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Investigating the Structures of Turbulence in a Multi-stream, Rectangular, Supersonic Jet

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Abstract

Supersonic flight has become a standard for military aircraft, and is being seriously reconsidered for commercial applications. Engine technologies, enabling increased mission capabilities and vehicle performance, have evolved nozzles into complex geometries with intricate flow features. These engineering solutions have advanced at a faster rate than the understanding of the flow physics, however. The full consequences of the flow are thus not known, and using predictive tools becomes exceedingly difficult. Additionally, the increasing velocities associated with supersonic flight exacerbate the preexisting jet noise problem, which has troubled the engineering community for nearly 65 years. Even in the simplest flows, the full consequences of turbulence, e.g. noise production, are not fully understood. For composite flows, the fluid mechanics and acoustic properties have been studied even less sufficiently. Before considering the aeroacoustic problem, the development, structure, and evolution of the turbulent flow-field must be considered. This has prompted an investigation into the compressible flow of a complex nozzle.

Experimental evidence is sought to explain the stochastic processes of the turbulent flow issuing from a complex geometry. Before considering the more complicated configuration, an experimental campaign of an axisymmetric jet is conducted. The results from this study are presented, and guide research of the primary flow under investigation. The design of a nozzle representative of future engine technologies is then discussed. Characteristics of this multi-stream rectangular supersonic nozzle are studied via time-resolved schlieren imaging, stereo PIV measurements, dynamic pressure transducers, and far-field acoustics. Experiments are carried out in the anechoic chamber at Syracuse University, and focus primarily on the flow-field. An extensive data set is generated, which reveals a detailed view of a very complex flow. Shear, shock waves, unequal entrainment, compressibility, and geometric features of the nozzle heavily influence the development of this jet plume. In the far-field, the acoustic radiation is found to be highly directional. Noise spectra contain high-frequency tonal signatures, and relations to the turbulent structures are made in an effort to explain the physics responsible for such acoustic generation.

Analysis of the flow is made possible by the carefully planned experiments. By acquiring a large number of simultaneous data points, the stochastic processes are studied through statistical approaches. First- and second-order moments are used to describe the steady-state behavior of the flow. The wide array of sensors used in
the tests allows for cross-moments to be computed, which provide evidence linking
different phenomena. Proper orthogonal decomposition (POD) is used to separate
flow-field quantities into temporal and spatial pieces, which are then further uti-
lized in conjunction with other sensors. Through these methods, a high-frequency
instability is discovered in the near-field of the jet, which pervades the flow-field and
propagates ubiquitously throughout the acoustic domain. Additionally, the complex
shock structure is found to play a vital role in redistributing disturbances throughout
the flow. Finally, several POD modes in the side shear layer of the jet are found to
be correlated with acoustic production.
Investigating the Structures of Turbulence in a Multi-stream, Rectangular, Supersonic Jet

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<tbody>
<tr>
<td>AFRL</td>
<td>Air Force Research Laboratory</td>
</tr>
<tr>
<td>BBSAN</td>
<td>Broadband Shock-Associated Noise</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>cRIO</td>
<td>compact Reconfigurable Input/Output chassis</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
</tr>
<tr>
<td>LES</td>
<td>Large Eddy Simulation</td>
</tr>
<tr>
<td>LSE</td>
<td>Linear Stochastic Estimation</td>
</tr>
<tr>
<td>MARS</td>
<td>Multi-Aperture Rectangular SERN</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>Neodymium-doped Yttrium Aluminum Garnet</td>
</tr>
<tr>
<td>NI</td>
<td>National Instruments</td>
</tr>
<tr>
<td>NPR</td>
<td>Nozzle pressure ratio</td>
</tr>
<tr>
<td>NTR</td>
<td>Nozzle temperature ratio</td>
</tr>
<tr>
<td>OASPL</td>
<td>Overall Sound Pressure Level</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>PLC</td>
<td>Programmable Logic Controller</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
</tr>
<tr>
<td>POD</td>
<td>Proper Orthogonal Decomposition</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds Average Navier-Stokes</td>
</tr>
<tr>
<td>SERN</td>
<td>Single-Expansion Ramp Nozzle</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>TKE</td>
<td>Turbulent Kinetic Energy</td>
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Chapter 1

Introduction

Supersonic flight sees widespread use in military aircraft, and is a distinctive design of future commercial vessels. Many of these vehicles utilize advanced engines with nozzles far removed from the basic configurations studied in classical aerodynamics. Complexity in jet nozzles, whether geometric, fluidic, or thermodynamic, has emerged in many supersonic aerospace applications over recent decades. The engineering solutions, while commendable, have developed more quickly than our understanding of the flow physics. The full consequences of the flow, e.g. acoustic production and unsteady loading, are thus not known. This uncertainty is due to the turbulent nature of these flows, where computational tools simply cannot resolve all of the scales which the complex physics span. One of the most challenging phenomena to simulate is aeroacoustics, a field that has challenged the aerospace community for over 65 years. For the problem of jet noise, experimental test campaigns play a vital role by providing invaluable data and analyses that are otherwise unattainable. In increasingly complex jet flows, the nonlinear fluid dynamic processes must first be studied before moving to the more difficult problem of aeroacoustics.
1.1 Motivation

Understanding and reducing jet noise are difficult problems due to the inevitable turbulence encountered [1, 2]. The aerospace industry continues to invest considerable effort into mitigating jet noise [3] because the intense sound creates unwanted acoustic pollution near airports [4] and generates negative health consequences to flight deck crews [5]. In the modern jet engine, acoustic production results from the moving parts (i.e. fan, compressor and turbine stages), the combustion process, and the jet flow exhausting from the nozzle. Engine technologies have advanced such that the largest source of noise during takeoff in low-bypass and supersonic engines is a result of the external jet flow, which increases with speed.

Despite the tremendous advances in aeroacoustics, the problem of jet noise is growing because supersonic flight is becoming more popular. In the defense industry, these supersonic speeds are well known among fighters and reconnaissance aircraft. Currently, the technological obstacles that must be overcome to enable civilian transports revolve around increased fuel efficiency and reducing noise generation, which includes sonic booms and jet noise[6]. The sonic boom problem must be solved for the cruise portion of the flight envelope, and many efforts are focused on this[7–9]. Meanwhile, two technological examples work to increase affordability. Rectangular nozzles yield improved airframe integration, which can significantly reduce wave drag [10]. Three-stream engine designs [11] offer greater efficiencies at supersonic by adaptively changing to the flight conditions. While these approaches show promise, many fundamental questions of aeroacoustics remain unanswered. Furthering the understanding of turbulence in supersonic flow is a critical step toward noise source identification and reduction for future aircraft.

To complicate the problem, the flow exhausting from the engines may travel through intricate features. Complex nozzles, which have evolved from alterations to basic jets, incorporate a variety of canonical flows to achieve specific performance gains in the system at little cost to the propulsion. Rectangular nozzles have been
studied for some time[12–14]. Because of their ease in varying the internal geometries, they are adaptable to different flight scenarios. SERNs (Single-Expansion Ramp Nozzles) are one type of rectangular nozzle which can efficiently utilize translational throats[15] and can operate robustly over a range of nozzle pressure ratios (NPRs) off design condition[16]. Furthermore, SERNs are an excellent choice for use with thrust vectoring and reversing technologies[17], thereby increasing the agility of aircraft. Finally, the addition of multiple streams in nozzles have been shown to reduce noise while maintaining thrust requirements[18], and show great promise of optimizing performance, increasing efficiency, and aiding in thermal management [11]. These distinct configurations form the cornerstones of the current design under study, which aims to further reduce noise production while maintaining superior flight performance.

1.2 Historical Perspective

Increased payloads, mission capabilities, and flight envelopes require enhanced powerplants, and propulsion systems have met these goals by providing adequate thrust. One approach is to make the engine bigger, as commonly seen in commercial airliners. But an enlarged engine size is costly due to the associated drag and weight penalties, and is practically impossible for supersonic vessels. Therefore, greater thrust is achieved by increasing the exit velocity of the gas, which scales as \( u^2 \). Commendable progress has been made on noise reduction[19], such as greater bypass ratios, the implementation of chevrons, and active flow control techniques. However, even with these technologies available, meeting noise regulations is challenging [20], and aircraft regularly get louder. For example, the 65 dB noise footprint of the F-35 has nearly doubled in size as compared to the F-16 [21]. (This value, which reflects a day-night average, limits the number of flights that can be performed in a 24 hour period, and operating costs rise as additional airstrips are built to spread out the flight demand.) The technical challenge of noise reduction in advancing technologies is clearly a result
of the increasing fluid velocities.

Historically speaking, engines are typically designed around thrust requirements. Power requirements drive the engine design, from which other considerations, e.g. inlet and nozzle geometries, are secondary. Noise production is usually not considered in the design process. Chevrons are a good example of this; the General Electric GE90 turbofan was originally developed with a clean nacelle, and the chevron modification was then implemented a decade later. The process may not produce the most optimal solution, however. Particularly in supersonic aircraft, airframe integration is a more crucial step of the design, as the drag increase plays a larger role at higher speeds.

The supersonic engine has changed significantly over the years, mostly from lessons learned through military aircraft. The focus here will be on exhaust systems, but note that inlet geometries are equally important. Afterburning engines were introduced in the 1940’s, which used strictly convergent nozzles. With the advent of this technology, the need for variable exhaust geometry became apparent. The exit area must be enlarged to allow for increased mass flow rate, else the back pressure increases which adds drag and can lead to engine stall [22]. The convergent-divergent (CD) nozzle wasn’t introduced until the 1950’s, until after the first supersonic fighter, the F-100D, had been produced. The F-100 Super Sabre achieved supersonic flight through the Pratt & Whitney J-57, which had a two-position convergent nozzle. This meant the nozzle was inefficient though, and its maximum speed was limited. The CD nozzle, first seen in the F-4, expands the exit flow further in the diverging portion of the nozzle, which results in more thrust and thus greater speeds to be achieved. Improvements on the CD nozzle were sought as airframe integration was considered. By bringing the nozzle closer to the body, which requires a departure from the axisymmetric nozzles used in the F-4, aft-end drag can be reduced [23]. Today, CD nozzles are almost exclusively employed for supersonic propulsion. Specific fighter jets, e.g. the F-22, have moved away from the axisymmetric model. This aircraft, along with the Concorde, also demonstrates the technology of supercruise. This con-
dition of supersonic flight does not require the use of an afterburner, and is thus much more efficient. Supercruise is achieved through careful airframe integration: a high fineness-ratio fuselage and a low aspect-ratio, highly-swept wing can supercruise \[24\]. Both the F-22 and the Concorde, perhaps the most technologically advanced vehicles of their kind, show vastly different propulsion systems than the axisymmetric convergent jet originally used for supersonic flight.

As the need to travel even faster, e.g. Mach 4 - Mach 6, becomes more likely among future vehicles, engineers look to designs which can operate robustly over a wide range of NPRs. The axisymmetric configuration faces the challenge of mechanical complexity in its variable nozzle geometry, and so two-dimensional designs have been considered. The two-dimensional CD (2DCD) nozzle is one common configuration, and the single-expansion-ramp nozzle (SERN) is another that has received attention. Both geometries are designed to operate robustly over a range of off-design conditions, which many future vehicle will require as they fly faster and higher. The SERN, however, appears to be advantageous because of its smaller size; reduced weight and lower skin friction drag \[15\] make this an ideal candidate for future designs. One potential disadvantage of the SERN is a thrust-dependent pitching moment, which requires advanced controls and careful design to handle. While the operation is meant to be robust, it is not necessarily the most efficient over all conditions. To achieve this adaptive operation, the use of additional flow configurations is necessary \[11\].

Moving toward a more efficient nozzle design, the consideration of noise should be included. As the noise problem is not fully understood, a goal of designing an engine which is optimally powerful and quiet is likely overly ambitious. Instead, noise prediction capabilities are desired with engine design. If the acoustic characteristics of nozzles are documented, noise trends may be observed over a range of different geometries. With time, designers could relate specific noise features to engine configurations, and work toward a quieter propulsion system that does not sacrifice thrust.
1.3 Objectives

The objective of this research is to understand the structures of turbulence of a complex supersonic jet flow. In the context of aeroacoustics, this flow is of interest for physics-based low-dimensional models being developed. The nozzle under investigation has been designated *MARS* - Multi-Aperture Rectangular SERN, where SERN is the acronym for a Single-Expansion Ramp Nozzle. The jet is rectangular, possesses multiple streams, expands asymmetrically, is exhausted over a partial boundary (i.e. representative an aft deck), and has only a single plane of symmetry; thus, the flow from this source is extremely complex, and will test the fidelity and robustness of numerical models. In the broadest sense, experimental evidence is gathered and utilized to describe this flow. Specifically, the scope of the work can be broken into three distinct parts; the work will:

- Establish a high-quality dataset, which consists of:
  - Far-field acoustics
  - Near-field pressures
  - Time-resolved schlieren
  - Stereo PIV

- Provide a three-dimensional description of the jet via:
  - Mean flow-field statistics
  - Turbulence components

- Investigate flow physics unique to:
  - Formation of turbulent structures
  - Aeroacoustics and noise generation
To study this problem, experimental testing is performed. Conditions are created in the Syracuse University anechoic chamber to model real-world flow conditions as closely as possible. This physical experiment provides invaluable data which cannot be produced by any other means, and that can be provided to aid modelers and analysts. The jet rig has been designed, manufactured, and installed in the chamber for extensive testing. Facility upgrades have been implemented to allow for this powerful jet exhaust. Furthermore, the development of specific instrumentation is critical to capturing the fluid mechanics at the required levels of fidelity important to this flow. From the simultaneously sampled data, analytical techniques are performed which extract meaningful conclusions from the cumbersome datasets.

1.4 Overview

The document contains five more chapters and additional appendices. Chapter two discusses the technical background of turbulence and aeroacoustics before presenting some recent findings on the round jet. This experiment on the round jet was conducted exclusively for this thesis, and was necessary for transitioning to the MARS jet. However, as it is a fundamental configuration, the results have been placed in the chapter on Background, before discussing in detail some advanced nozzle configurations. Chapter three describes the experimental setup used to study this flow, and lays out the test conditions under investigation. Chapter four presents the results of the density gradient field, as acquired through schlieren imaging. Chapter five explores in greater detail the structure of the jet through stereo PIV measurements. The final chapter ends with important conclusions that have been learned from this work and suggests some areas that future efforts may focus on.
Chapter 2

Background

The major difficulty in investigating high $Re$ flows is the inevitable turbulence encountered. Turbulence has been studied for over 130 years\cite{25, 26}, and remains today one of the last unsolved problems in classical mechanics. These flows are irregular, highly three-dimensional, rotational, and evolve nonlinearly, thus leading to the general description that they are chaotic. Even in the most basic flows, deterministic solutions have not been found to date, and turbulent flows are therefore typically described via stochastic approaches\cite{27}. This chapter demonstrates the difficulty of the turbulence problem, especially when dealing with real, complex, engineering flows. Some examples of previous endeavors are covered, and finally the groundwork for the approach taken herein is presented.

2.1 Turbulence & Compressibility

One particular complication in the treatment of turbulent flows is the extensive range of scales encountered. The largest scales are determined from the greatest geometric parameter, i.e. the characteristic length, $l$, and the mean flow properties. Coherent structures (discussed later) breakdown through the so-called “cascade of scales” into smaller events until molecular diffusion dominates the processes at the smallest level
and eddies dissipate through heat (due to viscosity). Kolmogorov’s famous theory [28, 29] arrived at the conclusion that these smallest scales decrease in size (nonlinearly) with increasing $Re$. Thus, the range of scales increases with $Re$, making the more momentum-driven flows the most challenging. (Note that, despite the tremendous advances in both theory and computational power, this inherent large range of scales is one of the greatest obstacles facing CFD for high $Re$ flows.)

The full Navier-Stokes equations, which govern the motion of linear (newtonian) fluids, are given in indicial notation by

$$\rho \frac{Du_i}{Dt} = \rho g_i - \frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j}. \quad (2.1)$$

Here, $D$ is the material derivative, $u_i$ is the velocity, $\rho$ is density, $g$ is gravity, $p$ is pressure, and $x_i$ represents Cartesian coordinates. The viscous stresses are given by

$$\sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \delta_{ij} \lambda \frac{\partial u_k}{\partial x_k}, \quad (2.2)$$

where $\mu$ is the dynamic viscosity, $\lambda$ is the coefficient of bulk viscosity, and $\delta_{ij}$ is the Kronocker delta. Closure is achieved by including conservation of mass,

$$\frac{D\rho}{Dt} + \rho \frac{\partial u_i}{\partial x_i} = 0, \quad (2.3)$$

and the first law of thermodynamics, which for fluid motion is written as

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right) + \sigma_{ij} \frac{\partial u_i}{\partial x_j}, \quad (2.4)$$

where the enthalpy is given by $h = e + p/\rho$, $e$ is internal energy, and $k$ is thermal conductivity (introduced through Fourier’s law). These equations, Equation 2.1, Equation 2.3, & Equation 2.4, form a full description of fluid motion. The primary unknown variables are $p$, $u_i$, and $T$. Auxiliary relations (e.g. an equation of state) are required to relate the remaining variables (i.e. $\rho = f(p,T)$, $\mu = f(p,T)$, $h = f(p,T)$, 


\[ k = f(p, T). \]

Together with conservation of mass and energy, the Navier-Stokes form a system of coupled nonlinear partial differential equations. While they exactly describe Newtonian fluid flow, the existence of a solution has never been proven to always exist, i.e. conditions arise (e.g. turbulence) where a mathematical singularity may occur and the equations are not possible to solve. With the extreme difficulty of such equations, assumptions are made to arrive at more useful forms. Requiring the flow to be incompressible allows for the thermal-fluid relations to be decoupled, which greatly simplifies the problem. In this case, the energy equation can be solved separately, and the Navier-Stokes equations reduce to

\[
\rho \frac{Du_i}{Dt} = \rho g_i - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j}.
\]

The incompressible continuity equation is

\[
\frac{\partial u_i}{\partial x_i} = 0.
\]

However, even with the restriction of incompressibility, solutions are still not attainable due to the onset of turbulence. As discussed, increasing the Reynolds number increases the range of scales that occur, which makes the more inertially dominated flows the most difficult. Reynolds’s took an approach to work around the many scales of turbulence [30] using a decomposition method, now honored with his name. By breaking the flow quantities into mean and fluctuating variables, the Reynolds decomposition allows one to time average the incompressible non-linear Navier-Stokes equations. The unknown variables become averaged quantities, which in many cases can be more useful information than the instantaneous variables governed by the Navier-Stokes. The instantaneous variables are decomposed into
where the \( \langle \cdots \rangle \) indicates the time-averaged quantity and \( \langle \cdots \rangle' \) is the fluctuating value. Upon substituting Equation 2.7 into Equations 2.5 and Equation 2.6 and performing a time-average, one arrives at the Reynolds-Averaged Navier-Stokes (RANS) equations. After manipulation, these are expressed in the convenient form [31]:

\[
\frac{\rho D\bar{u}_i}{Dt} = \rho g_i - \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \tag{2.8}
\]

where

\[
\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho u'_i u'_j. \tag{2.9}
\]

In 2.9, the viscous term represents the laminar contribution to the stress tensor, and the time-averaged, fluctuating quantity is the "apparent stress" introduced by the inertia of turbulence. The six quantities that make up this symmetric tensor are referred to as the Reynolds stresses, and often abbreviated as \( R_{ij} \). These are very useful quantities that can be measured experimentally. However, these additional variables are unknown \textit{a priori}. Without a type of mathematical model (which in many cases use over-simplifying assumptions), more unknown variables than available equations exist. This forms the infamous closure problem of turbulence.

When compressibility effects cannot be neglected, thermal-fluid coupling is active and the full equations must be considered (Equation 2.1 - Equation 2.4). In this regime, some very nonintuitive mechanics arise, such as nonlinear interactions between vortices, acoustic waves, and shock/expansion waves. For example, Xu \textit{et al.} [32] showed that from a field of random shocklets, dispersive shock waves associated with locally supersonic turbulent eddies, the formation of 'giant collective incoherent' shock waves can occur. In compressible turbulent flow, the flow variables
are not the only quantities fluctuating. The thermodynamic properties such as density and enthalpy, and thermophysical fluid properties such as specific heat capacity, viscosity, and thermal conductivity, also vary in space and time. Therefore, these variables cannot be treated as constant and many cross-correlations term arise. In this case, performing Reynolds averaging results in an unmanageable number of unknown terms. One way to treat these additional complications is to decompose the fluctuations into vorticity, acoustic, and entropy modes [33]. A more compact way was shown by Favre [34, 35], who introduced density-weighted variables, defined by

\[ \tilde{f} = \frac{\rho f}{\bar{\rho}}, \]  

(2.10)

where \( f \) is any flow variable and the \( (\cdots) \) again indicates the Reynolds-averaging operation. With this definition, a dependent variable, \( f \), can be written as

\[ f(x_i, t) = \tilde{f}(x_i, t) + f''(x_i, t), \]  

(2.11)

where \( f'' \) is the unresolved part of the variable. Note that this unfiltered variable, \( f'' \), is not the same as the fluctuating component in Reynolds’ approach, i.e. \( f'' \neq f' \). It can be shown [36] that the two variables are instead related by

\[ f'' = f' - \frac{\rho' \tilde{f}}{\bar{\rho}}. \]  

(2.12)

Substituting in the density-weighted variables and time-averaging the governing equations, the Favre-averaged equations are arrived at. Conservation of mass is given by

\[ \frac{D\bar{\rho}}{Dt} = -\bar{\rho} \frac{\partial \tilde{u}_i}{\partial x_i}. \]  

(2.13)

The momentum equations are
\[
\rho \frac{D\bar{u}_i}{Dt} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} - \frac{\partial \bar{R}_{ij}}{\partial x_j}, \tag{2.14}
\]

where the viscous stress tensor can be approximated by neglecting viscosity fluctuations. The strain rate, \( S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \), is formed by approximating the gradients with Reynolds-averaged variables rather than the true Favre-mean velocity. This gives

\[
\sigma_{ij} = 2\mu \left( S_{ij} - \frac{1}{3} S_{kk} \delta_{ij} \right) \approx 2\tilde{\mu} \left( \tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{kk} \delta_{ij} \right). \tag{2.15}
\]

The turbulent stress tensor is

\[
\bar{R}_{ij} = \rho \bar{u}_i \bar{u}_j - \bar{\rho} \bar{u}_i \bar{u}_j = \rho (\tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j). \tag{2.16}
\]

Substituting the density-weighted decomposed variables, Equation 2.11, into the turbulent stress tensor yields

\[
\bar{R}_{ij} = \rho \tilde{u}_i \tilde{u}_j = \rho \left( \tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j \right) \tag{2.17}
\]

Substituting the definition of the unresolved quantity, the relation \( \bar{\rho} u_i'' = 0 \) can be shown. Thus, Equation 2.17 simplifies to

\[
\bar{R}_{ij} = \rho u_i'' u_j'' = \rho \tau_{ij}^c. \tag{2.18}
\]

where the compressible turbulent stress tensor is

\[
\tau_{ij}^c = u_i'' u_j''. \tag{2.19}
\]

Equation 2.18 is the form commonly found in literature and represents the turbulence that arises from the unfiltered variables leftover from Favre-averaging, analogous to the fluctuating terms in Equation 2.9.
Though the derivation is outside the scope of this work (interested readers should refer to Gatski & Bonnet [36]), the energy equation cannot be ignored. It is included here for completeness. Using the stagnation energy, \( E \approx e + u_i u_i / 2 \), and approximating the stagnation enthalpy as \( H \approx c_p T + u_i u_i / 2 \), where \( c_p \) is the specific heat at constant pressure, the Favre-averaged energy equation is written as

\[
\frac{\partial (\bar{\rho} \bar{E})}{\partial t} + \frac{\partial (\bar{u}_j \bar{\rho} \bar{H})}{\partial x_j} = \frac{\partial (\bar{u}_i \bar{\sigma}_{ij})}{\partial x_j} - \frac{\partial \bar{q}_j}{\partial x_j} - \frac{\partial \bar{Q}_j}{\partial x_j},
\]

(2.20)

where, if a perfect gas is used as the equation of state and Fourier’s law invoked,

\[
\bar{\rho} \bar{E} = \bar{\rho} e + \frac{\bar{\rho} \bar{u}_i \bar{u}_j}{2} + \frac{\bar{R}_{ii}}{2} = \bar{\rho} c_p \bar{T} + \frac{\bar{\rho} \bar{u}_i \bar{u}_j}{2} + \frac{\bar{R}_{ii}}{2},
\]

\[
\bar{\rho} \bar{H} = \bar{\rho} \left( \bar{E} + \frac{\bar{p}}{\bar{\rho}} \right) = \bar{\rho} c_p \bar{T}_t,
\]

\[
\bar{q}_j = - \left( k \frac{\partial \bar{T}}{\partial x_j} \right) \approx - \bar{k} \frac{\partial \bar{T}}{\partial x_j},
\]

\[
\bar{Q}_j = \bar{\rho} u_k H - \bar{\rho} \bar{u}_j \bar{H} = \bar{\rho} c_p \left( \bar{u}_j \bar{T}_t - \bar{u}_j \bar{T}_t \right),
\]

(2.21)

and \( \bar{T}_t \) is the stagnation temperature. Like Reynolds’ approach, the Favre-averaged equations require multiple closure models, as they form an open set of differential equations. In Equation 2.20 & Equation 2.21, there are multiple unknown correlation terms that must be treated, in fact many more than produced in the RANS approach. Note that in this process, the density fluctuations in the time-averaged equations of motion have been removed, with the exception of the turbulence terms. The viscous diffusion term \( \bar{u}_i' \sigma'_{ij} \), and scalar flux \( \bar{Q}_j \), have been created as a result of turbulence associated with thermal-fluid coupling. These require additional modeling or parameterization to close the equations. From these three governing equations, the fluctuating transport equations (e.g. for \( u''_i \)) are next derived by applying conservation laws to each term in question. These provide additional relations that can be imposed on the correlation terms, and making a few additional assumptions, closure can be
achieved. This is still an active area of research, and readers are again referred to Gatski & Bonnet [36] for a full discussion of this complex topic.

The compressible turbulent kinetic energy (TKE) equation can be derived in the same manner as the incompressible case. The process begins by acquiring the equation for the turbulent stress transport equations, which comes from taking the transport equation for fluctuating momentum, $u''_i$, multiplying it with $u''_j$, and then using a Strong Reynolds analogy to average the terms [37]. This yields the evolution equation for the compressible turbulent stress tensor, $\bar{\rho}\tau^{c}_{ij}$:

$$\frac{D\tau^{c}_{ij}}{Dt} = \frac{\partial}{\partial t} \left( \bar{\rho}\tau^{c}_{ij} \right) + \frac{\partial}{\partial x_k} \left( \tilde{u}_k \bar{\rho}\tau^{c}_{ij} \right). \quad (2.22)$$

When closure and modeling is pursued, Equation 2.22 is usually broken into components by their physical role: production $P$, redistribution $\Pi$, destruction $\epsilon$, mass flux contribution $M_f$, and transport/diffusion $D$. Grouping the terms in this manner also provides physical insight. Proceeding on to the TKE equation, the trace of Equation 2.22 is performed, which produces the transport equation for compressible TKE, $\bar{\rho}k = \bar{\rho}\tau^{c}_{ii}/2$, given by

$$\bar{\rho} \frac{Dk}{Dt} = \bar{\rho}P + \bar{\rho}\Pi - \bar{\rho}\epsilon + \bar{\rho}M_f + \bar{\rho}D \quad (2.23)$$

where
\[
\begin{align*}
\dot{\rho}P &= \frac{\dot{\rho}P_{ii}}{2} = -\dot{\rho}_{ij}^{\epsilon} \frac{\partial u_i}{\partial x_j}, \\
\dot{\rho}\Pi &= \frac{\dot{\rho}\Pi_{ii}}{2} = p' \frac{\partial u_i'}{\partial x_i}, \\
\dot{\rho}\epsilon &= \frac{\dot{\rho}\epsilon_{ii}}{2} = \sigma'_{ij} \frac{\partial u_i'}{\partial x_j}, \\
\dot{\rho}M_f &= \frac{\dot{\rho}M_{fi}}{2} = \frac{\partial p'}{\partial x_i} \left( \frac{\partial \bar{u}_i}{\partial x_i} - \frac{\partial \sigma_{ij}}{\partial x_j} \right), \\
\dot{\rho}D &= \frac{\dot{\rho}D_{ii}}{2} = -\frac{\partial}{\partial x_j} \left[ \frac{\bar{u}_i' \bar{u}_i'' \bar{u}_j''}{2} + p' u_i' \delta_{ij} - \sigma_{ij}' u_i' \right].
\end{align*}
\] (2.24)

By considering the mean energy equation, Equation 2.20, and the TKE equation, Equation 2.23, one can derive relations between specific terms for comparison. Lele [38] discusses the energy budget for compressible flows, and shows the complex interchange of energy that takes place. In comparison to incompressible flow, additional pathways exist in compressible turbulence that allow for the exchange of energy between the mean kinetic energy, TKE, and internal energy. The production term, \(\dot{\rho}P\), is of particular interest, because it is the direct and exclusive contribution from the mean kinetic energy equation that generates TKE. Note that this production term is similar in form to the incompressible case \(\left( u_i' u_j' \frac{\partial u_i}{\partial x_j} \right)\), but has density dependence hidden in the Favre-averaged and compressible turbulence stress tensor variables.

### 2.1.1 Morkovin’s Hypothesis

Morkovin [39] made an important contribution to compressible turbulence by proposing that the fluctuating density can be decoupled from the turbulence terms. He postulated that, for non-hypersonic flows, ‘... the essential dynamics of these supersonic shear flows will follow the incompressible form’. This postulate was based on experimental evidence from wall-bounded shear layers, where the heat transfer from the wall was related to the skin friction. The experiment revealed that turbulence-
induced heat and momentum transfer to the wall occurred in the same manner, from which the general hypothesis was made. Mathematically, this was stated as

\[
p'/p \ll 1, \quad T'/T_0 \ll 1 \quad \Rightarrow \quad \rho' / \rho \approx -T'/\bar{T} \approx (\gamma - 1) M^2 (u'/\bar{u})
\]  

(2.25)

In other words, the fluctuating velocity correlations are affected by the mean density only, which allows one to extract the unfiltered density from the correlation term. This essentially equates to

\[
\overline{R_{ij}} = \bar{\rho} \overline{u_i' u_j'} \approx \bar{\rho} \overline{u_i u_j'}.
\]

(2.26)

Bradshaw [40] states the Morkovin hypothesis, Equation 2.25 is valid based on the requirement that \( \sqrt{\rho'^2 / \bar{\rho}} \sim 0.1 \). Bradshaw estimated this occurs in boundary layers and wakes for a freestream Mach number less than 5, and in jets for a Mach number less than 1.5. For the MARS jet, operating at a Mach number of 1.6, the limit of the Morkovin hypothesis is tested.

This hypothesis, has very useful consequences. It allows experimental data, which typically acquires the Reynolds-averaged velocity terms only, to be used to study compressible turbulence. Panda and Seasholtz [41] have shown that in a shock free jet, the hypothesis holds as high as \( M = 1.8 \). They performed an experiment which captured both Reynolds-averaged and Favre-averaged quantities to compare against each other. Deviations in the axial turbulent stresses varied between a fraction of a percent to 1%. Morkovin’s hypothesis is extended in the present work to allow for calculation of the production term in Equation 2.23. The validity of this assumption is revisited in subsection 5.1.2. Looking at the axial components of the triple-product correlation terms in the energy equation, Panda and Seasholtz found errors up to 4%. It should be noted that only the axial terms were considered, due to experimental
limitations. Maeder et al. [42] looked at the shear stresses and found positive agreement between incompressible and compressible, which serves as general confirmation of the hypothesis. However, to the author’s knowledge, the presence of shocks has not been experimentally tested. Thus, in ideal jet conditions, Morkovin’s hypothesis appears justified for mild Mach numbers. But shocks and strong scalar fluxes may violate the proposed density decoupling in flows that are generally assumed to be valid.

As a final note, Morkovin’s hypothesis finds additional application in comparing experimental data with computational results from LES. Various relationships can be derived which equate the different variables. The correlation terms can be found in Chapter 4 of Gatski & Bonnet [36]. Researchers looking to compare experimental and numerical data are encouraged to explore these relations.

2.1.2 Structure Identification

Structure identification has played a crucial role in furthering our understanding of turbulence [43–47] and jet noise [48–55]. In an attempt to better understand the multifaceted nature of turbulent flows, coherent structures are identified as frequently occurring flow lattices. By that definition, these spatial organizations are representative of some important process unfolding in the flow. Since turbulence possesses a range of scales, structures of different size, velocity, and lifespan can exist at any given time. An obvious example of such arrangements are vortices, which are a major component of turbulent flow. After some observed intermittencies in shear layers [56] were found to influence global parameters (e.g. momentum and heat transfer), Liepmann [57] was one of the first to write about these “secondary” structures in the flow; this line of thought was furthered by Townsend [43]. Crow and Champagne [44] found additional experimental evidence of such features through flow visualization (e.g. schlieren and spark photography). In the axisymmetric jet that they studied, particular modes (i.e. coherent structures of similar scales) were linked to oscillatory
behavior in the shear layers and waves in the jet column. More physical evidence [46] subsequently found vortex pairing to be an integral part of the turbulent mixing process, and Brown and Roshko [45] noted how entrainment rate is intertwined with the large scale structures of the flow. These earliest examples of coherent structures spurred countless more works in turbulence. Today, coherent structures are a well-established feature of turbulent flows.

During the onset of the experimentally-observed coherent structures, Lumley began to formulate his theoretical approach to recognizing flow lattices. Based on principal component analysis, proper orthogonal decomposition (POD) was introduced in 1967 [58] as an objective means to identify coherent structures. It wasn’t for another 20 years until this approach was utilized in an experimental setting through hot wire probes [59]. This mathematical approach, discussed in subsection 2.1.3, is a much more rigorous treatment of the turbulent quantities, rather than the flow visualization techniques as previously discussed. Specific energy modes can be solved for to identify important features at scales of interest. Through both visualization methods and POD, different canonical flows are shown to possess coherent structures characteristic of that flow. Free shear flows, e.g. the jet, have some of the most well-defined structures and therefore are an excellent application of POD, as first successfully shown by Glauser [60].

Reduced-order models, such as POD, are useful tools which can simplify highly complex systems. POD is routinely selected in fluid mechanics because it optimizes a basis set with respect to the turbulent kinetic energy, and these bases can then be carefully interpreted as coherent structures. In the context of jet noise, POD has proven valuable in relating flow physics to acoustic production. Much work has been carried out in the $r - \theta$ planes of the axisymmetric jet for the subsonic case [61–64] and supersonic case [63, 65]. However, there is additional flow organization in the streamwise direction. For example, the reviews by Ho and Huerre [66] and Liu [67] illustrate that the mixing layer consists of primary (spanwise) structures, superimposed
by secondary (streamwise) vortical structures. Similarly, one would not expect the
transverse planes of the axisymmetric jet to fully characterize its three-dimensional
turbulent structures. In fact, by studying the $r - z$ plane, Berger [68] identified
through POD an event in the subsonic jet that was linked to high acoustic output.
Ukeiley et al. [69] considered both transverse and streamwise orientations of the jet
as they built spatial correlations to locate noise mechanisms, noting the differences
in structures that each view revealed. For the supersonic jet, Edgington-Mitchell et
al. [70] similarly looked in this orientation, focusing on the helical instability mode
of a screeching jet. Using POD as a means to phase-average the data, they were able
to form a triple decomposition to identify regions of high stresses and shock-vortex
interactions that varied with streamwise position.

