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We study numerically the dynamical triangulation formulation of two-dimensional quantum gravity using a restricted class of triangulation, so-called *minimal triangulations*, in which only vertices of coordination number 5, 6, and 7 are allowed [1]. A real-space RG analysis shows that for pure gravity (central charge $c = 0$) this restriction does not affect the critical behavior of the model. Furthermore, we show that the critical behavior of an Ising model coupled to minimal dynamical triangulations ($c = 1/2$) is still governed by the KPZ-exponents.

1. INTRODUCTION

In the discrete formulation of two-dimensional quantum gravity, known as *dynamical triangulations*, the micro–canonical, or fixed area, partition function is defined as the sum over all possible ways of gluing A equilateral triangles together to form a piecewise linear manifold (with an appropriate topology):

$$Z_{\mathcal{T}}(A) \;=\; \sum_{T \in \mathcal{T}} Z_M(T). \qquad (1)$$

In the case of matter coupled to gravity, the triangulations T are weighted with the partition function Z_M for the corresponding matter fields.

In Eq. (1) $\mathcal{T}$ represents the class of triangulations (of area A) included in the partition function. Different classes correspond to different discretization of the manifolds. Two commonly used classes are: *combinatorial* triangulations $\mathcal{T}_C$, and *degenerate* triangulations $\mathcal{T}_D$. In the former a vertex is not allowed to connected to itself, nor can any two vertices be connected by more than one link — this excludes tadpole and self-energy diagrams in the dual graph. This restriction is eased for degenerate triangulations. Clearly $\mathcal{T}_C \subset \mathcal{T}_D$. In cases where the model Eq. (1) has been solved, it has been shown that $Z_{\mathcal{T}_C}$ and $Z_{\mathcal{T}_D}$ are in the same universality class [2].

But how general is this universality, i.e. how much restriction can be imposed on the triangulations before the critical behavior of the model changes — clearly including only one triangulation would produce different result. In the work presented here, we investigate this by considering a new class of triangulations $\mathcal{T}_M$, the class of *minimal triangulations*, in which only vertices with coordination number 5, 6, and 7 are allowed. This is as far as we can go in suppressing curvature fluctuations, while still retaining the fluid nature of the surfaces. We will demonstrate, by studying the critical behavior of pure gravity and an Ising matter coupled to gravity, that this restriction does not change the critical behavior of the model.

An additional motivation for studying this class of triangulations is that they more closely resemble conventional condensed matter systems. Previously studied models of dynamical triangulations allow vertices with arbitrary high curvature, whereas a real system places a cut-off on the number of interaction for a single particle. That minimal dynamical triangulations are in the usual universality class of $2D$ gravity, implies that the KPZ-exponents might be realized in a real system (yet to be discovered).

2. PURE GRAVITY

We start by investigating this model in the absence of matter ($Z_M = 1$). To determine the critical behavior we study the fractal structure of the surfaces. More precisely we measure the string susceptibility exponent γ_s, which governs the critical behavior of the grand canonical partition function $Z(\mu) \sim (\mu_c - \mu)^{2-\gamma_s}$. In numerical simulations, γ_s is obtained from the size distribu-

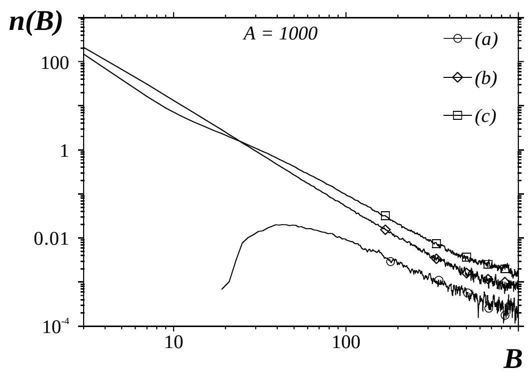

Figure 1. The measured distributions of baby universes for *(a)* the minimal class of triangulations, *(b)* combinatorial triangulations and *(c)* triangulations obtained by node decimation from *(a)*. All measurements are for triangulations of 1000 vertices.

tion of minimal neck baby universes $n(B)$ [3];

$$n_A(B) = [B(A-B)]^{\gamma_s - 2} \,. \tag{2}$$

For pure gravity $\gamma_s = -\frac{1}{2}$.

We have measured the distribution $n(B)$ for minimal triangulations, this is shown in Fig. 1 (curve a). For comparison we also show the corresponding distribution measured on combinatorial triangulations (curve b). In addition Fig. 1 includes a distribution measured on triangulations obtained from minimal triangulations via node decimation. We will discuss this later.

