, \tau_0 + \Delta\tau_0]
\]

Following the procedure, the possible event candidates that satisfy all the criteria have been extracted. An example is shown in Fig. 3.27. The event candidates follow the
positive correlation coefficients (the two-point correlation peak is positive), while there may be some ‘false negatives’ due to the criterion of local correlation \( (C_c) \), e.g. the red patch around \( t = 160\, ms \). The selection of the criterion values will be discussed later. Some event candidates appear to belong to the same group, such as those around \( t = 150\, ms \) and \( St = 0.3 \). It is desirable to separate the candidates based on which groups they belong to and keep one ‘best’ event from each group.

This is achieved by first recording the local maxima in the scalogram. A local maximum is the point at \((t_c, f_c)\) with the largest coefficient (absolute value) within a box of size \( \Delta t = T_c/2 \) and \( \Delta f/f_c \approx 1/2z_0 \). Here \( T_c \) is the local period. \( z_0 \) is the envelope factor of Morlet wavelet and it can be used to estimate the frequency resolution. The local
maximum is used as an estimate of the center of a given group (Fig. 3.28). The distance between each event candidate and a specific local maximum is calculated, which is defined as:

$$L = \sqrt{\left(\frac{t_0 - t_c}{T_0}\right)^2 + \left(\frac{f_0 - f_c}{f_0}\right)^2 z_0^2}. \quad (3.10)$$

The center with the smallest distance is assigned to the event candidate and thus the candidates are sorted into groups based on their distance from the center. The ‘best’ event of a certain group is chosen as the one with the largest correlation coefficient $|\xi(t_0, f_0)|$ (absolute value), i.e. contributing the most to the D-M correlation. Fig. 3.29 is an example of the extracted events (marked as green circles) that relate the NF kinematics ($|\dot{\omega}'|\) to FF acoustics ($15^\circ$ M signal). The reason that the algorithm didn’t start with the local maxima from the beginning is that about half the events extracted are not local maxima.

Figure 3.29: The events relating $|\dot{\omega}'|$ and $15^\circ$ M signals (marked by green dots)

So far the event extraction algorithm has been explained. During the process, several quantities are introduced as the criteria for event selection, the values of which have not yet been specified. These include: $C_\xi$ (correlation coefficient criterion), $C_M$ (M signal magnitude criterion) and $C_c$ (local correlation criterion). In addition, a value for $N_T$ is
needed, which defines half the length of local D/M excerpt. For Fig. 3.29, the values of
the criteria is set as $\bar{p} + 1.5\sigma_p$, where $p$ stands for the quantity in question. For example,
$p$ stands for the correlation coefficient for the $C_\xi$ criterion. This value is chosen as an
initial guess in compliance with the choice of some other researchers, who have looked
at event selection based on FF pressure magnitude [47, 67]. For Fig. 3.29, the value of
$N_T$ is set as 2.5, which is half the number of oscillations of the Morlet continuous
wavelet ($z_0$).

![Figure 3.30: The dependency of FPER on different criteria (the values used for Figure
3.29 are represented by the red dotted lines)](image)

The effectiveness of the algorithm is partly controlled by the values of the criteria.
Stricter criteria may lead to fewer ‘false-positives’ but may also result in more ‘false-negatives’. To investigate the values of the criteria and to obtain an estimate of the false-positive events, the algorithm is applied to D-M signal pairs that belong to the same or different datasets. The number of ‘events’ extracted by signals from different datasets is an estimate of the false-positives or ‘noise’. Its ratio with the number of events from the ‘right’ data (the same dataset) defines the False-Positive-to-Event Ratio (FPER):

\[
FPER = \frac{N_{false-positive}}{N_{event}}.
\] (3.11)

Using \(|\dot{\omega}'|\) and 15° M signals as an example, a range of criteria is tested by comparing the FPER (Fig. 3.30). As expected, the number of events decreases as the criterion is stricter (larger \(N_T\) leads to longer excerpts and smaller correlation values, i.e. stricter criteria). However, when looking at the trend of FPER, there seems to be an optimal criterion value. Also taking into account the observation that when signals are not correlated, the coefficient magnitude decreases, e.g., Fig. 3.22, a fixed set of values is preferred over a varying one, so as to better compare the influencing factors, such as D signal location. Based on the FPER trends, the criteria are set as \(C_{\xi} = 0.1, C_M = 0.43, N_T = 3, C_c = 0.68\). This produces 166 events with \(FPER = 52\%\) for \(|\dot{\omega}'|\) and 15° M signals.

When only one D and one M signals are compared, the smallest FPER is about 50%, which is less than satisfactory. To reduce FPER, the algorithm is extended to take advantage of the multi-correlation. This way, the patterns we are looking for are in common among more signals, reducing the number of chance matches or false-positives.
3.3.3 Evaluation of Algorithm Performance

In the last section, the algorithm performance has been evaluated firstly by signals from the wrong dataset. Later the algorithm evaluation is performed by lagging the D-M signals incorrectly. These two methods are basically the same: they use experimental signals that do not correlate to estimate the level of chance matches. In this section, the
algorithm performance will be evaluated more systematically by two other different approaches.

The basic idea of the event extract algorithm is to look for local patterns that are in phase and shared by the NF and the FF. It may be assumed that the phases of the original signals are a key component to the NF-FF communication. One approach of algorithm evaluation is to randomize the phases of the experimental data. There are two ways of achieving this. The first is to replace the original phase by that of a White Gaussian Noise (WGN) signal:

\[
p_W(t) = \mathcal{F}^{-1}\{\mathcal{F}(\text{wgn}) \frac{||\mathcal{F}(f)||}{||\mathcal{F}(\text{wgn})||}\}. \tag{3.12}
\]

Here the WGN signal is generated by the Matlab WGN generator. The \( p(t) \) signal may be either D or M signal or both. Using the same algorithm and signal combination with that of Fig. 3.29, less than 5 ‘events’ are extracted when all the D and M signals are replaced by WGN, i.e., \( FPER < 2\% \).

The second approach for phase randomization is to generate a set of random phase angles and combine this with the magnitudes of the original signal:

\[
p_r(t) = \mathcal{F}^{-1}\{||\mathcal{F}(p)|| \cdot e^{i2\pi\cdot\text{rand}}\}. \tag{3.13}
\]

Here \( \text{rand} \) stands for the random number generator of Matlab and this can be applied to both D and M signals. After randomizing all the D and M signals, the algorithm with the same criteria produces similar level of false-positives with the WGN approach.
The second approach of algorithm evaluation is through constructing synthetic signals of which the events are known in advance. The events / noise sources are simulated as Morlet wave packets:

\[ f_s(t) = S_0 \cdot A \cos[2\pi(t - t_s)f_s]e^{-2|\pi f_s(t-t_s)/\gamma_0|^2} \]  

(3.14)

Here \( S_0 \) ensures the simulated signals have the same signs with those of the original. \( A \) specifies the amplitude of the source and it is set as the largest around \( St = 0.2 \), simulating the spectra of actual data. The time and frequency of occurrence of the sources \( (t_s \text{ and } f_s) \) are recorded. \( f_s \) varies between \( St = 0.005 \) and \( St = 1.13 \), which is the frequency range of analysis with the experimental data. At each frequency band, one source is generated at a random time \( t_s \). Then all the sources are collected to generate a ‘clean’ signal, which is duplicated and shifted by \( \tau_0 \) for six ‘clean’ signals for event extraction (4 D signals and 2 M signals are used for Fig. 3.29).

\[ \text{Figure 3.32: Left plot: simulated signals at different SNR values; right plot: the Fourier spectra of the simulated source signal and of the background noise (SNR } \approx 1) \]
An actual signal is composed of both the events of interest and background noise, which is simulated by WGN signal:

\[
    f_n(t) = \int_{f_s-\Delta f}^{f_s+\Delta f} \mathcal{F}(wgn)e^{j2\pi ft}df, \quad \text{where} \quad \Delta f = f_s/z_0.
\]  

(3.15)

For each source, a WGN signal is generated and band-pass filtered around the frequency of the source \(f_s\). The WGN signals for all the sources are combined as the background noise signal. For each simulated signal, a noise signal is generated independently. In the left plot in Fig. 3.32, the sources and the noise are combined to yield the simulated signal, while the right plot shows the Fourier spectra of the sources and the noise. The Signal (source) to Noise ratio is estimated by the energy level of the source and noise signals: \(\text{SNR} = \sigma_{\text{source}}^2/\sigma_{\text{noise}}^2\). With \(\text{SNR} \approx 1\), the simulated signal is rather chaotic apart from some oscillations at very low frequencies. As the SNR increases, the large oscillations gradually disappear.

Figure 3.33: Algorithm evaluation by synthetic signals: the actual sources are marked as magenta crosses while the extracted are shown as green dots

Fig. 3.33 shows the extracted events (green dots) and the sources (magenta crosses) for the synthetic signals of \(\text{SNR} \approx 1\). There exist about 7% false positives, e.g. around
\([t, St] = [210ms, 0.5]\). About half of these are very close in time and frequency to the extracted events, such as the one around \([t, St] = [200ms, 0.5]\), or the one around \([t, St] = [450ms, 0.15]\). This may be due to the frequency resolution of the wavelet analysis and the results may be further improved with better algorithm for event categorization by group. The level of false negatives, or the percentage of sources that the algorithm fails to catch is only about 5%. Most of these are at very high frequencies. This is expected since the energy level of the noise is much higher than that of the source at high frequencies (the right plot in Fig. 3.32).

Figure 3.34: The variation of the percentage of false events (blue curve) and the percentage of missed sources (orange curve) when the SNR of synthetic signals changes

In Fig. 3.34, the algorithm performance is evaluated over a range of SNR values. For each SNR, one set of six synthetic signals are generated and the algorithm is applied for event extraction. The list of extracted events are compared with the list of sources and those close enough (\(\Delta t < 2T_0\) and \(\Delta f/f_0 < 2/z_0\)) are regarded as a match. As the SNR decreases, the source is gradually contaminated in greater extent by the background noise and as expected, the percentage of false-negatives decreases. However, the number
of false-positives remains smaller than 10% regardless of the SNR. This provides further proof to the effectiveness of the event extraction algorithm and confirms that the majority of the events are the link between NF-FF communication.

3.4 Properties and Features of the Noise-Associated Near-Field Events

3.4.1 The Statistical Properties of the Acoustic Events

In this section, the properties and features of the events will be investigated, which are extracted using six signals ($Ma$, $|\bar{\nu}|$, $|\bar{\omega}|$, $\dot{Q}$, $15^\circ M$ and $30^\circ M$ signals). The D signals are spatial-filtered using the thinner window setting in Fig. 3.8 (red dashed lines). The events extracted using the larger window (black solid lines in Fig. 3.8) are very similar and thus omitted.

Starting with the events extracted with the $Ma = 0.6$ data set, the left plot in Fig. 3.35 shows the probability distribution of the events’ frequency of occurrence. The majority of the events occur around Strouhal 0.2, which is consistent with the spectral peak of the FF acoustics (Fig. 3.13) and it also agrees with the finding of many researchers [25, 12]. Assuming that the events last for similar amount of period ($2 \cdot N_T$), the events at higher frequencies will cover a shorter time compared with those at lower frequencies. Thus it is beneficial to look at the distribution over frequency of the time coverage instead of the number of events. The time coverage also provides an estimate for the events’ level of intermittency. The right plot in Fig. 3.35 is generated by multiplying the number of
events by the event length: $2 \cdot N_T \cdot T_0$, where $N_T = 2.5$ (the value used in event extraction algorithm) and $T_0$ is the local period. The intermittency level distribution is shifted slightly towards lower frequency and the peak is found at $St = 0.15$, which is the same with the spectral peak of the Filtered D signals (Fig. 3.8). Across the range of dominant frequencies, around $0.13 \leq St \leq 0.25$, the intermittency level is slightly above 10%, which is consistent with some other researchers’ estimation [64].

![Frequency Distribution of Events](image1)

![Estimation of Level of Intermittency](image2)

Figure 3.35: Frequency distribution of the extracted events and the level of intermittency at each frequency level ($Ma = 0.6$, filtering window for D signals at $[X/D,Y/D] = [6.87, -0.08]$)

![2D Property Distribution of Events](image3)

![2D Property Distribution of Events](image4)

Figure 3.36: 2D distribution of frequency and magnitude ($|\tilde{p}_{Ma}|$) of the extracted events ($Ma = 0.6$, filtering window for D signals at $[X/D,Y/D] = [6.87, -0.08]$)
The Morlet coefficient magnitude ($|\tilde{p}_M|$) of the M signals at the occurrence time of the extracted events provides an estimate of the FF ‘loudness’ of the events. Fig. 3.36 shows the 2D probability distribution of the magnitude and frequency of the extracted events. When the frequency is coupled with the magnitude of the $15^\circ$ M signal, the first peak is found around $St = 0.15$ and the second peak is around $St = 0.18$ with larger magnitude. The general shape of the 2D distribution resembles that of the FF spectra (Fig. 3.13). When the $30^\circ$ M signal magnitude is used instead, the frequency peak becomes slightly higher and the magnitude becomes smaller. These again correspond to the trend observed with the FF spectra (Fig. 3.13).

