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# AN INVESTIGATION OF US AND CHINESE PROSPECTIVE ELEMENTARY TEACHERS' PROBLEM POSING WHEN INTERACTING WITH PROBLEM-SOLVING ACTIVITIES

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**Abstract**

This study examined the patterns of problem posing shown by United States (US) and Chinese prospective elementary teachers during problem-posing processes when problem-solving activities were involved in an alternating manner. It further explored the features of the relationship between problem posing and problem solving. Data were collected by asking 32 US and 55 Chinese prospective elementary teachers to pose problems for Translating, Comprehending, Editing, and Selecting processes (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005) before a problem-solving task, and then to pose two more problems after that problem-solving task, namely, problem posing after the problem solving process. All participants completed the first set of tasks, and 43 of them completed the similarly structured second set of tasks.

Participants' posed and solved problems were quantitatively analyzed. For problem solving, the results showed that (1) 25% of the 32 US participants and 98% of the 55 Chinese participants completely solved the given problem during the first task administration, and (2) 19% of the 16 US participants and 89% of the 27 Chinese participants correctly solved the given problem during the second task administration. In their problem posing, Chinese participants posed a much higher percentage of solvable mathematical problems in the Comprehending and Selecting processes compared to their US counterparts, while their US counterparts were challenged most by these two processes. Additionally, both US and Chinese participants' best performance in problem posing did not occur during problem posing after the problem solving process.

In qualitative analysis, US and Chinese participants' problem-posing performance shared some similar patterns regarding (1) features of posed problems, (2) capability of posing problems

with creative ideas, and (3) progression of problem-posing performance throughout all five problem-posing processes. The US and Chinese participants also showed some differences during the problem-posing process as follows: (1) figure visualization; (2) calculation interpretation; (3) habitual preference of posing a sequence of problems; (4) perception of a given answer based on previously posed or solved problems; and (5) problem-posing strategy selection for integrating given information. This study further examined the features of the relationship between problem posing and problem solving. Different types of problem-posing tasks needed different amounts of problem-solving effort and they had different impacts on problem-solving performance. In addition, problem solving before problem posing had a positive influence on participants' subsequent problem-posing performance.

This study suggests recommendations for future research to understand other forms of interactions between problem posing and problem solving, explore specific impacts of cultural or academic background on problem-posing performance, and develop models or frameworks that could help problem posers overcome the difficulties involved in posed problems that were ill-structured, unsolvable, or not mathematical problems. For teacher preparation, this study advocates that prospective elementary teachers need more exposure to multiple types of problem-posing tasks, practices involving interactions between problem posing and problem solving, opportunities to work with ill-structured mathematical problems, as well as opportunities to recognize and analyze different types of mathematical problems before posing their own problems.

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## Definition of Terms in This Study

- **A mathematical problem** is a task that involves mathematical concepts and principles, with a solution that is not able to be assessed using immediate knowledge or straightforward means at hand.
- **Problem solving** is defined as a complex activity that demands cognitive thinking to assess a solution rather than just a simple recall of facts and procedures.
- **Problem posing** refers to either generate new problems according to a given situation or reformulate a given problem.
- **A solvable mathematical problem** is a mathematical problem with sufficient or extraneous given information that is enough for finding a solution.
- **An unsolvable mathematical problem** is a mathematical problem that has insufficient information to find a solution, or that is impossible to be solved usually due to infinite numbers of answer to that problem.
- **Not a mathematical problem** refers to a problem that does not require mathematical computational or reasoning steps, or a description or a phrase that is not a problem.
- **A close-ended problem** has only one correct answer and only one way to reach that answer.
- **An open-middled problem** has multiple paths for solving, but with only one answer.
- **An open-ended problem** has multiple paths for solving and more than one correct answer.
- **Mathematical creativity in problem posing** is viewed as rare mental feats with respect to the core dimensions including (1) fluency, referring to the number of posed problems, (2) flexibility, referring to the number of different categories of posed problems, and (3) novelty, referring to how rare one posed problem was in the set of all posed problems (Guilford, 1950).

- **A creative idea** involved in a posed problem refers to a new relationship among mathematical concepts or procedures that is implicit in given figures and situation.
- **High-level cognitively demanding tasks** involve procedures with connections and doing mathematics. In other words, these tasks are challenging, and require problem solvers to understand mathematical concepts and look for the underlying mathematical structure. On the contrary, low-level cognitively demanding tasks include memorization tasks and tasks involving procedures without connections.
- **The Billiard Ball Mathematics Task** involves a ball that is shot on a billiard table at a 45-degree angle from the lower left corner of a rectangular table with whole numbers as its side lengths. The ball travels in straight lines. Each time the ball hits a side of the table, it bounces off at a 45-degree angle, until it gets to another corner of the table.

# Chapter 1: Introduction

## Overview

People ask questions every day. Questions encourage us to think, to explore, and to understand the world better. Similarly, mathematical problems involving one or more questions play a larger role in mathematics teaching and learning. A mathematical problem is defined as a task involving mathematical concepts and principles, with a solution that is not able to be assessed using immediate knowledge or straightforward means at hand (Kantowski, 1977; Schoenfeld, 1985). A problem that does not involve mathematical concepts or principles is not a mathematical problem. Problem solving therefore, is considered a complex activity that demands cognitive thinking (e.g., applying prior knowledge, making connections, visualizing, estimating, reasoning, self-questioning) rather than a simple recall of facts and procedures. Lesh and Zawojewski (2007) developed an even broader definition of problem solving from a models-and-modeling perspective that emphasized problem solving as “a process of interpreting a situation,” during which a problem solver is usually required to refine clusters of mathematical concepts from various topics within and beyond mathematics, and is usually engaged in “several iterative cycles of expressing, testing, and revising mathematical interpretations” (p. 782). Simply speaking, successful problem solving requires deep perception into the structure of a problem. Studies have shown that problem perception was one of the major reasons that lead to the difference in problem-solving skill and performance between experts and novices while, in turn, problem-solving training and experience resulted in improving solvers’ problem perception (Schoenfeld & Herrmann, 1982).

Mathematical problem solving usually focuses on non-routine problems (i.e., problems that involve productive thinking and the application of a certain strategy), instead of fixed computations or solution procedures (English, 1996). It requires learners to discover mathematical structures, make meaningful conjectures, test possible ideas, and formulate reasonable solutions. In other words, “mathematical problem solving cannot be construed merely as knowledge to be received and learned” but as a “process of making sense of particular phenomena” (O’Shea & Leavy, 2013, p. 297). In the long run, problem solving can gradually change learners’ beliefs about mathematics, especially beliefs that can have negative influence on student learning (Lesh & Zawojewski, 2007).

Problem posing is another cognitively demanding activity that asks people to either generate new problems according to a given situation or reformulate a given problem (Kilpatrick, 1987; Silver, 1994). For example, if talking about the relationship between multiplication and division with whole numbers, given a situation about groups of flowers, one could ask: How many flowers are there for four groups of three flowers? This refers to the generation of a new problem from a given situation. After solving this problem with the answer of 12, a following problem could be: If 12 flowers were evenly sorted into six groups, how many flowers in each group? This problem is reformulated from a given or, more accurately, a solved problem.

In light of this definition, problem posing contains two forms of activities: (1) Problem posing is a comparatively independent learning activity that requires exploration of a given situation, which usually is a real-world and/or open-ended situation and involves a cluster of mathematical concepts or ideas; and (2) Problem posing is a tool of problem solving or a further step after problem solving. In this case, problem posing that occurs before or during problem solving helps learners understand the structure of the given problem and solve that problem in the first place. This is referred to by Brown and Walter (1990a) as the “prior effect” (p. 114). Problem posing that occurs after problem solving demonstrates posers’ initiative and creativity of mathematical thinking, especially in approaches or solutions that they are surprised or confused by, where Brown and Walter labeled the "after effect" (p.112). Regardless of the form, problem posing requires the exploration of the mathematical problem's structure and therefore is an applicable practice for developing learners' perception in mathematical problems.

Existing studies show that problem posing is beneficial for improving learners' conceptual understanding, problem-solving skills, dispositions, and creative thinking in mathematics (English, 1997a; Greer & McCann, 1991; Leung & Silver, 1997; Silver & Cai, 1996; Silver, Mamona-Downs, Leung, & Kenney, 1996; Siswono, 2014; Singer, Ellerton, & Cai, 2013; Tichá & Hošpesová, 2013). Silver (1994) also demonstrated that problem posing could be used for assessing student learning and designing inquiry-oriented instruction. These findings illustrate that, when learners are engaged in problem posing, they do not only have similar learning opportunities created by the problem-solving process, but also gain opportunities, particularly brought by problem-posing activities, such as opportunities to improve their initiative and creativity.

Numerous studies have found that learners' problem-posing performance is closely related to their problem-solving performance, and this is true for both students and teachers (Cai et al., 2013; Crespo, 2003; Kar, Özdemir, İpek, & Albayrak, 2010; Silver & Cai, 1996). Kilpatrick (1987) argues that the quality of a problem posed by a subject may serve as an index of his/her problem-solving performance. There is evidence from existing studies to support this. For example, Silver and Cai (1996) found that more able students in mathematical problem solving posed more, and more complex problems than the problems posed by the less able students. Cai and Hwang (2002) found a much stronger link between Chinese sixth-grade students' variety of posed problems and their problem-solving performance, more specifically, the abstract problem-solving strategy they used, than their US counterparts. These studies quantitatively evidenced the relationship between problem posing and problem solving, and examined the magnitude of this relationship. However, more exploration about the nature and the features of this relationship needs to continue.

While some researchers put the issue that formulating a problem is more important than solving a problem (Einstein & Infeld, 1938; Lavy & Bershadsky, 2003), other researchers argue that problem posing is not independent from problem solving, but rather an important companion to it (Cai & Hwang, 2002; Kilpatrick, 1987; Silver & Cai, 1996). Shuk-kwan (2013) suggests that the first phase of Polya's (1945) problem-solving process (i.e., understand the problem, make a plan, carry out the plan, and look back) could be considered as a problem-posing phase, yet Gonzalez (1998) describes problem posing as the fifth phase of Polya's problem-solving process. Silver (2013) considers looking back at the statement and solution of a problem as "posing a simpler problem" (p. 160). Shuk-kwan actually focused on problem posing before problem solving while Gonzalez and Silver talked about problem posing after problem solving. These perspectives indicate that problem posing can actually occur at any stage (i.e., before, during or after) of problem solving. Furthermore, problem posing may take turns appearing in Polya's four problem-solving stages because individuals usually cannot solve a problem by just passing through this procedure once, but by going back and forth.

In the past several decades, great advances have been achieved in research on problem solving all over the world. Problem solving is now placed at the heart of school mathematics (Ellerton, 2013). Even so, Lesh and Zawojewski (2007) pointed out that the body of knowledge of instruction in problem solving was enlarging, and the "approximately 10-year cycles of pendulum swings between emphases on basic skills and problem solving" (p. 763) had started to swing toward the problem-solving side, despite the apparent decrease in the amount of research on problem solving. This shows that problem solving will continue to play an important role in teaching and learning mathematics. This trend of problem solving and the models-and-modeling perspective are demonstrated by the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). More specifically, among the Standards for Mathematical Practice, MP1 formulates that mathematical proficient students should be able to explain to themselves the meaning of a problem and persevere in solving it, while MP4 claims that mathematical proficient students should be able to model with the mathematics they know to solve problems arising in everyday life, society, and the workplace.

On the contrary, although many studies have explored the nature of problem posing such as problem-posing strategies and processes, problem posing is rarely seen as part of the school mathematics curriculum. In addition, although teachers and students are capable of posing mathematical problems, they have many difficulties in problem posing, such as posing open-ended problems and problems related to specific mathematics concepts, and distinguishing different types of posed problems (Chapman, 2012; Shuk-kwan, 2013). Researchers propose that both teachers and students should have more exposure to problem-posing practices (Contreras, 2007; Singer, Ellerton & Cai, 2013). Ellerton (2013) claimed:

> Perhaps the only way that problem posing has a chance of being seriously introduced into school mathematics curricula and classroom practices would be for young teachers to acquire problem-posing skills and confidence in problem posing themselves to the point where they would be capable and willing to help their students to pose problems. The simplest way to move towards achieving this would be to focus attention on this issue in

early childhood, primary, and secondary mathematics teacher education programs. (p. 100)

Considering the appeal of incorporating problem posing into educational programs (Ellerton, 2013), especially in mathematics courses (Lavy & Bershadsky, 2003), together with the close relationship between problem posing and problem solving, it is reasonable to expect that the involvement in both problem posing and problem solving would be more beneficial for prospective teachers' mathematics learning as opposed to only engaging them in one of those activities. Siswono (2014) claimed that "problem posing as a stand alone activity makes less sense in mathematical activity than a situation when it is combined with problem solving" (p. 22). That was because learners who had not experienced problem posing before usually had a hard time understanding what they were doing. Therefore, the ways of situating problem posing within the large body of problem solving, the ways of interaction between problem posing and problem solving, as well as patterns of problem-posing skills and performance while problem solving is involved are worthy of further research.

To better understand these research topics, an international comparison study can make unique contributions. A widely acknowledged justification says that a comparison study contributes by "making the strange familiar and the familiar strange" (Xenofontos & Andrews, 2014, p. 280). So, first, an international comparison study allows us to understand the extent to which the educational activity is culturally situated. Second, an international comparison study warrants challenges to the ways in which a specific system constructs the educational opportunities for the students. To summarize, international comparison studies allow for exploring the similarities and differences of cultural-based learning environments and the outcomes of cross-national students. Further suggestions can be made based on discovered similarities and differences in educational opportunities for students from different countries. Therefore, it is worthwhile to conduct international comparison studies for investigating students' problem posing, especially when such investigation is rare.

Among existing international comparison studies on problem solving and problem posing, the majority have helped researchers to understand and differentiate students' interpretation of mathematical problems, problem-solving strategies, conceptual understanding and reasoning, as well as some beliefs in mathematical problem solving (Cai, 2004; Ma, 1999; Mayer, Tajika, & Stanley, 1991; Xenofontos & Andrews, 2014). A few studies attempt to explore problem-posing related topics (Cai, 1998; Cai & Hwang, 2002; Yuan & Sriraman, 2011), but these attempts focus mainly on comparing different groups of students' problem-posing performance and mathematical creativity, and exploring the relatedness among problem posing, problem solving, and mathematical creativity.

Taking into account the close relationship between problem posing and problem solving, specifically the sequential effects (i.e., prior and after effects) demonstrated by Brown and Walter (1990a) between these two activities, it is discouraging to note that the existing literature reveals little about specific reasons or patterns of those sequential effects, let alone ways of efficiently utilizing those effects on developing learners' problem-posing and problem-solving performance.

## Aims of the Study

In this study, I recruited prospective United States (US) and Chinese elementary teachers and aimed to investigate the patterns of problem-posing performance when they were alternatively engaged in problem-posing and problem-solving activities, as well as the features of the relationship between problem solving and problem posing under this particular circumstance.

I examined the differences in problem-posing and problem-solving performance shown between US and Chinese participants. I also investigated problem posing in a cross-cultural context, specifically with prospective elementary teachers in this particular circumstance.

I chose to engage prospective elementary teachers in alternating problem-posing and problem-solving activities based on the research literature. I theorized that US and Chinese prospective elementary teachers would be able to pose problems, even though they had not had formal problem-posing experience before. In order to fulfill the aim of examining their direct reaction and performance in problem posing, I engaged them initially in problem posing rather than problem solving. I then planned to design a problem-solving task that was closely related to previous problem-posing tasks, with regard to continuously analyzing the mathematical structure of given real-world situation and figures. This process was beneficial in investigating specific prior effects of problem posing on problem-solving activities. Finally, my participants would be required to pose problems again. This process allowed me to explore the specific after-effects of problem-solving experiences on their problem-posing thinking.

In addition, since my participants had no prior problem-posing experience, I would not utilize a free problem-posing situation (i.e., posing problems without any restriction) developed by Stoyanova and Ellerton (1996), in case they had no idea about what they were doing (Siswono, 2014). Instead, I chose to utilize a semi-structured situation (i.e., posing problems based on an open-ended situation, given pictures, or diagrams) and a structured situation (i.e., posing problems by reformulating solved problems) (Stoyanova & Ellerton, 1996). I designed these types of problem-posing tasks with specific requirements in order to scaffold them to pose problems from different perspectives. These decisions of task design allowed me to look closely into the US and Chinese prospective elementary teachers' performance in specific types of problem-posing tasks. To summarize, this study could be placed in the intersecting part of the related topics, including mathematics learning, problem posing, problem solving and international comparison studies, as indicated in Figure 1.1.

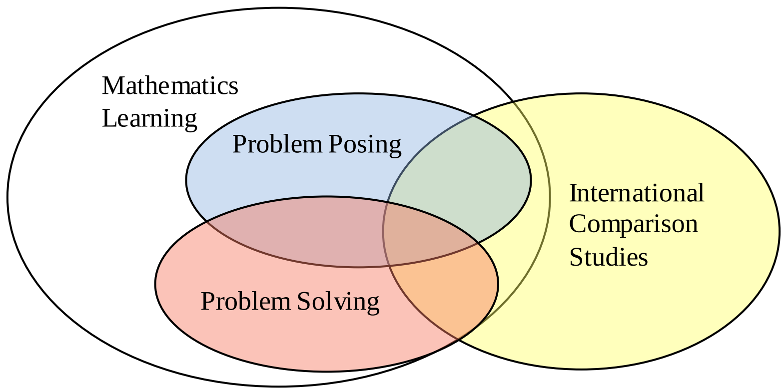



Figure 1.1: All Involved Topics of This Study

**Research Questions**

The following research questions guided this study:

1. What are the similar patterns of problem posing shown by US and Chinese prospective elementary teachers during their problem-posing processes when problem solving is involved in an alternating manner? Are there any differences in the patterns shown by these two groups of participants?
2. What are the connections between US and Chinese prospective elementary teachers' problem-posing and problem-solving performance? Are there any differences in the connections between these two groups of participants?

**Guiding Frameworks**

To answer the research questions, this study discussed three related bodies of literature about problem posing, problem solving and international comparison studies. More specifically, the related literature includes (1) the meaning and features of problem posing, (2) ways that problem posing is situated in the large body of problem solving, and (3) particular contributions that international comparison studies have made on problem solving and problem posing. These bodies of literature illustrated the research gaps regarding of further exploration in cross-cultural context, impacts of different problem situations on problem-posing performance, as well as nature of the relationship between problem posing and problem solving.

As a consequence, in this study I aimed to investigate the patterns of problem posing shown by US and Chinese prospective elementary teachers when problem solving was involved in an alternating manner, as well as the features of the relationship between problem posing and problem solving under this particular circumstance. In this study I first utilized a guiding model with four specific problem-posing processes proposed by Christou, Mousoulides, Pittalis, Pitta-Pantazi, and Sriraman (2005). Each process of this model assesses problem posers' way and justification of analyzing quantitative information in a given problem or situation from different perspectives. Therefore, it allows investigation of prospective elementary teachers' performance in each process. In this study I also adapted the Active Learning Framework developed by Ellerton (2013) for integrating problem-posing and problem-solving activities in a reasonably systematic process. The use of this framework enabled the investigation of the features of the relationship between problem posing and problem solving.

**Summary**

Both problem solving and problem posing are cognitively demanding activities and core elements of mathematical proficiency. Particularly, problem posing is beneficial to improve both teachers' and students' conceptual understanding, problem-solving skills, creative thinking, and dispositions towards mathematics. Research on problem solving has achieved great advances in the past several decades and problem solving is now placed at the heart of school mathematics. On the contrary, problem posing is rarely seen as part of school mathematics curriculum. At the same time, there are still many unanswered research questions about problem posing. Researchers propose that both teachers and students should have more exposure to problem-posing practices, and they appeal for further investigation about the relatedness between problem posing and problem solving.

In this study, I aimed to investigate the patterns of problem posing shown by US and Chinese prospective elementary teachers when problem solving was involved in an alternating manner, as well as the features of the relationship between problem posing and problem solving under this specific circumstance. This study could contribute to the body of cross-national research, as well as the body of research about the relationship between problem posing and problem solving, problem-posing task design, and teacher preparation for enhancing their problem-posing capability.

This dissertation is organized into five chapters. Chapter 1 includes an overview of the research about problem posing, aims of the study, research questions and theoretical considerations. Chapter 2 discusses related literature, and existing guiding frameworks used to (1) integrate problem posing and problem solving together, and (2) design specific problem-posing tasks. Chapter 3 discusses the research methodology of this study, while Chapter 4 reports the results and findings, and answers to the research questions. Finally, Chapter 5 summarizes the findings, makes conclusions, and also discusses the contributions, limitations, and implications for future research and teacher preparation practice.

# Chapter 2: A Review of the Literature

This literature review synthesizes the research on three related topics, including (1) meaning and features of mathematical problem posing, (2) ways of situating problem posing in the large body of problem-solving activities, and (3) particular contributions that international comparison studies have made for exploring the roles of problem posing and problem solving in mathematics learning. At the end of each of these bodies of literature, I provide discussions and summaries to link to the insights from each body of literature regarding whether they helped develop or hinder prospective teachers' mathematics learning and understanding. In addition, these bodies of literature implied available research methods and guidelines that have been utilized in problem-posing research area. However, besides numerous empirical results, this area "remains ripe for theoretical work that will provide a cohesive framework for understanding these empirical results and the overall phenomenon of problem posing," as Cai (2015, p. 29) claimed. As a consequence, I selected two guiding frameworks, instead of theoretical frameworks, that were good fits for this study. The use of those two guiding frameworks in existing studies and why they were good fits for this study are discussed in the fourth section of this chapter.

## Meaning and Features of Problem Posing

Among existing studies on problem posing, much emphasis has been placed on problem-posing strategies, processes of posing one or a series of problems, categories of posed problems, and task selection for problem posing. For each perspective, the synthesized results focus on problem posers' performance, disposition and understanding of problem posing, cognitive reasoning patterns and difficulties, as well as inspirations for future study design or instructional design.

**Problem-posing strategies.** Researchers have found many strategies that problem posers used for generating new problems. The "what-if-not" strategy developed by Brown and Walter (1990b) initially aimed to suggest a way to engage students in problem solving by posing problems. It provides a means of posing new problems by manipulating the attributes of a given problem in two stages (Lavy & Bershadsky, 2003). In stage I, all attributes of a given problem are listed. In stage II, for each listed attribute, alternatives and new questions for the negated attributes are developed by asking, "What if not this attribute?" This strategy has been used to develop a new framework, or as part of a new framework. For example, Silver, Mamona-Downs, Leung and Kenney (1996) summarized the following problem-posing strategies by synthesizing their findings: (1) constraint manipulation, where the "what-if-not" strategy is a particular example; (2) goal manipulation, where only the goal of given problems is manipulated but the conditions are kept the same; (3) symmetry, where the givens and goals of a problem are symmetrically exchanged for posing new problems; and (4) chaining, where one is required to solve prior problems in order to pose new problems. These strategies are available for both posing a single problem and posing a cluster of problems.

Furthermore, by utilizing the "what-if-not" strategy to solve geometric problems with prospective mathematics teachers, Contreras (2007) developed a framework that aimed to improve teachers' abilities to generate problems by systematically modifying attributes of a given problem. This model focuses on the ways of posing the following kinds of problems: (a) a proof problem; (b) a converse problem (i.e., a new problem created by reversing a known and unknown attribute); (c) a special problem (i.e., a new problem created by substituting an attribute with a particular example or case, such as adding additional restrictions and making an implicit relationship among mathematical concepts explicit); (d) a general problem (i.e., a new problem created by substituting an attribute with another for which the initial one is a specific example or case); (e) an extended problem (i.e., a new problem created by substituting an attribute with another similar or analogous attribute, where no one attribute is a specific case of another one); and (f) a further extended problem (i.e., a new problem extended from a special, general, or extended case by modifying other attributes). The fundamental idea involved in these different problems is to discover new possibilities, patterns, or relationships among the mathematical concepts of the original problem. Contreras concluded that, first, without adequate experiences, students rarely used these prototypical strategies to generate problems. Second, all of the difficulties the students had, and the errors they made during problem posing, indicated that they needed a broad variety of experiences in problem posing.

Abu-Elwan (1999) used two sources for 60 pre-service middle school teachers to pose mathematical problems: textbook problems and semi-structured situations. For the textbook problems, a group of pre-service teachers were trained to pose problems by adding more, or new conditions to the original problems and removing some initial conditions for four weeks. Abu-Elwan asked his participants to use the constraint manipulation problem-posing strategy according to Silver, Mamona-Downs, Leung, and Kenney (1996). His participants posed special problems, general problems, extended, and further extended problems, according to Contreras (2007). For the semi-structured situations, the other group of pre-service teachers was also trained for four weeks, but to complete a given situation in order to have an integrated mathematical structure, and then to pose problems. This way of engaging pre-service teachers in problem posing is quite different from previously described ways. It involves both constraint and goal manipulation strategies, and participants can pose proof problems, special problems, general problems, and extended problems according to Contreras. The same pre- and post-test that included both of the aforementioned problem-posing skills were conducted before and after four weeks' training. By quantitatively comparing participants' pre- and post-tests in problem-posing skills, Abu-Elwan found that both ways had significantly positive impacts on developing pre-service teachers' problem-posing skills. It indicates that the source and the characteristics of selected tasks/situations could influence people's problem-posing performance.

To summarize, researchers have found that using problem-posing strategies is an effective way to develop pre-service teachers' problem-posing skills, especially when they are able to systematically pose new problems using different strategies so they can better understand mathematical concepts and involved structure. However, without formal training or adequate experiences, pre-service teachers will not use these strategies frequently or effectively. This is likely because problem posers who lack problem-posing experience may randomly select strategies to help them pose new problems. Their use of problem-posing strategies is usually haphazard, with no predictable pattern involved. In other words, they lack the experience of using different problem-posing strategies systematically or efficiently. Brown and Walter (1990a, 1990b) called for incorporating types of problem-posing strategies in standard mathematics courses (cited from Lavy & Bershadsky, 2003). Twenty years later, this need has still not been fulfilled (Lavy & Bershadsky, 2003; Leung & Silver, 1997; Silver, Mamona-Downs, Leung, & Kenney, 1996).

**Problem-posing processes.** Koichu and Kontorovich (2013) referred to problem posing as a student-centered and process-oriented activity. They framed a case study with two pre-service teachers and their problem-posing experience with the following components: (1) the notion of mathematically meaningful problems evaluated by a problem poser involving positive characteristics such as simplicity, brevity, and clarity; and (2) a set of attributes characterizing the processes of formulating meaningful problems, such as a mathematical knowledge base, problem-posing strategies, and individual considerations of aptness. The researchers developed a general process of problem posing: (1) warming-up by associating the given task with particular types of problems, (2) searching for interesting mathematical foundations or phenomena, (3) hiding the problem-posing process when formulating problems which is not transparent to the problem solvers, and (4) reviewing and reshaping posed problems to make them more constructive. This framework allowed the researchers to explore problem posers' skills while searching for interesting mathematical phenomenon.

Olson and Knott (2013) addressed the gap of the somewhat limited literature on problem-posing episodes in higher education settings. They examined a college instructor's didactics by using specific episodes to develop students' mathematical conceptual reasoning. These episodes included setting up a problem, stating that problem, and preparing follow-up questions as scaffoldings. In each episode, the researchers focused on two main observations: (1) the teacher's role in making instructional decisions, and (2) both teachers' and students' cognitive demands associated with the problem. Olson and Knott found that the teacher's mindset influenced her choice of ways to engage students, questions she posed to students, and the select responses students provided. On the other hand, students' mindsets, guided by these episodes, led to their active participation. This include frequently asking questions, doing mathematics, and choosing comparatively difficult problems to solve when given multiple choices. These results show that clear problem-posing procedures and explicit lesson plans can inspire an approachable teaching and learning environment.

As mentioned before, problem posing can occur before, during, or after problem solving. Silver, Mamona-Downs, Leung, and Kenney (1996) found that teachers posed more problems before problem solving than during or after problem solving. In addition to this, there are few studies that focused on other features of posed problems at different problem-solving stages. Therefore, the effects of clear problem-posing procedures on prospective teachers' problem-posing performance and the principles of developing clear problem-posing procedures need further examination.

**Categories of posed problems.** Classifying the posed problems is an important method for researchers to measure problem posers' problem-posing capability, reasoning patterns, and mathematical content knowledge. Stein, Smith, Henningsen, and Silver (2000) classified the problems used by in-service teachers, some of which were selected from other sources while some were generated by the teachers, during their teaching in two lower level and two higher level categories. The lower levels included memorization tasks and tasks involving procedures without connections while the higher levels included tasks involving procedures with connections and doing mathematics. Olson and Knott (2013) explored the impacts of different levels of posed problems on students' learning. They found that providing multiple levels of problems for students strengthened teachers' belief in students' growth mindset. In one example, after the instructor carefully selected the problems and posed them to the class, students usually chose the higher level problems to work on. This indicated the significance of posed problems that covered multiple levels of difficulty for the multiple needs of students' mindsets.

Some studies have explored the categories of posed problems from other perspectives. Silver, Mamona-Downs, Leung, and Kenney (1996) asked 53 in-service and 28 pre-service teachers to pose problems for the Billiard Ball Mathematics Task. They classified 334 posed problems into the following categories: (1) a goal problem posed by manipulating the problem-solving goals but fixing the constraints; and (2) two types of constraint-manipulation problems that changed (a) initial conditions (i.e., the explicitly stated conditions), and (b) implicit assumptions (i.e., the underlying assumptions). Data analysis also displayed some relationship among the posed problems, which included (1) chaining - a series of posed problems that have a sequentially linked character, (2) systematic variation - a critical aspect of a problem held constant while others are varied systematically, and (3) symmetry – where goals and conditions are symmetrically exchanged. This study suggested that, although both in-service and pre-service teachers had personal capacity to pose problems of different categories, they had a disappointing percentage of inadequate problems because many posed problems were ill-structured or poorly stated.

Shuk-kwan (2013) examined 60 in-service elementary teachers' techniques, challenges, and strategies when implementing child posed problems or any asked questions in the classroom. Shuk-kwan classified children's posed problems into five categories: (1) not a problem (i.e., a description or a phrase, but not a problem), (2) a non-math problem (i.e., a problem but not a mathematical problem), (3) an impossible problem (i.e., a problem in mathematical form but unsolvable), (4) an insufficient problem (i.e., a mathematical problem with insufficient information and could not be solved), and (5) a sufficient or extraneous problem (i.e., a solvable mathematical problem). Although the in-service teachers were trained in a seminar on implementing child posed problems, the coding methods of posed problems, and the use of coding results in teaching, they were still confused when distinguishing the following paired categories: (1) not a problem and non-math problem and (2) impossible and insufficient problems.

During the second seminar, for collecting teachers' feedback on implementing problem posing in class, 33% of the teachers said that they still needed help from colleagues when classifying students' responses, and 28% found it hard to use the code categorization scheme, while 15% had no opinion. Shuk-kwan (2013) suggested that more examples should be provided to those teachers who felt it was difficult to use the coding scheme to get a deeper understanding of the categories of non-math and impossible and insufficient problems. The researcher also claimed that children's posed problems helped teachers notice their understanding or misunderstanding of mathematics concepts, while the exercise of classifying children's responses enabled the teacher educator to see to what extent the teachers understood the categorization scheme. These findings indicate the valuable role of a categorization scheme in posed problems in both teaching and learning areas.

In addition, Shuk-kwan's (2013) study provides evidence for integrating a categorization scheme for posed problems into teacher preparation programs. First, children generated a higher percentage of plausible mathematics problems when a teacher educator was working closely with teachers and having the teachers practice problem posing before enacting tasks in the class. In this case, the teacher educator did not directly work with the children. In contrast, children could not pose reasonable mathematics problems when the tasks were directly presented to them by the teacher educator. Second, teachers felt challenged because children were capable of posing similar problems with the problems posed by themselves, and they sometimes had trouble dealing with those problems. These results impart the necessity of collaboration between teacher educators and teachers for integrating problem posing in classroom teaching. Finally, Shuk-kwan found that about half of the 60 teachers had negative feelings about the implementation of mathematical problem posing in class, while only seven teachers reported negative reactions from their students about this implementation. This demonstrates that it is also necessary to enhance pre-service teachers' beliefs and confidence in problem posing.

**Task selection.** Two main divisions of task design for problem posing have emerged in the research literature. The first division involves specific mathematical, or daily-life situations and focuses on answering the “what-to-pose” question. For example, Chen, Van Dooren, Chen, and Verschaffel (2011) provided division-with-a-remainder items for 128 pre-service and in-service elementary teachers from China to pose story problems; Contreras (2007) used a task related to the median of an isosceles triangle; and Toluk-Uçar (2009) conducted a study for pre-service teachers’ in-depth understanding of fractions through problem posing. In addition, the Billiard Ball Mathematics Task is a popular task utilized in many studies (e.g., Koichu & Kontorovich, 2013; Silver, Mamona-Downs, Leung & Kenney, 1996) because it is rich enough for posers to generate interesting problems or conjectures, but it is accessible enough for posers with only basic mathematical conceptual knowledge.

Leung and Silver (1997) conducted a study examining the impacts of different types of situations on pre-service teachers’ problem-posing performance. They created the Test of Arithmetic Problem Posing by providing real-world situations with and without specific numerical information for 63 elementary pre-service teachers. The results demonstrate that the pre-service teachers’ mean performance was better on tasks with specific numerical information than their performance on tasks without specific numerical information. More specifically, they posed more plausible and fewer non-mathematical problems on tasks with specific numerical information. Furthermore, the differences were statistically significant for the measures of quality and the complexity of posed problems. This study provides evidence for the necessity of testing the impacts of task variables on participants’ problem-posing performance.

Similarly, Isik and Kar (2012) considered pre-service elementary teachers’ problem-posing performance within formal symbolic contexts related to real world situations that could be solved by using linear equations or a system of linear equations. They found that pre-service teachers were challenged by several types of difficulties: (1) a conceptual understanding difficulty, such as incorrectly translating the meaning of a mathematical operation in the equation into verbal problem statements, or posing separate problems for each equation in a system; (2) contextual difficulties, such as assigning unrealistic values to the unknowns; and (3) violations of the conventions of word problems, such as using symbolic representations in the posed problems. These difficulties may or may not occur in other problem-posing situations, such as posing a problem involving division-with-a-remainder consideration (Chen, Van Dooren, Chen, & Verschaffel, 2011) and posing a problem according to a daily-life situation without specific numerical information (Leung & Silver, 1997). This indicates that problem posers will encounter specific difficulties in different problem-posing tasks.

The second division references specific principles for subjects to follow when posing mathematical problems and focusing on answering "how-to-pose" question. A typical example is the three problem-posing situations (i.e., a free situation, a semi-structured situation and a structured situation) developed by Stoyanova and Ellerton (1996). Another example is the nine categories of problem-posing tasks developed by Chapman (2012). Prospective elementary teachers are asked to pose a problem "(a) of their own choice, (b) similar to a given problem, (c) that is open-ended, (d) with similar solution, (e) related to a specific mathematics concept, (f) by modifying a problem, (g) using the given conditions to reformulate the given problems, (h) based on an ill-formed problem, and (i) derived from a given picture" (p. 138). Chapman found that the participants were challenged most by posing problems that were (1) open-ended, (2) related to specific mathematics concepts such as the meanings of multiplication, and (3) derived from a given picture of a mathematics concept such as a picture describing comparison subtraction.

These findings indicate that different tasks have different difficulty levels and this should be considered when designing problem-posing tasks for a specific group of participants.