### 2.1.3 Analytical Tools

Proper orthogonal decomposition produces optimal basis functions for a field by maxi-
mizing energy of the system through a two-point spatial correlation. Lumley’s original
method lends itself well to velocity data because it effectively sorts structures based
on their kinetic energy content. For spatially high-resolved data, such as PIV, a more
computationally tractable formulation was made by Sirovich [71] and is therefore em-
ployed with large data sets. He simplified the calculation by assuming ergodicity,
which allows the spatial correlation kernel of the classical POD optimization problem
to be restated in terms of a temporal correlation tensor in an eigenvalue problem.

A brief summary of the snapshot simplification is now discussed. The goal of
POD is to breakdown a spatiotemporal field quantity, generally $u_i(\vec{x}, t)$, into time-
dependent coefficients, $a_n(t)$, and spatial basis functions, $\phi_i^{(n)}(\vec{x})$. The method initi-
ates with an integral eigenvalue problem defined by:

$$
\int_T C(t, t')a_n(t')dt' = \lambda^{(n)}a_n(t),
$$

(2.27)
where $T$ is the integration time and $C(t, t')$ is the two-time correlation tensor calculated from

$$C(t, t') = \frac{1}{T} \int_\Omega u_i(\vec{x}, t)u_i(\vec{x}, t')d\vec{x}.$$  

(2.28)

To maintain consistency between the snapshot and classical approaches, the temporal coefficients must be scaled such that the eigenvalues are equal to the norm of the time-dependent coefficients,

$$\langle a_m(t), a_n(t) \rangle = \lambda^{(m)} \delta_{mn},$$  

(2.29)

where $\langle \cdots \rangle$ is the inner product and $\delta_{mn}$ is the Kronecker delta. Finally, the orthonormal basis functions, or spatial eigenfunctions $\phi_i^{(n)}(\vec{x})$, are obtained from

$$\phi_i^{(n)}(\vec{x}) = \frac{1}{T\lambda^{(n)}} \int_0^T a_n(t)u_i(\vec{x}, t)dt.$$  

(2.30)

Reconstruction of the flow field can then be accomplished through the combination of the temporal coefficients and basis functions. Thus, the instantaneous velocity field is equal to the following expansion:

$$u_i(\vec{x}, t) = \sum_{n=0}^{N} a_n(t)\phi_i^{(n)}(\vec{x}).$$  

(2.31)

Because the integral in Equation 2.27 is across all time, the total number of modes in the decomposition ($N$) is equal to the number of snapshots used (e.g. 2500 snapshots yields 2500 modes). In this formulation the zeroth mode is the average flow-field quantity. The flow may be reconstituted exactly, by letting $n = 0, 1, 2, \ldots, N$, or selective reconstruction may be used by choosing specific modes of interest for $n$. When time-resolved data is used, downsampling becomes necessary to ensure statistical independence between snapshots [72]. Once the field is decomposed into its spatial and temporal components (using the downsampled data), the original time-resolved
velocity field is then projected onto the basis functions, \( \phi^{(n)}_i(\vec{x}) \), to recompute the temporal coefficient, \( a_n(t) \). This preserves the time resolution of the temporal coefficients, while using statistically independent snapshots. Typically, this method is employed with velocity selected as the field quantity as the energy analogy is straightforward. However, it can also be applied to other field quantities, such as pressure or density, and the vector computations then reduce to scalar ones. This is used on the time-resolved schlieren data, and the results are presented in chapter 4.

An approach for quantifying similarities between spatial structures must also be established. Since POD is an objective tool which relies on statistical information, the deterministic results, i.e. the spatial basis functions, should be reproducible under ideal conditions. Comparing the similarities in the eigenfunctions of two separate flows is a way to address this reproducibility. To compare two eigenfunctions, \( \phi^{(n_1)}_i \) & \( \phi^{(n_2)}_j \), a cross-correlation method is used, which is computed by

\[
\rho_{n_1,n_2} = \langle \phi^{(n_1)}_i(\vec{x}), \phi^{(n_2)}_j(\vec{x})^T \rangle.
\]  

(2.32)

where \( i \) & \( j \) refer to the eigenfunctions from different ensembles, and \( n_1 \) & \( n_2 \) are in reference to the particular modal number from each data set.

Because the spatial functions are previously normalized (Equation 2.30), there is no need to include the \( L^2 \) normalization as is conventionally done; the maximum value of Equation 2.32 is unity and implies a perfect correlation. Furthermore, because the basis functions are orthonormal, Equation 2.32 can be viewed as a measure of orthogonality between any two modes in question. As such, computing these values for one test case against itself, i.e. \( i = j \), returns the identity matrix, \([\vec{I}]\). When two separate studies are being compared, a square matrix of size \([n_1 \times n_2]\) is generated, describing the relative similarity in shape between each mode.

With the velocity data decomposed, further analyses can be executed on each component. In the current approach, the most statistically significant basis functions that govern the large-scale dynamics of a multi-degree-of-freedom system are quan-
titatively selected. The temporal coefficients, $a_n(t)$, are treated as any time signal, allowing for cross-correlations to other simultaneously sampled diagnostics, e.g. far-field pressures, $P_i(t)$. This technique is a type of stochastic estimation, which was first proposed by Adrian [73]. Bonnet et al. [74] originally used this concept to estimate the POD coefficients from another signal. By using a conditional pressure signal, the method was furthered by Taylor and Glauser [75], fully demonstrated by Tinney et al. [62], with more recent efforts by Low et al. [64] and Berger et al. [76]. The multi-time stochastic estimation [77–79] is used throughout a few of these studies and is presently employed. In this technique, a range of time lags ($\tau$) are created around the average acoustic propagation time, $t$, and then checked for correlation. To determine acoustically significant events, the covariance, $R$, must first be computed by

$$R_{n,k}(\tau) = \frac{1}{N} \sum_{n=1}^{N} \langle a_n(t), P_k(t + \tau) \rangle,$$

where $n$ is a POD mode number and $k$ refers to a specific pressure signal (e.g. from a microphone). Thus, a matrix is generated for each time lag. The Pearson correlation coefficient is then normalized by the standard deviation of the time-dependent coefficients, related by $\sigma_{a_n} = \lambda^{(n)(1/2)}$, and the standard deviation of the far-field pressure, $\sigma_P$,

$$\rho_{n,k}(\tau) = \frac{R_{n,k}(\tau)}{\lambda^{(n)(1/2)}\sigma_P}.$$

Correlation values in each time lag series, $\rho_{n,k}(\tau)$ are then extracted based on a $\rho_{n,k} > r\sigma_\rho$ threshold, where $\sigma_\rho$ is the standard deviation of the correlation coefficient defined in Equation 2.34 and $r$ is defined by user input. The statistically significant data can provide considerable insight into flow states which correlate well with other diagnostics (e.g. a loud flow representation correlating with a microphone).
2.2 Jets

Jets are one of the most widely studied physical phenomena, and an overwhelming amount of literature exists on them. In the most fundamental geometries, assumptions can be made to simplify the governing equations and learn important results about the flow. As a first order approximation, the MARS jet is treated as a rectangular one. Even the basic rectangular jet is a challenging flow to deal with. However, it can be thought of as a point in between the round (axisymmetric) and planar (two-dimensional) jet. While theoretical approaches to the rectangular case is very limited [80], extensive theory has been developed for the other two cases. All jets eventually become round with downstream flow evolution, so the theory of round jets is of significance for the far-field behavior of the complex nozzle. Additionally, because the MARS jet can be viewed as a fusion of multiple rectangular flows, the planar jet behavior serves as an adequate starting point along each stream’s centerline in the near-field.

2.2.1 Jet Dynamics

Even under the most restrictive circumstances (e.g. two-dimensional, incompressible), reducing the full Navier-Stokes (Equation 2.5) or RANS (Equation 2.8) equations to a workable form is no trivial task. Similarity solutions are one powerful approach used to reduce the nonlinear partial differential equations to a (series of) differential equation(s). This reduction of complexity is usually achieved through a coordinate transformation, where the independent variables are collapsed into dimensionless groups that typically scale with a flow parameter such as velocity. For the laminar cases, Schlichting [81] found exact solutions for a narrow round jet and infinite planar jet. In most scenarios, a high-speed flow transitions from laminar to turbulent early on, so that the assumptions used by Schlichting to simplify the Navier-Stokes equations do not hold. Similarity solutions which include the fully-compressible turbulence terms
are extremely limited [82], and so the present discussion is limited to incompressible turbulent jets.

A schematic of a turbulent jet’s formation is shown in Figure 2.1. While the growth rates differ, \( b(x_1) \), planar \( (x_1 \sim x, x_2 \sim y) \) and axisymmetric \( (x_1 \sim z, x_2 \sim r) \) jets have the same basic developing features. Seen in Figure 2.1, high velocity flow issues from an orifice at \( U_j \) into an ambient fluid. Immediately downstream, this fluid is unperturbed by the mixing layers formed along the jet boundary, so it can be treated as nearly inviscid and thus a potential flow. As the mixing layers grow, aided by the unstable development of Kelvin-Helmholtz vortices, they eventually merge, ending the potential core region. This length, \( L_c \), depends on the exit velocity. Significant turbulent activity occurs here, as the coherent structures from each shear layer collide and produce high stresses. As the role of viscosity increases, the plateau in the velocity profile diminishes and self-preservation is approached. 

Figure 2.1: Features of a real developing jet.
describes a state of the flow when mean and turbulent fields are determined solely by local quantities. Mathematically, this is expressed by writing

$$\frac{\bar{u}}{U_{\text{max}}} \sim f\left(\frac{x_2}{b}\right)$$

(2.35)

Generally, the length to self-preservation, $L_s$, occurs downstream of $L_s \sim 20l$, where $l$ is the diameter or slot height of the jet. Beyond this, similarity analyses can be applied.

Görtler [83] first used an eddy-viscosity model to close the RANS boundary-layer equations and performed a similarity analysis to obtain a profile for the planar and axisymmetric jet. He assumed a mixing-length model which varies as $x^{1/2}$, and defined the following similarity variables for the planar jet,

$$\bar{u} = U_o \left(\frac{x_0}{x}\right)^{1/2} F'(\eta)$$

$$\nu_t = K U_0 b_0 \left(\frac{x}{x_0}\right)^{1/2}$$

$$\eta = \frac{\sigma y}{x}$$

where the subscript $0$ denotes initial conditions at $x_0$, $K$ and $\sigma$ are constants to be determined, $F(\eta)$ is the similarity function, and the transformed coordinates permitting similarity are $\nu_t$ and $\eta$. After a respectable amount of mathematics, he was able to obtain a solution for $F$,

$$F = \tanh(\eta)$$

(2.37)

and ultimately the growth rate, $b(x)$. (Note that a true boundary of the jet is not well-defined, since the velocity asymptotes to zero as shown in Figure 2.1.) From his
experimental data, Görtler found $\sigma_{\text{planar}} \approx 7.67$. Using the half-velocity boundary condition ($\bar{u} = U_{\text{max}}/2$ and $\eta_{1/2} = \text{sech}^{-2}(0.5)$), the growth rate is found to be

$$b_{\text{planar}} = x \tan(13^\circ). \quad (2.38)$$

For the round jet, the geometry suggests a $1/z$ scaling of $U_{\text{max}}$. Görtler also solved the round jet problem, and found that

$$\frac{\bar{u}}{U_{\text{max}}} \approx \left(1 + \frac{\eta^2}{4}\right)^{-2} \quad \eta = \sigma_{\text{round}} \frac{y}{x} \quad (2.39)$$

where in this case, $\sigma_{\text{round}} \approx 15.2$. Again, using the half-velocity boundary condition, one can calculate the growth rate as

$$b_{\text{round}} = z \tan(9.6^\circ). \quad (2.40)$$

Mean velocity data, as measured from experiments, match well with this relationship. However, turbulent quantities do not have good agreement; this is likely from the assumed $1/z$ dependencies of the similarity variable. Since the mean growth rate matches well with data, a value of $\sigma_{\text{round}}$ was recalculated using the planar similarity variables. Letting $\sigma_{\text{round}} \approx 10.4$ and substituting into Equation 2.39, excellent agreement is seen with experimental data [31]. While the estimations are indeed close, this ad hoc approach has not treated the physics appropriately, and the solution obtained is partly luck. Nonetheless, equations Equation 2.38 & Equation 2.40 are hypothesized to be the upper and lower bounds on the growth rate of the rectangular jet.

When the turbulence quantities are investigated more thoroughly (e.g. the Reynolds stresses, Equation 2.9), one sees that self-preservation is not achieved until much further downstream than $L_s > 20l$. The dominant turbulent quantity in the near-field
of jets is $u'$, and the transverse components do not collapse to the similarity profiles until $x_1 > 70l$ [84]. Even beyond this length, there is scatter in the data from different experiments [85]. These discrepancies led to questioning of the approach taken, and brought about a new method of analysis. Traditional similarity analysis, as the above section falls under, assumes a priori a scaling of the amplitude in the similarity variables [86–88]. This is a restrictive assumption, because it does not allow for different initial conditions between jets (e.g. $Re$).

George argued [85] that the jet behavior is not universal, and the fully-developed region depends on the initial conditions. His conjecture was that differences in the observed far-field data are dependent on the large-scale structures of the near-field; the breakdown process of the largest structures into smaller ones, leading to a self-preserving turbulence state, differs based on initial conditions. In other words, turbulence does not forget. Since then, experimental evidence has been brought forth supporting this for the jet, [89, 90], along with shear layers and wakes [91–95]. To complicate the problem, Wygnanski and Fiedler [84] point out that even far downstream in a jet ($L_s > 100l$), turbulence may not be isotropic. The single-point statistics (e.g. $u'_i u'_j$) used for similarity do not consider any variation in turbulence as functions of length scale. Energy is contained and distributed among different scales of motion in the flow, which single-point quantities cannot capture. Recent work by Ewing [96] suggests an ‘underlying equilibrium’ may be at play, and uses a two-point similarity analysis to improve the estimations of the jet behavior. This work provides additional evidence for use of the equilibrium similarity analysis of George. Ewing concludes that “there is not a universal solution that describes the large-scale structures in the far field of the round jet, so that the structures present may depend on how the flow is generated.” The ultimate far-field effect of these turbulent structures can be found in the acoustic generation. As will be seen, the spectral signatures of jets are very sensitive to the initial flow conditions.

Finally, the mechanics of supersonic jets are considered. In addition to the turbu-
lence problem, these class of flows are compressible, and often posses discontinuities in the flow-fields in the form of shock waves. Figure 2.2 shows the classical behavior of a highly expanded jet and will serve as the example to describe the flow.

![Figure 2.2: Structure of the highly underexpanded jet.](image)

Because pressure waves are limited by the local speed of sound, the supersonic jet may be exhausted at pressures unequal to the ambient fluid, as in Figure 2.2. This scenario allows for oblique/normal shocks and expansion fans to occur in the fluid (shown in gray), which initiate at the nozzle lip. This series of steps permit the static pressure of the jet to gradually match that of the surrounding medium. The subsonic mixing layer acts as a boundary. As the shocks from the nozzle lip propagate across the centerline into the ambient air, they encounter the shear layers and are reflected back inwards to the core. Upon this incidence of reflection, the expansion fans are forced into compression waves. Depending on the strength of the jet, the compression waves may coalesce and form oblique shocks. When the shocks meet again in the centerline, the region downstream of their intersection is lower than the surrounding environment, forming a “cell” of pressure. If the pressure difference at the exit is great enough, normal shocks are necessary to make up the mismatch and the presence of Mach disks may form. In this case, the oblique shocks are reflected
here rather than the centerline. Again, the waves continue onto the mixing layer and are again reflected, this time reverting back to expansion fans. As the pressures try to stabilize, additional oblique shocks propagate downstream just as before and the process repeats itself. Depending on the Re of the jet and it’s over/under-pressurization, this continues downstream until the shock strengths decrease (due to molecular and turbulent dissipation) and the pressures eventually equalize. The presence of these shocks create additional interactions with flow structures that complicate the turbulence problem previously discussed.

2.2.2 Aeroacoustics

Acoustic Analogies

Acoustic analogies are useful tools that assess the overall noise generation from the flow and corroborate the observation that faster exhaust speeds generate more noise. The aeroacoustic problem is reduced by representing complex sources as simple field emitters (e.g. from a multipole expansion). In 1952, Lighthill’s acoustic analogy [1, 97] was the first theoretical treatment of aerodynamically-generated noise, and showed that sound power scales as high as the 8th power of velocity.

To calculate acoustic radiation from a region of turbulent flow, Lighthill [1] developed an analogy, derived from the Navier-Stokes equations. The acoustic analogy is named so because it models a complex fluid mechanical process as a point source rather than allowing feedback, but it is exact; i.e. no approximations are made in its derivation. The formulation rearranges the conservation of mass Equation 2.3, and momentum Equation 2.1, equations into a wave equation, so that the acoustic field at the listener can be accurately be described by:

$$\frac{\partial \rho'}{\partial t^2} - c_0^2 \nabla^2 \rho' = 0$$  \hspace{1cm} (2.41)

where the density fluctuation, or acoustic variable, is given by \( \rho' \equiv \rho - \rho_0 \), and the
ambient fluid properties in an infinite homogeneous fluid are density, $\rho_0$, and the speed of sound, $c_0$. One begins the derivation with mass and momentum equations for an ideal gas. Neglecting body forces, these are

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \quad (2.42)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2.43)$$

By multiplying continuity Equation 2.6 with velocity, $u_i$, one adds it to momentum to obtain

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2.44)$$

where, for conciseness, use of the viscous stress tensor has been made, given in Equation 2.2. Differentiating continuity, Equation 2.6, with respect to time,

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\partial^2 \rho u_i}{\partial t \partial x_i} = 0. \quad (2.45)$$

Taking the divergence of Equation 2.44,

$$\frac{\partial^2 \rho u_i}{\partial t \partial x_i} + \frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j} = -\frac{\partial^2 p}{\partial x_i \partial x_i} + \frac{\partial^2 \sigma_{ij}}{\partial x_i \partial x_j} \quad (2.46)$$

and subtracting Equation 2.46 from Equation 2.45, one obtains

$$\frac{\partial^2 \rho}{\partial^2 t} - \frac{\partial^2 p}{\partial x_i \partial x_i} + \frac{\partial^2 \sigma_{ij}}{\partial x_i \partial x_j} = \frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j} \quad (2.47)$$

The equation is rearranged into

$$\frac{\partial^2 \rho}{\partial^2 t} = \frac{\partial}{\partial x_i} \left( \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial \sigma_{ij}}{\partial x_j} \right) \quad (2.48)$$

Finally, the acoustic pressure, $c_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i}$, is subtracted from Equation 2.48 to yield
\[
\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\partial p u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial \sigma_{ij}}{\partial x_j} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} \right).
\]

(2.49)

Substituting in the decomposed pressures and velocities \( \rho' \equiv \rho - \rho_0 \) & \( p' \equiv p - p_0 \), using the relation \( \rho_0 = p_0/c_0^2 \), and rearranging again, one arrives at the famous Lighthill equation. For a listener at a distance far away from the source, the sound can be exactly modeled as:

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

(2.50)

where the Lighthill turbulence stress tensor is given as

\[
T_{ij} = p u_i u_j - \sigma_{ij} + (p' - c_0^2 \rho')\delta_{ij}.
\]

(2.51)

In equation Equation 2.51, \( p u_i u_j \) is the Reynolds stress tensor, describing the nonlinear convective forces, \( \sigma_{ij} \) is the viscous stress tensor ( Equation 2.2), and the term \( (p' - c_0^2 \rho')\delta_{ij} \) is the deviation from isentropic sound propagation, or nonlinear acoustics. Due to the far distance from the source, viscosity can be neglected. And in all but the most intense noise-generation scenarios, the nonlinear acoustic term can be ignored. Thus, Lighthill’s equation is often written as

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} \approx \frac{\partial^2 p u_i u_j}{\partial x_i \partial x_j},
\]

(2.52)

and the contribution from turbulence is immediately apparent. While Equation 2.50 is exact, and Equation 2.52 a justified approximation, it is no easier to solve than the original Navier-Stokes equations. As seen in Sec. subsection 2.2.1, solving for the Reynolds stresses in a turbulent flow is not feasible. Nonetheless, if this tensor is somehow known, for example through experimental data, the density fluctuations in the radiation field can be calculated at a location, \( \vec{x} \). Using Lighthill’s equation in conjunction with the appropriate Green’s function and Kirchoff’s theorem, for
decaying turbulence such that \( T_{ij}^{-3} < y^{-3} \) for large \( y \), it is possible to show [98]:

\[
\rho(\vec{x}, t) - \rho_0 \sim \frac{1}{4\pi c_0^2} \frac{x_i x_j}{x^3} \int \frac{1}{c_0^2} \frac{\partial^2 T_{ij}}{\partial \tau^2} \left( \vec{y}, \tau - \frac{R}{c_0} \right) d\vec{y}
\]

where \( \tau \) is the propagation time and the separation vector is \( \vec{R} = \vec{x} - \vec{y} \). Using this formulation, the axisymmetric jet is considered. After considerable effort, Lighthill was able to derive an order-of-magnitude estimate, given by

\[
\bar{\rho}^2 \sim \left( \frac{\rho_0 D}{4\pi|x|} \right)^2 M_0^8,
\]

where the intensity of the sound is \( \bar{\rho}^2 \). In terms of total sound power, \( W \sim p^2 / (\rho_0 c_0) \), this is

\[
W = K \left( \frac{\rho_0 D^2}{c_0^3} \right) U^8.
\]

the Lighthill constant has been verified to be \( K = 3 \times 10^{-5} \). Equation 2.55 shows excellent agreement with experimental data for a subsonic jet[98], and is now typically viewed as the mixing noise of a jet. More rigorous treatments of the turbulence quantities results in the well-known directivity pattern of the radiation field [99]. Refraction in the jet flow is generally attributed to this heart-shaped spatial redistribution of acoustic energy [100–102]. The “cone of coherence” is situated along the centerline, and the most intense noise is radiated at a shallow angle off the downstream direction, thus making the heart shape. As the velocity of the jet approaches Mach 1, the above relation breaks down (as a result of compressibility and a variation in the acoustic wave’s intensity). Above Mach 1, shock waves appear, which interact with turbulence and coherent structures. The non-isentropic behavior of the near-field results in a reduced rate of sound production, and experimental data shows the following relations [103],
\[ W \propto \begin{cases} 
U^8, & \text{for } M < 0 \\
U^6, & \text{for } 1.0 < M < 1.5 \\
U^4, & \text{for } 1.5 < M < 2.5 \\
U^3, & \text{for } 2.5 < M < 5 
\end{cases} \quad (2.56) \]

Since Lighthill’s theory, many findings have revised the relation for more specific conditions [104], but the sound power scaling is always found to exceed the thrust rate. Concerning jets, Viswanathan [105] extensively investigated the temperature dependency, and Tam et al. [106] looked at a large number of data sets to generalize the analogy further, including Mach number effects and polar angle. Their study found that the power scaling varies between \( n = 5.3 \) and \( n = 9.9 \), where \( n \) is the generalized exponent. For low bypass ratio engines found in supersonic applications, experience shows that improved performance correlates with higher sound production, and the various analogies account for this trend (see Karabasov et al. [107] for a further discussion on current acoustic analogies). Acoustic analogies perform remarkably well, but their idealized treatment of noise sources obscures the generation process, which will be of concern in more complex jet configurations.

**Vortex Sound**

One criticism of the acoustic analogy is the lack of feedback involved. The flow is assumed to be known, and only producing sound. In reality, the noise generated interacts with the flow itself, thus changing the previously assumed solution of \( T_{ij} \). A formulation more appropriate for the feedback is based on vortex-sound [108], which works well at low Mach numbers and was originally proposed by Howe [109]. The likes of [110] are followed, starting with Crocco’s form of the Euler equation:

\[
\frac{\partial \vec{u}}{\partial t} + \nabla (h + 1/2u^2) = -\vec{\omega} \times \vec{u} \quad (2.57)
\]
where the specific enthalpy is $h = e + p/\rho$, and the vorticity is $\vec{\omega} = \nabla \times \vec{u}$. As with the Lighthill approach, one subtracts the time derivative of the continuity equation, Equation 2.6, from the divergence of Equation 2.57 to obtain

$$\frac{\partial}{\partial t} \left( \frac{1}{\rho} \frac{D\rho}{Dt} \right) - \nabla^2 (B) = \nabla \cdot (\vec{\omega} \times \vec{u}).$$ \hspace{1cm} (2.58)

Here, use has been made of the substitution $B = h + 1/2u^2$. Using the differential equation of state relating pressure, density, and entropy $(s)$,

$$dp = c^2 d\rho + \left( \frac{\partial p}{\partial s} \right)_\rho ds,$$ \hspace{1cm} (2.59)

the fundamental thermodynamic relation,

$$dh = \frac{dp}{\rho},$$ \hspace{1cm} (2.60)

and recognizing that for low Mach numbers entropy is constant (i.e. $ds = 0$), this can be written as

$$\frac{\partial}{\partial t} \left( \frac{1}{c^2} \frac{Dh}{Dt} \right) - \nabla^2 B = \nabla \cdot (\vec{\omega} \times \vec{u}),$$ \hspace{1cm} (2.61)

or,

$$\frac{1}{c^2} \frac{D_0^2 B'}{Dt^2} - \nabla^2 B' = \nabla \cdot (\vec{\omega} \times \vec{u}) + \frac{1}{c^2} \frac{D_0^2 B'}{Dt^2} - \frac{\partial}{\partial t} \left( \frac{1}{c^2} \frac{Dh}{Dt} \right),$$ \hspace{1cm} (2.62)

where the decomposition $B' = B - B_0$ is utilized and a reference flow $\vec{U}_0$ is used for the material derivative, $\frac{D_0}{Dt} = \frac{\partial}{\partial t} + \vec{U}_0 \cdot \nabla$. This finally leads to an inhomogeneous wave equation demonstrating the vorticity dependency at low Mach number where $c \sim c_0$:

$$\frac{1}{c_0^2} \frac{D_0^2 B'}{Dt^2} - \nabla^2 B' = \nabla \cdot (\vec{\omega} \times \vec{u}) + \frac{1}{c_0^2} \frac{D_0^2 B'}{Dt}.$$ \hspace{1cm} (2.63)
The quantity $B' = p'/\rho_0 + \vec{u}_a \cdot \vec{u}_0$ is the chosen acoustic variable in this instance, and $\vec{u}_a$ is the acoustic velocity. Since $B'$ accounts for the fluid velocity, $u_0$, it allows for feedback to occur, as Lighthill’s analogy does not. Additionally, the concept of vortex sound is tied back to turbulence as such flows are highly rotational. Equation 2.63 demonstrates how the acoustic source terms are associated exclusively with regions of the flow where non-zero vorticity vectors and entropy-gradient vectors exist. In the low Mach number regime, where this equation is valid, analytical techniques exist [109] to describe certain problems accurately.

**Noise Source Identification**

Identifying noise sources has been rigorously studied for a number of years [101, 111–116]. Jet noise is a fluid dynamic phenomenon, resulting from shear and turbulence, which give rise to intense pressure fluctuations in the near-field and ultimately radiate noise to the far-field. Within the mathematical framework, noise sources are represented as quadrupoles of the Lighthill stress tensor [98]. However, these are applicable only in the context of the acoustic analogy, and experimental evidence, as extensively reviewed by Tam et al. [63], has shown that coherent structures are the true sources. Both fine- and large-scale turbulence contribute to the broad spectrum of noise observed, though their importance varies depending on the jet conditions. Within the plume, large-scale activity, associated with interacting structures downstream of the potential core’s collapse [117], is a significant source of the radiated noise [49, 52, 118, 119]. Much evidence points to the post-potential core area as the region of peak noise emission [120, 121], where maximum acoustic production has been identified across a range of Mach numbers [122, 123] and further confirmed when nonlinear propagations are accounted for [124–126]. Therefore, considerable attention has been devoted to studying this region of the flow.
Supersonic Aeroacoustics

Supersonic flow conditions introduce additional source mechanisms. Typical spectra of an imperfectly-expanded jet are found in Figure 2.3.

![Figure 2.3: a) Narrow-band acoustic spectra showing the different characteristics of supersonic jet noise. $M_a = 1.5$, $M_j = 1.67$, Ref. Norum and Seiner (1982).](image)

Tam’s review paper [127] breaks supersonic jet noise into three categories based on their formation:

- Turbulent mixing noise, subdivided into:
  - Fine-scale structures
  - Large-scale structures
• Broadband shock-associated noise (BBSAN)

• Screech tones

Screech tones, first identified by Powell [128], are produced from a feedback loop: instability waves form off the nozzle lip and give rise to turbulence downstream within the jet; pressure fluctuations from these structures travel upstream through the subsonic surrounding fluid until reaching the nozzle lip, and the instability waves are then amplified, thus achieving resonance [129]. BBSAN is the result of turbulent structures in the shear layers interacting with the shock cell structure encountered in imperfectly expanded jets [130]. Finally, turbulent mixing noise is further divided into fine scale turbulence and the convection of supersonic large-scale structures associated with Mach wave radiation [131]. While the general mechanisms of these noise sources are believed to be known, many specifics are not fully understood, and these are still active areas of research.

Turbulent mixing noise is subdivided because of the means in which the noise propagates. The acoustic frequency has long been tied to the scale of the turbulence causing them [98]; however, different propagation physics seem to be dependent on that frequency. Fine-scale structures produce noise as described by the Lighthill and Howe analogies, and the observed experimental data is generally consistent across all angles. However, the large turbulence structures’ noise in transonic jets emits Mach wave radiation in a directional pattern, indicating different propagation properties. This large-scale activity, associated with interacting structures near the collapse of the potential core, is the largest contributing factor to OASPL in subsonic jets [52, 132]. These structures are seen to grow in size downstream of the potential core collapse and are believed to be linked to the magnitude of OASPL [49].

Tam [131] describes the large-scale noise generating mechanism in supersonic flows as follows: “As a simple model of large turbulence structures, one may regard them, in a stochastic sense, as traveling instability waves. When the wave speed is supersonic
relative to the ambient speed of sound, the near pressure field of the large turbulence structures, which extends outside the jet, develops into Mach waves very similar to that of supersonic flow past a wavy wall.” The wavy-wall analogy offers a simple calculation for determining the peak emission of Mach wave radiation:

\[
\phi_{\text{peak}} = \pi - \cos^{-1}\left(\frac{c_0}{u_{\text{conv}}}\right),
\]

(2.64)

where \( u_{\text{conv}} \) is the convective velocity of the jet. The convective velocity can vary significantly based on the turbulent structure [62], and researchers typically use the gross approximation of \( u_{\text{conv}} = 0.8U_j \) as the average value when no other information is available [131]. Equation 2.64 describes the theoretical angle of the Mach cone of maximum radiation, and typically aligns well with data. The distinction between large- and small-scale supersonically convecting structures has been somewhat difficult to observe in the far-field, but as shown in Figure 2.4, these features can be seen clearly in the near-field. As discussed in subsection 2.4.1, the implementation of chevrons typically see a shifting of the low-frequency to high-frequency noise mixing noise.

Figure 2.4: a) Spark schlieren photograph tuned to capture fine-scale turbulence. b) Turbulent mixing noise of a high-speed jet. c) Shadowgraph highlighting Mach wave radiation at roughly 45° from the jet axis. Ref. Tam (2009).
Unpredicted Jet Phenomena

The remaining components of supersonic jet noise are associated with nonlinear phenomena occurring as a result of acoustic-flow interactions and/or cumulative nonlinear waveform distortion. Since analytical descriptions of these are unlikely (and none have yet been found), qualitative explanations are given below. These components of jet noise have significant consequences in application. The jet noise example thus motivates further studies into the nonlinear behavior of aeroacoustics.

BBSAN occurs when quasi-periodic shock cell structures are present in the jet. As a coherent structure is convected downstream, it must pass through one such shock cell. This interaction gives rise to a disturbance consisting of a traveling wave with components of the structure and the shock in all directions. A common line of thought in supersonic jets is to treat the mixing layer surrounding the jet column as a waveguide. Along this idea, each shock has a wavenumber corresponding to its cell length. The frequency of each disturbance emitted from these turbulent-shock interactions is therefore slightly different, and the broadband nature is seen in the far-field [127]. Many stochastic models based on empirical data exist [133, 134] for the prediction of BBSAN.

Finally, screech tones are considered, and as with BBSAN, are only present in imperfectly expanded jets. Powell [128] was the first to observe this phenomenon in jets. Many different ideas have been proposed for their mechanism, and today this remains an active area of research. Much evidence has been presented over the years to support the idea of a feedback mechanism in the jet. Figure 2.5 is modified from Tam [127] and shows the feedback loop as it is understood today.

Large-scale instability waves in the center of the jet column are thought to interact with the shock cells at a distance downstream and generate disturbances. Acoustic waves are generated from the shock cells here (likely due to shock-structure interactions as in BBSAN) and propagate back to the nozzle lip. As the core is supersonic, the only medium permitting this extension back towards the nozzle lip is the mixing
layer. When the waves reach the nozzle lip, they are reflected back downstream. Because the mixing layer is thin and unstable here, it is susceptible to further excitation and these reflected waves amplify the waves. The jet column’s instability wave (which is in the core and separate from Kelvin-Helmholtz associated waves) grows in strength in the downstream direction with this constructive interference. With this stronger wave, more intense radiation is emitted during the shock-structure interaction in the downstream shock cells. The feedback loop continues to grow in strength until the jet’s available energy for this process is balanced by molecular diffusion. The excess energy is then emitted in the form of acoustic radiation, and the signature peaky spectrum observed is directly related to the spacing of these shock cells [127]. As with BBSAN, many empirical models exist to predict the screech tone frequency based on the Mach number of the jet. Recently, evidence has been found [70, 135] to support the idea that large-scale coherent structures interact with shocks at the boundary of the potential core to generate the initial disturbance. This is expanded on in detail in section 2.3. Depending on the pressure ratio and geometry, the structure of the feedback loop can change significantly, giving rise to additional harmonics in the far-field[136].

Figure 2.5: Feedback mechanism of an underexpanded axisymmetric jet.
2.3 Recent Findings in the Supersonic Round Jet

This section, taken from a recent publication [135], focuses on identifying important flow features in a cold jet issuing from an axisymmetric, convergent nozzle; in particular, sonic and supersonic flows are investigated to further recognize differences in noise generation associated with shocks. Particle imaging velocimetry (PIV) is simultaneously sampled with far-field pressures to allow for analyses that can provide insight into the relationship between the near-field jet plume and far-field noise. Reduced-order modeling via POD of the jet plume is executed in the streamwise plane (complementing the work done in the transverse planes by Caraballo et al. [65]), the directivity and magnitude of acoustic radiation are calculated, and correlations between the two measurements are further carried out. By interpreting the POD eigenfunctions and selectively rebuilding the flow-field, flow physics are then related to far-field noise signatures. Previously, high subsonic test campaigns at Syracuse University’s Skytop Turbulence Laboratory by Low [64, 137] and Berger [76, 138] have presented evidence correlating lower-dimensional spatial structures in the near-field flow with far-field noise. Both showed, Berger using observable inferred decomposition (OID) [139], that these structures are not necessarily the most energetic modes in terms of the turbulent kinetic energy. The highly-correlated ‘loud’ modes were instead found to be intermittently energetic. At present, data sets containing choked and underexpanded jets ($M_j = 1.0 \& M_j = 1.1$) are analyzed to similarly locate noise generation mechanisms relevant to shock noise.