![Figure 3.37: The variation of the extracted events by D signal filtering window location ($Ma = 0.6$): left plot - variation of the number of events; right plot - variation of intermittency level](image)

For Fig. 3.35 and 3.36, the filtering-window of the D signals is placed along the jet centerline and downstream of the end of the potential core ($X/D = 6.87$, see Fig. 3.8). When the location of the filtering window changes, the number of extracted events varies as well (the left plot in Fig. 3.37). The X-axis in the plot shows the window center location in the streamwise direction and the Y-axis is the center location in...
radial direction. The number of events increases as the filtering window moves
downstream and towards the centerline, consistent with the trend observed with
two-point NF-FF correlation (Fig. 3.15). More events are extracted on the side of the
centerline that is farther away from the FF microphones (negative $Y$ locations), the
reason of which is not yet understood. When the windows are located at $Y/D = -0.44$,
the number of events first increases and then decreases in the downstream direction,
which could be due to the collapse of the potential core between $X/D = 6$ and 7. The
right plot in Fig. 3.37 is obtained with filtering windows at the same streamwise
location ($X/D = 6.87$) but different radial locations. The same active frequency range is
found with all the windows.

Figure 3.38: The dependency of the extracted events on Mach number: left plot - the
number of events (all filtering windows for D signals at $X/D = 6.87$); right plot - the
intermittency level (all filtering windows at $[X/D, Y/D] = [6.87, -0.08]$)

The datasets acquired at different Mach numbers enable the investigation whether the
noise-associated events are of the same nature. Firstly compared is the number of
events extracted (the left plot in Fig. 3.38). The filtering windows of the D signals are
at the same streamwise location while the radial location varies. The same trend is
Figure 3.39: The dependency on Mach number of the magnitude ($|\tilde{p}_M| / Ma^4$) distribution of the extracted events ($Ma = 0.6$) observed at all Mach numbers: more events along the centerline and at the side further away from the FF microphones. The right plot in Fig. 3.38 shows the intermittency level distribution of the extracted events across Strouhal number. The same filtering window is used for all Mach numbers. Events at higher Mach number occur at higher frequencies, but they are non-dimensionalized by the jet stream velocity ($Ma \cdot c_0$) and nozzle diameter to the same range of Strouhal numbers (around $0.14 \leq St \leq 0.26$).

Lastly, the magnitude distributions of the events are compared to see if the Mach number is an influencing factor (Fig. 3.39). The magnitude of the Morlet coefficients of M signals actually increases with Mach number, which is to be expected since the FF acoustic spectra contains more energy at higher Mach numbers. However, when the FF magnitude is normalized by the fourth order of velocity ($Ma^4$), the magnitude distribution collapses to the same trend at both $15^\circ$ and $30^\circ$ observation angles. This supports one of the conclusions from the Lighthill’s analogy, which reasons that the FF acoustic energy scales with the eighth order of velocity [110]. From the statistical
distributions of the event properties, we can conclude that the sources of FF noise to aft angles are placed at the end of the potential core at the centerline. The sources of aft angle noise are large scale structures and they are present and of the same nature across a range of Mach numbers.

### 3.4.2 Event-Based Filtering and Conditional Averaging

![Figure 3.40: One example of the extracted events: the solid and dotted lines show the real part and the norm of the \( \tilde{p}_M \) (\( Ma = 0.6 \), filtering window for D signals at \([X/D, Y/D] = [6.87, -0.08]\))](image)

In the last section, we will focus on the D and M signals around the event occurrence time and look for the features shared by most events. For all the figures in this section, the dataset is at Mach number 0.6 and the filtering window for D signals is placed along the jet centerline after the end of the potential core (\( X/D = 6.87 \)). Fig. 3.40 shows one example of the extracted events. The local excerpts around the event occurrence time are shifted by the optimal local lag and they extend 2.5 period on each side of the occurrence time. The solid lines represent the real part of the Morlet coefficients of the signals while the dotted lines represent the norm or the envelope. The six signals are in
phase with each other and show highly similar oscillatory patterns.

Figure 3.41: The correlations between condition-filtered D and M signals based on events ($Ma = 0.6$, filtering window for D signals at $[X/D, Y/D] = [6.87, -0.08]$)

Having identified the time and frequency of occurrence of the events, we can condition-filter the D and M signals based on the events and filter out the ‘noise’ that do not contribute to NF-FF communication [31, 75]. This is performed by taking the Morlet transform of the signals and setting the coefficients outside the range of the events to zero. The extent of an event is set as five local periods in time and five scales in frequency ($f_0 \pm \Delta f$, where $\Delta f/f_0 = 2/z_0$). The filtered Morlet coefficients are inverse-transformed to obtain the filtered signals. The upper plot in Fig. 3.41 shows the correlation between the condition-filtered D signals and the condition-filtered 15° M signal. A correlation peak can be found with all the D signals at the acoustic propagation time (about 10ms). Comparing this with the correlation between the original D and M signals (Fig. 3.17), the correlation levels are greatly increased from 20% to nearly 70%. The lower plot in Fig. 3.41 shows the correlation between the condition-filtered M signals at aft-angles. As a result of removing the non-contributing
components, the correlation level is also greatly increased comparing with that of the original signals (Fig. 3.14).

![Figure 3.42: Condition-averaged raw signals (shifted by the best mean lag $\tau_0$) based on the extracted events ($Ma = 0.6$, filtering window for D signals at $[X/D, Y/D] = [6.87, -0.08]$)](image)

Since various evidence has indicated that the majority of the extracted events are the link between near-field structures and far-field acoustics, condition-averaging is applied on the raw signals in the hope of discovering the shared features of the events. The upper plot of Fig. 3.42 shows an excerpt of the raw D and M signals that are shifted by their best mean lag $\tau_0$. At some instants, the signals show roughly similar patterns, if the sign difference is ignored, such as around $t = -2ms$, while in most cases, it’s difficult to spot the shared pattern. In the lower plot of Fig. 3.42, the raw signals are truncated around the identified event occurrence time (extending 7ms on each side) and averaged. The signal excerpts that have negative magnitude at the occurrence time are flipped for better comparison. The condition-averaged signals show similar patterns that contain one strong peak and two weaker troughs. Other two secondary peaks may also be present. The frequency of the oscillation is approximately 950Hz or $St = 0.24$. 
Figure 3.43: Condition-averaged band-pass filtered signals (shifted by the best mean lag $\tau_0$) based on the extracted events ($Ma = 0.6$, filtering window for D signals at $[X/D, Y/D] = [6.87, -0.08]$)

The raw signals contain the information at all frequencies while the most dominant structures contributing to far-field noise are mostly at relatively low frequencies. In Fig. 3.43, the raw signals are band-pass filtered between $0.14 \leq St \leq 0.26$ before being condition-averaged. This frequency range has been found as the most active for the extracted events in the last section. The top plot in Fig. 3.43 shows the excerpts of the band-pass filtered and shifted ($\tau_0$) D and M signals. There seem to exist lots of random oscillations whose shared patterns are brought out through condition-averaging (the bottom plot in Fig. 3.43). The condition-averaged signals excerpts assume the pattern of wave packets that oscillate around the occurrence time for a few periods. This wave pattern has some similar features with those identified by some other researchers [64, 56].
3.4.3 Event-Conditioned Flow Field

In the previous section, event-conditioned D signals showed strong patterns associated with FF noise. We build on this by mapping the filtered signals back into snapshots of the spatial-filtered flow field. Firstly the time series around the identified events are inspected. When looking at the local excerpts of the raw signals, over half of the D and M signals show similar large oscillatory patterns without filtering. About 40% events are at the local maxima of FF pressure signals and correspond to local extrema of D
Figure 3.45: One example of the identified event that tracks the local minimum of acoustic pressure: upper plots - excerpts of raw / filtered signals around event occurrence time; lower plots - the snapshot of spatial-filtered Mach number / vorticity at the event occurrence time.

Signals (Fig. 3.44); about 35% are at the local minima of M signals and local extrema of D signals (Fig. 3.45); about 15% is half-way around the zero-crossings; for the rest, the signals seem scrambled and may be falsely captured. In the upper-left plot of Fig. 3.44, the shared patterns of D and M signals are interrupted by scrambled intervals around 11ms to 14ms. After applying Fourier bandpass filtering, the M signals show a packet of waves before 13ms that is not present with the D signals (upper-right plot of Fig. 3.44). The snapshots of the velocity and vorticity field (spatial filtered to exhibit large-scale structures) that correspond to an event with pressure maxima show slanted
and stretched jet stream passing across the jet centerline at or around the event occurrence time (lower plots of Fig. 3.44). However, when coupling with the events at pressure minima, the flow patterns are the opposite: very tranquil and without apparent distortion along the centerline (black box, Fig. 3.45). The raw and bandpass filtered signals on the other hand, exhibit similar localized oscillatory shapes with those at the pressure maxima. It is possible that the noise radiation is not related to the flow activities at a specific instant, but rather the interruption of quasi-periodic flow movements, thus the truncated oscillation patterns. Further analysis is needed to test if the hypothesis is valid.

### 3.5 Discussion

In this chapter, an experimental dataset of subsonic axisymmetric jet has been analyzed, which includes NF PIV snapshots, NF pressure and FF pressure signals. The main results include:

- Several Diagnostic signals are constructed from the PIV snapshots and the NF-FF correlation level is increased from 10% to 20%.
- The events are isolated in time and frequency which contribute most to the NF-FF information propagation.
- The event extraction algorithm is improved using multi-correlation, and the percentage of false matches is less than 10% as evaluated using synthetic signals.
- Through event-conditioned filtering, the shared patterns of the D signals are identified and the flow activity possibly related to noise production is depicted.
The stated goal of the analysis was to identify the sources of noise production. Time constraints stop us short of unraveling the mechanism; it is possible that a three-dimensional view of the near-jet or an alternative flow representation is needed to reveal the answer. However, the properties and patterns of the events are highly consistent with existing theories and the flow patterns have much similarity after being categorized based on event features. The results in this chapter have exhibited the value of simultaneous and time-resolved data in the near- and far-field, of the time-frequency analysis and related non-linear filtering, and of expanding conventional correlation techniques to include frequency resolution and multiple signals.
Chapter 4

Near-Field Flow Structures and
Acoustic Radiation of a Two-Stream Supersonic Jet

4.1 Description and Validation of the Numerical Datasets

In this chapter, we turn our attention to a supersonic two-stream jet, which is a simplified model of the flow coming from a three-stream nozzle. Fig. 4.1 is an image of the three-stream engine released by the U.S. Air Force. The red line represents the hot air that goes through the core of the engine, which is the typical path for a turbojet engine. The yellow line represents the fan flow (the traditional bypassed flow), and the blue line shows the flow path of the third stream. In the test configuration of the data,
the red and yellow streams are mixed and form the core jet. The exit of the jet is
flowing over a surface due to integration of the propulsion system into the aircraft. A
wall jet is formed (blue stream) and it combines with the core jet to form a two-stream jet from the three-stream engine.

Figure 4.1: Example of a three-stream engine (courtesy of C. J. Ruscher)

The data was obtained through a Large Eddy Simulation (LES) by Ruscher [111]. Fig. 4.2 shows the nozzle geometry (single expansion ramp nozzle) and the computational domain. As mentioned earlier, the core stream and the fan flow were assumed as perfectly mixed; the simulation focuses on the influence of the wall jet and the aft deck. Two sets of numerical data were available: the first set was sampled at 80 kHz and had 40 million nodes (referred to as Test 80K40M), while the second set was sampled at 200 kHz and had 60 million nodes (Test 200K60M).

The simulation was performed using unstructured tetrahedral mesh. The domain extended 70 diameters downstream, 17 diameters upstream, and 30 diameters in the radial direction. A hydraulic diameter was defined for the rectangular nozzle using the following equation

\[ D_h = \frac{2HW}{H+W}, \]  

\[ 101 \]
where $H$ and $W$ are the height and width of the nozzle exit. In the thesis, the Near-Field (NF) region of the jet is analyzed, which extends 27 diameters downstream. The resolution was about 200 nodes per diameter at the lip and about 30 nodes per diameter at the end of the NF region.

The flow field was simulated using the Naval research laboratory’s (NRL’s) Jet Engine Noise Reduction (JENRE) code. JENRE used LES to solve for the near-field flow and employees an unsteady compressible flow solver. The solver used an edge-based formulation to handle the flux integration and limiting algorithms. The Taylor-Galerkin finite element method with second order accuracy for tetrahedral cells was used. Finite element flux corrected transport was used as an implicit subgrid stress model. More details about JENRE and its application to jet noise can be found in the works by Liu et al. [81] and Ruscher et al. [111]

Two separate stagnation conditions were set upstream of the jet exit, one for the core jet and one for the wall jet. The nozzle pressure ratios are 4.25 in the core jet and 1.89 in the wall jet, giving Mach numbers of 1.6 ($Ma_1$) and 1.0 ($Ma_3$) respectively. Both
streams are unheated. The walls of the nozzle are treated with a slip wall condition and the outer boundaries are set to match ambient conditions. To mitigate reflections from the boundaries, a sponge zone is inserted $10D_h$ from the far-field boundaries, which is well outside of the resolved region.

The data were interpolated from the simulation onto a three-dimensional Cartesian mesh. The interpolated mesh has a spacing of 32 nodes per diameter. In the thesis, we analyze the near-field pressure and velocity data extracted at various planes (Fig. 4.3). The pressure signals are available at the plane of symmetry ($XY0$), the horizontal plane ($XZ0$) and 4 transverse planes ($YZ1, 2, 4$ and $8$); the velocity data is extracted at the $XY0$ plane. $YZ1$ is located $1D_h$ downstream of the nozzle exit; $YZ2$ is $2D_h$ downstream of the nozzle, around the end of the deck edge; and $YZ4$ and $YZ8$ are named likewise. The planes extend $\pm 2D_h$ in the y and z directions and $0D_h$ to $10D_h$ in the x direction. For each data plane, 10,300 samples were acquired ($51.5ms$ of duration). For the entire resolved region ($X = 27D_h$, Fig. 4.2), the flow through time is about $10ms$ (5 dimensionless time). Based on the grid resolution, the cutoff frequency is about $50kHz$ for Test 200K60M for the whole resolved region.