Crespo (2003) designed a comparatively different problem-posing task for 34 pre-service teachers by asking them to either participate in small group teaching for grade six/seven pupils or exchange mathematics letters with one or two fourth graders weekly. She found that the features of posed problems at the beginning of the study were (1) easy-to-solve, (2) familiar with given story problems or computational exercises, and (3) blind, namely, posing problems without solving or deeply understanding, or asking questions that lack awareness of the potential and scope of problems. However, the latter posed problems had significantly different characteristics: (1) unfamiliar problems were tried; (2) posed problems could challenge pupil's thinking; and (3) posed problems were used to study a pupil's thinking by pre-service teachers. These results showed that the interaction between pre-service teachers and students were beneficial to develop pre-service teachers' problem-posing initiative and performance. In addition, such problem-posing tasks had positive impacts on pre-service teachers' academic and professional development. Meanwhile, this study showed that the changes in pre-service teachers' views and skills on problem posing did not happen overnight. In short, this study developed a model for measuring pre-service teachers' development of problem-posing ability, and it could be used regularly to examine the effectiveness of incorporating problem posing into teacher preparation programs.

In summary, existing studies have evidenced the significance of selecting different types of problem-posing tasks for preparing pre-service teachers' problem-posing abilities. First, the listed difficulties indicate that the difficulty level of a problem-posing task is related to the type of a mathematical problem, mathematical concepts involved in the task, ways of representation, and other specific characteristics. Second, in order to pose high-quality problems that are based on formal symbolic contexts, teachers need to build their conceptual understanding of the underlying mathematics and their pedagogical understanding. In addition, the Test of Arithmetic Problem Posing, developed by Leung and Silver (1997), can serve as a model for measuring subjects' mathematics content knowledge and exploring the influence of other variables of tasks on problem-posing performance.

Finally, the nine categories of problem-posing tasks developed by Chapman (2012) are especially beneficial for new problem posers or problem posers who are challenged. This is because these tasks are " presented one at a time in an intentional sequence to minimize the influence of one task on participants' thinking of another" (p. 138). Therefore, it is a good strategy to provide these tasks to pre-service teachers at the beginning of their problem-posing practice.

**Discussion and implication for mathematics learning.** Generally speaking, pre-service teachers are able to pose mathematical problems, and their problem-posing capability can be improved by a certain amount of practice and systematic training. According to this review of literature, the use of different problem-posing strategies asks posers to manipulate attributes of a given problem, givens and goals, as well as both implicit and explicit relationships involved in the structure of given problems or situations. Clear problem-posing procedures can lead learners to active participation in doing mathematics. It can also be helpful in making connections between problem posing and problem solving. The categorization scheme provided in a problem-posing task can bring different perspectives of interpreting mathematical problems. Different types and levels of problems encourage learners' creativity and meet their different mindsets. Finally, problem-posing tasks with distinct variables or features place emphasis on building different mathematics skills and aspects of mathematical knowledge. Therefore, prospective teachers' conceptual knowledge can be developed by properly selecting problem-posing tasks, to some extent. In short, all the perspectives of problem posing discussed above can help learners discover the patterns and relationships involved in mathematics, make connections among different operations or representations, and gradually change their attitude on mathematics learning.

**Problem Posing Situated in the Problem-solving Literature**

This section describes how problem posing is situated in the large body of problem-solving activities or classes, and how engaging students in both problem-posing and problem-solving activities supports student learning. Approaches can include (1) situating problem posing in different stages of the problem-solving process, or in problem-solving classes, (2) situating problem posing, together with problem solving, in different contexts, usually involving multiple types of artifacts, namely, real or reproduced materials that students typically see in real-life situations, and (3) adding technology as scaffolding to problem-posing and problem-solving activities. These approaches are categorized into subsections while the theoretical foundations and guiding frameworks are discussed within each subsection.

**In different stages of the problem-solving process or problem-solving classes.** Silver (1994) states that problem posing can occur before, during, and after a problem-solving activity. More specifically, he suggested that problem posing could be applied to three different forms of mathematical activity: (1) pre-solution posing, in which students are required to generate original problems from a given situation; (2) within-solution posing, in which students are required to reformulate a problem that is being solved; and (3) post-solution posing, in which students are required to generate new problems by modifying the goals or conditions of a solved problem.

Silver and Cai (1996) administered a pre-solution problem-posing task with a distance problem to 509 sixth and seventh grade students in order to investigate their problem-posing performance and its relationship with their problem-solving performance. Silver and Cai did not ask students to solve their own posed problems, but focused on measuring students' original problem-solving capability because, their problem-solving capability could possibly be affected by their problem-posing performance. By doing this, Silver and Cai drew conclusions about the direct relationship between students' problem-posing and problem-solving performance.

More specifically, Silver and Cai (1996) divided all the participants into three groups and required each group of students to complete one problem-posing task and eight problem-solving tasks. Three groups of students followed different arrangements of those nine tasks in order to level-out the effect of task order on students' problem-posing performance. One third of the students completed the problem-posing task as the second task of nine tasks, another third completed it as the fifth task, and the rest of the students completed it as the eighth task. Silver and Cai found that middle school students were capable of generating appropriate mathematical problems and, more importantly, "a considerable number of students were able to generate syntactically and semantically complex mathematical problems" (p. 534). By exploring the features of each set of three problems posed by individual student, Silver and Cai found that nearly half the students showed either symmetric responses (i.e., responses that reflected relationship between or among given information or other imputed mathematical objects), or chained responses (i.e., responses that require the use of information provided by the solution of earlier posed problems), or both. This indicates that middle school students have the capability to use the process of association, which was argued by Kilpatrick (1987) as one of the basic cognitive processes when posing problems.

Silver and Cai (1996) also found that "good problem solvers generated more mathematical problems and their problems were more mathematically complex" (p. 535) than the less successful problem solvers. However, they did not discuss whether those three groups of students performed differently or similarly in either problem posing or problem solving. But, the three groups of students completed the problem-posing task at different stages in eight problem-solving tasks. Therefore, it is unclear whether the order or amount of problem-posing and problem-solving practice affected students' performance. Among existing studies, only the study conducted by Silver, Mamona-Downs, Leung, and Kenney (1996) with in-service and pre-service teachers concluded that teachers posed more problems before problem solving than during or after problem solving. The researchers did not further expound on this finding. Therefore, we do not know what variables may have influenced the teachers to pose more problems before problem solving.

Mathematicians, mathematics educators, and educational institutions consider problem posing as one of the core elements of mathematical proficiency. The Common Core State Standards for Mathematics Practice, MP5: Use appropriate tools strategically, claims that mathematically proficient students should be able pose or solve problems by identifying and using relevant external mathematical resources (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) advocated for teachers to regularly provide students problem-posing opportunities according to a wide variety of situations. Problem posing has been more or less incorporated in mathematics courses by experienced teachers, or by using textbooks with certain problem-posing tasks. However, only a few studies have investigated the effects of problem posing when incorporated in problem-solving classes, and the studies on long-term effects are even fewer.

Contreras (2007) implemented his problem-posing framework in a class with prospective secondary teachers. His framework consisted of five fundamental mathematical processes including proving, reversing, specializing, generalizing, and extending based on a given problem. Contreras employed an instructor-centered approach to implement this framework when his students had no prior problem-posing experience. He then used a student-centered approach to ask the prospective teachers to independently pose problems under the guidance of this framework. The problem-posing activity in his study lasted more than two class periods (about three hours). Contreras found that each step of the framework helped prospective teachers discover mathematical patterns and relationships from different perspectives. The teachers' errors and the difficulty with the material were easily observed.

Another study that integrated problem posing in a mathematical classroom for a comparatively long period of time was conducted by Beal and Cohen (2012). Their study lasted for twelve weeks, and was based on a web system in which middle school students posed mathematical and scientific problems and then solved problems posed by their peers. Beal and Cohen concluded that students were able to create problems and both teachers and students had positive responses to such an activity. At the same time, they found that teachers found it challenging to review and approve students' posed or solved problems. However, Beal and Cohen's study did not assess for student learning. Therefore, although the students themselves in this study stated that they learned the most from solving their peers' posed problems, it is difficult to determine whether their mathematical understanding was actually improved or developed more by posing or solving problems.

To summarize, researchers have integrated problem posing in either problem-solving activities or mathematics classes in order to further understand the effects of these two activities on students' mathematics learning. However, few studies have explored the different impacts of problem posing (problem solving) at different stages of the problem-solving (problem-posing) process, let alone the underlying causes of those differences. Only a few studies have incorporated problem posing in problem solving, or regular mathematics classes in a comparatively long period of time. Therefore, more research is needed to investigate the ways of incorporating problem posing for a long time and the methods of assessing students' academic development during a long exposure to problem-posing activities.

**In different contexts that usually involve multiple artifacts.** As discussed previously, all three problem-posing situations developed by Stoyanova and Ellerton (1996) require problem posers to actively explore the structure of given information and connect to the mathematical knowledge and problem-solving skills that they have learned prior. Within these situations, problem solving is usually used as scaffolding to investigate the relationship between problem-posing performance and other mathematical performance indicators, such as conceptual understanding or creativity.

Bonotto and her colleagues (Bonotto, 2010a; Bonotto & Dal Santo, 2014) arranged for groups of students to visit the Italian amusement park "Mirabilandia" and used a semi-structured situation to investigate the following questions in two separate studies: (1) What were the impacts of problem posing on student learning when it was implemented by using the aforementioned situation which involved both suitable artifacts and mathematics knowledge (Bonotto, 2010a)? (2) What was the relationship between students' creativity and their problem-posing and problem-solving performance when engaged in that situation (Bonotto & Dal Santo, 2014)? All students were familiar with the park which ensured the attraction of the tasks to them.

Eighteen fifth-grade students participated in the study. They answered the first question and completed an individual problem-posing activity, a problem-solving activity in pairs, and a whole group discussion on possible errors and incoherencies of posed problems. Bonotto (2010a) found that, "contrary to the practice of traditional word problem solving, children do not ignore the relevant and plausible aspects of reality, nor did they exclude real-world knowledge from their observation and reasoning (p. 24)." This shows that problem posing based on a familiar situation, and supported by problem-solving thinking, provides students opportunities to explore, compare, and select meaningful information.

In addition, this study created an environment for problem critiquing in which students were encouraged to analyze the structure of mathematical problems, make corrections for ill-structured problems, and discover new possibilities. Problem solving alone does not always allow students access to these learning opportunities. Bonotto (2013) claimed that "the children showed remarkable originality due to their wide variety of experiences outside of school, which involved different and complex aspects" (p. 44). This shows that problem posing as a novel learning approach, together with suitable artifacts (Bonotto, 2013), is not only able to attract students' attention, but also to facilitate their creativity (i.e., three components including fluency, flexibility, and originality) enhanced by their real-life experience.

Bonotto and Dal Santo (2014) conducted an additional study to further examine the relationship between students' creativity and their problem-posing performance. The same semi-structured situation was used with 71 fifth grade students from two primary schools in northern Italy. They further evidenced that the pupils were able to deal with real-life situations. The results revealed that students who had prior problem-posing experience did not show higher levels of creativity within posed problems compared to students who had no prior problem-posing experience. The researchers claimed that students' creativity in mathematics might not be highly correlated to their prior problem-posing experience, but is related to their academic performance in mathematics.

Bonotto and Dal Santo (2014) further claimed that problem solving after a problem-posing activity had a series of positive effects on student learning. More specifically, problem solving after problem posing allowed students to get a better understanding of the initial situation, analyze the structure of different mathematical problems, improve their control of the quality and types of posed problems, and therefore explore new possibilities, i.e., pose further problems. In a word, engaging students in problem posing and problem solving with rich cultural-based artifacts can build a bridge between students' school mathematics and their out-of-school experiences.

The two studies above use a theory of situated cognition. As Driscoll (2005) claimed, situated cognition demonstrates the adaptability of human thought to the environment, and the development of what people perceive, think, and do in a fundamentally social context. More specifically, students from the two studies showed their understanding of the amusement park "Mirabilandia" in both daily-life experience and mathematics perspectives. They also had numerous opportunities to develop their mathematical knowledge and skills, take control of the quality of mathematical problems, make connections inside and outside of mathematical classes, and improve their creative thinking in mathematics.

Similar to Bonotto and Dal Santo's (2014) study, Siswono (2014) also used a semi-structured situation to investigate students' creative thinking during problem-posing and problem-solving processes. Siswono's task asked about the area and perimeter of geometric figures rather than a real-world situation. The study aimed to develop and justify a hierarchical framework to measure students' levels of creative thinking instead of only investigating the relationship between students' creativity, problem-posing, and problem-solving performance. Siswono defined three components of creative thinking (i.e., fluency, flexibility, and novelty), for problem posing and problem solving, and then created five hierarchical levels of creative thinking including very creative, creative, quite creative, almost not creative and not creative.

By conducting task-based interviews with thirteen eighth grade students, Siswono (2014) found that the students who tended to say that posing a problem was more difficult than solving one usually had higher levels of creative thinking than the students who considered problem posing was easier. This reaches common sense validity, because when a student says that problem posing is easier, he/she usually poses problems by manipulating given information or posing problems in which he/she already knows the solution. The problems usually contain a low level of creativity. Siswono claimed that, one possible reason for the relationship between students who said problem posing was easier and their lower level of creativity was that those students rarely faced problems in the classroom that required them to be creative. These problems were common to their level of schooling. Therefore, Siswono therefore suggested that teachers should value the opportunity to justify and promote students' creative thinking using indicators such as novelty, fluency, and flexibility via problem-posing and problem-solving tasks.

In order to provide a coherent picture of students' problem-posing thinking, Christou, Mousoulides, Pittalis, Pitta-Pantazi, and Sriraman (2005) proposed a theoretical model that described four processes of young students' cognitive thinking when dealing with the quantitative information, of a given situation, during the problem-posing process. The four processes include: (1) editing the information in order to pose problems without any restriction;

(2) selecting the information in order to pose problems that are appropriate to specific answers; (3) comprehending information in order to pose problems from given equations or calculations; and (4) translating information in order to pose problems from given graphs, diagrams or tables. Christou et al. designed a quantitative study to validate this model with 143 sixth-grade students. Although specific problem-solving activities were not included, students experienced problem solving because the majority of the tasks asked students to write problems with specific answers or mathematical operations.

Christou et al. (2005) found that the four processes not only significantly represented four distinct functions of student thinking, but also measured students' problem-posing performance. More specifically, they found that "both the editing and selecting process characterized the most able students" (p. 156) while students with lower problem-posing performance were usually able to respond to only comprehending and translating tasks. They argued that students who tended to use the Comprehending process to begin posing a problem may be influenced by their classroom experiences or problems from textbooks that emphasized the algorithmic ways of thinking at the expense of the other three types of thinking processes. In a word, the model explains how problem posing happens and what students think about during problem-posing processes. Therefore, this model can help teachers and teacher educators determine what should be observed during students' problem-posing processes.

To summarize, problem posing, together with problem solving, situated in different contexts, including both real-life and mathematical situations, which usually involve multiple types of artifacts, is beneficial for sustaining students' learning interests. This process deepens their mathematical knowledge, enhances real-life experiences, and encourages their creativity in mathematics. Problem posing can also help teachers and teacher educators justify how student learning happens within specific contexts, and what tasks or learning opportunities are more appropriate to provide at certain times.

**In technology-based situations.** Leung (1993) adapted Polya's (1945) four problem-solving steps to four problem-posing steps: pose problems, plan, solve posed problems, and look back. Chang, Wu, Weng, and Sung (2012) developed and implemented Leung's model (see Table 2.1) in a game-based computer system in order to investigate the effects of this system on students' problem-posing and problem-solving performance as well as their flow learning experience (i.e., learning engagement with personal satisfaction and/or motivation). The steps of the model indicate that problem-posing and problem-solving activities can easily form a cycle to continuously foster students' analyzing and synthesizing abilities in mathematics.

Table 2.1

*Problem-posing Steps*

| Steps | Problem-solving phases (Polya, 1945) | Problem-posing phases (Leung, 1993) | Proposed problem-posing procedures |
|---|---|---|---|
| 1 | Understand | Pose problem | Self-posed problems |
| 2 | Plan | Plan | 1. Attempt to solve self-posed problem<br>2.Obtain feedback from the teacher<br>3. Judge whether the solution is reasonable<br>4. Refine problems |
| 3 | Carry out | Carry out | Solve posed problems in the game-based setting |
| 4 | Look back | Look back | 1. More feedback from the teacher<br>2. Get new ideas and be prompted to create new problems |

In a quasi-experimental study with 92 elementary students with an average age of 11 years old, Chang, Wu, Weng, and Sung (2012) compared the performance of an experimental group that experienced the game-based problem-posing system and a control group that experienced the traditional paper-based instruction. They found that problem posing encouraged students to integrate their prior knowledge and become aware of their cognitive processes. Problem solving provided further opportunities for them to reflect on, and adjust their problem-posing skills.

In addition, this game-based system brought a statistically significant higher state of flow experience for students in the experimental group than students in the control group. More specifically, the system helped sustain students' interests by offering potential challenges in problem-posing and problem-solving activities, and immediate feedback from teachers and the system. System feedback included the time that individual students spent on activities, scores on correctly solved problems, etc. This system allowed students to easily revisit the materials they had encountered, which helped them to make connections between givens and goals and among different posed problems. The system was even more effective in improving the performance of students with low problem-solving scores.

Abramovich and Cho (2006) developed a spreadsheet-based environment by using Microsoft Excel 2004 to help pre-service elementary teachers and their students pose and solve open-ended mathematical problems, more specifically, money sharing and changing problems. They found that the use of a spreadsheet within the context of problem posing helped to facilitate pre-service teachers' sense-making process, including connecting different layers of the problem structure, systematically reasoning about the varying parameters of a situation, and giving additional attention to the solvability of posed problems. Abramovich and Cho additionally claimed that this technology was commercially available and had almost no financial constraints. Therefore, it is potentially easy to adopt in classrooms.

Elwan (2007) conducted an experimental study to investigate the impacts of using WebQuest on pre-service teachers' problem-posing ability and beliefs. The results showed that pre-service teachers from the experimental group performed significantly better on a problem-posing achievement test developed by Elwan than pre-service teachers from the control group (i.e., the participants in the experimental group got significantly higher mean score, 27.8>16.6, out of 36 in total). Additionally, pre-service teachers from the experimental group showed significant changes in their beliefs toward the role of problem posing in mathematics education after using WebQuest (i.e., the participants in the experimental group showed a significant difference in mean scores of beliefs between pre- and post-WebQuest use).

The studies discussed in this section show that technology-based scaffoldings have positive impacts on problem-posing performance and mathematics learning for both students and teachers. The uses of those technology scaffolds should be taught to students, especially the students with a low level of problem-solving performance, to help them monitor their learning, promote their self-perception, and sustain their motivation to learn. Such strategies should also be taught to prospective teachers because they will utilize such strategies in classroom teaching in the future.

**Discussion and implications for mathematics learning.** This body of literature examimined three different approaches that situated problem posing within the large problem-solving context. The approaches are beneficial for developing students' problem-posing and problem-solving skills, sustaining their learning interests, and improving their academic performance in mathematics. More specifically, these approaches can help learners to develop an understanding of mathematical problems (e.g., quality and complexity levels of mathematical problems, structure and different types of mathematical problems), the capability to use the process of association (e.g., posing problems that are related or in a sequence, connecting relevant and plausible aspects of reality to mathematical knowledge and problem-solving skills), the capability of mathematical reasoning (e.g., comparing different situations and problems, selecting suitable information and discarding irrelevant information), and creativity in mathematics (e.g., flexibly manipulating the attributes of a given problem, posing innovative problems according to given information).

Regarding the interaction between problem posing and problem solving, Silver, Mamona-Downs, Leung and Kenney (1996) found that teachers usually posed more problems before problem solving than during or after problem solving. Bonotto and Dal Santo (2014) indicated that problem solving after problem posing had a sequence of positive effects on student learning. It fostered their capability to synthesize many perspectives of a given problem, including its structure, quality, type, and new possibilities (i.e., new problems that can be posed). But beyond that, few studies have explored the impacts of different arrangements of problem-posing and problem-solving activities on student mathematics learning and understanding, let alone the underlying causes of those impacts.

**International Comparison Studies**

The international comparison study in teacher education is a product of the main focus of educational reforms for teacher quality improvement during the last 50 years (Akiba, LeTendre, & Scribner, 2007). It has provided unique opportunities to better understand teaching and learning. Ma (1999) claimed that "comparative research allows us to see different things, and sometimes to see things differently" (p. xx). This indicates that the international comparison study helps to describe the outcomes of different educational systems and investigate explanatory factors that affect the quality of education (Gustafsson, 2008). With this information, a better understanding of the issues involved in a functional educational system is captured and evidence for decision making is gathered to improve the learning environment for students. These benefits also apply to teacher preparation. By analyzing the 2003 Trends in International Mathematics and Science Study (TIMSS) data involving 46 countries around the world, Akiba, LeTendre and Scribner (2007) indicated that the international comparison study could reveal the level of teacher quality in different countries, deficiencies in teacher knowledge, and gaps in their perception of their own needs. All of these were closely related to students' achievement.

This section of the literature review focused on international comparison studies with prospective teachers. Different approaches in prospective teachers' preparation in educational programs were synthesized as well as their academic performance including their mathematics content knowledge (MCK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK) (Shulman, 1986), their beliefs in mathematics and mathematics teaching, and comparison studies particularly related to problem posing.

**Prospective elementary teachers' preparation.** In order to gain a broad view of the challenges in teacher preparation across different countries, Comiti and Ball (1996) examined the trends of teacher preparation in England, France, Germany, and the United States. After closely comparing French and US educational program curriculum, the researchers concluded that four crucial issues were central to teacher preparation. The first two issues involved challenges teacher educators faced in both countries, including (1) making significant changes in perspectives, such as prospective teachers' views of how mathematics is taught and learned and how students think about mathematics, and (2) improving prospective teachers' pedagogical understanding in mathematics with limited hours for pedagogical practice and a poor structure of the content addressing their needs.

The third crucial issue was the challenge of finding proper ways to prepare teachers, especially those in the US who could teach but had little teaching experience, and those methods were not widely used in elementary or secondary schools. The last crucial issue concerned the connection of the program components across institutions. More specifically, program courses and field experiences were usually unbound. French programs did a better job of structuring those settings. These four crucial issues illustrate that comparison studies can help illuminate both common and specific problems in specific countries.

While the proposed issues were considered by comparing education programs in different countries, some studies have investigated more detailed indicators and educational program outcomes. Yuan and Han (2009) used prospective elementary educational programs in Shanghai Normal University and City University of New York as examples and compared their mathematics content and methods courses. They found that the two programs were different in credit requirements, teaching objectives, teaching methods, and course content. Overall, the researchers claimed that, although Chinese prospective elementary teachers were taught more advanced mathematics knowledge, those knowledge had little relatedness with elementary mathematics curriculum; although Chinese prospective elementary teachers learned basic teaching theory and methods as well as technology use comprehensively and systematically, they lacked field experience for effectively coaching elementary students. The researchers suggested Chinese teacher educational programs to provide prospective elementary teachers more opportunities to understand children's thinking and corresponding teaching methods.

Chen and Mu (2010) compared the mathematics teacher education and curriculum structure in China, Singapore and the US. They observed that Chinese prospective elementary teachers were usually trained in junior college programs while Singaporean prospective teachers were required to obtain a higher-level postgraduate diploma, and US prospective teachers needed to attend an education program for a bachelor's or even master's degree. In addition, the teacher training in China was usually independent from local schools while both US and Singaporean modes showed a closer partnership between universities and school districts. This information suggested that the training mode in China was at a comparatively low level.

Regarding curriculum structure, both Singaporean and US courses “have evolved in the last few decades from being rather theoretical and psychology-based to a combination of theory and practice” (Chen & Mu, 2010, p. 133). However, the Chinese courses still focused on academic learning, but lacked educational studies and practices. Finally, in consideration of content selection and teaching method, Chen and Mu found that, in order to combine theory and practice instead of over-emphasizing the content knowledge and learning theory, both Singapore and the US provided prospective teachers with multiple-dimensional learning opportunities such as an education internship in classrooms and elective courses to link to primary and secondary school curricula. These approaches were helpful for turning the training mode “from a teaching-training model to a reflective practice model” (p. 134). All these approaches were deficient in the Chinese teacher educational system.

To summarize, teacher preparation has been investigated from many perspectives via comparison studies involving different countries. The findings can be used to make decisions about how to improve educational programs’ curriculum structure and content, or gradually overcoming the challenges that each educational program faces by learning from other institutions around the world.

**Prospective elementary teachers’ academic performance.** Shulman (1986) classified the knowledge that a teacher needed to know into three categories: (1) mathematics content knowledge (MCK), which refers to mathematical concepts, ideas, and facts as well as the relationship among them, and the ways of creating and evaluating new knowledge of this discipline; (2) pedagogical content knowledge (PCK), which refers to the understanding and knowledge of ways to make mathematics content comprehensible to learners; and (3) general pedagogical knowledge (GPK), which refers to the knowledge of teaching and learning theories and principles, knowledge of students, and knowledge of classroom behavior management principles and techniques. This section synthesized international comparison studies about these three categories.

***Comparison studies on MCK.*** Teachers' MCK has a significant influence on designing and teaching high quality lessons and therefore plays a large role in student achievement (Kahan, Cooper & Bethea, 2003). In order to better prepare teachers' MCK, international comparison studies have tried to identify contributing factors that lead to different levels of content knowledge for teachers from different countries. Luo, Lo, and Leu (2011) investigated 89 US and 85 Taiwanese pre-service elementary teachers' fundamental fraction knowledge by assessing with 15 multiple-choice items that covered the meaning of fractions, equivalent fractions, and the meaning of fraction operations.

Luo, Lo, and Leu (2011) found that Taiwanese pre-service teachers performed much better than their US counterparts with a statistically significant difference on 12 out of 15 items between the two groups. The US pre-service teachers were more challenged by linear model problems, especially when working with a number line. In addition, both US and Taiwanese pre-service teachers needed more preparation on the meaning of multiplication and division with fractions. In light of this information, we now know more about a specific group of pre-service teachers' difficulties and conceptual misunderstanding that may cause issues in their understanding of fractions.

***Comparison studies on PCK***. Blömeke, Suhl, Kaiser, and Döhrmann (2012) examined the influences of teacher background, opportunity to learn (OTL) in educational programs, and teacher intake on their MCK and PCK preparation. About 14,000 prospective primary teachers from 527 education programs in 15 countries participated in a 60-minutes paper-and-pencil assessment that covered 106 items during the last year of their programs. The researchers found that significant differences existed not only between different countries, but also between programs within the same country. First, teacher background was shown to partially influence the outcomes of teacher education while gender turned out to be the most important indicator among all demographic factors in the acquisition of MCK, but not with PCK. Second, participants' prior knowledge and motivation in mathematics had a significant impact on both MCK and PCK, while teacher intake and OTL played an even more important role. This study indicated the variables that influenced the outcomes of teacher education on different levels. This information assists in adapting the beneficial features of a teacher educational program from other countries and, probably more easily, from other programs in the same country because of the similar cultural background.

***Comparison studies on GPK.*** Schmidt et al. (2008) also investigated OTL, but focused on OTL in MCK and GPK with lower secondary prospective teachers. They asked 1127 participants from 34 institutions in six countries two types of questions, one about their MCK preparation and the other about their GPK preparation. The results illustrate that the prospective teachers experienced different opportunities and ways of preparation across the six countries. Three components emerged, the coverage of mathematics content, theoretical pedagogy, and practical pedagogy. The biggest difference across countries was practical pedagogy. The researchers claimed that the way of structuring these three components was related to teacher preparation outcomes.

Brown and Remesal (2012) investigated 996 prospective teachers' perceptions of the assessment inventory in Spain and New Zealand, focusing on four major intentions: (1) assessment improves teaching and learning, (2) assessment is irrelevant to teaching and learning, (3) assessment indicates school accountability, and (4) assessment indicates student accountability. The participants included prospective elementary and secondary teachers, infant school teachers, and other prospective teachers.

By analyzing participants' responses to 27 items that covered all four intentions, Brown and Remesal (2012) found that there was a big difference between teachers from two countries. Prospective teachers from New Zealand were more likely to agree that assessment measured school quality and grade students, and therefore improved teaching and learning. On the contrary, Spanish prospective teachers were more likely to agree that assessment was irrelevant or bad. The researchers claim that such big difference was consistent with each country's use and purpose of assessment. More specifically, the researchers concluded four possible reasons for the difference. These included cultural factors, context factors, instructional effect, and demographic differences. The findings indicate that prospective teachers' perceptions towards assessment are culturally situated. Although it is possible that there are some other factors that impact prospective teachers' perception on assessment, this study has made one step towards "a better understanding of cross-national elements related to the psychology of assessment" (Brown & Remesal, 2012, p. 14) of educational programs.

**Prospective elementary teachers' beliefs.** Research studies have shown that "what teachers believe is a significant determinant of what gets taught, how it gets taught and what get learnt in the classroom" (Xenofontos & Andrews, 2014, p. 283). What teachers believe includes their beliefs in both mathematics and mathematics teaching. This section discusses the international comparison studies in regard to these two areas separately.

***Beliefs in mathematics.*** Cross-national learners' beliefs in mathematics are culturally situated and their conceptual understandings are cross-nationally different. By considering mathematical problems and problem solving in particular, Xenofontos and Andrews qualitatively compared Cypriot and English prospective primary teachers' beliefs via semi-structured interviews right as they entered the education program (2012) and as they were exiting the program (2014). The area of mathematical problem and problem solving was selected because "both Cyprus and English present mathematical problem solving as a major curricular goal" (p. 279). This fact made the two groups more comparable.

At the point that the prospective primary teachers entered their programs, Xenofontos and Andrews (2012) found that both Cypriot (N=13) and English (N=14) participants showed consistent beliefs in the nature of mathematical problems and problem solving and the characteristics of problem-solving experts. But they focused on different interpretations. More specifically, Cypriot participants expected that mathematical problems should be well defined, without ambiguity, related to real-world context in some sense, and difficult for problem solvers, while English participants did not show such expectations but interpreted a mathematical problem as task that required number operations.

In addition, although both groups of participants described mathematical problem solving as a process, Cypriot participants described the problem-solving process as a sequence of actions while English participants considered it as a way of reducing the initial task down to small steps. Finally, Cypriot participants considered a problem-solving expert as someone who could solve given problems quickly and accurately while English participants described a problem-solving expert as someone who could apply his or her prior knowledge effectively. In consideration of the fact that these prospective teachers were at the point of entering their education programs, the researchers highlighted the possible influence of the prospective teachers' past learning experience on their beliefs.

At the point that the prospective primary teachers (N=12 for each country, not the same groups of teachers from the previous study) were exiting their programs, Xenofontos and Andrews (2014) again compared their beliefs about mathematical problems and problem solving. More specific themes of similarities and differences in prospective primary teachers' beliefs emerged. First, both cohorts expressed that mathematical problems were related to real-life situations. In addition, over half of the Cypriot participants described schema theory (e.g., four arithmetic operations) and a need for problems with generic strategy while English participants emphasized the feature that a mathematical problem has no direct solution. For mathematical problem solving, most Cypriot participants believed that problem solving was a cyclic process while their English counterparts emphasized the important role of basic and/or prior knowledge in problem solving. In consideration of the features of curricula and textbooks of each educational program, researchers concluded that prospective teachers' beliefs in mathematical problem and problem solving were culturally situated and highly related to curricular systems.

These two international comparison studies have partially demonstrated how the curriculum system of an educational program can enhance and/or hinder prospective teachers' understanding of mathematical problem and problem solving. Curriculum designers as well as textbook writers therefore could be inspired to clearly clarify the meaning of mathematical terms and create rich opportunities to develop prospective teachers' problem-solving capabilities. In short, comparing cross-national prospective teachers' beliefs in mathematics offers insights for developing curriculum systems.

***Beliefs in mathematics teaching.*** The ultimate goal of teaching is to make student learning possible. Therefore, teachers' beliefs in mathematics teaching can be evaluated by their efficacy belief, which is defined as a teacher's judgment in his/her capability of engaging students in learning and bringing about desired student learning outcomes, even with low-level academic performance students or unmotivated students (Tschannen-Moran & Hoy, 2001). Existing studies show that teaching efficacy beliefs can impact teachers' retention in the profession, persistence when facing difficulties, openness to new teaching methods, as well as students' motivation and therefore, their learning efficiency (Coladarci, 1992; Ross, 1998; Stein & Wang, 1988).

Cakiroglu (2008) compared 141 Turkish and 104 US pre-service elementary teachers' efficacy belief by using one questionnaire with two subscales, of which one was about pre-service teachers' personal teaching efficacy beliefs while the other was about their teaching outcome expectancy. Cakiroglu found that the two cohorts both had positive teaching efficacy beliefs and high teaching outcome expectancy. However, Turkish pre-service teachers showed a statistically higher teaching outcome expectancy than their US counterparts. This showed that Turkish pre-service teachers believed more that teaching could influence student learning.

Although both US and Turkish participants easily agreed that teachers should welcome students' questions and students' inadequacy of mathematics learning could be overcome by good teaching, Cakiroglu (2008) found that the US participants were mostly concerned about teaching skills in the future while Turkish participants were mostly concerned about ways of effectively teaching concepts of mathematics. The US participants disagreed most that students' low achievement was due to teachers' teaching while Turkish participants disagreed most that extra attention given to low-achieving students could make progress in their learning.

All of these similarities and differences inspired the researcher to further compare their coursework of each program. Due to the fact that Turkish pre-service teachers had a less intense field experience when taking their methods courses than their US counterparts, Cakiroglu (2008) claimed that the significantly higher expectancy of Turkish pre-service elementary teachers "should not be immediately interpreted as a 'positive' aspect" (p. 41) of their program. In light of the above discussion, Cakiroglu concludes that new insights for enhancing pre-service teachers' efficacy beliefs could be generated by exposing one educational program to other countries' educational programs and being questioned by those programs. In other words, international comparison studies offer opportunities for teacher educators to better understand the different levels of pre-service teachers' efficacy beliefs, factors that enhance or hinder their efficacy belief development, as well as the effective approaches to enhance their beliefs.

**Comparison studies particularly related to problem posing.** In this section, I synthesized international comparison studies that were particularly about students' problem-posing performance and the relationship between problem posing and other mathematical proficiency indicators, including problem-solving performance and creativity in mathematics.

***Relationship between problem-posing and problem-solving performance.*** The problem-posing performance of both teachers and students has been investigated in many countries such as the US, China, Japan, Malaysia and Indonesia (Chen, Van Dooren, Chen, & Verschaffel, 2011; Kojima & Miwa, 2008; Rosli, Goldsby, & Capraro, 2013; Silver & Cai, 1996; Siswono, 2014). However, only a few cross-national comparison studies examined problem posing, and the majority of those studies were between the US and China. These studies were conducted mainly by Cai and his colleagues, and none of them were conducted with prospective teachers. In light of the cross-national studies on students' mathematical knowledge and problem-solving skills that reported that the finding that Asian students usually outperformed US students was not always the case when, for example, the tasks were designed for assessing relatively novel or complex problem solving (Cai, 1995; Silver, Leung, & Cai, 1995), Cai started to conduct cognitive analysis to explore more profound rationale behind the different performances by using both problem-solving and problem-posing tasks. Since then, the area of cross-national studies has been expanded because problem-posing activities were involved.