2.3.1 Experimental Setup

The experimental results were acquired in the Skytop Turbulence Laboratory at Syracuse University. The jet can be seen in Figure 3.1. The 206 m$^3$ anechoic chamber is acoustically treated with fiberglass wedges, achieving a cutoff frequency of 150 Hz. Within the chamber is an axisymmetric $5^{th}$-order-polynomial stainless-steel conver-
gent nozzle \((M_d = 1.0)\) with a diameter of \(D = 50.8\text{mm}\), described by Tinney et al. [140]. The jet rig is configured in a blow-down facility with a co-annular co-flow moving at approximately 5 m/s. A 100 hp reciprocating Joy compressor powers the jet by pressurizing air in a 45 m³ storage tank array. This discharges air at approximately 1.0 kg/s in the axisymmetric jet configuration for a nozzle pressure ratio of 2.3. A pneumatically controlled Fisher® butterfly valve is used to control the nozzle pressure ratio. This system is capable of producing high Reynolds number supersonic jet flow for short (e.g. 30-60 sec) test durations. A 900 kW Chromalox heater is available to run the jet under increased nozzle temperature ratios; however, the experiments of this research operate under unheated conditions.

Figure 2.6: Anechoic chamber showing far-field microphone array (left), near-field pressure ring and PIV instrumentation (center), and axisymmetric jet (right).

A National Instruments™ PXI-1000B controller chassis was used as the central data acquisition hub of the experiments. Through a LabVIEW-controlled program operating in real-time, PIV and far-field microphones synchronization was achieved. A PXI-6070E digitizer interfaced with a SCXI signal conditioning chassis, which housed five daisy-chained SCXI-1531 modules for microphone acquisition. Upon test initialization, PIV triggers were sampled through NI-4472 cards. With each trigger time-stamped, individual PIV snapshots can later be correlated with far-field microphone signals. To reduce unwanted acoustic reflections, instrumentation inside the
The chamber was acoustically treated as seen in Figure 3.1.

**Particle Image Velocimetry**

Near-field velocity measurements were acquired using two different PIV setups. A 10 kHz PIV system was used to gather the $M_j = 1.0$ data in 2012. Results from this study additionally looked at the characteristic frequencies of the flow through simultaneous near-field pressure measurements [141]. The spectra revealed peak frequencies in the 1-2 kHz range, with roll-off from 3-4 kHz and a flat response from 5 kHz and up. From the results of this test case, research was motivated to look at a slightly faster speed, at $M_j = 1.1$. To capture an uninterrupted view of the near-field structure, the campaign at the supersonic speed was executed using a 4 Hz, multi-camera PIV system. While the two test cases used different instruments, each setup was found to have its advantages. The 10 kHz PIV produces valuable temporal resolution, but sacrifices the field of view due to large laser power requirements. In contrast, the multi-camera PIV can capture a much greater interrogation region, but lacks temporal information for this flow. Both sets of experiments were carried out along the jet centerline (i.e. the $r-z$ plane).

To achieve adequate particle density in the test section, both the core jet and surrounding co-flow are seeded. This is a necessity, as failure to seed the entrained fluid results in erroneous measurements throughout the mixing layer. Olive oil particles were introduced into the high-pressure jet through a PIVTEC seeder with twelve Laskin nozzles, aerosolizing the oil into particles of a mean diameter of 1-2 $\mu$m [142]. The co-flow was seeded using two commercial, theatrical foggers; these evaporate and then condense a glycerin-based fluid to form smoke particles. Kähler et al. [143] have shown these smoke particles have a narrow size distribution around a mean diameter of 2 $\mu$m. A schematic of the jet and seed injectors is shown in Figure 2.7a.

The 10 kHz PIV system operated a 10 kHz Quantronix® Hawk-Duo Nd:YAG laser. In combination with a Photron® FASTCAM CCD, two-component velocity
measurements were taken with a field of view of $1.9 \times 1.9$ jet diameters, $D_k$. Seven different streamwise locations were sampled in this campaign, but presently the furthest downstream window is focused on. Samples were acquired from $(z/D = 0)$ to $(z/D = 7.8)$ with approximately 0.5 diameter overlap.

The multi-camera PIV system utilized a 15 Hz New Wave® Nd:YAG Gemini laser as the light source, triggering the Dantec Dynamics acquisition hub. Three 12-bit, 1 MP HiSense cameras simultaneously imaged the flow at different locations. The acquisition rate of the cameras limited sampling to 4 Hz. The laser sheet and cameras were oriented to take two-component measurements along the centerline of the jet axis. Each camera’s field of view slightly overlapped the neighboring one’s to ensure appropriate merging of velocity vectors. Using a least-squares algorithm as described in [144], the PIV vectors from each image were stitched together to achieve a single streamwise velocity window from $2.5 < z/D < 9.4$ and $-1.0 < r/D < 1.0$.

PIV data acquisition and processing were carried out using Dantec Dynamics’ Flow Manager and LaVision’s DaVis software. In the multi-camera 4 Hz PIV experiments, the average particle size was approximately 3 pixels with a mean displacement of 15 pixels in the core. For the 10 kHz experiment, the mean particle size was between

Figure 2.7: a) Diagram of the seeding arrangement and jet coordinate system, b) 10 kHz PIV setup, and c) 4 Hz multi-camera PIV setup.
1 & 2 pixels, and an average displacement of 7 pixels. Despite the small particle size, peak-locking was not found to be an issue. Operating in dual-frame mode, snapshots are processed using adaptive correlations with an iteratively decreasing interrogation window, starting at $64 \times 64$ pixels and $32 \times 32$ pixels for the 4 Hz and 10 kHz experiments respectively, using a 50% overlap between interrogation windows.

Uncertainty calculations on PIV data are challenging, as many experimental sources contribute to the error. Particle density, concentration, displacement, gradients, and signal-to-noise ratio influence the tracking of particles [142]. Additionally, uncertainty contributions are specific to the flow under study, vary spatially throughout a given flow, and are a result of the number of images that can be acquired in an experiment. Turbulence measurement errors are proportional to the turbulence intensity [145], and supersonic flow conditions produce artificial velocity fluctuations due to the existence of shocks. Mitchell et al. [146] demonstrated that values as high as 35% of the freestream velocity can be generated behind a strong normal shock, and 10% downstream of the weaker oblique shocks. The Mach 1.1 data set was not found to have any normal shocks, so it is assumed that a 10% error from the oblique shocks is possible in this flow. Large velocity gradients, as in the high-shear region of the mixing layers, are typically the regions with the highest errors [147], and are therefore used as the upper bound of displacement uncertainties. The mean velocity gradients were found to be as high as $23 \text{ m/s}$ between adjacent vectors. Thus, following the approach described in §5.5 of *PIV: A practical guide* [148], the highest displacements error were calculated to be 6.7% for the two setups. From the variance formula, the different sources of uncertainty result in the conservative estimates of instantaneous velocities containing a 12% margin of error in the Mach 1.1 (4 Hz PIV) data set, and a 7% error in the Mach 1.0 (10 kHz) data set.
Far-field Acoustics

Far-field acoustics were simultaneously sampled through a series of SCXI-1531 boards with each PIV configuration. The setup used two arrays of six G.R.A.S. 46BE free-field condenser microphones positioned in an arc, \((z/D = 75)\) away from the nozzle lip. One arc of six microphones is located in-plane with the jet at an elevation angle of 90° (as shown in the left of Figure 3.1), while the second arc is positioned approximately one meter above, at an elevation angle of 75°. Both arcs are centered about the jet centerline. The microphones are spaced in 15° increments from 90° to 165° with respect to the jet axis, where the jet axis, \(+z\), corresponds to 180°. These pressure transducers have a nominal accuracy and frequency range of ±1 dB for 10 Hz to 40 kHz, ±2 dB for 4 Hz to 80 kHz, ±3 dB for 4 Hz to 100 kHz, and a dynamic range of 166 dB. Overall sound pressure levels (OASPLs) reported herein are taken with reference to the 20 \(\mu\)Pa. Calibration is achieved through a G.R.A.S. 42AB Class 1 sound calibrator, outputting a 1 kHz sine wave at 114 dB (OASPL). Uncertainty in the OASPLs at the SU facility are estimated at ±1 dB with a repeatability of ±0.2 dB [64].

2.3.2 Axisymmetric Jet Results

Data were acquired using the same convergent nozzle, \(M_d = 1.0\), at two different flow conditions: choked, \(M_j = 1.0\), and mildly underexpanded, \(M_j = 1.1\). Due to hardware constraints, the acquisition parameters of each case are different. The number and frequencies of PIV snapshots and microphone samples of each case are found in Table 2.1. 10 kHz PIV experiments were limited to less than one second (due to bandwidth limitations from the large amount of data acquired), while the multicamera 4 Hz PIV setup allowed for much longer simultaneous acoustic sampling. For a full description of the experimental setup, interested readers are referred to reference [135].
Table 2.1: Data acquisition parameters.

<table>
<thead>
<tr>
<th>$M_j$</th>
<th>$M_d$</th>
<th>$Re_j$</th>
<th>$f_{PIV}$</th>
<th>$N_{PIV}$</th>
<th>$f_{Mic}$</th>
<th>$N_{Mic}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>$1.26 \times 10^6$</td>
<td>10 kHz</td>
<td>8623</td>
<td>40 kHz</td>
<td>$4 \times 10^9$</td>
</tr>
<tr>
<td>1.1</td>
<td>1.0</td>
<td>$1.45 \times 10^6$</td>
<td>4 Hz</td>
<td>2500</td>
<td>25 kHz</td>
<td>$2.5 \times 10^7$</td>
</tr>
</tbody>
</table>

The results of the choked jet are presented first, followed by the under-expanded flow and then the similarities across Mach numbers. The similar Reynolds numbers suggest that the turbulence spectra are largely alike, and analyses reveal specific modes are associated with turbulent mixing noise in the sonic case. However, moving to the supersonic plume represents a shift in regimes where different flow features and noise emission exist. The data presented from that case illustrate the emergence of screech modes. Finally, resemblances are sought between the basis functions of the two in an effort to identify potential modal structures representative of turbulent mixing noise.

**Choked Jet**

Data from the $M_j = 1.0$ case were originally reported by Berger *et al.* [141]. This section focuses on results from the choked jet that are comparable to the underexpanded jet, and identifies a flow structure correlated with noise production.

Seven separate tests were conducted at a target nozzle pressure ratio of 1.89, each taken at different streamwise locations and designated W1-W7. The PIV setup was traversed to a new position after each previous test, so the data are not phase-locked. Averaged velocities from four of the seven camera locations, W1, W3, W5, & W7, are shown in the top of Figure 2.8, while the overlapping ones, W2, W4, W6, are omitted. The centerline velocities are extracted from each of the seven windows, normalized for cases W4 & W5 when the experimental conditions were slightly off, and merged using a piecewise cubic Hermite interpolating polynomial (PCHIP in Figure 2.8). The deviations in the mean profile are a result of the scatter in the nozzle conditions between separate experiments, and are zero-phase digitally filtered.
to produce a smooth curve for defining the potential core (Data Fit).

Figure 2.8: Streamwise velocity component of the $M_j = 1.0$ jet. (Top) Ensemble average of time-resolved data from four separate window locations, W1, W3, W5, & W7. (Bottom) Centerline velocities from all seven cameras merged (PCHIP) and filtered (Data fit) used for one definition of a potential core length.

The length of the potential core is an important parameter for scaling the developing region of jets. One definition of this length is given in Figure 2.8. From the decaying velocity, a potential core length is defined using a 95% threshold and found to be $z/D \sim 7.7$. This is consistent with the results from a very similar experiment, consisting of a cold Mach 0.9 jet of $D_j = 0.0508$ m, which estimated the potential core length to be $z/D \sim 7.8$ [117].

The furthest downstream camera location of the TRPIV setup, covering $6 < z/D < 7.8$, is selected to investigate. Interest is in noise generation from coherent structures, and with the potential core ending here, large turbulent structures from the merging shear layers interact with each other, significantly influencing the flow. The measured velocities in the potential core prior to its decay indicate an actual Mach number of $M_j = 1.006$. As a final note on the mean profile, note that the minor velocity deficit
in the upper left corner is the artificial result of a transducer blocking the camera’s view.

Snapshot POD is applied as a reduced-order model to the data near the collapse of the potential core. The resulting streamwise POD modes are shown in Figure 2.9. The high-speed ($0.95U_j$) and low-speed ($0.10U_j$) sides of the mixing layer are overlaid to help accentuate regions in the flow where these modes reside. These first 20 modes account for approximately 45% of the data’s energy. Modes 1 & 2 contain large spatial coherence, indicative of the greatest structures. As the subsequent less energetic modes are examined, smaller structures are observed. Modes 3 & 5 appear to have similarities across the centerline, approximately 180° out of phase, and modes 4 & 6 are symmetric about the centerline. The column mode in the jet ($m = 0$ in the $r - \theta$ plane) is likely associated with modes 4 & 6. Additionally, one of the first Fourier-azimuthal modes ($m = 1, 2$) may be linked to modes 3 & 5. The less energetic modes lose symmetry and the smaller structures become more poorly organized (e.g. 9-13), while modes 14 & 15 appear to regain some symmetry. Presumably, these capture the collapsing mixing layers which introduce further turbulence in the center of jet.

Far-field data are explored next. Spectral content, as originally reported by Berger et al.\cite{141}, are given in Figure 2.10a. Typical acoustic signatures are observed, with turbulent mixing being the prime source of noise and spatially-varying magnitudes obeying refraction \cite{101}. The time-varying pressure signals from the 165° microphone were then used to compute correlations with the POD velocity modes. As discussed in the Experimental Setup section, correlations are computed to draw conclusions between the spatial POD functions representing the velocity in the near-field and the far-field acoustics. A lag of approximately 10 ms is confirmed to be the average propagation time of a sound wave from the jet to the microphones in the anechoic chamber setup at Syracuse University. These calculations yielded significant correlations between a select few POD velocity modes and the 165° microphone, shown in Figure
Figure 2.9: First 20 spatial modes of the $M_j = 1.0$ data, taken near the collapse of the potential core.

2.10b. A threshold of $\rho > 3\sigma_\rho$ was chosen as the minimum value of significance, where $\sigma_\rho$ is the standard deviation of the maximum correlation coefficients.

Figure 2.10b demonstrates that POD velocity mode 15 correlates the most with the 165° microphone, and modes 16, 10, & 12 subsequently follow with substantial correlations. No other significant correlations were found between other velocity modes or microphones. Since turbulent mixing noise is presumably the only source of noise at this flow state of $M_j = 1.0$, these four modes are referred to as "turbulent mixing modes." Note that these four modes contain a small percentage of the average turbulent kinetic energy, less than 5%. However, in a time-dependent sense they can have instances of high energy. Identifying these flow events is desired. But, because there are multiple POD velocity modes which correlate with the acoustics, interpretation of the flow structure and its contribution to the far-field noise is not straightforward.
(a) Far-field spectra of the $M_j = 1.0$ jet. The downstream jet centerline is defined as 180°.

(b) Correlations between velocity POD coefficients and far-field pressures from the 165° microphone.

Figure 2.10: Far-field pressure signatures (a), and correlation coefficients of loud modes (b) for the $M_j = 1.0$ jet.

Additionally, while Figure 2.10b identifies specific POD velocity modes, it does not tell to what degree each one influences the acoustic production.

The 10 kHz PIV measurements provide suitable temporal resolution to observe a time-resolved evolution of the low-dimensional flow representation. To sort through the many snapshots and identify acoustically-energetic instances in time, the stochastic pieces of the POD result are considered, $a_n(t)$.

The sums of the squares of the temporal coefficients are computed and shown as functions of time in Figure 2.11. Figure 2.11a accounts for the total turbulent kinetic energy in all of the POD velocity modes $n = 1, \ldots, N$, and Figure 2.11b shows the contribution specifically from the highly correlated POD velocity modes with the far-field acoustics, as found through the previous analysis, i.e. $n = 10, 12, 15, 16$. One observes that the flow randomly contains large turbulent kinetic energy events, for example at $t_1$, while the four loud modes have very little energy output. Conversely, a presumptively acoustically energetic event, $t_2$, sees a large output in kinetic energy.
Figure 2.11: Time-dependent kinetic energy of a kinematically energetic flow event \( (t_1) \), and acoustically energetic instance \( (t_2) \).

by the four loud modes \( (2.11b) \), yet the rest of the flow hides this \( (2.11a) \). In fact, the energy of the four loud modes increases from its 5\% average energy contribution to approximately 25\% of the total turbulent kinetic energy at time \( t_2 \). By simply comparing the specific energy, these loud modes appear critical to the flow state as relevant for acoustic production. These two instances in time are considered more thoroughly by selectively reconstructing the velocity fields.

Using selected low-dimensional modes, reconstructed fluctuating velocity fields are shown in Figure 2.12. In these plot, velocity vectors overlay magnitudes. The first 30 modes are considered, as these account for more than 50\% of the total energy. Plots 2.12a & 2.12b use the first 30 POD velocity modes to rebuild the flow-field. Plots 2.12c & 2.12d use the first 30 modes excluding the loud modes \( (n = 10, 12, 15, 16) \). And charts 2.12e & 2.12f only utilize the loud modes. Additionally, the two time instances identified in Figure 2.11 are compared against one another, with a kinematic event, \( t_1 \), shown in 2.12a, 2.12c & 2.12e, and the acoustic event, \( t_2 \), in 2.12b, 2.12d & 2.12f.
Figure 2.12: Selective flow-field reconstruction. Fluctuating velocity vectors overlay magnitudes using the first 30 POD velocity modes, (a) & (b), the first 30 uncorrelated velocity modes, (c) & (d), and the acoustically-correlated modes, (e) & (f). Two times are considered (see Figure 2.11): an energetic flow event at time $t_1$ in (a), (c) & (e), and an acoustically energetic instance at time $t_2$ in (b), (d) & (f).
A loud flow state is represented by 2.12f. A ring-like structure is interpreted from this, with vortex cores centered about \( z/D \sim 6.6, \ r/D \sim 0.6 \). Interestingly, the rotation appears to pull flow upstream in the core, before radially ejecting it (note these are fluctuating velocities, so the instantaneous flow is still downstream). The rotation of this vortex ring is of opposite sense compared to what is generated on the free side of the shear layer (as famously shown by Brown & Roshko [45]). This suggests that it may be a secondary flow, driven by the primary vortices in the shear layers. Furthermore, a stagnation region appears along the centerline, near \( z/D = 7.0 \), where mass is flowing inward. One can imagine how this process could easily generate large pressure fluctuations that result in acoustic production. As a final note, the full time trace, \( t = 0, \ldots, 0.862 \) ms, reveals at least 10 equally acoustically-energetic events, along with many more snapshots capturing at least half of the energy shown at \( t_2 \) in 2.11b.

Consider the comparisons of Figure 2.12. At time \( t_2 \), the ‘full’ low-dimensional representation of flow, 2.12b, shows drastically different behavior than what is reconstructed from the loud POD velocity modes. Large structures exist; however, aside from their local coherence, they exhibit little organization across the window. The same is true for the residual of the flow, 2.12d; clearly the uncorrelated POD velocity modes possess the majority of the energy and dominate the flow-field. The vortex-feature of 2.12f is essentially suppressed by the dominant velocities of these other structures. At the kinematically-energetic instance, \( t_1 \), the vortex-ring structure is much quieter (2.12c), while the first 30 modes (2.12a) and the remaining POD velocity modes (2.12c) see increased turbulent activity on the bottom shear layer. These reconstructions and modal-pressure correlations indicate that the most energetic modes are not necessarily tied to the highest acoustic production. This is in contrast to what has been learned from the \( r - \theta \) plane, where the low-order Fourier modes are most important.
Underexpanded Jet

The multiple-camera setup of the 4 Hz PIV setup captures simultaneous velocity data at three separate location and allows the merging of instantaneous vector fields from $2.5 < z/D < 9.4$. Figure 2.13 shows a snapshot of the jet’s streamwise mean (top) and centerline (bottom) velocities. Spatially varying mean velocities indicate the existence of shock cells typical of underexpanded jets. The potential core length, defined first using a 95% threshold, is calculated as $7.1D$. Since this value is less than the $M_j = 1.0$ jet, which is counterintuitive, additional verification is sought. A model proposed by Witze [149] is based on the modified Oseen approximation for a turbulent axisymmetric jet [150, 151], which yields, for a supersonic jet,

$$
\frac{z_{\text{core},M>1}}{D} = \frac{1.111(M_j^2 - 1)^{1.15}}{(\rho_e/\rho_\infty)^{0.5}},
$$

(2.65)

where $\rho_e$ is the density of air at the jet exit plane and $\rho_\infty$ is the density of the ambient air. This approach gives an upper value of $7.9D$, close to that of the $M_j = 1.0$ jet. The majority of the turbulence activity is anticipated to be within these limits.

Again, POD is implemented and the velocity is decomposed into basis functions and temporal coefficients. The first 24 modes contain approximately 43% of the total energy, and their eigenfunctions are shown in Figure 2.14. Again, the contours of the bases’ $u$-components are plotted, with the mixing layer boundary overlaid. Some distinct features of the jet are highlighted in these spatial modes, and they are classified into three main groups based on specific energy content and shape. The lowest modes (1-3) exhibit large-scale structures near the collapse of the potential core, modes 6-9 possess strong streamwise oscillations within the supersonic core, and the higher modes (i.e. 11-14 & 17-20) contain smaller organized structures embedded in the mixing layers. Each grouping is believed to be associated with a particular physical feature of underexpanded flow, discussed momentarily. Modes 15 & 16 appear to capture smaller events at the collapse of the potential core.
Figure 2.13: Streamwise velocity component of the $M_j = 1.1$ jet. (Top) Phase-locked instantaneous flow-fields from three cameras are merged and averaged. (Bottom) Centerline velocity decay, used to calculate the potential core length.

The first group of modes in Figure 2.14 is now examined. Near the collapse of the potential core, structure sizes diminish from mode 1 to 2. In mode 3, two apparent vortices are 90° out of phase with one another in the $z$-direction, leaving a wave-like feature near the centerline. Further structure breakdown is observed in the 4th and 5th modes, along with the emergence of some spatial variations in the potential core. Finally, mode 3 is noteworthy, because the enlarged length scale found in a less energetic mode is atypical of POD results. One possible explanation for this is that the structures are anisotropic and inhomogeneous \cite{152}; however, more data (i.e. $w'$) are necessary to support this claim.

Regions of compression and decompression are thought to be associated with the shock cell-like modes 6-9. Fluctuating velocities contained within the potential core are apparently tied to axisymmetric structures in the shear layer, likely vortex rings. Furthermore, the core oscillations are 180° out of phase with the structures in the
shear layers. These high gradients suggest there are strong interactions along the high-speed side of the shear layer. One theory of screech production points to this location; energy leakage from shocks occurs through the shear layer boundary when local vorticity is weakened [153, 154], and the escaped energy then radiates as acoustic waves. Edgington-Mitchell et al. [70] explored this further via PIV measurements, and proposed that regions of high axial velocity kurtosis and through-plane vorticity fluctuations near shock reflections were indicative of acoustic wave production. Following this work, similar locations are selected here.

Based on the locations in the jet identified by Edgington-Mitchell et al. (see 17 of [70]), slices of the velocity and 6\textsuperscript{th} eigenfunction are extracted, shown in \ref{fig:2.15a}. Clear oscillations emerges from the eigenfunction in \ref{fig:2.15b}. Accounting for the coordinate rotation of the chosen path, $z^*$, the Fourier transform is straightforward from this,
(a) Selected pathways based on previously identified regions of high kurtosis [70].

(b) Extracted amplitudes of mode 6 along transformed coordinates, \( z^* \), identified in (a).

(c) Mean velocity along transformed coordinates, \( z^* \), identified in (a).

(d) Mode 6 wavenumbers along selected pathways identified in (a).

Figure 2.15: Screech tone extraction.
and a dominant wavenumber is calculated as $k_z = 26.4 \, m^{-1}$, shown in 2.15d. An estimate for screech frequency is then attempted.

$$f_{\text{screech}} = k_z (c - u_{\text{conv}}). \quad (2.66)$$

Equation 2.66 is employed using the average quantities along the paths shown in Figure 2.15. The local speed of sound, $c$, is estimated to be an average between the ambient speed of sound and that at the centerline, calculated to be 325 m/s. Next, the convective velocity is estimated, which is challenging because Tinney et al. [62] has shown that specific POD modes convect at different speeds. Additionally, the convection velocity is a function of radial location. With time-resolved measurements, it is possible to obtain estimates of these velocities, and convective Mach numbers have been shown to vary significantly in transonic jets, from $M_c = 0.3$ [55] to $M_c = 0.64$ [62]. However, this 4 Hz PIV data set does not have the time support. The authors acknowledge this shortcoming, and are left with estimating the convective velocity based on structure evolution found in previous literature [74, 155] as $u_{\text{conv}} \sim 0.6 \bar{u}$. The quantity $\bar{u}$ is the mean velocity along the selected paths, shown in 2.15c. With this estimation in place, which gives a convective Mach number of $M_c = 0.45$, one obtains the frequency $f_{\text{screech}} \sim 4500$, or a Strouhal number of $St_{\text{screech}} \sim 0.67$.

Consider again the structure of the POD velocity modes 6-9 in Figure 2.14. Axisymmetric structures in the shear layer are clearly tied to downstream fluctuations in the core. Following the leakage theory proposed by Suzuki et al. [153], a pathway appears to exist in these modes along the edge of the shear layer that could allow for energy to pass from a shock-vortex interaction radially outward and partially upstream in the shear layer. These modes help illustrate the feedback mechanism responsible for screech tones [127]. The slower-convecting shear layer acts as a waveguide, allowing acoustic radiation at specific frequencies, generated from vortex-shock interactions, to propagate upstream to the nozzle lip and close the feedback loop. This idea is also consistent with the model proposed by Henderson et al. [156] and
supported through time-resolved schlieren images by Mitchell et al. [157].

Finally, the mixing layer modes \( (n > 10) \) capture smaller eddies. Specific spatial functions qualitatively match well with the lower modes of subsonic jets [155], and are believed to be associated with turbulent mixing noise as in the \( M_j = 1.0 \) case. More evidence is sought to reinforce the relations to noise production, hence the far-field noise data are next considered.

![Figure 2.16: Far-field pressure signatures (a), and correlation coefficients of loud modes (b) for the \( M_j = 1.1 \) jet.](image)

As in the \( M_j = 1.0 \) case, far-field data, presented as spectra in Figure 2.16a are used to compute correlations between acoustics and POD modes. Supersonic noise is well documented, see [127], and the results found here concur with previous findings. Narrow bands of intense SPLs (e.g. \( St \sim 0.67 \)) are identified as screech tones \( (f_{s_0} = 4545Hz & f_{s_1} = 9101Hz) \), consistent with the estimation provided from POD mode 6. The peak from \( 0 < St < 0.3 \) directed towards the 165°& 150°microphones is characteristic of turbulent mixing. Broadband shock-associated noise (BBSAN) is quite weak for this jet, and thus does not show up strongly in the spectra.

Correlations to the 165°microphone are considered first, shown in Figure 2.16b. Five loud
modes are plotted for this microphone, though eight total modes pass the minimum statistical threshold. Mode 6, believed to be associated with screech, has an oscillatory correlation pattern, as do modes 7-9 (omitted for cleanliness). This is because screech is very tonal, which is a result of the resonance in the jet structure. These tones produce very large correlation coefficients (near 50% in some cases), and are highly directional.

The acoustic spectra of the $M_j = 1.1$ are presented in 2.17a as contours in polar coordinates. Sound pressure levels (SPLs) are plotted as a function of Strouhal number ($St = \frac{fD}{U_j}$) and microphone angle relative to the jet. As with the $M_j = 1.0$ data, correlation coefficients are computed for each microphone, POD mode, and time lag. In contrast to the sonic case, however, the underexpanded jet has significant correlations which change as a function of microphone angle. To display the multi-dimensional data, the maximum values of the correlations (for example from 2.16b) are plotted in 2.17b. The orientation is the same as the frequency spectra presented in 2.17a, with the radial coordinate exchanged for the normalized cross-correlation coefficient.

Figure 2.17b demonstrates the directional dependence of the strongest velocity-acoustic correlations. In this test case, a $5\sigma$ threshold is used to identify significant correlations. Comparing Figure 2.14, 2.17a, and 2.17b, one can immediately see which modes overlap particular noise signatures. The high correlation coefficients of the shock-containing modes (6-9) align with the screech tone propagation to the microphones at 90°, 105°, and 165°. While the fundamental screech frequency is greater at the shallower angle, the presence of the 1st harmonic at the 90° and 105° microphones apparently results in higher correlations. In addition to the similarity in frequencies, this provides evidence that modes 6-9 correspond to screech production. The angles where turbulent mixing noise exist (i.e. 150° and 165°) also correlate well with higher modes ($n = 15, 16, 18 \& 23$), associated with the collapse of the potential core. These data are in good agreement with the results of numerous research on subsonic axisym-
Figure 2.17: Directional dependence of spectra and correlations of the $M_j = 1.1$ jet.

Figure 2.17: Directional dependence of spectra and correlations of the $M_j = 1.1$ jet.

(a) Frequency and directional dependence of SPLs. The max intensity is approximately 83 dB.

(b) Directional dependence of spectra and correlations of the $M_j = 1.1$ jet.

Figure 2.17: Directional dependence of spectra and correlations of the $M_j = 1.1$ jet.

metric jets, where turbulent mixing is the primary component of noise generation and the traditional directivity patterns show the most intense acoustics at shallow angles [53, 101]. The intermediate microphones, where screech tones and turbulent mixing are not present, have relatively little correlation to the POD modes. Though not presented, data collected from a second microphone array (positioned above the one shown in Figure 3.1) are directionally consistent with the current results. Regions of higher and lower acoustic patterns in the offset array (e.g. screech tone intensity) result in the expected change in correlation coefficients.

Similarities across Mach numbers

Comparison between the two cases is somewhat complicated due to differences in instrumentation used. Nonetheless, some similarities are expected in flow structure and noise generation. To make such a comparison, the Mach 1.1 data, which utilized the multi-camera setup, must be truncated and interpolated to match the spatial locations of the smaller window of the Mach 1.0 data before executing POD. This is
performed on the flow field, and the principal modes are then compared via a spatial cross-correlation coefficient (Equation 2.32). Maps of the lowest POD velocity modes’ spatial correlations are shown in Figures 2.18a and 2.18b and each are discussed in detail below. Dark red contours indicate high levels of correlation, while white represents no correlation. Thus, a red spot indicates that the associated modes have comparable structure. Once similarly-shaped basis functions are found, they are cross-referenced with the results from the correlations between the POD velocity modes and far-field pressures to learn if any similarities in turbulent mixing noise modes exist across different Mach numbers.

Figure 2.18a shows how higher modes of the large-window data correlate to the lowest POD velocity modes of the truncated set. This agrees with the window dependent effects found by Shea et al. [144] and Berry et al. [158]. Consider Mode 23 of the original $M_j = 1.1$ velocity data; it correlates with Mode 10 of the truncated velocity data by 60%. This truncation process is analogous to a spatial filter in that the
Table 2.2: Turbulent mixing modes identified through temporal cross-correlations between POD velocity modes of each data set and far-field pressures.

<table>
<thead>
<tr>
<th>Data set</th>
<th>ModeNo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_j = 1.1$</td>
<td>15, 16, 18, 23</td>
</tr>
<tr>
<td>$M_j = 1.1</td>
<td>_{\text{trunc}}$</td>
</tr>
<tr>
<td>$M_j = 1.0$</td>
<td>10, 12, 15, 16</td>
</tr>
</tbody>
</table>

large-scale structures are filtered out in the operation. For example, shock-containing regions are predominantly upstream of this window location, so that the new POD velocity modes essentially have no signatures of them.

Since a cropped velocity field is now being used and the velocities’ eigenfunctions have clearly changed, correlations between the POD modes and the far-field pressures must be recomputed. The high temporal correlation values between the screech-containing modes and the far-field noise that were previously obtained (see 2.16b) are entirely lost in this process because the phenomenon occurs upstream of this new window ($6 < z/D < 7.8$). However, POD velocity modes linked to turbulent mixing noise remain, and these are identified as in the previous manner. The recomputed eigenfunctions are then compared to the $M_j = 1.0$ jet.

Figure 2.18b demonstrates the similarity of the lowest modes between the truncated $M_j = 1.1$ data and the $M_j = 1.0$. The lowest modes (1-6) correlate spatially well with one another, and the trend overall appears symmetric, with spreading/mode-switching occurring at higher values. For example, mode 10 of the $M_j = 1.1$ coincides spatially with mode 12 of $M_j = 1.0$ by a correlation coefficient of 50%. Additionally, modes 12 & 14 interchange, 15 remains the same, and 17 & 16 reciprocate. Upon recomputing the temporal correlations between the microphones and the POD velocity modes of the truncated data set, $M_j = 1.1|_{\text{trunc}}$, these switching modes are found to have far-field correlation coefficients, $\langle a_n(t), P_j(t + \tau) \rangle$, close to those of their siblings. This suggests similar events are recurrent across the two different Mach numbers and are captured by unique POD modes.

With the far-field correlations between POD velocity modes and far-field pressures
of the truncated $M_j = 1.1$ jet data stored, the most statistically significant values are again retained as loud modes. These are summarized in Table 2.2. The spatially similar POD velocity modes are then cross-referenced against the velocity-acoustic correlations of the truncated data set. As indicated by the bold-faced font in the table, a common structure appears to be identified by mode 23 of the unaltered $M_j = 1.1$ data and mode 10 of the cropped velocities, $M_j = 1.1|_{\text{trunc}}$. Table 2.2 also shows that POD velocity mode 12 from the $M_j = 1.0$ case is similar to mode 10 from the truncated $M_j = 1.1$ data. This is significant, because it evokes the possibility that a particular flow configuration is associated with turbulent mixing noise; a specific shape appears to emerge with noise production for, at least, two different Mach numbers. One shortcoming of the spatial correlations occurs among the higher, and more randomly distributed, modes. This results from the algorithm lacking robustness against an increasing number of structures, each of which possesses its own amount of variability.

Because the data were acquired using two different setups, experimental uncertainties between the two data sets could play a large role in comparing the structures between two different Mach numbers. For example, if displacement uncertainties in the velocity fields constructively interfere, two very similar structures could appear out of phase with each other, thus resulting in a low spatial correlation value. However, two additional experiments were performed under the same conditions. While these are not directly comparable to the two jet cases studied here, they do demonstrate the utility of this spatial correlation technique. Figure 2.19 shows the similarities in the POD spatial modes that occur across different Mach numbers by removing the variability associated with separate experimental setups.

Figure 2.19a used the same 10 kHz PIV system at two different Mach numbers, the $M_j = 1.0$ jet studied here, and the $M_j = 0.6$ jet researched by Berger [155]. Berger showed that POD velocity mode 6 had the highest correlation to far-field noise. From the present study, it has been shown that POD velocity mode 12 is a ‘loud’ eigenfunction.
φᵢⁿ(⃗x), M = 1.0

φᵢⁿ(⃗x), M = 0.6

(a) Spatial correlations between the first modes of the $M_j = 0.6$ jet and $M_j = 1.0$ jet using the 10 kHz PIV setup.

(b) Spatial correlations between the first modes of the $M_j = 0.6$ jet and $M_j = 1.1$ jet using the multi-camera 4 Hz PIV setup.