The validation study for Test 80K40M was performed using both experimental and numerical datasets by Magstadt et al. [86] and Ruscher et al. [111]. The experimental dataset was collected at the Syracuse University anechoic chamber by Magstadt et al. for the same flow conditions. The locations of strong shock structures were obtained from the simulations and from Schlieren images. The largest location difference in both the $XY0$ and the $XZ0$ planes is less than $5%D_h$. The far-field acoustics of the
simulation were also compared with the noise measurements from the experiment. The same trend was observed from the OASPL and less than 1\text{dB} difference was found in the peak noise direction, with the largest discrepancy less than 3.5\text{dB}. Test 80K40M missed a strong peak around 34kHz which showed up in the experiment. This high-frequency information is recovered in the near-field using Test 200K60M, which will be the focus of this chapter.

The grid independence study for Test 80K40M was first performed by Ruscher et al. and the results showed good agreement with that of the same sampling frequency and 12 million grids [111]. The mean velocity distributions showed less than 0.1\% mean error and about 1\% RMS error for all three velocity components. The higher order statistics were compared using proper orthogonal decomposition (POD). The large scale structures extracted by the first six POD modes showed no observable difference. In addition, the SPL of far-field acoustics had less than 1\text{dB} difference, which is less than the 2\text{dB} uncertainty level.
The validation for Test 200K60M is first conducted by comparing its results to those of Test 80K40M [65]. The difference observed is the better resolution of certain finer structures, especially those at very high frequencies, above \(30kHz\) approximately.

Secondly, the data of Test 200K60M is low-pass filtered at \(30kHz\) and the results of the filtered signals are compared to those of Test 80K40M. The comparison covers all the statistical and correlation results in this chapter and no obvious difference has been observed. The emphasis of the thesis is put on the information below or around \(30kHz\), since even finer grid and higher sampling frequency may be needed to fully resolve the information at even higher frequencies.

### 4.2 General Description of the Near-Field Flow

#### 4.2.1 General Flow Description and Definitions of Key Structures

From the mean pressure distribution (left plot in Fig. 4.4) and the pressure fluctuation distribution (right plot in Fig. 4.4), some organizing features of the flow emerge. In the figures of this chapter, the locations of the nozzle and the deck are indicated by solid or dotted brown lines. The pressure fluctuation distribution is plotted in logarithmic scale (the color scaling is the exponent with the base equal to 10) since the pressure fluctuations vary greatly in the order of magnitude. The shock structures are very clear in both the mean and fluctuating pressure distributions, while the fluctuations in the shear layers are highlighted by the pressure fluctuations. The large pressure fluctuations
in a shock can be due to the fluctuations of its intensities or its actual position. As the
flow evolves downstream, the shock structures gradually weaken and the shear layers
expand. As the flow reaches the YZ8 plane, the pressure fluctuations indicate no
distinct structures and the whole flow is engaged in apparent mixing.

Figure 4.4: Shocks and other structures in the near field: mean pressure (left figure) and
pressure fluctuations (logarithmic scale, right figure) at XY0, XZ0 and YZ planes

The instantaneous snapshots of the flow field conveys a consistent story. The left plot in
Fig. 4.5 shows one instantaneous snapshot of the three components of velocity. The
velocity is non-dimensionalized to Mach number using the speed of sound around the
nozzle exit: \( c_0 \approx 290 \text{ m/s} \). The transverse velocity \( V \) highlights the shock structures
while the shear layers are more clearly observed with the spanwise velocity \( W \). The right
plot in Fig. 4.5 shows the mean vorticity (spanwise) field and one snapshot of the
fluctuating vorticity. The vorticity is non-dimensionalized using the speed of sound \( c_0 \)
and the thickness \( D_p \) of the splitter plate (between the core jet and the wall jet). Strong
vorticity can be seen from the nozzle lips and from the deck edge in the mean field. The
fluctuating field shows the expansion of the shear layer, both from the splitter plate and
from the nozzle top lip and the vorticity is in larger scales in the downstream direction.

For the sake of clearer referencing of the flow structures, here we give the definitions of the shear layers (blue paths in Fig. 4.6) and the shock structures (red paths in Fig. 4.6). We also define selected points of interest, which are regions with large pressure fluctuations (Fig. 4.4), including weaker but possibly important locations such as the nozzle lip region. Over a dozen candidates are selected for each plane. Some of them (black stars with abbreviations in Fig. 4.6) display distinct features and are called the hot-spots, while the others (annotated by black circles in Fig. 4.6) provide no additional information.

First focusing on the XY0 plane, from the upper lip of the nozzle, we have a first expansion oblique shock and the Lip Shear Layer (SL). The shock is reflected by the Inner SL as a compression Shock (Sh) and an active region (possibly a normal shock) also forms normal to the wall jet. The Inner SL starts from the splitter plate and separates the core jet from the wall jet. The hot-spot ISL tracks the evolution of this

Figure 4.5: One example of the instantaneous velocity fields (left figure) and the mean fluctuating vorticity field (right figure) at XY0 plane; all the quantities are non-dimensionalized
shear layer. The Lip SL and Sh intersects at the Hub (Hb), yielding an unnamed expansion oblique shock and the Evolutionary SL. Either as a reflection of the oblique shock or as an effect of the trailing edge of the deck, a second compression shock forms and is approximately parallel to Sh. This shock interacts with the Evolutionary SL at the Fulcrum (Fl) to yield the Main SL. Extending from the Fl along the weak shock is the Appendix (Ap). The Lip SL, Evolutionary SL and Main SL are connected to each other and form the Upper SL. Furthermore, the shear layer from the deck’s trailing edge is named as the Deck SL. A hot-spot with the same notation (DSL) tracks the evolution along this path. These features will all play a role in the analysis. A summary of the definitions of the hot-spots can be found in Table 4.1.

Figure 4.6: Flow structures in the near field pressure (left - XY0 plane; middle - XZ0 plane; right - YZ plane; please see text for definitions)

Following the same procedure with that of the XY0 plane, several other sets of hot-spots and shear layers are defined for the XZ0 plane. The XZ0 plane is centered at mid-height of the nozzle, i.e. not coinciding with shear layers defined in XY0 plane. Starting from the nozzle lips, shear layers are formed and extend downstream, moving towards the jet centerline. These are called the Converging Side SL. The Trench (Tr) is formed between the Converging Side SL and the first oblique shock. Moving
downstream, the second expansion oblique shock from the Hb meets the Converging Side SL at the Intersect (Int). The shock is reflected inward and the Ridge (Rg) is selected along the weak shocks, preceding the Ap (located arround the second crossing at the centerline). In later sections, it will be shown that the flows converge after the deck edge and around Ap. Thus the region between the weak shocks after the Ap is named as the Converge (Cv). After the blending around Cv, the Converging Side SL starts to evolve away from the centerline, developing into the Diverging Side SL, which evolves into more turbulent mixing downstream (YZ8 plane in Fig. 4.4). In addition, the Outer (Ou) is placed outside the Converging Side SL upstream of the deck edge.

<table>
<thead>
<tr>
<th>XY0 plane</th>
<th>Hb Hub</th>
<th>Fl Fulcrum</th>
<th>Sh Shock</th>
<th>ISL Inner Shear Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ap Appendix</td>
<td>DSL Deck Shear Layer</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>XZ0 plane</th>
<th>Ou Outer</th>
<th>Tr Trench</th>
<th>Int Intersect</th>
<th>Rg Ridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cv Converge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YZ planes</th>
<th>Up Upper</th>
<th>US1 Upper Shear One</th>
<th>US2 Upper Shear Two</th>
<th>UCc Upper-Corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sd Side</td>
<td>ThS Third-Stream</td>
<td>LCc Lower-Corner</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Definitions and abbreviations of the hot-spots

Finally, in the YZ sections, in particular at $X/D_h = 2$ (deck trailing edge), the SPL distribution (Fig. 4.4) emphasizes the shear layers and possibly the corner vortices. In the last plot of Fig. 4.6, the blue lines are the cross sections of the Upper SL (top), Side
SL (left and right) and Inner SL (bottom) respectively. The red line represents the cross section of the shock that passes through the transverse planes. Some of the hot-spots follow the shear layers, such as US1 and US2 (US for Upper-Shear); some keep track of the corner vortices, i.e., UCn and LCn (abbreviation for Upper- and Lower-Corner). The ThS displays the evolution of the Third-Stream (ThS). The Upper (Up) and Side (Sd) are a summary for the activities in their respective regions.

4.2.2 The Spectral Features and the Definitions of Frequency Bands

Before moving on to the main contents, the definitions to the terms and parameters relevant to the spectral analysis will be introduced. Firstly the scaling parameters are explained which will be used to non-dimensionalize the frequency into Strouhal number. In this chapter, two sets of scaling parameters will be needed, the reason for which will become evident later. The first set consists of the core jet parameters, i.e. the hydraulic diameter $D_h$ and the core jet speed $Ma_1 \cdot c_0$. The dimensionless quantities thus obtained will be annotated with subscript 1, such as Strouhal number $St_1$. The second scaling choice is the parameters of the small-scale structures coming off the splitter plate between the core and the wall jet [86]. They include the splitter plate thickness and the average speed $(Ma_1 + Ma_3)/2 \cdot c_0$ and the dimensionless quantities are subscripted by 3, such as $St_3$.

Secondly, a wide range of frequencies will be analyzed in this chapter and they are separated into six frequency levels based on the flow features. Four of these levels are
defined while the other two are not as relevant for the current analysis:

- Below $1kHz$ ($St_1 < 0.1$);
- Low-Frequency Range: between $1kHz$ and $2kHz$ ($0.1 \leq St_1 \leq 0.2$);
- Mid-Frequency Range: between $2kHz$ and $6kHz$ ($0.2 \leq St_1 \leq 0.55$);
- High-Frequency Range: between $6kHz$ to $18kHz$ ($0.55 \leq St_1 \leq 1.8$ or $0.05 \leq St_3 \leq 0.15$);
- Ultra-Frequency Range: between $18kHz$ to $43kHz$ ($1.8 \leq St_1 \leq 4.2$ or $0.15 \leq St_3 \leq 0.36$);
- Above $43kHz$ ($St_1 > 4.2$ or $St_3 > 0.36$).

The flow features of each frequency band will investigated using spectral analysis and continuous wavelet transform.

![Compensated Morlet Spectra](image)

Figure 4.7: Compensated Morlet spectra of the pressure signals at the hot-spots in XY0 (left) and XZ0 (right) planes (at $1kHz$, $St_1 = 0.096$; at $10kHz$, $St_1 = 0.96$ and $St_3 = 0.084$; at $50kHz$, $St_1 = 4.80$ and $St_3 = 0.42$)

Using the Morlet wavelet transform, we can calculate the Morlet compensated spectrum at each hot-spot. The Morlet spectrum is preferred over the Fourier spectrum because
with the short signal length, the Fourier spectrum is too oscillatory to show the
dominant spectral ranges. First shown here are the spectral characteristics of pressure
signals at the hot-spots in the XY0 and XZ0 planes (Fig. 4.7). In the XY0 plane (left
plot in Fig. 4.7), the Hb, Sh and ISL are located more upstream and closer to the deck
(please refer to Fig. 4.6 for their relative locations). They display two similar features.
Firstly, there are two peak frequencies, i.e., $2.6 kHz \ (St_1 = 0.25)$ and $32.4 kHz
(St_1 = 3.11 \text{ or } St_3 = 0.27)$. Note that these two frequencies are non-dimensionalized to
approximately the same Strouhal if different scaling parameters are used. Secondly, the
spectra of hot-spots Hb, Sh and ISL are more broadband, or energetic over wider range
of frequencies than the other hot-spots. As the hot-spot locations move downstream,
the peak of the spectra of Fl, Ap and DSL shift toward lower frequencies. The low
frequency peak in the Mid-Frequency Range is also present downstream at the Ap. The
spectra distributions of the velocity signals are very similar and thus omitted. In the
XZ0 plane, a similar trend can be observed (right plot of Fig. 4.7). The peak at $2.6 kHz$
is present for most of the hot-spots, while the peak at $32.4 kHz$ shows up for Tr and Ou,
which are located further upstream than the others. These two hot-spots are also
further away from the jet centerline. This indicate the side mixing layers may have
different features with the shocks.

An examination of the mixing layers in the YZ planes reveals some of their evolution
patterns (Fig. 4.8). For reference, YZ1 plane passes through the hot-spot Sh
approximately; YZ8 plane is well downstream of the hot-spots in Fig. 4.7, in what can
be viewed as the turbulent mixing region of the near jet. At the YZ1 plane (left plot in
Figure 4.8: Compensated Morlet spectra of the pressure signals at the hot-spots in YZ1 (left) and YZ8 (right) planes (at 1kHz, $St_1 = 0.096$; at 10kHz, $St_1 = 0.96$ and $St_3 = 0.084$; at 50kHz, $St_1 = 4.80$ and $St_3 = 0.42$).