Cai (1998) conducted a study with 181 US and 223 Chinese sixth-grade students in order to investigate the similarities and differences between these two groups of students' cognitive thinking in complex problem-solving activities and student-generated problem-posing process. There were four tasks in total. The first problem-solving task was about computational exercises and the fourth problem-solving task was a division-with-a-remainder problem. The second one was a problem-posing task and the third one was also a problem-solving task. The second and third tasks were developed from the same group of figures in a pattern. For problem-solving tasks, Cai found that, although a significantly larger proportion of the Chinese than US students found a correct answer, they performed similarly "on the sense-making phase" (p. 47). In other words, Chinese and US students performed similarly in providing appropriate interpretations to their solutions. For the problem-posing task, the US and Chinese students posed almost the same proportion of extension problems, namely, problems that questioned the pattern beyond given figures. However, the proportion was small. Cai claimed that one possible interpretation of this result was that problem-posing activities were rarely incorporated in classrooms and therefore students had little experience in problem posing.

In the meantime, Cai (1998) found that students from both countries who posed at least one extension problem performed better on problem-solving tasks than the students who posed only non-extension problems, namely, problems that only questioned the given figures. This finding demonstrates that students' problem-posing performance is related to their problem-solving performance. Cai further claimed that the types of students' posed problems were seemingly related to their problem-solving strategies, and this was true for both US and Chinese students. For example, when given a pattern problem with shapes made of dots, the students who solved the initial problem by focusing on the total number of dots, but not the shape of each figure posed problems mostly about the total number of dots in different shapes of the given pattern. This finding provides one possible way of specifically looking at the connections between students' problem-posing and problem-solving thinking.

By further using pattern-based tasks, Cai and Hwang (2002) conducted a study with a total of 98 US and 155 Chinese sixth grade students to explore their generalization skills in problem solving, generative thinking in problem posing, and the relationship between their generative performance in problem-posing and problem-solving activities. Two pattern-based tasks with a similar structure asked students to first solve a sequence of problems, of which the last one needed generalization skills, and then pose three problems, beginning with an easy problem first, then a moderate one, and finally a difficult one. All collected data were students' written responses, there were no verbal protocols. As the authors claimed, collecting only participants' written responses brought some practical advantages for conducting cross-national studies. The study was fit for answering their research questions because they focused on students' generative performance in problem solving and problem posing instead of their deeper cognitive reasoning behind the problem-solving and problem-posing strategies that they chose.

Comparing to Cai's (1998) findings, Cai and Hwang (2002) further confirmed that the US and Chinese students had quite different tendencies in choosing problem-solving strategies. More specifically, the US students tended to choose concrete strategies and visual representations while Chinese students preferred to use abstract strategies and symbolic representations. This tendency was consistent with the findings of previous studies that examined US and Asian students' thinking in problem solving (Becker, Sawada, & Shimizu, 1999; Silver, Leung, & Cai, 1995). Cai and Hwang claimed that this difference might be able to explain why US students had lower rates of success when solving mathematical problems than Chinese students. They argued that concrete strategies "were more prone to errors of execution, and thus were less likely to succeed" (p. 418). This argument makes sense because, compared to abstract strategy, concrete strategy usually involves more computational steps. For example, in a pattern problem, the later an item is in the pattern, the more computational or counting steps it requires, and the higher the chance of error.

For problem-posing performance, Cai and Hwang (2002) found that the types of problem posed by US and Chinese students were quite different. More specifically, the US students posed a large group of rule-based extension problems (i.e., problems that asked about the pattern beyond the given figures or items) while their Chinese counterparts tended to pose more non-extended problems (i.e., problems that constricted within the first several given figures or items). In addition, Chinese students showed a clear trend of posing problems in a sequence. They usually started with a problem using the information given in the task, followed by an attempt to generate the pattern, and finally tried to integrate further applications of the given pattern. US students did not display this trend. In terms of the relationship between problem posing and problem solving, Cai and Hwang indicated that Chinese students showed a strong link between their problem-solving and problem-posing performance while US students showed a much weaker link. They claimed that this strong link shown by Chinese students might be attributable to a possibility that “posing a variety of problem types seems to be strongly associated with abstract strategy” (p. 419) used for problem solving, which was almost never used by US students.

Both Cai’s (1998) and Cai and Hwang’s (2002) studies provide evidence about the similarities and differences between US and Chinese students’ problem-posing and problem-solving performance, as well as the possible relationship between problem posing and problem solving. More specifically, the relationship between the types of posed problems and the problem-solving strategies used by students, from a cross-national perspective. However, these two studies only collected students’ written responses in problem-posing and problem-solving tasks, while no verbal protocols were considered. Therefore, these two studies did not dig into students’ cognitive thinking processes or ways of reasoning during the problem-posing and problem-solving processes.

***Relationship between students’ problem-posing performance and creativity in mathematics.*** Yuan and Sriraman (2011) also conducted a cross-national study with US and Chinese students, but from a different perspective from Cai and his colleagues’ work. They investigated the relationship of students’ creativity and their problem-posing performance using a quantitative approach. Three tests were administered to 55 Chinese and 30 US high school students measuring their creative thinking, problem-posing ability, and mathematics content knowledge, respectively. For students’ creativity in mathematics, Yuan and Sriraman found that the US students were more capable of using words to express their ideas than Chinese students, while their capability to express ideas by drawing pictures were similar. They also found that

Chinese students showed a statistically significant correlation between their creativity and two perspectives of problem-posing ability (i.e., fluency and flexibility) while no significant correlation was found between the US students' creativity and any perspective of their problem-posing ability (i.e., fluency, flexibility, and originality).

For students' problem-posing performance, Yuan and Sriraman (2011) found that both the mathematics content areas involved in US and Chinese students' posed problems and the types of posed problems were different. Chinese students mainly posed problems about combination and permutation while the US students mainly posed problems about Arithmetic. Chinese students' posed problems were categorized into ten different types while US students' posed problems were categorized into eight types, excluding transformation and proof problems. By taking students' performance on the test for measuring their mathematics content knowledge into consideration, Yuan and Sriraman claimed that "the superior performances of Chinese students in the mathematics content test and the mathematical problem-posing test suggest that there might be some correlation between the two" (p. 26). Bonotto and Dal Santo (2014) claim that students' creativity in mathematics might have a correlation with their academic performance in mathematics, but not highly correlated to their prior problem-posing experience. In consideration of both aforementioned statements, it is possible that students' creativity, problem-posing ability and their content knowledge are correlated, while the content knowledge may be the most fundamental reason behind students' levels of creativity and problem-posing capability.

Some limitations about international comparison study were discussed in Yuan and Sriraman's (2011) study. Those limitations were brought about by the different mathematics curriculum in different countries, a non-random sample from either US or Chinese student population, the translation of the instruments between English and Chinese, and the time and distance restrictions which made the US and Chinese participants complete all four tests in different periods of time. These limitations are seemingly inescapable when conducting a comparison study between the US and China.

In short, the studies discussed above indicate that international comparison studies can serve to investigate explanatory factors that cause different outcomes of problem posing with students from different countries, instead of just describing the similarities or differences of those outcomes. US and Chinese students' different performance in problem posing are demonstrated by the types of their posed problems, mathematical contents involved in their posed problems, relationships between their posed problems and problem-solving strategies, and relationships between their creativity and their problem-posing capability. Since few international comparison studies on problem posing have been done with prospective elementary teachers, such studies have a vast potential to be able to provide new insights into ways to prepare prospective teachers for problem posing.

**Discussion and implications for mathematics learning.** In general, international comparison studies contribute by investigating explanatory factors that are able to improve the student learning environment, teacher academic and pedagogical preparation, and teacher beliefs in both mathematics and mathematics teaching. These factors provide well-grounded evidence for making decisions to enhance the quality of education in each country. In particular, international comparison studies on problem posing have investigated some explanatory factors that cause different outcomes of problem posing and have further confirmed the close relationship between students' problem-posing and problem-solving performance. For example, students who were able to pose extension problems usually outperformed on problem-solving tasks compared to students who posed only non-extension problems. In addition, the types of students' posed problems were seemingly related to their problem-solving strategies for both the US and Chinese students (Cai, 1998). These findings indicate the particular contribution that international comparison studies make to the research area in mathematical problem posing.

Although international comparison studies can either quantitatively describe the outcomes of different educational programs or qualitatively investigate explanatory factors that affect the quality of teacher preparation, the majority of the studies discussed above used a quantitative approach rather than a qualitative approach. This phenomenon is understandable, because it is consistent with the shift of focus in the International Association for the Evaluation of Education Achievement (IEA) from phase one between 1950 and 1990, during which the goal was "to generate knowledge about determinants and mechanisms behind educational achievement", to phase two since 1990, during which the goal was "to describe the outcomes of different education systems, leaving it to the different participating countries to find the explanations" (Gustafsson, 2008, p. 2). The consequence of the aforementioned shift is the need for more international comparison studies to explain the patterns that have been discovered by previous comparison studies.

## Guiding Frameworks

In consideration of the aims of this study, in this section I discuss the guiding frameworks that I used to (1) integrate problem posing and problem solving together in order to enable the justification of the features of the relationship between problem posing and problem solving, and (2) design specific problem-posing tasks for investigating problem posers' performance on different types of problem-posing tasks. Ellerton (2013) developed the Active Learning Framework for locating problem posing after problem-solving practices in mathematics classrooms. This framework was utilized in this study for developing a reasonably systematic way of engaging the participants in alternating problem-posing and problem-solving activities. Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman (2005) developed a theoretical model with four specific problem-posing processes of analyzing quantitative information in a given problem or situation. Those four processes were utilized in this study for designing specific problem-posing tasks.

**Active learning framework.** Ellerton (2013) developed the Active Learning Framework (see Figure 2.1) for locating problem posing after certain problem-solving practices in mathematics classrooms. This framework places emphasis on the process of engaging learners in problem posing, rather than only on the posed problems. In addition, this framework aims to move students from being passive receivers to active learners, and offers a promising direction for incorporating problem posing into instruction (Silver, 2013). As Ellerton (2015) claimed:

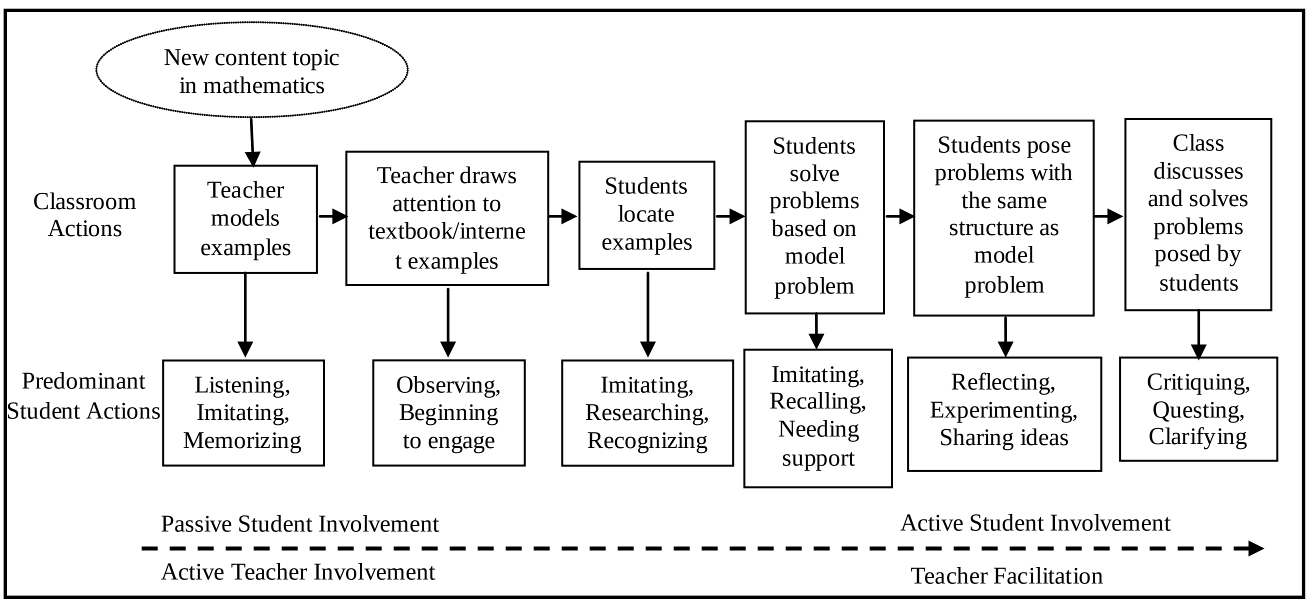



Figure 2.1: Active Learning Framework (Cited from Ellerton, 2013, p. 99)

Central to this framework is the *active* involvement of students in posing problems that not only demonstrates their understanding of the structure of the mathematical concepts

> they have been learning, but also gives students the opportunity to solve and critique the problems of others, and to reflect on and improve their own problems. (p. 516)

Ellerton (2015) utilized this framework with seven preservice middle-school teachers and four practicing middle-school teachers who were pursuing their Master degree in Mathematics in a modeling course. Ellerton usually provided students a problem-posing project that was related to real-world situations, and asked them to individually plan and draft mathematical problems, then discuss with peers and revise drafted problems, and finally present final version of problem and find a solution. The students were often involved in problem posing and problem solving in many cycles. Ellerton found that each student showed attempt to pose his/her unique, interesting and solvable problems, and they took through logic thinking and time during the alternating problem-posing and problem-solving process. The four practicing teachers also tended to put consideration of their particular students when posing problems. Ellerton (2015) claimed that this way of integrating problem posing in classroom "represents an untapped opportunity to transform routine tasks into exciting and refreshing discoveries and teachers alike" (p. 527). In conclusion, the Active Learning Framework enables students to be actively involved in both problem posing and problem solving.

**Four cognitive processes.** There are a variety of ways to classify problem-posing tasks and each way of classification focuses on a different perspective of a problem-posing activity (Chapman, 2012; Silver, 1994; Stoyanova & Ellerton, 1996). Stoyanova and Ellerton (1996) defined three problem-posing situations: (1) a free situation, in which the subjects are asked to pose problems without any restrictions or guidelines; (2) a semi-structured situation, in which the subjects are provided open-ended problems or asked to write similar problems with given problems or pose problems based on specific pictures or diagrams; and (3) a structured situation, in which the subjects are asked to reformulate solved problems or manipulate conditions and goals of given problems. All of these situations provide problem posers the opportunities to actively explore the structure of a given situation and connect to their prior knowledge, skills, and conceptual understanding. Because of these advantages, Stoyanova and Ellerton's classification has become a popular framework. Many studies use this framework, but mainly focus on investigating participants' problem-posing performance, creativity, and the relatedness between problem posing and problem solving in different situations (Bonotto, 2013; Siswono, 2014; Yuan & Sriraman, 2011).

Few studies use Stoyanova and Ellerton's (1996) framework to investigate the coherent picture of students' cognitive thinking while posing problems. This is because their framework was not purposefully designed to capture the nature of problem posing. In order to enable young students' cognitive process of problem-posing thinking to be described or measured directly, Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman (2005) proposed a model with the following processes: (1) editing quantitative information, called Editing process for short, which requires students to pose problems according to provided information such as a real-life situation or a story but without any other restriction; (2) selecting quantitative information, called Selecting process for short, which requires students to pose problems that are appropriate to specific given answers; (3) comprehending qualitative information, called Comprehending process for short, which requires students to understand the meaning of given operations before posing mathematical problems; and (4) translating quantitative information, called Translating process for short, which requires students to pose problems based on given graphs, diagrams or tables. The researchers claim that this model can be applied to many mathematical areas such as algebra and geometry.

Although the Editing process asks students to pose problems without any other restriction, it is different from a free problem-posing situation because it asks students to pose problems according to a given situation. All of the processes in Christou et al.'s (2005) model ask students to pose problems in a semi-structured situation. More specifically, those processes ask students to pose problems according to given real-world situations, graphs or tables, specific operations and answers. Therefore, Christou et al.'s model actually defines specific types of problem-posing tasks. In addition, Christou et al. speculated that those four problem-posing processes corresponded to specific problem-solving tasks that were presented in iconic, tabular, or symbolic form. In other words, different features of problem-solving tasks and potential problem-solving strategies are implicitly involved in each process of this model. The four problem-posing processes require students to pose problems and justify the solutions at the same time. Meanwhile, different problem-posing processes involve different problem-solving strategies.

Christou et al. (2005) claim that this theoretical model is able to help educators understand students' thinking process in posing problems, and each of those processes indicates specific and important components of problem-posing abilities. Existing studies have agreed that this model helps to investigate specific aspects of problem-posing process, pose subsidiary questions in a constant process, and demonstrate the close relationship between problem posing and problem solving (Bonotto, 2010b; Kontorovich & Koichu, 2009; Singer, Ellerton, & Cai, 2013).

**Discussion and implication for mathematics learning.** This section first discussed the Active Learning Framework developed by Ellerton (2013) that helps involve students in active problem-posing and problem-solving activities. However, both problem solving and problem posing are inquiry-based activities. These activities require time for students to understand the task, and then establish and revise mental models. More time should be given to problem posing, especially when students have not had such experiences before. Since my task administration would be conducted outside of classrooms, and the Active Learning Framework implies different ways to engage students in alternating problem-posing and problem-solving activities, this framework was revised for this study (see Figure 2.2). The revised framework can also continuously move students from being passive receivers to active learners. In order to examine specific prior and after effects between problem posing and problem solving, my participants were engaged in problem posing first as they were familiar with problem solving, and we still lacked understanding about their direct reaction and performance on problem-posing tasks.

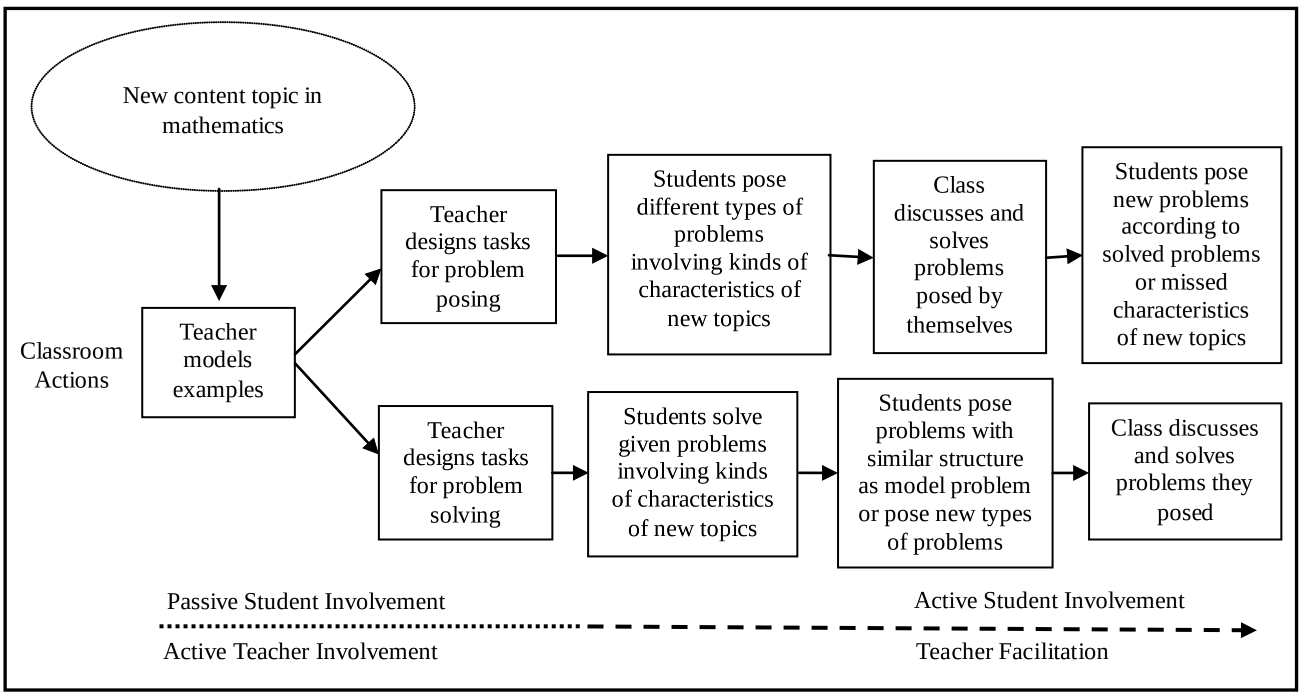



Figure 2.2: Revised Active Learning Framework

I then discussed a theoretical model with four specific problem-posing processes, proposed by Christou et al. (2005). This model helps investigate specific aspects of problem-posing abilities, and all four problem-posing processes are not too open-ended to be approached. From this viewpoint, Christou et al.'s model was a good fit for this study because, on the one hand, I could examine the participants' performance of specific types of problem-posing tasks. On the other hand, my participants would not grope aimlessly or had no idea about what to do exactly when encountered problem-posing tasks at very beginning. In addition, each process of this model corresponds to specific problem-solving tasks, as the researchers speculated. From this viewpoint, Christou et al.'s model was an even better fit for this study because it enabled further exploration of the relatedness between problem posing and problem solving.

# Chapter 3: Research Methodology

The purpose of this study was to understand the patterns of prospective elementary teachers' problem-posing performance, particularly when problem solving was involved in an alternating manner. Additionally, in this study I aimed to further investigate the features of the relationship between problem posing and problem solving. In order to achieve these goals, in this chapter I first discuss methodological considerations from the literature review, then provide a description of participants and sample selection, task design considerations and pilot study implications, data collection procedures, and data analysis procedures.

## Methodological Considerations from the Literature Review

The review of the literature suggests further exploration of students' problem-posing performance with particular components of problem-posing activities, such as specific problem-posing strategies, procedures, and categories of posed problems, as well as particular variables of problem-posing tasks, such as involved content topics, the format of representation, and ways of engaging problem posers. My study focused on exploring the impacts of specific types of problem-posing tasks on problem posers' performance, while all types of problem-posing tasks utilized in this study were closely related because they used the same real-world situation as well as its corresponding figures.

The rationale for using real-world situations is also supported in the review of the literature. Existing studies show that real-world situations involving rich mathematical content knowledge and multiple artifacts are beneficial to mathematics learning from many perspectives, including sustaining learners' interests, helping connect classroom mathematics content knowledge with out-of-school experience, as well as changing their disposition and beliefs in mathematics.

The review of the literature appeals for further exploration of the relatedness between problem posing and problem solving. In consideration of the lack of adequate investigation about the impacts of problem posing (problem solving) at different stages of problem-solving (problem-posing) processes, participants in this study engaged in alternating problem-posing and problem-solving tasks. This strategy ensured certain interactions between participants' problem-posing and problem-solving thinking. In addition, all of the tasks were designed according to the same situations and figures. This strategy ensured a particular relatedness of the two activities.

Finally, Ellerton (2013) and other researchers (e.g., Osana & Pelczer, 2015) appeal for problem-posing integration into teacher preparation programs. There are still a lot of unanswered questions about the methods and principles of doing this. By further considering the benefits of an international comparison study, and the lack of international comparison studies, particularly related to problem posing with prospective teachers, I chose to conduct an international comparison study with prospective elementary teachers. Participants from China and the US were recruited in this study. Existing studies evidence a large difference between US and Chinese teacher preparation systems. This study therefore was able to provide further insights about these two groups of participants' problem-posing performance as well as the relationships between problem posing and problem solving.

**Participants and Sample Selection**

I used a convenience sampling technique (Creswell, 2012) to recruit potential participants in selected institutions that had a specialized educational program for preparing prospective elementary mathematics teachers in the US and China. I selected six institutions in total. Three were from the northeastern part of the US. The first US institution is a private, student-focused global research university. This university recruits both undergraduate and graduate students from across the US, and international students from more than 100 countries. The second US institution is a four-year, public coeducational college that draw 95% of its students from the state. The third is also a public institution, but a two-year community college. These three institutions are ranked on different levels according to US News, which is an American media company well known for its Best College rankings.

The other three institutions were from the southeastern, southwestern, and northwestern parts of China, respectively. The first Chinese institution is a comprehensive public university located in a developed urban area. It recruits both national and international students. The second institution is also a public university where the majority of its students are from the southwestern part of China. Many of them are minority students, namely, students that are from a group of people who differ racially from the largest group of China, which is the Han nationality. The third institution is located in a rural area of China with the majority of students coming from the same province. The first two Chinese universities are normal universities, which attempt to prepare elementary school teachers while the third one is a non-normal four-year college. Similar to the three institutions from the US, the three universities from China are also ranked on different levels according to the *People's Daily,* which is an official newspaper of the Chinese Communist Party, published worldwide with high social influence.

I contacted the prospective elementary teachers' instructor in each of the selected institutions through email and received permission for visiting his/her class for 10 to 15 minutes either at the beginning or at the end of the class to recruit participants. I went to the class and explained the purpose of my study. In consideration of respect for each prospective teacher, I recruited participants according to their willingness. There were a few prospective teachers who agreed to participate in my study, but withdrew from the study later. I excluded their data from the data analysis. The data analysis was based on data collected from 87 first- and second-year (the majority were first year) prospective elementary teachers enrolled in teacher education programs. For a qualitative study, a sample of 87 participants was a large size, and this helped to ensure that the sample size was defensible (Mason, 2010).

I recruited 32 US participants. Eight of them were from the private university and in a four-year educational program, 20 were from the public university and also in a four-year program, and four were from the community college and either in a two-year degree program or a one-year certificate program. These US participants were required to take liberal studies courses such as Literature and History, major courses about both advanced and elementary level content knowledge in mathematics, as well as education courses related to elementary level mathematics. The remaining 55 participants were from Chinese programs and all in four-year educational programs, of which 20 were from the comprehensive public normal university, 17 from the public normal university located in southwestern part of China, and 18 from the non-normal four-year college. The Chinese prospective elementary teachers mainly took general courses such as English, Sports and Computer, and advanced mathematics courses such as Calculus, Linear Algebra and Analytic Geometry, but no education or practice courses related to elementary level mathematics, during their first- and second-year study in the programs.

Due to the fact that the US participants were from two different types of universities (i.e., private and public) while the Chinese participants were from three quite different areas of China, I expected the study findings to be more reliable than if only one university in each country was selected. Gender difference was not considered during participants' recruitment process due to the purpose of my study as well as the fact that prospective elementary teachers are predominately female, which is true in both the US and China. And according to their short oral responses, no participants had formal experience in problem posing prior to this research study. Each participant was assigned a unique serial number consisting of the country initial followed by a number. For example, U07 represented the seventh participant and he/she was from the US; C55 represented the fifty-fifth participant and he/she was from China. Therefore, no actual names of participants were used in this study.

## Task Design

In this section, I first considered four principles when designing problem-posing and problem-solving tasks. I then conducted a pilot study by administrating initially designed tasks to a class of 22 US prospective elementary teachers and to three individual Chinese freshmen who had just came to the US after graduating from a Chinese high school. I finally revised my tasks according to the pilot study results.

**Four considerations.** Two sets of tasks with a similar structure were designed according to four considerations.

*First consideration.* The first consideration was mathematical topics or concepts that involve rich mathematical thinking, multiple ways of exploration and, in the meantime, are related to real-life situations. Fractions seemed to fit this consideration. According to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), US students start to learn fractions in Grade 3 and operations with fractions in Grade 4. In China, students start to learn fractions in Grade 4 and operations with fractions in Grade 5. Fractions are a challenging topic for US elementary teachers and students (Ball, 1990; Toluk-Uçar, 2009), and they can be seen almost everywhere in our daily life. Problems with fractions usually require multiple ways of cognitive thinking, especially problems that are contextualized. Mack (1990) referred to this type of knowledge constructed by individual students from applied, real-life situations as informal knowledge. Mack found that students usually possessed a rich store of informal knowledge about fractions and that knowledge played a role in developing their understanding of fraction symbols and algorithm procedures.

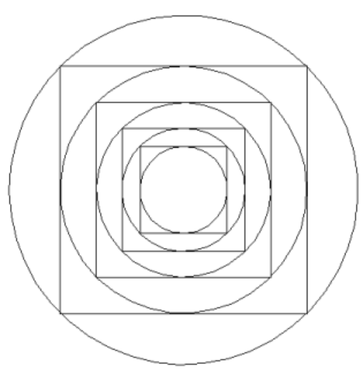
I selected geometry as the topic for the second set of tasks. According to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), US students start to learn two-dimensional shapes in Grade 1 and will learn the Pythagorean Theorem in Grade 8. In China, the students start to learn geometric shapes in Grade 3 and will learn the Pythagorean Theorem either in Grade 7 or Grade 8. Jones (2002) claimed that, "Geometry provides a culturally and historically rich context within which to do mathematics" (p. 125). More specifically, the study of geometry provides students opportunities to visualize, conjecture and reason, argue and justify, solving problems, and develop their intuition. Additionally, the study of geometry helps individual students for a better understanding of other content areas such as fractions, functions and statistics. However, the National Center for Education Statistics (2012) reported that students' mathematical performance was consistently lagging in geometry and measurement. They posed that one of the reasons was that teachers had limited knowledge related to these two content strands (Steele, 2013; Van der Sandt & Nieuwoudt, 2003).

Existing international comparison studies have documented that "Chinese students consistently outperformed US students across grade levels and mathematical topics" (Cai & Nie, 2007, p. 460), including numbers and operations, measurement, geometry, and so on (Lapointe, Mead, & Askew, 1992; Robitaille et al., 1992). Even so, researchers have found that Chinese students showed relatively weak performance on visual representation items or using pictorial representations (Brenner, Herman, Ho, & Zimmer, 1999; Cai, 2000). In addition, Chinese students did not shown explicit "higher performance on complex, open-ended tasks measuring creativity, problem posing, and non-routine problem solving" (Cai & Nie, 2007, p. 460). In consideration of the challenges for US and Chinese students respectively, as well as potential learning opportunities involved in the topics of fraction and geometry, I expected to investigate whether problem posing could help or hinder prospective elementary teachers' learning of these topics that they found challenging. I finally selected two prototypical problems (see Table 3.1), both of which involved pictorial representations. Additionally, it is worthwhile to point out that it was not necessary to use the Pythagorean Theorem to solve the Geometry problem.

*Second consideration.* The second consideration concerned the developmentally proper

Table 3.1

*Prototypes of Fraction and Geometry Problems*

| | |
|---|---|
| Fraction problem* | Diana bought a piece of cloth 4 feet wide and 5 feet long. It cost $16. She cut off a piece that was $1\frac{3}{4}$ feet wide and 4 feet long to make a scarf. Her sister saw Diana's cloth and really liked the material. She asked for a piece that was $1\frac{3}{4}$ feet wide and $1\frac{2}{3}$ feet long to also make a scarf. |
| Geometry problem** | The radius of the smallest circle is one unit. What is the ratio of the area of the largest circle to the area of the smallest circle?  |

* This fraction problem was adapted from Masingila, Lester and Raymond (2011).
** This geometry problem was adopted from Pershan (2013).

way of engaging participants in alternating problem-posing and problem-solving activities. I adapted the Active Learning Framework developed by Ellerton (2013). The participants began by posing mathematical problems individually according to given figures and a situation, followed by solving a mathematical problem related to the same figures and situation. Finally, they were asked to pose two more problems that were different from all previously posed or solved problems. The purpose of requiring the participants to continue focusing on the same figures and situation was to provide them the opportunities to investigate the givens and goals from multiple perspectives and extending their understanding about the mathematical patterns involved to a large extent, if possible.

*Third consideration.* The third consideration of task design concerned the order of specific problem-posing and problem-solving tasks so that they would better match with prospective elementary teachers' natural thinking processes when encountering a real-life situation. The four problem-posing processes (i.e., Comprehending, Translating, Editing, and Selecting) were first ordered according to the levels of cognitive demand and the levels of difficulty for six-grade students discussed by Christou et al. (2005). According to Christou et al., the Comprehending process requires students' understanding of the meaning of different operations while the Translating process requires more of students' understanding of different mathematical representations. Editing is a higher cognitively demanding process because it requires students to extract the information they specifically needed for posing a reasonable problem. Selecting is an even higher cognitively demanding process because it requires students to focus most on the structure and the relationship amongst provided information while Editing has no such requirement. At the same time, Christou et al. found that "both the editing and selecting processes characterized the most able students" (p. 156).

In consideration of the findings above, I initially ordered the four problem-posing processes as Comprehending, Translating, Editing, and Selecting. I did not ask the participants to solve the problems they posed. According to Christou et al. (2005), problem solving was actually involved in those problem-posing processes. In other words, the participants were required to justify their solutions before writing down a specific problem in order to fit a given answer or equation. In consideration of the complexity of the given figure in Geometry Problem, I was not sure whether the way of the arrangement of those circles and squares would increase the difficulty of interpreting the figure or using Pythagorean Theorem. I prepared a second figure that would not change the answer to the initial problem by rotating two squares 90 degree clockwise (see Figure 3.1).

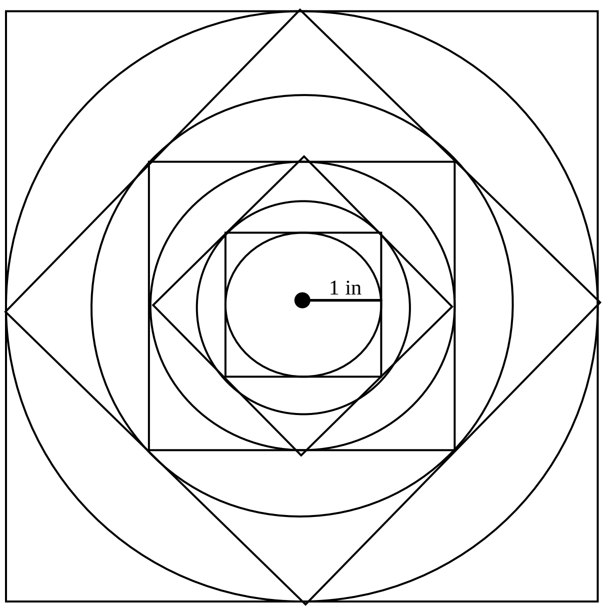



Figure 3.1: Figure of Geometry Problem with Rotated Squares

*Fourth consideration*. I finally considered the features and number of problems that the participants should pose for each process. Cai and Hwang (2002) asked students to pose three problems with three levels of difficulty: easy, moderate, and difficult. However, anticipating that students may have different criteria for difficulty levels due to their academic background and performance, this guideline might influence their selection of the type, or the nature of each problem they were going to pose. Therefore, I decided not to ask my participants to pose problems with this restriction. In addition, instead of asking the participants to pose as many problems as they could, I chose to ask individual prospective teachers to pose one problem for each problem-posing process and two more problems after problem solving. In other words, each participant was expected to pose six problems in total for each task administration. In doing this, they were able to pay more attention to the quality of each problem that they posed instead of rushing to pose many similar problems or problems with a low level of quality. I assumed that those problems could provide more information about their problem-posing and problem-solving performance, which correspondingly, could serve to better answer the research questions.

Four problem-posing processes followed a problem-solving task. The task was not too easy for the participants since they already had a certain understanding of the given situation and figures through the problem-posing exploration. On the other hand, this task was formed so that it was not too difficult or completely irrelevant to problem-posing exploration. Otherwise, it could have elicited feeling of frustration or participants would have no incentive to make connections between problem posing and problem solving. Therefore, I used the Plus One strategy suggested by Biggs and Collis (1982) to design my problem-solving tasks. This strategy can help students raise their levels of thinking appropriately and does not bring students too far away from their zones of proximal development (Vygotsky, 1987).

For the Fraction problem, the added piece of information for the problem-solving task was the cost of a piece of cloth. I selected this information because none of the previous four problem-posing processes specifically required the participants to pose a problem involving a cost issue. Even if some participants integrated the cost information into their posed problems and considered it, it was possible that solving that kind of problem would be challenging for them. For the Geometry problem, the added piece of information for the problem-solving task was the concept of ratio. Similar to the first situation, none of the previous four problem-posing processes specifically required the participants to pose a problem involving a ratio. In addition, research has documented the difficulties that both students and teachers have with this concept (Cramer, Post, & Currier, 1993; Fuson & Abrahamson, 2005). Therefore, it was reasonable to assume that the problem developed for the problem-solving task had the potential to raise their thinking. I finally designed two sets of tasks before conducting the pilot study (see Appendix A).