Figure 2.19: Correlation maps of POD basis functions for two sets of experiments performed using the same PIV setups.

Consulting 2.19a, one observes a 50% correlation between the two structures. Clearly, some resemblance between structure is identified for loud modes at two significantly different jet conditions.

The modes acquired from the same multi-camera 4 Hz PIV setup at $M_j = 1.1$ and $M_j = 0.6$ are shown in 2.19b. During the $M_j = 0.6$ experiments, acoustics were not acquired. Unfortunately, this denies the usage of the present analysis on this data set and identifying loud modes is impossible. Nonetheless, large similarities in the structures are apparent at these two Mach numbers. Note the highly uncorrelated POD modes from $φᵢ^6, \ldots, φᵢ^9$. Because the subsonic jet does not contain any shocks, there is essentially zero correlation to the modes representing shock fluctuations in the $M_j = 1.1$ data. Aside from this, there is excellent agreement between many of the velocity modes extracted by POD at the two different Mach numbers, with spatial correlation coefficients up to 80%.
2.3.3 Summary of Axisymmetric Jet Findings

POD and velocity-acoustic correlations have been carried out on data taken from an axisymmetric jet operating at choked and underexpanded conditions. Specific POD modes were found to contain important flow structures relevant to jet noise production. In the Mach 1.0 case, four POD velocity modes were identified and used to reconstruct the flow-field. The rebuilt flow-field revealed an underlying structure similar to a vortex ring, with rotation such that the fluid along the centerline moves slower than what is in the mixing layer. In an average sense, these velocity modes possess very little turbulent kinetic energy; however, during an acoustic event, their energy output was found to contribute over 25% of the total turbulent kinetic energy in the streamwise direction.

In the supersonic case, shock-related fluctuations, and turbulent mixing regions of the flow were isolated. By computing cross-correlations and considering the spatial distribution of both the pressure signals and correlation coefficients, particular modes were related to noise spectra. POD velocity modes 6-9 were associated with screech production by considering the directional dependence of the cross-correlations. Additionally, wavenumber calculations from POD mode 6 were used to estimate a specific frequency from the velocity data, which yielded a close approximation of the fundamental screech tone as observed in the far-field. Turbulent mixing noise was isolated in the $M_j = 1.1$ case. By comparing to the sonic case, where turbulent mixing is the sole source of noise, one similar mode shape was found across both Mach numbers. Furthermore, by comparing to the work by Berger [155], an additional loud mode was found to exist between the Mach 0.6 jet and the Mach 1.0 jet, suggesting that recurrent structures may be responsible for noise generation at different Mach numbers.

The distinct features of supersonic flow create an opportunity to classify particular sources of acoustic spectra through reduced-order models. For example, POD modes 15 and 16 of the 10 kHz PIV and the truncated multi-camera 4 Hz PIV data sets
presented herein appear to represent structures indicative of noisy turbulent flow events. These higher modes are not the most energetic in an average sense; however, they are found to have instances of increased activity where their energy content is amplified. These occur near the collapse of the potential core, yet the cause of such events is still unknown. Time-resolved data allow one to watch the evolution of the flow and identify such a particularly noisy event. The larger window can then be consulted using similarities between POD modes to learn more about the large-scale flow events that may have been filtered by the smaller window. This approach is believed to be useful to ascertain the behavior of the specific modes which may influence acoustic production.

2.4 Advanced Configurations

While the axisymmetric nozzle is the correct choice to carry out fundamental studies, current and future aircraft have evolved to more exotic designs. Two conflicting goals motivate engine technologies: increased vehicle performance and decreased noise emissions. As seen in Section 2.2.2, for supersonic axisymmetric jets, sound pressure levels are related to velocity by a power-law relation with velocity. Thus, one must utilize engineering methods to overcome the flow physics and meet the desired goals. This section discusses some current approaches to achieving improved flight performance and decreasing the acoustic propagation.

2.4.1 Noise Suppression Techniques

As the axisymmetric jet has many features that make practical implementation favorable, it’s been widely used over the years. Thus, approaches have been developed to retroactively reduce the noise in this geometry. In subsonic engines, noise reduction has been achieved by moving towards high-bypass fans and ejectors, where thrust can be maintained by increasing the cross-section of the engine while decreasing the
exhaust velocity. However, this is impractical for supersonic flight since the engine profile must be minimized to reduce the large increase in wave drag. Two designs that lower noise output in both regimes are chevrons fluidic injectors. While effective, these generally come at a cost to thrust and/or efficiency.

Among the passive flow control techniques attempted over the years, chevrons have become the most popular as they have shown the greatest noise reduction to thrust penalty ratio [159]. In chevrons, the nozzle lip is replaced with a serrated edge, which enhances the mixing throughout the shear layer. Bridges [160] performed a parametric investigation of the effectiveness of chevrons, and since then much research has been carried out on variations of chevron geometry to find the most optimal configuration [161–165]. Through this work (and many more), an increase in streamwise vorticity has been related to a reduction of OASPLs. Peak turbulence levels near the jet exit, where large coherent structures dominate, are lessened as the enhanced mixing enables faster vortex breakdown. The far-field effect of the chevrons is a shift from low-frequency mixing noise (associated with the largest coherent structures, see 2.2.2) to high-frequency noise; experiments routinely observe these shifts in both the turbulence and acoustic spectra. In supersonic jets, chevrons force the shock cells closer together, thereby reducing the BBSAN. As the nozzle lip of a chevron is not smooth, the feedback mechanism responsible for screech tones is additionally inhibited. Though chevrons are the best choice of passive techniques, the associated thrust penalty still leaves much room for improvement.

Another favored method of noise reduction is through fluid injection, developed with the idea that the amount of mixing could be controlled at different flight times. Aqueous injection has substantial control over noise reduction[166], however is impractical for in-flight use. In this method, the jet velocity is drastically slowed due to momentum transfer between the fluid and droplets, and temperature drops as evaporation occurs, both contributing to lower noise production. Gaseous injection is much more practical, but is limited to the less-effective process of introducing streamwise
vorticity (as chevrons do). Conflicting results have been observed for the effectiveness of mass flow injection for supersonic noise\cite{167, 168}. Despite the variation in results, Henderson’s review paper \cite{166} indicates that fluidic injection reduces BBSAN and OASPL. This spread in results indicate that the method may not be robust enough for implementation into flight vehicles.

Technologies reducing jet noise have shed light on the importance of streamwise vorticity. Gutmark \cite{169} developed a hybrid method using chevrons and fluidic injectors to further noise reduction by introducing the optimal amount of streamwise vorticity in the shear layer. He found that the optimal configuration, arrived at through parametric studies, reduced the screech tones, BBSAN, and turbulent mixing noise, while contributing minimally to higher-frequency noise generation as chevrons do. The associated flow-field changes included reducing the shock cell strength, shortening of the shock cell spacing, and decreasing the potential core length. It was observed that enhanced mixing produced a more uniform turbulence field, increasing levels in the shear layers and reducing the centerline values.

From the successes of chevrons and fluidic injectors, it has become apparent that increasing streamwise vorticity has the potential to reduce noise. If introduced early on, axial vorticity can disrupt the growth of large-scale structures near the exit plane. Because the azimuthal rotation is so strong in the early development of the jet, the sound production is dominated by these events (see vortex sound in 2.63). While the increased mixing inevitably generates small scale structures, they are more quickly dissipated. Alkislar \cite{170} noted that the low-speed side of the shear layer is a more favorable region to generate vorticity than the inner side. He states that streamwise vorticity “disrupts the natural coherence of the initial layer and alters the distribution of azimuthal vorticity and associated shear stresses. It is also clear that streamwise vorticity directly increases entrainment, shear-layer growth, and ultimately the production of turbulent kinetic energy.”

A more exotic approach to further noise reduction is through the use of additional
exhaust streams. This concept is somewhat similar to high-bypass fans used in civilian transports. However, because supercruise vehicles cannot reduce the overall velocity (i.e. specifically below Mach 1) and maintain thrust through a larger cross-section, the aim of multi-stream engines is to simply reduce the shear rate that occurs in the mixing layer between the core and ambient air. A three-stream nozzle, as studied by Henderson [18], is shown in Figure 2.20.

From previous research showing that high shear rate is tied to sound production, it is not surprising to learn that Henderson reported the high-frequency noise as being reduced through use of additional streams. Not only does the presence of the additional streams reduce the small-scale vorticity generated between the core and ambient fluid, but the core jet structure can be modified as well. Mid-frequency levels were additionally observed to be impacted, though to a less extent. The structure of the high velocity core, however, still likely generates large-scale turbulence responsible for the low-frequency noise and the bulk of the OASPL. Nonetheless, because this design maintains thrust capacity, it is a promising step towards noise reduction in future technologies.
2.4.2 Nozzle Performance Enhancements

Aside from the noise issue, efforts continuously try to enhance nozzle performance. Geometric perturbations are made to the round jet to achieve various desired characteristics, and sometimes entirely new approaches are taken. Figure 2.21 shows four different nozzle shapes implemented in military aircraft. The F-117 (top left) utilizes high-aspect ratio, tapered rectangular nozzles. The B-2 (top right) incorporates an aft deck to reduce the profile and improve airframe integration. F-22s (bottom left) have rectangular nozzles with variable exhaust angles (i.e. thrust vectoring). A concept design, the X-47B (bottom right) utilizes tapered, elliptical geometry, a beveled exit plane, and is body-integrated. Not only is there military interest in such geometries, but future civilian transport vehicles must also consider moving away from the axisymmetric jet if supersonic flight is to be achieved [171].

Figure 2.21: Variation of nozzle geometry with military aircraft.

Rectangular nozzles have been studied for some time [10, 12, 13, 172–175]. Due
to their favorable integration with supersonic cruise vehicles over traditional pylon
designs[176], they have recently gained increased attention. Rectangular internal ge-
ometry is also more easily modified than round configurations, and enables thrust
vectoring to be straightforwardly implemented [177]. A translational throat, for ex-
ample, is an attractive feature as the engine can change its choked flow properties to
operate more efficiently across all ranges of the flight regime. This in turn enables
increased fuel efficiency and thrust, which yields an overall improvement in mission
capabilities.

As foreshadowed in 2.2, the flow-fields of rectangular jets are complex. The three-
dimensionality (as opposed to planar and round jets) and presence of strong corner
vortices greatly limits the simplifications available and makes theoretical analysis
exceedingly difficult. Previous work on rectangular jets experimentally investigated
the near-field flow arrangement. Sforza [178, 179] originally looked at the flow-field
when considering an array of geometries. He was followed by Krothapalli [12], who
gave it a more thorough investigation and divided the flow into three regions: a
potential core, a two-dimensional type, and an axisymmetric type. Since the vertical
and horizontal shear layers are separated by different distances, they meet at separate
locations, thus creating the first two regions. Further downstream, the jet decays
such that it eventually looks axisymmetric. A sketch by Krothapalli [12] of the three-
dimensional flow field is given in 2.22.

Upon compiling data from a number of tests, Krothapalli concluded the following
about the rectangular jet. Similarity is observed in mean variables and shear stresses
after roughly 30 widths downstream (30L in Figure 2.22), though this value varies
based on aspect ratio of the nozzle. This occurs in the second region, where the
flow is similar to two-dimensional jets. One common characteristic of rectangular
jets is that the mean velocity profile in the major axis of the second region (i.e. the
x − z plane from A − B of Figure 2.22) resembles a saddle shape [180]. The third
region however, which appears axisymmetric in the mean, lacks self-preservation in
Figure 2.22: Flow-field of a rectangular jet. The insert gives the decay of the mean axial velocity along the centerline, $U_C$, normalized with the exit velocity, $U_0$.

the higher-order moments (e.g. r.m.s.). While all jets appear to eventually evolve into a mean axisymmetric flow, whether or not the rectangular jet ever becomes truly self-preserving has not been established.

2.4.3 Three-stream Engine

The design of the MARS jet currently under investigation is based on AFRL’s three-stream engine, shown in Figure 2.23. The architecture of this powerplant has been developed through a system-level integrated design with the airframe, and offers a number of benefits to future aircraft. Using adaptive architecture, the three-stream engine claims to have optimized performance over variable operating conditions, reduced specific fuel consumption (SFC) related to an increased range and thrust, and improved thermal management.

Figure 2.23 is based on a variable-geometry engine cycle by Simmons [11]. The design clearly incorporates features of advanced nozzle configurations, including the
rectangular geometry and multi-stream flows. From the multi-stream axisymmetric experiments, the third stream may help to reduce noise generation. As previously discussed, the rectangular geometry gives the ability to easily implement thrust vectoring. And from a thermodynamic perspective, the third stream gives an added level of control to the engine, including: “providing a cool heat sink for dissipating aircraft heat loads, cooling turbine air, and providing a readily available stream of constant pressure ratio air for lift augmentation.” [11]. Propulsive efficiency is also thought to be increased, because the additional streams can be utilized to reduce spillage by matching the inlet’s airflow demands. The attractive noise, thermodynamic, and propulsive characteristics of this engine have thus placed it as a leading candidate for future technologies.
Chapter 3

Experimental Setup

3.1 Facilities

The jet rig under investigation is installed in the Skytop Turbulence Laboratory at Syracuse University. The lab consists of an anechoic chamber in which the primary testing occurs, a large reciprocating compressor and storage tanks in a separate building, the jet itself, and the control room. Each of these components are detailed in the following sections.

3.1.1 Facility

The anechoic (i.e. echo-free) chamber sits inside a concrete building on the Skytop Campus. Originally constructed in the 1970’s, the facility is described in detail by Ahuja [181]. Recently overhauled to handle the complex nozzle, the 7.9 m x 6.1 m x 4.3 m chamber is treated with fiberglass wedges, yielding a cutoff frequency of \( \sim 150 \text{ Hz} \), to allow for aeroacoustic experiments. The jet and chamber are seen in Figure 3.1.

With the addition of the new nozzle, various facility limitations have been encountered in the original build. One such problem was flow recirculation inside the chamber. The high velocity of the jet and increased mass flow rate created conditions
such that the exhaust blower could not remove air quickly enough from the chamber, introducing unwanted pressure gradients in the chamber and altering the far-field boundary conditions of the experiment. To overcome this, a higher capacity blower was installed (Harztell Fan, Model A54-G-367VA-STFCP 54 in. Belt Drive Vaneaxial Fan, rated for 20,000 cfm (9.4 m$^3$/s) at SP: 3.5 in. w.g. / TP: 4.0 in. w.g.) with a variable frequency drive (Yaskawa Z1000 Series Drive). Using the HVAC system previously retrofitted [182] and the controller as discussed later in subsection 3.1.3, the pressure inside the chamber can be controlled. Additionally, a plume catcher was designed to help capture the jet exhaust. By matching the contour to a $5^{th}$ order polynomial, its shape mimics the contraction section of a wind tunnel. The exhaust area was also increased from approximately 0.6 m$^2$ to 1.5 m$^2$, reducing the pressure differential to the outside air. The plume catcher, installed with anechoic foam around its body, is visible in the center of Figure 3.1.

The HVAC system serves a dual purpose: in addition to balancing the pressure difference between the chamber and ambient conditions, it also supplies the co-flow air around the jet. An Industrial Commercial Equipment (ICE) make-up air unit is fitted to the roof and provides 14,000 cfm (6.6 m$^3$/s) of intake air, which is temperature controlled from 50 °F - 95 °F (10 °C - 35 °C). This air is forced through a circular duct enclosing the jet pipe, where flow straighteners remove turbulence. While the flow around the jet is relatively slow (approximately 7 m/s), it is essential for adequate
and reproducible particle seeding in the ambient fluid. Without this, accurate PIV measurements in the shear layer are not possible.

### 3.1.2 Jet Rig

The jet is powered by a 2-stage, 100 hp reciprocating Joy compressor. Compressed air is stored in a tank array of 45 m$^3$ up to pressures of 3.4 MPa (500 psig) to run the jet. A butterfly valve controls the output of the tanks, which is in turn controlled by the user’s input to the LabVIEW master program (subsection 3.1.3). A $\sim$50 m steel pipe connects the butterfly valve to the anechoic chamber, where an in-line heater resides. A Chromalox heater is used to heat the jet when necessary. This is discussed in subsection 3.1.4. Pressure, flow, and temperature sensors are distributed throughout this system to monitor and feed back information for control. This arrangement is capable of producing supersonic exit velocities in the MARS jet at $Re > 10^6$ for test durations up to 2 minutes.

Downstream of the heater, the jet rig is attached to the pipe through an azimuthally symmetric coupling, allowing it to rotate along the centerline, $z$-axis, of the jet. Using a small crane, experimenters can hoist, spin, and reposition the jet to acquire data at different orientations of interest. A contraction section converts the circular flow to rectangular, at which point the MARS nozzle is fastened. During heated operation the entire pipe and jet expand. Thus, the rig is fastened to four expansion rollers to allow for the exit to slightly move with axial expansion. A rendering of the jet rig is seen in Figure 3.2

**Nozzle Design**

MARS was designed for use in Syracuse University’s blow-down, anechoic chamber. The core geometry of the MARS studied herein is the SERN, from which additional features are then integrated. The core and fan flows are assumed perfectly mixed entering the SERN, and the third-stream flow is introduced into the diverging portion
of the SERN at the appropriate location based on pressure. *The MARS rig therefore only has two streams, but its flow configuration represents a three-stream engine.* Finally, a removable/configurable aft deck is incorporated into the nozzle to represent airframe integration.

The nozzle is designed for supersonic operation and is optimized for a *NPR* of 4.25. As the SERN is the primary geometry, it is the initialization of the design process. Employing the method of characteristics[183], the contour of the expansion ramp is calculated. The inputs to the routine can be found in Table 3.1. Nozzle pressure ratio and nozzle temperature ratios (*NTR*) are taken from an idealized stagnation chamber upstream of the nozzle, *R* is the radius of the expansion portion of the nozzle, and *hₜ* is the height of the throat. Knowing the aft deck is to be included later, it was assumed that the bottom side of the nozzle would be enclosed, unlike a traditional SERN. Using isentropic relations, the computation begins where the flow is ideally choked, i.e. the nozzle throat. The Mach number is calculated at 1.612, and the computed design of the SERN, optimized based on thrust, is shown in Figure 3.3.

Once the methods of characteristics have generated the curve, the location of the
third stream is considered. The asymmetry of the nozzle requires some elementary computational fluid dynamics (CFD) to determine a suitable position at which the third stream is introduced into the nozzle. The 2D Euler equations are therefore solved using the MacCormack method, and initialized from the isentropic relations as computed at the throat of the nozzle (resulting from the previous operation). The conditions given as inputs to the solver may be found in Table 3.1. As a check of solver accuracy, the Mach numbers and NPRs are compared against the isentropic relations along $y = 0$, shown in the bottom of Figure 3.3. No diverging portion will likely exist in the third stream, so the maximum (ideal) velocities correspond to choked flow. Thus, the pressure ratio in the third stream stagnation chamber is

<table>
<thead>
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<th>M.o.C. Inputs</th>
<th>CFD Parameters</th>
</tr>
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<tbody>
<tr>
<td>$NPR$</td>
<td>$N_{cell}$</td>
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<td>2040</td>
</tr>
<tr>
<td>$NTR$</td>
<td>$N_t$</td>
</tr>
<tr>
<td>5.42</td>
<td>1000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>CFL</td>
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<td>0.8</td>
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<tr>
<td>$R/h_t$</td>
<td>$h_t$ (mm)</td>
</tr>
<tr>
<td>6</td>
<td>18.8</td>
</tr>
</tbody>
</table>
taken as 1.893. The pressures at the exit plane of the third stream and the SERN are then matched by varying the exit location. The converged CFD solution in the nozzle suggested placing the third stream exit plane between 16.5 & 17 mm, as indicated in Table 3.2 by the normalized flap length, \( L_f/h_t = 0.89 \).

Machining and structural concerns determined the vertical offset of the third stream, and the throat height was arrived at with input from AFRL personnel. As resonant effects are known to occur from fluid-structure interactions [184], the deck plate thickness has been over-designed to avoid such motions. Deck plate length was chosen by hypothesizing oblique shock locations from the trailing edge of the SERN. Three exit heights, \( h_e \), were taken as the estimate so that reflecting oblique shocks from the SERN would coalesce with the one leaving the deck plate, ideally generating a cleaner, quieter flow. The final design of the MARS is shown in Figure 3.4. 3.4a is a schematic of the nozzle, and the nomenclature used to describe it can be found in 3.4b, with the corresponding values for these parameters in Table 3.2. Computational results provided by Ruscher et al. [185] from the original LES estimate the thrust of this geometry to be 659 N, approximately 94% of the ideal thrust calculated by isentropic relations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
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<tr>
<td>( w )</td>
<td>Nozzle width</td>
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</tr>
<tr>
<td>( h_t )</td>
<td>Throat height</td>
<td>18.8 mm</td>
</tr>
<tr>
<td>( h_e )</td>
<td>Exit height</td>
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</tr>
<tr>
<td>( w/h_t )</td>
<td>Throat aspect ratio</td>
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<td>( h_e/h_t )</td>
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</tr>
<tr>
<td>( L_f/h_t )</td>
<td>Normalized flap length</td>
<td>0.89</td>
</tr>
<tr>
<td>( L_r/h_t )</td>
<td>Normalized ramp length</td>
<td>3.69</td>
</tr>
<tr>
<td>( L_d/h_t )</td>
<td>Normalized aft deck length</td>
<td>4.80</td>
</tr>
<tr>
<td>( D_h )</td>
<td>Hydraulic diameter, ( \frac{2h_e w}{h_e + w} )</td>
<td>44.5 mm</td>
</tr>
</tbody>
</table>

Note that the value reported for the hydraulic diameter reflects the physical measurements from the assembled jet, which slightly differs from the theoretical value provided from the CAD files. Machining tolerances and the addition of protective
surface coatings are thought to account for this discrepancy. For the experimental work presented here, the value provided in Table 3.2 is used.

![Diagram](image)

**Figure 3.4:** Cross-section and nomenclature of MARS.

The baseline parameters are given in Table 3.3. Since the jet is rectangular, a characteristic length (i.e. diameter) must be defined to scale variables. However, in a flow with multiple characteristic lengths, there is not necessarily a universal value that collapses all data. In the case of rectangular jets, the hydraulic diameter works well for near-field data, while the effective diameter more appropriately aligns far-field data. The hydraulic diameter is given in Table 3.2. Far from the jet, one makes the argument that the flow eventually becomes axisymmetric, so the effective diameter, $D_e$, is used. This is defined as the area-equivalent diameter of an axisymmetric jet:

$$D_e = 2\sqrt{\frac{h_t w}{\pi}},$$

where $h_t$ is the throat height and $w$ is the nozzle width. For the MARS, these values are nearly identical, and are given in Table 3.3.

The hypothesized flow-field from this jet is shown in Figure 3.5. The multi-faceted nozzle creates a complex flow, even in the mean sense. Mixing layers develop top and bottom, along with one between the bulk and tertiary flows. The deck plate and mixing layers create a skewed potential core. Oblique shocks are thought to
Table 3.3: MARS jet specifications.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_e$</td>
<td>Effective diameter</td>
<td>44.4 mm</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter</td>
<td>44.5 mm</td>
</tr>
<tr>
<td>$Re_{De}$</td>
<td>Reynolds number</td>
<td>$2.76 \times 10^6$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Mach number, bulk flow</td>
<td>1.6</td>
</tr>
<tr>
<td>$M_w$</td>
<td>Mach number, tertiary stream</td>
<td>1.0</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Total mass flow rate</td>
<td>1.97 kg/s</td>
</tr>
</tbody>
</table>

form off the top and reflect through the shear layer down the length of the core. Superimposed on this skeletal structure are the turbulent structures that interact with each of these features. The asymmetric nature of this flow raises questions about when self-preservation will be reached, so this is left open-ended. As this is a view about the plane of symmetry, corner vortices and the behavior in the direction of the major axis are not seen. The full three-dimensional nature of the flow adds additional complexity.

### 3.1.3 Jet Controller

Renovating the control system was essential for proper operation of the MARS jet. Due to the nonlinear isentropic relations of compressible flow [183], the higher velocities are more susceptible to disturbances fed into the PID controller. The existing scheme was deemed unacceptable for proper control of the jet, as its response time was $> 10$ ms. Additionally, the air requirements of the jet at target conditions consume an enormous amount of energy, making efficient operation a priority for testing to conclude in a timely manner.

The previous PLC-based control system for the facility was replaced with a more capable, robust, and responsive NI-based system. A cRIO was selected as the main chassis for the controller, from which task-specific modules were then chosen and installed. The inside of the control box is presented in Figure 3.6, and the hardware specifications are given in Table 3.4.
Figure 3.5: Hypothesized developing features of the MARS jet about the plane of symmetry.

The program responsible for controlling the NI hardware is coded using LabVIEW and then compiled to the FPGA on board the cRIO system. The primary advantages of the FPGA are faster response times, real-time operation, and reliable, uninterrupted performance of the controller. Once the software has been written to the FPGA, the end-user interfaces through the GUI front diagram. The software for the program is shown in Figure 3.6. The far left tree shows the program hierarchy and connectivity status, a portion of the block diagram in the left background, and the front panel (right) containing the user inputs and monitored outputs. Viewpoint Systems was contracted for the baseline coding of this control program.

Operation of the jet can either be manual, where the operator selects a % opening of the butterfly valve, or automatic. In automatic mode, a PID control is used to throttle the valve in a cascade loop, which increases the response of the controller.
Figure 3.6: NI hardware used for facility & jet control (top), front panel of the software program (middle), and back panel of the program (bottom).
Table 3.4: Summary of LabVIEW-based controller hardware.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>I/O Hardware</th>
</tr>
</thead>
<tbody>
<tr>
<td>cRIO-9082</td>
<td>High Performance Integrated System, 8-Slot, RT, embedded FPGA</td>
<td>PC &amp; task modules</td>
</tr>
<tr>
<td></td>
<td>4-Channel, 0-20 mA, 16-Bit Analog Current Output Module</td>
<td>Jet, HVAC, exhaust fan</td>
</tr>
<tr>
<td>NI-9265</td>
<td>16-Channel, ± 20 mA, 24-Bit Analog Input Module</td>
<td>Pressure &amp; flow probes</td>
</tr>
<tr>
<td>NI-9208</td>
<td>8-Channel, 100 ns, TTL Digital Input/Output Module</td>
<td>Industrial controls</td>
</tr>
<tr>
<td>NI-9401</td>
<td>4-Channel, 24-Bit, Universal Analog Input Module</td>
<td>Thermocouples</td>
</tr>
<tr>
<td>NI-9219</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The primary feedback node is a transducer inside the jet, which measures the pressure ratio of the nozzle (NPR) relative to the ambient air. With temperature additionally sampled in the jet, a Mach number is calculated through the isentropic relations \[183\] and the target operating condition is achieved. A second set of sensors is installed in the jet to monitor the status of the tertiary stream. Immediately downstream of the valve, a pressure transducer is used as an additional feedback node in the PID-cascade loop. This configuration quickly and efficiently ramps the jet up to speed and maintains a steady operational state for testing to take place.

### 3.1.4 Jet Heater

The jet facility is equipped with an electrical heater for experiments at increased NTRs. Two 470 kW Chromalox units are operated by the LabVIEW control system by providing an auxiliary set point to a set of industrial PID controllers. A flow sensor must be activated (which occurs near \(NPR = 1.03\)), to prevent the electrical rods from overheating. Although there is a considerable amount of power input to the system, and high heat transfer rates with almost 2 kg/s of air, the entire system still requires 1-2 hours to heat up to a “steady”-state. Due to the short test durations and significant thermal inertia of the jet facility, equilibrium is never truly
reached. Running the heated experiments for the MARS jet requires some practice and experience to achieve reproducible test conditions.

The requirement to run the jet heated is a result of the PIV instrument. Data are collected in the jet plume, and doing so requires the appropriate amount of seeding particles in the flow. This is expanded on in detail in subsection 3.2.3, but for now suffice it to say that water vapor interferes with seeding of the flow, which is a result of the air cooling upon expansion. Based on the 1-D isentropic flow equations Equation 3.2 - Equation 3.3, the exit temperature of the jet is predicted to be 197 K. The air leaving the compressor is dried through a desiccant drier, essentially removing all water vapor in the jet flow. Thus, the core of the jet is not problematic for seeding purposes. However, the ambient air is entrained and cooled, resulting in the formation of water droplets in the flow. The solution is to heat the jet, and raise the overall temperature of the jet so that it cannot cool below the ambient dew point. Predicting exactly when condensation occurs is difficult because the physics of the mixing layer are very complex, but nonetheless a prediction is made of raising the temperature by 100 K. Experimental observations are then used to refine the necessary stagnation temperature. Because the dew point changes with weather conditions, tests are performed over a range of different days to identify the minimum value expected in Syracuse, NY.

To evaluate the thermal response of the pipe and jet, a series of tests was performed. Initially, the thermocouple embedded in the jet (i.e. the one used for control purposes) was used to monitor the temperature of the jet. However, this is mounted in the wall and thus has a large measurement bias resulting from the thermoconductivity of the steel jet wall. As the air temperature is the real quantity of interest, an additional thermocouple was inserted into the core of the jet, just downstream of the jet exit. With the high velocities stagnating on the 1/4" probe volume, this was assumed to give a better estimate of the total temperature of the air (in reality it reads some type of average between static and stagnation temperature). This probe
can be seen mounted to a sting in Figure 3.7.

Figure 3.7: Thermocouple placed in the supersonic core of the jet. This experiment was performed to calibrate the thermal response of the jet facility when using the Chromalox heater.

The calibration tests are performed as follows. While the tanks were charging from the compressor, air is slowly trickled through the jet with the heater on, and set to 250 °F (121 °C). This is referred to as the idle state, and data is recorded through the Jet Controller program. Once a sufficient temperature in the air is reached, the control program is executed and the jet operates at design conditions. Temperature is monitored through four K-type thermocouples mounted in different locations: in the primary stream jet wall, the tertiary stream jet wall, the core of the jet exit, and the plenum temperature which records the pipe temperature just downstream of the heater. Data from seven different tests are shown in Figure 3.8. A total of 27 tests were actually performed to establish the appropriate laboratory practice for heated jet operation. With the MARS jet, the static temperature must not exceed 95 °C, or damage will occur to critical components.

The runs in Figure 3.8 show the response of the jet. The timescales are not consistent between tests, as some lasted longer than others. They are of the same order, however (200-300 seconds). During Idle operation, significant thermal energy amasses in the heater and pipe. Though the thermocouple readings indicate a steady state, components upstream of the probes are much hotter, which will release more
heat during operation. When the jet is turned on, the air temperature initially increases rapidly in the primary stream as heat transfer from the heater to the airflow.

Figure 3.8: Calibration results from seven different heater tests, showing the progression toward ideal operation (given by Case g).
is hastened. This is easily identified in each of the graphs by the jump in the primary stream temperature. The jet exit temperature decreases as the air expands through the nozzle. After the initial transient behavior, temperatures level out during the operational state. The primary stream temperature continues to increase slightly, due to contamination in the measurement from the jet wall which is constantly heating up during operation. However, the jet exit temperature stays quite steady, with deviations of 1-3 °C considered acceptable. Once the jet is turned off, the primary stream temperature decreases because the convective heat transfer has dropped, with the cooler jet wall acting as a heat sink. The jet exit temperature increases, as it essentially reads the total temperature again. To determine if a test operated at a steady total temperature, the initial and final temperatures are considered.

Referring to Figure 3.8, the seven tests were carried out over a few hours. Data were acquired on October 8, 2016, with 100% Relative Humidity and a dew point of 14 °C. Case a) was the first test, and g) the last. One immediately observes that the overall temperature of the probes increases with subsequent tests as all components of the system heat up. In the first three tests, temperature varied too significantly, indicating a quasi-steady state had not been achieved. By Case d), the initial and final temperatures were closer, but the operational state varied too much due to the overshoot in the heater. Case e) was a test where the heater was powered on during the test. This was to observe the amount of heat coming from warm components, which clearly indicates the requirement of the heater during operation. Case f) is suitable operation, and Case g) is ideal: the initial and final temperatures of the jet exit vary within 1 °, and the operational state is flat. Note the importance of the jet exit thermocouple. By simply observing the probes installed in the jet, one would have no indication that the air temperature is steady. This data from these runs are used to establish predictions of the air temperature through the primary and third stream thermocouples.

For ideal operation, these guidelines should be followed. The heater setpoint is
250 °F (121 °C), and the jet is warmed for 1-2 hours at an idle rate of $NPR_1 = 1.03$. The primary stream temperature is heated to 130 °F (55 °C). This corresponds to a jet total temperature of approximately 200 °F (94 °C). Additionally, the third stream temperature is 120 °F (49 °C). If the third stream section of the jet is not at the appropriate temperature, the pressure ratio $NPR_3$ will be incorrect. After a run, the jet has warmed up significantly. The experimenter must wait (usually 30-45 minutes) until components have cooled enough, at which point the jet can be run idle again and the target temperatures can be reached. Running too soon will result in an overshoot condition, such as Case d) in Figure 3.8. When these procedures are followed, the heated jet can be operated in a quasi-steady manner, with temperature deviations as little as 1-2 °C.

### 3.2 Instrumentation

The hub of the data acquisition system uses a National Instruments (NI) machine to synchronize and record various information in the lab. As with the jet/facility control system, LabVIEW software serves as the interface between the researchers and the various hardware for each measurement quantity. Throughout all of the experiments, far-field acoustics and near-field pressure are sampled directly through a NI PXIe system. Flow-field measurements, which alternate between schlieren and PIV, are acquired using task-specific machines, and synchronized through the NI machine. Each instrument is discussed in detail in the following sections.

#### 3.2.1 Pressures

Pressures are acquired through two separate instruments: near-field pressure transducers and far-field microphones. The hardware that powers and samples these sets of probes is a PC-based platform known as PXI, a National Instruments® product. Its robust and modular design allows for high-performance data acquisition to be spe-
cialized for a particular application - in this case the high-speed sampling of pressure signals.

The data acquisition (DAQ) chassis is a NI PXIe-1082, which has 8 express slots to mount specific acquisition cards. The chassis is controlled by a NI RMC-8354. This rack-mount controller is the PC which runs LabVIEW in a Windows environment, and has internal RAID 5 hard drive for protected storage. On here, experimenters generate the LabVIEW code necessary to acquire data. The high bandwidths of data require fast transfer rates, so a NI PXIe-PCIe8375 fiber optic data transfer module is used to port incoming data from the PXI chassis to the controller. The far-field microphones are sampled through a NI PXIe-4497 card. This module is a 24-bit A/D converter, capable of simultaneous sampling at 204.8 kS/s across 16 channels. Near-field pressures, acquired through Kulite® pressure transducers, are sampled with NI PXIe-4331 modules. Each card is a 24-bit bridge analog input, which acquires data at 102.4 kS/s over 8 channels. On both types of DAQ modules, channels can be configured in synchronization or independently. NI’s aliasing rejection technologies are built into each card, which essentially guarantee the digitized signals are free of aliased components and scales with the chosen sampling frequencies. On the NI PXIe-4331, this is achieved through the use of an oversampling architecture with a system of sharp digital and analog filters. In the NI PXIe-4497, a dynamic low-frequency filter rejects signals above 45.35% of the sampling frequency. With the DAQ hub configured for the lab, a considerable amount of data is generated across the multiple channels. Approximately 25 MB/s of data is acquired, up to a maximum acquisition limit of 10 sec, at which point data must be transferred from the PXIe on-board memory to the hard drive on the controller.

