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Anisotropic Membranes

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Abstract

We describe the statistical behavior of anisotropic crystalline membranes. In particular we give the phase diagram and critical exponents for phantom membranes and discuss the generalization to self-avoiding membranes.

1 Introduction

The statistical mechanics of phantom tethered membranes with bending rigidity is quite well understood if the elastic and bending moduli are isotropic. There is a continuous *crumpling* transition from the expected high-temperature *crumpled* phase to a low-temperature *flat* phase that has no analogue in conventional spin systems [1, 2]. This flat phase is characterized by long-range orientational order of the membrane in the embedding space and large longitudinal (in-plane) fluctuations. Two important generalizations of this class of membranes arise by incorporating *self-avoidance* and/or intrinsic *anisotropy*. In [3] Radzihovsky and Toner (RT) showed that intrinsic anisotropy generally leads to a much richer phase diagram for *phantom* (non-self-avoiding) tethered membranes. Although anisotropy is irrelevant in the flat and crumpled phases it leads to an entirely new *tubular* phase separated by distinct continuous phase transitions from both the conventional phases. This phase diagram is shown in Fig. 1.

¿From the applied point of view both isotropic and anisotropic membranes should be important. Isotropic membranes may be made by suitable random polymerization of fluid

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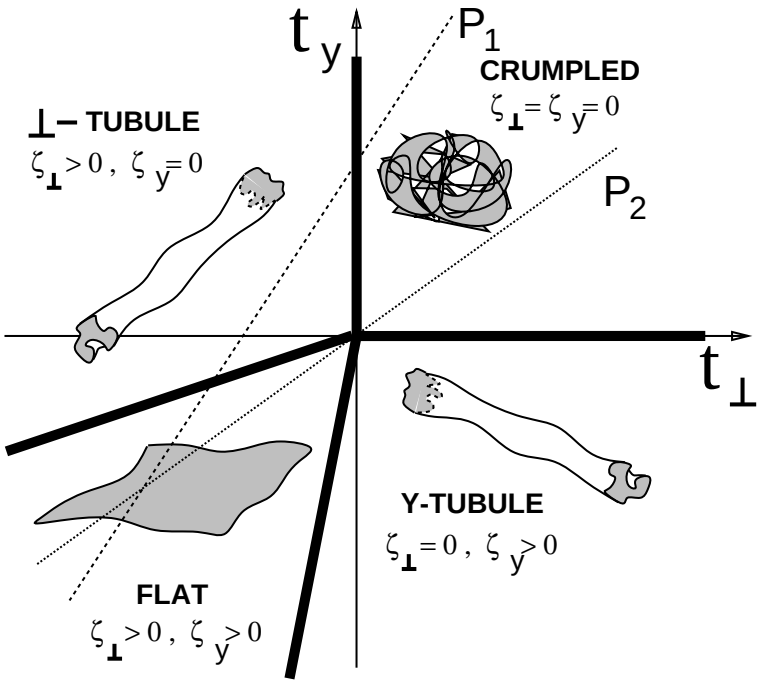

Figure 1: The mean field theory phase diagram for phantom anisotropic membranes from [3].

vesicles. On the other hand polymerization in the presence of an applied electric field should in principle yield an anisotropic membrane [4]. Polymerized membranes with in-plane tilt order have a natural anisotropy. The tubular phase is characterized by long-range orientational order in one (extended) direction only — in the remaining transverse direction the membrane is crumpled.

The tubular phase thus resembles a rough sausage with a crumpled cross-section. This existence of this totally tubular phase was confirmed by Monte Carlo simulations in [5]. In this paper the size (Flory) exponent ν_F, describing the growth of the tubule-diameter R_G with internal system size (L), and the roughness exponent ζ, characterizing the height fluctuations along the extended tubule axis, were also reported. The results $\nu_F = 0.305(14)$ and $\zeta = 0.895(60)$ were in rough qualitative agreement with the theoretical predictions $\nu = \frac{1}{4}$ and $\zeta = 1$ of [3]. More extensive subsequent simulations [6] have improved these first results – the best current results are $\nu_F = 0.269(7)$ and $\zeta = 0.859(40)$. Configurations characteristic of the various phases are shown in Fig. 2.