Fig. 4.8), we see three groups of hot-spots. The Sd and Up are outside the shear layers and they have the two frequency peaks identified before, around 2.6kHz ($St_1 = 0.25$) and 32.4kHz ($St_1 = 3.11$ or $St_3 = 0.27$). The LCn and UCn are around the corner vortices and have much smoother spectrum. The difference between these groups is a reminder that the plane of symmetry is not representative of the entire jet. The ThS, US1 and US2 keep track of the shear layers, and the peak around 32.4kHz is dominant as YZ1 plane is upstream where higher frequencies are more active. As the plane moves downstream to YZ8 (right plot in Fig. 4.8), the spectra become rather broadband and they superpose each other regardless of the hot-spots, indicating more turbulent mixing. The spectra at YZ2 and YZ4 are omitted here as they display features intermediate of YZ1 and YZ8, which is consistent with the general trend of lower frequency and less clear structure in downstream direction.

As the hot-spots seem to come in categories based on their locations, it may be
beneficial to take a field view of the spectral information. Firstly we focus on the frequency peaks that contain the most energy and Fig. 4.9 shows the peak frequency distribution. The contour levels are plotted in logarithmic scale so as to avoid over-emphasizing low frequencies. Admittedly, this ignores broad spectral distributions and/or multiple peaks, to which we will return later. However, the peak frequency distribution is informative. Most notably, we see that the Ultra-Frequency Range dominate only at certain upstream regions. They include the shear layers (Lip SL, Inner SL and Converging Side SL) upstream of $X/D_h = 2$, regions outside of Converging Side SL, and possible corner vortices at $X/D_h = 1$. As the shear layers extend downstream, the High-Frequency Range becomes dominant. This includes the Evolutionary SL, Deck SL and the downstream parts of Converging Side SL. The abrupt transitions from cold to warm shades reflects the shift in emphasis between competing peaks of the spectrum. The rapid changes between the YZ planes are indicative of strong interactions and of preferential frequency bands for the various flow structures involved. As the YZ planes
moves downstream, the shocks become more blurred and the mixing regions (shear layers) are clearly expanding.

Next we look at the spectrum distribution of the pressure signals at various frequencies; Fig. 4.10 shows the ‘normalized’ spectra. Using the same method with Fig. 4.9, the compensated Morlet spectra at each location are calculated and they are collected into one matrix $E_M(f, x, y)$. Then we can look at the contour plot of the spectrum distribution at specific frequencies, i.e. $E_M(f_0, x, y)$. However, since the shocks have much higher overall energy than the other regions, only the shocks stand out at all the frequencies. Thus instead of showing the spectrum, here we use the ‘normalized’ spectrum, which is the spectrum at each location $E_M(f, x_0, y_0)$ normalized to zero mean and unit variance:

$$\hat{E}_M(f, x_0, y_0) = \frac{E_M - \bar{E}_M}{||E_M||}.$$  \hspace{1cm} (4.2)

$\hat{E}_M(f, x_0, y_0)$ still keeps the energy distribution at different frequencies at a specific location $(x_0, y_0)$. Its field distribution (Fig. 4.10) also enables the comparison of active frequencies at different locations and shows the relative energy levels in space.

Comparing across frequency, we can see the trend of warm regions shifting upstream as the frequency increases. Different regions and structures are associated with different frequencies.

Below $1kHz$ ($St_1 < 0.1$), all the pressure sections are rather inactive; thus it’s normalized spectrum is omitted in Fig. 4.10 and also in the rest of the chapter. In the Low-Frequency Range (first plot in Fig. 4.10), the only active regions include the second oblique shock, the parts of Main SL, Deck SL and Diverging Side SL
Figure 4.10: Normalized Morlet compensated spectrum of the pressure signals at different frequency levels
downstream of $X/D_h = 4$. The Mid-Frequency Range is particularly interesting (the second and third plots in Fig. 4.10): the whole fields are highly active, except at the shear layers (Lip SL, Evolutionary SL, Inner SL and Deck SL) and possible corner vortices upstream of $X/D_h = 4$. We speculate this may be the frequency band that affects the flow development. As we move to the High-Frequency Range (the fourth plot in Fig. 4.10), the active regions move upstream. The shear layers including Evolutionary SL, Deck SL and Converging Side SL are marked as dark red areas; so are the corner vortices and a region seemingly originating from hot-spot Fl and expanding outside of Main SL. The active regions at the Ultra-Frequency Range are mostly upstream of the deck edge (bottom-left plot in Fig. 4.10). Apart from the lower shock cells and the Inner SL, two active regions seem to be radiating away from the centerline, one from hot-spot Hb and one from the deck edge. The side regions on either side of the nozzle are also active. Lastly above frequency $43kHz$ ($St_1 > 4.2$ or $St_3 > 0.36$), only limited regions on the sides of the nozzle and the Lip SL are active. As we recognize this range may be better resolved with higher sampling frequency and resolution, we won’t include this into the analysis in later sections. All the observed transitions happen gradually and most of them sustain continuously over a range of frequencies. We note that the regions that share the common active frequencies are not necessarily communicating, and the propagation between locations will be investigated using correlation technique in the next section.
4.3 The Pathways and Features of Near-Field Flow Structures

In this section, several techniques will be applied to extract the Near-Field flow features. For clearer referencing, the figures obtained with different techniques are presented with different color settings. The peak correlation distributions are colored in orange-black; the space-time correlations are shown in red-blue colors; the frequency-resolved correlations between two sensors are colored in cyan-purple; the frequency-resolved space-time correlations are in purple-black; lastly, the phase-averaged Near-Field are presented in green-black. Please refer to the following paragraphs for the definitions and examples of the various techniques.

4.3.1 The Main Flow Structures and Their Pathways

We proceed with the cross-correlations between the pressure signals at the hot-spots and the pressure fields. The correlations between the velocity signals show very similar patterns and thus omitted. One of the primary uses of the cross-correlation is to track the propagation of information between two points, or the differential propagation from some unspecified third location to the two points of interest. With the signals shifted by a variable lag relative to each other, and normalized (zero mean, unit variance), their mean dot product will vary depending on the lag; its peak is a good statistical measure of the differences in propagation times. In the following, it will be stated which signal is taken as the reference. The lag of peak correlation is the mean lag of the events at all
Cross-correlations between hot-spots

We start with the cross-correlations between hot-spots in the XY0 plane (Fig. 4.11). The plot on the top-left is between the Sh and other hot-spots in this plane. Between Sh and Hb, a negative peak with positive lag indicates the information reaches Sh earlier than Hb. Similarly, the peak with positive lag between Sh and Ap shows Sh receives the information first. Between Sh and ISL, there is one negative peak with time lag approximately zero. This could be caused by a common third source that has the same displacement from these two hot-spots. Still looking at the hot-spots in the same plane, the plot on the top-right (Fig. 4.11) is between Ap and the others. A positive peak with negative lag can be observed between Ap and Fl, while between Ap and DSL,
the peak is negative with a positive lag. These show a clear sequencing as the Sh and ISL come first, followed by Hb, then Fl and lastly Ap. Moving on to the hot-spots in the XZ0 plane (bottom plots of Fig. 4.11), the dominant peaks have negative lags between Sh and Tr, Ap and Tr, Ap and Rg, and Ap and Int. The peaks with positive lags are between Sh and Cv and Ap and Cv. All these are consistent with the propagation in downstream direction. Note the correlation between Sh and Tr and between Sh and Cv shows repetitive patterns, the period of which is roughly 0.39ms. This may correspond to a periodic propagating pattern at $St_1 = 0.25$ ($2.6kHz$) between these hot-spots; and indeed, this is confirmed using the frequency-resolved correlations and phase-averaging in later sections.

Figure 4.12: Cross-correlations between pressure signals at the hot-spots in the YZ2 (top plots) and YZ4 (bottom plots) planes and hot-spots in the XY0 plane

Next we look at the information propagation between hot-spots in the transverse planes and XY0 plane. Up is located above the core jet outside the Upper SL. When Up is in the YZ2 plane, it has dominant peaks with positive time lags with most of the hot-spots
in the XY0 plane (left-top plot in Fig. 4.12). Since Hb and Sh are located upstream of
the YZ2 plane, this seems to indicates that there is information propagating upstream
crossing the shear layers. Another possibility is that the information from a common
source propagates along different paths and reaches Up earlier than the others. As the
transverse plane moves downstream to YZ4, the time lags of correlation peaks become
negative, consistent with downstream propagation (left-bottom plot in Fig. 4.12). The
right plots in Fig. 4.12 is between Sd (located along the side of the core jet) and
hot-spots in the XY0 plane. The only hot-spot correlated with Sd is Ap. When Sd is
upstream of Ap (YZ2 plot), the best lag is positive, indicating information propagation
from the side to Ap; when Sd is downstream of Ap (YZ4 plot), the best lag is negative,
indicating information propagating to the side. As the region around Ap (and Cv in
XZ0 plane) is the only region correlated with the sides, it is concluded tentatively that
this region may be the link to the 3-D interactions in the near jet.

Peak correlation distributions between hot-spots and pressure fields

A more global view is obtained by cross-correlating all points in a given section (e.g.
XY0 plane) with various points of reference at the hot-spots. With spatial coordinates
as independent variables, the correlation level as a function of lag requires a reduction
of the data or animation.

First we look at the peak correlation levels regardless of the time lag (Fig. 4.13 to 4.15).
At each location, the largest absolute value of the correlation is recorded. Then the sign
is restored due to the consideration that consecutive positive and negative peaks
constitute one period of events / structures. The 2D distribution of peak correlations provides an idea of which regions of the field communicate with the selected hot-spots. The hot-spot positions are indicated by the blue crosses.

Fig. 4.13 shows the regions of communication between the pressure signals at the hot-spots in the XY0 plane and pressure fields. The regions correlated with the Hb constitute the Lip SL, Evolutionary SL, and a region outside the shear layers, expanding away from the core jet. The regions correlated with the Fl are the Evolutionary SL and the Main SL. The peak correlation patterns of US1 and US2 (at YZ planes) are similar to these two hot-spots. They together reveal the propagation paths of the Upper SL (first row in Fig. 4.13). Similarly, the paths and expansion of the Inner SL and the Deck SL are displayed by ISL and DSL (second row in Fig. 4.13).

With ISL, there is also a region that seems to originate and radiate from the deck edge. The Sh and Ap are highly correlated with very wide regions (last row in Fig. 4.13), but each with its distinctive patterns. The regions that resonate with Sh include the regions inside the shocks along the core jet, and regions outside the core jet upstream of the deck edge (above the Upper SL, below the deck and at the sides of the nozzle exit).

This pattern is repeated with hot-spots Up (at YZ planes), confirming the relationship between the above mentioned regions. The Ap (and Cv in XZ0 plane) is again the only hot-spot that is correlated with both Side SL in XZ0 plane, which is another evidence of its key role in 3D interaction. One interesting feature of these plots is that the shocks are marked by the sharp transition in color, and they are the interface between two separate flow regions.
Figure 4.13: Peak correlation level distribution between the pressure signals at the hot-spots (XY0 plane) and pressure fields.
Figure 4.14: Peak correlation level distribution between the pressure signals at the hot-spots (XZ0 plane) and pressure fields

Fig. 4.14 displays the peak correlation distribution between hot-spots in the XZ0 plane and pressure fields. With Ou (left plot in Fig. 4.14), the highlighted regions include the side regions at the nozzle exit and the region expanding away from the nozzle side lip. With the Int (right plot in Fig. 4.14), the paths of the Converging and Diverging SL are exhibited and limited region inside the core jet is also correlated.

Figure 4.15: Peak correlation level distribution between the pressure signals at the hot-spots (YZ2 plane) and pressure fields
Fig. 4.15 shows the regions that are correlated with the hot-spots in the YZ2 plane. At the corners of the core jet, the possible corner vortices are only expanding and convecting downstream, without much communication with the surrounding areas (left plot in Fig. 4.15). The Sd is correlated with the Side SL and regions under the deck (right plot in Fig. 4.15). Another correlated region is localized along jet centerline after the end of the deck, right at the location of the Ap and Cv and this is the where the cross stream communication starts.

**Space-time correlations between hot-spots and the pressure field**

The regions of peak correlations have omitted the lag information; a more comprehensive view is to look at the space-time correlations (Fig. 4.16 to 4.21). At each time instant, the correlation level between the hot-spots and pressure fields is shown in a 2D contour plot and different time lags are shown in successive frames. Thus the space-time presentation of the correlations is obtained. In this section, six representative hot-spots are used to convey the near-field structure evolutions. All the figures use the same color scaling, i.e. from correlation level $-0.15$ to $0.15$. Based on the observations, correlation levels smaller than $0.1$ (absolute value) can be regarded as the uncertainty level due to incomplete convergence. Thus the bright blue or red regions (most of them saturated) are significantly above the uncertainty level and represent regions propagating or convecting in various directions. Some of the regions could be attributed to evolving structures or radiating sound waves.

The Fl and other hot-spots located along the Upper SL (such as the Hb) show similar
space-time correlation patterns (Fig. 4.16). Localized patterns with shifting blue-red colors propagate along the Upper SL. As the time lag increases (plots shifting from left to right, top to bottom), their location shifts downstream and their sizes are increasing. This is consistent with the expansion of the shear layers and with the trend of decreasing frequency in the downstream direction. In the upper shock cells (above the shocks in the XY0 plane), elongated patterns shifting in red and blue move downstream. When these patterns reach the intersections of the shear layers and the shocks, circular patterns originate from the intersections, e.g., from the Hb (top-right plot in Fig. 4.16)
and from the Fl (bottom-left plot in Fig. 4.16). Circular waves can also be seen radiating from the edge of the deck (top-right plot in Fig. 4.16). These could be part of the sound waves since they originate from fixed points (shear-layer-shock intersections and deck edge) and the radius of the waves expands as time evolves.