Near-field pressures are sampled using Kulite® transducers, model XCE-093. These are high-temperature miniature pressure transducers. They have a 50 kHz dynamic response and are capable of operating in conditions up to 525 °F. The outer diameter is 0.095” so they can be installed in tight places. For these experiments,
they have been installed in the deck plate of the jet. The location of the Kulites can be seen in Figure 3.9

Figure 3.9: Kulite locations installed in the deck plate of the jet. The locations shown in a) were used for the schlieren campaign, while the positions in b) were used during PIV acquisition.

The initial design and location of the Kulites is observed in Figure 3.9 a). This deck plate was machined from stainless steel, and had a second plate mated to the bottom face which held the Kulites in place. This configuration was used for the duration of the schlieren experiments. When PIV was used, a transparent deck plate was required to minimize wall reflections and allow for measurements close to the surface. Thus, a polycarbonate plate was chosen, and the Kulites held in place by an O-ring compression-type fitting. The locations of the Kulites during the PIV measurements is found in Figure 3.9 b). The positioning was changed slightly to allow for slice of PIV data to be taken without reflecting off the transducer faces. However, the locations of Kulite F and 1 are at the same streamwise locations along the center plane of symmetry, and Kulites E and 5 can be assumed identical as 5 has simply been mirrored about the plane of symmetry.

The far-field acoustics were measured using 12 G.R.A.S. free-field condenser microphones located $r/D_e = 85.8$ from the exit plane. The 46BE microphone sets have a frequency response of $\pm 3$ dB in the range of 4 Hz - 100 kHz with a dynamic range
from 30dBA to 160 dB (re. 20µPa). These characteristics make the microphones good choices for the range of flows encountered in the lab, and have been successfully used extensively by previous experimenters. The microphones (and the transducer calibrator, G.R.A.S. 42AB) were recalibrated and certified by the manufacturer in August of 2015 to correct for drift in the transducers over the years. One in-plane array of the microphones is positioned in the horizontal plane of the nozzle exit. This circular array is spaced from 90° to 135° in increments of 15°, and then the resolution is increased to 5° increments until 165° is reached. Two more microphones are located approximately one meter above the first, angled down at 15° and positioned 135° and 150°. These are not used in the current study. For reference, the 165° is located 15° off the streamwise centerline of the jet axis, \( x \). Microphones were simultaneously sampled at 100 kHz in order to capture high-frequency acoustic phenomena in the flow. The far-field microphone array and their relation to the jet can be seen installed in the chamber in Figure 3.1, and their location is apparent in Figure 3.10.

Figure 3.10: Coordinate system for the acoustic data fixed relative to the jet (a) and as distributed in the chamber (b).

The experimental error of the OASPL from the MARS jet is estimated to be ±2.3
dB with a repeatability of ±0.4 dB. The method and calculations for this can be found in Appendix A.

3.2.2 Schlieren

A z-type schlieren system[187] was built to acquire density variations in the flow, part of which is seen in Figure 3.1, (b). Geometric constraints in the test chamber required the z-configuration to orient vertically. To allow for tunable optomechanics, a single structure of 80/20 extruded aluminum was built to support all of the components. By mechanically linking the mirrors, light source and camera together, low-frequency structural vibrations previously encountered were greatly reduced, which resulted in more consistent, higher-quality data.

The details of the schlieren setup are listed in Table 3.5 and Figure 3.11.

Table 3.5: Summary of schlieren setup, in order of the light path.

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light Source</td>
<td>Luminus CBT-120 green LED</td>
</tr>
<tr>
<td>Collimating Lens</td>
<td>50.8 mm achromatic doublet</td>
</tr>
<tr>
<td>Condensing Lens</td>
<td>75 mm aspheric condenser</td>
</tr>
<tr>
<td>Aperture</td>
<td>1 mm vertical slit</td>
</tr>
<tr>
<td>Twin Mirrors</td>
<td>318 mm parabolic, f/D = 8</td>
</tr>
<tr>
<td>Knife Edge</td>
<td>Vertical razor blade, 80% cutoff</td>
</tr>
<tr>
<td>Condensing Lens</td>
<td>25.4 mm achromat</td>
</tr>
<tr>
<td>Camera</td>
<td>Photron SA-Z</td>
</tr>
</tbody>
</table>

The components of the schlieren setup are now described. Two twin 318 mm diameter parabolic mirrors with 2.54 m focal lengths, loaned to SU by AFRL, provided the collimating elements. With offset angles minimized, this yielded a field of view of approximately 312 mm. A Luminus CBT-120 green LED light source was driven by a pulsed circuit at high frequencies, modeled after Willert [188] and Wilson [189]. By using short duty cycles, the LED was overcharged to produce high-intensity output (up to 550% of the manufacturer’s peak rating). A knife edge was vertically positioned at the second focal point along the optical axis and set to approximately 80% cutoff.
for the experiments herein. Achromatic condensing lenses were placed immediately behind the knife edge and the light was then directed straight onto the CCD chip in the camera. A Photron SA-Z high-speed camera (also on loan from AFRL) was selected to acquire the data, providing high-resolution images at frame rates up to 400 kHz. Finally, a LabVIEW subroutine was used to drive the light source, sync components, and trigger the data acquisition. This was integrated into the jet controller program, and sampled with the DAQ hub used for pressure acquisition. The configuration allowed for synchronized data acquisition between the pressure system and schlieren imaging at a range of different sampling frequencies.
3.2.3 PIV

Particle Imaging Velocimetry (PIV) is an optical measurement technique which measures the displacement of tracers in the flow to calculate velocity fields. The flow is seeded with particles of the appropriate size, and these are then illuminated, typically via a laser. By taking two sequential images of the particles, the displacement of each particle is determined, from which velocity is then calculated. While conceptually very simple, many complications are hidden in the details. As such, PIV is considered one of the more advanced experimental techniques, and has required appreciable research to develop into today’s state (Interested readers should are referred to review papers and books on the matter [142, 190–194]). With careful planning, experimenters are able to extract 3-component velocity fields of complex flow with a high degree of accuracy.

Particle Seeding

PIV seeding particles must be small enough that inertial effects are negligible, but still provide the minimum illumination to be captured by the cameras (ideally between 30% & 50% of the pixel saturation level). Laser power can only be increased so much, and is typically limited by the equipment as lasers are the most expensive component. Thus, choosing the particles primarily focuses on sizing to minimize lag.

To achieve adequate particle density in the test section, both the core jet and surrounding co-flow are seeded. This is a necessity, as failure to seed the entrained fluid results in erroneous measurements throughout the mixing layer. The jet core was seeded using a ViCount 1300 smoke generator, with a white mineral oil as the aerosol. This machine operates by pressurizing and superheating a mixture of oil and CO$_2$ to 370 °C. When the oil is exhausted, it rapidly cools and condenses into fine droplets. These are estimated to have a mean particle diameter of 600 nm based on experiments by Mitchell et al. [195]. The co-flow was seeded using two commercial, theatrical foggers; these evaporate and then condense a glycerin-based
fluid at constant pressure to form smoke particles. Kähler et al. [143] have shown these smoke particles have a narrow size distribution around a mean diameter of 2 µm.

When acquiring PIV data, one must avoid contamination of other particulates. With the MARS rig, this problem arises as condensation of water vapor in the surrounding air. As the ambient fluid mixes in the shear layer and cools, the water condenses and a vapor cloud is formed. In these experiments, this process is significant, and the large amount of condensation saturates the pixels in the CCD (charged couple device) of the camera. Experimental observations have shown these regions of saturation to grow proportionally with the mixing layer, so this mechanism is substantiated. When the appropriate heat is added to the jet (subsection 3.1.4), these regions of saturation are eliminated, and the desired particles are imaged as planned.

Optical Setup

PIV images were synchronized and acquired using Dantec Dynamics equipment. Two FlowSense EO 4MP cameras were setup in stereo configurations to capture 3-component velocity fields. The timing board, a NI PCIe-6612 acquisition card, synchronized the cameras and lasers to within nanosecond accuracy. Images were transferred through a NI PCIe-1430 vision acquisition card, which allows for data streaming. With these cameras, full-resolution images (2048 × 2048) can be streamed at 10 Hz. Data were stored on the available RAM and then written to the hard drive. Memory was expanded to 72 GB on the desktop computer used in these experiments. This allowed for over 800 image pairs from each camera to be acquired in a single run. A NewWave Gemini Nd:YAG laser, generating 200 mJ/pulse at 532 nm, was used as the light source. Dantec Dynamics sheet optics spread the laser into a plane prior to reflecting off a high-energy Nd:YAG laser mirror. This mirror was installed on a 6-axis opto-mechanical traverse, and used to fine-tune the laser alignment for PIV calibration. The PIV apparatus is shown in Figure 3.12.
Figure 3.12: PIV setup installed in the chamber, and acoustically treated.

The laser & optics, scheimpflug mounts, and cameras were fixed relative to one another via 80/20 extruded aluminum, and installed on a two-axis traverse system. Acoustic treatment can be seen covering the equipment in Figure 3.12. Additionally, an optical shield was installed behind the alignment mirror, to block any stray laser reflections from damaging the cameras. Dantec Dynamics’ Dynamic Studio 2015 was used to control the hardware and acquire data. A summary of the experimental parameters is given in Table 3.6.

Two PIV campaigns were executed: the first took data along the cross-planes (i.e. the $y - z$ planes), and the second acquired data along the streamwise direction (i.e. the $x - y$ planes). Additionally, the streamwise campaign included an auxiliary 5 planes downstream of the primary field of view to provide overlapping statistics. These had a field of view from $4 < x/D_h < 8$, and taken at $z/D_h = -1, -0.5, 0, 0, 51$. The rest of their parameters were unchanged from the streamwise planes’ listed here. A schematic of these should aid the reader, which can be seen in Figure 3.13.

Each orientation presented its own calibration challenges, and the final conditions of these setups are documented in Table 3.6. In the cross-plane configuration, the cameras view the lightsheet from one side (the horizontal laser line in Figure 3.13). Thus, a single-sided calibration target was used, as both cameras can see the same side
Table 3.6: Summary of two PIV setups.

<table>
<thead>
<tr>
<th></th>
<th>Cross-plane</th>
<th>Streamwise</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lens</strong></td>
<td>Nikon 60 mm, f/2.8D</td>
<td>Nikon 60 mm, f/2.8D</td>
</tr>
<tr>
<td><strong>f#</strong></td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Offset angle, ψ</strong></td>
<td>34°</td>
<td>45°</td>
</tr>
<tr>
<td><strong>Scheimpflug angle, θ</strong></td>
<td>2.6°</td>
<td>7.2°</td>
</tr>
<tr>
<td><strong>Magnification, M₀</strong></td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>Sheet thickness, Δz</strong></td>
<td>1 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td><strong>Depth of field, δz</strong></td>
<td>3.9 mm</td>
<td>1.45 mm</td>
</tr>
<tr>
<td><strong>FOV</strong></td>
<td>∆Y ~ 3.6Dₕ, ∆Z ~ 4.3Dₕ</td>
<td>∆X ~ 3.9Dₕ, ∆Y ~ 3.1Dₕ</td>
</tr>
<tr>
<td><strong>Locations</strong></td>
<td>x/Dₕ = 4, 5, 6, 7, 8</td>
<td>z/Dₕ = -1, 0, 0.25, 0.5, 0.75, 1</td>
</tr>
<tr>
<td><strong>Calibration Target</strong></td>
<td>Single-sided, 200 × 200 mm, h = 5 mm</td>
<td>Dual-level, δZ = 2.38 mm</td>
</tr>
<tr>
<td><strong>Calibration Fit</strong></td>
<td>3rd O polynomial</td>
<td>Direct Linear Transform (DLT)</td>
</tr>
</tbody>
</table>

of the image simultaneously. This is straightforward, and the calibration is easily performed by traversing the plate through a series of x–locations. The imaging modeling fit selected for this was a 3rd O polynomial, as the algorithm is robust and generally produces a smaller reprojection error as compared to other methods. When looking at the streamwise planes, the cameras are on opposite sides of the light sheet (the vertical laser line in Figure 3.13). Thus, a dual-sided calibration target is required. A custom plate was made for this with a second set of markers recessed/raised, as off-the-shelf items use coarser grids than what was required for these viewing parameters. A Direct Linear Transform was used to calibrate this configuration. While this algorithm is less stable and has a larger reprojection error than a 3rd O polynomial, this is the only method available with this configuration. Because there are only two z-locations (one from each set of markers), a 3rd O polynomial suffers from Runge’s phenomenon, which result in incorrect nonlinear distortions of the image plane. In both cases, a calibration refinement was performed using particles from the freestream, which minimizes disparity vectors. This accounts for any minor misalignment between the laser
sheet and the calibration target.

Uncertainty calculations on PIV data are a challenging topic. Many experimental sources contribute to the error. Particle density, concentration, displacement, gradients, and signal-to-noise ratio influence the tracking of particles [142]. Additionally, uncertainty contributions are specific to the flow under study, vary spatially throughout a given flow, and are a result of the number of images that can be acquired in an experiment. As these are still active areas of research [196–198], improved methods are being developed, and an exact procedure of determining uncertainty is not currently available. Experimenter must make assumptions to provide the best estimates. Note that this flow is a very challenging configuration due to the compressibility and high
Reynolds number. Turbulence measurement errors are proportional to the turbulence intensity [145]. The quantity for this flow was initially estimated using LES data as $Tu = 13\%$ [199], and later confirmed via PIV measurements. Supersonic flow conditions produce artificial velocity fluctuations due to the existence of shocks, which also must be considered.

A thorough discussion and calculations for PIV uncertainty are available in Appendix A. Shock-free average velocities are determined to have an uncertainty of $\pm 5\text{m/s}$ for the in-plane vectors. Out-of-plane velocities are estimated at $\pm 7.25\text{m/s}$ and $\pm 10.7\text{m/s}$ for the streamwise and cross-plane orientations, respectively. In the presence of shocks, the oblique shocks can introduce additional errors up to 10%, and normal shocks can cause artificial ripples resulting in a 35% error of the velocity. These uncertainties occur in the vicinity immediately downstream of a shock, and decay exponentially with distance from the shock.

### 3.3 Experimental Conditions

As discussed in section 3.1.2, the design conditions of the jet are $NPR_1 = 4.25$ and $NPR_3 = 1.89$. As the purpose of this study is to understand the fundamentals of this complex flow, all tests are carried out at these target pressure conditions. However, because of the vapor cloud problem interfering with PIV measurements (subsection 3.1.4, subsection 3.2.3), the core of the jet must be heated above the atmospheric dew point. Schlieren measurements were insensitive to this, and so they were operated with the jet unheated. When the PIV experiments were performed, requiring slight heat, the conditions of the jet were slightly changed.

The 1-D isentropic relations are used as a first-order approximation to evaluate the extent of the difference in flow conditions. The pressure ratios are matched, which is calculated as:

$$ \frac{P}{P_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-\gamma}{\gamma - 1}}, \quad (3.2) $$
where the subscript '0' is in reference to stagnation properties and $\gamma$ is the specific heat ratio. The stagnation temperature, however, is increased for the PIV experiments, thus raising the static temperature, $T$, of the jet:

$$\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}. \tag{3.3}$$

This of course causes a change in density as well,

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M^2, \right)^{\frac{1}{\gamma - 1}} \tag{3.4}$$

along with the speed of sound, $c = M/\sqrt{\gamma RT}$, exit velocity of the jet, viscosity, and finally, Reynolds number. The change in viscosity is calculated via Sutherland’s formula as

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{\frac{\gamma}{2} - 1} \left(\frac{T_0 + S}{T + S}\right) \tag{3.5}$$

where $\mu$ is the dynamic viscosity, and $S$ is Sutherland’s temperature, taken as 120 K. Using the kinematic viscosity at the jet exit, $\nu_J = \mu_J/\rho_J$, the well-known Reynolds number is then calculated as:

$$Re_D = \frac{U_J D}{\nu_J}, \tag{3.6}$$

where, following convention, the subscript $J$ refers to conditions at the jet exit. As an interesting aside, the nonlinear behavior of the Reynolds number as a function of Mach number and stagnation temperature is quickly looked at. A linear range of values for these two variables are explored using Equation 3.2 - Equation 3.5, and Figure 3.14 illustrate this counter-intuitive relationship.

Figure 3.14 shows the nonlinear dependence of Reynolds number on the exit Mach number and stagnation temperature. Curved isocontours help explain the growth rates of each variable in $Re_D$, and their upward concavity indicates that temperature is dominant. Holding temperature constant, the exit velocity, $U_J$, can be approximated
as a quadratic function. At a constant Mach number, however, the kinematic viscosity exhibits exponential decay. Thus, for the MARS jet, a decrease in Reynolds number can be expected for the heated case. While a faster exit velocity will be observed, the increased viscosity results in a decrease of inertial effects. Computing these relations at the two conditions under investigation, the following values are found:

<table>
<thead>
<tr>
<th></th>
<th>$M_{J,1}$</th>
<th>$M_{J,3}$</th>
<th>$T_0$ (K)</th>
<th>$T_J$ (K)</th>
<th>$c_J$ (m/s)</th>
<th>$U_J$ (m/s)</th>
<th>$\nu_J$ (m$^2$/s)</th>
<th>Re$_J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold</td>
<td>1.6</td>
<td>1.0</td>
<td>298</td>
<td>197</td>
<td>280</td>
<td>450</td>
<td>$7.23 \times 10^{-6}$</td>
<td>$2.74 \times 10^6$</td>
</tr>
<tr>
<td>Heated</td>
<td>1.6</td>
<td>1.0</td>
<td>367</td>
<td>243</td>
<td>312</td>
<td>499</td>
<td>$9.07 \times 10^{-6}$</td>
<td>$2.44 \times 10^6$</td>
</tr>
</tbody>
</table>

Table 3.7 shows the difference in thermodynamic and fluid dynamic quantities for the unheated jet case (i.e. the schlieren tests) and the mildly heated scenario (i.e. the PIV experiments). Because the Mach numbers are matched, one can expect the shock structure to be largely invariant to temperature. The Reynolds numbers are slightly different, and, as turbulence is sensitive to the initial conditions, dynamic flow variables likely change. The full implications of these different initial conditions must be thoroughly investigated. As a first inquiry, the pressure signals are compared.
3.4 Acoustic Characterization

Acoustic signals of the cold and hot jet are acquired. In the cold jet case, ten 10-second blocks of data were recorded at 100 kHz. For the hot jet, six 10-second blocks of data were recorded at 100 kHz. The spectral results are shown in Figure 3.15 as a function of Strouhal number, $St = fD_j/U_j$.

Figure 3.15: Sound Pressure Levels (SPL) w.r.t. $p_0 = 20\mu Pa$ taken along the sideline orientation.
Some features in Figure 3.15 are worth discussing. Most notably, there is a high frequency tone in both conditions. In the cold flow, this peak occurs at $St = 3.33$, while heated it is at $St = 3.58$. This tone is stronger and more widely distributed across the polar angles in the cold case. When heated, it diminishes to primarily the 120°microphone, though some resemblance of $St = 3.29$ can be seen at the 105°& 90°microphones. Additionally, two lower-frequency tones emerge at $St = 0.27$ and $St = 1.43$ under elevated temperatures. Overall, the peak frequencies shift to lower values in the heated jet, which is consistent with the findings of Viswanathan [105].

The tonal band, which is most distinguishable in the 120°microphone along $St \sim 3.3$ contributes significantly to the OASPL, and the below-deck data set have an even more pronounced spike at this location. Upon first inspection, this appears like the signature of screech tones that occur in improperly expanded supersonic jets [127]. However, these frequencies are much too high to be considered screech. Compared to axisymmetric jets, neither the frequency range nor spatial distribution suggest screech as shown in section 2.3. The nonlinear fluid phenomenon causing this tone is explored in chapter 4.

OASPLs are obtained by integrating the SPLs over the entire frequency domain. Figure 3.16 shows these values for both jet conditions. For the unheated jet, an extensive acoustic campaign was carried out at the three different orientations shown in Figure 3.10. These are originally reported by Ruscher & Magstadt et al. [199]. The heated jet run was focused primarily on acquiring PIV data, and so only looked at the sideline orientation.

In 3.16a, data from the cold jet are compared against an axisymmetric CD nozzle by Seiner & Ponton [200]. The axisymmetric nozzle was operated at $M_j = 1.46$ while the complex nozzle was run at the design condition $M_{j,1} = 1.6$ and $M_{j,3} = 1.0$. As Seiner’s acoustics were taken at a radial location of $r/D_e = 40$, the Syracuse data are scaled to match the location of the axisymmetric jet. Note that the effective diameter used here is based on exit plane height to match Seiner’s data, not throat.
(a) Cold MARS acoustic distribution scaled to $r/D_e = 40$ for comparison to an axisymmetric jet [200].

(b) Temperature effects on the acoustic field.

Figure 3.16: Overall Sound Pressure Levels (OASPLs) of the MARS jet.
height as used elsewhere in this work. Despite the higher Mach number, the maximum OASPL of the complex nozzle is less than the axisymmetric nozzle. The angle of this maximum OASPL is slightly shifted as well, moving from 160°-155°, which may be a result of the increased Mach number producing steeper Mach waves. At smaller angles, the axisymmetric jet appears generally quieter than the MARS.

The directionality of the MARS's acoustics has some interesting features not observed in axisymmetric jets. First, consider the behavior at the microphone angles closest to the jet axis, from 150°-165°. This area is generally regarded as the cone of coherence [100]. The below-deck orientation is loudest in this region, with the sideline and above-deck data interchangeably claiming the quietest directions. At angles more obtuse with respect to the jet axis (i.e. along 90°-150°), the average OASPL is increased relative to the axisymmetric jet, resulting in a less prominent cone of coherence. The sideline and below-deck directions approximately follow the same unusual trends throughout these smaller angles, while distribution of the above-deck noise appears to more closely match the axisymmetric data. The peculiar peak at the 120° microphone is the result of a high-frequency tone at $St = 3.32$, as revealed by the spectral content.

Sideline microphone data from the heated jet, 3.16b, are compared against the cold jet. These data have not been equivalently scaled like with the cold jet, and represent the acoustic pressures at the microphone location of $r/D_e = 86.6$. The most obvious feature in the heated jet is the increase in OASPL across all microphone angles, again consistent with [105]. The acoustic energy has also been distributed differently in the region of peak emission, with two maxima at 145° & 155°. Spectra indicate an increase in mixing noise at the 145° microphone when heated.
Chapter 4

Gradient of Density Field

4.1 Mean & Instantaneous Schlieren Fields

Schlieren data can provide significant insight into the flow. Although this optical measurement is a powerful technique to extract density variations in compressible flows, one must remember the caveat that is inherently associated with such an instrument: schlieren is a path integral of the density variations. Therefore, the image that is generated is always averaged across the direction normal to the image plane. Two-dimensional flow is thus viewed very clearly, while highly three-dimensional (i.e. turbulent) flow distorts the image. As a final note to orient the reader before considering the data, the vertical orientation of the knife edge means that horizontal gradients are observed. In these experiments, a bright spot indicates streamwise expansion ($\frac{\partial \rho}{\partial x} < 0$), while a dark spot indicates horizontal compression ($\frac{\partial \rho}{\partial x} > 0$).

Schlieren data are acquired at both above/below deck and sideline, shown in Figure 4.1. The sideline, $x - y$ orientation (bottom) is the same as the PIV data, while the jet is rotated 90°to scan through the $x - z$ planes and give the above/below deck perspective. In the sideline view, the deck plate is stainless steel and is instrumented with an array of high-speed pressure transducers (visible in the bottom-left corner). Rotating the jet, the deck plate is interchanged with a polycarbonate piece to view
the flow immediately exiting the jet. (Note on orientation: \( z = 0 \) is the center plane of symmetry, the same location that the PIV data are acquired at.) For these full-window images, the exposure time was set to \( t_{\text{shutter}} = 3.75 \, \mu s \) and the light was pulsed at 50 kHz. Approximately \( 1.2 \times 10^5 \) images were acquired in each direction for these two cases.

![Image](image.png)

Figure 4.1: Snapshot schlieren images showing complex shocks, turbulence, and acoustic propagation.

Considering the \( x - z \) field first, oblique shocks are observed leaving the jet exit in the left of the image, intersecting near \( x/D_h \sim 1.1 \), and traversing to the shear layers, approximately aligned with the edge of the deck plate. Trace reflected shocks can be seen further downstream. However, the flow appears much more turbulent once it has cleared the deck plate. The side shear layers, found at \( z/D_h \sim \pm 1D_h \) are
seen to grow in width, entraining ambient fluid and consisting of a range of coherent structures.

The bottom image of Figure 4.1, the $x-y$ plane, shows the strong density gradients integrated across the $z$-direction. Acoustic waves are clear in the surrounding air, with waves propagating from the nozzle exit (top) and deck plate (bottom). Fine-scale turbulent mixing noise appears at the steepest degree, approximately forming a vector from $(x = 0, y = 0.25D_h)$, angled at $110^\circ$ (recall the downward jet axis is defined as $180^\circ$). Mach wave radiation, acoustic waves generated from supersonically convecting structures, is also seen leaving the top shear layer, emanating at approximately $160^\circ$. Note that the theoretical Mach wave angle, using Equation 2.64, yields $\phi_{peak} = 163^\circ$.

A shock train is clear throughout the core of the jet, with an oblique shock forming off the upper lip and reflecting downstream. This creates regions of compression and expansions, associated with accelerating and decelerating flow, that persist until turbulence dissipates the shock structures. Fine shocks are found below the first oblique shock, and the third stream is observed just above the deck plate as a more disorganized flow.

The mean density variations are computed by averaging over $6.5 \times 10^4$ snapshots. (Note that this method gives results different than via long exposure images, because the light intensity can be maximized in each snapshot to capture finer detail.) The resulting density fields are found in Figure 4.2, and give a thorough representation of the mean density behavior by highlighting stationary features. Viewing both orientations together, one can see normal shocks at $x/D_h \sim 0.6, 1.4 & 2.8$, apparently coincident with the top and bottom shear layers. The oblique shocks also stand out much more, due to their relative stationarity. The angle of the primary oblique shock leaving the upper nozzle lip is calculated at $37^\circ$. The apparent 2D $\rightarrow$ 3D transition of the flow also becomes clear. Above the deck plate, the shocks are very crisp. However, once the flow leaves the deck, the shocks are blurred due to the more turbulent environment in which they reside. One may also take note of the vectored plume again.
Comparing to previous studies,[17], this feature is thought to be a consequence of the SERN. The top, third stream, and deck plate shear layers all grow at different rates, and the latter two appear to merge near $x/D_h \sim 3.5$.

![Figure 4.2: Density gradients, $(\int_{O.A.} \frac{\partial \rho}{\partial z} dz)$, time-averaged over $6.5 \times 10^4$ snapshots.](image)

The discontinuities immediately leaving the nozzle exit plane in the $x-y$ plane of Figure 4.2 are of considerable interest. Recall that a bright band of pixels indicates expansion, and a dark region is compression. However, because the gradient of density is acquired, the images can be slightly misleading. One must remember that the maxima and minima define boundaries rather than the centers of shocks. For example, there are a combination of expansion waves and oblique shocks leaving the nozzle exit. The bright line emanating from the nozzle lip is an expansion wave, and immediately downstream of it is an oblique shock. Just below that, beginning at $y/D_h \sim 0.25$, is a
strong compression shock, generated within the nozzle, which propagates downward in this field of view. Underneath this shock, is a complex arrangement of alternating oblique shocks and expansion waves. Tracing these back and reflecting them off the SERN, they are found to initiate from the splitter plate.

The measured angle of 37°provides vital information, from which additional conclusions may be drawn. Using oblique shock relations, e.g. see Figure 9.9 in Anderson [201], a value of 37°is not permitted in a $M = 1.6$ flow. This angle can only be achieved if the flow is faster than this. It follows that, because the geometry was generated for a $M = 1.6$ flow, the nozzle operates in an overexpanded condition. This means that the discontinuity leaving the nozzle lip must be an oblique shock, not an expansion wave. Figure 4.2 is puzzling then, because of the bright band that indicates an expansion wave. For this to exist, the upstream oblique shock must be stronger than the expansion wave, because there must be an overall increase in pressure from the core to the shear layer in the region leaving an overexpanded nozzle. So the shock traveling from the top of the SERN is quite complex. It actually consists of an oblique shock upstream, a region of locally high pressure, an expansion wave, and what appears to be a thin oblique shock on the downstream side. This multi-shock structure is reflected once off the deck and then once off the top shear layer, though its strength diffuses. Eventually, the shocks coalesce. Leaving the edge of the deck, the jump appears to be exclusively that of compression (i.e. an oblique shock) as one would anticipate. Downstream of this, typical shock reflection behavior of an imperfectly expanded jet is observed, where turbulent mixing diffuses the strengths of the oblique shocks and expansion waves. This description is consistent with LES results generated by Ruscher [199].
4.2 Low-Dimensional Representation

Typically, Proper Orthogonal Decomposition is employed with velocity selected as the field quantity, and the energy analogy is straightforward. However, scalar POD can work equally well on schlieren, and it has been previously demonstrated [202–207]. Instead of maximizing based on the kinetic energy of the flow, this least-squares problem uses the mean-squared value of density. Thus, the modes will not be ordered by energy content, but rather the fluctuating density gradient.

POD is performed on the schlieren data. Select spatial modes are plotted in Figure 4.3 for the $x - z$ orientation and in Figure 4.4 for the $y - z$ orientation. An integral time scale of $T_{int} = 100 \, \mu s$ is calculated via the LES data provided by Ruscher [185], and is used to downsample the images in an effort to obtain statistically independent snapshots.

Modes 1-9 are plotted in 4.3a. Structures appear downstream of the deck plate, starting at $x/D_h > 2$. The first two modes appear to be mostly noise, but also show some shock residuals. These are thought to arise because of a nonstationary mean. Slight fluctuations in the light source intensity cause the entire field to oscillate, which dominate the mean-squared energy ordering. Berry et al. [207] has recently investigated this, and worked to eliminate the noise. Mode 3 shows a column-type mode (similar to the first spatial Fourier mode of an axisymmetric jet[132]). Apparently, the deck plate delays this oscillatory behavior, which could be explained by the emergence of the bottom shear layer. Once both the top and bottom sides of the jet are free, their boundaries can interact with one another. Modes 4-9 transition to smaller structures in the side shear layers. These are thought to be associated with corner vortices and Kelvin-Helmholtz structures in the side shear layers. In each of modes 3-9, a shadow of the shocks is observed. Because POD extracts fluctuating quantities, the shocks must therefore be oscillating, or imparting some fluctuation onto the density field, at some level.

The convergence rate, 4.4b, is plotted on a log-log scale, to illustrate the relative
differences in energy between each mode. This shows that the first two modes are significantly higher than the remainder, and account for approximately 1.5% of the total energy. The convergence rate of the schlieren data is found to be slower than
past experiences with PIV data. Out of 2000 snapshots, the first 100 modes account for 21% of the total energy.

Spectra of the temporal coefficients, 4.3c, indicate broadband frequencies of the lowest modes. Looking at modes 3 - 6, a wide peak is centered around \( St \sim 0.5 \), which shifts to \( St \sim 0.7 \) by mode 6. Modes 7 - 9 show a second peak, occurring at \( St \sim 1.2 \). Linking the spectra to the spatial functions, the argument is made that each of these representations of flow structures behave at these frequencies. For example, the flapping mode (3) and the largest scales of corner vortices (4 & 5) are believed to be pulsing at \( St \sim 0.5 \),

The first spatial mode of the \( x - y \) planes is found in 4.4a. Acoustic waves are immediately apparent in the first two modes, emanating outward from the flow. The POD operation has isolated these features well, from which better propagation vectors can be calculated. Recall that 180° is directly downstream. The shallowest waves are found to propagate at 155°, followed by a second and third set at 148° and 135°, respectively. The steepest vector, which initiates at the nozzle lip rather than the shear layer as the other three directions, is calculated at 120°. Leaving the deck plate, the waves are found to propagate at 235°. Additionally, the first two modes show structures within the flow. A vortex train is formed along the top shear layer, and is tied to the Mach wave radiation. In phase with these structures are a series of pulses along, and downstream of, the first oblique shock. The third stream mixing layer, first seen leaving the nozzle exit at \((x/D_h \sim 0, y/D_h \sim -0.3)\), also shows a series of coherent structures. The influence of the oblique shock on these vortices is also seen, as they are slightly deflected near \( x/D_h \sim 0.8 \) before persisting downstream. Finally, small vortex shedding is found off the deck plate, at \((x/D_h \sim 2.1, y/D_h \sim -0.4)\). Downstream of the deck plate, the flow becomes more turbulent, and POD fails to extract large coherent structures, likely due to a loss of two-dimensionality.

The remaining modes in 4.4a similarly reveal interesting flow physics. Mode 3 highlights the oblique shocks, thus indicating something is fluctuating, either on a
fluid mechanical level, or due to light scattering across the sharp gradients. Modes 4 - 9 have extracted structures downstream of the deck. These organized regions progressively get smaller in size with increasing mode number. Modes 4 & 5 have structures which cross the centerline of the jet and look to be a flapping type mode, whereas 6 - 9 are mostly organized within the shear layers, suggesting corner vortices have been extracted.
The convergence rate of this window, 4.4b has a smoother roll-off than the $x-z$ orientation (4.4b), and also converges faster. Out of 2061 snapshots, the first 100 modes account for 25% of the total energy.

The spectra tie together some of the modes between these two orientations. Considering 4.4c, Mode 3 has content in the $0.3 < St < 0.5$ region, and Modes 4 & 5 have peaks centered around $St = 0.27$. Moving higher, Modes 6 - 9 group together, and show broadband responses focused around $St = 0.60$, with some a second peak arising around $St = 0.80$. Finally, a sharp peak at $St = 1.53$ is evident in almost all of the modes, most notably in the acoustic modes. However, with the microphones identifying content at $St = 3.32$, Figure 3.15, and these data acquired at 50 kHz, the signals are likely aliased. Thus, the window size is reduced by a factor of 2, and the sampling rate doubled. Schlieren is again acquired, this time at 100 kHz, and POD is performed. These results are shown in Figure 4.5.

Figure 4.5 shows very similar content to Figure 4.4, but the windowing effects are apparent. The structures downstream of the deck plate are truncated, and the solution is optimized based on content directly over the deck plate. Acoustic modes are still apparent (1 & 2), but some of these waves have also leaked into other modes. Mode 3 is likely due to light source oscillations because of the increased weight of the ambient field here. And rather than the flapping mode, apparent vortices in the top shear layer dominate Modes 5 - 9. The convergence rate has also increased, as the first 100 modes (of 4500) contribute to 33% of the total energy. Though the field of view has been sacrificed, the appropriate time resolution is now available, and the frequency of interest is resolved in 4.5c. The dominant peak is identified at $St = 3.36$, and it is strongest in the acoustic modes. This is very strong evidence that these representative flow states are linked to acoustic production at $St = 3.32$ (34 kHz). Some common features between them exist, e.g. the vortex train along the 3rd stream shear layer. The mechanism for this acoustic generation is of interest, and is investigated thoroughly in the following section, section 4.3.
(a) Select spatial modes.

(b) Convergence rate.

(c) Spectra of temporal coefficients.