**A pilot study.** In order to examine the feasibility of the initially designed tasks, as well as the reasonableness of the assumptions about those tasks, I conducted a pilot study with 11 pairs of US prospective elementary teachers. All 11 pairs completed the first set of tasks, while six pairs of them worked on the second set of tasks with the initial figure, and the other five pairs worked with the figure with rotated squares. I also conducted the pilot study with three individual Chinese students who were freshmen and had just come to the US after graduating from Chinese high schools. They completed the first set of tasks, and then the second set of tasks with initial figure. I finally asked their opinions about the second set of tasks with the figure with rotated squares. I recruited those US participants using a convenience sampling method and found the three Chinese participants using a snowball sampling method (Creswell, 2012).

The pilot study first showed that if both figures and situations were given at the very beginning, it was hard to justify whether the US and Chinese participants could differentiate the expectations of the Translating and Editing processes. In other words, it was not clear whether the participant posed a problem for the Translating process by interpreting the given figures, or posed a problem for the Editing process by analyzing the given story. In light of this, I placed the Translating process as the first task with designed figures, before showing the corresponding situation to the participants. In other words, the participants were allowed to freely create a real-life situation according to the given figures and pose a story problem for the Translating process. In addition, both the US and Chinese participants' performance indicated that there was no big difference between using the initial figure and using the figure with rotated squares in the second set of tasks. I decided to use only the initially designed figure.

Regarding the US participants' problem-solving performance, I found that 66% (7 out of 11) pairs of the US participants correctly solved the given problem during the first task administration, while only 9% (1 out of 11) pairs correctly solved the given problem during the second tasks administration. The three Chinese participants had no difficulty in solving the problem given in either the first or second tasks administration. Regarding the participants' problem-posing performance, I found that the US participants were challenged most by the Selecting process, and then the Comprehending process. More specifically, only 9% (1 out of 11) pairs of them posed a solvable mathematical problem in the Selecting process during the first task administration, while 19% (2 out of 11) pairs posed a solvable mathematical problem during the second task administration. In the Comprehending process, although 90% (10 out of 11) pairs of them posed a solvable mathematical problem during the first task administration, 30% (3 out of 10) of those problems did not represent the given calculation. During the second task administration, although 67% (7 out of 11) pairs of them posed a solvable mathematical problem, 86% (6 out of 7) of those problems did not represent the given calculation. The three Chinese participants did not show such a difference among those problem-posing processes. Overall, I concluded that the second set of tasks was more challenging than the first set of tasks for the US participants. I therefore assumed that it was unnecessary to ask all participants to complete the second set of tasks, and selecting the half who performed better on the first set of tasks could provide more reliable evidence of effective thinking that supported their problem-posing and problem-solving performance and understanding.

The final version of the designed tasks was in English (see Appendix B). I then translated both sets of the tasks into Chinese for the participants from China (see Appendix C). In order to verify the translation of the tasks from English to Chinese, I read through the tasks in English line by line and carefully compared to the Chinese sentences that I developed. I replaced one Chinese word with multiple synonyms in order to select the most suitable one in the given context. Chinese is my native language and I am confident in this translation. Additionally, one of the members of my dissertation is a native Chinese speaker and he also verified that the translation from English to Chinese was accurate. The only difference between the tasks in English and the tasks in Chinese was the length measurement and unit. I used the foot and inch as the length units of the tasks in English and used the meter and centimeter as the length units of the tasks in Chinese. I correspondingly used different length measurements for the tasks in English and in Chinese in order to make the given measurements comparable. This consideration produced a consequence that, in the first set of tasks, the fractions given in the task in English were mixed numbers, while the fractions given in the tasks in Chinese were proper fractions (i.e., fractions that were less than one). This difference may introduce a higher level of complexity of the tasks for the US participants than their Chinese counterparts. This different may influence the US and Chinese participants' performance in both problem-posing and problem-solving tasks.

**Data Collection Procedures**

Each task administration lasted one hour and twenty minutes. The participants from the same university were encouraged to participate at the same time. However, due to many different circumstances, I sometimes administered the tasks with a small group of participants (five or fewer), while other times with a large group (more than five). My participants completed each set of the tasks individually during administration. I asked participants to use pens instead of pencils in order to make sure that their work would not smudge, and to make reading easier. For each set of the tasks, I first asked the participants to pose a problem in the Translating process according to given figure/figures without real-life situation within 10 minutes on a separate sheet of paper. I then handed out the second sheet of paper and asked them to pose problems for the other three problem-posing processes, one problem for each process, using a real-life situation that corresponded to the previously provided figure/figures within 30 minutes. Next, they tried to solve a problem on the third sheet of paper in 20 minutes. Finally, I asked them to pose two more problems according to the same figure/figures and situation on a fourth sheet of paper within another 20 minutes.

Each time the participants received a new sheet of paper, they were required not to change anything they had completed previously. This was because, if they were allowed to do so, many participants were capable of making their previously posed problems better or different due to more or deeper understandings they got from the subsequent tasks. It was also plausible that some participants would change their previously posed problems to very similar ones after they read the problem given in the problem-solving task. Both of these situations could have influenced the data analysis and findings about prospective elementary teachers' performance of problem posing and problem solving.

All 87 participants completed the first set of tasks and I selected 43 of those to complete the second set of tasks. The 43 participants comprised about half of the participants from each institution. Shuk-kwan (2013) classified participants' posed problems into five categories: not a problem, non-Math, impossible, insufficient, sufficient or extraneous. In this study, I combined these five categories into three categories: (a) a solvable mathematical problem (i.e., a problem in mathematical form with sufficient or extraneous given information that is enough for finding a solution); (b) an unsolvable mathematical problem, including impossible and insufficient problems (i.e., a problem in mathematical form that is impossible to solve or does not have sufficient information to find a solution); and (c) not a mathematical problem, including not a problem and a non-Math problem (i.e., a problem not in a mathematical form, or a description or a phrase, but not a problem). If a participant asked two or more questions for one problem-posing process, each question was counted as one posed problem and categorized into one of those three categories.

Therefore, the criteria I used for selecting participants for the second task administration was that the majority of one participant's posed problems (usually more than 50%) during the first task administration were solvable mathematical problems. More specifically, in each institution, immediately after the first task administration, I analyzed participants' posed and solved problems. I then selected the participants who posed three or more solvable mathematical problems during the first task administration for the second task administration. If there were more than half of the participants from the same institution who had such a good problem-posing performance, I considered their problem-solving performance as well to generate the finalists. If there were less than half of the participants from that institution who met this criteria, I considered the participants who posed two solvable mathematical problems.

The 43 selected participants completed the second task administration within one week after completing the first one. As discussed before, the second set of tasks was structured similarly to the first set. The difference was that I videotaped the second task administration in order to capture the entire process clearly. Before the participants arrived, I set up a tripod with a video camera at a corner of the room making sure that the camera could catch all coming participants at that time. During the participants' work, I walked around and asked participants questions about their posed problems and/or the ways of their thinking. In doing so, I aimed to be clear about their handwriting, use of words that were vague to me, or their specific considerations, strategies, difficulties, and misunderstandings of each step in problem posing and problem solving, if possible. I carefully selected the prompt questions that I could ask during task administration (see Question Examples, Appendix D) in order to prompt their explanations, instead of leading them to make any changes to the work they had already done. I did not equally ask all the participants all those prompt questions. I asked some of those questions (e.g., What do you mean by this word?) to a participant only when I saw something confusing or interesting in his/her posed problems. I asked some questions (e.g., How did you come up with this problem?) when I saw a participant who had finished posing or solving a problem and sat there doing nothing. I asked those questions aiming to make them feel comfortable and not bored.

During both the first and second task administration processes, I asked the participants to write down whatever thoughts or feelings they had about their problem-posing or problem-solving experience, if they had completed, reread, and were satisfied with either the posed problems or problem-solving solutions, while the other participants were still working (see Question Examples for Prompting Thinking, Appendix E). I asked the participants who had not completed the assigned task to keep focusing on the task until the time was complete. I told my participants that even if they did not get a chance to write down their thoughts or feelings because of the limited time, their focus was the assigned tasks. This was to avoid causing them to feel disappointed or stressed.

## Data Analysis Procedures

The collected data of this study included participants posed problems and problem-solving solutions. I analyzed all collected data quantitatively first. I classified my participants' problem-solving performance into three categories: (1) completely solved, meaning that the problem solver implemented correct strategies and methods, and got a correct answer; (2) partially solved, meaning that the problem solver implemented correct strategies and methods, but got an incorrect answer due to errors made during the problem-solving process; and (3) incorrect/no solution, meaning that the problem solver implemented incorrect strategies and methods, and got an incorrect answer, or did not get an answer. I then coded these categories as "completely solved" = 2, "partially solved" = 1, and "incorrect/no solution" = 0. I then developed a table to record the frequency of each category of problem-solving performance for the participants from each country. If a participant provided more than one solution to a problem, I counted his/her solutions only once according to the best solution. For example, if a participant completely solved a problem using more than one method, or he/she provided both correct and incorrect solutions, I counted his/her solutions only once as "completely solved". This table describes different groups of participants' problem-solving performance and, more importantly, indicates each group of participants' problem-solving performance when closely related problem-posing activities were alternatively involved.

I then developed a table for recording both US and Chinese prospective teachers' problem-posing performance. I coded the posed problems as "solvable mathematical problem" = 2, "unsolvable mathematical problem" = 1, and "not a mathematical problem" = 0. I listed the frequency of each category of the posed problems for each problem-posing process (i.e., Translating, Comprehending, Editing, Selecting), as well as the problem posing after problem solving process, in this table. This table shows quantitatively the two groups of participants' problem-posing performance for each process. I discuss some illustrations about the differences of problem-posing performance between these two groups of participants, as well as the differences of participants' problem-posing performance before and after problem solving.

For the qualitative data analysis, I used a purely inductive reasoning approach to organize the data for analysis, explore and develop the codes and sub-codes, and then synthesized those codes to major descriptions and themes to answer the research questions (Creswell, 2012). Bogdan and Biklen (2007) asserted that "the first step involves a relatively simple house-cleaning task: going through all the files and getting them in order" (p. 184). Therefore, I first systematically arranged my participants' paper work into three piles: problem-posing work before problem solving; problem-solving work; and problem-posing work after problem solving. Each pile was then separated into two smaller piles: the US participants' work and Chinese participants' work.

The pile of my participants' problem-posing work before and after problem solving was analyzed one process at a time. More specifically, I started coding the problems posed by US and Chinese participants in the Translating process (see examples of codes for Translating process in Appendix F). I then compared the codes for US and Chinese participants' posed problems, developed coding categories according to similarities and differences of those problems, justified redundant codes that were irrelevant to my research questions, and finally synthesized the rest of the codes into major descriptions (see examples of final codes and descriptions for Translating process in Appendix G) about the patterns of my participants' performance. I coded the Translating process in the first and second set of tasks, respectively. I followed the same procedures and successively coded the problems posed in the other problem-posing processes (i.e., Translating, Comprehending, Editing, Selecting, and problem posing after problem solving processes).

After I finished coding all of the problem-posing processes, I read through the descriptions for each process and built more descriptions about the connections between my participants' performance in one problem-posing process and the previous problem-posing processes, as well as connections between their problem-posing and problem-solving performance (see description examples in Appendix H). I finally used all the descriptions to develop major themes (see theme examples in Appendix I) with evidence to answer my research questions.

# Chapter 4: Results and Findings

In this study, I aimed to (1) explore the patterns of US and Chinese prospective elementary teachers' problem-posing performance when problem solving was involved in an alternating manner, as well as the similarities and differences of their performance, and (2) further examine the features of the relationship between problem posing and problem solving under this specific circumstance. This chapter reports the results and findings, and is structured as follows: (1) descriptive statistics and interpretation of US and Chinese participants' problem-posing and problem-solving performance, (2) major descriptions built from qualitative analysis and interpretation for each problem-posing process, and (3) a summary and interpretation of all descriptions, aiming to answer Research Question 1 and Research Question 2, respectively.

## Descriptive Statistics and Interpretation

In this section, I first developed and interpreted a table that described the US and Chinese participants' problem-solving performance for both the first and second task administrations. I then described their performance for each problem-posing process including Translating (PP1), Comprehending (PP2), Editing (PP3), and Selecting (PP4) processes as well as problem posing after problem solving process (PP5), using a table and bar graphs.

**Problem-solving performance.** I list the frequency of US and Chinese participants' completely solved problems, partially solved problems, and incorrectly solved or no solution problems for each task administration in Table 4.1. This table shows that 25% (8 out of 32) of the US participants completely solved the problem given in the problem-solving task during the first task administration while 98% (54 out of 55) of the Chinese participants completely solved

Table 4.1

*Problem-solving Performance*

| | | Freq. of completely solved | Freq. of partially solved | Freq. of incorrect or no solution | Total |
|---|---|---|---|---|---|
| First Task Administration | US Participants | 8 | 15 | 9 | 32 |
| | Chinese Participants | 54 | 1 | 0 | 55 |
| Second Task Administration | US Participants | 3 | 5 | 8 | 16 |
| | Chinese Participants | 24 | 2 | 1 | 27 |

that problem. About half of the US participants partially solved that problem. They had accurate understanding, but either mistakenly dividing total area by total cost (i.e., $(4\times5)\text{ft}^2 \div \$16$) to represent the unit price of the cloth that Diana bought, or made mistakes while calculating with fractions. The remaining 28% (9 out of 32) of the US participants showed an incorrect understanding or set up calculations that did not make sense (e.g., $\frac{4}{5}=\frac{16}{100}$, $\frac{3}{4}+4$) in order to solve the problem.

During this task administration, some Chinese participants solved the problem much faster than others. When they were waiting for others to finish, I encouraged them to solve the same problem using different methods. In this group, 20 of the 55 Chinese participants correctly showed two different solutions and three participants showed three different solutions. Those solutions included setting up a proportion, multiplying the unit price of the cloth by an area, and multiplying the total cost by the fraction of the area of a smaller piece of cloth to the total area.

For the problem given in the problem-solving task during the second task administration, 19% (3 out of 16) of the US participants and 89% (24 out of 27) of the Chinese participants correctly solved it, while the majority of these Chinese participants used the Pythagorean Theorem. Thirty-one percent (5 out of 16) of the US participants either figured out that they could use the Pythagorean Theorem to calculate the radius of each circle one after another, from the smallest to the largest, or found that the ratio of the radius of the largest circle to the radius of the third largest circle was two, and so was the ratio of the radius of the third largest circle to the radius of the smallest circle. However, they either failed in computing or mistakenly thought that the ratio of the area of the largest circle to the smallest circle was equivalent to the ratio of their radii. The remaining 50% (8 out of 16) of the US participants had significant difficulties in solving that problem and the majority of them arbitrarily selected a number, for example, half of an inch, as the increase amount of the radii between any two neighboring circles. No US or Chinese participant showed more than one solution to the given problem during this task administration.

To summarize, a much higher proportion of Chinese participants compared to their US counterparts correctly solved the given problem in both sets of tasks. One possible reason is that the Chinese participants took more advanced mathematics courses than their US counterparts. In addition, during the first task administration, about half of the US participants (15 out of 32) partially solved the given problem; and half of those participants (8 out of 15) had difficulties in calculating with mixed fractions. Therefore, the use of mixed fractions contributed to the difficulty of the tasks given to the US participants.

**Problem-posing performance.** I coded each posed problem into one of the following three categories: (1) a solvable mathematical problem, (2) an unsolvable mathematical problem, and (3) not a mathematical problem. For each category, I refined these into more specific subcategories. Each solvable mathematical problem was classified into either (1) a solvable mathematical problem that was well written and had enough necessary and sufficient conditions, or (2) a solvable mathematics problem that had redundant information. An unsolvable mathematical problem could either be a mathematical problem that was (1) impossible to solve, meaning that it was unclear what the goal was due to either an ambiguous description or infinite choices of the answer, or (2) had insufficient conditions for problem solving, meaning that it did not have enough information to find an answer. Finally, the category of not a mathematical problem included (1) a problem but not a mathematical problem, meaning no mathematical calculation or reasoning steps were required, and (2) not a problem at all, meaning it was just a description or a statement. Specific examples from the collected data for each category and subcategory are listed in Appendix J. I aimed to make each category of the posed problems accurately and clearly defined.

I then summarized the US and Chinese participants' problem-posing performance for both the first and second task administrations in Table 4.2. The frequency of each category of posed problems for each problem-posing process, namely, Comprehending (PP1), Translating (PP2), Editing (PP3), and Selecting(PP4) as well as the problem posing after problem solving process (PP5) is listed in this table. Figure 4.1 shows bar graphs of each percentage category of the problems posed by the US and Chinese participants during the first and second task administrations, respectively. I aimed to make the interpretation of the patterns of their problem-posing performance clearer.

Table 4.2 shows that, on average, each of the US and Chinese participants posed almost the same number of problems for each task administration, as expected. More specifically, each participant posed about seven problems during the first task administration, and approximately six problems during the second task administration. This may be because the topics involved in the first set of tasks were more familiar or easier. In consideration of the average number of solvable mathematical problems, Table 4.2 shows that each of the US participants posed fewer

Table 4.2

*Problem-posing Performance: Number of Posed Problems for Each Category*

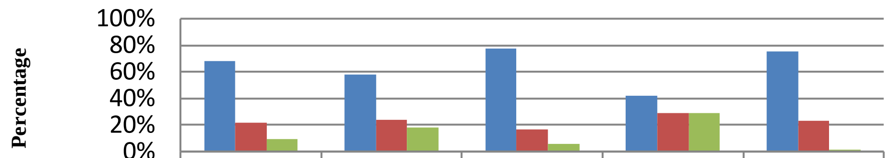
| | | | PP1* | PP2* | PP3* | PP4* | PP5* | Total | Average |
|---|---|---|---|---|---|---|---|---|---|
| First Task Admini-stration | US Participants (N=32) | Solvable math problem | 28 | 19 | 28 | 13 | 62 | 150 | 4.69 |
| | | Unsolvable math problem | 9 | 8 | 6 | 9 | 19 | 51 | 1.59 |
| | | Not a math problem | 4 | 6 | 2 | 9 | 1 | 22 | 0.69 |
| | | **Total** | **41** | **33** | **36** | **31** | **82** | **223** | **6.97** |
| | Chinese Participants (N=55) | Solvable math problem | 66 | 54 | 62 | 56 | 104 | 342 | 6.22 |
| | | Unsolvable math problem | 17 | 1 | 6 | 1 | 17 | 42 | 0.76 |
| | | Not a math problem | 8 | 0 | 2 | 0 | 10 | 20 | 0.36 |
| | | **Total** | **91** | **55** | **70** | **57** | **131** | **404** | **7.34** |
| Second Task Admini-stration | US Participants (N=16) | Solvable math problem | 13 | 10 | 14 | 6 | 22 | 65 | 4.06 |
| | | Unsolvable math problem | 9 | 3 | 4 | 6 | 10 | 32 | 2.00 |
| | | Not a math problem | 0 | 5 | 0 | 4 | 1 | 10 | 0.63 |
| | | **Total** | **22** | **18** | **18** | **16** | **33** | **107** | **6.69** |
| | Chinese Participants (N=27) | Solvable math problem | 29 | 27 | 27 | 26 | 55 | 164 | 6.07 |
| | | Unsolvable math problem | 2 | 0 | 3 | 1 | 5 | 11 | 0.41 |
| | | Not a math problem | 1 | 0 | 1 | 0 | 0 | 2 | 0.07 |
| | | **Total** | **32** | **27** | **31** | **27** | **60** | **177** | **6.56** |

* PP1 represents Translating process; PP2 represents Comprehending process; PP3 represents Editing process; PP4 represents Selecting process; and PP5 represents problem posing after problem solving process.

than five solvable mathematical problems during the first task administration, while each of the Chinese participants posed more than six solvable mathematical problems. In the second set of tasks, each of the US participants posed about four solvable mathematical problems, while each of the Chinese participants posed about six solvable mathematical problems. Accordingly, each Chinese participant posed fewer unsolvable mathematical problems and problems that were not mathematical problems than their US counterparts. These findings indicate that Chinese participants are more capable of posing solvable mathematical problems.

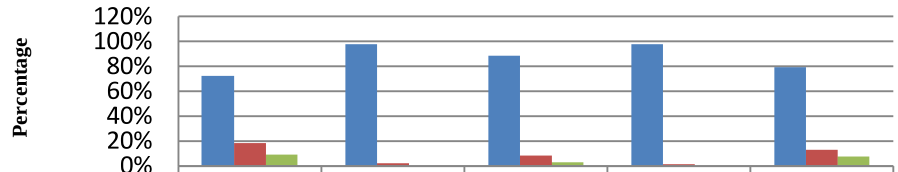
Table 4.2, together with Figure 4.1, shows that the US participants performance on different problem-posing processes can be ordered starting from the best as follows: (1) Editing process (PP3), (2) problem posing after problem solving process (PP5), (3) Translating process

US Participants' Performance out of Total Number of Posed Problems - First Task Administration



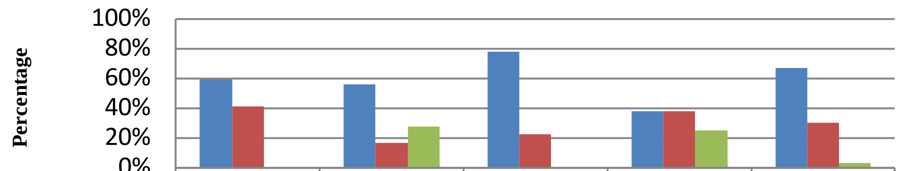

| | PP1 | PP2 | PP3 | PP4 | PP5 |
|---|---|---|---|---|---|
| Solvable math problem | 0.68 | 0.58 | 0.78 | 0.42 | 0.76 |
| Unsolvable math problem | 0.22 | 0.24 | 0.17 | 0.29 | 0.23 |
| Not a math problem | 0.10 | 0.18 | 0.06 | 0.29 | 0.01 |

Chinese Participants' Performance out of Total Number of Posed Problems - First Task Administration



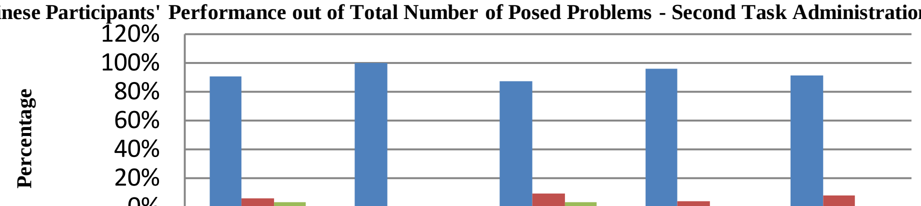

| | PP1 | PP2 | PP3 | PP4 | PP5 |
|---|---|---|---|---|---|
| Solvable math problem | 0.73 | 0.98 | 0.89 | 0.98 | 0.79 |
| Unsolvable math problem | 0.19 | 0.02 | 0.09 | 0.02 | 0.13 |
| Not a math problem | 0.09 | 0.00 | 0.03 | 0.00 | 0.08 |

US Participants' Performance out of Total Number of Posed Problems - Second Task Administration




| | PP1 | PP2 | PP3 | PP4 | PP5 |
|---|---|---|---|---|---|
| Solvable math problem | 0.59 | 0.56 | 0.78 | 0.38 | 0.67 |
| Unsolvable math problem | 0.41 | 0.17 | 0.22 | 0.38 | 0.30 |
| Not a math problem | 0.00 | 0.28 | 0.00 | 0.25 | 0.03 |

Chinese Participants' Performance out of Total Number of Posed Problems - Second Task Administration




| | PP1 | PP2 | PP3 | PP4 | PP5 |
|---|---|---|---|---|---|
| Solvable math problem | 0.91 | 1.00 | 0.87 | 0.96 | 0.92 |
| Unsolvable math problem | 0.06 | 0.00 | 0.10 | 0.04 | 0.08 |
| Not a math problem | 0.03 | 0.00 | 0.03 | 0.00 | 0.00 |

Figure 4.1: Percentage Bar Graphs of Participants’ Problem-posing Performance

(PP1), (4) Comprehending process (PP2), and (5) Selecting process (PP4). This order is true for both the first and second task administration. More specifically, Figure 4.1 shows that, for the first set of tasks, 78% of the posed problems for the Editing process, 76% for problem posing after problem solving, and 68% the Translating process are solvable mathematical problems, while only 58% of the posed problems for the Comprehending process and 42% of the posed problems for the Selecting process were solvable mathematical problems. For the second set of tasks, 78% of the posed problems for the Editing process, 67% for problem posing after problem solving, and 59% for the Translating process are solvable mathematical problems, while only 56% of the posed problems for the Comprehending process and only 38% of the posed problems for the Selecting process were solvable mathematical problems.

In addition, on the first set of tasks, the US participants posed a higher percentage of unsolvable mathematical problems and problems that were not mathematical problems in the Comprehending and Selecting processes than for other processes. In consideration of the use of the mixed fractions for US participants, it is possible that their performance in the Comprehending and Selecting processes was partially due to the complexity of the tasks for them. In the second set of tasks, the US participants posed a much higher percentage of problems that were not mathematical problems for the Comprehending and Selecting processes than for other processes. These results show that the US participants were most challenged by the Selecting process and then the Comprehending process.

The Chinese participants showed a quite different performance in problem-posing processes compared to their US counterparts. According to Table 4.2, although the Chinese participants posed almost the same number of solvable mathematical problems for each problem-posing process during both the first and second task administrations, they posed fewer unsolvable mathematical problems and problems that were not mathematical problems for the Comprehending and Selecting processes than Translating and Editing processes, particularly during the first task administration. This difference is evident according to Figure 4.1. More specifically, 98% of the problems for the Comprehending and Selecting processes posed by the Chinese participants during the first task administration were solvable mathematical problems, while only 73% for the Translating process, 89% for the Editing process, and 79% for problem posing after problem solving were solvable mathematical problems. During the second task administration, 100% of the posed problems for the Comprehending process and 96% for the Selecting process were solvable mathematical problems, while 91% for the Translating process, 87% for the Editing process, and 92% for problem posing after problem solving were solvable mathematical problems. These findings indicate that, contrary to their US counterparts' performance, the Chinese participants posed a higher percentage of solvable mathematical problems in the Comprehending and Selecting processes than other problem-posing processes.

It is understandable that both US and Chinese participants posed comparably fewer problems for the Comprehending and Selecting processes during both the first and second task administrations. The total number of posed problems for each problem-posing process could be partially influenced by the feature of each process. More specifically, the Comprehending process asked participants to write an appropriate mathematical story problem to represent a given calculation, while the Selecting process asked participants to write an appropriate story problem so that the answer to that problem was exactly the given number. Due to these specific requirements, participants usually chose to pose only one problem for each of those two processes. On the contrary, the other three problem-posing processes had fewer of such restrictions and therefore many participants posed more than one problem for each of those processes.

Thinking back to the study conducted by Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman (2005) with 143 six-grade students from urban district schools in Cyprus, the researchers found that students with a lower problem-posing performance were usually able to respond to only Comprehending and Translating tasks while the most able students could also respond to Editing and Selecting tasks. In other words, the Editing and Selecting processes were more demanding than the Comprehending and Translating processes for six-grade students in Cyprus. This conclusion is different from my findings about the US and Chinese participants' problem-posing performance. Although it is possible that the different performance shown by the US and Chinese participants was partially due to the use of mixed fractions for US participants in the first set of tasks, or the Chinese participants took more advanced mathematics courses than their US counterparts, it is still unclear whether these differences were exactly caused by different features of administered tasks, academic and cultural backgrounds (all involved participants came from three different countries), or professions (elementary students versus prospective elementary teachers), or any additional variables.

Finally, Figure 4.1 shows that both US and Chinese participants' best performance in problem posing did not occur during problem posing after problem solving process. For both the first and second sets of tasks, the US participants posed the highest percentage of solvable mathematical problems in the Editing process and then the problem posing after problem solving process, while the Chinese participants posed a higher percentage of solvable problems in the Comprehending and Selecting processes than the problem posing after problem solving process. Possible reasons for this finding are discussed later during the qualitative analysis.

In summary, a much higher proportion of Chinese participants compared to their US counterparts found a correct answer to the given problem during both the first and second task administrations. For problem posing, Chinese participants posed a higher percentage of solvable problems in the Comprehending and Selecting processes than other three problem-posing processes, while their US counterparts posed a higher percentage of solvable problems in the Editing and Translating processes as well as the problem posing after problem solving process. In other words, the US participants were most challenged by the Comprehending and Selecting processes. Such differences shown by the US and Chinese participants are possibly due to the use of mixed fractions for US participants in the first set of tasks, or the Chinese participants took more advanced mathematics courses than their US counterparts during their first- and second-year study in educational programs. Finally, both US and Chinese participants' best performance in problem posing did not occur during the problem posing after problem solving process.

**Qualitative Analysis and Interpretation for Problem-posing Processes**

In this section, I presented and interpreted major descriptions that I built for each of the Translating, Comprehending, Editing, Selecting, and problem posing after problem solving processes, respectively, according to my qualitative analysis. Each description either focuses on the patterns of the US and Chinese participants' performance in a particular problem-posing process, or the connections between problem-posing processes, or between problem posing and problem solving. A summary paragraph for each process is given at the end of each sub-section.

The phrase of a creative idea is often referred to throughout this section. I therefore want to define it clearly before moving to the major descriptions. A creative idea in one problem-posing process might not be a creative idea in next processes. More specifically, I defined a creative idea involving in a posed problem in the Translating process as a new relationship amongst mathematical concepts or procedures that was implicit or beyond given figures. For example, if a participant posed a problem in the Translating process during the first task administration involving only the concept of area or perimeter of rectangles, this problem was not a problem with a creative idea because those concepts were explicit in the given figures. If a participant posed a problem involving the concepts of ratio or proportion, I justified that problem as a problem with a creative idea because these concepts were implicit in the given figures. I defined a creative idea involving in a posed problem in the other four problem-posing processes (i.e., Comprehending, Editing, Selecting, and problem posing after problem solving processes) as a new relationship amongst mathematical concepts or procedures comparing to the entire set of posed problems in previous processes. In other words, if an idea or structure of a posed problem in one process was rarely occurred in previous processes, I justified this problem as a problem with a creative idea.

**Translating process.** I built three descriptions about US and Chinese participants' performance on the Translating process by qualitatively analyzing each problem that they posed. The first description discusses their direct reaction to the given figures and their preferences to select attributes of given figures when they were asked to pose problems based on those figures. The second description focuses on creative ideas that occurred in the posed problems and were implicit in the given figures. The third description interprets the reasons that led the posed problems to be unsolvable and not mathematical problems. The first description is built on solvable mathematical problems that involved no creative ideas, the second description is built on solvable mathematical problems that incorporated creative ideas, and the last description is built on unsolvable problems and those that were not mathematical problems.

***Description 1: The explicit concepts or ideas involved in given figures, such as the area***

***or perimeter of a rectangle for the first set of tasks and the diameter or area of a circle for the second set of tasks, were directly used by many prospective elementary teachers.*** During the Translating process in the first task administration, 47% (15 out of 32) of the US participants and 36% (20 out of 55) of the Chinese participants only used either the area or perimeter of a rectangle and posed solvable mathematical problems. For example:

> Carla's garden is 4 feet by 5 feet. She wants to make her garden larger. If she were to add a $1\frac{3}{4}$ feet by 4 feet area at the bottom of her garden, how big would her garden than [then] be? (U09, 1st, PP1)

This participant only used the concept of the area of a rectangle in his/her problem. Some of those solvable mathematical problems involved only one of three given rectangles and therefore required only one or two computational steps due to the nature of the area and perimeter formulas of a rectangle. More specifically, the area formula (i.e., $\text{area}=\text{length} \times \text{width}$) of a rectangle involves only one computational step while the perimeter formula (i.e., $\text{perimeter}=(\text{length} + \text{width})\times 2$) of a rectangle involves two computational steps.

The problems about the area or perimeter of rectangles that involved more than two computational steps asked for either the sum or difference of areas or perimeters. A noticeable difference was that 33% (5 out of 15) of those US participants and 70% (14 out of 20) of those Chinese participants incorporated all three given rectangles. The solvable problems that involve all three given rectangles usually require more computational steps. However, regardless of the number of rectangles involved, the operations needed to solve all of those problems are addition, subtraction, and multiplication, and not division.

For the given figure in the second set of tasks, the commonly used concepts included radius, diameter, circumference, area of a circle, and side length, perimeter, and area of a square.

Among a total of 42 solvable mathematical problems posed by US and Chinese participants, 29% (12 out of 42) needed fewer than four computational steps and mainly asked for the radius, perimeter or area of the smallest circle, or side length or area of the smallest square. For example:

> One cookie was placed in the center of the plate for snack. Given that the cookie's radius is 1 inch, what is the diameter? What is the circumference of the cookie? (U29, 2nd, PP1)

Among these 12 problems, half were posed by Chinese participants while the other half were posed by US participants. In the rest of the solvable mathematical problems, 71% (30 out of 42) needed four or more than four computational steps. For example:

> Using the radius given in the center circle. Help Jen to figure out the area of the outer most square. Jen knows that the area of the inside square is equal to 4 sq feet. (U26, 2nd, PP1)

Three of those 30 problems were about the area difference between the smallest square and the smallest circle, and all posed by Chinese participants. Ten of those 30 problems asked questions about the biggest circle, while 17 asked questions about the biggest square. Among the 10 problems that asked about the biggest circle, nine were posed by Chinese participants and only one was posed by a US participant. Among the 17 problems that asked about the biggest square, 11 were posed by Chinese participants while the other six were posed by US participants.

To summarize the Translating process in the second set of tasks, fewer than half of the US participants posed problems that extended their questions from the smallest circle to the biggest circle or square, while more than half of Chinese participants did this. In addition, none of the US or Chinese participants asked questions about either the circles or squares between the smallest and the biggest ones.

***Description 2: Multiple creative ideas using mathematical concepts and knowledge***

***occurred in prospective elementary teachers' posed solvable mathematical problems.*** I defined a creative idea involved in a posed problem as a new relationship amongst mathematical concepts or procedures that was implicit or beyond given figures and real-life situation.

*Division with fractions and division with a remainder.* One creative idea involved in both US and Chinese participants' posed problems during the Translating process in the first set of tasks was about the maximum number of times a small rectangle could fit into a big rectangle. Here is an example:

> Joe's backyard is $4[\mathrm{ft}] \times 5\mathrm{ft}$. Paul's pool is $1\frac{3}{4}[\mathrm{ft}] \times 1\frac{2}{3}[\mathrm{ft}]$, how many pools can Paul essentially fit in Joe's backyard? (U10, 1st, PP1)

Three out of 32 US participants and six out of 55 Chinese participants posed solvable mathematical problems with this idea. I justified these problems as creative problems because, first, they involved procedures of division with fractions in order to be mathematically solved. Second, these problems also involved the idea of division with a remainder in order to be realistically answered. Existing studies document that prospective teachers have difficulties with the meaning of division with fractions (Ball, 1990), and they were challenged by posing word problems representing a division with fractions (Rizvi, 2004; Toluk-Uçar, 2009). These studies provided only an expression of division with fractions and asked the participants to pose a story problem representing that expression.

Studies have also explored prospective and in-service teachers' capabilities of posing division-with-a-remainder problems (Chen, Van Dooren, Chen, & Verschaffel, 2011; Silver, & Burkett, 1994). However, these studies administered tasks that involved only whole numbers. In consideration of the designed tasks and the participants' problem-posing performance in my study, it may be easier for prospective elementary teachers to pose division-with-fractions problems and/or division-with- a-remainder problems starting with a figure instead of only a mathematical expression and/or with only whole numbers. It may also be more efficient for deepening their understanding of the meaning of division with fractions and division with a remainder.