Moving towards the deck, the ISL and other hot-spots at neighboring locations track the same structures (Fig. 4.17). Localized pulsing in blue-red is propagating along the Inner SL above the deck. After they enter the lower shock cells (second plot in Fig. 4.17), they split tracks. The first part continues along the Inner SL, while the second part become elongated structures, positioned with a sharp angle from the normal shock and propagate along the side of the shock. As the first part of the structure reaches the edge of the deck, circular waves originate and radiate away from the edge. Similarly, possible acoustic waves radiate when the second part of structure reaches the shear-layer-shock intersection (Hb in this instance). The propagation inside the lower shock cells can also be seen in their cross-sections in the XZ0 plane.

Figure 4.17: The evolution of space-time correlation between the pressure signal at the ISL (XY0 plane) and pressure fields
When the hot-spots are located along the Deck SL (such as DSL), alternating red-blue patches are also found propagating and expanding in the downstream direction. The pattern is rather similar with that of the Upper SL and Inner SL and is thus omitted.

Figure 4.18: The evolution of space-time correlation between the pressure signal at the Sd (YZ1 plane) and pressure fields

With the previous hot-spots, there is not much activity in the XZ0 planes since they are located along the top or bottom of the core jet. The Sd and some other hot-spots (such as Ou and Int) are located at the side of the jet stream and activity shows up in the horizontal plane (Fig. 4.18 and 4.19). When the Sd is upstream at $X/D_h = 1$, circular waves radiate from the nozzle lip in both XZ0 and YZ planes (Fig. 4.18). These are acoustic waves propagating in spherical planes from the nozzle lips. Note the regions around the corners of the jet stream are left as white, which is consistent with previous findings that the possible corner vortices are not communicating with neighboring regions (Fig. 4.15). When the Sd is downstream at $X/D_h = 4$, structures are observed moving along the Side SL (Fig. 4.19). They first move towards the centerline along the Converging Side SL. After the deck edge and at the location of the Ap and Cv (left plot
in Fig. 4.19), they cross the centerline. Activity starts to show up in XY0 plane and cross-stream mixing commences. The structures continue to propagate downstream, while expanding in size and moving away from the centerline along Diverging Side SL (right plot in Fig. 4.19). These structures of alternating colors are of the same nature with those along the shear layers in the XY0 plane. Also apparent with the Sd in YZ4 plane is the propagation at the side of the shock cells.

Figure 4.19: The evolution of space-time correlation between the pressure signal at the Sd (YZ4 plane) and pressure fields

To further illustrate the cross-stream activity, Fig. 4.20 shows the correlation evolution between Cv and pressure fields. A very similar pattern can be observed with hot-spot Ap. These two hot-spots probably belong to two adjacent shock cells, on each side of the first weak oblique shock after the deck edge. With these hot-spots, structures moving along both Side SL can be observed, first towards (left plot in Fig. 4.20) and then away from (right plot in Fig. 4.20) the jet centerline. In the transverse planes, the communication with the sides is also evident. This is not present with any other hot-spots and confirms the location where cross-stream communication occurs.
In some of the previous figures of this section, there seem to be large areas propagating upstream outside the core jet. This activity is most obvious with the hot-spot Sh. In Fig. 4.21, three areas propagate in apparent upstream direction. They include the regions above the Upper SL, below the deck and on both sides of the nozzle exit (XZ0 plane) and all these happen before the deck edge at $X/D_h = 2$. Very wide regions are engaged and they seem to come in repetitive patterns. Another repetitive pattern is the
movement inside the shock cells in both XY0 and XZ0 planes. By counting their instant of occurrence, the periodicity of these activities is estimated as 0.39ms or 2.6kHz \((St_1 = 0.25)\). This will be further discussed using phase-averaging later.

4.3.2 The Frequency-Specific Features of the Flow Structures

One possibility with regards to the propagation paths is that they are associated with different frequencies. In this section, frequency-resolved cross-correlations are used to isolate the activities at specific frequency levels. The Morlet wavelet is used to decompose the signals and the dot products of the Morlet coefficients are calculated at the same frequency level. Before calculating the correlations, we may choose to normalize the Morlet coefficients to zero mean and unit variance, such that the resulted correlations have levels varying between \(-1.0\) and \(1.0\). Or we may choose to use non-normalized Morlet coefficients so as to capture actual contributions to the overall correlation coefficient (see Equation 2.31 and discussions in Section 2.2.2).

**Frequency-resolved correlations between hot-spots**

Fig. 4.22 to 4.24 show the frequency-resolved correlations obtained with normalized Morlet coefficients. The strong patches (cyan or purple in the figures) indicate high correlation level at the specific frequency (y-axis) and time lag (x-axis). Fig. 4.22 is between pressure signals at the hot-spots in the XY0 plane. In the left plot, the Hb is correlated with Fl and DSL respectively. The Hb is highly correlated with Fl around 12.5kHz \((St_1 = 1.20\) or \(St_3 = 0.11)\) with a positive time lag, indicating the information
Figure 4.22: Frequency-resolved cross-correlations between the pressure signals at the hot-spots in the XY0 plane

propagates downstream and reaches Hb earlier than Fl. This group of activity
disappears with the Hb-DSL correlation, while the activity around $2.6kH\,z\, (St_1 = 0.25)$
persists for both hot-spot pairs. The right plot in Fig. 4.22 is the correlations between
hot-spots Sh and ISL and Ap. With the Sh-ISL pair, very wide frequency range shows
high level of communication, and the time lag of peak correlation is around 0 for most
of the frequencies. This is consistent with the result from conventional correlation (Fig.
4.11) and with the observation that the flow splits paths, reaching these hot-spots
around the same time (Fig. 4.17). Around $32.4kH\,z\, (St_3 = 0.27)$, a very strong patch
can be seen when Sh is correlated with ISL, and it becomes a lot dimmer when
correlated with Ap. Since Fl is located upstream of DSL and ISL upstream of Ap, the
frequencies of activities concur with the previous observation (Fig. 4.9 and 4.10), that
the frequency level decreases in downstream direction. Note the activity around $2.6kH\,z$
is present with most of the hot-spots. This again agrees with the spectral analysis (Fig.
4.10) that this frequency level prevails in most of the flow regions. Its periodic feature
also reappears (Fig. 4.11), as the patches alternate in orange and black without much
variance in magnitude or an obvious ‘best’ lag.

![Correlation: Tr (XZ0) and Hot-Spots (XZ0)](image)

**Figure 4.23**: Frequency-resolved cross-correlations between the pressure signals at the hot-spots in the XZ0 plane

The frequency-resolved correlations between the pressure signals at the hot-spots in the XZ0 plane is shown in Fig. 4.23. The top plot is between hot-spots Tr and Ou. In the High- and Ultra-Frequency Ranges, the dominant activity has a positive time lag. In the Low- and Mid-Frequency Levels, the dominant activity seems to have a negative time lag, or it possibly belongs to the periodic patterns. Although clearly at least two groups of activity are present, a clearer and more intuitive presentation is needed to better understand their respective paths. The bottom plot in Fig. 4.23 is between hot-spots Tr and Cv. The periodic pattern can be seen around $2.6kHz$ ($St_1 = 0.25$) and in the Low-Frequency Range as well.

Next we look at the correlation contributions between the pressure signals at the hot-spots in the XY0 plane and pressure signals on the sides of the jet stream (XZ0 plane). The left plot in Fig. 4.24 is between hot-spot Sd in YZ2 plane and hot-spots Sh and Ap. With Sh, there is only the periodic patterns around $2.6kHz$ ($St_1 = 0.25$). With Ap, another activity is present in the Mid-Frequency Range and the lag is positive,
Figure 4.24: Frequency-resolved cross-correlations between pressure signals on the sides of the jet stream (YZ2 plane) and hot-spots in the XY0 plane representing the information converging towards Ap from the sides (or Cv as in Fig. 4.20). In the right plot in Fig. 4.24, the sensor is located at the symmetric position of Sd about the jet centerline. The correlation patterns are rather similar with that of the other side (left plot in Fig. 4.24), indicating the statistical convergence is achieved with this data. While comparing the frequency-resolved correlations of different sensor locations, the correlation pattern is found rather sensitive to location. This may be due to the existence of the multiple structures, such as Side SL and possible sound waves. A more global and more comprehensive presentation of the frequency-resolved correlations is needed, which will be explored in the next section. Another thing to be noticed with all the figures in this section is that high correlation levels can be found below 1kHz ($St_1 < 0.1$), even though all the signals are very inactive at this level (Fig. 4.10). This is due to the normalization of the frequency-resolved signals to unit variance. In the next section, the filtered signals are not normalized to avoid over-emphasizing highly-correlated but low-energy activities. The non-normalized coefficients have much smaller magnitudes and the level of significance (absolute value 0.01) is determined by
Frequency-resolved space-time correlations

Using the same idea with the temporal correlations, the space-time version of the frequency-resolved correlations can provide us a more comprehensive view of the evolutions of possible structures. For each time lag and at a specific frequency, a 2D distribution of the correlations will highlight the regions that are correlated at that instant. In the sequence of increasing time lag, the evolution of the correlation patterns can be observed frame-by-frame. In this section, the correlations are obtained with non-normalized Morlet coefficients and those with absolute values above 0.01 are regarded as significant. All the figures in this section have the same color scaling of $-0.025$ to $0.025$. Also, the plots are presented in the sequence of increasing frequencies and increasing time lags (in some cases only one time frame is presented). The few instances shown are the more representative ones, and keep in mind that most of the structures evolve with time and frequency rather gradually.

Starting with structures in the XY0 plane, Fig. 4.25 shows the frequency-resolved correlation evolutions obtained with hot-spots along the Upper SL. The hot-spot locations are indicated by the red crosses. From the left plot to the right and top to bottom, the frequency increases. The first plot is in the Mid-Frequency Range and is between hot-spot Fl and the pressure fields. Localized structures with alternating colors emerge along the Main SL and propagate downstream. Below these structures, elongated shapes with alternating colors move inside the upper shock cells in
Figure 4.25: The evolution of frequency-resolved space-time correlations between pressure signals at the hot-spots (first three plots with Fl and the last with Hb, both in the XY0 plane) and pressure fields downstream direction. Their cross-sections show up in the XZ0 plane as elliptical shapes moving downstream along the jet centerline. Structures also originate from the intersection of the weak oblique shock after Fl and the Deck SL. They radiate from the intersection while expanding (YZ4 subplot in Fig. 4.25). In the High-Frequency Range (the next three plots), the structures reduce in size and emerge more upstream. At 10.51 kHz ($St_1 = 1.01$, the second and third plots in Fig. 4.25), structures move along the Evolutionary SL towards the Main SL. When they pass the deck edge (the second
plot in Fig. 4.25) and the shock-shear-layer-intersections (the third plot), possible acoustic waves originate. Note in the YZ4 subplot of the third plot, the upper half of the waves come from Fl, while the bottom half originate from the oblique shock and Deck SL intersection. The waves radiate from the origins and are concentrically circular. At $14.87kHz (St_1 = 1.43$, the last plot in Fig. 4.25), the structures are further reduced in size and emerge along the Lip SL. The structures propagating along the shock sides are also smaller, and their cross-sections move downstream in two branches in the center shock cells in XZ0 plane.

Fig. 4.26 still focuses on structures in the XY0 plane, but are mostly along the Inner SL and Deck SL. The first plot is between hot-spot DSL and the pressure fields and is in the High-Frequency Range. Similar structures with those along the Upper SL now propagate along the Deck SL. Their neighboring regions in the lower shock cells also propagate downstream along the jet centerline (XZ0 plane). Possible radiating waves can be observed from the oblique shock and Deck SL intersection. The next three plots in Fig. 4.26 are between hot-spot ISL and the pressure fields and are at higher frequency levels. In the High-Frequency Range (second plot in Fig. 4.26), structures emerge from the downstream half of Inner SL and propagate downstream along the Deck SL. In the Ultra-Frequency Range (last two plots in Fig. 4.26), the activity along the Deck SL disappears. Instead, strong activities come out of the nozzle exit and move along the deck. When they enter the second shock cell, they split path, one propagating along the shock side and one continuing downstream along the Inner SL. When the structures reach the shear-layer-shock intersection at the Hb and the deck edge,
Figure 4.26: The evolution of frequency-resolved space-time correlations between pressure signals at the hot-spots (the first plot with DSL and the rest with ISL, both located in the XY0 plane) and pressure fields concentric circular waves radiate. There also seems to be circular radiation from the side lips of the nozzle (XZ0 and YZ1 subplots in Fig. 4.26), which will be further illustrated later. In the XZ0 plots (Fig. 4.26), the evolution of the structures can also be found in the lower shock cells.

With the velocity signals at the hot-spots in the XY0 plane, very consistent patterns show up in the Low- and Mid-Frequency Bands. In the High- and Ultra-Frequency Bands, only the regions inside the jet stream are active with the velocity signals,
indicating the activities outside the jet with the pressure signals could contain more acoustic rather than hydrodynamic information. Especially at $32.4 kHz$ ($St_3 = 0.27$), only the hot-spots ISL and Sh are correlated with the regions along the Inner and the Evolutionary Shear Layers.