Figure 4.5: POD of 100 kHz schlieren data in the $x - y$ orientation.
4.3 3rd Stream Interactions

With the capabilities of the schlieren setup, sampling rates up to 400 kHz were possible using reduced window sizes. A campaign was carried out at this maximum speed of 400 kHz, with an exposure time of $t_{\text{shutter}} = 0.89\mu s$. This sampling rate is of the same order of magnitude as the Kolmogorov scales in this flow. Five regions of the flow were scrutinized at these speeds: the upper nozzle lip, the exit plane above the deck plate, the reflection point of the first oblique shock on the deck, the reflection point of the second oblique shock along the upper shear layer, and the edge of the deck. Processing these data sets is computationally demanding, and a detailed account of this is given in Magstadt et al. [208]. Considering the first window, the upper nozzle lip, a sequence of snapshots are shown in Figure 4.6.

![Figure 4.6: Close-up schlieren of the upper nozzle lip, taken at 400 kHz.](image)

The short time differences lets one observe the evolution of flow structures. Attention is drawn to the region immediately downstream of the oblique shock leaving the upper nozzle lip. A wave-like pattern can be seen moving left-to-right, as indicated by light & dark bands. Looking at the slow side of the shear layer, regions of expansion appear to move in phase with the waves downstream of the oblique shock. This
suggests the fluid oscillations at the lip, likely a Kelvin-Helmholtz instability, sends disturbances both into the core along the oblique shock and outward as acoustic radiation. The wave in the core is angled slightly, possibly due to the sheared velocity in the core, \( \frac{\partial U}{\partial y} \) (section 5.1). Using the full time record of this window, spectral content can be extracted.

Figure 4.7: Close-up schlieren of the upper nozzle lip, taken at 400 kHz with timeseries extracted at select locations and corresponding spectra computed.

Figure 4.7 shows a snapshot (left) of the 400 kHz data at the nozzle lip. Timeseries, \( 6.5 \times 10^4 \) samples of pixel intensities, are extracted from each snapshot at the locations indicated by the colored markers, and the spectra are then computed (right). The naming convention of the markers is simply used for indexing purposes in data processing. The spectra at all specified locations show the familiar, distinguishable frequency. The dominant peak is calculated at \( St_{max} = 3.33 \), and the subsequent peaks are its associated harmonics. These probes provide further evidence of this mechanism, and its presence is ubiquitous. The signal is found in the shear layer (N = 4160), traveling along the first oblique shock (N = 6054), downstream of the oblique shock (N = 14757), an upstream shock (N = 4122), and in the core of the flow (N = 1543, N = 4100). The ambient fluid can be assumed stationary, and is investigated
with three probes \((N = 4701, N = 11613, N = 9465)\), which show weakened amplitudes relative to the other probes. As these do not capture any fluid dynamics of the jet, the oscillations are due to traveling acoustic waves at \(St_{\text{max}} = 3.33\), which is confirmed by the microphone signals Figure 3.15.

The high frequency of this peak makes structural vibrations and oscillations in electronic equipment lousy candidates. Additionally, resonators were sought, but a similar length scale for this frequency could not be identified in the geometry. The pervasive signal indicates that its source must be upstream in this supersonic flow - the top shear layer alone can not be exclusively responsible. Because the flow is supersonic, upstream propagation of information is not permitted, and transverse communication is limited by the convection speed. For example, there is no way that the marker in the bottom of Figure 4.7 \((N = 1543)\) could be caused from something at the top shear layer \((N = 4160)\). The source must be upstream. Based on the structures in Figure 4.4, an oscillation likely occurs where both the upstream shock and third stream shear layer are previously in contact with one another. Note that the other four windows acquired at 400 kHz give consistent results, but have been omitted here due to their redundancy (see [206]).

POD is performed on these smaller windows. With much of the structures identified through POD of the larger windows, a region is chosen that excludes the acoustic field, but focuses on specific dynamical features of the flow. Therefore, these results will not include the acoustic waves that played such a large role in the larger windows. The reflection of the first oblique shock is selected to investigate. Not only does this window span the majority of the vertical dimension of the core, but some very interesting fluid mechanics occur at this location. The 3\(^{rd}\) stream shear layer interacts with the oblique shocks here, which has significant consequences on the flow. Additionally, a normal shock is formed in the 3\(^{rd}\) stream below the reflection point, which modifies the vortex train downstream. Applying POD to this area, which is rich in shock-turbulence interplay, reveals a great amount of information of this flow,
and the results are shown in Figure 4.8.

![Figure 4.8: POD of 400 kHz schlieren data in the x−y orientation.](image)

(a) Select spatial modes.

(b) Convergence rate.

(c) Spectra of temporal coefficients.

Figure 4.8: POD of 400 kHz schlieren data in the x − y orientation.

The first spatial functions, 4.8a, show primarily structures associated with shocks and wave-like oscillations. Note that there are two sets of oblique shocks. Consulting Figure 4.2, the most upward shock (e.g. highlighted by Mode 2) stems from the upper nozzle lip, while the lower shock (e.g. Modes 1 & 4) appears to be coming from inside the nozzle. Particularly interesting features are the wave-like structures observed in Modes 2, 3 & 5 - 8. The majority of their content is in the expansion region of the core, but they are tied to features elsewhere in the field. In Modes 2 & 3, it is easy to
see their link to the 3rd stream vortex train. Specifically, take note of their periodicity. The jet core waves have nearly double the wavenumber of the structures in the shear layer. Modes 6 - 8 show pulses not only in the jet core, but also within the shocks. This indicates the shocks carry some information. The shock modes, 1 & 4, suggest they are oscillating, but in the direction perpendicular to the shock front rather than transversely as 2, 3 & 8 show. This is thought to be a consequence of the shock’s position fluctuating. Mode 9 shows larger structures. As the schlieren images are from path integrals over z, Mode 9 may be a result of structures at other z locations in the jet. Comparing with PIV results, these appear to be from the formation of corner vortices. Modes 10-12 isolate the vortex train, and show its vectoring away from the deck plate as it passes through the shock.

With the truncated window size, the POD converges much more rapidly (similar to that of PIV data): the first 100 modes, of 2000, contain 46% of the energy, shown in Figure 4.8a. Spectral content, resolved to nearly $St \sim 20$, are also available in Figure 4.8b. Again the dominant frequency is found at $St_{max} = 3.33$. Note that, in contrast to the temporal probes (Figure 4.7), the POD operation has eliminated the harmonics in these lowest modes. The temporal coefficients uncover much more than the familiar $St = 3.3$ signal though. For example, the larger structure size in Mode 9 has a broadband response from $0.4 < St < 1.6$, which can be related to the frequencies found in other diagnostic tools to identify features of the flow section 4.4. Higher spectral content is discussed at the end of this section, in subsection 4.3.2.

4.3.1 Formation of the Dominant Signal

Based on the time-resolved schlieren data, a Kelvin-Helmholtz (K-H) instability is believed to be the mechanism responsible for generating the widespread signal at $St \sim 3.3$. While experimental data is not available inside the nozzle, one can infer what may be happening by considering the downstream flow and the internal geometry. Furthermore, since this data were acquired, high-fidelity LES (Large-Eddy...
Simulations) have been performed [199, 209, 210]. The schlieren data have provided valuable insight, which have partially guided the LES. With their finely resolved data, regions inside the flow have been further analyzed which support this mechanism.

Consider the flow sketch in Figure 4.9. A schematic of the nozzle is given, along with features identified by schlieren data. The multiple shocks seen in the mean density gradients Figure 4.2 and POD modes Figure 4.4 arise from reflected shocks which are formed internally. A splitter plate exists between the core and wall jet, which acts as a bluff body. An expansion fan is formed off each corner (the bottom one is omitted for cleanliness), and a separation/recirculation region exists immediately downstream of the splitter plate. In this wake, alternating vortices are formed, as occurs in bluff body vortex shedding. Further downstream, a recompression shock coalesces at the location where the free shear layers meet [211]. What would be a typical von Kármán vortex street is complicated by the presence of shear across this fluid interface. The schlieren POD results do not give any indication of which direction the vortices are rotating, i.e. whether the vortex train is a series of counter- or co-rotating structures. Stack et al. [210] have recently shown that the alternating vortices are quickly dissipated, and shear dominates downstream of the recompression shock. Therefore, a series of clockwise rotating vortices is present in the third stream shear layer.

The jet Strouhal number of $St = 3.33$ gives a frequency of 34 kHz, which is then used to recalculate a Strouhal number based on the splitter plate dimension. This
value, $St_{\text{splitter}} = 0.26$, is slightly higher than the typical value of 0.20 for a von Kármán vortex street. However, this is not a uniform wake, as shear is present between the two stream. Given the compressibility of this flow, unequal pressures and densities of the merging streams, and reported Strouhal numbers of $St = 0.4$ for large-scale structures in supersonic shear layers [212], $St_{\text{splitter}} = 0.26$ is well within a reasonable range.

Initiating from the K-H instability, a disturbance propagates along the recompression shock to the top boundary layer in the SERN. From here it splits and travels to the top shear layer, and is reflected down along the next shock. Traveling along these propagation paths (i.e. the recompression shock), a pulse excites the flow as fluid is convected through the shock. Kan et al. [213] have shown strong evidence from LES data that supports this propagation mechanism. Because shear layers are unstable, they are susceptible to a range of inputs. Therefore, the K-H instability in the top shear layer is likely amplified by the $St = 3.33$ signal. Vortex formation, from which acoustic production follows, occurs at this Strouhal number because of the energetic input oscillations, and the signal spreads throughout the domain.

### 4.3.2 High Spectral Content

The ability to acquire long time records at 400 kHz provides excellent spectral resolution of this flow. The 400 kHz data over the first shock reflection (Figure 4.8) are again looked at, but this time information beyond the K-H instability frequency are investigated. Figure 4.10 shows these results.

Selected spectra of the temporal coefficients are plotted in Figure 4.10. Starting at Mode 10 and increasing by increments of 10, some of the first 100 modes are looked at. Experience shows that increasing the POD mode number generally leads to higher frequency content in the spectra, which is why the range of temporal coefficients has been increased here. Note that the modes in 4.8c have largely filtered frequencies above the dominant 34 kHz signal. Mode 10, however, shows a second spike at
Figure 4.10: High spectral content of POD coefficients over the first shock reflection.

$St = 3.69$ before decaying. The higher modes also illustrate the emergence of the first harmonic, at $St = 6.68$. Mode 20 is fairly broadband around this, but as mode number increases the peak becomes sharper. Second and third harmonics can even be identified at $St = 9.91$ and $St = 13.4$.

Two features of Figure 4.10 are significant findings. First, LES data [210] have shown the presence of structures in the origin of the 3rd stream shear layer oscillating at the first harmonic, $St \sim 6.6$. The fact that the experimental and computational studies are identifying the same harmonic at these very high frequencies gives excellent confidence in the data. This subtle fluid phenomenon exists as a fine scale of turbulence, but it has large implications on the flow given its relation to the formation of the dominant 34 kHz signal. Without the close communication between the experiments and simulations, this event would likely have been overlooked. Second, and perhaps more profound, is the identification of the second peak, at $St = 3.69$. As will be seen in the case of the heated jet, the dominant frequency in the flow and acoustics (which are identified here as a product of the 3rd stream instability) shifts to $3.6 < St_{heated} < 3.7$. Because its signature has been located in the cold jet means that a similar flow event occurs in both cases. What gets damped out in the cold jet
is apparently allowed to grow with the presence of heat, which enables its formation to dominate the lower $St = 3.33$.

### 4.4 Deck Pressures

Deck pressures are acquired on the cold jet through Kulite probes. During the schlieren test campaign, these data were acquired at 50 kHz due to software technicalities. After the schlieren and acoustic data revealed the strong 34 kHz signal, the Kulite signals were realized to be undersampled. The software was then rewritten and upgraded to acquire at 100 kHz. These spectra, which use $3 \times 10^7$ samples, are shown in Figure 4.11.

The responses of the pressure probes, whose locations are given in Figure 3.9, are shown in Figure 4.11. Note that the y-locations are arbitrary, as the data are repositioned to more clearly show the different frequencies captured by the probes. Again, the well-known frequency shows up in all of the spectra, but this dominant signal is found to be slightly lower than the schlieren and acoustics, at $St = 3.26$. A specific explanation for this is currently unknown, but it is thought to be associated...
with the difference in temperature between the core and ambient air. While discussing this signal once again may seem insistent, the identification via another independent instrument is important. Three separate measurements have located this frequency, giving high credibility to its existence and reach.

The behavior of each probe is discussed. Kulite A, located at the most downstream location, shows a steady decay of scales. The inertial subrange, the scales at which dissipation balances energy flux, is characterized by $k^{-5/3}$ and is marked in the upper right of Figure 4.11. Kulite A appears to approach this slope, which suggests Kolmogorov turbulence, and thus a more homogeneous field. The other probes have shallower slopes, indicating a downwarnd energy transfer to the viscous scales. The presence of the K-H instability is also weakest in this transducer. Kulites C, D & G are located downstream of, centered in, and at the initiation of the normal shock in the third stream, respectively. These show similar spectra. However, note the creation of a tone at $St = 0.26$ as moving downstream. Kulite G has no signature, a small emergence shows in D, and it is well-defined in C. The side shear layers are probed with Kulites H & E, where E is downstream of H. Relatively wide peaks are centered around $St = 0.33$ & $St = 0.38$ in H & E, which are thought to represent corner vortices. A minor presence of the $St = 0.26$ is also found in these probes. Finally, the strength of the 34 kHz signal is strongest inside the nozzle, at Kulite F, which agrees with the formation of the K-H instability. Evidence of events at $St = 0.40$ is also present in this probe.

### 4.4.1 Density-Pressure Relations

The synchronized data taken during the schlieren campaign allows for the computation of cross-correlations between signals to perform stochastic estimation (**Equation 2.27** - **Equation 2.34**). Because of the large datasets, calculating cross-correlations in the time domain is computationally expensive. Thus, the results presented here are limited to the 50 kHz schlieren data, as spectral computations require matching sampling
Figure 4.12: Cross-correlations between the 50 kHz pressure signals of Kulite F and the first nine POD coefficients of the 50 kHz schlieren data in the $x - y$ orientation.

The seven Kulites are cross-correlated with the time-dependent coefficients of the schlieren POD, as seen in Figure 4.4. Significant correlations exist at all of the Kulites and will be discussed shortly. However, only the detailed results from Kulite F are shown in Figure 4.12. Here, the first nine modes are checked for correlations over a range of time lags, $\tau$.

Figure 4.12 shows an increase in activity between the signals around 1.25 - 1.5 ms, the average convection time from Kulite F to a region just beyond the deck plate. Modes 4 - 7 are the most identifiable, 8 & 9 have visible (though not necessarily strong) correlations, while 1 - 3 show essentially zero correlation. Referring back to 4.4a, Modes 1 & 2 are associated with the $St = 3.33$ K-H instability, which occurs at 34 kHz. Because the transducer was acquired at 50 kHz, the Nyquist theorem prevents identification of this signal. Thus, it is no surprise that these modes do not correlate.

The correlation with Modes 4 - 7 is interesting: the large structures related with the flapping mode apparently have a connection with the flow inside the nozzle. Their
Figure 4.13: Map of maximum correlation peaks as a function of Kulite position and modal coefficient.

spectra, 4.4c, show the strongest frequency content around $0.27 > St > 0.6$. This Kulite spectrum, Figure 4.11, also shows a weak signature at $St = 0.40$. Remarkably, a low-frequency disturbance is generated in the nozzle and grows into the flapping mode. This same discussion directly extends to Modes 8 & 9, which have slightly smaller structures and weaker correlations.

The same analysis is performed across all of the Kulites, and the first 30 POD modes are considered. As this generates a large amount of data, the maximum correlation values are retained from each cross-correlation time series and then plotted as a 3-D bar graph, which is a function of modal coefficient and Kulite location. The results are given in Figure 4.13.

The map of correlation coefficients, Figure 4.13, allows one to quickly identify which Kulites correlate best with particular modes. The data from Figure 4.12 have been collapsed to a horizontal line across the Kulite location F, which shows the peaks associated with modes 4 & 5. Note these peaks are relatively weak when compared to some other modes. Kulite A, the transducer located near the edge of the deck
plate, has the strongest relation to the POD coefficients. This is likely because the propagation distance from Kulite A to the majority of modal activity (observed to occur downstream of the plate) is the shortest here. The shorter distance implies a greater possibility for efficient information transfer. Additional factors may contribute to this as well, such as this sensor’s proximity to the reflecting shocks. Perhaps much information is transferred along these paths. Kulites C & D, the next two upstream probes, show similar behavior with weakened cross-correlations. The side shear layer Kulites, E & H, appear to have stronger ties to the smaller structures in Modes 8 & 9, which are hypothesized to represent corner vortices. Considering the spectra, these corner vortices likely oscillate around $St = 0.35$ (3.5 kHz). Kulite G shows little correlation to any of the modal coefficients. Given its location just upstream of the normal shock, one may infer that the shock is destroying any information between it and the downstream flow. Across all Kulites, zero activity is observed for the K-H instability modes, 1 & 2, due to undersampling.
Chapter 5

Velocity Field

The PIV campaign acquired data at five cross-plane orientations and 11 stream-wise orientations, as given in Table 3.6. As discussed in section 3.3, PIV requires a mild temperature increase, so the jet has been heated to \(NTR_1 = 1.23\) for these experiments. At each of the locations scanned, 3-component velocity vectors are acquired. A minimum of 2400 snapshots are captured at each of the locations shown in Table 3.6. In addition to these planes, five more streamwise plane are acquired downstream with a field adjusted to \(4 < x/D_h < 8\). These are scanned at \(z/D_h = -1, -0.5, 0, 0.5, 1\), with a minimum of 800 snapshots at each location, and were acquired simply to provide overlapping statistics with the cross-plane measurements. Examples of the vector fields are shown in Figure 5.1 and Figure 5.2.

The three-dimensional instantaneous velocity field from the most downstream location, \(x/D_h = 8\), is given in Figure 5.1. The \(x\)–axis has been reversed to match the right-handed coordinate system of the jet. The \(u\)–component is plotted as the contour field, and the velocity vectors use the \(v\) & \(w\) components. The vector field has been downsampled by a factor of 2 in each direction to more clearly view the data. From this instance in time, one recognizes the approximate center of the jet is not positioned at the \(y_0\) location, consistent with the plume-vectoring observation of the schlieren data Though structure identification is qualitative with one snapshot, a wide
range of scales is also apparent. Low-velocity structures exist as far as \( z/D_h < -2 \). Higher-speed eddies, found in the center of the jet, appear to be as large as \( 1D_h \), and decrease to the resolution of the image.

One of the streamwise planes, \( z/D_h = 1 \), is exemplified in Figure 5.2. This plane sits just beyond the edge of the nozzle walls, which are located at \( z/D_h = \pm 0.92 \). Velocity vectors are \( u - v \) and the scalar map is the out-of-plane component, \( w \). The flow nearest to the exit plane is nearly zero, as PIV here captures the ambient fluid beginning to be entrained. The top shear layer grows rapidly along this location as the upper corner vortices take shape. The majority of structures in the upper shear layer have a positive \( w \)-component, indicating that fluid moves outward (seen from \( 0.5 < x/D_h < 4 \) and \( 0.1 < y/D_h < 0.5 \)). Near \( y = 0 \), the majority of structures have a negative component. Downstream of the deck plate, (\( y/D_h \sim -0.25 \)), the \( w \)-components return positive. These observations suggest that the side shear layers

Figure 5.1: Instantaneous velocity taken at \( x/D_h = 8 \).
pull fluid inward along the centerline of the jet, and eject fluid along the upper and lower corners. As will be seen in the average velocities, specifically the cross-planes of the streamlines Figure 5.7, this is the case, and in fact part of the corner vortices’ formation. Scales in this field of view are seen to increase in size with downstream direction, as the side shear layer grows more turbulent. Around $x/D_h \sim 3$, two vortices can be see in the core of the jet, which have large out-of-plane components.

Statistics provide a quantitative approach to evaluating the turbulence and evolution of the jet. Data are first substantiated using a correlation-peak validation method and a universal outlier detection algorithm, followed by a $N - \sigma$ validation filter, which rejects vectors outside of the $N$ limit. This results in gappy data, where rejected vectors have been removed from the field at certain snapshots. Statistics are computed from these vector fields by ignoring the rejected data points, which requires extensive bookkeeping for the higher-order moments. Once computed, the statistical variables from each plane are then interpolated onto a volumetric grid and probed at locations of interest, to give insight into the development of this complex flow. An intricate jet structure is revealed by the different statistical variables. First, single-
point statistics are considered. Second, the spatial resolution of the PIV is exploited to compute multi-point quantities.

5.1 Single-Point Statistics

5.1.1 1st Order Moments

Combining the data from each plane, the jet can be viewed volumetrically. Data from streamwise campaign over the plate, given in Table 3.6, are mirrored about the center plane, \( z/D_h = 0 \). The five downstream planes are not mirrored and used to populate the volumetric grid as is. Statistical variables are fit to multiple hypersurfaces, \( v_i = f(x, y, z) \), which pass through the points from each PIV plane. Data are then linearly interpolated using a Delaunay triangulation scheme onto a uniformly spaced grid. The mean \( u \)-component of velocity is first considered, given as exit Mach number, and is shown in Figure 5.3.

![Figure 5.3](image)

Figure 5.3: Mean velocity field. Isosurfaces represent a 10% and 90% maximum velocity threshold.

The evolving structure of the jet’s velocity is seen in Figure 5.3. The contour slice is along the plane of symmetry and shows Mach number. The isosurfaces are defined using a 10% (blue) and 90% threshold (red) to illustrate the bounds of the
mixing layers. The minor rib-like structures in the outer isosurface are a result of measurement uncertainty, as the hypersurface bends to match the data acquired at the different planes. Additionally, note that the Mach number ranges from $M = 0$ to $M = 1.7$. A discussion on this range is first discussed before moving to the complex field of the jet.

The measured exit velocity is higher than what is calculated based on the one-dimensional isentropic equations Equation 3.2 - Equation 3.4. At Mach 1.7, target conditions are $U_{j,\text{ideal}} = 520\text{ m/s}$, but acquired data calculate the maximum velocity to be $U_{j,\text{exp}} = 545\text{ m/s}$. Due to the large number of samples ($N = 2400$), the uncertainty on the mean values are approximately $10\text{ m/s}$ Appendix A. Additionally, LES experiments of the jet at matched conditions find similar results, with $U_{j,\text{LES}} = 550\text{ m/s}$. Thus, the discrepancy is not due to experimental uncertainty. The disagreement in expected value is likely due to the assumptions made with the one-dimensional equations. There are clearly three-dimensional effects. While one may argue that the $z$-direction can be neglected along the plane of symmetry, certainly the $y$-component cannot be ignored. The introduction of the third stream complicates the problem, such that we can no longer assume one-dimensional flow. Because the third stream is at a slower velocity, its effect is to act as a reduced throat area. (In addition there is the finite thickness of the splitter plate which has zero velocity). The ‘effective’ throat area is reduced, and the flow therefore expands more which accelerates the flow to a greater speed.

The reduced effective throat area helps explain two other observations in the flow. Because the $NPR$ and exit area are fixed, and the nozzle designed using the method of characteristics, overexpansion of the flow should exist. Shock diamonds only exist when the flow has been under- or over-expanded. As their presence is noted in Figure 5.3 by the islands of isosurfaces in the core downstream, the flow does not undergo ideal expansion. A reduced throat area results in overexpansion, which forms the shock diamonds. This is somewhat of a coarse argument; in actuality,
the overexpansion appears to have manifested along the recompression shock within the nozzle, and the shock train is a series of prism-shaped regions, rather than the traditional diamond. Secondly, LES data [209] have shown a small region of separation along the SERN ramp near the exit. If a flow has been expanded too much, it will separate from the wall. With the impinging recompression shock, a large adverse pressure gradient could possibly exist locally along the wall, which may then trigger separation.

As a final note on the matter of observed exit velocity, the PIV results extend to the case of the cold jet. The one-dimensional isentropic estimates of the exit velocity for the unheated flow are inaccurate as well. Revisiting computational data performed by Ruscher and provided in Ruscher & Magstadt et al. [199], the maximum exit velocity is found to be $U_{j,\text{LES}} = 500$ m/s, roughly 10% higher than predicted. This is consistent with discrepancy in the heated jet.

The outer jet structure is discussed by considering the mean flow behavior in Figure 5.3. The flow leaving the jet travels over the deck plate until approximately $x/D = 2$. The shape remains largely rectangular over the deck, and then rapidly changes once convecting beyond the edge. This is a result of the asymmetric entrainment, as the deck plate restricts ambient mixing along the bottom of the jet which would normally contribute to the formation of corner vortices. Conversely, the top shear layer is more along the jet corners; this can be partially seen in Figure 5.3 as the outer boundary departing upward while still over the deck plate. Considering the lobe-shaped end profile, the jet clearly transitions from the original rectangular area in a complex manner. The top half of the downstream profile has spread more than the bottom, because the corner vortices have not been obstructed and thus allowed to grow more.

The jet core, again referring to Figure 5.3, has a complex topology. Note the conventional term ‘potential core’ is avoided here; the schlieren results have revealed significantly turbulent structures throughout the core, so it cannot be treated as po-
potential flow. The series of oblique shocks sets up regions of acceleration and deceleration throughout the core. The three-dimensionality of these regions is also apparent. Zooming in on the core region reveals this shape even more.

![Figure 5.4: Mean velocity field over the deck plate, showing shock cells. Isosurfaces represent a 50%, 90%, & 95% maximum velocity threshold.](image)

In Figure 5.4, the camera angle is from underneath, to give a better view of the shock behavior. The three isosurfaces are defined at 50%, 90%, & 95% of the maximum velocity. The fastest regions, identified by the orange and yellow surfaces, indicate pockets of acceleration, which are bounded by the oblique shocks. Immediately leaving the nozzle, the flow is fastest near the deck plate. Interestingly, the region also extends upward slightly and through the primary oblique shock leaving the nozzle lip, from $0.5 < x/D < 1$. Shocks leaving the side walls of the jet (see Figure 4.2) confine this fastest region inward until meeting at $x/D \sim 1$. Beyond these first oblique shocks, the flow decelerates, indicated by the volume enclosed in blue. Upon reflection of the primary oblique shock with the top shear layer and the side-wall shocks merging in the plane of symmetry, which both occur at nearly the same downstream location of $x/D \sim 1.3$, the flow is again accelerated. This is the pocket situated above the end of the deck plate, at $x/D \sim 2$. The process continues downstream, until the shear layers
of the jet have grown enough that mixing has occurred and the pressures equalize.

More quantitative results are presented as velocity profiles. One-dimensional ex-
tracts of the mean velocities are shown in Figure 5.5. The $y$—dependency is explored
in two directions. First, the streamwise direction, $x$, is varied along the plane of sym-
metry. Second, the horizontal direction, $z$, is varied at a fixed streamwise location of
$x/D_h = 1$.

Figure 5.5: Variation of the mean $u$—component of velocity at select locations.
The velocity profiles change significantly with position. Consider the variation in the downstream direction, in 5.5a. Immediately leaving the jet, at \( x/D = 0 \), the jet has somewhat of a top-hat profile. The flatness along the center is instead curved due to the accelerating flow in the core. Also note the presence of the third stream shear layer, as indicated by the shallower climb from \( M = 1 \) to the core velocity. As one moves downstream, the primary oblique shock is encountered, which is identified by the inflection point in the \( x/D = 1 \) curve. At the edge of the deck plate, \( x/D = 2 \), the third stream shear layer has actually accelerated beyond the velocity of the core. This is only possible through a region of expansion within the shock train. Beyond the deck plate, the center of the jet is observed to move upward, and the third stream shear layer is mixed with the core. Deficits in the core at different locations are thought to be associated with regions of compression in the shock train. By the end of the interrogation region, \( x/D = 8 \), the profile appears nearly symmetric about its offset center. The jet is still clearly supersonic here, despite the shear layers merging.

The streamwise velocity dependence of the horizontal direction, 5.5b, demonstrates the asymmetric development of the jet. The data are taken at \( x/D_h = 1 \), so the centerline plot, \( z/D = 0 \), matches the curve of \( x/D = 1 \) in 5.5a and again shows the third stream shear layer and oblique shock. Stepping away from the center, to \( z/D_h = 0.25 \), the deficit in the third stream shear layer is smaller, and the core approaches a more uniform velocity. This effect is more pronounced at \( z/D_h = 0.5 \). By \( z/D_h = 0.75 \), the presence of the side shear layer has become clear, as the maximum velocity has reduced slightly. The outer-most position, \( z/D_h = 1 \), is especially informative because it sits just beyond the horizontal boundaries of the jet. This location shows the asymmetric entrainment of the jet. Along the top of the jet, the flow is not restricted and ambient fluid has been drawn in beneath it. The rotating fluid results in corner vortices. The sharp peak at \( y/D_h \sim 0.25 \) uncovers this, as the PIV plane has sliced through it. Along the bottom half, very little momentum transfer has taken place outside of the jet, and the velocity is nearly zero. Blockage from the deck
restricts the formation of corner vortices, and fluid is entrained much more slowly. However, the inflection point in this curve is interesting. As this location is outside of the jet, a shock is not possible. It shows that, despite the deck plate, momentum is still transferred from the jet to the ambient fluid. Thus there is a structure (or are multiple structures) here contributing to the increased velocity (e.g. Kelvin-Helmholtz vortices). Their description is unknown at this point, but they certainly contribute to the final dual-lobed profile of the jet shown in Figure 5.3.

The $v-$component of velocity is next considered, shown in Figure 5.6. With the velocities an order of magnitude smaller than the $u-$component, the shocks are easily identified. Regarding the plane of symmetry, 5.6a, the well-known oblique shock structure over the deck plate is observed. As the velocities presented are in the vertical direction, the pressure gradients driving the flow in this dimension are also in the $y-$direction. There appears to be one significant region of downward flow, upstream of the first reflected shock from $0.5 < x/D_h < 1.25$. Downstream of the deck plate, the shock cells continuously drive the flow upward. While this may seem counterintuitive, as a balance in velocities might be expected, it is consistent with the observation that the plume vectors upward. At first, $x/d \sim 3$, vertical flow occurs in regions outside of the shock. However, downstream of this the shocks cannot be clearly identified, and pockets of high vertical velocity are found mostly in the top shear layer. By the end of the domain, the vertical velocity has decayed. Nonetheless, evidence is still found that the flow has not fully equalized to ambient pressure because regions of mean acceleration exist.

Knowing the three-dimensionality of the jet, another $z-$location is briefly investigated, via the contour plot in 5.6b. As opposed to the plane of symmetry, the region of downwash along the first reflected shock is missing in this plane. This is because the side oblique shocks have moved inward at this location (i.e. they are confined to $-0.5 < z/D_h < 0.5$). The shock off the deck plate induces a higher velocity at this plane than along the plane of symmetry, but downstream the effects are diminished.
Additionally, the spatial distribution of the vertical velocity in the plume is more complex than the center plane, which is clearly due to the three-dimensionality of the jet.

The three-dimensional structure is considered by looking at the development of the corner vortices. Streamlines are computed at multiple cross planes (i.e. \( z - y \) planes) using the average transverse velocities, \( V, W \). These highlight vortex cores, and the evolution is observed with downstream propagation by selecting planes in \( 1D_h \) increments. Figure 5.7 shows these streamlines, overlaying streamwise Mach number.

Beginning at \( x/D_h = 1 \), in Figure 5.7, corner vortices in the upper shear layer are
Figure 5.7: Streamlines generated from the $V, W$ components, overlaying streamwise Mach number.
prominent. These grow in size by \( x/D_h = 2 \), but note that none are present along the bottom of the jet. The deck plate inhibits entrainment here, which prevents the formation of these vortices. At \( x/D_h = 3 \), bottom corner vortices have strongly developed. These views clearly show the entrained fluid through the side walls of the jet, which was suggested previously by streamwise instantaneous velocity, Figure 5.2. (Note the wing-like shapes of the streamwise Mach number in the upper corner vortices at this location are a result of ill-conditioned hypersurfaces because data have been extrapolated here.) By \( x/D_h = 4 \), the top vortices are seen to have spread more than bottom, as they have had an additional \( 2D_h \) to develop over the deck plate.

In \( x/D_h = 5 \), a secondary vortex is apparent between the primary ones, located at \( z/D_h = -0.6 \), \( y/D_h = -0.2 \). The lack of symmetry here is believed to be a result of a slight Coanda-like effect in the chamber, and unevenly seeded streamlines. However, corner vortices are also present in the other side of the jet, which will be seen in Figure 5.9. Moving downstream, the flow becomes less organized, though the vortices can still be identified. In \( x/D_h = 7 \) the bottom primary vortices have moved toward one another, and secondary vortices in the upper shear layer have formed. By the final location, \( x/D_h = 8 \), the jet has spread significantly, and regions of lateral entrainment and ejection can be identified. Because the jet is pulled toward the chamber wall (the \(-z\) direction), the streamlines are slightly biased, which do not clearly pick up vortices in the \(+z\) direction. Computing the vorticity, plotted in Figure 5.9, captures these structures.

Using the shock angles, validity of the PIV measurements can be further assessed with 2-D gas dynamic theory [201]. The so-called \( \theta - \beta - M \) equation is given by

\[
\tan(\theta) = 2 \cot(\beta) \frac{M^2 \sin^2(\beta) - 1}{M^2 (\gamma + \cos(2\beta) + 2)}, \tag{5.1}
\]

where \( \theta \) is the deflection angle of a streamline passing through an oblique shock and \( \beta \) is the shock angle. The shock locations are defined by the maximum velocity gradients. In this case, \( \frac{\partial V}{\partial y} \) gives the best results. The contour of this is given in
Figure 5.8: Shock locations, defined by the velocity gradient, with streamlines overlaid.

Streamlines overlay the velocity gradient in Figure 5.8, and are used to calculate the deflection angle of the flow. With a value of $M = 1.74$ and a shock angle calculated at $37^\circ$ (in excellent agreement with the schlieren results), a theoretical deflection angle is determined to be $\theta = 2.2^\circ$. Due to the complexity of this primary shock (see section 4.1) and the resolution of the PIV grid, significant uncertainty exists in measuring the deflection angle. Results range from 1-6° over the length of the shock. Nonetheless, a best estimate of 3° is found if one restricts the change in streamline direction to a domain around the center of the shock based on a minimum gradient threshold. While a rigorous uncertainty analysis of this has not been performed (due to the complexity in the PIV measurements), the closeness of these values gives further confidence in the velocity data. More in-depth analyses are next considered.

The PIV data acquired downstream provides detailed resolution of the scalar derivatives in the cross-plane orientation. Vorticity, $\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$, is calculated at each of the cross-planes. This is shown in Figure 5.9.
Contour lines of vorticity are presented in Figure 5.9. Considering the first plane, \( x/D = 4 \), four pairs of vortices are found, symmetric about the center plane. The strongest set are the second pair from the top. These are presumably the corner vortices observed forming over the deck, as their rotation would pull flow upward from the jet and back in through the side shear layers. The corner vortices along the bottom of the jet are the set just below this, rotating in the opposite direction. The center of these loosely form a rectangle, with an aspect ratio similar same as the jet. From these primary sets, counter-rotating, secondary vortices are formed. Moving downstream, the 4 pairs of vortices weaken. The rectangular shape also dissipates, as the jet transitions to a more axisymmetric-like jet. Minor asymmetry about the center plane is thought to be a consequence of unequal entrainment in the chamber. Because the one side of the jet (i.e. the +z axis) is closer to the wall than the other, a Coandă-like effect occurs. The jet is pulled closer to this wall, and the acquired PIV cross-planes are thus a slight projection of the truly perpendicular planes. This effect becomes more apparent by considering the higher-order statistics.
5.1.2 2nd Order Moments

With the structure of the velocity field represented by mean statistics, the second order moments are next considered to gain a better understanding of the energy distribution. The variance and covariance of the velocity field are considered. These are in fact the six Reynolds stresses, given by $R_{ij}$ in Equation 2.25. The three normal stresses, $R_{ii}$, are presented in Figure 5.10.