*Interpreting a figure as a dynamic process*. For both sets of tasks in my study, the majority of US and Chinese participants interpreted the given figures as static objects, such as three pieces of paper with different measurements for the given rectangles in the first set of tasks and a tablecloth with a decorative pattern for the given figure in the second set of tasks. Conversely, some participants treated the given figures as dimension-changing results of a dynamic process. More specifically, one out of 32 US participants and three out of 55 Chinese participants used two smaller rectangles to continuously change the dimensions of the biggest rectangle. Here is an example:

> A given rectangle has a height [width] of 4ft and length of 5ft. The rectangle is altered so that the height [width] remains the same, but the length is reduced by $3\frac{1}{4}$ ft. Next, the rectangle is changed again so that the length now stays the same, but the height [width] is reduced by $2\frac{1}{3}$ ft. What is the area of the rectangle in the end? (U07, 1st, PP1)

For the given figure in the second set of tasks, one out of 16 US participants and 11 out of 27 Chinese participants thought of it as a dynamic process. And among those 12 problems, 10 were solvable mathematical problems. For example:

> 现有一块正方形纸板，边长不知，有人将此正方形裁剪，取出其内切圆，又从内切圆中剪出一个最大的正四边形，如此循环4次得到一个正方形，因为需要，他复[又]从这个正方形中掏出其内切圆，得到一个半径为 3cm 的圆。试算出原正方形的面

积。(C73, 2nd, PP1)

[Translation] There was a square paperboard, the side length of which was unknown. Someone cut this square paperboard and got its inscribed circle. He then cut out the biggest possible square from that circle. He repeated this process four times and finally got a square. For some reason, he cut that square again and got its inscribed circle, of which the radius was 3cm. Find the area of the original square.

These type of problems are special because the participants clarified the relationship among the three rectangles given in the first set of tasks and the relationship between neighboring circles and squares in the second set of tasks during the problem-posing process. Therefore, no figures need to be provided for problem solving. In addition, this types of problem has application in our daily lives.

*More creative ideas*. In the first set of tasks, my participants posed story problems for the Translating process that also involved the following creative ideas: (1) setting up a walking speed for a specific walking distance – perimeter of rectangle(s) – in an one-dimensional situation, or providing a lawn clean-up rate for specific section(s) of lawn – area of rectangle(s) – in a two-dimensional situation, or giving a water injection rate for filling a fishpond – volume of a cuboid generated by using the measurements of given rectangle(s) – in a three-dimensional situation; (2) adding price information for each unit area of given rectangles and asking for total cost; (3) ratio idea between areas or volumes, ratio idea presented in a fraction form, and scale idea for reading a map; (4) unit conversion idea, for example, between foot and inch; and (5) rate of change idea involving in a self-created functional image.

For the Translating process in the second set of tasks, besides the 10 solvable problems that interpreted the given figure as a dynamic process, I found only two solvable mathematical problems posed by two Chinese participants that involved creative ideas. One problem asked for the ratio of the area of the biggest circle to the area of the biggest square. The other problem utilized the relationship between the circumferences of the smallest and biggest circles:

> 小明与小红在玩大转盘的游戏，看到转盘的圆形图案后 [如所给图形]，他们突发奇想进行一个测试，他们分别拿了两个[一个]棋子，小明在最大的圆圈上移动，小红在最小的圈上移动，假设他们的移动速度都是 1cm/秒，小明走完1圈时，小红走完了4圈，问，最大圆的半径与周长，以及转盘盒子的边长。(C41, 2nd, PP1)
>
> [Translation] Xiaoming and Xiaohong were playing a spinning game. When they saw the circular pattern [see the given figure] on the board, they came up with an idea and wanted to test it. They each got two chess pieces [one chess piece each]. Xiaoming moved his chess piece along the biggest circle while Xiaohong moved her chess piece along the smallest circle. Assuming that they moved the chess pieces at the same speed, 1cm per second, when Xiaoming finished one round, Xiaohong had just finished four rounds. Question: What are the radius and circumference of the biggest circle? What is the side length of the box of the game board [the biggest square]?

This problem involves multiple mathematical relationships including the relationships between speed and distance, radius and circumference of a circle, as well as the radius of an inscribed circle and the side length of its circumscribed square. In addition, it shows that this participant understood the relationship between the smallest circle and the biggest circle before posing this problem. He/She correctly assumed that “when Xiaoming finished one round, Xiaohong just finished four rounds” according to the given figure.

***Description 3: Both US and Chinese participants posed a number of unsolvable and not mathematical problems, which demonstrated similar difficulties that they had.*** The major reasons that caused US and Chinese prospective elementary teachers' posed problems to be unsolvable included: (1) vague or improper wording in a problem; (2) a lack of sufficient conditions in order to solve the posed problems, and (3) a contradiction between the posed problem and the given figure particularly in the second set of tasks. The problems that were categorized as not mathematical problems were due to no questions asked at all, or no mathematical computational or reasoning steps needed. These reasons illustrate the difficulties that my participants had when posing a solvable mathematical problem.

*Vague or improper wording*. Specific cases of vague or improper wording included: (1) an unclear description of a condition or a question, which usually generates confusions, and (2) the incorrect use of mathematical vocabularies, which sometimes leads to infinite answers. For example:

> A garden is 5ft $\times$ 4ft. One section of the garden is filled w/ [with] tulips. This section is 4ft $\times$ $1\frac{3}{4}$ ft. Another section of the garden is filled w/ [with] lilacs. This section is $1\frac{2}{3}$ ft $\times$ $1\frac{3}{4}$ ft. The gardener wants to fill the rest w/ [with] roses and lavender. If she/he split up the remaining section evenly b/t [between] these two types of flowers, what would be the dimensions of the rose section? (U08, $1^{st}$, PP1)

The Oxford Dictionary defines dimension, usually dimensions, as a measurable extent of some kind, such as length, breadth, or height. Therefore, the use of "dimensions" in this problem refers to all the side lengths of the "rose section". However, the dimensions of the rose section depend on the arrangement of the "tulip section" and the "lilac section", as well as the ways that the gardener would split up the rest of the garden between the "rose section" and the "lavender section". There are actually an infinite number of answers to this problem; therefore, there is not one specific answer.

*Lacking sufficient conditions*. The second major reason that led posed problems to be unsolvable was the lack of sufficient conditions. This mistake was more commonly made by participants during the first task administration. Nine US and Chinese participants made this mistake during the first task administration while one US participant made this mistake during the second task administration. Among those nine participants, eight created a scenario with three-dimensional objects, but provided information for only two dimensions. For example:

> Jane's grandfather is building her a toy box. He wants her to be able to fit all her toys in the box. If the toy box has a length of 5 feet and a height of 4 feet, what is the area of the toy box Jane's grandfather is building? (U13, 1st, PP1)

This problem had two difficulties. First, it was not clear about which face's area of the toy box this participant was asking for. Second, a toy box is a three-dimensional object while only two dimensions' measurements were provided. If this participant meant to ask for the volume of this box, there would not be sufficient information to find a solution. Both of these gaps led this problem to be classified as an unsolvable mathematical problem.

*Contradiction involved*. Some posed problems in the second set of tasks were unsolvable because there was a contradiction between the posed problem and the given figure. Here is an example:

> The farmer has a plot of land that consists of circles with squares inside of them. As each circle gets bigger, the radius of that circle is $\frac{1}{2}$ of the length of one side of the square within it. What is the radius of the largest circle of land the farmer has? (U21, 2nd, PP2)

According to the given figure, the radius of each circle is $\frac{1}{2}$ of the side length of the square outside of it, rather than "within it". It is actually impossible to have a circle with its radius that is half of the side length of any square inside of it. Due to this contradiction, I categorized this problem as an unsolvable mathematical problem.

Among the 13 posed problems that I categorized as not mathematical problems in the Translating process, two asked no question at all. One of these was from the first task administration while the other was from the second task administration. The remaining 11 problems were all from the first task administration. Seven of those 11 problems asked questions that had no mathematical concepts or relationships (e.g., "小明可以算出答案吗？[Translation] Could Xiaoming find the answer?", C79, 1st, PP1), while the other four problems asked questions about mathematical concepts or relationships, but no computational or reasoning steps were necessary in order to solve those problems. Both US and Chinese participants made these mistakes in posing problems that were categorized as not mathematical problems.

***Summary.*** In the Translating process, many US and Chinese participants directly used apparent concepts or ideas involved in given figures to pose problems during both the first and second task administrations. More than half of the Chinese participants incorporated all three given rectangles during the first task administration and they asked questions about the biggest circle or square during the second task administration, while fewer than half of the US participants did this. In addition, both US and Chinese participants were capable of posing problems with creative ideas that were implicit or invisible in given figures, and many of the ideas utilized by these two groups of participants were similar. Finally, US and Chinese participants made similar mistakes that led the posed problems to be unsolvable or not mathematical problems. This indicates the US and Chinese participants had similar difficulties in posing solvable mathematical problems.

**Comprehending process.** Due to the fact that the Chinese participants performed very well in the Comprehending process during both the first and second task administrations, I made two descriptions for this process that mainly discussed the US participants' performance. The first description examines the obstacles that US participants had when posing a problem representing a given calculation. The obstacles include knowing the properties of arithmetic, recognizing notation, and connecting the given calculation back to the initial figures and story, in other words, thinking backwards from solution to givens. The second description further discusses the reasons that led US participants' posed problems to be unsolvable or not mathematical problems.

***Description 4: Many US participants experienced obstacles interpreting the given calculations according to initial figures and story, while a few of their Chinese counterparts experienced such obstacles.*** Comparing to the Translating process, the prospective elementary teachers had less freedom in posing problems for the Comprehending process because this process was confined to both the initial story and the given calculation.

During the first task administration, the given calculation represented the total area of Diana and her sister's scarves. Thirty-two US participants posed 33 problems; one participant posed two problems for this process. Fifty-eight percent (19 out of 33) of those problems were solvable mathematical problems. Among those 19 solvable problems, 36% (7 out of 19) asked a question that was about the initial story and represented the given calculation, as expected. In 32% (6 out of 19) of those problems, although the given calculation was correctly represented, participants added or generated new scenarios for the initial story. For example, Diana wanted to sew her scarf and her sister's scarf together, or Diana's sister wanted to make her scarf 4 feet longer. This group of participants actually interpreted the given calculation $1\frac{3}{4}\times(4+1\frac{2}{3})$ as length × width, where $1\frac{3}{4}$ was the width and $(4+1\frac{2}{3})$ was the length. They did not recognize that the given calculation applied the distributive property from the initial story.

The remaining 32% (6 out of 19) were solvable mathematical problems, but were not about the initial story, or did not represent the given calculation. More specifically, four of those six problems started from the initial story but asked questions that did not represent the given calculation. For example:

> How many more ft of fabric does Diana's sister need in order to make the length of her scarf as long as Diana's? (U27, 1st, PP2)

The other two problems developed brand-new scenarios and asked questions that were irrelevant to the given calculation. For example:

> I have a piece of fabric that is $1\frac{2}{3}$ ft longer than 4 ft, in width. The fabric is $1\frac{3}{4}$ ft, in length. What is the dimension of my piece of fabric. (U05, 1st, PP2)

For this group of participants, it is possible that, first, they had difficulties in understanding the meaning of addition in one-dimensional space (i.e., sum of lengths) and the meaning of multiplication in two-dimensional space (i.e., area of a rectangle). It is also possible that they were challenged in connecting the given calculation back to the initial figures or story.

Regardless of the given situation, in total, 39% (13 out of 33) of the posed problems correctly represented the given calculation while the remaining 61% of the posed problems did not. I kept thinking that, first, to what extent that the use of mixed fractions hindered their performance of posing a solvable problem as expected in the Comprehending process. Second, if the US participants were asked to solve the corresponding problem to the given calculation as usual, the majority of them would be able to do it because it only involved the idea of the area of rectangles. In a way, it possibly shows a gap between the US participants' problem-solving capability and their problem-posing capability, specifically for the Comprehending process.

Compared to their US counterparts, 55 Chinese participants posed 55 problems for the

Comprehending process, one for each, during the first task administration. Only one was classified as an unsolvable mathematical problem because it had ambiguity in its wording that made it hard to understand what exactly it was asking. The remaining 54 problems were solvable mathematical problems, among which 80% (43 out of 54) directly represented the given calculation from the initial story, 19% (10 out of 54) either added new conditions or created new scenarios, but asked a question that correctly represented the given calculation. Only one solvable problem did not represent the given calculation. This participant asked for the area of a rectangular cloth that was $\frac{3}{4}$ meter wide and $\frac{7}{4}$ meter long, while $\frac{7}{4}$ was the result of $1+\frac{3}{4}$ in the given calculation for Chinese participants.

During the second task administration, the given calculation represented the radius of the mouth of the second smallest bowl in the set. Sixteen US participants posed 18 problems, of which 56% (10 out of 18) were solvable mathematical problems. Among those 10 solvable mathematical problems, one problem correctly asked for the radius of the second smallest bowl, while another imagined the squares in the figure as boxes and asked for “the distance from the center of the smallest bowl to the corner of the smallest box” (U03, 2nd, PP2), which also represented the given calculation. The remaining eight solvable mathematical problems did not represent the given calculation but asked questions about the diameter of the smallest bowl, the radius of the third smallest bowl, or the side length of the smallest square. During the task administration process, a few US participants asked what the notation of square root was for. Some figured out that $\sqrt{1^2+1^2}=\sqrt{2}$ and then had no idea what $\sqrt{2}$ represented in the figure or the story, and the majority did not recognize that $\sqrt{1^2+1^2}$ was in the form of Pythagorean Theorem.

Twenty-seven Chinese participants posed 27 solvable mathematical problems, one each, for Comprehending process during the second task administration. Among those problems, 67% (18 out of 27) asked for the radius of the second smallest bowl and 22% (6 out of 27) asked for half of the diagonal of the smallest square, both of which correctly represented the given calculation. In the remaining 11% (3 out of 27) of the solvable problems, one asked for the diameter of the second smallest bowl, another asked for the diagonal of the inscribed square of the smallest bowl, and the third added a new condition to the initial story that provided a upper boundary for the radius of the second smallest bowl and asked whether the second smallest bowl in the given figure met the criteria.

In consideration of the US and Chinese participants' performance in the Comprehending process of both the first and second sets of tasks, especially the percentage of solvable mathematical problems that did not represent the given calculation and the percentage of unsolvable mathematical problems and the problems that were not mathematical problems, I concluded that, overall, the US participants struggled to interpret the given calculation and/or think backwards, in order to connect the calculation back to the given figures and initial story. It is possible that the US participants have had fewer opportunities to think backwards when doing mathematics. In particular, it is not clear to what extent that the use of mixed fractions for US participants during the first task administration hindered their performance in the Comprehending process.

***Description 5: The difficulties that the US participants had in posing problems for the Comprehending process were similar to the difficulties they had in the Translating process.*** During the first task administration, the US participants posed 24% (8 out of 33) unsolvable mathematical problems and 18% (6 out of 33) problems that were not mathematical problems. Among the eight unsolvable mathematical problems, five were worded unclearly. Occasionally, it was unclear which piece of cloth a smaller piece was cut from; sometimes it was unclear as to which piece of cloth the question was asking about. Here is an example:

> Diana needs at least a four foot long piece of fabric. Diana's sister needs at least $1\frac{2}{3}$ foot long piece to make her scarf. They both need a $1\frac{3}{4}$ foot wide piece to complete it. Using distribution of $1\frac{3}{4}$, how would you obtain an answer? (U11, 1st, PP2)

It is clear that this participant recognized the use of distributive property in the given calculation. However, it is not clear what "an answer" was about in his/her question. It is not clear how to use the "distribution of $1\frac{3}{4}$", either. The other three problems lacked information to find a solution. One example follows:

> George is hanging up a new poster and needs giant tape to hold it up. He needs tape that is $1\frac{3}{4}$ ft wide. Carson also got a new poster and needs tape 4ft long plus $1\frac{2}{3}$ feet wide. How much tape all together do George and Carson needs to hang up 2 posters. (U24, 1st, PP2)

In this problem, the missing information is the length of tape that George needed.

Among the six problems that were categorized as not mathematical problems, three asked no question at all, while the other three asked questions that could be answered without computational or reasoning steps. For example:

> Diana's sister wants to add her piece onto Diana's piece to make one big scarf for them to share. How could she do it? (U22, 1st, PP2)

No mathematical computation or reasoning is needed in order to answer this question. Therefore, it is not a mathematical problem.

Sixteen US participants posed three unsolvable mathematical problems and five problems that were categorized as not mathematical problems in the Comprehending process during the second task administration. Four of those problems only described the computational process of the given calculation, for example, “What is the square root of the two radii of the smallest bowl?” (U15, 2nd, PP2). These participants used the given information that the radius of the smallest bowl was one inch and ignored the “ $1^2$ ” part because $1^2=1$. In this case, no mathematical reasoning step is needed. The fifth and sixth problems had the improper use of mathematical terms (e.g., “the radius of a square”, U13, 2nd, PP2), while the seventh problem had a contradiction with the given figure (i.e., “the outer most box [square] area is equal to $\sqrt{10}$”, U26, 2nd, PP2). The last problem did not ask a question and the participant admitted that he/she could not figure out what the given calculation represented. However, this participant discussed what the given calculation could not represent:

> My thinking was that this calculation [$\sqrt{1^2+1^2}$] should have something to do with either the area of [or] the circumference of the smallest circle/bowl [,] but both the those calculations would involved pi($\pi$). And I don’t think it has to do with the area of the smallest square because the length is 2, not 1. I do not have any problems for this. Sorry. (U28, 2nd, PP2)

Although this participant did not pose any problems, he/she made an effort to write the mathematical reasoning.

***Summary.*** The US participants were challenged more than their Chinese counterparts in posing a problem representing given calculations. This is possibly because they experienced difficulties in either interpreting the given calculation, connecting the given calculation back to initial figures or story, wording the sentences of a problem clearly, or working with mixed fractions.

**Editing process.** This process asked the participants to pose a new problem by editing the initial story. Similar to the Translating process, the participants had more freedom to select useful information and discard unnecessary information in order to generate new problems than in the Comprehending process. Due to this feature of the Editing process, I again explored the mathematical concepts and ideas that the participants selected to use (Description 6), as well as new creative ideas that occurred in their posed problems compared to the creative ideas that I had discussed for the Translating and Comprehending processes (Description 7). In addition, since the Editing process was the third problem-posing process during each task administration, I examined the influence of participants' prior experiences on their performance during the Editing process (Description 8 and Description 9).

***Description 6: Although a large number of posed problems for the Editing process used similar mathematical concepts during the Translating and Comprehending processes, many of the problems involved more computational steps and/or a different structure.*** During the first task administration, 53% (17 out of 32) of US participants posed 64% (18 out of 28) solvable mathematical problems that also focused on the area of rectangles, including the idea of the sum or difference of different area pieces (13 out of 18), and fit-in ideas (i.e., the maximum times that a small piece of cloth could fit in a big piece, 5 out of 18). However, no US participant posed a problem about the perimeter of given rectangles. Among the five problems that involved the fit-in idea, both underestimation and overestimation cases for solving division-with-a-remainder problems occurred. Underestimation means that, in some cases, the remainder is discarded in order to answer a question realistically (see example U04, 1$^{st}$, PP3), while overestimation means that, in other cases, the remainder needs to be taken as one no matter how big it is, in order to answer a question realistically (see example U26, 1$^{st}$, PP3).

If Diana's sister didn't want a scarf, how many scarves could Diana have made from the [initial] piece of cloth, assuming all of her scarves are the same size ($1\frac{3}{4}$ feet×4 feet)? What if only Diana's sister had wanted scarves, not Diana. How many scarves of the same size ($1\frac{2}{3}$ feet×$1\frac{3}{4}$ feet) could Diana's sister have made? Show all of your work. (U04, 1st, PP3)

Jeffrey is having a roof leak and needs to get this fixed before the weather gets bad. Jeffrey knows that 2 roofing titles equals 1ft by 1ft. Jeffrey must cover a space that is $1\frac{3}{4}$ feet wide and 4 feet long. How many titles will he need to purchase? (U26, 1st, PP3)

Thinking back to Translating process in the first set of tasks, the eight solvable mathematical problems that involved the fit-in idea were all about underestimation, and no overestimation cases occurred. In addition, the group of participants who used the fit-in or division-with-a-remainder idea to pose problems during the Translating process were completely different from the group of participants who did this during the Editing process. Although, this was predictable because all the participants were asked to pose different problems for each process from the problems they had posed previously. It shows that this way of engaging prospective elementary teachers encourages them to keep applying different mathematical concepts and constructing multiple mathematical connections.

Fifty-one percent (28 out of 55) of Chinese participants posed 48% (30 out of 62) solvable mathematical problems that incorporated the ideas of the area sum or difference of different pieces of cloth (15 out of 30), area ratio of different pieces (5 out of 30, see example C56, 1st, PP3), area fit-in with only underestimation case (2 out of 30), extreme area value of a square/rectangular/circular piece of cloth (6 out of 30, see example C36, 1st, PP3), and some creative ideas (2 out of 30) related to the minimum side length of a rectangle (see example C67, 1st, PP3) and the superficial area of an uncovered box (see example C55, 1st, PP3).

李娜买了一块布料。她先裁剪了一块长1米宽 $\frac{3}{4}$ 米的布料做了围巾，此时布料面积为原来的 $\frac{5}{8}$ 。后来她的妹妹在余下布料裁剪了一块长宽各 $\frac{3}{4}$ 米的布料。请问妹妹所裁布料占在李娜剩下的布料的几分之几？ (C56, 1st, PP3)

[Translation] Na Li bought a piece of cloth. She first cut off a piece that was 1 meter long and $\frac{3}{4}$ meters wide to make a scarf, the area of which was $\frac{5}{8}$ of the initial cloth. Her sister then cut off a piece that was $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide from the rest of the cloth. What is the fraction of Na Li's sister's cloth to the rest of the initial cloth after Na Li cut off her scarf?

李娜花12元买了一块长2米宽1米的布料，她先剪了一块长1米宽 $\frac{3}{4}$ 米的布料做了围巾，后来又从余下的布料又剪了一块长宽各 $\frac{3}{4}$ 米的布料给妹妹做了围巾。若李娜还想做一条长方形围巾，请问该长方形围巾的最大面积是多少？(C36, 1st, PP3)

[Translation] Na Li spent 12 yuan (RMB) and bought a piece of cloth that was 2 meters long and 1 meter wide. She first cut off a piece that was 1 meter long and $\frac{3}{4}$ meters wide to make a scarf. She then cut off another piece that was $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide to make a scarf for her sister. If Na Li wanted to make a third rectangular scarf, what would be the maximum area of that scarf?

已知李娜买了一块布料，她先裁剪了一块长1米宽 $\frac{3}{4}$ 米的布料做了围巾，然后又从剩下的布料上剪了一块长宽各 $\frac{3}{4}$ 米的布料，求李娜买的这块布料的长和宽的最小整数米。(C67, 1st, PP3)

[Translation] Na Li bought a piece of cloth. She first cut off a piece that was 1 meter long and $\frac{3}{4}$ meters wide to make a scarf. She then cut off another piece that was $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide. What are the minimum integral meters of the length and width of the initial cloth bought by Na Li?

小明想做1个长为1m、宽为 $\frac{3}{4}$ m、高为 $\frac{3}{4}$ m的矩形无盖盒子，但家里只剩下一张长为2m、宽为1m的卡纸，问：材料够吗？如果不够，至少还要买几张这样的卡纸。

(C55, 1st, PP3)

[Translation] Xiaoming wanted to make a rectangular uncovered box that was 1 meter long, $\frac{3}{4}$ meters wide and $\frac{3}{4}$ meters high. However, there was only one paperboard that was 2 meters long and 1 meter wide left at home. Question: Was that paperboard enough to make the box? If not, at least how many more paperboards with the same size of the paperboard that was left at Xiaoming's home should he buy?

According to Stein, Smith, Henningsen, and Silver (2000), these four problems are all high-level cognitively demanding tasks. In other words, these problems are challenging and require problem solvers to understand mathematical concepts and look for the underlying mathematical structure. The problems posed by C56 and C67 had an unknown beginning, "Na Li bought a piece of cloth", which increased the difficulty level of problem solving. The problem posed by C56 required the problem solvers to understand the meaning of a fraction in order to correctly find the answer. The problems posed by C36 and C67 were similar in that both of the problems required the problem solvers to try different arrangements of two cut-off pieces of cloths in order to answer the questions. Finally, the problem posed by C55 talked about the surface area of a rectangular box with one missing face.

For the second set of tasks, 75% (12 out of 16) of the US participants and 74% (20 out of 27) of the Chinese participants posed solvable mathematical problems that focused on the concepts of radius, diameter, area, or circumference of a circle, area or perimeter of a square, and area sum or difference between two shapes. For example:

> Using the smallest bowl with the radius of 1 inch, calculate the area of the bowl. The [Then] calculate the area of the smallest box [square]. How much area is not taken up by the bowl [from the smallest square]? (U03, 2nd, PP3)

However, compared to their performance in the Translating process that no participant asked questions about the circles or the squares between the smallest and the biggest ones, five of those 12 US participants and eight of those 20 Chinese participants asked questions about the circles or squares that were not the smallest or the biggest ones for the Editing process. For example:

> Find the diameter of the bowl labled [labeled] #4 [the second biggest bowl]. (U14, 2nd, PP3)

More creatively, two US participants asked questions about the measurement of an unknown circle that was inscribed in a square with either the side length or diagonal length of that square given. Another US participant asked for the radius of the tenth circle if the same pattern in the initial figure was extended. A Chinese participant provided a radius of an unknown circle and asked which circle was in the initial figure. All of these meaningful mathematical ideas and problem structures show that it is worthwhile to provide prospective elementary teachers more time and to encourage them to pose problems with new ideas. They are able to continuously innovate.

***Description 7: Both US and Chinese participants were capable of posing more creative, high-level cognitively demanding problems in the Editing process. Some of those problems***

***had similar mathematical ideas and/or structure with the problem given in later problem-solving process, especially during the first task administration.*** During the first task administration, 25% (8 out of 32) of the US participants and 40% (22 out of 55) of the Chinese participants posed solvable mathematical problems for the Editing process that introduced money as a new constraint. One of those 22 Chinese participants provided the prices of different scarves and asked for the total cost. It was a simple addition problem. One US participant and four Chinese participants posed problems that used the relationship among the cost, unit price of the cloth and area of that piece of cloth (i.e., cost = unit price × area), as well as all four operations. The remaining seven US participants and 17 Chinese participants all posed a higher-level cognitively demanding problems that involved similar ideas and/or structure with the problem given in later problem-solving process. For example:

> Diana bought a piece of cloth that was 4 ft wide and 5 ft long. The total cost of the cloth was $16. How much money would a piece of 2 ft wide and 3 ft long cloth cost at the same price? (U09, 1$^{st}$, PP3)

This problem was worded clearly and this type of problem required a high-level of cognitive thinking in order to be solved. This type of problem has multiple ways to be solved, but has only one correct answer, namely, open-middled problems according to Johnson and Herr (2001). Problem solvers could use the proportion idea, multiply the unit price of the cloth with a specific area, or multiply the total cost with the fraction of the specific area to the total amount of the area.

Among those problems, a particular example involved even more mathematical reasoning:

> 李娜花12元买了一块长2米宽1米的布料用来做衣服。后来她发现，做衣服，李娜还需长$\frac{3}{4}$ m宽 $\frac{3}{4}$ m的布料。如果做裤子还需长1m宽1m的布料。而李娜还剩5元。请问李娜剩下的钱买做衣服的布料够吗？如果做裤子则李娜还需要问妈妈拿多少钱？

(C44, 1st, PP3)

[Translation] Na Li spent 12 yuan and bought a piece of cloth that was 2 meters long and 1 meter wide to make clothes. She then found that, if she wanted to make a coat, she also needed a piece of cloth that was $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide. If she wanted to make a pair of pants, she also needed a piece of cloth that was 1 meter long and 1 meter wide. However she only had 5 yuan left. Did she have enough money to buy the needed cloth to make a coat? If she wanted to make a pair of pants, how much more money should she ask her mom for?

In consideration of this participant's selection of "5 yuan" and the questions that he/she asked, it was clear that he/she knew that five yuan was enough for the needed piece of cloth to make a coat but not enough for the needed piece of cloth to make a pair of pants. He/She had the control of the mathematical structure that was involved in this problem.

Despite these findings, there were some differences in US and Chinese participants' posed problems that had similar ideas and/or structure with the given problem in the problem-solving process. First, seven out of 17 Chinese participants posed problems that asked for the cost of either the sum or difference of more than one piece of cloth, which usually involved more computational steps than problems asking about the cost of only one piece of cloth. Only one out of seven US participants posed such a problem. Second, eight of those 17 Chinese participants asked at least two questions in a sequence, while six of them started with a question about the unit price of the cloth and then asked a question for the cost of specific pieces. Here is an example:

李娜花12元买了一块长2米宽1米的布料，她剪裁了一块长为1米宽$\frac{3}{4}$米的布料做围巾。①请问1平方米的布料需花费多少元？②在商场里李娜同款的围巾买[卖]6元，

那么李娜自己买布料做围巾是否划算？若划算，则可以省下多少元钱？(C50, 1st, PP3)

[Translation] Na Li spent 12 yuan and bought a piece of cloth that was 2 meter long and 1 meter wide. She cut off a piece that was 1 meter long and $\frac{3}{4}$ meter wide to make a scarf. ① How much was each square meter of this cloth? ② If in the market the price of the same scarf with the one that Na Li made was 6 yuan, was that a better deal for Na Li to make it by herself? If yes, how much money could she save?

This group of problems involves the sequence of problem solving. In other words, the answer to the first question could serve to help answer the next. No US participants posed problems in this format.

For the second set of tasks, 19% (3 out of 16) of the US participants posed solvable mathematical problems about the biggest circle, while one of them asked for the area difference between the smallest and the biggest circles. No one asked questions about the concept of ratio. Forty-eight percent (13 out of 27) of Chinese participants posed solvable problems about the biggest circle, while three of them asked for the ratio between either the radii or the areas of the biggest circle and the smallest circle, which had the similar idea and structure with the problem given in the problem-solving process.

***Description 8: Despite being asked to pose a problem related to the sum of the areas in the Comprehending process, some US participants were not successful. However, they posed that expected problem in response to the Editing process. On the other hand, some Chinese participants anticipated what the next step would be and posed a problem that was expected in the Selecting process.*** During the first task administration, 13% (4 out of 32) of US participants posed a problem in the Editing process that correctly represented the given calculation in the

Comprehending process. No Chinese participants did this. One of those four participants posed exactly the same problem for both processes. I counted once for those two problems in the total number of posed problems by US participants in the Comprehending process, but did not count in the Editing process. The other three US participants either asked no questions or posed a solvable mathematical problem that was irrelevant to the given calculation in the Comprehending process. It is beneficial to consider why these participants could not pose a proper problem for the given calculation but could successfully pose a problem representing that calculation when it was not given immediately afterwards. In addition, it shows that, although these US participants were able to pose solvable mathematical problems, they might not know how to solve those problems that they posed.

In contrast, 5% (3 out of 55) of Chinese participants posed problems in the Editing process, during the first task administration, that had exactly the same answer as the given number in the subsequent Selecting process. Since they posed those problems without solving them, they might not have known the specific answer to those problems. Once they moved to the Selecting process, they realized that the number given in the Selecting process was the exact answer to the problem that they posed in the Editing process. I counted once for those problems to the total number of posed problems by the Chinese participants for Editing and Selecting processes respectively. No US participants posed problems during the Editing process that had exactly the same answer as the given number in the Selecting process.

***Description 9: Many Chinese participants posed problems for the Editing process that could be solved using the solution of the problem they posed for the Comprehending process, and this was true for both the first and second sets of tasks.*** During the first task administration, 31% (17 out of 55) of Chinese participants posed a problem in the Editing process that could be solved using the solution to the problem they posed in the Comprehending process. More specifically, all 17 Chinese participants posed a problem asking for the total area of two cut-off pieces of cloth, which correctly represented the given calculation in the Comprehending process. They then posed problems for the Editing process asking for the area of the rest of the cloth, cost of two cut-off pieces of cloth, cost of the rest of the cloth, or the ratio of the area of the cut-off cloths to the total area of the initial cloth. Here are the posed problems by C60 for Comprehending and Editing processes, respectively:

> 已知如上，求李娜与妹妹总共裁剪的布料的面积？(C60, 1st, PP2)
>
> [Translation] Known as above [given story and figures], find the total area of the cut-off cloths for making scarves by Diana and her sister.
>
> 已知如上，请求出李娜与妹妹裁剪下来的布料价钱为多少元？(C60, 1st, PP3)
>
> [Translation] Known as above [given story and figures], find the total cost of the cut-off cloths for making scarves by Diana and her sister.

The relatedness of these two problems are obvious. One out of 32 US participants posed problems for Comprehending and Editing processes during the first task administration that had such a relationship.

In the second set of tasks, six Chinese participants asked questions about either the circumference or area of the second smallest circle in the Editing process, while all of them asked the same question in the Comprehending process about the radius of the second smallest circle which correctly represented the given calculation. None of 16 US participants' posed problems in the Comprehending and Editing processes during the second task administration showed such a relationship, including two US participants who had posed problems that correctly represented the given calculation in the Comprehending process. This indicates that

Chinese participants had more of an aptness to pose problems in a sequence or that were connected, while their US counterparts did not show such aptness.

***Summary.*** For the Editing process, although both US and Chinese participants posed many problems that involved similar concepts and relationships to the problems they posed for the Comprehending process, a number of those problems involved either more computational steps or a different mathematical structure. Meanwhile, more creative ideas comparing to prior processes occurred in US and Chinese participants' posed problems. These findings indicate the extension of their problem-posing ideas and skills throughout the problem-posing process. The problems posed in the Editing process also showed that Chinese participants were more apt at posing problems in a sequence, or that were connected, than their US counterparts. This finding is consistent with previous studies (e.g., Cai & Hwang, 2002).

**Selecting process.** This process asks the participants to pose a problem that has a specific answer as given. It is somewhat similar to the Comprehending process. Therefore, the participants' problem-posing performance in the Selecting process was comparable to their performance in the Comprehending process. In this section, Description 10 and Description 11 discuss the US and Chinese participants' performance in the Selecting process compared to their performance in the Comprehending process. Description 12 and Description 13 analyze the patterns shown in unsolvable and not mathematical problems, respectively.

***Description 10: The US participants were more challenged in the Selecting process than the Comprehending process, while their Chinese counterparts showed few such difficulties.*** Compared to their performance in the Comprehending process during the first task administration, 82% (45 out of 55) of the Chinese participants posed a problem purely according to the initial story and satisfied with the requirement (i.e., the answer to that problem was $\frac{11}{16}$

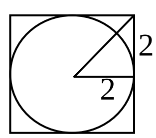
square meters) for the Selecting process. Fourteen percent (8 out of 55) of the Chinese participants posed problems that had the answer as the given number, but with added information to the initial story or they created new scenarios. More specifically, two participants created completely different real-life scenarios with a similar mathematical structure to the initial story, three participants changed the measurements of two cut-off pieces of cloth in order to get a total area as the given number, and the remaining three participants incorporated creative ideas in their posed problems (e.g., “请问李娜和李娜妹妹做围巾共用布料的 $\frac{11}{21}$ 是多少？ [Translation] What was $\frac{11}{21}$ of the total area of Na Li and her sister’s scarves? (C63, 1st, PP4)”). Apart from these, only one Chinese participant posed a solvable mathematical problem for this process that had a different answer from the given number and one Chinese participant posed an unsolvable mathematical problem due to the ambiguity in the wording of its question.