Figure 4.27: The evolution of frequency-resolved space-time correlations between pressure signals at the hot-spots (the first plot with LCn in YZ2 plane, the second and the third with Int in XZ0 plane, the last with Ou in XZ0 plane) and pressure fields

Switching to the structures in the XZ0 plane, Fig. 4.27 is the frequency-resolved correlations between hot-spots along or outside the sides of the jet streams and the pressure fields. Firstly, using hot-spot LCn in YZ2 plane and filtered in the
Mid-Frequency Range (first plot), structures are observed moving along the Diverging Side SL. As they move downstream, they expand and engage more regions into turbulent mixing. In the side shock cells, structures with alternating colors also move in downstream direction. Secondly, hot-spot Int is correlated with the pressure fields at higher frequencies (the second and third plots in Fig. 4.27). The structures emerge more upstream and propagate along the Converging Side SL. When they pass the shear-layer-shock intersections at the side of the jet stream, possible acoustic waves radiate in circular patterns (YZ2 subplots in Fig. 4.27). Lastly, the last plot in Fig. 4.27 shows the correlation obtained using hot-spot Ou and is in the Ultra-Frequency Range. Strong circular waves originate from the side lip of the nozzle. Structures with much more reduced sizes are also found propagating along the Converging Side SL and along the shock sides in the center shock cells (XZ0 subplot in Fig. 4.27).

Figure 4.28: The evolution of frequency-resolved space-time correlations between pressure signals at the hot-spot Ap (located in the XY0 plane) and pressure fields

As have been shown a few times earlier, the regions around hot-spots Ap and Cv are where 3D mixing occurs (Fig. 4.12, 4.13, and 4.20); and around the frequency level
2.63kHz ($St_1 = 0.25$), very wide regions in the near-field are highly active and there seem to be periodic activities (Fig. 4.10, 4.11 and 4.21). Fig. 4.28 is the frequency-resolved correlations between hot-spot Ap and pressure fields at frequencies 2.63kHz ($St_1 = 0.25$) and 3.72kHz ($St_1 = 0.35$) (both in the highly-active Mid-Frequency Range). In the XZ0 subplots (Fig. 4.28), both Side SL are highly correlated. In the YZ2 subplots (Fig. 4.28), regions propagate from Ap to the sides, which are not present with any other hot-spots. On the other hand, the very wide regions of activity in periodic patterns are observed with most of the hot-spots at the two selected frequencies (their figures are omitted here due to repetition). The areas with periodic patterns include: regions upstream of the deck edge outside the jet stream propagate upstream; regions downstream of the deck edge outside the jet stream propagate downstream; regions along sides of the shocks propagate downstream.

Figure 4.29: The evolution of frequency-resolved space-time correlations between hot-spot UCn (left plot - YZ2 plane, right plot - YZ1 plane) and pressure fields

At last, Fig. 4.29 illustrates the evolution of possible corner vortices at the upper corners of jet stream. In both plots, hot-spots UCn are located at the top-right corners
of the jet stream; but they are at different transverse planes and filtered at different frequencies. In both plots (Fig. 4.29), the corner regions do not interact much with the surroundings and they are convected downstream while expanding slightly in size.

When UCn is located downstream in the YZ2 plane (left plot in Fig. 4.29), the active frequency of the structures is in the High-Frequency Range. Its cross-section in the XY0 plane resembles the radiation pattern from shear-layer-shock intersections (Fig. 4.25). But the two structures are active at different frequencies and their cross-sections in the transverse planes originate from different locations. The frequency-resolved correlation enables us to isolate the different structures in their respective frequencies and propagation paths, which are otherwise buried together in the original correlations.

When UCn is located upstream in the YZ1 plane (right plot in Fig. 4.29), the active regions are finer and the active frequency is in the Ultra-Frequency Range. This is consistent with the general trend of increasing frequency in upstream direction.

### 4.3.3 Phase-Locking of The Main Flow Structures

Movies of the instantaneous pressure fluctuations in the near jet showed a regular pulsing that seems to initiate along the shocks (so far unsubstantiated), triggering activity that travel along the Upper SL and the Deck SL. From the correlations and frequency-resolved correlations, the existence of periodic patterns has been shown repetitively. The pulsing and the periodicity suggest that phase averaging might be an effective way to reduce the complexity of the flow.

In the absence of externally imposed phase (no periodic forcing), the Morlet transform
provides an objective way to assign phase to any signal. At any time and frequency, the phase angle of the complex coefficient is well defined, and it corresponds to the conventional definition of phase when the signal is a cosine. An example is given in Fig. 4.30. For a given time interval, there will be more periods at high than at low frequencies, therefore dislocations of the phase in time-frequency are topologically unavoidable. Such dislocations are typically simultaneous with low-amplitude fluctuations at that frequency. However there are also frequencies at which dislocations are rare, indicating a sustained cosine-like pulse. Such is the case along the red line in Fig. 4.30, with the phase plotted as a function of time. The single dislocation at time 2.6 ms shows on the time trace; elsewhere, one can see a widening and shrinking of the period (sawtooth spacing). The corresponding signal is used to define the phase applicable to conditional averaging of other signals at the same frequency.

Figure 4.30: Phase of pressure signal excerpt as function of time and frequency (top), grey shades range is $[-\pi, \pi]$; (bottom) section at 2.1 kHz (red line) showing the phase variation in time

Depending on the signal selected for this purpose, the phase angle might be more or less
relevant to the other signals. If less relevant, phase averaged signals at the other location will scramble positive and negative fluctuations to a larger extent, leading to smaller phase-averages than would be obtained with physically relevant triggers. This effect is illustrated in Fig. 4.31. A number of cosine waves of unit frequency were assigned random amplitudes, as a model of modulated wave packets in the wavelet transform of some signal. All signals are then normalized based on their collective standard deviation, and the averages are calculated for the coherent and scrambled groups. The coherent reference corresponds to the same phase for all signals and the averaged phase is the largest and of order one for the normalized signals. Whereas the scrambled phases are random, to model the effect of an irrelevant phase reference. The scrambled average amplitude is much smaller and of order 0.2 or lower. It varies from sample to sample for the small number of signals used in this example. For very long signals without phase coherence, the averaged phase would tend to zero.

![Figure 4.31: Effect of phase coherence or scrambling on the phase-averaging of signals](image)

Figure 4.31: Effect of phase coherence or scrambling on the phase-averaging of signals

We first look at the possible phase-locking at $2.63kHz$ ($St_1 = 0.25$), with the reference
Figure 4.32: Snapshots of the phase-averaged Sound Pressure Level (left plot, the weak fluctuations at $SPL \leq 100$ are whitened out) and the dimensionless velocity and vorticity (right plot) at $2.63 kHz$, in reference to the sensor located at the red cross signals in the XY0 plane. This frequency has displayed the strongest periodicity and it plays an important role for the majority of pressure and velocity fields (Fig. 4.10, 4.11 and 4.21). For a given reference signal location and at the selected frequency, its phase is calculated as a function of time. Then the pressure and velocity fields are band-pass filtered (Morlet transform) and conditional-averaging is conducted on the filtered signals based on the phase of the reference signal. Thus a phase-averaged pressure / velocity fields is obtained for the reference signal. The left plot in Fig. 4.32 shows one snapshot of the Sound Pressure Level calculated from the phase-averaged pressure fluctuations. The top plots are the phase-averaged SPL distribution in the XY0 plane, and the bottom plots are the XZ0 planes. The reference signal is placed outside the Upper SL (the red cross); the choice of location will be explained later. As the phase angle increases, the flow pattern evolves and highly resembles the pattern observed with the frequency-resolved correlation at $2.63 kHz$ (Fig. 4.28). The right plot in Fig. 4.32 shows
one snapshot of phase-averaged velocity and vorticity field using the same reference signal. The patterns in the XY0 plane include: the flow propagates along the shock (right plot), while large areas of pressure propagate downstream along the shock sides; as they reach the shock-shear-layer intersections and the deck edge, the flow erupts and continues downstream along two paths, one along the Main SL and one along the Deck SL; meanwhile, large areas of pressure are propagating upstream outside the jet stream upstream of the deck edge. In the XZ0 plane, we observe structures propagating along the Side SL and areas propagating upstream at the sides of the nozzle exit.

Figure 4.33: Distribution of the reference sensor’s effectiveness at 2.63kHz, with the mean of the phase-averaged Sound Pressure Level as the evaluation parameter; the dashed line shows the possible Kulite sensor locations

Phase averaging is likely to be an asset when collecting and analyzing experimental data. Some reference locations are more ‘effective’ and the pressure fields are more in sync with them, e.g. the sensor in Fig. 4.33. Some reference locations are less effective and the averaged pressure field has lower amplitudes and the patterns are much dimmer. To evaluate the effectiveness of the reference signals, the Sound Pressure Level
of the phase-averaged pressure fields is calculated and then their mean value is recorded. The mean of the averaged SPL field is then assigned to the selected reference point. Performing this procedure for all reference points gives a distribution of the mean amplitudes in space (Fig. 4.33). Large mean amplitude indicates that a large fraction of the field is in a coherent phase relationship with that sensor location, while small mean amplitude indicates a lack of coherence. In Fig. 4.33, very wide regions both inside and outside the jet stream are very effective reference locations (dark red regions). Although the regions inside the jet stream are not experimentally reachable, the signals outside the jet streams could be measured and guide and synchronize the measurements of the core jet. One set of possible Kulite sensor positioning for phase locking is indicated by the black dashed line in Fig. 4.33 (the sensor in Fig. 4.33 is positioned along this line). It is obtained through linear fitting of the local maxima of the mean value distribution above the Upper SL.

Figure 4.34: Distribution of the reference sensor’s effectiveness at 32.4 kHz and possible Kulite sensor locations (dashed lines)
Figure 4.35: Snapshots of the phase-averaged Sound Pressure Level (left plot, the weak fluctuations at $SPL \leq 100$ are whitened out) and the dimensionless velocity and vorticity (right plot) at 32.4kHz, in reference to the sensor located at the red cross.

The phase-locking at other frequencies are also investigated and Fig. 4.34 and 4.35 are the results of phase-averaging at 32.4kHz ($St_3 = 0.27$). This frequency level is another dominant peak in many hot-spots and near-field regions, especially those along the Inner SL. Also as mentioned earlier, the two dominant frequency levels (2.6kHz and 32.4kHz) are non-dimensionalized to approximately the same Strouhal number 0.26, if their respective scaling parameters are used. Fig. 4.34 shows the effectiveness (mean values) of the reference sensors. Two regions outside the jet streams display high coherence with the pressure field and two sets of possible Kulite positioning are indicated by the dashed lines. The left plot in Fig. 4.35 is the phase-averaged Sound Pressure Level, with the reference signal positioned along the dashed line above the Upper SL. The structures with high coherence are similar to those observed using frequency-resolved correlation (Fig. 4.26). The right plot in Fig. 4.35 is the
phase-averaged velocity and vorticity. Structures move downstream along the Inner SL (both plots) and along the side of the shock (left plot); possible acoustic waves erupt as the structures reach the shear-layer-shock intersection and the deck edge (left plot).

### 4.3.4 Summary of Near-Field Interactions

In this section, we have observed five kinds of NF activities using space-time correlations. Their pathways are summarized in Fig. 4.36:

- Along the shear layers, there are trains of localized pulsing, propagating and expanding downstream along the blue paths. The shear layers include: the Upper SL at the top the core jet, the Inner SL as the interface between the core and the wall jet, the Side SL on both sides of the core jet and the Deck SL starting at the edge of the deck. Note the merging of the shear layers start just after the deck ends.
- Apparent sound waves are radiating at two types of locations: the intersection of the shock and the shear layers (dark green waves), and the edge of solid surface.
(bright green waves), such as the nozzle lip and the deck edge.

- There are large areas that show periodicity with upstream propagation. These are located outside the jet core upstream of the deck edge, i.e., above the Upper SL, below the deck and on both sides of the jet streams.

- The shocks act as the interface of two flow regions. Inside the shock cells, the flow moves with sharp angles from the shock (red paths). This is manifested by the overall downstream propagation along jet centerline and the zigzag of the possible weak shocks downstream.

- The possible corner vortices at the upper corners of the jet stream are highly localized. They move downstream and start to mix with the surroundings after reaching the more turbulent mixing regions downstream of the deck edge (orange waves).

The frequency-specific features of these activities have been investigated using spectral analysis, frequency-resolved correlations and phase-averaging. Fig. 4.37 summarizes the dominant frequency levels of the observed structures. Some structures are active across more than one frequency range; here only the most energetic level is shown for clarity.

- The general trend for all the activities is that, in streamwise direction, the active frequency is the highest at the nozzle exit and decreases gradually downstream; in spanwise direction, the active frequency decreases moving away from the jet centerline; in transverse direction, the highest frequency occurs around the deck and moving away from it, the frequency decreases.

- The structures along the Upper SL are active in similar frequencies with those
Figure 4.37: The dominant frequency levels (color coded) of the near-field structures moving in the upper shock cells, while those in the lower shock cells follow the same frequencies with the structures along the Inner SL and Deck SL.

- The possible acoustic waves (shown up with pressure signals) emerge at higher frequencies than the structures along the shear layers (with both pressure and velocity signals) that the waves originate from.
- Two frequency levels are found as important to the majority of the flow activities: around 2.63kHz, most of the pressure and velocity fields show periodicity, moving downstream inside and moving upstream outside the jet streams; the frequencies around 32.4kHz seem the active level for most of the acoustic waves (pressure signals) and the structures moving along the lower sides of shocks, and along the Inner SL and Deck SL (pressure and velocity signals).
4.4 Near-Field Acoustic Events and Their Associated Flow Structures

One potential advantage of the three-stream engine design is the reduction of far-field noise [15, 55]. In this section, we focus on the near-field acoustic radiations, more specifically the waves most active around 32.4kHz ($St_3 = 0.27$). These acoustic waves are observed originating from two kinds of sources, the shock-shear-layer intersection and the edge of solid surfaces (Fig. 4.26). The radiations from solid edge have some similar features with the noise from jet-surface interaction, such as the low Strouhal number and the directivity [16, 95]. However, more detailed analysis is needed for a more definitive judgement, while the topic of the thesis is about the noise from flow structures. On the other hand, the radiations from the shock-shear-layer intersection around 32.4kHz are apparently related to the structures along the Inner SL, while its dominant frequency (non-dimensionalized) is in the same range with that of the noise from coherent structures for subsonic jet flows [25, 12]. The structure of this section is similar to that of Chapter 3: after selecting several sensors for tracking acoustic information propagation, an event extraction algorithm will be applied and the acoustic events will be analyzed.