In Figure 5.10, the contours of each component are plotted along the plane of symmetry, $z = 0$, and at the five downstream cross-plane PIV locations, $x/D_h = 4, 5, 6, 7, 8$. While data is available upstream of these locations, they were taken only by streamwise measurements. Their distributions therefore rely largely on interpolation from a relatively coarse grid, and so confidence in the accuracy of the transverse gradients is not as high. The magnitudes are normalized by the jet’s ideal exit velocity. This normalization is performed to show what percentage of the total energy in the flow is contained in each of these components. The coloring of the contour levels of each variable are calculated based on the maximum and minimum values of that variable in the entire flow volume. Thus, the spectrum may not appear fully utilized, due to the plane of symmetry possibly not intersecting with the maxima.

As expected, the streamwise normal component, $\overline{uu}$, given in 5.10a, is the most energetic component of the Reynolds stresses. The shear layers are observed to grow more turbulent with downstream distance. The top mixing layer has the greatest velocity difference across it and the most distance to develop, so it has the highest stresses. The bottom shear layer does not initiate until after the deck plate, at $x/D_h = 2$, and is slightly slower due to the presence of the third stream. The third stream mixing layer can be seen right above the deck plate as a mild increase in $\overline{uu}$. Remarkably, this stream can be identified apart from the bottom shear layer until about the first cross-plane, beyond which it has merged with the bottom shear layer. Subtle regions of increased activity in the top shear layer, at $x/D_h \sim 1$ and $x/D_h \sim 3$, appear to approximately coincide with reflection points of the shock train. Down-
Figure 5.10: Reynolds normal stresses, $R_{ij}\delta_{ij}$.
stream, a wavelike pattern in the amplitude is apparent for both the top and bottom shear layers, which appears to be linked to the shock cells observed in Figure 5.3.

The cross-plane distributions of 5.10a reveal specific quadrants of high stress. In the side shear layers, $x/D_h = \pm 0.5$, areas of intense velocity variation exist. At $x/d_h = 4$, these precisely coincide with the vortex cores identified in Figure 5.9. However, with downstream propagation, the vortices migrate outward. Conversely, the strong regions of $uu$, as seen in $x/D_h = 5, 6, 7$, work inward with downstream direction. By $x/d_h = 8$, these have merged, and the most intense areas are essentially four quadrants along the $y$ and $z$ axes. This process clearly shows the inhomogeneity of the jet.

The variance of $v$, found in 5.10b, similarly is found to increase with $x$. However, the magnitudes of this vertical component are approximately half of what were found in the $uu$ component. This is the first evidence that the turbulence field of the jet is anisotropic, but the theme is a reoccurring one. The transverse distribution of this component is concentrated in two vertical lobes on horizontal sides of the symmetry plane. The third stream shear layer can again be identified above the deck plate, and trace outlines of the oblique shocks from $0.5 < x/D_h < 4$ are observable. The maximum value of $vv$ is found in the upper corners of the jet at $x/D_h = 4$, which then decays with downstream direction.

The $vv$ component of the stress tensor indicates a discrepancy in the data $x/D_h = 5$ that previously wasn’t noticed. Vertically skewing of the isolines is spaced by $z = 0.5D_h$, which is the grid resolution of the streamwise scans. Upon further investigation, the values of $vv$ are found to be slightly lower at this plane than neighboring regions. Thus, the contorted curves are a result of the interpolation scheme. But the question remains: why are $vv$ reportedly lower at this plane? A worst-case scenario of poor seeding conditions is thought to be the reason. Due to the significantly lower displacement vectors in the vertical direction of the CCD chip on the camera, the velocity calculation in the $y$ direction relies more on subpixel refinement, which has
a higher uncertainty. While the mean variables mask this, the effect becomes more apparent in the second moments (e.g. $\overline{vv}$). Note that $x/D_h = 7$ also exhibits similar, though much reduced, behavior, again due to a minor seeding issue. For more on the uncertainty of the PIV measurements, see Appendix A.

Finally, consider the horizontal normal component of the Reynolds stress, $\overline{ww}$, given in $5.10c$. Again, the jet is seen becoming more turbulent with downstream distance. The magnitude of this horizontal component is of the same order as the vertical component. However, note the growth rates of the horizontal normal stress relative to the vertical component along the plane of symmetry. $\overline{ww}$ doesn’t merge as quickly as $\overline{vv}$. This may seem trivial, but consider the rectangular origin of this flow. The top shear layers is strictly horizontal leaving the exit, so the structures generated immediately downstream of the nozzle lip are vortex sheets extending in the $z$ direction. Before these break up, the dominant fluctuation is in the $y$—direction, with little variation in $z$. Though the structures are entirely three-dimensional by $x/D_h = 8$, the flow still has a memory of this predominantly two-dimensional shear $(\partial u/\partial y)$, as there is a fuller profile of $\overline{vv}$ than $\overline{ww}$. The symmetry plane is one example that shows the complex development of this turbulent field. Looking at the off-diagonal components of the Reynolds stress tensor, shown in Figure 5.11 provide a deeper look at this inhomogeneous, anisotropic flow-field.

Figure 5.11 shows the transverse planes acquired by the cross-plane PIV measurements. The streamwise plane of symmetry is omitted here because it is uninformative. With the exception of $\overline{uv}$, the values are universally zero along it. Contours are colored along each of the five planes, $x/D_h = 4, 5, 6, 7, 8$, and the values have again been normalized by the jet exit velocity. The $\overline{uv}$ & $\overline{ww}$ components have similar energy magnitudes, while $\overline{vw}$ is significantly lower.

In $5.11a$, the shear stress in the $x – y$ direction, $R_{uv}$, is plotted. $x/D_h = 5$ again shows a loss of correlation (because of the $v$ component), but its shape is consistent with the other planes’. The top and bottom shear layers are dominant, with additional
Figure 5.11: Reynolds shear stresses, $R_{ij}, i \neq j$.
structures in the side shear layers. In \( x/D_h = 4 \), the regions of maximum stress coincide with the vortices, similar to the \( \overline{uw} \) component. Moving downstream, these regions of maximum shear move radially inward, again in the opposite direction of the vortex cores. The top and bottom zones merge inward with the shear layers, but activity in the side walls of the jet pinch it inward, giving a ‘v’ and ‘^’ shape. In \( x/D_h = 8 \), the center of the jet, seen here as the zero shear region in the core, can be seen slightly shifted upwards, consistent with the plume vectoring previously observed. Finally, note that the shear stress in the top half of the jet is approximately 30\% greater than what is measured in the bottom. This is likely a consequence of the increased velocity difference across the top shear layer relative to the bottom one.

5.11b shows the evolution of the shear stress in \( x-z \) direction. Two lobes are apparent at each slice of data, confined to the side walls of the jet. The strength of the \( +z \) lobe is less than the \( -z \) lobe, because the jet has reduced entrainment in this direction due to the presence of the chamber wall, i.e. the Coandă effect. This covariance term is moderately similar to \( R_{uv} \), however rotated 90\° about \( x \). Notice the resemblance as the regions grow inward in the same manner as \( R_{uv} \). However, by \( x/D_h = 8 \), they have not merged as closely simply because side shear layers are spaced further apart.

The final Reynolds stress component, \( R_{vw} \), is shown in 5.11c. This component represents shear in the transverse direction of the jet, which does not have a streamwise component. Hence, its magnitude is much less, as the turbulence in the near-field of the jet is still anisotropic. As fluid convects downstream, one observes the increase in this component. This demonstrates the decaying anisotropy of the turbulence, as energy is transferred from the mean \( u \) component into the transverse components, \( v \) \& \( w \). In contrast to all other terms, the covariance of \( vw \) has a stronger presence in the bottom half of the jet than the top half.

Finally, the total turbulent kinetic energy (TKE) is considered. The TKE, \( k = 1/2u_iu_i \), sums the three normal components of the Reynolds stress tensor. The production of
TKE is approximated by \( \frac{\partial u_i}{\partial x_j} u_i u_j \). From planar PIV, this value cannot be entirely calculated, because the derivative in the out-of-plane direction is unknown. For the jet, which is dominated by \( u \) component of velocity, one can argue \( \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \). Thus, the cross-plane orientations give the best approximation of the production term, as one can ignore them without losing significant contributions. However, shocks clearly violate this assumption, so the streamwise orientation over the deck plate provides valuable information. The spatial derivatives are calculated along each plane with the available data, and then interpolated onto the grid volume, from which TKE production is then determined. Alongside the sum of the normal components, which yields the TKE, the production of TKE is shown in Figure 5.12.

Figure 5.12 reveals a very complex turbulent structure of the MARS jet. Transparent slices at each of the PIV planes are plotted, to give a three-dimensional perspective of the turbulence. The mean TKE, shown in Figure 5.12, is confined to the shear layers as expected. A 50% threshold is selected for the isosurface, and the maximum practical value, again normalized by the exit velocity \( U_J \), is found to be approximately 4% of the flow’s kinetic energy. As mixing grows with downstream direction, so does the turbulence. A steady rise is observed in the TKE, with the most energetic region found in the top shear layer at \( x/D_h = 8 \). The redistribution of energy is interesting, as in \( x/D_H = 4 \), the peak values are located in the upper corner vortices. At the most downstream plane, the maximum TKE regions are located within the central areas of each jet wall. This shift in TKE appears to be linked to the corner vortices. Considering Equation 2.23, multiple pathways of energy transfer exist, which the experimental data cannot entirely account for. The one quantity that is available, however, is the production term, and is considered next.

TKE production is given in 5.12b. The colors represent the quantity \( \frac{\partial u_i}{\partial x_j} u_i u_j \), and black isolines of TKE are overlain at the cross planes. By convention, see Equation 2.23 & Equation 2.24, a negative value here indicates that energy is extracted from the mean flow and introduced as TKE. Consider the cross planes first.
(a) TKE, $k = \frac{1}{2}u_i u_i / U_j^2$. 

(b) TKE production, $\frac{\partial u_i}{\partial x_j} u_i u_j / U_j^2$, with isolines of $k$ overlain at the cross planes.

Figure 5.12: Turbulent kinetic energy distribution and production.
Looking at $x/D_h = 4$, the peaks of production are tied to the four sets of corner vortices. Specifically, they are on the ‘return-side’ of the vortices; fluid is entrained and pulled radially inwards, where it then interacts with the high-velocity flow. In contrast to this, regions of radial ejection have very little production, leaving a rough ‘X’ shape in the cross planes. As this process unfolds, production in the side shear layers diminishes, and the top shear layer is observed to have the highest generation rate by $x/D_h = 8$. Generally, regions of high production are seen to coincide with areas of maximum TKE, which lie in the shear layers and is linked to the corner vortices. The primary corner vortices appear to play a key role in turbulence production by redistributing TKE. When counter-rotating vortices work to pull fluid radially inward, regions of high stress are encountered, and turbulence production spikes.

Turbulent generation from the shocks is also apparent, found in the core of the jet in 5.12b. Focusing on the flow over the deck plate, the oblique shock pattern is found in the production quantity. Where these shocks reflect off the shear layers, additional rises in production are seen. Notice the increase in magnitude downstream of these reflections in the shear layers. While the direct turbulence terms, $R_{ij}$, have not shown any measure of the shocks, these current results suggest that energy leaks into the turbulence terms through shock interactions.

The quantity computed in 5.12b is not the true production term, because it does not take into consideration the density weight of the Favre-averaged mean velocities. It has been approximated by arguing that the density dependency may be uncoupled from the turbulent stresses (see subsection 2.1.1). To qualitatively examine what effect this approximation has made, the regions of maximum density fluctuations are compared to the areas of production. The center plane is taken from 5.12b. As the flow is symmetric about this location, the out-of-plane spatial derivatives that are missing from the data can be ignored, i.e. $\frac{\partial}{\partial z} \sim 0$, and the incompressible production term is accounted for. The third POD mode from the largest window of the schlieren data is selected to represent the fluctuating density field. This mode was found to best
Figure 5.13: Comparison of TKE production with POD mode 3 from the schlieren data.

isolate the variations in density associated with the shock modes. The comparison is shown in Figure 5.13.

Data from the two separate experiments and analyses collapse remarkably well, seen in Figure 5.13. TKE production is plotted in the background, with the color scheme changed from the previous figure to contrast with the POD eigenfunction. The highest generating regions are given in white, intermediate in red, and mild defined by black. Spatial mode 3 from the schlieren data superimposes these values, extending from $0 > x/D_h > 6$. These data are extracted from 4.4a. Therefore, recall that some error in this perspective exists, because schlieren takes the path integral of light, rather than a single plane. By considering this center plane of production, the top and bottom shear layers stand out. Hidden in the complex three-dimensionality of 5.12b, the third stream shear layer is also visible above the deck plate. But the shock structure is the interesting feature of this chart. POD Mode 3 defines these boundaries by red and blue contours, which overlap nearly identically with the black regions defined by the production term. Similarities in the two quantities can be seen as far as five diameters downstream. In the shear layers, a marked increase in
production is visible downstream of each shock reflection. (Note that the sudden decrease in production at $x/D_h = 4$ is a result of the interpolation process.) These regions are further validation that the shocks act to transfer information. Production occurs along a shock front, but not downstream of it. Also, TKE generation increases at the end of the shock. For instance, notice that the reflection of the primary oblique shock with the deck plate is surrounded by a production increase.

Finally, an evaluation of the production term’s accuracy is sought. If density-weighted velocities were available, a direct comparison could be made. As they are not, Figure 5.13 provides at least a qualitative indication of this. For events tied to Mode 3, the incompressible TKE production appears to be correlated with density fluctuations. This suggests that, along these regions, uncoupling of the density and velocity terms is not justified. The current data slightly capture production here, but it is due exclusively to the contribution of the velocity gradient, as that is all that is available through PIV. Production is calculated upstream of the shocks, and schlieren clearly shows turbulence increases downstream of them. Comparing the fluctuating gradients along the shocks, the velocity gradient is several orders of magnitude weaker (the evidence for this lies in the POD modes of each variable). As the fluctuations should be taken into account via the Favre-averaged variables of the compressible production term, Equation 2.23, the direct velocity estimates ignore the density variation. Figure 5.13 clearly shows that these cannot be ignored. Thus, density-weighted variables would likely contribute to an increase in production along these shock regions. While this would simply change the calculated production quantity reported here, this may have profound impacts for modelers. Depending on the model selected, turbulence production may be significantly filtered in a simulation. The shock-turbulence interactions appear to have tightly coupled compressibility and turbulence effects, which may warrant additional attention in modeling.
5.2 Multi-Point Statistics

The strength of PIV is its ability to acquire a spatial velocity field at an instance in time. Not only does this provide detailed resolution, as shown in section 5.1, but this allows for the computation of multi-point statistics. As with the schlieren data, POD is the method selected to analyze the data. Note that, in contrast to the scalar operation of the schlieren data, vector POD is necessary to capture the additional energy content in the shear terms. Multiple length scales are computed by correlating spatial locations in the grid, and then optimized based on energy content through POD.

Before considering the POD results, a discussion on the quality of the data is pertinent. Because of the difficulty in acquiring PIV data in this complex flow, much of the velocity fields are imperfect, and ‘bad’ vectors are present. The advanced PIV algorithms provided by Dantec can fill in individually absent vectors, but cannot handle a large cluster of missing information. As a result of this experimental shortcoming, additional care must be taken in certain statistical analyses, such as POD. With single-point statistics, the vectors can simply be ignored, as the normalization of each spatial point is independent of its neighbors. In multi-point statistics, however, dimensional consistency is required, or the matrix operations cannot be performed over the entire field. Thus, for traditional POD (i.e. the classical & snapshot methods), one cannot reject bad data points at different realizations. The entire field must be used or ignored. As a result, noisy data sets bias traditional POD results, because the decomposition fits the modes to account for all pieces of data. Nevertheless, traditional POD serves as an appropriate starting point for further analyses of the complex flow, before considering more advanced techniques such as gappy POD.

Snapshot POD is performed on each of the unfiltered data sets. The first eigenfunctions in the \( x \)-direction are included in Appendix B at each plane. In general, shocks are not observed in the \( u \)-component, but they can be seen in the \( v \)-components. Because the PIV is acquired at 10 Hz, the coefficients are essentially
series of independent realizations. Thus, they lack temporal information and they appear as random signals. (Note that these coefficients provide critical information that will be used in later analyses.) Convergence of POD is summarized below, in Table 5.1 and Figure 5.14.

Table 5.1: Summary of 50% POD energy content amongst PIV planes.

<table>
<thead>
<tr>
<th>Streamwise planes</th>
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<tbody>
<tr>
<td>$z/D_h$</td>
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<tr>
<td>$n_{50%}$</td>
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<table>
<thead>
<tr>
<th>Transverse planes</th>
</tr>
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<tbody>
<tr>
<td>$x/D_h$</td>
</tr>
<tr>
<td>$n_{50%}$</td>
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</tbody>
</table>

Figure 5.14: POD convergence rates of PIV planes.

Table 5.1 lists the mode number at each PIV plane that accounts for the 50% energy threshold in the flow. For instance, at $x/D_h = 0.25$, the first 59 modes constitute 50% of the total kinetic energy. As each plane uses 2400 total modes, this indicates the relative convergence rate.
In Figure 5.14, the first 50 modes are considered (beyond this, the convergence rates are indistinguishable from one another). The smoothness of the curve in the logarithmic scale allows for the identification of potentially inaccurate modes \([214]\). Ideally, convergence should roll off smoothly, with no inflection points. In 5.14a, \(z/D_h = 0\) and \(z/D_h = 0.25\) have especially high content in the first mode. Considering the eigenfunctions in Appendix B, these are found to essentially represent the core. Moving away from the center plane of symmetry, these jumps in the rates diminish, possibly due to the changing oblique shock structure in \(z\) (see Figure 4.2). Additional variances is seen in the first lowest modes, which are thought to be a result of erroneous vectors. Because of unwanted reflections from the deck plate, incorrect vectors are randomly introduced. The regions directly next to the plate and Kulites are the most susceptible to this, and their outlines can clearly be seen in these lowest modes’ basis functions.

The transverse planes, 5.14b, show similar behavior in the first modes. The most upstream three planes all have relatively high energy content in the first mode. These modes are again associated with the core. However, in this case, the fluctuations in the data are a result of seeding difficulties in the core. (This is determined by comparing the eigenfunctions with regions of low data count.) Thus, the first mode of these planes should be ignored in further analyses. Moving downstream, to \(z/D_h = 7, 8\), the energy is more widely distributed, as the shear layers have merged and the seeding was not as problematic here.

Note that some of the modes have been biased by poor data points. The amount of bias is dependent on the PIV plane, as some locations were more prone to errors than others. To handle the erroneous vectors, gappy POD is used to improve the data quality.
5.2.1 Gappy POD

Gappy POD, first suggested by Everson and Sirovich [215], is a modal-based procedure which is useful to repair ‘gappy’ data. First, the process requires knowledge of which data points are erroneous. For the velocity data acquired here, multiple filters have been applied, which flag a vector if they do not meet the filter requirements. This provides the information of ‘good’ and ‘bad’ vectors. Using the correct data only, modal bases are constructed through POD and then used to estimate the missing data. As these eigenfunctions are biased by the holes in the field data, the entire process must be iterated to smooth out the basis functions. The velocity field is reconstructed using the original data and the gappy data as estimated by a modal reconstruction. This is repeated until convergence is met, and is therefore a very expensive operation for large data sets. In the fluid dynamics community, Gappy POD has been employed by a number of researchers [216] [217] [218] [219] to improve the quality of data. Following their success, it is implemented here to fill in missing velocity vectors.

Before performing the gappy POD operation, a discussion on the quality of data is necessary. Resulting from seeding difficulties in the jet, the distribution of erroneous vectors varies throughout the flow domain. This is considered by computing the normalized accepted number of vectors, which is given in Figure 5.15. The distribution of acceptable vectors, Figure 5.15, shows the most problematic seeding areas. 2400 snapshots were acquired, so a value of 1 indicates that all of the data points are usable. For the downstream streamwise planes (4 < x/D_h < 8 scanned at z/D_h = −1, −0.5, 0, 0.5, 1), 800 snapshots were acquired, hence the majority of these data have values near 1/3. Jet boundaries are computed using a 10% and 90% velocity threshold, given by black isolines. Immediately over the deck plate, near x/D_h = 1, much of the data are lost due to reflection from the plate. (This particular streamwise location accumulates oil due to the presence of a Kulite sensor.) Additionally, a loss of data can be observed in the second shock cell above the edge.
of the deck plate. In the cross-plane direction, the core of the jet clearly has the highest dropout rates, which is a consequence of the difficulty in seeding the core. The dropout diminishes with downstream location, as the shear layers mix and more seeding is introduced. The $x/D_h = 5$ cross plane has significant loss of vectors, which was a result of poor seeding conditions for that particular run. Even at this plane, however, approximately 1600 data points are still usable.

The convergence of the gappy POD algorithm is a function of the available data in the flow-field. Fewer points requires additional iterations, and if a low-dimensional representation cannot be achieved, the algorithm will never converge. A uniform distribution of gaps is desired. In the event that insufficient points exist, e.g. the reflections over the deck plate, the region should be masked out to avoid corrupting the lowest modes. More complex distributions of gaps requires more input from the user. For the current data, the most furthest downstream plane is the most straightforward. As will be seen, even a relatively minor amount of missing data points can contribute to significantly incorrect POD modal reconstruction.

Gappy POD is demonstrated on the most downstream location, $x/D_h = 8$. For these computations, the entire series of snapshots (i.e. $N = 2400$) is used for recon-
struction in each iteration. The first 22 modes, accounting for 25% of the total energy, are used for the flow reconstruction, and six iterations are performed. Convergence is assumed achieved by this last step, as the residuals of the $U, V, W$ components report 2.6%, 1.0%, and 1.4%, respectively. To illustrate the utility of this method, consider two snapshots, given in Figure 5.16.

![Instantaneous velocity field at $x/D_h = 8$ through Gappy POD.](image)

(a) Snapshot of a filtered field, showing missing data. (b) Reconstructed snapshot via Gappy POD data.

Figure 5.16: Instantaneous velocity field at $x/D_h = 8$ through Gappy POD. $v, w$ are shown as vectors, and $u$ is the contour, with a maximum color scale of 520 m/s.

Considering 5.16a, there are multiple regions in the flow which have missing data due to the filtering process. Here, conglomerations of seeding oil occurred, which obscured whole regions. The gappy POD process has filled in these regions very well, 5.16b. All three vector components have been estimated by their modal representation, producing a continuous, and smooth vector field. Visual inspection verifies that the process works, and so statistical values are sought for quantitative comparisons. As demonstrated by the residuals, the averages are nearly identical, and so these are skipped. The second-order moments are therefore considered. Not only do verification in the higher-order statistics prove the method is effective. But, from the discussion in section 5.1, these are known to be important quantities for
turbulence. Thus, if gappy POD reconstructs the Reynolds stresses correctly, it can be said to be an effective representation of the flow. The variance of the $u$ component is first considered, shown in Figure 5.17.

![Figure 5.17](image.png)

(a) Standard deviation of $u$, calculated by ignoring bad vectors.  
(b) Estimated standard deviation of $u$, using the 50% most energetic modes from Gappy POD.

Figure 5.17: Correlation coefficients at $x = 8D_h$.

In Figure 5.17, the RMS($u$) is displayed, with the contours ranging from 0-80 m/s. 5.17a shows the quantity computed by ignoring bad data points, and a local normalization. 5.17b, however, uses the first 50% most energetic POD modes from the repaired flow-field to reconstruct another data set, from which the statistics are then computed. In this chart, data from every snapshot has been used in the calculation, i.e. no ‘bad’ vectors have been ignored. The distribution is remarkably similar. Maxima occur in the center of the shear layers, and a distinct ‘X’ shape can be seen along the corners of the jet. The magnitudes differ because we have chosen to only use 50% of the TKE to rebuild the velocity field. The loss of energy in the core is not fully understood at this point. Note that this area has the highest signal loss, and so it requires the most reconstruction. This makes both approaches, the traditional statistical approach where bad data points are ignored and the gappy POD method,
susceptible to error. Therefore, the gappy POD approach may actually provide a better estimate of the second order moment than what is taken here as the ‘true’ RMS. Or, structures in the core may be far less energetic here, and thus have been filtered out by selecting the lowest modes only.

The likeness in structure of the \( \overline{uu} \) component is encouraging, because the second-order quantities are difficult to acquire. This is confirmation that POD is capturing the correct physics of the flow: with only 69 of 2400 modes (these account for 50% of the energy), the largest-scale structures are effectively captured. These contain some of the most important energetics in the flow, and this shows the low-dimensional representation can represent a very large data set with a small amount of data. For further verification, the remaining 5 Reynolds stresses are considered. Significant energy lies in the shear and transverse components, so these cannot be ignored. Additionally, a convergence study, based on the number of POD modes, is desired. To quantify the estimation provided by the reconstructed velocities, a scalar value is computed. Here, the error is defined as:

\[
Error = \frac{|\text{mean}(R_{ij,\text{true}}^2) - \text{mean}(R_{ij,\text{Recon}}^2)|}{\text{mean}(R_{ij,\text{true}}^2)},
\]

where \( \text{mean}(\cdots) \) is the spatial average of the entire field, \( R_{ij,\text{true}} \) is the Reynolds stress computed from the original flow-field ignoring erroneous data, and \( R_{ij,\text{Recon}} \) is the Reynolds stress as computed through reconstructed data based on various energy limits using the gappy POD modes. Note that this definition of error is not a perfect representation of how closely the Reynolds stresses are reconstructed. Because it’s a spatial average, the distribution is not relevant; only the magnitudes are. Nonetheless, this provides a decent first approximation.

Velocities are rebuilt reconstructed using 25%, 50%, 75%, and 100% of the most energetic modes. To compare to the unrepaired (i.e. the gappy) data, the traditional snapshot method is used. These are similarly reconstructed with the above thresholds. The results are shown in Figure 5.18.
Figure 5.18: Convergence of $R_{ij}$ through gappy and snapshot POD reconstructions using the lowest modes to account for 25%, 50%, 75%, and 100% of the energy.

The $x$-axis represents each component of the Reynolds stress tensor, and the $y$-axis is the error given by Equation 5.2. Gappy POD estimates of the Reynolds stresses are given by the solid blue lines, and snapshot POD use dashed green lines.

Consider first the remaining components of $R_{ij}$ from the 50% reconstruction used above. Because the dominant velocity is in the $u$ direction, this is the lowest of the normal stresses. The reconstruction uses the most energetic modes, and because $u$ is dominant, most of the lowest modes have been chosen through the optimization process to represent the streamwise component. Therefore, the reproduced structures reflect these modes, dominated by $u$. Now consider the increasing energy cases of the gappy POD. When a larger number of modes is used, the error is reduced and the flow is more exactly reconstructed. When all of the modes are used, accounting for 100% of the energy, the Reynolds stresses are modeled to within a few percent error. This is a significant result. Because the gappy POD results do not rely on rejecting,
flagged vectors, and instead use the repaired data, this proves the method’s success. It has essentially removed and repaired regions of the flow, and estimated it to the best of the available data. Through all of this additional processing and, the original Reynolds stresses, as reported in subsection 5.1.2, have been almost exactly recovered.

Traditional snapshot POD demonstrates a stark difference to the gappy POD. With this method, the bad vectors are included, and the basis functions may become corrupted. Using only 25% of the energy for reconstruction, the snapshot method gives similar results to the gappy approach. (In fact, the snapshot method is closer in some components, likely due to numerical errors accrued in the gappy operation.) However, a discrepancy in the most energetic component, $R_{uu}$, is apparent at the 25% level. With an increasing number of modes used in reconstruction, the error in this component soars. Using all of the modes, the snapshot estimate is off by over a factor of 2. This means excessive energy lies in the higher modes of the streamwise velocity, because these are generally the incorrectly computed vectors from PIV. As this quantity is used in many other terms, e.g. the TKE, this large error is unacceptable, because it will dominate every subsequent calculation. However, the agreement in the lower-energy reconstruction indicates that these have not been as heavily influenced, thus giving confidence in the choice of modes used to perform the gappy POD operation. Interestingly, the remaining components of $R_{ij}$ have favorable agreement with the gappy results. The shear components asymptote to an approximately 10% error, but the transverse components’ errors are minimized. This is believed to be a result of the experimental acquisition: errors are more prone in the $u$ direction.

An intriguing result of Figure 5.18 is the convergence of the streamwise shear components, $R_{uw} & R_{uw}$. Surprisingly, the shear stress terms provide better estimates than the normal stresses. These consistently have the lowest error, despite their energy content being much lower than the streamwise normal stress. This result reaffirms the POD approach, because it indicates that the correct physics are being modeled. Even the dynamics of the less-energetic shear components have been
accurately captured by these low-dimensional models.

Again, the reader should be reminded that calculating a scalar to represent the fit of two vector fields hides a great deal of information. Thus, some of the minor discrepancies in these results may be a result of the error definition selected.

**Additional PIV Planes**

The $x/D_h = 8$ plane presented for the gappy POD produced the most straightforward results. Some of the other planes have been looked at, but due to the extensive processing required, not all have been considered. Additionally, successfully implement gappy POD becomes more complicated at different planes. For example, reflections from deck plate recurred in the same location. This drastically limits the capabilities of the gappy POD algorithm, because a lack of statistical information at these regions of reflection results in unconverged basis functions here. Thus, repairing these areas is near impossible, and they need to be masked out to prevent corruption of the eigenfunctions. Also note that for some of the other planes, the lowest modes are not necessarily the best selection to rebuild the gappy regions. One may need to do selective reconstruction, which relies heavily on the input of the user.

Following the success of the $x/D_h = 8$ cross plane, gappy POD is tested on three other planes, $x/D_h = 4$, $z/D_h = 0$ and $z/D_h = 0.75$. Gappy POD parameters are held constant: six iterations, using the 25% most energetic modes and all 2400 snapshots in reconstruction. These planes are selected as limits of the data. For example, $x/D_h = 4$ has significant dropout in the jet core, while $x/D_h = 8$ was already seen to be manageable. Rather than performing convergence studies on each of these planes, which would be expensive, the final accuracy of gappy POD is simply assessed by considering the reconstructed flow using 100% of the energy. The results for these planes are shown in Figure 5.19.

The reconstructed Reynolds stresses of four different PIV planes are shown in Figure 5.19. Reconstructed flows using the snapshot method, given by dashed lines
Figure 5.19: Convergence of $R_{ij}$ through gappy and snapshot POD reconstructions comparing at four different planar locations.

with open symbols, consistently show divergence in the second order moments, indicating a poor job of capturing the flow dynamics. The $x/D_h = 8$, which was already looked at, is given in green. For $x/D_h = 4$, the gappy POD does not reconstruct the flow as closely - likely due to incomplete convergence from the number of iterations selected. In the streamwise direction, the plane of symmetry $z/D_h = 0$ is repaired well in the normal components. However, the shear stresses do not show any improvements using the gappy POD approach. This is likely due to the reflections off the deck plate. The flow has less usable statistics in these regions, and the cross-moments apparently suffer more. For the shear layer, $z/D_h = 0.75$, reflections were much less of an issue, and gappy POD worked well. With the exception of $x/D_h = 4$, all components of the Reynolds stresses are captured to an error within 5%. The dynamics follow these quantities, so an accurate repair is necessary for utilizing the POD results further. The cleansed decomposition of the shear layer, $z/D_h = 0.75$, is next used to cross-correlate against pressure signals.
5.2.2 Velocity-Pressure Relations

Following the experiment described in section 2.3, stochastic estimation is carried out using the gappy POD results. Though the temporal POD coefficients have a sampling rate of 10 Hz, they are simultaneous with the 100 kHz Kulite and acoustic data. The sampling frequency mismatch is dealt with by computing everything in the time domain, and the process is described in subsection 2.1.3. These coefficients are cross-correlated with each of the 7 Kulite probes and 10 microphones. This process produces an enormous amount of data, and there is much analysis to be performed. Due to bandwidth limitations in the hardware, the number of simultaneous data points from the experiments is limited to approximately 600 for each plane. In this complex flow, this is found to generally be an insufficient number of points for statistical convergence. As such, there are many instances that ‘suggest’ specific POD modes are related to particular sensors. However, in a select few cases, the cross-correlations are definitive.

Before considering the correlation between the velocity field and the deck pressures, the spectral characteristics of the Kulites must be revisited. Recall that the jet requires the addition of mild heat to acquire PIV data, which slightly changes the condition of the jet. Pressures each of the Kulites are reacquired, and the spectra are presented in Figure 5.20.

The characteristics of the deck pressures, given in Figure 5.20, are largely the same as in the cold jet, as in Figure 4.11, with a few notable differences. (Note that the positions of the Kulites have changed, which are given in Figure 3.9). The dominant frequency associated with the Kelvin-Helmholtz instability has shifted slightly to $St = 3.58$, and in some locations split into two peaks. Some additional tones are now observed as well, at $St = 0.27$ & $St = 1.28$. The side shear layers, probed by Kulites 5 & 7, have a stronger $St = 0.32$ component with the addition of mild heat. For now, these serve as a suitable reference for the discussion relating to the velocity field.

In the streamwise orientation, the $z = 0.75D_h$ location is considered. This PIV
Figure 5.20: Spectra of deck pressure for the heated jet.

plane is found to have the highest correlations with the Kulite sensors. As three instruments are simultaneously measured, each of which represents an additional dimension of data, common points of interest are required to compare results. Kulite 5 and Microphone 2 are chosen to investigate thoroughly, before all sensors are considered. Cross-correlations between these probes and the POD modal coefficients are performed over a range of time lags, and the results are presented in Figure 5.21 - Figure 5.24.

Figure 5.21 shows consistencies between the modal representation of the velocity field and pressures on the deck plate. Kulite 5 is located in the side shear layer, which is the closest probe to this particular PIV plane. For consistency with the far-field cross-correlations, the Kulites and microphones are assumed to occur at time $t$, and the velocity signal is delayed. Thus, the propagation time in this case is negative, because an event must occur in the Kulite before it is measured in the convected downstream flow. Modes 1& 2 are found to have the highest correlation values with Kulite 5. Their modal shapes are considered later, in Figure 5.25, but for now suffice it to say they represent vortices in the upper and lower shear layers. This result
is not particularly revealing (vortices in the shear layer should have a relation to an upstream probe), but it does prove the method. Additionally, one can surmise that these specific length scales found in modes 1 & 2 are ‘important.’ The average propagation time, approximately 0.2 ms, is appropriate given the convection speeds and distance between the Kulite and the region of activity in the mode shapes. The other modes appear as mostly random, with hints of correlations in modes 4 & 9. The difference in cross-correlation magnitudes as compared with Figure 4.12 is due to the reduced number of statistics, and a much lower SNR is observed here. Extending this method to the far-field microphones, relations are sought for noise-producing events.

In Figure 5.22, the same correlation algorithm is employed, this time between the velocity modes and the pressure signals from the 160° microphone. In this case, the average propagation time is much longer, around 10 ms. Note that it is positive too, as this is the time required for an acoustic wave to travel from the jet to the microphone. The SNR is again low, but Mode 2 clearly emerges from the noise around 10.5 ms. Given the structures exist in the shear layers, and are still easily supersonic (see 5.5b),
Figure 5.22: Cross-correlations between the 100 kHz acoustic signals of Microphone 2 (i.e. at 160°) and the first nine velocity gappy POD coefficients.