During the second task administration, 89% (24 out of 27) of the Chinese participants posed solvable mathematical problems purely according to the initial story, to which the answer was the given number. Only 7% (2 out of 27) of the Chinese participants posed solvable mathematical problems according to the initial story that did not have the answer as the given number, and the remaining 4% (1 out of 27) of the Chinese participants posed an unsolvable mathematical problem due to the vagueness in the wording of its question.

On the contrary, during the first task administration, only 9% (3 out of 32) of the US participants posed a solvable mathematical problem according to the initial problem that had the answer as the given number. Thirty-eight percent (12 out of 32) of the US participants posed solvable mathematical problems that had different answers from the given number. Among those 12 participants, seven created new scenarios and the remaining five used only the initial story. Four out of the five participants posed a problem that successfully represented the given calculation in the Comprehending process and two participants posed exactly the same problem for both the Selecting and Comprehending processes. Among the remaining 53% (17 out of 32) of the US participants, 11 participants posed unsolvable mathematical problems and six posed problems that were not mathematical problems. According to Christou et al. (2005), the Editing and Selecting processes are more demanding than Comprehending and Translating processes. In addition, if the use of mixed fractions for US participants increased the level of complexity of the tasks, specifically in the Comprehending process, it is understandable why the US participants were challenged more in the Selecting process than in the Comprehending process.

During the second set of tasks, 19% (3 out of 16) of the US participants posed solvable mathematical problems that had the answer as the given number. However, it was not clear whether those participants knew which shapes in the given figure they were talking about. One of the three participants posed the following problem:

> One of the bowls in the set has a radius of 2 inches. Calculate the distance from the center of this bowl to the corner of the box that is packaged in (see graph [picture] below). (U03, 2nd, PP4)




The answer to this problem was the given number $2\sqrt{2}$. This participant successfully generated this problem because he/she found that $\sqrt{2^2+2^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$, where the structure of $\sqrt{2^2+2^2}$ was exactly the same as $\sqrt{1^2+1^2}$ given in the Comprehending process. Therefore, he/she simply manipulated one constraint of the problem that he/she posed in the Comprehending process (see the problem below) and posed above problem in the Selecting process.

Using the given radius of 1 inch, calculate the distance from the center of the smallest

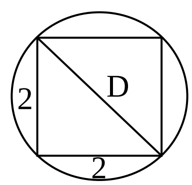
> bowl to the corner of the smallest box. (U03, 2nd, PP2)

This participant actually applied the what-if-not problem-posing strategy (Brown & Walter, 1990b) and asked, what if the radius of the inscribed circle was two inches, instead of one inch. Because of the use of this strategy, it was not clear whether he/she knew that the bowl that had a radius of 2 inches was the third biggest bowl or if the question he/she asked was about the radius of the second biggest bowl.

The second participant also noticed that $\sqrt{2^2+2^2}=2\sqrt{2}$ but focused on a circumscribed circle of a square with a given side length of two inches:

> If the square that goes inside of a circle has side lengths of 2 inches, what is the diameter of the circle that houses that square? (U04, 2nd, PP4)

This participant drew the following picture:




Although this participant understood the relationship between a square and its circumscribed circle, it was not clear whether he/she knew that he/she actually was asking for the diameter of the second smallest circle in the initial figure. The third participant posed exactly the same problem for both the Comprehending and Selecting processes asking for the diameter of the second smallest circle in the initial figure. Again, it was hard to tell whether he/she knew that the diameter of the second smallest circle was $2\sqrt{2}$, instead of $\sqrt{1^2+1^2}$ .

Another 19% (3 out of 16) of the US participants also posed solvable mathematical problems in the Selecting process, but had answers different from the given number. One of them asked for the diameter of the third smallest circle, another asked two questions about the biggest circle, while the third asked for the area of the third smallest square. Among the remaining 62% (10 out of 16) of the US participants, half of them posed unsolvable mathematical problems and the other half posed problems that were not mathematical problems.

***Description 11: The Chinese participants were more capable of perceiving the relationship between the given calculation in the Comprehending process and the given number in the Selecting process during both task administrations compared to their US counterparts.*** For both sets of tasks, the calculation given in the Comprehending process was closely related to the given number in the Selecting process. For the first set of tasks, the Comprehending process expected problem posers to ask a question about the total area of two cut-off pieces of cloth while the Selecting process asked for the remaining area after cutting off those two pieces. For the second set of tasks, the result of the given calculation in the Comprehending process was half of the given number in the Selecting process and the participants actually had multiple choices for the Comprehending and Selecting processes. More specifically, the calculation given in the Comprehending process represented (1) the radius of the second smallest circle, (2) half of the diagonal of the smallest square, or (3) half of the side length of the second smallest square, while the number given in the Selecting process could be (1) the diameter of the second smallest circle, (2) the diagonal of the smallest square, (3) the side length of the second smallest square, (4) the radius of the second biggest circle, (5) half of the diagonal of the third biggest square, or even (6) half of the side length of the second biggest square. The participants had the chance to flexibly combine those possibilities to pose reasonable problems.

The US participants' performance in both processes during both task administrations indicated that the majority of them had difficulty in perceiving and applying those relationships for problem posing. Conversely, almost all of the Chinese participants posed solvable mathematical problems as expected, and many of them utilized those relationships. As evidenced in the results, 44% (12 out of 27) of the Chinese participants correctly used the relationship that the radius of a circle was half of its diameter (6 out of 12), the radius of a circle was half of the diagonal of its inscribed square (4 out of 12), and the radius of a circle was half of the side length of its circumscribed square (2 out of 12). There was one US participant who used the relationship between the radius and diameter of the same circle in the Comprehending and Selecting processes, but mistakenly believed that $\sqrt{1^2+1^2}$ represented the radius of the third biggest circle while $2\sqrt{2}$ represented the diameter of that circle.

***Description 12: Many participants tried to either construct the number given in the Selecting process according to its appearance or directly use the given number as one condition of their posed problems rather than considering it as the answer to their posed problems. These two circumstances occurred especially in problems posed by US participants.*** Nine US participants posed nine unsolvable mathematical problems in the Selecting process during the first task administration. Four of them tried to construct the answer $\frac{121}{12}$ by either looking at this fraction as a division (e.g., "break up the material" that was 121 square feet "into 12 pieces" but asked for the length of each piece which was unsolvable. U21, 1st, PP4), or integrating new fractions which had or would make a denominator of 12 (e.g., "She must trim off $\frac{19}{12}$ of a foot extra fabric that she didn't need." This participant meant to use $\frac{19}{12}$ as the area of a piece of fabric according to the computations he/she made on the margin of the page, but worded ambiguously. U20, 1st, PP4). Similarly, five out of six US participants who posed unsolvable mathematical problems in the Selecting process during the second task administration tried to construct $2\sqrt{2}$. Four of them treated this number as $2\times\sqrt{2}$, where the "2" represented the number of circles/bowls and the "$\sqrt{2}$" represented the diameter or area of a circle/bowl. The fifth participant treated this number as $2\times\sqrt{1+1}$ because the radius of the smallest circle in the initial figure was one inch.

Of the remaining US participants who posed unsolvable mathematical problems in the Selecting process, five during the first task administration and one during the second task administration, directly used the given number, $\frac{121}{12}$ square feet for the first set and $2\sqrt{2}$ for the second set, in their posed problems. For example:

> Carl wants to have a piece of material with the area of $10\frac{1}{12}\text{ft}^2$ area[,] he has a piece of scrap that is $4 \times 5$ft in length[,] what is the missing side? (U16, 1st, PP4)

This participant converted $\frac{121}{12}$ to $10\frac{1}{12}$ and used it as one constraint of the posed problem. However, his/her problem was phrased ambiguously. According to this participant's posed problems in the Comprehending and Editing processes, he/she used a × b ft to represent the area of a piece of cloth. Therefore, the use of "4×5ft in length" was confusing. The strategy of using the given number as a constraint in posed problems was not applied in solvable mathematical problems posed by US or Chinese participants during both task administrations.

The strategy of trying to construct $\frac{121}{12}$ in the Selecting process during the first task administration also occurred in solvable mathematical problems posed by 12 US participants. All of those problems failed to make $\frac{121}{12}$ the correct answer. Eight of those participants misunderstood the operations with fractions such as $2\times\frac{60.5}{6}=\frac{121}{12}$, $\frac{110}{3}+\frac{11}{9}=\frac{121}{12}$, and $\frac{121}{12}=10\frac{1}{12}=5\frac{1}{6}\times2\frac{1}{2}$. This strategy was not used by US participants who posed solvable mathematical problems during the second task administration.

As discussed in Description 10, eight of 55 Chinese participants posed solvable mathematical problems during the first task administration that had the answer of $\frac{11}{16}$ square meter, but created new real-life scenarios or added new condition to the initial story. Five of the eight participants tried to construct $\frac{11}{16}$, but had different pathways from their US counterparts. Three of them decomposed $\frac{11}{16}$ into $\frac{9}{16}+\frac{2}{16}$ or $\frac{9}{16}+\frac{1}{8}$, which was also $\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}$ or $\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{2}$, while the other two incorporated new fractions that had denominator of 16. For example:

> 小美有一块长1米、宽 $\frac{3}{4}$ 米的布料。她用这块布料做了一条裙子，裙子做好后，小美发现还剩了 $\frac{1}{16}$ $m^2$ 布。请问做这条裙子总共花了多少布？(C58, 1st, PP4)
>
> [Translation] Xiaomei had a piece of cloth that was 1 meter long and $\frac{3}{4}$ meter wide. She used this piece of cloth to make a skirt. After making that skirt, Xiaomei found that there was still $\frac{1}{16}$ $m^2$ of cloth left. How much cloth was used for that skirt?

No Chinese participants used this strategy during the second task administration.

***Description 13: The problems that were categorized as not mathematical problems posed by US participants either asked no questions at all or purely described the procedures to get the given number.*** Five out of eight and three out of four US participants who posed not mathematical problems did not ask any questions in the Selecting process during the first and second task administration, respectively. Some admitted that they could not figure out what the given number was for, and some described what they had noticed, for example, “When you multiply that [those] fractions together 12 is the common factor [multiple] in all of them. (U25, 1st, PP4)” and “I think that the square root of 8 is $2\sqrt{2}$, so somehow the question must guide the students to use $\sqrt{8}$. (U21, 2nd, PP4)”. The remaining participants, three in the first set of tasks and one in the second, asked procedural questions such as “Show Sam how to turn it [$\frac{121}{12}$] into an improper fraction. (U11, 1st, PP4)” and “What is the diameter of the smallest bowl multiplied by it’s square root? (U15, 2nd, PP4)”. These problems required no mathematical reasoning.

***Summary.*** The US participants were most challenged in the Selecting process than in other processes. One possible reason that influenced the US participants’ performance in this process of the first set of tasks is the use of mixed fractions that may have increased the level of complexity of the tasks. In spite of this, many of them showed endurance in posing a problem for this process. More specifically, many of them tried to construct the given number according to its appearance or directly used the given number as one condition of their posed problems. One possible reason that the Chinese participants were much less challenged by the Selecting process was that they were more capable of perceiving the relationship between different problem-posing processes.

**Problem posing after problem solving process.** I discussed the features of posed problems after problem solving in this section. I focused on the different features of posed problems before and after problem solving (Description 14), the impacts of the problem-solving experience on participants’ problem-posing performance (Description 15), and a specific feature of problems posed by Chinese participants (Description 16).

***Description 14: The US and Chinese participants posed many creative problems after problem solving. More importantly, some of those creative ideas or mathematical structure were not shown in problems posed before problem solving.*** During the stage of problem posing before problem solving, the posed problems that involved money information in the first set of tasks and ratio relationship in the second set of tasks, and had a similar structure to the problem in problem-solving task were classified as creative problems. In this stage, those kinds of posed problems were not classified as creative problems because the problem posers had seen an example during the problem-solving process. Even so, the categories of creative problems posed after problem solving were expanded compared to the problems posed before problem solving.

During the first task administration, 44% (14 out of 32) of the US participants posed 31% (19 out of 61) solvable mathematical problems and 44% (24 out of 55) of the Chinese participants posed 35% (35 out of 100) solvable mathematical problems that were classified as creative problems. Four of the 19 problems posed by US participants involved the meaning of a fraction (e.g., "If she used $\frac{3}{5}$ of the cloth how much would that cost her to replace?", U29, 1st, PP5) and three of them used the concept of percentage (e.g., "Diana bought a piece of cloth that was 4 feet wide and 5 feet long for 16 dollars. The store was selling the cloth at a sale of 20% off.", U06, 1st, PP5). Eleven of the 35 problems posed by Chinese participants involved the concepts of fractions (5 out of 11), ratios (2 out of 11) and percentages (4 out of 11).

Five of those 19 problems posed by US participants focused on the area of a piece of cloth, unit price and total cost, but involved more complex reasoning processes. Here is an example:

> Suppose Diana wanted to make three blankets, each 20 sq ft, but each of a different material. The fabric store has a different price per sq ft for each of the materials she has chosen. The first material is 0.8¢ per sq ft. The second is $1.25 per sq ft. If Diana spent a total of $60 in the fabric store on materials, what was the cost of the last material per sq ft? (U01, 1st, PP5)

This problem was special in hiding the unit price of the third type of material and having just enough conditions to determine the price. This problem also involved unit conversion between cents and dollars which made it more challenging, although it could possibly be an unintentional mistake. In order to correctly solve this problem, problem solvers need to be clear about the overall mathematical structure and each part of this story. Five out of 35 problems posed by Chinese participants involved the relationship of the area of a piece of cloth, unit price and total cost. Three of these problems asked questions about profit and loss, and two problems involved unit convertion between square centimeters and square meters.

The remaining seven out of those 19 problems posed by US participants and 13 out of 35 problems posed by Chinese participants involved the idea of how many times a small number would go into a big number. Two of those problems posed by US participants involved a higher level of cognitive thinking:

> Diana is starting a scarf making business. She has a budget of $500. If each square foot cost $1.25 and her scarves are $1\frac{2}{3}$ ft by $1\frac{3}{4}$ ft long, how many scarves can she make, assuming she spends all $500 on scarf making? (U08, 1st, PP5)
>
> [A university] is buying new bricks to fix a roof on the buildings. One side of the roof is 4 ft wide and 5 ft long. Another side is $1\frac{3}{4}$ ft wide and 4 ft long. If each brick is 1 dollar, how much money will the school have to spend if each brick is 1 ft long and $\frac{3}{4}$ ft wide? (U24, $1^{st}$, PP5)

In addition to asking how many times a small number would go into a big number, the first problem also involves total cost, unit price, and dimensions of a rectangular piece of cloth. These ideas actually provide multiple choices for problem solving. A problem solver could either focus on how many times the cost of "one scarf" could go into the total cost (i.e., the "budget"), or focus on how many times the area of "one scarf" could go into a maximum area of cloth that the "budget" could afford. Regardless of the solution, the problem solver has to underestimate at the end to realistically answer the question.

The second problem also involved more than one solution. A problem solver could first determine the number of bricks the school needed for each side of the roof and then add the results together in order to find the total cost. He/she could also calculate the total area of both sides of the roof that needed to be fixed and then determine the number of bricks in order to finally compute the total cost. In the end, the problem solver needs to overestimate the number of bricks. However, if the problem solver choose to use the first method, he/she may overestimate twice, one for each side of the roof, which could cause unnecessary waste in reality. Therefore, different problem solvers may get different answers to the second problem. According to Johnson and Herr (2001), the first problem was an open-middled problem while the second one was an open-ended problem.

A third example was from a Chinese participant who incorporated three-dimensional cubes:

> 这些[矩形]是由 $\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}$，即长宽高都是 $\frac{1}{4}$ m 的正方体拼出来的图形的三视图 [俯视图]，问最少几个正方体能摆出这样的图形。(C68, 1st, PP5)
>
> [Translation] Suppose these [given rectangles] are top views of three-dimensional cuboids formed by cubes of $\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}$ (i.e., cubes with side length of $\frac{1}{4}$). How many cubes will be needed in order to form these top views?

Although this problem talks about three-dimensional cubes and cuboids, only the areas of two-dimensional rectangles and squares are needed in order to solve it. In addition, the word "最少[at least]" in this question ensured the solvability of this problem. It was clear that this participant knew what would make his/her problem unsolvable.

The remaining three solvable problems posed by US participants involved the relationship between the change of dimensions in proportion and the change of area of a rectangle. For example:

> Dylan is installing a new door to his garage. Dylan has purchased a piece of the garage door at $250.00. His piece was 4 feet by 5 feet. If the total space is 12 feet by 15 feet, how much money will Dylan need to spend to finish the garage door? (U26, 1st, PP5)

A problem solver could use the idea of multiplying the unit price of the door material by the area, or set up a proportion to find the total cost of the garage door. It is possible that some problem solvers would be able to recognize that the side lengths of the door and the corresponding side lengths of the bought piece are in proportionate, and the ratio is three. In this case, they would have to multiply nine (i.e., three squared) to $250 in order to get the total cost of the garage door, instead of multiplying three to $250. This type of problem helps to challenge thinking and deepen learners' understanding of the relationship between side length and the area of a rectangle, or the relationship between one- and two-dimensional measurements. No Chinese participants posed this type of problem.

The remaining three problems posed by Chinese participants during the first task administration asked questions about a circular shape that was restricted by a rectangle from two different perspectives. No US participants used such an idea. Two of the problems asked for the biggest possible circle that could be cut off from a rectangle with a given width and length (see example C38, 1st, PP5), while the other problem made the diameter of a circle only restricted by the length of the initial rectangle (see example C39, 1st, PP5).

> 已知：李娜花 12 块买了一块长 2 米宽 1 米的布料，她先裁剪了一块长 1 米宽 $\frac{3}{4}$ 米的布料做了围巾，李娜的妹妹看到后非常喜欢，便从余下布料上裁剪了一块长宽各

$\frac{3}{4}$米的布料。问：若李娜妹妹发现圆形围巾更创新，那么她最大能通过裁剪她原先的围巾得到半径为多少米的圆形围巾？(C38, 1st, PP5)

[Translation] Given: Na Li spent 12 yuan and bought a piece of cloth that was 2 meters long and 1 meter wide. She first cut off a piece that was 1 meter long and $\frac{3}{4}$ meters wide to make a scarf. Na Li's sister saw that cloth and she really loved it. She then cut off a piece that was $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide from the rest of the cloth. Question: If Na Li's sister found that a circular scarf was more attractive, what would be the radius of the biggest possible circular scarf that she could cut off from her initial scarf?

李娜花 12 块买了一块长 2 米宽 1 米的布料。她先裁剪了一块长 1 米宽 $\frac{3}{4}$ 米的布料做了围巾。李娜的妹妹看到李娜买的布料后说她非常喜欢，而且她想做个方形的围巾。于是她从余下布料上裁剪了一块长宽各 $\frac{3}{4}$ 米的布料。将李娜裁剪的围巾两端宽缝上，制成一条围脖。将这条围脖摆成一个圆形，求问这个圆的半径、面积。(C39, 1st, PP5)

[Translation] Na Li spent 12 yuan and bought a piece of cloth that was 2 meters long and 1 meter wide. She first cut off a piece that was 1 meter long and $\frac{3}{4}$ meters wide to make a scarf. Na Li's sister saw the cloth and said that she really loved it. And she wanted to make a square scarf. She then cut off a piece that was $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide from the rest of the cloth. She sewed two width ends of Na Li's scarf together and made into a neck warmer, then arranged into a circular shape. Find the radius and the area of this circular shape.

These two problems were slightly different. However, both of them provide problem solvers the opportunity to think about the concepts and features of a circle and a rectangle.

For the second set of tasks, 31% (5 out of 16) of the US participants posed 23% (5 out of 22) solvable mathematical problems and 70% (19 out of 27) of the Chinese participants posed 42% (24 out of 57) solvable mathematical problems that were classified as creative problems. Among the five problems posed by US participants, one problem generated the smallest square in the given figure into a cube and asked for its volume. The second problem provided the price of the smallest bowl and asked problem solvers to set up a proportion between the price and area of the mouth of a bowl in order to find the price of the second smallest bowl. The third problem asked for a general rule about the relationship between the radius of a circle and the area of the next circle in the initial figure, of which the answer would be a formula. The remaining two problems required an even higher level of cognitive thinking for problem solving:

> A factory that specializes in producing dinnerware decides to produce a set of five bowls in different diameter sizes. The sizes are constrained using the following model diagram and the radius of the smallest bowl is one inch. The factory wants to know how many of the smallest bowls they could place side by side and fit into a box with an area of 1 sq foot? (U28, 2nd, PP5)
>
> How many of the smallest bowls (r = 1) could you fit into the largest bowl? Assume that the bowls will not overlap. (hint → use diameter) (U04, 2nd, PP5)

Strictly speaking, both of the problems are “unrealistic” because no information about the body shape of the bowls was provided and the different body shapes of the bowls would cause these problems to either solvable or unsolvable. However, this misconception might come from the poorly worded initial story. The given figure was actually the top view of the model of the set of five bowls, instead of a “model” of the “sizes” of those five bowls; meanwhile, the given number “1 inch/3 centimeters” was the radius of the mouth of the smallest bowl, instead of “the radius of the smallest bowl”. In consideration of this limitation in the designed task, the above two problems were classified as solvable problems and only the size of the mouth of each bowl was taken into consideration for problem solving.

It is possible that many problem solvers would divide the big area by the area of the smallest bowl to get an answer for solving the above two problems. This method will not work because of the inevitable space between bowls when arranging them side-by-side. Furthermore, the unit conversion between inch and foot required by the first problem and the circular shape of the mouth of the biggest bowl in the second problem increase the difficulty level of the two problems.

Among the 24 creative problems posed by Chinese participants in the second set of tasks, five problems generated three-dimensional objects from the circles and squares given in the figure and asked for the surface area of the cubic boxes, volume difference between two boxes, or whether a bowl or multiple bowls were able to fit into a specific box. Six problems added new information about either the cost of producing/decorating/cleaning a bowl, or the profit in selling a bowl, and then asked different questions. Three problems asked for a general rule involving the radii of all five circles, and two problems asked for a recurrence formula of the sequence consisting of the area of five circles. Another three problems creatively included the idea or conceptual understanding of counting skills (e.g., asking for the number of storing ways when bowls were paired with one left over and only putting a smaller bowl inside of a bigger one, C37, 2$^{nd}$, PP5), perimeter and area of a sector of the biggest circle, and the mass ratio between the biggest and the smallest bowl given the relationship between their density and the ratio of volumes.

The remaining five solvable problems posed by Chinese participants involved similar ideas with the problems posed by U28 and U04 discussed above. In spite of this, three of those problems had very unique features. Here is an example:

> 一个工厂制造出一套五个大小不一的圆形碗具，按从小到大排列，小碗半径的$\sqrt{2}$倍是比它稍大一点碗的半径，现在工厂要给这一套碗的碗口绕一圈彩带来装饰。现在已知最小碗的半径是 3 厘米。请问：如果将最大碗碗口的彩带拆下来，可以装饰在其它大小碗的碗口，有几种装饰方法，各分别可以装饰多少碗？(注：这里所指装饰在其它碗的碗口，是指恰好用完彩带，没有剩余) (C48, 2nd, PP5)
>
> [Translation] A factory that specializes in producing dinnerware produced five circular bowls in different sizes. If the five bowls were arranged from smallest to biggest, $\sqrt{2}$ times of the radius of a smaller bowl is the radius of the next sized bowl. Now this factory plans to decorate the mouth of each bowl using ribbon and the radius of the smallest bowl is 3 centimeters. Question: If the ribbon on the mouth of the biggest bowl was taken off for decorating other bowls, how many ways can this be done? And how many bowls could be decorated in each case? (Annotation: In this story, decorating the mouth of other bowls requires the use of all the ribbon that removed from the biggest bowl, no residue.)

This problem used all five bowls and asked for the possible combinations of the circumference of the biggest circle made from the circumferences of the smaller sized circles. It is an open-ended problem. Similar to this problem, the other two problems also involved all circles or squares, and were open-ended problems. All of these problems require high-level cognitive thinking for problem solving.

***Description 15: Many US and Chinese participants utilized the mathematical ideas and structure of the problem given in the problem-solving task in later problem-posing process, and the majority of their posed problems were solvable mathematical problems.*** In the first set of tasks, 44% (14 out of 32) of the US participants posed 31% (19 out of 61) solvable mathematical problems while 44% (24 out of 55) of the Chinese participants posed 29% (29 out of 100) solvable mathematical problems that had similar ideas and structures with the problem given in the problem-solving task (i.e., giving the price of a piece of cloth and asking for the cost of another piece of cloth that had the same unit price). No such problem posed by US participants was unsolvable or not a mathematical problem, and only two such problems posed by Chinese participants were unsolvable mathematical problems. One of those two Chinese participants posed an unsolvable problem because he/she mistakenly thought that the product of 1 and $\frac{3}{4}$ was smaller than $\frac{11}{16}$.

For the second set of tasks, one US participant posed a problem that was exactly the same as the problem given in the problem-solving task while all other US participants did not pose any problems regarding the concept of ratio between any two shapes. This may have influenced by their prior problem-solving performance. On the contrary, 38% (12 out of 32) of the Chinese participants posed 28% (15 out of 54) solvable mathematical problems that had similar ideas and structures to the problem given in the problem-solving task. More specifically, those 15 problems asked for the ratio between radii, diameters, circumferences and areas of circles, circumferences of two squares, and masses of two bowls. For this set of tasks, no unsolvable or not mathematical problems posed by US or Chinese participants had similar ideas or structure to the problem given in the problem-solving task.

This may indicate that, first, many US and Chinese participants' problem posing was influenced by the mathematical idea and structure of the problem given in the previous problem-solving task. Second, due to the fact that almost all the posed problems that had the similar idea and structure to the problem given in the problem-solving tasks were solvable mathematical problems, it is reasonable to claim that the problem-solving experience had positive impacts on US and Chinese participants' problem-posing performance later on.

***Description 16: Many participants started to pose a problem by completely copying the initial story, and then chose the option of adding new conditions if needed before posing problems. This phenomenon was particularly common in Chinese participants' posed problems.*** The biggest difference in the problems posed by US and Chinese participants occurred during the stage of problem posing after problem solving during the first task administration. There were 3% (1 out of 32) of the US participants who posed two solvable mathematical problems that completely utilized all the given information from the initial story. In contrast, 64% (35 out of 55) of the Chinese participants wrote down all the given information from the initial story, then added new conditions if needed, and finally posed 54% (54 out of 100) solvable mathematical problems, 53% (9 out of 17) unsolvable mathematical problems, and 70% (7 out of 10) problems that were not mathematical problems.

Among those 54 solvable mathematical problems, 18 problems involved redundant information. In other words, those 18 problems had information that was unnecessary for problem solving. Using the problem posed by C39 (see above example, C39, 1st, PP5) as an example, the total cost (i.e., 12 yuan), the measurements of the initial piece of cloth (i.e., 2 meters long and 1 meter wide), and the measurements of Na Li's sister's piece of cloth (i.e., $\frac{3}{4}$ meters long and $\frac{3}{4}$ meters wide), was all redundant or unnecessary information for problem solvers. It indicates that US participants usually selected only the condition that they would use from the initial story or figures to pose new problems, while their Chinese counterparts preferred to generate new problems by building on all given information. This difference was not shown during the second task administration. Almost all US and Chinese participants posed problems starting from the given figure and the condition that the radius of the smallest bowl was one inch/three centimeters. This can be explained by recognizing that the given information in the second set of tasks was limited, and therefore the participants had little choice but to incorporate all the information to generate new problems.

***Summary.*** Although both US and Chinese participants had posed at least four problems and solved one problem before getting to the last problem-posing process, they were still able to pose new solvable mathematical problems. And more importantly, they were able to continuously pose problems with expanded categories and creative ideas. In addition, many US and Chinese participants (e.g., 30% of the US and 30% of the Chinese participants during the first task administration) treated the problem given in the problem-solving process as a model and utilized the idea or structure of that problem to pose new problems. More importantly, the majority of those problems were solvable. This indicates that both US and Chinese participants' problem-posing performance was positively influenced by their previous problem-solving experience. Finally, US and Chinese participants showed quite different preferences in selecting given information from the initial situation to generate new problems. The Chinese participants preferred to completely copy all information from the given situation before asking a question, and therefore they posed a number of solvable mathematical problems that had redundant information for problem solving.

**Summary and Interpretation of the Findings**

In order to summarize and synthesize US and Chinese participants' problem-posing and problem-solving performance, I developed a table (see Appendix K) to list the major descriptions that I made for each problem-posing process, together with participants' performance during the problem-solving process. This table helped me to justify the similar and different patterns of problem posing, as well as the relatedness of problem posing and problem solving, shown by US and Chinese participants when they were engaged in alternating problem-posing and problem-solving activities. In this section, I answered my research questions according to the results shown in that table.

**Answering Research Question 1.** Research Question 1 is as follows: What are the similar patterns of problem posing shown by US and Chinese prospective elementary teachers during their problem-posing processes when problem solving is involved in an alternating manner? Are there any differences in the patterns shown by these two groups of participants? The answer to the first part of this research question can be found by focusing on similar patterns of problem posing shown by US and Chinese participants. To answer the second part of this research question, the different patterns of problem posing shown by US and Chinese participants are discussed.

***Similar patterns***. First, the problems posed by US and Chinese participants shared some similar features regarding interpreting given figures and initial stories, and similar reasons that led a posed problem to be an unsolvable or classified as not a mathematical problem. Second, the US and Chinese participants showed similar progression in their problem-posing performance throughout all five problem-posing processes.

*Similar features of posed problems.* Both US and Chinese participants had no formal problem-posing practice before participating in this study. Although they came from different countries with quite different academic and cultural backgrounds, they showed similar starting points and preference or aptness during the first problem-posing process (i.e., the Translating process), where they were asked to pose a story problem according to given figures. In the first set of tasks, they were initially given three rectangles and many of them posed problems that only asked for the area or perimeter of one rectangle. In the second set of tasks, they were initially given a pattern with continuously inscribed circles and circumscribed squares, and almost all of them asked only for the basic measurements of a circle or a square such as the area or circumference/perimeter. More particularly, no participant asked a question about the middle-sized circles or squares. They only focused on the biggest and smallest.

In addition, US and Chinese participants had similar creative ideas at the beginning of the problem-posing process. For example, both US and Chinese participants posed problems asking for the maximum number of times that a small rectangle could fit into a big rectangle during the first task administration. This type of problem involves a division with fractions and a division with a remainder, and requires higher levels of cognitive thinking as well as more computational steps than the problems that only ask for the area or perimeter of a rectangle. Both US and Chinese participants also posed problems that interpreted given figures in a dynamic process, during both the first and second task administrations. As discussed before, this type of problem is special as it clearly describe the mathematical structure and relationship of involved figures and therefore no figures are necessary to be given during the problem-solving process. In this case, problem solvers are allowed to develop figural representations by themselves, if needed.

Finally, US and Chinese participants had similar difficulties that led their posed problems to be unsolvable or considered not mathematical problems during the Translating, Editing, and problem posing after problem solving processes. These three problem-posing processes are different from the other two processes (i.e., the Comprehending and Selecting processes), because problem posers usually have more freedom to selecting and/or edit given information in the initial figures or situations, while the other two processes were confined to the initial situation and a particular calculation/answer. According to the problems posed by US and Chinese participants for these three processes, the reasons that led posed problems to be unsolvable problems included vague wording in either givens or questions, the improper use of mathematical terminology, a lack of sufficient information for problem solving during both task administration, and contradiction involvement particularly in the second set of tasks.

The reasons that led posed problems to be classified as not mathematical problems included that no question was asked at all, usually due to difficulties in posing expected problems, and no computational or mathematical reasoning steps were needed. This was usually due to misconceptions of the definition of a mathematical problem. In other words, they were not clear how a mathematical problem was defined. These findings show that both US and Chinese prospective elementary teachers find it challenging to clearly word a mathematical problem, accurately apply mathematical vocabularies, thoroughly consider the structure of a mathematical problem, and completely interpret the definition of a mathematical problem.

*Similar progression in problem-posing performance*. Along the five problem-posing processes, both US and Chinese participants' problem-posing performance improved. First, their focus on given figures and stories were extended. More specifically, during the first task administration, starting from asking questions about only one rectangle, many of my participants smoothly moved to ask questions involving more than one rectangle, which usually required more computational steps for problem solving. During the second task administration, starting from focusing on only the smallest or biggest circle or square, many of them also asked questions about middle-sized shapes during later problem-posing processes. There were also some participants who extended the given figure to even smaller or bigger shapes that followed the pattern but were not shown in the initial figure.

Second, more creative ideas continuously occurred in problems posed by both US and Chinese participants. The following concepts were used by many participants through all five problem-posing processes: fraction, ratio, percentage, proportion, multiple, surface area and volume of a three-dimensional object. Many problems with the use of these concepts were categorized as creative problems. At the same time, problems with more computational steps and/or different mathematical structure also occurred frequently in this process. For example, during the first task administration, both overestimation and underestimation cases of division-with-a-remainder problems occurred in the Editing process while only underestimation case occurred in the Translating process. In addition, there were more participants who posed problems using money information during the Editing process than during the Translating process. For the second set of tasks, both US and Chinese participants posed problems that began with an unknown circle in the Editing process, and there were a large number of posed problems that were closely tied to real-life situations (e.g., there will be inescapable space between circular bowls when they were arranged side by side) in the problem posing after problem solving process.

Finally, more open-middled and open-ended problems occurred during the Editing and problem posing after problem solving processes. All these findings show that prospective elementary teachers are capable of posing solvable mathematical problems. In addition, they are able to continuously pose problems with more and more creative ideas when given a certain amount of time and engaged in different types of problem posing as a systematic process. In a word, they are able to continuously innovate.

***Different patterns.*** Due to the different features of all five problem-posing processes, US and Chinese participants also showed quite different performances in each process. More specifically, I found the different patterns shown by US and Chinese participants' posed problems from the following perspectives: (1) figure visualization; (2) calculation interpretation; (3) habitual preference in posing a sequence of problems; (4) perception of a given answer based on previously posed or solved problems; and (5) problem-posing strategy for integrating given information.

*Figure visualization.* The Translating process is unique as it asks problem posers to generate a mathematical problem according to given figures. It is a semi-structured situation of problem posing that shows problem posers' focus, preference, and/or aptness when interpreting geometric figures. During the first task administration, fewer than half of US participants posed problems that involved all three given rectangles while about 70% of the Chinese participants posed problems involving all rectangles. During the second task administration, more than half of US participants posed problems focusing on the smallest circle in the given figure while the majority of Chinese participants asked questions about the biggest circle or square in that figure. These findings show that Chinese participants are more likely to integrate all information involved in the given figures into their posed problems, while their US counterparts usually chose to integrate only the information involved in the given figures they need for posing new problems.

*Calculation interpretation.* The Translating process gives problem posers a calculation and asks them to pose a mathematical problem representing that calculation. In other words, this process asks problem posers to generate a corresponding problem, while its solution is given. We traditionally learn mathematics through finding solutions to a given problem, instead of generating a problem for a given solution. Fewer than half of the problems posed by US participants during the first task administration and only about ten percent of the problems posed by US participants during the second task administration correctly represented the given calculations. On the contrary, more than 90% and about 90% of their Chinese counterparts' posed problems correctly represented the given calculations during the first and second task administration, respectively. This indicates that Chinese participants are capable of interpreting either the structure or properties of given calculations, and then making connections back to the initial story. It is possible that US participants were not used to this particular way of mathematical reasoning, or they were challenged when they had to interpret the meaning, structure, or properties of the given calculations.