4.4.1 Propagation Speed and Direction

In the previous section, the waves around 32.4kHz ($St_3 = 0.27$) have been categorized as acoustic radiations. This is based on several qualitative description of the waves:
they radiate from a localized center and assume the form of concentric circular waves; they are observed outside the jet stream and seem to radiate with the same speed in a wide range of directions. Before jumping into their analysis, more quantitative evidence will be provided to confirm their acoustic nature, the result of which will also be of use for the placement of near-field pressure sensors.

The waves of interest are the strongest with the frequency-resolved correlations at 32.4kHz obtained by correlating the pressure signal at the Inner SL and the pressure fields (Fig. 4.26). Fig. 4.38 shows one snapshot of the frequency resolved correlation in the XY0 plane, zooming in on the waves from the hot-spot Hb, which is located at \([X/D_h, Y/D_h] = [1.23, 0.36]\). To track the waves, the regions with correlation values (absolute value) larger than 0.02 are first identified. These are the strong purple or cyan patches in Fig. 4.38. The middle points of the patches are recorded as an estimate of the wave front locations (represented by the black crosses in the left plot). Then a curve fitting algorithm is applied to simulate the wave fronts (right plot in Fig. 4.38). Some
waves can be modeled as part of a circle and algebraic circular fitting is applied to obtain the optimal function (the red dashed curves). Some curves contain too few data points for successful circular fitting and a linear fitting is applied instead (the green dashed curves). Note that around the edges of the waves (purple or cyan patches), the shape of the wave does not overlap with the circular function. This could be due to the interaction of waves from multiple sources, such as those from the nozzle top lip and those from the shock-shear-layer intersection downstream (hot-spot Fl).

**Figure 4.39:** Calculating the wave propagation speed: left plot - the perpendicular distance between neighboring waves (black lines), which is used for speed calculation; right plot - propagation speed of the waves

Having obtained the wave front locations, we can proceed to calculate the wave propagation speed. From the simulated wave fronts, the center of the waves can be estimated. The blue dashed lines in the left plot in Fig. 4.39 represent the slope perpendicular to the wave center. Measuring the perpendicular distance of adjacent slopes (the short black lines, $\Delta L$) provide an estimate of the wavelength $\lambda$ ($\lambda = 2 \cdot \Delta L$). Thus the propagation speed of the waves can be calculated from $v = \lambda f_0$. The right plot
in Fig. 4.39 shows the probability distribution of the wave propagation speed. Over 50% of the waves have propagation speed between 340m/s and 380m/s. The mean speed of all the waves is 358m/s, which gives 4% discrepancy from the sound propagation speed \( c_0 = 343m/s \). The propagation speed of the waves from the hot-spot Hb is also calculated at other frequencies (these waves can be observed between 25kHz and 42kHz). The result shows the wave propagation speed is not diminished as the waves move away from the center, nor is frequency an influencing factor, thus confirming the waves’ acoustic nature.

### 4.4.2 NF Sensors for Acoustic Event Extraction

With the simulation data, we have access to time-resolved pressure (or velocity) distribution both inside and outside the jet stream. This is often not the case with experimental measurements. To simulate the experimental setting, the possible sensor location is looked for outside the jet stream and it is optimized through the statistics of the wave fronts. The left plot in Fig. 4.40 shows the probability distribution of the wave front slopes. The location of the wave center is estimated is the same fashion. Utilizing these information, the direction perpendicular to the wave fronts and extending from the wave center is obtained and a set of eight sensors is placed outside the jet stream (the black crosses in the right plot in Fig. 4.40).

The sensor closest to the jet stream is numbered as Sensor 1 and the number increases as the distance from the jet stream increases. The information propagation through these sensors can firstly be viewed in the statistical sense by calculating their two-point
cross-correlations (the left plot in Fig. 4.41). From the left subplot to the right and from the top to bottom, the distance between the sensor pairs increases. The dashed black line represents the expected acoustic propagation time between the sensors. The correlation between all the sensor pairs show large oscillations and the time lag of peak correlation is usually different from the acoustic propagation time. This is due to the periodic information propagation in the Mid-Frequency Range outside the jet stream (Fig. 4.28) and manifests the need of resolving the correlations in frequency. The right plot in Fig. 4.41 shows the frequency-resolved correlations between the same sensor pairs with those of the left plot. Around $32.4kHz \ (St_3 = 0.27)$, strong correlation can be found with all the sensor pairs and the correlation level is the largest and positive (purple shade) at the acoustic propagation time. These are shadowed by the strong periodic oscillations around $2.6kHz \ (St_1 = 0.25)$.

If the time lag between the sensors is fixed at the acoustic propagation time, the
Figure 4.41: The two-point (left plot) and frequency-resolved (right plot; the color scale is $[-0.2, 0.2]$) cross-correlations between NF pressure sensors (the sensor number increases as the sensor moves away from the jet stream)

time-frequency contributions to the peak correlation at $32.4 kHz$ ($St_3 = 0.27$) can be investigated using continuous wavelet. Morlet wavelet transform is applied to the sensors and Fig. 4.42 shows the dot products of the sensors’ Morlet coefficients in time and frequency domain. Here the real part of the complex products is used to smooth out the oscillations within and event and focus on the envelope (Equation 2.30). Strong blue-red patches are found at three frequency levels: $3kHz$, $12.5kHz$ and $32.4kHz$. The activity around $3kHz$ are sustained over a wide range of time and they contribute to the periodic patterns observed with temporal correlations. Some strong activity can be found around $12.5kHz$; however, they are not maintained through all the sensor pairs. Strong correlation contributors are found around $32.4kHz$ for all the sensor pairs. Some of these are maintained through all the sensors (e.g., around $13.2ms$ and $13.45ms$) while some gradually disappear (e.g., around $13.05ms$). The former will be extracted as acoustic events that are propagated from the NF to the FF, while the latter will be
discarded as background noise.

Figure 4.42: Excerpts of the correlation coefficients in time and frequency domain between NF sensors (all the sensors are shifted by the acoustic propagation time)

4.4.3 Event Extraction Algorithm

During the analysis of the subsonic jets, an algorithm has been developed to extract the events that contribute to the information propagation between multiple sensors. In this section, this algorithm will be adjusted to extract the acoustic events propagated through the eight pressure sensors. Fig. 4.43 shows the instantaneous time-frequency contributions to the multi-correlation between the eight sensors. Here the Morlet coefficient of the signal is normalized at each frequency level such that the
multi-correlation coefficient is in the range between $[0, 1]$, with 1 being perfectly in phase and 0 being completely un-correlated. The multi-correlation coefficients are then Gaussian filtered to focus on the envelope rather than individual oscillations (Equation 2.33). In Fig. 4.43, the sensors are shifted by the acoustic propagation time ($\Delta s/c_0$) and the frequency range between $21kHz$ and $50kHz$ is focused on. Large correlation contributors are found around $13.2ms$ and $13.4ms$ (dark grey patches), consistent with those in the pair-wise correlations (Fig. 4.42).

![Multi-Correlation between 8 Sensors](image)

Figure 4.44: The extracted events (blue markers) using the multi-correlation algorithm and eight pressure sensors

The strong multi-correlation contributors are recorded as possible event candidates. This is combined with several other criteria to make the ‘best’ selection. Firstly, since the structures around $32.4kHz$ have been revealed as quasi-periodic using both correlation and phase-averaging technique, one selection criterion ensures that the events assume sustained wave patterns. Extending from the occurrence time of the event candidate, the duration of this event with large enough correlation coefficient ($\xi(t_0, f_0) \leq C_\xi$) is required to be longer than a few period ($N_T \cdot T_0$). Secondly, the optimal time lag ($\tau_l$) between the local pressure excerpts (Equation 3.7) is adjusted using local cross-correlation. The requirement that the waves assume similar shapes at
different sensor locations is ensured by the threshold $C_c$ on pair-wise local correlation and the threshold $C_x$ on local multi-correlation (replace the $p_i$ signal in Equation 2.25 by the local excerpts). An additional criterion is on the band-pass filtered (around the event occurrence frequency) pressure magnitude at the event occurrence time. This makes sure that all the extracted waves are at the peak at the identified instants and are thus better synchronized. Thus a list event candidates is acquired and the last step is to keep one ‘best’ candidate from each group. This process is very similar to that of event extraction for subsonic jet and is omitted here. Fig. 4.44 shows the acoustic events (61 events in total) thus extracted. The majority of the events overlap with the strong correlation contributors and their frequency is around $32.4kHz$, as would be expected.

![Figure 4.45: One example of the extracted events: top plot - the band-pass filtered (32.4kHz) pressure field at the event occurrence time; lower plots - the raw (left plot) and band-pass filtered (right plot) pressure signals at the event occurrence time](image)

One example of the extracted events is shown in Fig. 4.45. The upper plot shows one instantaneous snapshot of the band-pass filtered pressure field during the duration of the event. For reference, the black crosses represent the sensor locations. The lower left
plot shows the raw pressure signals at the sensors around the event occurrence time and
the lower right plots shows the band-pass filtered pressure signals. The band-pass
filtered signals highly resemble each other, and the instantaneous snapshots of the
band-pass filtered pressure field show wave peaks (red) and valleys (blue) propagating
and expanding from the shock-shear-layer intersections to the far-field.

![Conditional Average of Raw Pressure](image)

**Figure 4.46:** Condition-averaged raw pressure signals shifted by the optimal local lag:
top plot - conditioned based on event occurrence time; bottom plot - conditioned based
a randomly generated list of time instants

To test the effectiveness of the event extraction algorithm, the raw pressure signals are
condition-averaged based on the extracted events (Fig. 4.46). Extending a few periods
on both sides of the event occurrence time (represented by the black vertical line), the
excerpts of the raw pressure signals at different sensor locations are extracted (different
colored curves). The signal excerpts are shifted by the optimal local lag ($\tau_l$) and then
averaged. The condition-averaged signals show very good overlap between different
sensors. They assume the shape of sustained wave packets that extend three and a half
period on each side of the occurrence time. This is estimated as the event duration,
beyond which, the signals scramble and become out-of-phase. For comparison, a randomly generated list of events are created. The raw pressure signals averaged based on the random list do not exhibit any apparent in-phase relation (the bottom plot in Fig. 4.46).

4.4.4 The Possible Source of Acoustic Events

![Figure 4.47: The frequency-resolved cross-correlations at 32.4kHz between the pressure sensor (red marker) and the velocity field (non-dimensionalized to Mach number)](image)

The existing noise mechanisms of supersonic jets include Mach wave radiation (and crackle), turbulent mixing noise (coherent structures and isotropic turbulence), BSAN and screech (Section 1.2.2). The waves in discussion are not Mach waves, since the convection speed along USL is subsonic and the radiation angle is too large (over 90°). Crackle is also ruled out since it is present with supersonic convection and often observed with full scale, very hot jets [43]. In addition, the criteria for crackle detection, i.e., the high skewness and sharp pressure gradient [35], has not been found in the NF pressure. These waves have some similarities with BSAN, such as the apparent source
location and the omnidirectional radiation. However, the active frequency for radiation around 90° should be approximately $5kHz$ (Equation 1.7). $32.4kHz$ is clearly too large for BSAN, as well as for screech noise [133]. The general trend of BSAN, i.e., increasing frequency in downstream direction, also contradicts the trend observed previously (Fig. 4.37). In previously analysis, the waves at $32.4kHz$ are observed along with the coherent structures along Inner SL using both frequency-resolved correlation (Fig. 4.26) and phase-averaging (Fig. 4.35). Their relation is confirmed by cross-correlating the velocity field (Mach number) and the pressure sensor outside the jet stream (Fig. 4.47). The patterns are very similar with Fig. 4.26, apart from the pressure waves propagating along the shock side.

Figure 4.48: The cross-correlations between the velocity signals along the Inner SL (their locations are indicated by the cyan markers in the left plot) and the pressure signal along the acoustic propagation path (the red marker)

To further analyze these noise-related coherent structures, four velocity sensors are placed along the Inner SL (left plot in Fig. 4.48). The right plot in Fig. 4.48 shows the frequency-resolved correlations (without normalization) between the velocity sensors
(top plot) and between the velocity and pressure sensors (bottom plot). All the sensor pairs are strongly correlated at $32.4\,kHz$ and the correlations are sustained for over five periods. Velocity sensor 3 is also correlated with the pressure sensor at $2.6\,kHz$ with a negative time lag. Whether this relates to the acoustic information needs to be further investigated. Based on the time lag of peak correlation, the convection speed between sensor 1 and 3 is estimated as $447\,m/s$, which corresponds to $Ma = 1.54$ using the acoustic speed in the jet. The convection speed between Sensor 2 and 4 is $434\,m/s$ or $Ma = 1.50$. Since the acoustic speed is lower inside the jet stream than the ambient, the eddies between the two streams could be convected at supersonic speed and radiate waves that are reflected by the deck and propagated along the shock structures.