Mach wave radiation is likely being detected. As will be seen, the 160° microphone has the highest detection rate of these modes. The wavy-wall analogy predicts a peak angle using Equation 2.64 as \( \phi_{\text{peak}} > 30^\circ \), where the convective velocity is estimated as 0.8\( U_J \). Again, 0.8\( U_J \) is a gross approximation, and this discrepancy suggests a slower convective velocity of the modes is likely. This is consistent with the findings and discussion on modal convection in section 2.3.2. For further validation, correlations between the Kulites and microphones are considered.

As a final component of this sensor configuration, the correlations between the 160° microphone and the seven Kulites are presented in Figure 5.23. Because the microphones and Kulites are simultaneously acquired at 100 kHz, a large number of data points is available and spectral computations are possible, which results in clean cross-correlation signals. The average propagation time is found to be almost identical to the velocity measurements, giving confidence in the correlation results from both cases. The jet is in the sideline orientation relative to the far-field acoustics, so the side shear layer closest to the microphones, probed by Kulites 5 & 7, gives the
Figure 5.23: Cross-correlations between the 100 kHz acoustic signals of Microphone 2 (i.e. at 160°) and the 100 kHz signals of Kulite 5.

Kulite 2, mostly hidden here, also shows significant correlation (it can be seen more clearly in Figure C.3). This probe is further upstream on the deck plate, and inside of the side oblique shocks. Due to the presence of the shock, the interpretation of this is not as straightforward. More information is needed to ascertain the physics responsible for this correlation.

The $z = 0.75D_h$ plane has positive agreement between instruments. POD mode 2 of the velocity data correlate with both deck pressures and far-field acoustics. The Kulites and microphones correlate with each other as well, i.e. everything is linked. The correspondence between all three sensors indicates that this particular velocity mode is tied to noise production. The PIV data provides some evidence for the physics, and the sensor on the deck plate represents a practical implementation, where stochastic estimation may be applied to predict noise generation directly from the nozzle. Note that the results presented here are the best-case scenario, and the other instruments do not provide such clear correlations. Additionally, many of the relations between probes are obscured in these views. Therefore, multidimensional
plots are considered, to observe the correlations between all sensors.

As with the schlieren results, the maximum correlation coefficient between each PIV mode and each sensor are stored, and then plotted as a contour. Since both the Kulites and microphones are considered, two plots are necessary. These are shown in Figure 5.24. The first 30 POD modes are considered, given along the $x$–axis as $a_u$. In this view of the data, information has clearly been lost. Thus, these contour plots only serve as a guide, and help for pattern recognition. If a significant peak in the correlation coefficient, it must still be verified by considering its time-series as a function of $\tau$. Due to the low number of data points, false-positives exist, and one must manually filter these by considering realistic time lags. Nonetheless, this is a useful way to sort through the large datasets, which lets one observe similarities across different sensors that would be otherwise difficult to see.

From 5.24a, the dominant communication is clearly between PIV modes 1& 2 and Kulite 5. The other Kulite sensors, given by the $y$–axis, show very little correlation to this particular PIV plane. As the PIV is acquired between Kulites 5 & 7, this result is not unusual. Considering the spectral content of these probes in Figure 5.20, it is likely that these structures leave a signature centered around $St = 0.33$. The correlations do not provide direct evidence of this frequency. However, a spatial frequency analysis of the second eigenfunction can be performed, as in section 2.3 (see Figure 2.15 in particular). This uses the local velocity and a range of convective Mach numbers between $0.3 < M_c < 0.64$, where $M_c = \frac{U_1 - U_2}{c_1 + c_2}$. These are based on observed ranges that have reported in literature for supersonic mixing layers [36], and bound the possible frequencies of Modes 3 & 4 between $0.25 < St < 0.5$. Also note that this is nearly the same frequency range identified by the time-resolved schlieren POD, which suggests similar structures (see 4.4c). While the wavenumber analysis yields an admittedly large range, the consistent overlap with spectra in Figure 5.20, the schlieren analysis, and high correlations between the PIV and Kulites indicate that the structures in Modes 1 & 2 are centered around the $St = 0.33$ signal. Note that
(a) Map of maximum correlation peaks as a function of Kulite position and velocity modal coefficient.

(b) Map of maximum correlation peaks as a function of microphone position and velocity modal coefficient.

Figure 5.24: Maximum correlation coefficients at $z = 0.75D_h$. 

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some of the higher PIV modes also appear to have some communication with Kulite 5, given by the rise in correlation coefficients along x at this probe. Specifically, Modes 4, 9 and 15 appear. Mode 15 also looks to have some communication with Kulite 2. As stated previously, more data points are need to clean up the cross-correlations and reach conclusions about how these flow structures relate to the deck pressures.

The 160°microphone data of Figure 5.23 are similarly collapsed, and compared against the other nine far-field sensors in 5.24b. Mode 2 is seen to correlate well with the first 6 microphones, which span from 165°to 140°. This region is thought to be dominated by Mach wave radiation and turbulent mixing noise. Starke et al. [220] has performed an investigation of the acoustic-Kulite relations using data from the cold jet. They have shown that the majority of this activity, excluding the dominant St = 3.3 signal, is in the 0.1 < St < 0.5 range. With the relation to the Kulite suggesting a St = 0.33 link and the acoustic correlations discussed here, the flow state represented by Mode 2 can be said with confidence to contribute to acoustic production in this frequency range. Mode 15 also stands out as a potential candidate, along with Mode 4. Cross-referencing with 5.24a, modes 4 & 15 share activity with the Kulites as well. Consulting the time-lag plots of the cross-correlations, these are realistic and significant correlations. Modes 4 & 15 therefore also likely contribute to noise production. With the guidance provided by these cross-correlations, these modes are investigated in Figure 5.25.

The structures of these ‘loud’ modes are now considered, in Figure 5.25. Because of the strong three-dimensionality of the jet, all three components of the basis functions are plotted. The u & v components are vectors, and w is the scalar map. The mixing layer is outlined using 10% and 90% isolines of U. Mode 1 has a strong structure in the core, at (x/D_h = 3.7, y/D_h = 0). A saddle point is also observed near (x/D_h = 3.5, y/D_h = 0.5). Mode 2 similarly has a strong vortex, shifted up from Mode 1, and a saddle point in the upper shear layer, slightly upstream of the position in Mode 1. These saddle nodes appear similar to the idealized large-scale
Figure 5.25: Significant POD modes of $z/D_h = 0.75$ identified through stochastic estimation.
structure in turbulent shear layers, as seen in the convective frame of reference [221], so POD could be extracting these. Mode 4 appears dominated by a diagonal structure, extending upward from the edge of the deck plate, possibly linked to an oblique shock leaving the plate. The highest mode, 15, has much smaller length scales, and could be capturing Kelvin-Helmholtz structures in the side and bottom shear layers. While these flow states have been associated with noise production, the mechanism of generation is not explained. Flow-field reconstruction using these modes sheds some light on this, but with a lack of time-resolved data, observing the evolution and formation is inconceivable. From the modes though, the event responsible is clearly a high-strain event which involves the formation of structures off the deck plate. As the noise-producing events are intermittent, performing statistical approaches also loses significant information.

**Additional PIV planes**

While the results from $z/D_h = 0.75$ have been guided by the relatively straightforward cross-correlations, the remaining PIV planes do not give such clear results. This is believed to be influenced by the data quality; velocities from $z/D_h = 0.75$ are the simply cleanest dataset. With only 600 points available for cross-referencing, noisy PIV signals result in a large loss of correlation. Because data are missing, the traditional snapshot POD method results in corrupt basis functions and temporal coefficients. Gappy POD should be performed first, but as previously discussed, this can be a demanding, and expensive task. This area is left for future efforts.
Chapter 6

Conclusions

The turbulent flow of a multi-stream, supersonic, rectangular jet has been studied. The MARS nozzle, based on engine designs of future aircraft, was designed and installed in the anechoic chamber at Syracuse University to gather experimental data. An extensive test campaign was then planned and executed, which has provided a large, multi-dimensional data set that can be used by modelers and aeroacoustic analysts. Analyses of the data were performed, which revealed a highly three-dimensional, turbulent, and compressible jet plume. This complex flow was dissected using stochastic analytical methods, allowing for quantitative conclusions to be reached.

Two aeroacoustic events were identified in a preliminary experiment of a supersonic axisymmetric jet, guiding much of the research for the MARS. The round jet study, which was performed in anticipation of the MARS campaign, discovered some aeroacoustic mechanisms linked to noise generation. In the case of the sonic jet, a particular flow state identified through POD was found to contribute to acoustic production. The 10 kHz PIV data indicated that these dynamics were, on an average sense, not the most energetic structure. However, during an acoustic event, they could contribute to over 25% of the total turbulent kinetic energy. For the supersonic jet studied here, the phenomenon of screech-generation was investigated by linking a flow state, again through POD, to screech tones. Flow structures associated with
turbulent mixing noise in this case were found to be consistent with the results of Berger [155]. The success of this study motivated the need for detailed PIV data, synchronized with multiple pressure sensors, of the MARS jet.

Significant changes required of the facilities were met with in-depth engineering solutions. The MARS jet rig was designed through an iterative process which used simple 1-D isentropic relations, the method of characteristics, numerical solution of the 2-D Euler equations, and basic machine design principles. Integrating the nozzle into the chamber required facility upgrades to provide suitable test conditions. The HVAC system was improved with a more powerful fan and a more efficient exhaust duct was designed, which handled the increased mass flow rates and velocities of the MARS jet. To operate the jet in a steady state, a National Instruments control system was engineered. Instrumentation upgrades were also necessary, as the flow events of this jet happen at a faster scale. Pressure acquisition capabilities were increased, a time-resolved schlieren system was developed, and a stereo PIV was implemented. Finally, because the PIV measurements require the jet to operate at elevated temperatures, a heater calibration was performed.

The data provided through the experimental measurements revealed a complex flow, rich in turbulent structures. Most obvious is a series of oblique shocks throughout the jet. Mach wave relations indicate that the jet operates slightly overexpanded, and the oblique shocks form a complex train of shock cells. The rectangular shape and asymmetric expansion throughout the nozzle results in three-dimensionally varying shock cells. The plume is also observed to vector away from the deck plate in the jet, which is thought to be a consequence of the SERN. The introduction of the additional stream above the deck plate results in complex interactions at a high frequency, which pervade the entire flow. Where the primary oblique shock travels through the third stream and reflects off the deck, a normal shock is observed. This shock appears to interact with the flow in a way that permutes the vortex train in the shear layer here. A summary of the experimentally-determined macroscopic and mi-
crosscopic flow conditions is given in Table 6.1, which are taken under mildly heated conditions. Turbulence intensity, $T u$, varies drastically throughout the domain, so the jet exit $T u_j$ and most downstream location $T u_{8Dh}$ are given at the centerline of the jet. Kolmogorov values are estimated using the macroscopic quantities, and are provided as bounds on the turbulence.

Table 6.1: Flow conditions.

<table>
<thead>
<tr>
<th>Operational Parameters</th>
<th>Large Scale Estimates</th>
<th>Kolmogorov Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{j,1}$</td>
<td>$U_j$ 520 m/s</td>
<td>$\eta$ 3.2 $\mu$m</td>
</tr>
<tr>
<td>$M_j, w$</td>
<td>$Re_j$ 2.88 x 10^6</td>
<td>$u_\eta$ 2.8 m/s</td>
</tr>
<tr>
<td>$D_h$</td>
<td>$Tu_j, Tu_{8Dh}$ 2%, 13%</td>
<td>$\tau_\eta$ 1.1 $\mu$s</td>
</tr>
</tbody>
</table>

With the fast sampling rates and high sensitivity of the schlieren campaign, a potent, high-frequency signal was found throughout the entire domain, which was linked back to a Kelvin-Helmholtz instability generated in the third stream. Traveling acoustic waves, also imaged through schlieren, were found to have this frequency of $St = 3.3$, also verified through the far-field microphone array. Proper orthogonal decomposition was used to extract structures in the flow specifically related to this event, and the Kelvin-Helmholtz-like mechanism responsible for this dominant frequency, which forms in the wake of the splitter plate in the nozzle, was determined. Stochastic estimation between the POD modes of the schliere n data and pressure sensors on the nozzle point to the dominant corner vortices in the side shear layers centered around $St = 0.33$. One important finding from this section of work was the waveguide observation; disturbances are found to propagate along the oblique shocks, which act as conduits to redistribute turbulence throughout the jet.

Stereo PIV measurements acquired planar slices of three-component velocity vectors. Using a large number of data points, a statistical description of the jet was presented. Data from the planar locations were interpolated onto a volumetric grid, enabling unique views of the plume and its intricate shock structure. All three components of velocity allow for a complete description of first- and second-order statistical
moments, which include the incompressible Reynolds stresses. These quantities displayed an anisotropic, inhomogeneous turbulence field, with specific features linked to the geometry of the nozzle. An interesting result was related to the corner vortices. Regions of peak turbulent stress initially align with the vortex cores, and then move radially inward as the centroids spread outward. As these corner vortices diffuse, smaller scales are generated, and the interacting structures are concentrated in the high-speed side of the shear layer, resulting in the highest Reynolds stresses.

Turbulence production was calculated using the six Reynolds stresses and velocity gradients determined through PIV measurements. Production was found to be primarily confined to regions of high shear, with one notable exception. Oblique shocks manifested in this quantity, which were not seen in any of the other turbulence terms. Compressibility effects along shocks are argued to be important here. Morkovin’s hypothesis, which decouples density fluctuations from turbulence, appears to break down along these shock-containing regions. This is believed to result in an underestimation of production, though some amounts were still captured through the velocity calculations. Given that waves were observed to ‘shed off’ these shocks in the schlieren experiment, high levels of turbulence in the density clearly exist, and this argument is well supported. Elsewhere in the flow, Morkovin’s hypothesis appears to hold. The compressibility effects show up in the first-order statistics rather than the second-order, which are the turbulence terms. This essentially validates its application here.

A modal decomposition via POD was performed on the velocity field, from which correlations to additional sensors were made. Gappy POD was shown to be a necessary and effective process to improve the data quality, as erroneous vectors generated by the PIV algorithms have been rejected through filtering, leaving regions of missing information. Traditional POD interprets these holes as variant data, which results in corrupted basis functions and temporal coefficients. By repairing the flow-field with gappy POD, statistical quantities, which rejected the ‘bad’ data points, were recovered.
to within a few percentages. With this encouraging result, a convergence study was performed using the gappy POD modes. Using reconstructed velocity fields, many of the essential energetics in the flow, e.g. the shear components, can be recovered with a relatively low number of modes. An important finding was this rapid convergence of the cross-moment Reynolds stresses, which shows how POD effectively captures the dynamics of even the more obscure structures.

Using the repaired data sets, cross-correlations between the velocity field and pressure sensors were investigated. The side shear layer over the deck plate was considered, and significant correlations were discovered. Specifically, structures captured by the first and second modes of the velocity data were linked to the transducers in the deck plate. The second mode was also found to be highly correlated with far-field acoustics, along with the same transducers in the deck plate. These three sets of correlations give high confidence that noise production is closely related to these POD modes, and Mach wave radiation is proposed as the dominant source. A high-strain event formed off the deck plate appears to be responsible for this. However, without the temporal resolution, the exact mechanism cannot be observed. The broadband nature of this process also makes prediction of a distinct frequency of this mode difficult, but reasonable overlap in the spectral ranges confirms the observed signals. The identification of these 'loud' POD modes is an important find, as these cannot be obtained with current computational tools. Because the aeroacoustic problem is so difficult to simulate, only experiments can provide the necessary statistics for these calculations. But with these results, simulations may be able to utilize the information and learn more about the flow. This is discussed further in section 6.1.

As a final comment on stochastic estimation, the velocity cross-correlations identified here are perceived as 'minimally-significant.' In other words, scarcely enough PIV data points were available to quantify correlations at the location investigated. Complex structures, three-dimensionality, and strong shocks seem to mitigate the communication between the velocity field and other sensors. In many other locations
of the jet, which have perhaps more complicated physics, correlations could not be identified above the noise floor. However, much of this is likely due to the noisy data sets, which still require performing gappy POD analyses on. Additionally significant loud modes may be hiding elsewhere in the data. For the schlieren data, only correlations with the Kulite sensors were identified. Evidently, the schlieren field is not a good estimator of the far-field acoustics. This strengthens the need for high-quality, and complete, datasets of such flows. For this jet, the three-dimensional velocity is a critical aspect of the correlations. A significant amount of fluid dynamics and energy would otherwise be overlooked and the modes not accurately represented.

6.1 Future Work

The present work was an initial study on the flow-field of the MARS jet at target conditions, which has opened up many more questions. Obvious extensions of this work include parametric studies of the nozzle, requiring extensive fluid dynamic and aeroacoustic experiments to understand the flow. One example would be focusing on the low-noise region found with a reduced tertiary stream velocity, reported in [199]). Another might look at removing the third stream altogether; the jet rig is configured to easily block off this flow and shift the deck plate up to study the SERN portion only.

Narrowing our attention to the data acquired for the design conditions, much can still be learned about this flow. The dataset is extensive, and analyses of a specific phenomenon such as noise production can be a demanding task. Many of the PIV planes can be explored more thoroughly via stochastic estimation, which will require careful implementation of gappy POD to improve data quality (likely using selective modal reconstruction). A most intriguing topic that has been discovered occurred with the addition of mild heat in the jet. While the large-scale dynamics of the flow appear largely unchanged, such as the side corner vortices, the finer-scale events seem
to have been altered.

The link between the cold & mildly heated jet may provide additional insight into the role of compressibility in aeroacoustic applications. Appendix C summarizes the changes in spectra and cross-correlations for the two cases. The strong Kelvin-Helmholtz instability that was found to dominate so much of the flow in the cold case was greatly diminished with a minor addition of heat. The frequency also shifted to a higher Strouhal number with heat. This frequency, \( St = 3.69 \), was also identified in the cold jet, but was very subtle. Why the heat amplified this event and damped the \( St = 3.33 \) event is currently an unanswered question. Because the energy equation is tightly coupled with momentum in compressible flow, the answer may be tied to the interaction of compressibility and turbulence, which would be particularly important along the oblique shocks where this phenomenon occurs.

One immediate extension of this work would involve collaborating with modelers working on the aeroacoustic problem. The identification of significant noise-related flow states is intractable for simulations in the foreseeable future. However, the strengths of both approaches to this complex problem could be utilized to further understand the flow. Experiments lack the time resolution, and simulations lack the spatio-temporal domain. Using the POD results from the experiments and the flow-field of a LES dataset, one may be able to observe the evolution of specific structures. For example, the ‘loud’ modes may be reconstructed with temporal resolution, which may give further insight into the mechanism of acoustic production.

If future experiments are to be performed, a few recommendations are noted. The acquisition of schlieren data at mildly heated temperatures is a natural next step. An even more useful dataset would result from simultaneous PIV and schlieren systems, specifically background-oriented schlieren. While this would be an ambitious undertaking, it is not infeasible in the anechoic chamber at Syracuse University, and the procedure has been performed at other facilities. This could provide simultaneous knowledge of the density and velocity fields, which would be useful for studies into
compressible turbulence. In any case, more statistics would be helpful, to improve the quality of the cross-correlations. This could be achieved with the current hardware by reducing the number of pressure channels acquired, which would reduce data transfer rates and allow for longer simultaneous record times.
Appendix A

Uncertainties

The primary focus of this chapter is the uncertainty in pressure and PIV data. As schlieren are used primarily for identification of flow structures, their uncertainty is not discussed.

A.1 Pressure Uncertainties

Experimental uncertainty of pressure is determined following methods described by Coleman and Steele [222]. Traditionally, measurement uncertainty is calculated using

\[ U = \sqrt{U_{sys}^2 + U_{ran}^2}, \]  

(A.1)

where \( U \) is the total uncertainty associated with the data and the subscripts refer to \textit{systematic} and \textit{random} per Coleman and Steele. The random error is calculated using a 99\% confidence interval of the Student’s \( t \)-distribution, or \( t_{99\%} \),

\[ U_{ran} = t_{99\%} \frac{\sigma}{\sqrt{N}}, \]  

(A.2)

where \( \sigma \) is the standard deviation of the data and \( N \) is the number of independent samples taken. For experimental data, the systematic error sums in quadrature the
significant biased error sources as

\[ U_{sys} = \sqrt{U_{sys,1}^2 + U_{sys,2}^2 + \cdots + U_{sys,m}^2} \]  
(A.3)

For pressure data, a digitization error must be included in addition to the sensor error and calibration error as an A/D converter is used in the acquisition process:

\[ U_{sys,prs} = \sqrt{u_{cal}^2 + u_{sens}^2 + u_{dig}^2}, \]  
(A.4)

with \( u_{cal} \) determined from Equation A.2 using \( 1 \times 10^6 \) data points at 114 dB and 1 kHz, generated through the G.R.A.S. 42AB sound calibrator. The sensor uncertainty is based on the specifications provided by the manufacturer. For the sampling speed of the G.R.A.S. microphones, this is given as

\[ u_{mic} = \pm 2\text{dB}. \]  
(A.5)

The Kulites uncertainty is given as a percentage of the full scale output (FSO). The pressure exposed to these sensors place this value at

\[ u_{Kul} = \pm 0.1\%\text{FSO}. \]  
(A.6)

The digitization uncertainty is taken from

\[ u_{dig} = \frac{1}{2} \frac{\Delta V}{LSB}, \]  
(A.7)

where a 95% confidence interval has been assumed, \( \Delta V \) is the voltage range of the A/D converter, and LSB is the least significant bit of the converter (i.e. \( LSB = 2^n \), and \( n = \) number of bits). With Equations A.1 - A.7, the experimental uncertainty can be calculated for pressure data.

As the Kulites are only utilized for spectral data here, and were not calibrated to
acquire amplitude data, their only uncertainty is repeatability. Under fixed experimental conditions, the repeatability is observed to be, in terms of Strouhal number

\[ U_{Kul} = \pm 0.003St. \] (A.8)

Previously, the OASPL uncertainty from the microphones in the SU facility was estimated at \( \pm 1 \) dB with a repeatability of \( \pm 0.2 \) dB\cite{64} for a Mach 0.6 axisymmetric jet. With the higher sampling rates and the increased OASPL observed, the experimental error of the OASPL from the MARS jet is recalculated using Equation A.1 - Equation A.7. Propagating values through, the microphone uncertainty is estimated to be \( \pm 2.3 \) dB with a repeatability of \( \pm 0.4 \) dB. Note that this conservative value reflects a worst-case scenario and assumes that the uncertainty has constructively interfered.

### A.2 PIV Uncertainties

Uncertainty calculations on PIV data are very challenging, and are still active areas of research\cite{196–198} as many experimental sources contribute to the error. Particle density, concentration, displacement, gradients, and signal-to-noise ratio (SNR) influence the tracking of particles\cite{142}. Additionally, uncertainty contributions are specific to the flow under study, vary spatially throughout a given flow, and are a result of the number of images that can be acquired in an experiment. Turbulence measurement errors are proportional to the turbulence intensity\cite{145}, and supersonic flow conditions produce artificial velocity fluctuations due to the existence of shocks. This section focuses on the main sources of error for this flow.

Of primary concern are the particle size, or inertial effects, and velocity calculation errors. Timing resolution is on the order of ns, and is typically not a concern. Thus, displacement errors are the prime source of uncertainty in the velocity calculation. Here, a discussion of tracer response due to turbulence and shock waves is presented,
followed by a look at the displacement gradients. As discussed in subsection 3.2.3, two separate seeds are used in the flow. In the core, white mineral oil droplets are estimated to have a mean particle diameter of 600 nm. The ambient fluid uses glycerin smoke with a mean diameter of 2 µm.

The Stokes number, \(Sk\), is a measure of tracer fidelity, and is defined as the ratio of particle relaxation time, \(\tau_p\), to the smallest fluid timescale, \(\tau_f\):

\[
Sk = \frac{\tau_p}{\tau_f}
\]  

(A.9)

For \(Sk \ll 1\), tracer particles follow streamlines closely [223]. To create a drag force on the particle, a difference between the particle velocity and a fluid parcel is required. In terms of particle dynamics, this translates to a frequency response and slip velocity. Following Chapter 2 of Adrian [142], an estimate for the error due to particle slip can be calculated. This takes into account Reynolds number effects and the unsteady motion in turbulent flows. For small \(Sk\), the particle slip is approximated as

\[
\vec{v}_p^* \approx \frac{u - v_p}{\tau_p} + g,
\]  

(A.10)

where \(v_p\) is the particle velocity, \(u\) is the fluid velocity, and \(g\) is acceleration due to gravity. The particle relaxation time is calculated from

\[
\tau_p = \frac{\rho_p/\rho_f - 1}{\rho_p/\rho_f} \tau_0 \phi.
\]  

(A.11)

where the time constant, \(\tau_0\), is calculated as

\[
\tau_0 = \frac{\rho_p d_p^2}{18 \nu_f \rho_f}.
\]  

(A.12)

Here, the subscript \(f\) refers to the fluid, and \(p\) is the particle. \(\rho\) is density, \(d\) is diameter, and \(\nu\) is kinematic viscosity. The correction function for particle dynamics, \(\phi\), is given by Mie [224] as a function of Reynolds number:
\[ \phi = \frac{2}{3} + \left[ \frac{12}{Re_p} + 0.75 \ast \left( 1 + \frac{3.315}{Re_p^{1/2}} \right) \right]^{-1} \] (A.13)

For the MARS experiments, a white mineral oil is aerosolized and injected in the core of the jet. A ViCount 1300 smoke generator, by Concept Engineering, was used as the seeder. The particle size distribution varies as a function of temperature and pressure, and the manufacturer’s specification place the mean particle diameter for this application between 300 & 600 nm. (Note that Mitchell et al. performed a series of experiments, finding the mean particle size was closer to 600 nm for a very similar environment). Using the most conservative value, the Reynolds number based on particle slip, \( Re_p \) for a 10 m/s slip is less than unity. Together with equations Equation A.10 Equation A.13, a particle relaxation time, \( \tau_p \) is approximated as 1 µs. This can be used to calculate a Stokes number via Equation A.9 for a range of turbulent fluctuations. Using an estimate of the Kolmogorov scale as 1.3 µs, this results in a high Stokes number. However, this approach assumes a step input, which does not in fact occur at that level. To incorporate more realistic changes, the Stokes number can be calculated in an alternative way based on frequency sinusoidal oscillations, \( \omega \), as

\[ Sk = \sqrt{\frac{9 \omega \tau_p}{4 \rho_p/\rho_f}}. \] (A.14)

Using Equation A.14, \( Sk \) is found to be less than 0.1 at frequencies up to \( 10^6 \) and asymptotes to zero with decreasing unsteadiness. Thus, the particles injected into the core can be considered to follow the flow with a high level of confidence, even for fluctuations at the Kolmogorov scale. The smoke particles present in the ambient fluid are not entrained into shock-containing regions. Their larger sizes will have slower responses. However, as the velocities remain subsonic, confidence in their measurements is found by considering the results of previous experiments in the SU anechoic chamber [155, 225–227].

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Additional treatment of the core seeding is necessary, because in transonic flows, particles travel through normal and oblique shocks. Melling et. al. [228] suggested a particle relaxation time when subjected to a step change in flow velocity. A factor of $1/e$ is used, and the particle relaxation time is calculated as

$$
\tau_p = \frac{\rho_d d_p^2}{18 \mu} (1 + 2.7 Kn_d), \tag{A.15}
$$

where $Kn$ is the Knudsen number, defined as the ratio of the molecular mean free path to the mean particle diameter. In this experiment, it is approximated as $Kn_d \approx 0.4$, which gives a relaxation time of $\tau_p = \approx 0.5 \mu s$. In the plume, oblique shocks are the prime concern. An estimate of the fluid time scale is based on the length of the shock cells (taken as $l \sim 0.5 D_h$) relative to the exit flow velocity, which yields $\tau_f = \approx 86 \mu s$. Thus, an estimate of the Stokes number is found to be $Sk = 0.006$, and the oblique shocks are thought to have little effect on the response of the particles.

Mitchell et al. [146] demonstrated that an artificial velocity is introduced into the interrogation window, which corrupts the cross-correlation functions. They reported values as high as 35% of the freestream velocity can be generated behind a strong normal shock, and 10% downstream of the weaker oblique shocks. For the vast majority of the MARS’ jet plume, normal shocks are not present. The one exception is in the third stream, where a normal shock is observed at the reflection of the primary oblique shock. Thus, the PIV measurements immediately downstream of this shock may have uncommonly high uncertainty. Elsewhere, a 10% uncertainty is assumed to be possible in the velocities immediately downstream of an oblique shock, and are taken as the upper bound of uncertainties.

With confidence in the fidelity of the tracer particles, attention is turned to the remaining portions of the flow. An estimate of the uncertainty due to displacement error is considered using classical approaches. This again uses the formulas Equation A.1 - Equation A.3. Following the approach described in §5.5 of PIV: A practical guide [148], the system uncertainty is given by
\[ U_{sys, PIV} = \sqrt{u_{dp}^2 + u_{\Delta x_p}^2 + u_{N_1}^2 + u_{Quant}^2 + u_{noise}^2 + u_{x}^2}, \tag{A.16} \]

where the subscripts represent particle diameter, particle image displacement, and particle image density, image quantization, background noise, and displacement gradients, respectively. Estimates for these quantities are taken from Monte Carlo simulations. For a particle image size of 1.25 pixels and a 64 × 64 pixel interrogation window, an RMS uncertainty value is found to be 0.03 pixels. Image displacement is similarly found to be 0.015 pixels, using an average displacement of 6-7 pixels. With an image density of \( N_1 > 50 \), \( u_{N_1} < 0.01 \) pixels. The cameras were set to 8 bits/pixel, and given the high image density, quantization levels are assumed negligible. Investigating the cross-correlation functions, the SNR typically varies between 10 and 20, and the background noise is estimated as \( u_{noise} \approx 0.03 \) pixels.

Using these values, the system in-plane uncertainty of 5m/s is found. The random error is included, which is calculated as 5.25m/s. This yields the overall in-plane PIV uncertainty of 7.25 m/s. Note that the random error varies significantly due to its dependence on the standard deviation. Especially when vectors have been rejected in particularly noisy locations, discussed at the end of this section, the uncertainty significantly increases. These values reflect an average uncertainty. The velocity gradient has been neglected, and this rationale is discussed next.

Traditionally, large velocity gradients, as in the high-shear region of the mixing layers, have dominated the uncertainty \[147\]. However, the advanced PIV algorithms available in Dantec’s DynamicStudios essentially eliminates this uncertainty. Not only is the interrogation window adapted to the particle density, the shape of the window is iteratively modified to treat the velocity gradients, and remove this bias. For the present study, the window is modified until the sum of the squares of each gradient term is less than 0.4, and the norm of the gradients is less than 0.1. An additional challenge arises when calculating vectors near a wall, such as over the deck plate. The mean velocity gradients become very large and a portion of the window
is empty resulting in large bias. To account for this, when adaptive PIV and a mask is used, a wall-windowing function can be utilized, which nearly eliminates the bias toward the wall.

Peak locking occurs when the imaged particle size is smaller than an individual pixel, and the correlation peak becomes strongly affected. Displacement vectors are biased toward discrete pixel values. This is avoided by adjusting optical properties such that the imaged particle size stays between 1 & 2 pixels. In the present study, the mean particle size was approximately 1.25 pixels, and evaluation of the histogram of particle displacements indicate that peak locking is generally not an issue. However, as the particles are in fact a distribution of sizes, some do lie below the 1 pixel threshold. This is thought to be the cause of the erroneous vectors in the data, which are later rejected through post-processing filters. A low-pass Gaussian filter function and top-hat window function are used to pre-process the particles, which helps reduce the peak locking effect.

Bridges & Wernet [229] studied the variation of error in stereo measurements. They showed the in-plane velocity error is inversely proportional to particle displacement. The out-of-plane error scales as $\tan^{-1}(\theta)$, where $\theta$ is the coupling half angle, given in Table 3.6. For this reason, the offset angle is aimed to be as close to 45° as possible. With the values provided in Table 3.6, out-of-plane uncertainties are 10.73 m/s and 7.25 m/s, for the cross-plane and streamwise orientations, respectively.

Finally, spatial uncertainty in the calibration plate were minimized by performing a calibration refinement. This process uses a least-squares approach to determine the actual location of the light sheet by reprojecting the coordinates onto illuminated particles and minimizing the error. A 0.1 pixel uncertainty is achieved in this process. This translates to 0.024mm in the streamwise orientation, and 0.03mm in the cross-plane direction. Note that these values reflect vector position relative to one another, and the overall grid location has an uncertainty in space of 1 mm. Much of the data
have been realigned through post-processing to provide matching overlapping values.

Looking at the higher order moments, one can further assess the accuracy of the measurements, as these require more statistics for convergence. The cross-plane measurements proved difficult upstream of $x/D_h = 6$, which became apparent by looking at the variances of the velocities. With the limited laser intensity in this orientation (due to a thickened light sheet), seeding issues in the core resulted in more spurious vectors being generated. This is a limitation of the experimental facilities, particularly the capabilities of the ViCount machine for this jet. Seeding in the shear layers was acceptable due to increased particle density along the walls of the jet and entrainment from the ambient smoke. However the sparseness of the core particles and relatively low light intensity contributed to the peak-validation algorithms (e.g. the SNR cutoff) allowing more vectors through. To remove bad vectors, various post-processing filters were applied to the data (e.g. range validation, $N - \sigma$ validation). The final data were additionally subjected to a thresholding based on the data from streamwise planes at the same location. (Because the light sheet thickness is reduced in this orientation, the laser intensity is greater and the peak-detection algorithms are more effective.) If a value of the standard deviation in the cross-plane data was found to exceed that of the streamwise direction, it was removed. More accurate approaches are certainly available, and encouraged for future work. However, this thresholding was found effective for removing the majority of the erroneous vectors in the core, with minimal efficacy noticed elsewhere.
Appendix B

Eigenfunctions from each PIV plane

The following charts are the basis functions from each of the different PIV planes. Snapshot POD was performed on each one, and so erroneous vectors have not been ignored in the computations. The $u$ component is shown, at is the dominant mode, and the contour scales are consistent for a given orientation.
Figure B.1: POD eigenfunctions from $z/D_h = 0$. 
Figure B.2: POD eigenfunctions from $z/D_h = 0.25$. 
Figure B.3: POD eigenfunctions from $z/D_h = 0.5$. 
Figure B.4: POD eigenfunctions from $z/D_h = 0.75$. 
Figure B.5: POD eigenfunctions from $z/D_h = 1$. 
Figure B.6: POD eigenfunctions from $x/D_h = 4$. 
Figure B.7: POD eigenfunctions from $x/D_h = 5$. 
Figure B.8: POD eigenfunctions from $x/D_h = 6$. 
Figure B.9: POD eigenfunctions from $x/D_h = 7$. 
Figure B.10: POD eigenfunctions from $x/D_h = 8$. 
Appendix C

Summary of pressure signals between the cold & mildly heated jet

Figure C.1: Comparison of deck spectra

(a) Spectra of deck pressure for the cold jet. (b) Spectra of deck pressure for the heated jet.

Figure C.1: Comparison of deck spectra.
Figure C.2: Sound Pressure Levels (SPL) w.r.t. $p_0 = 20\mu Pa$ taken along the sideline orientation.
Figure C.3: Map of maximum cross-correlation peaks as a function of microphone position and velocity modal coefficient.
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Vita

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