*Habitual preference of posing a set of problems.* US and Chinese participants showed different preferences in posing problems when they got to the Editing process during both task administrations. More specifically, Chinese participants showed a preference of posing problems in a sequence. For example, a Chinese participant posed a problem asking for the radius of the second smallest circle in the figure for the Comprehending process during the second task administration. He/she then posed a problem asking for its area in the Editing process. There were also some Chinese participants who asked at least two questions in a sequence during the Editing process. For example, a Chinese participant first asked a question about the unit price of the cloth that Na Li bought and then asked for the cost of Na Li's scarf in the Editing process of the first task administration. Their US counterparts did not show this preference in problem posing. This difference is also shown in that, during the first task administration, there were a few Chinese participants who posed problems in the Editing process that were expected next in the Selecting process. Instead, some of their US counterparts posed problems in the Editing process that exactly presented the calculation given in the previous Comprehending process.

*Perception of a given answer based on previously posed or solved problems.* Similar to their performance in the Comprehending process, US and Chinese participants performed quite differently again in the Editing process. More specifically, less than 10% of the US participants posed problems that had the answer as a given during the first task administration, while more than ninety-five percent of their Chinese counterparts posed problems that had the answer as a given number. During the second task administration, less than 20% of US participants and almost 90% of Chinese participants posed problems that had the answer as a given number. One possible reason for why Chinese participants performed well in the Selecting process is that they were capable of perceiving and applying the relationship between the calculation given in the Comprehending process and the answer given in the Selecting process for each set of tasks. Meanwhile, the majority of their US counterparts had difficulties of doing this.

Even so, US participants came up with two ways of using the given number in problem posing. First, they broken down the given number into parts, then developed new scenarios representing each part and finally posed a problem. Some Chinese participants also used this strategy to pose a problem that had the answer as the given number during the first task administration. Second, some US participants used the given number as one condition in their posed problems, while no Chinese participants used this strategy. These two strategies indicate that, even if the participants were challenged at some points, they did not give up but insisted on posing new problems. It indicates that this way of problem-posing practice helps train prospective elementary teachers' resilience in mathematics learning.

*Problem-posing strategy for integrating given information.* Another problem-posing strategy that was particularly used by Chinese participants occurred in problem posing after problem solving process during the first task administration. In order to pose new problems after problem solving, the majority of the Chinese participants utilized all given information from the initial story and then asked new questions after adding other conditions if needed. Only one US participant did this during the problem posing after problem solving process. This becomes an issue because many of the Chinese participants' posed problems had redundant information that was unnecessary in problem solving. It again shows that Chinese participants preferred to integrate all given information when posing new problems while their US counterparts were more likely to select only the information that they needed. In addition, the Chinese participants are lacking awareness of checking the structure of their posed problems. More specifically, checking for redundant information that is included in their posed problems. In other words, they lack an awareness about what makes a mathematical problem concise.

**Answering Research Question 2.** Research Question 2 is as follows: What are the connections between US and Chinese prospective elementary teachers' problem-posing and problem-solving performance? Are there any differences in the connections between these two groups of participants? To answer this research question, I first discussed the general connections between problem posing and problem solving among US and Chinese participants. I then discussed the differences shown by US and Chinese participants.

***Connections between problem posing and problem solving.*** The general connections between problem posing and problem solving, during the problem posing before problem solving process, the problem-solving process, and the problem posing after problem solving process, are discussed respectively below.

*Problem posing before problem solving*. Christou et al. (2005) speculated that specific problem-solving thinking and ideas presented in iconic or symbolic form should include in Translating, Comprehending, Editing, and Selecting processes. Due to the features of each of those processes as well as participants' performance, it can be argued that different types of problem-solving thinking are involved in different problem-posing processes and, furthermore, different amounts of problem-solving effort are needed in each problem-posing process. More specifically, in order to pose an appropriate problem to a given answer according to the initial story or figures during the Selecting process, problem posers need to solve a series of problems through trial and error. The Comprehending process actually provides problem posers with the solution to an expected mathematical problem and asks them to find out that specific problem. It requires problem poser to make connections between problem solving and problem posing by themselves, and to view a mathematical problem from different perspectives.

Since Translating and Editing processes do not have specific requirements as Comprehending and Selecting processes do, many problem posers only pay a little attention to the solvability of their posed problems, especially the ones who had little experience in posing problems. In a word, problem posing and problem solving are not always inseparable. Different amounts of problem-solving efforts or practices is required depending on the form of the specific problem-posing processes. From this perspective, problem posing could be either a comparatively independent learning activity or a learning activity that is closely related to problem solving, according to the definition of problem posing generated by Kilpatrick (1987) and Silver (1994).

*During the problem-solving process*. It is clear that, overall, Chinese participants performed very well in the Comprehending and Selecting processes as well as the later problem-solving process for both sets of tasks. At the same time, their US counterparts had difficulties in all of those three processes. Furthermore, during the Editing process of the first set of tasks, more than twenty percent of US and Chinese participants posed problems that had similar ideas and structures with the problem in the problem-solving task. More than 10% of Chinese participants asked at least two questions in a sequence, starting with a question about the unit price of the cloth and then asking a question about the cost of specific pieces. Later on, during the problem-solving process, they all solved the problem following the steps that were exactly consistent with the questions they asked during the Editing process. During the second task administration, no US participant posed a problem about ratios in the Editing process, while more than 10% of Chinese participants posed problems with the concept of ratio that had similar ideas and structures to the problem in the problem-solving task. These findings indicate that, first, better problem posers in the Comprehending and Selecting processes are usually better problem solvers, under the condition that all of these processes are according to the same story and/or figures. Second, if a problem poser is able to pose a problem involving specific concepts or structures, or pose a problem or a sequence of problems involving problem-solving procedures, he/she is likely highly capable of solving similar problems with those concepts, structure, or problem-solving procedures.

*The problem posing after problem solving process*. After problem solving during the first task administration, about 30% of both US and Chinese participants posed problems that had similar ideas and structures to the problem given in the problem-solving task. After problem solving during the second task administration, there were still about 30% of Chinese participants who posed problems that had similar ideas and structures compared to the problem given in the problem-solving task, while only about 5% of US participants did this. It is possible that many US participants were challenged by solving the problem given in the problem-solving process and, therefore, they may not have been confident in posing similarly structured problems after problem solving. Regardless, the majority of those posed problems were solvable mathematical problems, and many of them involved even more reasoning and computational steps.

To summarize, the similar performance of the US and Chinese participants in problem posing after problem solving during the first task administration indicates that the problem-solving experience had a positive impact on the participants' problem-posing performance later. The difference in performances between those two groups of participants in problem posing after problem solving during the second task administration further indicates that the problem-solving performance may impact decision making in later problem-posing process.

***Different ways of connections shown by US and Chinese participants***. The differences in the ways of connections between problem posing and problem solving shown by US and Chinese participants are stated in the following three perspectives, all of which have been discussed previously: (1) Chinese participants performed well in the Comprehending and Selecting processes as well as the problem-solving process in both the first and second sets of tasks, while their US counterparts had more difficulties in all of those three processes. (2) Many Chinese participants showed an aptitude for posing problems in a sequence. More specifically, many of the problems they posed in earlier steps could serve in solving later posed problems. The US participants did not show such an aptitude for posing a sequence of problems. And (3) the order of the sequential problems posed by some Chinese participants were consistent with the procedures they used later in the problem-solving process. Their US counterparts did not show this way of thinking when posing and solving problems. Instead, they preferred to pose problems from multiple perspectives that were quite different.

**Summary.** In conclusion, this chapter first quantitatively described and interpreted the US and Chinese participants' problem-solving and problem-posing performance. The results indicate that, overall, a higher proportion of the Chinese participants than their US counterparts correctly solved the given problem in both sets of tasks. For problem posing, the Chinese participants posed a higher percentage of solvable mathematical problems in the Comprehending and Selecting processes than the other three processes. Contrarily, the US participants posed a higher percentage of solvable mathematical problems in the other three processes than in the Comprehending and Selecting processes. One possible reason that influenced the US participants' performance in the Comprehending and Selecting processes as well as problem-solving process during the first task administration is the use of mixed fractions that may have contributed to the complexity of the tasks.

I then qualitatively synthesized the patterns of problem posing shown by my participants in each of the Translating, Comprehending, Editing, Selecting, as well as problem posing after problem solving processes, as well as the relatedness between problem posing and problem solving during these processes. I found that the US and Chinese participants shared some patterns of problem posing during the entire process. The similar patterns include (1) similar features involved in posed problems, and (2) similar progression in problem-posing performance. They also showed some different patterns of problem posing during this process. Those different patterns were shown from the following perspectives: (1) figure visualization, (2) calculation interpretation, (3) habitual preference of posing a sequence of problems, (4) perception of a given answer based on previously posed or solved problems, and (5) problem-posing strategy for integrating given information.

Finally, my study evidenced some specific ways of relatedness between problem posing and problem solving including: (1) depending on the form of specific problem-posing task, it requires different amounts of problem-solving efforts or practice; (2) a better problem poser in the Comprehending and Selecting processes is usually a better problem solver, under the condition that all the Comprehending, Selecting and problem-solving processes are based on the same situation (e.g., story or figures); and (3) problem-solving experience has positive impacts on subsequent problem-posing process. Regarding of the different ways of connections between problem posing and problem solving shown by the US and Chinese participants, I found that (1) the US and Chinese participants performed quite differently in the Comprehending and Selecting problem-posing processes as well as subsequent problem-solving process; (2) the Chinese participants showed an aptitude for posing problems in a sequence, while their US counterparts did not show such an aptitude; and (3) the sequential problems posed by some Chinese participants were consistent with their procedures for solving the given problem, while their US counterparts did not show this way of thinking for posing and solving problems.

# Chapter 5: Discussion

In this study, I aimed to investigate the patterns of problem-posing performance and the features of the relationship between problem posing and problem solving shown by US and Chinese prospective elementary teachers. In order to do so, I designed two sets of tasks in both English and Chinese to engage participants in alternating problem-posing and problem-solving activities. My participants were 87 prospective elementary teachers from three US and three Chinese universities. The participants completed the tasks using paper and pens between October 2015 and January 2016. The collected data included participants' posed and solved problems based on two real-life situations and corresponding figures. I later answered my research questions by analyzing the patterns of my participants' performance in different types of problem-posing tasks, and the interactions between their problem-posing and problem-solving activities. This chapter includes the following parts: (1) conclusion of this study; (2) contributions and limitations; and (3) implications for future research and teacher preparation practice.

## Conclusion

Generally speaking, both US and Chinese participants were able to pose solvable mathematical problems. This is consistent with the findings of many previous studies (e.g., Chen, Van Dooren, Chen, & Verschaffel, 2011; Crespo, 2003; Silver, Mamona-Downs, Leung, & Kenney, 1996). My study also shows that the US and Chinese prospective elementary teachers were able to pose solvable problems that contained creative ideas and quite different mathematical structures from previously posed problems. Furthermore, many of those creative ideas were real-life related. For example, many participants brought the idea of sales promotion during the first task administration and some participants integrated the cost of decorating or producing a certain sized bowl during the second task administration. This evidences that, first, prospective elementary teachers possess a rich store of knowledge from applied, real-world situations, namely, informal knowledge defined by Mack (1990). Second, this indicates that engaging problem posers in real-world situations provides them with opportunities to make connections between mathematical knowledge inside and outside the classrooms, flexibly explore, compare, and select meaningful information, and facilitate their creativity. These benefits have been documented in previous studies (e.g., Bonotto, 2010a; Bonotto & Dal Santo, 2014).

My participants did not have formal problem-posing experience before participating in this study, and they were initially asked to pose a story problem according to given figures in the study. I found that both US and Chinese participants were able to start posing problems with basic mathematical concepts and ideas, and they had a similar preference of posing problems according to given figures. For example, many of them posed problems that only asked questions about one rectangle during the first task administration, and most of them integrated only the basic concepts of the smallest or biggest circle or square during the second task administration, no question was asked about the middle-sized circles or squares. However, as they had more and more problem-posing opportunities, they posed problems containing more and different mathematical concepts, creative ideas, and mathematical structures. Also, some problems were more complicated than others with more computational and reasoning steps. During both task administrations, I provided no scaffoldings and each task administration lasted for about one hour and twenty minutes. Because of these reasons, I would argue that prospective elementary teachers have the initiatives and capability to pose higher-level cognitively demanding problems when given enough time or opportunities. Meanwhile, their problem-posing performance could be developed and they are able to continuously create new ideas, even in a short amount of time, when they are engaged in a systematic process of problem posing.

Similar to the findings of existing studies (e.g., Cai & Hwang, 2002; Crespo, 2003; Silver, Mamona-Downs, & Leung, 1996), both the US and Chinese participants in this study posed many unsolvable problems and those that were not mathematical problems in nature. The main reasons that led posed problems to be unsolvable problems included unclear wording and the lack of sufficient information for problem solving. The main reasons that led posed problems to be classified as not mathematical problems included problems that had no computational or reasoning steps needed, and when no questions were asked. Both US and Chinese participants displayed these mistakes and misunderstandings. Additionally, these errors are possibly because (1) there was a lack of problem-posing experience, (2) they have difficulties in selecting proper mathematical concepts or terminologies, (3) they are not clear on how a mathematical problem is defined, (4) they are challenged by the requirements of the task itself, and (5) more likely, they have limited conceptual understanding of specific mathematical topics. Therefore, when prospective elementary teachers are engaged in problem posing, they should be given space and scaffoldings to pose different problems according to aforementioned perspective.

There were also some different patterns in the problems posed by US and Chinese participants. Among all five problem-posing processes, Chinese participants posed a higher percentage of solvable mathematical problems during the Comprehending and Selecting processes (i.e., goal-oriented problem-posing tasks) than other three problem-posing processes (i.e., Translating, Editing, and problem posing after problem solving processes, which are open-ended problem-posing tasks). The US participants posed a higher percentage of solvable mathematical problems during the Translating, Editing, and problem posing after problem solving processes than the Comprehending and Selecting processes. This indicates that Chinese participants were more capable of interpreting the given calculations, perceiving the given answers, and making connections between the given calculations or answers back to the initial story and figures than their US counterparts. Since the Comprehending and Selecting processes provide either a solution or an answer and ask for a corresponding problem, it is also possible that the Chinese participants were more capable of thinking in reverse than their US counterparts.

Neither US nor Chinese participants' performance in the four cognitive problem-posing processes was the same with other groups of participants' performance documented in the literature. For example, the six-grade students in Cyprus were challenged more in the Editing and Selecting processes than in the Comprehending and Translating processes (Christou et al., 2005), while the prospective primary teachers in Turkey had low success in all problem-posing processes and had the highest difficulty in the Comprehending process (Işık, Kar, Yalçın, & Zehir, 2011). Such differences among cross-national problem posers indicate the possible impact of cultural and academic backgrounds on their problem-posing performance.

Additionally, my study found that the Chinese participants preferred to pose problems in a sequence, while their US counterparts did not show such a preference. More specifically, Chinese participants either sequentially posed at least two problems for one problem-posing process, or posed problems for different problem-posing processes that the answer to one problem could server to solve the other. This habitual preference of posing problems by Chinese participants was displayed during both goal-oriented and open-ended problem-posing processes. On the one hand, it is a problem-posing strategy that Chinese participants found useful. This problem-posing strategy is called *chaining* in many existing studies (e.g., Koichu, & Kontorovich, 2013; Silver, Mamona-Downs, Leung, & Kenney, 1996). Chaining illustrates a problem-posing strategy where an existing problem is expanded in a way that a solution to a new problem would require the individual to first solve a previously posed problem. On the other hand, for both sets of tasks, the calculation given in the Comprehending process and the answer given in the Selecting process were closely related. Therefore, it is possible that the Chinese participants were better able to perceive and apply those relationships when posing expected problems.

Regarding problem posing, I also found that the Chinese participants preferred to utilize all given information from the initial story and figures, and then ask new questions after adding some other conditions if needed. This preference was particularly evident during the first task administration and more commonly occurred in the problem posing after problem solving process. But, many of those problems that involved all the givens had redundant information for problem solving. Alternatively, only a few US participants did this, and the majority of them selected only the condition they would use from the initial story or figures to pose new problems. This indicates that the Chinese participants lacked an awareness of checking the structure of their posed problems, while their US participants were better able to make their posed problems concise.

In investigating the interaction between problem posing and problem solving, my study found that these two activities are closely related, which is consistent with the findings from existing studies (e.g., Bonotto & Dal Santo, 2014; Silver & Cai, 1996). This conclusion was particularly shown in the US and Chinese participants' performance during the Comprehending and Selecting problem-posing processes and problem-solving process. It was true for both sets of tasks. More specifically, the majority of Chinese participants posed solvable mathematical problems in the Comprehending and Selecting processes as expected, and successfully solved the problem given in the later problem-solving task. Contrarily, more than half of the US participants did not pose solvable problems in the Comprehending and Selecting processes as expected. Meanwhile, more than half did not completely solve the given problem. For the other three problem-posing processes (i.e., Translating and Selecting processes as well as problem posing after problem solving process), although quantitatively Chinese participants posed a higher percentage of the solvable mathematical problems than their US counterparts, the difference was not large. These findings indicate that, within the same context, different types of problem-posing tasks will have different levels of interaction with problem-solving tasks.

I finally found that problem solving had positive impacts on later problem-posing performance. About 30% percent of both US and Chinese participants posed problems after problem solving that had similar ideas and structures to the problem given in the problem-solving task during the first task administration. About 30% of Chinese participants also did this during the second task administration. Only a few US participants did this during the second task administration, but this may be because they did not perform very well on the problem-solving task. Regardless, the majority of those posed problems were solvable mathematical problems, and many of them incorporated even more reasoning and computational steps. Therefore, I concluded that, first, a problem given before problem posing played a role as a model, especially when problem posers could successfully solve the problem, they may be more confident to utilize its ideas and/or structure to pose new problems. Second, a problem given before problem posing may provide problem posers a new perspective in applying it to the initial real-life situation. If this new perspective was interesting enough to maintain problem posers' curiosity and/or interests, it may be beneficial to enhance both their problem-solving and problem-posing performance. From this viewpoint, it is crucial to consider which type or level of problem is more appropriate to give problem posers during the problem-solving processes before problem-posing tasks.

**Contributions and Limitations**

This study contributes to the problem-posing research area in the following ways. First, the majority of existing studies examined the magnitude of the relationship between problem solving and problem posing. The researchers usually examined participants' problem-solving and problem-posing performance around the same topic and then predicted the relatedness between those two learning activities (e.g., Cai, 1998; Silver & Cai, 1996). They found that problem solving and problem posing were closely correlated. Simply speaking, a good problem solver was usually a good problem poser, and vise versa. However, my study provides a different perspective and expands our understanding of the features of the relationship between problem solving and problem posing. More specifically, this study adapted the Active Learning Framework developed by Ellerton (2013) and engaged the participants in alternative problem-posing and problem-solving activities. This way of engaging participants was not exactly the same as the procedures designed in the Active Learning Framework. As Ellerton claimed, this framework has not been broadly used and further research is needed before it can be extended. It indicates that we are still not sure which way of utilizing this framework is more efficient on mathematics learning than others. From this perspective, this study is also a contribution of the use of Ellerton's framework.

Second, this study shows that problem posing has specific impacts on later problem-solving activities and the problem-solving experience impacts the subsequent problem-posing performance. This indicates that the order of engaging students in problem solving and problem posing can possibly lead to different learning outcomes. Therefore, my study implies other choices of exploring the relationship between these two activities, for example, beginning with problem-solving tasks instead of starting with problem-posing tasks; and alternatively engaging students in problem solving and problem posing in a longer period of time, like one academic semester, rather than just a few hours.

This study also contributes to the body of literature because US and Chinese prospective elementary teachers' performance on particular problem-posing processes was explored, including the Translating, Comprehending, Editing, and Selecting processes as well as problem posing after problem solving processes. There are a few studies that integrate similar problem-posing processes, but either with elementary school students, or with no problem posing after problem solving process, and those participants were from different countries other than the US and China (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005; Işık, Kar, Yalçın, & Zehir, 2011). And more importantly, all the participants from different countries showed quite different performances on the specific problem-posing processes. This indicates that the integration of problem posing in classrooms for different grade levels and demographic groups of students could be quite complicated and need multiple considerations.

Third, existing studies focused on investigating problem-posing strategies and performances, the magnitude of the relationship between problem posing and problem solving, and the role of problem posing in creativity development, mathematics conceptual understanding as well as learners' disposition on mathematics learning. Few of those studies discussed the patterns involved in participants' posed unsolvable and not mathematical problems, let alone the reasons behind those problems. However, this study examined the reasons that led posed problems to be unsolvable and not mathematical problems. The reasons provide evidence for purposefully designing problem-posing tasks in order to overcome certain difficulties in problem posing and to develop students' problem-posing performance.

There are three limitations in this study. First, this study lacks advanced statistics. This study was designed as a qualitative study and I only provided quantitative descriptive analysis for the participants' problem-posing and problem-solving performance. I did not further justify the statistical significance among participants' performance on different problem-posing processes, between problem posing and problem solving, or between US and Chinese participants. Therefore, although I drew conclusions about US and Chinese participants' problem-posing and problem-solving performance according to the most obvious descriptive data, those conclusions may not fully describe their real capability.

The second limitation of this study concerns the different languages used by US and Chinese participants. The US participants worked on designed tasks in an English version, while their Chinese counterparts worked on designed tasks in a Chinese version. I first translated the designed tasks in English into Chinese, and analyzed the collected data in English and Chinese at the same time. Additionally, I translated some Chinese participants' posed problems back to English if needed. I did not use an external reviewer to check all of these translations. It may be a limitation of my findings. It was also noteworthy that the US participants were more familiar with the inches and feet as length units while Chinese participants were more familiar with centimeters and meters as length units. I therefore used different length units and measurements for both sets of tasks in order to make the numbers used comparable. However, this consideration produced a consequence that, in the first set of tasks, the fractions given in the tasks in English were mixed fractions while the fractions given in the tasks in Chinese were proper fractions (i.e., fractions that were less than one). This difference may have contributed to the complexity of the tasks for US participants, as well as their performance in problem-posing and problem-solving processes.

The last limitation concerns participant selection. I approached three universities from each country in order to achieve a comparatively representative group of participants for the entire population. The three universities from each country were ranked on different levels according to famous institutions with high social influence. However, all three US universities are from the same state while the three Chinese universities are located in different areas of China. This difference may impact the generalization of my findings to a larger population to an extent.

## Implications for Future Research and Teacher Preparation Practice

This section discusses the implications of this study for future research and teacher preparation practices. The implications for future research touches on further investigation into the relatedness between problem posing and problem solving, problem-posing task design, and outcomes of different groups of learners' problem-posing activities. The implications for teacher preparation practice refer to prospective elementary teachers' needs before being engaged in problem posing, matters that need attention when integrating problem posing in problem-solving activities or classrooms, and the use of unsolvable and not mathematical problems.

**Future research.** In this study, both US and Chinese participants had no formal problem-posing experience before they were asked to pose four problems with four quite different requirements before problem solving. It is possible that the first stage of problem posing helped them to analyze the initial story and figures from different perspectives, raised their curiosity and interests, and prepared them before completing problem-solving activities. It is also possible that the participants were frustrated because of this unfamiliar learning activity. Since they were not given a follow-up interview, we do not know how they felt during the task administrations. Therefore, the next step of this study could be conducting interviews to look closer into prospective elementary teachers' thinking during this process in order to know more about their preferences and difficulties when beginning problem posing.

This study provides more evidence in support of the positive correlation between problem posing and problem solving. More specifically, during both the first and second task administration, Chinese prospective elementary teachers posed a higher percentage of solvable mathematical problems in the Comprehending and Selecting processes than other problem-posing processes, while their US counterparts were most challenged by those two processes. In the meantime, a much higher proportion of Chinese participants than their US counterparts found a correct solution to the given problem during each task administration. The mathematical idea and structure involved in the problem-solving task in turn, impacted participants' problem-posing thinking after problem solving. Many US and Chinese participants utilized the ideas and structure in their own posed problems and, more importantly, the majority of the problems were solvable mathematical problems.

Even so, it would be different if the participants engaged in problem solving first, then problem posing, and finally in problem solving again. Future study can explore the specific impacts of the order of engaging participants in problem posing and problem solving, and of problem examples given before problem-posing processes on developing problem-posing capabilities. By answering these questions, we could argue which way is more beneficial for engaging prospective elementary teachers in problem posing, getting used to problem posing, and improving in mathematics learning through problem posing. On the other hand, answering these questions will help in selecting a better way of integrating problem posing into prospective elementary teachers' problem-solving classes. In other words, it will provide further evidence about the interaction between problem posing and problem solving.

Different groups of problem posers showed distinct performances on closely related problem-posing and problem-solving tasks, and this may be due to cultural and academic backgrounds, or other reasons. Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman (2005) found that, for six-grade students in Cyprus, “the editing and selecting process characterized the most able students” (p. 156) while the students with lower problem-posing performances only approached the Comprehending and Translating processes. Christou et al. did not investigate Grade six students’ problem-solving performance in their study. Işık, Kar, Yalçın, and Zehir (2011) integrated the four problem-posing processes with 80 prospective primary teachers in Turkey. They found that, overall, participants had low success in all problem-posing processes and had the highest difficulty in the Comprehending process. In contrast, this study worked with US and Chinese prospective elementary teachers and found that Chinese prospective elementary teachers out-performed in the Translating and Selecting processes, while their US counterparts were most challenged by these two processes. This shows that problem posers with different cultural or academic backgrounds have different reactions to different types of problem-posing tasks. The possible reasons that lead to those differences need further investigation. In other words, future studies could explore why a specific group of participants posed more solvable mathematical problems for certain types of problem-posing tasks than others. Further study should also focus on exploring different considerations when designing problem-posing and problem-solving tasks for participants with particular cultural and academic backgrounds. In doing this, we are able to design rich tasks and order those tasks in a meaningful way for each particular group of problem posers accordingly.

Some studies have pointed out that many students and teachers pose ill-formulated and unsolvable mathematical problems (e.g., Crespo, 2003; Silver, Mamona-Downs, & Leung, 1996). However, few have investigated the possible reasons that led posed problems to be ill-formulated or unsolvable, or the reasons that led posed problems to be classified as not mathematical problems. This study showed that the main reasons that led posed problems to be unsolvable problems included unclear wording and a lack of sufficient information for problem solving, while the main reasons that led posed problems to be classified as not mathematical problems included a lack of computational or reasoning steps needed and no questions asked. Further study can focus on developing models or frameworks that help problem posers overcome those difficulties of posing ill-formulated, unsolvable, or not mathematical problems. Crespo and Sinclair (2008) found that providing prospective teachers opportunities to explore mathematical situation and aesthetic criteria to judge the quality of mathematical problems before problem posing improved their problem-posing performance and their understanding of the quality of posed problems. More specifically, problem posers with previous exploration opportunities posed more problems involving mathematical reasoning (i.e., answers were not immediately obvious, needed cognitive thinking to figure things out and explain why questions) than problems that only involved some facts or were too open-ended (i.e., with too many answers to promote mathematical reasoning, and therefore either lack of interests or unsolvable). Many of the participants posed problems that only contained facts and were too open-ended. Crespo and Sinclair's interventions could serve as a model for improving problem-posing performance in a specific way. However, more particular and/or comprehensive models are needed to help problem posers overcome the difficulties present in problem posing.

**Teacher preparation practice.** Researchers have agreed that problem posing, similar to problem solving, is a worthwhile intellectual activity that can serve to promote mathematical understanding. They appeal for integrating problem posing into problem-solving classrooms and teacher preparation programs (Crespo, & Sinclair, 2008; Ellerton, 2013). The first question is:, What do prospective teachers need to be prepared before engaging in problem posing? In this study, there were a number of posed problems that I classified as not mathematical problems because they did not involve either questions, or mathematical computational or reasoning steps. This result is in consistent with existing findings (e.g., Cai & Hwang, 2002; Crespo, 2003; Silver, Mamona-Downs, & Leung, 1996). Cai (2015) therefore asked, “Why do students pose nonmathematical, trivial, or otherwise suboptimal problems or statements?” (p. 9). For prospective elementary teachers in this study, it was possible that they lacked understanding of the definition of a mathematical problem, as well as its features, types, and possible structures. For example, during the first task administration, there were participants who asked a question about how to sew two pieces of cloth together. This was not a mathematical problem. In addition, in the four cognitive problem-posing processes, although it was clearly stated that they were required to pose story problems, some of the participants posed pure mathematical problems, involving no real-life situations.

In order to prepare prospective teachers to be better problem posers, we can start by developing their understanding of the definition of a mathematical problem. The countless well-structured mathematical problems in textbooks or online are all learning resources. In the meantime, it is important to expose prospective teachers to ill-formulated or unsolvable mathematical problems (Crespo, & Sinclair, 2008; Silver, 1994). By continuously comparing high-quality (i.e., well-structured and solvable), mathematical problems with ill-structured or unsolvable mathematical problems, they will have a better understanding and more experience in developing their own aesthetic criteria for judging the quality of mathematical problems. This helps them improve their problem-posing performance correspondingly, as Crespo and Sinclair found (2008).

This study further provided evidence that problem posers coming from a particular cultural and academic background performed quite distinctively in different problem-posing processes. In addition, prospective elementary teachers' performance in different problem-posing process related differently to their problem-solving performance. These results contribute to answer another question developed by Cai (2015) about further exploration of the feature of the relationship between problem posing and problem solving, especially in cross-cultural context. In spite of this, the way or the order that prospective teachers should be engaged in problem-posing and problem-solving tasks needs particular consideration. By carefully thinking about which types of problem-posing tasks that a particular group of posers are good at, we can consider whether we should engage them in easier or more challenging ones first. We should also consider whether engaging them in problem posing before problem solving or problem solving before problem posing could help deepen their conceptual understanding more efficiently. In one word, problem-posing and problem-solving task design and selection depend on the particular group of learners' needs and characteristics.

When integrating problem posing into problem-solving classrooms and teacher preparation programs, it is highly possible that prospective teachers will pose many unsolvable and not mathematical problems. They may become confused and frustrated at times. In this case, extra patience may be helpful. In other words, they can be given enough time to go through productive struggles, especially at the beginning stage when introducing problem posing. This study shows that, when prospective elementary teachers were given enough time and engaged in well-organized or designed tasks, they were able to pose different problems, many of which were solvable or creative problems. Different types of problem-posing played a role in scaffolding problem posers to explore the given situation or figures from multiple perspectives.

During the data analysis process, I found many entry points for teaching according to my participants' posed problems, including both solvable and unsolvable ones. For example, both underestimation and overestimation cases for solving division-with-a-remainder problems were posed by my participants, and those problems were with fractions. In teacher preparation classrooms, we could expose them to both of those cases and ask them to solve the problems at the same time. These problems help maintain their interests and insistence of problem solving, because those are their own posed problems. Studies show that students showed more interests in solving their own posed problems (Beal, & Cohen, 2012; Silver, 1994). Yet, in comparing the two cases, they could better understand the difference of the two cases and pay more attention to reality when solving and posing these kinds of mathematical problems. Just as Levin and Calcagno (2008) claim, without the opportunity to solve real-world problems which reveal the usefulness of the mathematics being taught, students are unable to extend and apply their new knowledge outside the context of the mathematics classrooms.

Another example is as follows. As discussed previously, a number of participants misunderstood operations with fractions such as $2\times\frac{60.5}{6}=\frac{121}{12}$, $\frac{110}{3}+\frac{11}{9}=\frac{121}{12}$, and $\frac{121}{12}=10\frac{1}{12}=5\frac{1}{6}\times2\frac{1}{2}$ during the Selecting process of the first task administration. When this happens during classroom teaching, it could be an opportunity to ask students to solve those problems. By comparing the correct answers to the given number in the Selecting process, problem posers who had those misunderstandings may start questioning themselves and a whole class discussion would help them further clarify their misunderstandings. This is an opportunity to help prospective teachers make sense of the interaction between problem solving and problem posing. In addition, it is an opportunity to help them see the usefulness of mistakes or misunderstandings in developing their mathematics learning. Similarly, in teacher preparation classrooms, we can purposefully ask students to solve the ill-structured problems that they have posed. Prospective teachers are expected to investigate the mistakes involved in unsolvable or not mathematical problems by themselves, such as implicit contradiction, lacking sufficient condition, and missed goal of a problem. They could be asked to correct and edit those ill-structured problems and generate solvable and concise problems. This would help them learn from mistakes and, at the same time, start to think like an expert in mathematics.

Cai (2015) asked, “What are the key features of effective problem posing and problem-posing instruction in classrooms?” According to above discussion, although this study is not about classroom teaching through problem posing, it may provide implications to answer this question. More specifically, this study indicates that effective problem posing is able to create many entry points for making teaching decisions. Depending on a specific group of students’ performance and needs, a teacher could help expand their understanding by utilizing either solvable or unsolvable problems posed in the class.

This study clearly showed that prospective elementary teachers were able to pose solvable mathematical problems, many of which were creative. However, it is possible that they could not solve all of their posed problems. Evidence suggest that there were a number of participants who posed problems with similar ideas and structures to the problem in problem-solving process and did not solve that problem during problem-solving process. In the case of either generating new problems from ill-structured problems or solving their own posed problems, peer supports and peer teaching may be helpful for prospective elementary teachers to overcome fear and frustration brought about by the unfamiliarity of the way of mathematical learning, namely, problem posing.

Contreras (2007) concluded that, first, without adequate experiences, students rarely used prototypical strategies to generate problems. Second, all the difficulties the students had and the errors they made during problem posing indicated that they needed a broad variety of experiences in problem posing. I will further claim that prospective elementary teachers need more exposure to multiple types of problem-posing tasks, practices involving interactions between problem posing and problem solving, chances to deal with ill-structured mathematical problems, as well as opportunities to recognize and analyze different types of mathematical problems before posing their own problems.

In conclusion, although it has been repeatedly evidenced that problem posing and problem solving are closely related, more descriptive, qualitative research on the nature of their relationship is needed. In addition, even though Ellerton (2013) and other researchers (e.g., Osana & Pelczer, 2015) appeal for problem-posing integration into teacher preparation programs, we rarely see problem posing as part of teacher preparation programs. Although Ellerton (2015) further evidenced the benefits of using the Active Learning Framework (Ellerton, 2013) in a mathematical class for preservice and practicing teachers, and proposed specific characteristics of a pedagogy for problem posing, it is still unknown whether a teacher preparation program that engages prospective teachers in problem posing produces stronger teachers. All of these unanswered questions indicate that the application of significant research results and findings in teacher preparation programs needs to continue. A critical point may be that, as Cai (2015) claimed, the research area of problem posing is still atheoretical at this point. It therefore appeals for more work in order to build a theory for better understanding all empirical results in the literature as well as the overall picture of problem posing.