Although, for large baby universes, the slope of the distributions a and b is similar, Fig. 1 clearly shows much bigger finite size effects for minimal triangulations. This is easily understood, as the restrictions on minimal triangulations effectively smoothens the surfaces locally — in fact there is a lower cut-off on the size of the baby universes. An estimate of γ_s for the minimal triangulations yields $-0.64(5)$ for $A = 1000$ and $-0.53(3)$ for $A = 4000$, compared to $\gamma_s = -0.501(4)$ for combinatorial triangulations and the exact value $-\frac{1}{2}$.

The same result is also obtained by using a recently proposed real-space RG method for dynamical triangulations; *node decimation* [4]. Applying node decimation we have blocked the minimal triangulations repeatedly, down to 250 nodes, using a blocking factor of 2. Note that under blocking, the minimal triangulations flow into the wider class of combinatorial triangulations. We then measure the distribution $n(B)$ on the ensemble of blocked triangulations. This is shown in Fig. 1. Clearly the distribution obtained from minimal triangulations via blocking is much closer to the distribution for combinatorial triangulations than the original — this implies that the finite size effects are reduced in the blocking. This also implies that under blocking the model flows towards a non-trivial fixed point, corresponding to $2D$ gravity.

The reductions of finite size effects is also apparent in the value of γ_s, measured on the blocked triangulations. This is shown in Table 1. Under blocking the measured value of γ_s approaches the exact value.

Table 1
Measured values of γ_s for minimal dynamical triangulations (without matter) after applying varying levels of node decimation with a blocking factor of $b = 2$.

	$A^{(0)} = 1000$		$A^{(0)} = 4000$	
$A^{(k)}$	k	γ_s	k	γ_s
4000			0	-0.530(26)
2000			1	-0.544(32)
1000	0	-0.644(48)	2	-0.530(18)
500	1	-0.619(26)	3	-0.504(9)
250	2	-0.574(49)	4	-0.478(36)

3. THE ISING MODEL

We have also investigated the critical behavior of an Ising model coupled to minimal triangulations, for lattice sizes $A = 250$ to 8000. In this case

$$Z_M(\beta) = \sum_{\{\sigma_i\}} e^{\beta \sum_{<ij>} \sigma_i \sigma_j}, \tag{3}$$

where σ_i is an Ising spin placed on vertex i. The Ising spins were updated using a Swendsen-Wang

Table 2
The measured critical exponents. νd_H and α are obtained from the scaling of the peaks in $\partial g_r/\partial\beta$ and C_V respectively. β and γ are measured from scaling at β_c, in which case the errors are dominated by the uncertainty in the location of β_c.

A	νd_H	α	β	γ
250-8000	2.857(24)	-0.806(41)	0.474(20)	2.117(54)
500-8000	2.890(33)	-0.922(58)	0.488(21)	2.095(52)
1000-8000	2.907(25)	-0.977(87)	0.500(18)	2.070(55)
Onsager	2	0(log)	1/8	7/4
KPZ	3	-1	1/2	2

cluster algorithm and approximately 10^7 sweeps performed per data-point.

We must first determine the infinite volume critical coupling β_c. This is done by locating the peaks in the specific heat C_V and the derivate of Binders cumulant $\partial g_r/\partial\beta$; both are expected to approach β_c as

$$|\beta_c - \beta_c(A)| \sim A^{-1/\nu d_H}. \tag{4}$$

The fit to Eq. 4 is made easier by an independent determination of νd_H — the height of $\partial g_r/\partial\beta$ scales like $A^{1/\nu d_H}$. This procedure yields $\beta_c = 0.2663(3)$ for the Ising model coupled to minimal dynamical triangulations.

Using this estimate of β_c, we then determine other critical exponents using finite size scaling. We have the magnetization; $M \sim A^{-\beta/\nu d_H}$, the magnetic susceptibility; $\chi \sim A^{\gamma/\nu d_H}$, and the specific heat; $C_V \approx c_0 + c_1 A^{\alpha/\nu d_H}$. The exponents are shown in Table 2 together with the KPZ and Onsager predictions. We indicate the finite size effects by imposing different lower cut-off's on the lattice size used. But the conclusion is clear — the measured exponents agree very well with the KPZ-exponents.

4. DISCUSSION

In this paper, we have investigated the critical behavior of a so-called minimal dynamical triangulation model of $2D$ gravity. In this model only vertices of coordination numbers 5, 6, and 7 are allowed — this effectively suppresses the high curvature fluctuations. By studying the fractal structure of pure gravity, and the critical properties of an Ising model coupled to gravity, we conclude that the critical behavior is not affected by this restriction to minimal triangulations — the model is in the same universality class as with combinatorial or degenerate triangulations. This implies that large curvature fluctuations do not play an important role in determining the continuum structure of $2D$ quantum gravity.

This result agrees with recent studies of a matrix-model formulation of R^2-gravity, where similarly the R^2-term can be used suppress the curvature fluctuations locally. In [5] it was shown that the R^2 operator is irrelevant in the continuum limit — at large length scales the model always reduces to that of pure gravity.

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