Finally, the flow snapshots related to wave radiation are isolated using the time lag information. Fig. 4.49 shows one snapshot before (top-left plot) and three snapshots immediately after the instant that corresponds to an acoustic event. In each frame, the top plot shows the instantaneous Mach number, zooming in on the Inner SL. The center plot shows the fluctuating vorticity distribution, filtered around $2.6\,kHz$ and focusing on the same region. The bottom plot is the distribution of Q criterion and the signals are filtered around $32.4\,kHz$. From the low-frequency vorticity distribution, the flow moves downstream rather slowly; around the event occurrence time, strong vorticity can be found where the shock is reflected at the deck; beyond the reflection point, the flow is dissolved into smaller structures. The Q criterion at $32.4\,kHz$ show very periodic behavior: the vortex is pulled towards the reflection point and becomes intensified; smaller structures are spun off, some follow the shock side and some continue.
Figure 4.49: The cross-correlations between the velocity signals along the Inner SL (their locations are indicated by the cyan markers in the left plot) and the pressure signal along the acoustic propagation path (the red marker) downstream.

4.4.5 Summary of Near-Field Acoustic Events

In this section, the NF acoustic waves radiating from the shock-shear-layer intersection at 32.4kHz are explored. By tracking the wave fronts, the propagation speed is calculated, which is very close to the speed of sound (4% error). The wave propagation speed is not influenced by the frequency, nor is it diminished as the waves move away.
from the radiation center. These confirm the acoustic nature of the radiating waves.

Based on the modeling of the wave fronts, a set of eight sensors are placed outside the jet stream perpendicular to the acoustic waves. The event extraction algorithm developed in Chapter 3 is adapted to look for the NF acoustic events that are propagated through all the eight sensors. The acoustic events assume sustained wave patterns that are maintained from the NF to the FF. The event-based flow snapshots relate the acoustic radiation to the supersonically convected eddies along the mixing layer between the core jet and the wall jet.
Chapter 5

Concluding Remarks

5.1 Conclusions and Discussions

This thesis presented the analysis on one experimental and one numerical dataset on high speed jet flows. The purpose was to learn more about the sources of acoustic fluctuations, with the broader goal of mitigating the environmental and medical problems traced back to jet noise. In this thesis, new presentations of the statistical analysis, combined with bandpass filtering, enable the frequency-specific pathway extraction of near-field flow structures. By combining the conventional statistical techniques with continuous wavelet transforms, events associated to jet noise production are identified in time and frequency. The event-conditioned signals and flow field reveal the noise-related patterns of flow activity.

One aspect of contribution of the thesis is the data extraction and information presentation of big data. The experimental dataset consists of two-dimensional
Time-Resolved PIV snapshots and multiple pressure signals. The numerical dataset is of the three-dimensional distribution of pressure and of three components of velocity. The essential part of the analysis is the pattern recognition in time and frequency, which expands the data size by another order of magnitude. To reduce the computational effort, we need to sift through the data and construct representative signals (Diagnostic signals for subsonic and Hot-spots for supersonic), and make selections of the representative signals by conducting statistical analysis systematically. The results involve multiple variables, i.e., about a dozen representative signals, five dimensions (X, Y, Z for space, time and frequency), additional dimension like time lag with correlation technique. Innovative presentations are developed to provide straightforward visualization of the results, which otherwise require much mental effort for information extraction and condensation. The new presentations include: peak frequency distribution, normalized spectral distribution, peak correlation distribution, animation of space-time correlation (with or without frequency resolution). In the thesis, snapshots of the animations are presented.

Although the selected representative signals help narrow down the subject of interest, the event identification between multiple signals is rather complex. It involves various different signal combinations, multiple criteria for event selection, and three dimensions, i.e., time, frequency and time lag. The extension of multi-correlation to time-frequency domain helps reduce the complexity, and the algorithm is devised such that it can be executed within very short timeframe. Based on the extracted events, the new presentation of the flow field is produced through event-based non-linear filtering and is
an effective tool in linking flow activity to noise production.

5.1.1 Subsonic Axisymmetric Jet

The first dataset analyzed in the thesis is of a fundamental flow condition, i.e., the axisymmetric subsonic jets. The noise sources consist of coherent structures, which are the focus of our analysis, and isotropic turbulence. Despite over sixty years of research, no full theories have been established on the noise mechanism of subsonically convected coherent structures. On the one hand, we have some existing source models, such as modulated wave packets, or growing-decaying instability waves. They have shown reasonable agreement with experiments regarding the far-field acoustic spectra. On the other hand, the very turbulent near-jet makes the direct pinpointing of the source a formidable task. The existing evidence gathered from experimental or numerical data is mostly in the statistical sense, e.g. the source distribution region, active frequency, estimated intermittency level and radiation directivity. No direct link has yet been established between the models and flow structures.

We tackle this problem by firstly constructing near-field Diagnostic signals which are post-processed and selected such that their correlation level with the far-field acoustics is the highest. Then the events that contribute to their correlation most significantly are identified in time and frequency, by extending the cross-correlation technique to multiple signals and combining it with continuous wavelet analysis. The event extraction algorithm combines several optimized criteria and is evaluated as capturing less than 10% false matches using various testing approaches. The event properties are
consistent with the existing evidence on noise source and the event-conditioned raw
signals exhibit modulated wave patterns. The filtered flow fields at the event occurrence
time are categorized and show similar patterns, and we speculate the noise source
consists of interrupted quasi-periodic flow activity.
The event algorithm is the first to date that is able to identify the main individual
contributors to NF-FF correlations. The wave forms of the event-conditioned signals are
very similar to the wave packet model while the event-based flow activity is consistent
with the growing-decaying instability theory. Although further analysis is needed before
unravelling the exact noise-producing activity, we have considerably narrowed the gap
between near-jet properties and far-field acoustics and have isolated the parameters
within which the source mechanisms may one day be found.

5.1.2 Two-Stream Supersonic Rectangular Jet

The second dataset is of a more novel and advanced flow configuration. It consists of
two supersonic jet streams coming off a rectangular nozzle, with a flat plate extending
from one nozzle side. This is the simplified model of the three-stream aircraft engine
that may be pursued by the United States Air Force. It only has one plane of
symmetry, and the 3D interactions of shocks, mixing layers and corner vortices seem
daunting. Existing research on the flow physics of multi-stream jets and jets with
complex nozzle geometries mostly involves qualitative flow characterization and many
focus on the shocks and corner vortices.

By introducing new presentations of the statistical analysis, such as normalized
spectrum distribution and space-time correlations, and by utilizing bandpass filtering (continuous wavelet), the interactions of NF structures are categorized and their frequency-specific propagation pathways are identified. The flow activities include the coherent structures along shear layers, shock structures, corner vortices, acoustic radiations from shock-shear-layer intersections and from deck edge, and the large regions propagating upstream periodically outside the jet stream. The NF regions where three-dimensional interaction takes place are localized just after the end of the deck along the jet centerline. Two frequency levels (2.6kHz and 32.4kHz), or one Strouhal number ($St_1 = 0.25$, $St_3 = 0.27$), are found to play a vital role in the NF physics. Using continuous wavelet analysis, the flow and acoustic fields are phase-averaged. Regions outside the jet stream are found in good phase-coherence with the majority of the NF regions at this Strouhal number, which could be used to guide the sensor placement in experiments.

The second part of the analysis focuses on the NF acoustic waves at 32.4kHz that radiate from the shock-shear-layer intersection. Multiple sensors are placed outside the jet stream and the event extraction algorithm for subsonic jet is adapted to isolate the sustained acoustic waves. The existing noise mechanisms for supersonic jets are compared with the acoustic events. The event-conditioned NF snapshots link the noise production to the supersonically convected coherent structures along the shear layer between the two jet streams.
5.2 Future Work

The complexity of the jet noise problem is caused by many factors: the turbulent flow is highly complex and not yet fully understood; the acoustic energy is of much smaller magnitude comparing with the flow kinetic energy, which requires very high resolution and produces very large datasets; the noise-producing flow activity is very intermittent while no widely-accepted indicator has yet been established to guide us about where to look. In this thesis, we have narrowed down the parameters of searching for the source. One part of the future work could be devoted to inspecting the flow fields with alternative presentations or in a three-dimensional view. Low-order presentation of the flow fields, such as POD analysis, can be combined with the event-based analysis. Other filtering approach of the flow fields can be explored, since the current analysis is constrained to selected settings of spatial filtering or band-pass filtering. Two-dimensional Fourier transform and wavelet transforms may be effective filtering techniques to isolate the event-related flow structures. It is also possible that the three-dimensional flow fields need to be looked into for the noise-producing structures. For the current analysis, the subsonic data is only available in one horizontal plane and no out-of-plane information is available. Similarly for supersonic case, the analysis on velocity fields is only performed for the plane of symmetry, while the flow is highly three-dimensional in nature. Another possible direction of future work is to combine the event extraction algorithm with existing noise source models. Firstly, the existing source model can be used as a template for event detection. This could lead to more objective event selection criterion
and could simplify the event selection process. However, since this depends on the validity of the source model, extensive analysis will be needed on the event features for validation purposes. Secondly, modeling using the wave packet and instability wave models can be investigated. Their results can be compared with those found using the time-frequency analysis and provide better understanding to the source properties.

Lastly, one aspect that sets apart the algorithm in the thesis with those in existing literature is the various algorithm testing approaches. This can be further improved by constructing synthetic signals that use the existing noise source models as the source. This may lead to better evaluation of the algorithm performance and could assist in the criterion selection for event detection.

The algorithm developed for noise-related event extraction could be applied to other kinds of event detection or information tracking. For supersonic jet, only one type of acoustic waves have been looked at, and the analysis could be extended to the waves from solid edges or those radiating from the downstream shock-shear-layer intersections. For subsonic jet, the NF pressure signals have shown varying features as their location moves downstream. They are not included in the current event extraction algorithm due to the existence of both hydrodynamic and acoustic pressure component. A filtering algorithm could be developed for separating the hydrodynamic and acoustic information using the idea of the event extraction algorithm. We could track the information that is convected downstream with the expected convection speed and this would be presumably the hydrodynamic component. Or we could isolate the acoustic components by isolating those that are also present in FF acoustic signals, or those that are
propagated to the FF with the speed of sound.

In the thesis, the velocity-related signals and velocity-derivatives have exhibited different features, such as the regions that they are correlated with the FF or the convection speed estimated from their correlation with NF pressure signals. Their difference is smoothed out using the spatial-filtering and the current event algorithm looks for those that are in common to all the Diagnostic signals. It would be interesting to explore their difference in more details. One possible approach is to look at the NF-FF communication in smaller regions and see what leads to their difference at the same locations. We could also compare the correlation contribution between the Diagnostic signals across different NF regions and look for those instants that are changed due to the spatial-filtering.

The experimental and the numerical data sampling each has its merits and more limited areas. The experiments can be performed with almost all kinds of flow conditions and do not involve any modeling. Longer data records can usually be obtained comparing with simulations. However, the measurements are usually at selected locations or selected plane sections and with time-resolved flow measurements, the advantage of better statistical convergence may be lost. The problem of flow intrusion is not present with the simulations, which can often provide better spatial and temporal resolutions and can measure properties inaccessible experimentally. The joint analysis of experimental and numerical datasets could be highly beneficial with regards to jet noise research. The identification of regions of interest from one dataset could guide the design of the other. The flow features obtained from numerical datasets, such as the
phase relation, could greatly reduce the expense of experimental setup. The parameters obtained from experimental data that help narrow down the instants and flow regions related to noise-production, can feed back to numerical data collection, whose three-dimensional and highly-resolved flow regions may provide more complete view on the noise-related flow activities.
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SUMMARY:
❖ Excellent proficiency in algorithm development for information extraction and problem solving
❖ Over seven years’ experience in data processing and pattern recognition
❖ Strong knowledge in CFD and FEA (self-written codes and ANSYS / Fluent)

EDUCATION

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SELECTED RESEARCH PROJECTS

STRUCTURE DETECTION AND EVOLUTION ANALYSIS
JUN 2015 - PRESENT SYRACUSE UNIVERSITY, SYRACUSE, NY
❖ Extracted the structures and depicted their interactions in a supersonic three-stream jet
❖ Identified the evolution paths and analyzed the structure properties using signal processing techniques (spectral analysis, correlation, continuous wavelet, etc.)
❖ Simulated the acoustic radiation from distributed sources
❖ Tracked and analyzed the events contributing to acoustic radiation

ALGORITHM DEVELOPMENT FOR NOISE SOURCE DETECTION
JUN 2011 - PRESENT SYRACUSE UNIVERSITY, SYRACUSE, NY
❖ Devised and tested algorithms to extract potential noise sources in subsonic jets using signal processing and pattern recognition techniques
❖ Analyzed experimental and numerical datasets in collaboration with researchers at Syracuse University and Institut Pprime (France)
❖ Simulated acoustic propagation with unsteady refraction from coherent structures

NUMERICAL SIMULATION OF FLUID / THERMAL MODELS
JAN 2012 - DEC 2014 SYRACUSE UNIVERSITY, SYRACUSE, NY
Simulated flows from 2D diffusers using self-written codes (Matlab and C) and investigated different time stepping schemes and artificial viscosity models

Using commercial software: a. flow around an airfoil; b. thermal conduction of laptop

JOURNAL AND CONFERENCE PUBLICATIONS


AND 5 OTHER PEER-REVIEWED CONFERENCE PUBLICATIONS, 9 CONFERENCE PRESENTATIONS AND 5 POSTERS (DFD-APS, SWBLI-TIM, TIM, ETC.)

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JAN 2011 - JUN 2011 COATES CHIA INTERNATIONAL TRADING LTD

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2011 - 2013 Fellowship in Mechanical and Aerospace Dept. at Syracuse University

2010 Award of Outstanding Thesis
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2007 First-class University-level Scholarship at Tianjin University