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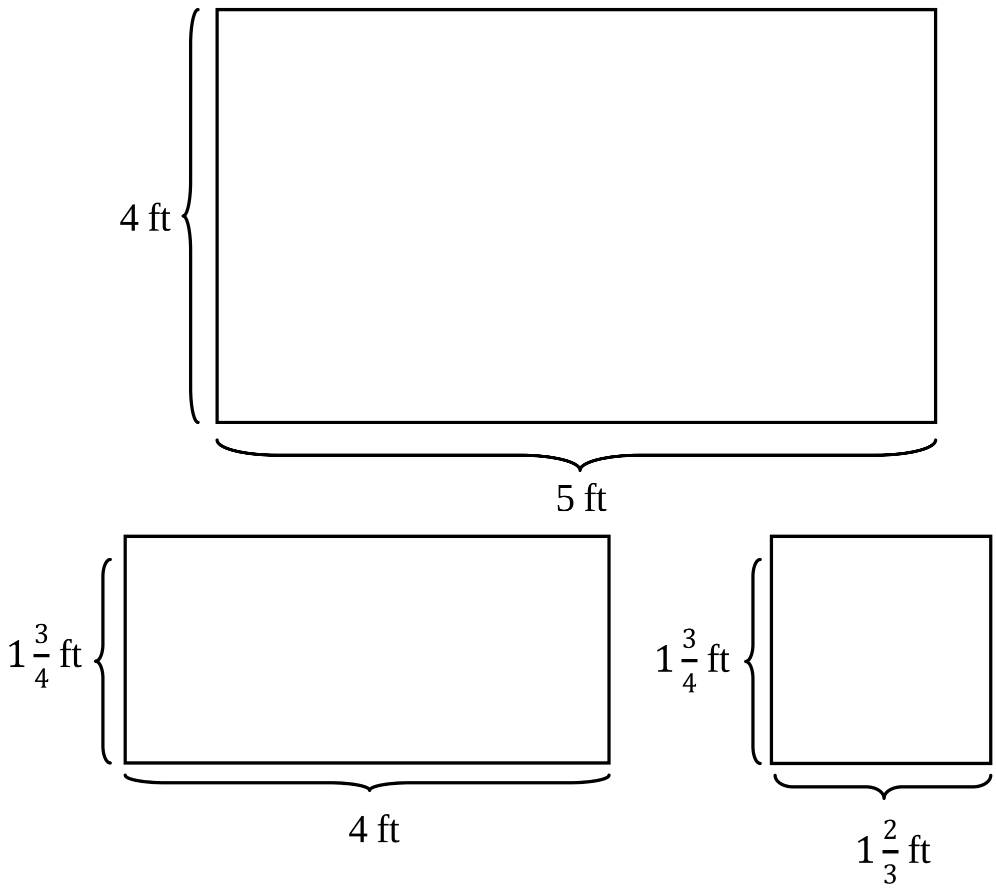
## Appendix A: Two Set of Tasks for Pilot Study

### First Set of Tasks

**<u>Here is a real life story:</u>**

Diana bought a piece of cloth 4 feet wide and 5 feet long. It cost $16. She cut off a piece that was $1\frac{3}{4}$ feet wide and 4 feet long to make a scarf. Her sister saw Diana's cloth and really liked the material. She asked for a piece that was $1\frac{3}{4}$ feet wide and $1\frac{2}{3}$ feet long to also make a scarf.



**Task I: Problem posing [20 min]**

***Comprehending:*** Write an appropriate mathematical story problem representing the following calculation: $1\frac{3}{4} \times (4 + 1\frac{2}{3})$, according to the given situation and figures.

***Translating:*** Write a mathematical story problem according to the given figures.

***Editing:*** Write a mathematical story problem according to the given story, which should be different from the problems you posed before.

***Selecting:*** Write an appropriate mathematical story problem according to the given story so that the answer to your problem is $\frac{121}{12}$ square feet.

**Task II: Problem solving [10 min]**

***Solve this problem:*** In this story, if Diana's sister wanted to pay Diana, how much should Diana charge her?

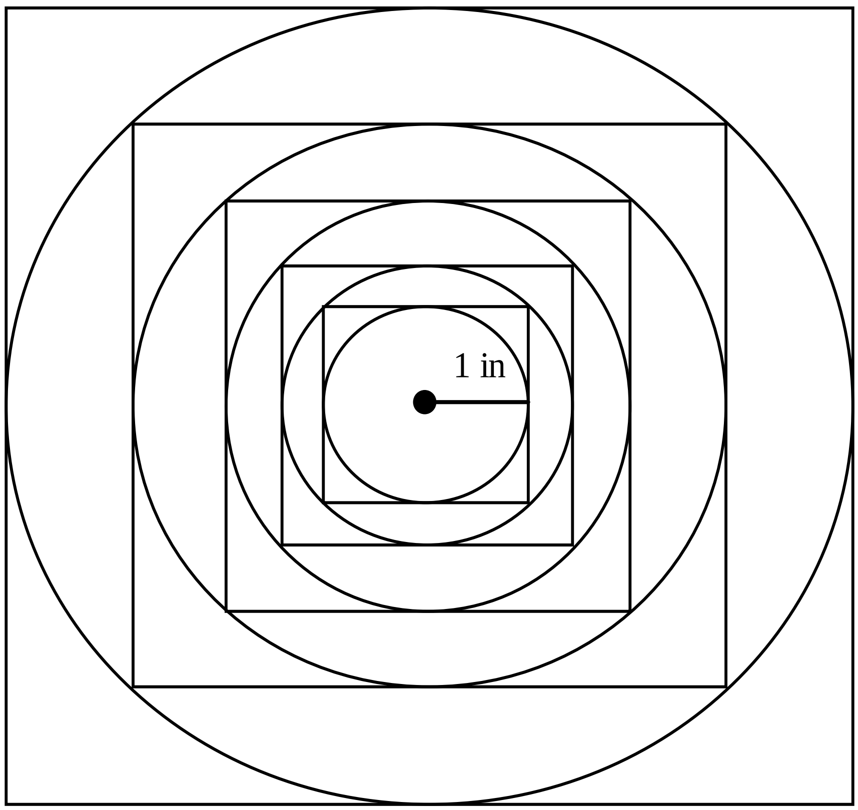
**Task III: Further problem posing [10 min]**

Write two more mathematical story problems that are different from all problems you posed or solved previously according to the same story.

**Second Set of Tasks**

**<u>Here is a real life story:</u>**

A factory that specializes in producing dinnerware decides to produce a set of five bowls in different diameter sizes. The sizes are constrained using the following model diagram and the radius of the smallest bowl is 1 inch.



## Task I: Problem posing [20 min]

***Comprehending:*** Write an appropriate mathematical story problem representing the following calculation: $\sqrt{1^2 + 1^2}$, according to the given situation and figure.

***Translating:*** Write a mathematical story problem according to the given figure.

***Editing:*** Write a mathematical story problem according to the given story, which should be different from the problems you posed before.

***Selecting:*** Write an appropriate mathematical story problem according to the given story so that the answer to your problem is $2\sqrt{2}$.

## Task II: Problem solving [10 min]

***Solve this problem:*** Given that the radius of the smallest circle is 1 inch, what is the ratio of the area of the largest circle to the area of the smallest circle?

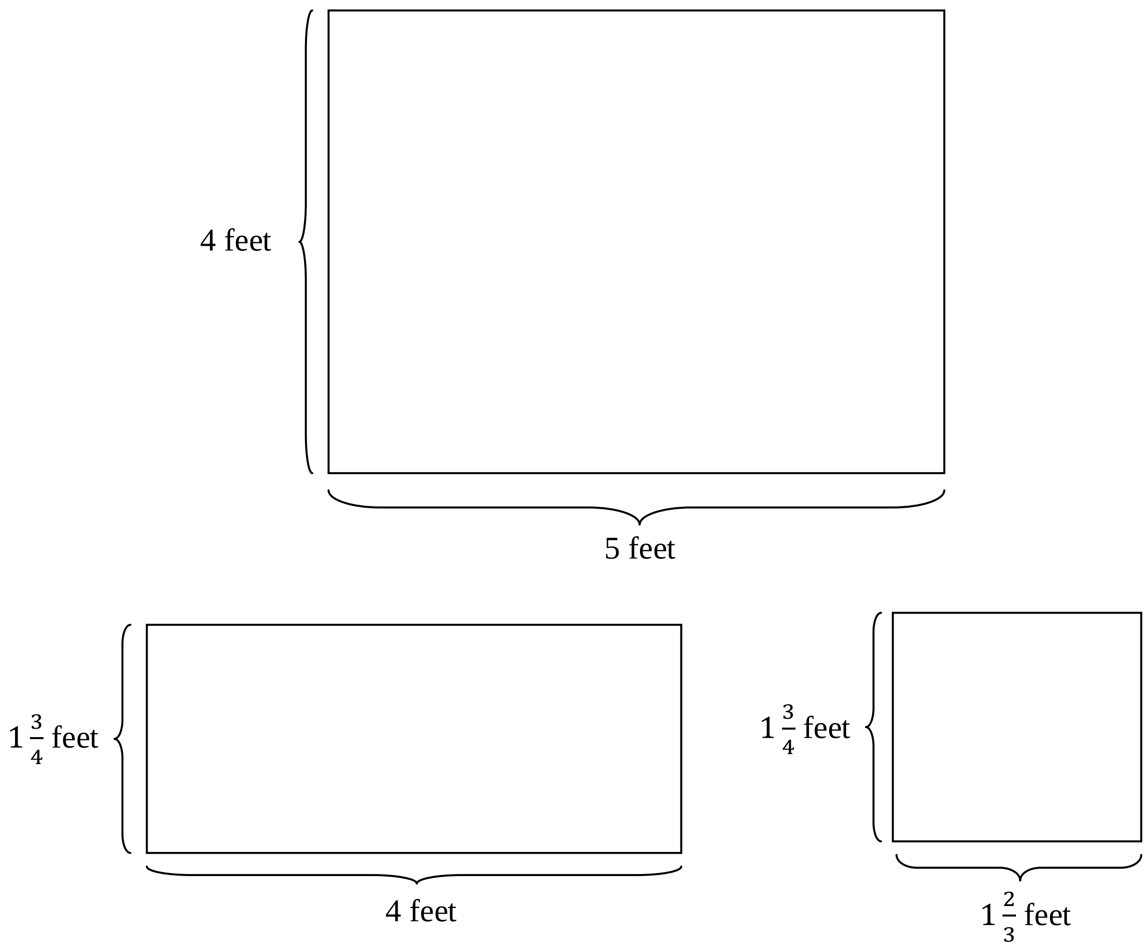
## Task III: Further problem posing [10 min]

Write two more mathematical story problems that are different from all problems you posed or solved previously according to the same story.

## Appendix B: Two Sets of Tasks in English

### The First Set of Tasks

Given the following figures:




### Task I: Problem posing [40 min]

***Translating:*** Write a mathematical story problem according to the given figures.

<u>Here is a real life story:</u>

Diana bought a piece of cloth 4 feet wide and 5 feet long. It cost \$16. She cut off a piece that was $1\frac{3}{4}$ feet wide and 4 feet long to make a scarf. Her sister saw Diana's cloth and really liked the material. She asked for a piece that was $1\frac{3}{4}$ feet wide and $1\frac{2}{3}$ feet long to also make a scarf.

***Comprehending:*** Write an appropriate mathematical story problem representing the following calculation: $1\frac{3}{4} \times (4 + 1\frac{2}{3})$, according to the given situation and figures.

***Editing:*** Write a mathematical story problem according to the given story, which should be different from the problems you posed before.

***Selecting:*** Write an appropriate mathematical story problem according to the given story so that the answer to your problem is $\frac{121}{12}$ square feet.

## Task II: Problem solving [20 min]

***Solve this problem:***

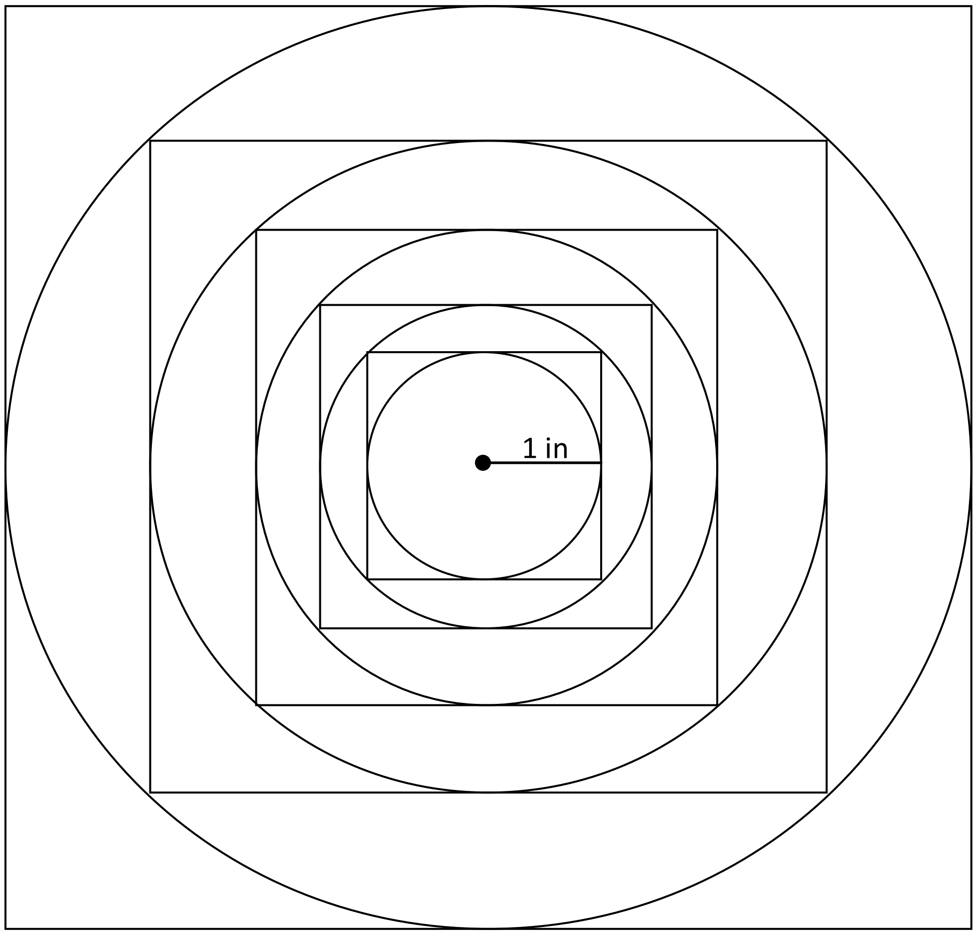
In this story, if Diana's sister wanted to pay Diana, how much should Diana charge her?

## Task III: Further problem posing [20 min]

Write two more mathematical story problems that are different from all problems you posed or solved previously according to the same story.

**The Second Sets of Tasks**

Given the following figure:




**Task I: Problem posing [40 min]**

***Translating:*** Write a mathematical story problem according to the given figure.

Here is a real life story:

A factory that specializes in producing dinnerware decides to produce a set of five bowls in different diameter sizes. The sizes are constrained using the following model diagram and the radius of the smallest bowl is 1 inch.

***Comprehending:*** Write an appropriate mathematical story problem representing the following calculation: $\sqrt{1^2 + 1^2}$, according to the given situation and figure.

***Editing:*** Write a mathematical story problem according to the given story, which should be different from the problems you posed before.

***Selecting:*** Write an appropriate mathematical story problem according to the given story so that the answer to your problem is $2\sqrt{2}$.

**Task II: Problem solving [20 min]**

***Solve this problem:***

Given that the radius of the smallest circle is 1 inch, what is the ratio of the area of the largest circle to the area of the smallest circle?

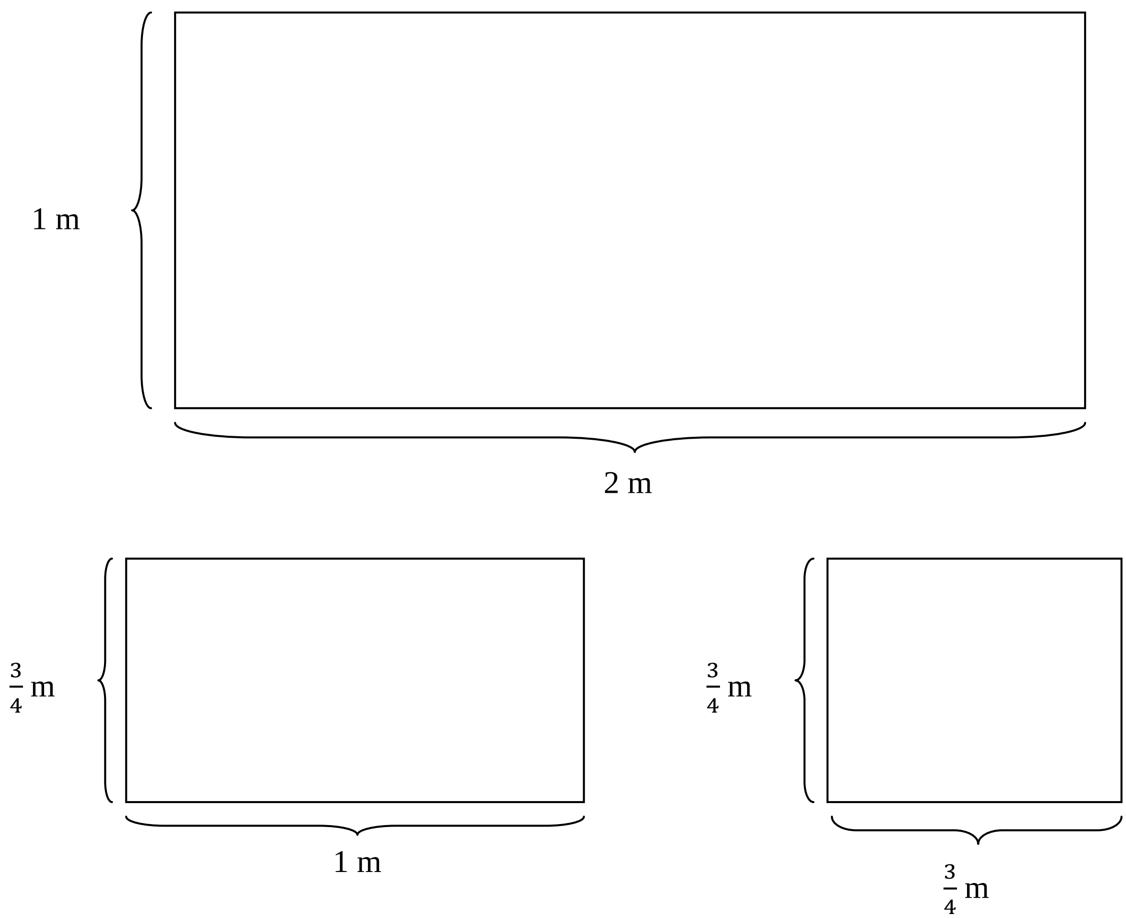
**Task III: Further problem posing [20 min]**

Write two more mathematical story problems that are different from all problems you posed or solved previously based on the same story.

# Appendix C: Two Sets of Tasks in Chinese

## 第一套练习题

<u>观察下图：</u>




### 任务 I：问题提出［40 分钟］

***转换过程：*** 请根据所给的图形编写一个数学应用题。

生活故事实例：

李娜花 12 元买了一块长 2 米宽 1 米的布料。她先裁剪了一块长 1 米宽 $\frac{3}{4}$ 米的布料做了围巾。李娜的妹妹看到李娜买的布料后说她非常喜欢，而且她想做个方形的围巾。于是她从余下的布料上裁剪了一块长宽各 $\frac{3}{4}$ 米的布料。

***理解过程：*** 请根据所给故事和图形编写一个合适的数学应用题， 要求该应用题能表示以下的运算过程： $\frac{3}{4}\times(1+\frac{3}{4})$。

***编辑过程：*** 请根据所给的故事实例编写一个数学应用题， 要求该应用题不同于你编写的前两个问题。

***选择过程：*** 请根据所给的故事实例编写一个数学应用题， 要求该应用题的答案是 $\frac{11}{16}$ 平方米。

**任务 II：问题解决［20 分钟］**

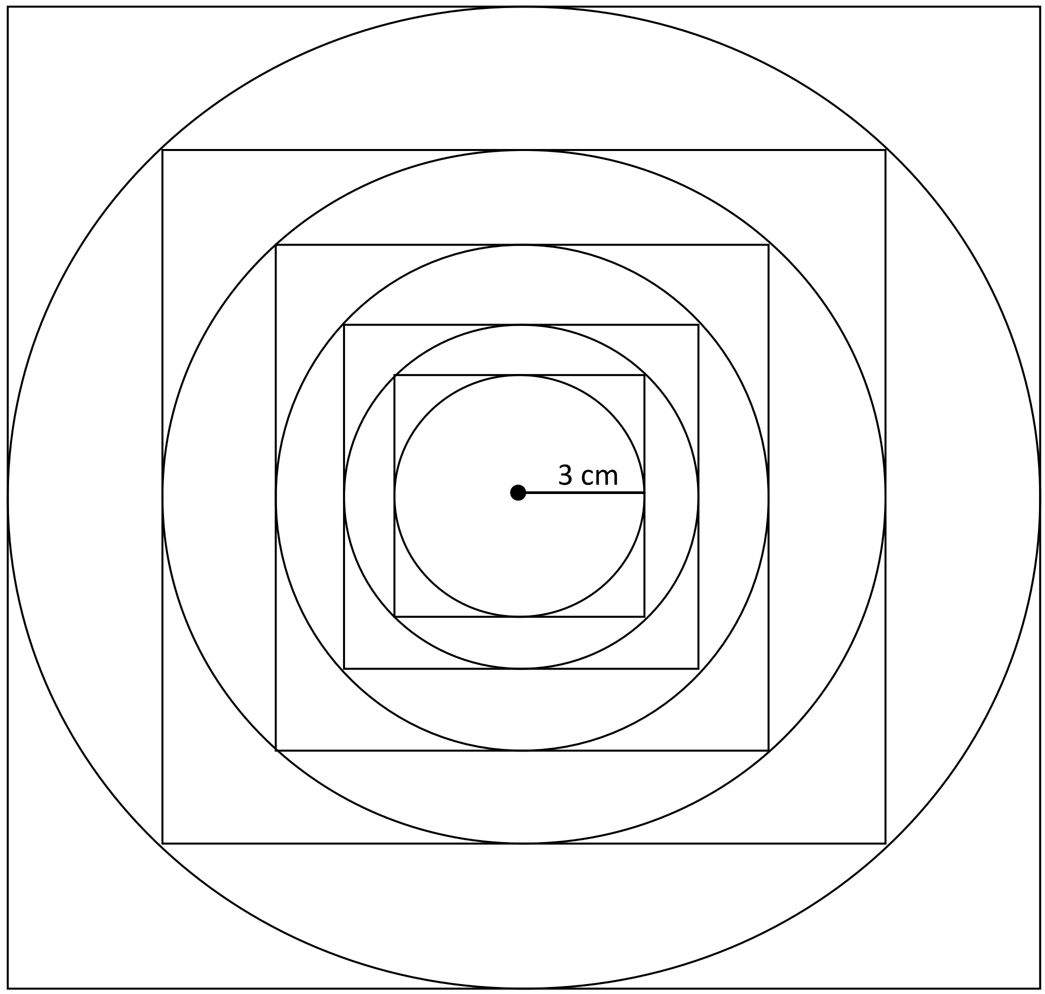
***请解决以下问题：*** 在这个故事中，如果李娜的妹妹想付钱给李娜，她应该付多少钱？

**任务 III：提出更多问题［20 分钟］**

请根据所给的故事实例再编写两个数学应用题，要求这两个问题不同于你在前面编写或者解决的所有问题。

## 第二套练习题

观察下图：




### 任务 I：问题提出［40 分钟］

***转换过程：*** 请根据所给的图形编写一个数学应用题。

生活故事实例：

一个专门制造餐具的工厂计划出产一套有五个大小不一的圆形碗具。上图显示了这套碗具的模型。最小碗的半径是 3 厘米。

***理解过程：*** 请根据所给故事及图形编写一个合适的数学应用题，要求该应用题能表示以下的运算过程：$\sqrt{3^2+3^2}$。

***编辑过程：*** 请根据所给的故事实例编写一个数学应用题，要求该应用题不同于你编写的前两个问题。

***选择过程：*** 请根据所给的故事实例编写一个数学应用题，要求该应用题的答案是 $6\sqrt{2}$。

**任务 II：问题解决［20 分钟］**

***请解决以下问题：*** 已知所给故事实例中最小碗的半径是 3 厘米， 那么最大碗和最小碗的碗口面积之比是多少？

**任务 III：提出更多问题［20 分钟］**

请根据所给的故事实例再编写两个数学应用题，要求这两个问题不同于你在前面编写的或者解决的所有问题。

**Appendix D: Question Examples during the Second Task Administration**

1. What do you mean by _______ (a word, a phrase, a sentence, a question, etc.)?
2. How did you come up with this problem?
3. How did you come up with this problem-solving solution?
4. Why did you draw this picture?
5. What are you calculating for?
6. What difficulties do you have for posing a problem for this process?
7. What difficulties do you have when solving this problem?
8. How satisfied are you with your posed problem? Are you confident?
9. How satisfied are you with your problem-solving solution? Are you confident?

## Appendix E: Question Examples for Prompting Thinking

1. How do you feel about problem posing in general?
2. Which one do you think is more difficult, problem posing or problem solving? Why?
3. Among all four problem-posing processes, which is most difficult to you? Why?
4. How do you feel about problem posing before and after problem solving? Are there any differences? Which process is more difficult to you? Why?
5. How do you feel about problem solving after problem-posing exploration?
6. How do you feel about the difference of this experience from your prior mathematics learning experience?

**Appendix F: Codes and Sub-codes for Translating Process**

| Major Codes | Sub-codes | Explanation |
|---|---|---|
| Category of posed problem | SMP | Solvable mathematical problem |
| | UMP | Unsolvable mathematical problem |
| | NMP | Not a mathematical problem |
| Visualization | VTE | Thinking that extends |
| | VTC | Thinking that creates |
| | NVT | No thinking that extends or creates |
| Features of posed problem | FCT | Contextualized |
| | FNC | Not contextualized |
| | FSP | Static process |
| | FDP | Dynamic process |
| | F2D | Two-dimensional situation/object |
| | F3D | Three-dimensional situation/object |
| | FCE | Close-ended problem |
| | FOE | Open-ended problem, including open-middled (FOE-M) and open-ended (FOE-E) |
| Complexity of solvable problem | COS 1-2 | One or two computational steps needed for problem solving |
| | COS 3-4 | Three or four computational steps needed for problem solving |
| | COS >=5 | Five or more computational steps needed for problem solving |
| Mathematical terminology use | TU-E-U | Efficiency of terminology use, used mathematical terminologies |
| | TU-E-NU | Efficiency of terminology use, did not use mathematical terminologies |
| | TU-A-C | Accuracy of terminology use, correctly used |
| | TU-A-IC | Accuracy of terminology use, incorrectly used |

**Appendix G: Final Codes and Descriptions for Translating Process**

| Final Codes | Sub-codes | Explanation |
|---|---|---|
| Category of posed problem | SMP | Solvable mathematical problem |
| | UMP | Unsolvable mathematical problem |
| | NMP | Not a mathematical problem |
| Visualization | VTE | Thinking that extends |
| | VTC | Thinking that creates |
| | NVT | No thinking that extends or creates |
| | FSP | Static process |
| | FDP | Dynamic process |
| | F2D | Two-dimensional situation/object |
| | F3D | Three-dimensional situation/object |
| | FCE | Close-ended problem |
| | FOE | Open-ended problem, including open-middled (FOE-M) and open-ended (FOE-E) |
| Complexity of solvable problem | COS 1-2 | One or two computational steps needed for problem solving |
| | COS 3-4 | Three or four computational steps needed for problem solving |
| | COS >=5 | Five or more computational steps needed for problem solving |
| Mathematical terminology use | TU-E-U | Efficiency of terminology use, used mathematical terminologies |
| | TU-E-NU | Efficiency of terminology use, did not use mathematical terminologies |
| | TU-A-C | Accuracy of terminology use, correctly used |
| | TU-A-IC | Accuracy of terminology use, incorrectly used |

By analyzing posed problems for the Translating process using above codes, I built the following three descriptions only for this particular process:

**Description 1:** The apparent concepts or ideas that involved in given figures, such as area or perimeter of a rectangle for the first set of tasks and diameter or area of a circle for the second set of tasks, were directly used by many prospective elementary teachers.

**Description 2:** Multiple creative ideas of using mathematical concepts and knowledge occurred in prospective elementary teachers' posed solvable mathematical problems.

**Description 3:** Both US and Chinese participants posed a number of unsolvable and not mathematical problems, which demonstrated similar difficulties that they had.

## Appendix H: Description Examples of Connections between Different Processes

The Editing process was the third problem-posing process during each of my task administration. Because of this, US and Chinese participants had had problem-posing experience before they got to Editing process. I therefore examined the influence of their prior experiences on their performance during the Editing process and built the following descriptions:

**Description 6:** Although a large number of posed problems for the Editing process used similar mathematical concepts during the Translating and Comprehending processes, many of the problems involved more computational steps and/or a different structure.

**Description 7:** Both US and Chinese participants were capable of posing more creative, high-level cognitively demanding problems in the Editing process. Some of those problems had similar mathematical ideas and/or structure with the problem given in later problem-solving process, especially during the first task administration.

**Description 8:** Despite being asked to pose a problem related to the sum of the areas in the Comprehending process, some US participants were not successful. However, they posed that expected problem in response to the Editing process. On the other hand, some Chinese participants anticipated what the next step would be and posed a problem that was expected in the Selecting process.

**Description 9:** Many Chinese participants posed problems for the Editing process that could be solved using the solution of the problem they posed for the Comprehending process, and this was true for both the first and second sets of tasks.

## Appendix I: Examples of Major Themes

By synthesizing all the descriptions that I built for each problem-posing process, I developed major themes for answering my research questions. Below are examples of major themes that I developed for answering my Research Question 1: What are the patterns of problem posing shown by US and Chinese prospective elementary teachers during their problem-posing processes when problem solving is involved in an alternating manner? Are there any differences in the patterns shown by these two groups of participants?

**Similar patterns**

- similar features of posed problems
- similar development in problem-posing performance

**Different patterns**

- figure visualization;
- calculation interpretation;
- habitual preference of posing a sequence of problems;
- perception to a given answer based on previously posed or solved problems;
- problem-posing strategy selection for integrating given information.

## Appendix J: Specifics Examples of Each Category and Subcategory of Posed Problems

| Category | Examples |
|---|---|
| Solvable mathematical problem | *A solvable mathematics problem with necessary and sufficient condition*: Diana paid $16 for a piece of cloth that is 4 feet wide and 5 feet long. Diana cut off a piece of cloth that was $1\frac{3}{4}$ feet wide and 4 feet long. How much money did the piece that Diana cut off cost her? (U03, 1st, PP3)* <br><br> *A solvable mathematics problem with redundant information*: Diana's sister was upset that Diana changed the dimensions of their scarves, so she went to the store herself and bought a 6 ft wide by 3 ft long piece of fabric. From that piece of fabric, she cut another scarf that was $5\frac{1}{6}$ ft wide by $2\frac{1}{2}$ ft long. What was the area of Diana's sister's new scarf? (U07, 1st, PP4) |
| Unsolvable mathematical problem | *A mathematical problem that is impossible to solve*: A garden is $5\text{ft} \times 4\text{ft}$ . One section of the garden is filled w/ [with] tulips. This section is $4\text{ft} \times 1\frac{3}{4}\text{ft}$. Another section of the garden is filled w/ [with] lilacs. This section is $1\frac{2}{3}\text{ft} \times 1\frac{3}{4}\text{ft}$ . The gardener wants to fill the rest w/ [with] roses and lavender. If she/he split up the remaining section evenly b/t [between] these two types of flowers, what would be the dimensions of the rose section? (U08, 1st, PP1) <br><br> *A mathematics problem with insufficient condition*: George is hanging up a new poster and needs giant tape to hold it up. He needs tape that is $1\frac{3}{4}$ft wide. Carson also got a new poster and needs tape 4ft long plus $1\frac{2}{3}$ feet wide. How much tape all together do George and Carson needs to hang up 2 posters. (U24, 1st, PP2) |
| Not a mathematical problem | *A problem but not a mathematical problem*: Diana's sister wants to add her piece onto Diana's piece to make one big scarf for them to share. How would she do it? (U22, 1st, PP2) <br><br> *Not a problem at all*: Diana and her sister laid both of their new scarfs [scarves] end to end and sewed them together so that now they had a single scarf that was $1\frac{3}{4}$ft wide by $5\frac{2}{3}$ft long. (U07, 1st, PP2) |

* U03, 1st, PP3 represents the problem posed by a US participant who was numbered as 03 for the third problem-posing process, i.e., Editing process, during the first task administration.

## Appendix K: Summary and Synthesis of Participants' Problem-posing and Problem-solving Performance

| | General Patterns | | Different Patterns | |
|---|---|---|---|---|
| | 1st Set of Tasks | 2nd Set of Tasks | 1st Set of Tasks | 2nd Set of Tasks |
| PP1 | • About 40% US and Chinese participants used only area or perimeter of one rectangle<br>• Similar creative ideas involved in solvable problems<br>• Similar reasons that led problems to be unsolvable or not math problems | • Almost all posed problems used only basic concepts of a circle/square<br>• No middle-sized circle/square was considered<br>• A specific reason that lead problems to be unsolvable was contradiction involvement | • 33% US vs. 70% Chinese participants involved all three rectangles in their posed problems | • More than half of US participants focused on the smallest circle vs. the majority of Chinese participants extended their consideration to the biggest circle or square<br>• 6% US vs. 48% Chinese participant posed problems with creative ideas |
| PP2 | • None | • None | • 39% of the posed problems by US participants vs. 96% of the posed problems by Chinese participants represented the given calculation | • 11% posed problems by US participants vs.89% posed problems by Chinese participants represented the given calculation |
| PP3 | • More and new creative ideas occurred<br>• Many solvable problems involved more computational steps and/or different structure<br>• Many problems had similar idea and structure with the problem given in problem-solving task | • Participants started to pose problems about middle-sized circles/squares<br>• Some posed problems starting from an unknown circle | • No US participant vs. 15% Chinese participants asked at least two questions in a sequence<br>• 3% US participants vs. 31% Chinese participants posed problems that could be solved using the answer to the problems they posed in PP2 | • No US participant vs. 11% Chinese participants posed problems about the concept of ratio, similar idea and structure with problem in problem-solving task<br>• No US participant vs. 22% Chinese participants posed problems that could be solved using the answer to the problems they posed in PP2 |
| PP4 | • None | • None | • 9% US participants vs. 96% Chinese participants posed problems as expected<br>• The majority of US participants had difficulties in vs. many Chinese participants | • 19% US participants vs. 89% Chinese participants posed problems as expected<br>• The majority of US participants had difficulties in vs. many Chinese participants |

| | | | | |
|---|---|---|---|---|
| | | | were able to perceive and apply the relation between the calculation in PP2 and the answer in PP4<br>• 16% US participants vs. no Chinese participant used the given number as a condition<br>• 50% US participants vs. 15% Chinese participants tried to construct the given number, the majority of those problems were solvable | were able to perceive and apply the relation between the calculation in PP2 and the answer in PP4<br>• 6% US participants vs. no Chinese participant used the given number as a condition<br>• 16% US participants vs. no Chinese participant tried to construct the given number, all those problems posed by US participants were solvable |
| PS | • None | • None | • 25% US participants vs. 98% Chinese participants correctly solved the problem<br>• 47% US participants vs. 2% Chinese participants partially solved the problem | • 19% US participants vs. 89% Chinese participants correctly solved the problem<br>• 31% US participants vs. 7% Chinese participants partially solved the problem |
| PP5 | • Similar percentage of posed problem with creative ideas<br>• About 30% solvable problems had similar idea and structure with the problem given in problem-solving task | • No unsolvable problem or problem that was not a math problem had similar idea or structure with the problem given in problem-solving task | • 3% US participants vs. 64 % Chinese participants posed two solvable problems that completely utilized all given information | • 5% solvable problems posed by US participants vs. 28% solvable problems posed by Chinese participants had similar idea and structure with problem in problem-solving task |

* PP1 represents Translating process; PP2 represents Comprehending process; PP3 represents Editing process; PP4 represents Selecting process; PS represents problem-solving process; and PP5 represents problem posing after problem solving process

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