2 Model

A tethered membrane is described by a 2-dimensional regular triangulated net, with the topology of a disk. Each node of the net has six neighbors, except at the boundary. The Hamiltonian of the system is

$$\mathcal{H}[\mathbf{r}] \;=\; \sum_{\langle\sigma\sigma'\rangle} |\mathbf{r}_\sigma - \mathbf{r}_{\sigma'}|^2$$

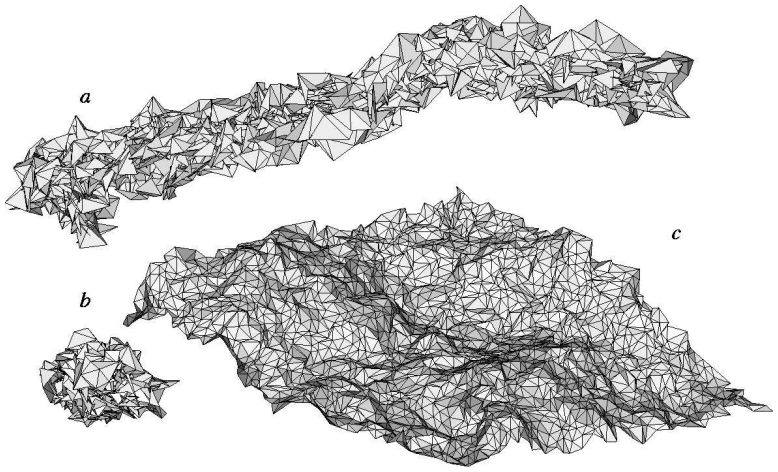

Figure 2: The three phases of anisotropic membranes:(a) tubular (b) crumpled and (c) flat.

$$- \ \kappa_1 \sum_{\langle ab \rangle}^{(x)} \mathbf{n}_a \cdot \mathbf{n}_b - \kappa_2 \sum_{\langle ab \rangle}^{(y)} \mathbf{n}_a \cdot \mathbf{n}_b \, . \tag{1}$$

where $\mathbf{r}_\sigma$ is the position in 3-dimensional space of the node labelled $\sigma = (\sigma_x, \sigma_y)$. The first sum runs over all nearest-neighbor pairs (bonds) of the membrane, and is the tethering potential. The second and third term are the bending energies in the x and y intrinsic directions. The bending energy is a ferromagnetic interaction between the unit normals to the faces of the membrane. The strength of this interaction is anisotropic: if two adjacent faces share a bond parallel to the x direction the coupling is κ_1; otherwise it is κ_2.

The canonical partition function for a membrane of fixed number of nodes N is

$$Z \ = \ \int [\mathrm{d}\mathbf{r}] \, \delta(\mathbf{r}_{\mathrm{cm}}) \, \mathrm{e}^{-\mathcal{H}[\mathbf{r}]}. \tag{2}$$

The Hamiltonian of Eq.(1) was simulated using Monte Carlo methods for triangular lattices ranging from 25^2 to 175^2 nodes. Further numerical details are given in [5, 6].

3 Global Phase Diagram

To determine the global phase diagram of anisotropic membranes we explored the line $(\kappa_1, \kappa_2) = (3\kappa, \kappa)$. We have also simulated the lines $(\kappa, 0)$ and $(2, \kappa)$ on smaller lattices. Distinct signatures of phase transitions were found in the specific heats C_V^i associated with the variance of the two bending energy terms in the action E_x and E_y;

$$C_V^i(\kappa) = \frac{\kappa^2}{L^2} \, \frac{\partial}{\partial \kappa} \langle E_i \rangle. \tag{3}$$

In Fig. 3 we show the total specific heat along the line $\kappa_1 = 3\kappa_2$ for lattice sizes up to 175^2. There are two distinct peaks. We have confirmed that the two peaks occur at *different* values of the bending rigidity, signaling the existence of *two* distinct transitions.

Performing a similar analysis along the other two lines in the (κ_1, κ_2) plane yields a phase diagram consistent with that of Fig. 1. This confirms the three phase scenario.

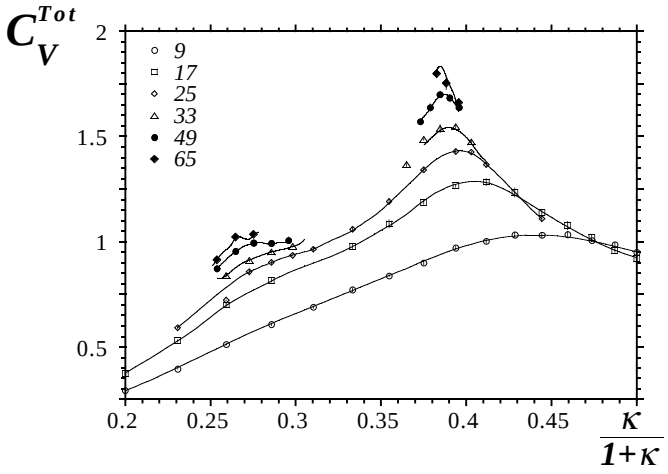

Figure 3: The value of the total specific heat C_V for various lattice sizes. The interpolating lines are obtained using multi-histogramming methods.

4 Tubular Exponents

A more detailed understanding of this tubular phase is obtained by looking at the fluctuations of the zero-mode of the tubule height h_{rms}, analogous to the height fluctuations of a flat membrane, and the scaling of the width of the tubule $R_\perp^G$ [5, 6]. These are expected to scale as $h_{rms} \sim L^\zeta$ and $R_\perp^G \sim L^{\nu_F}$.

We find $\zeta = 0.859(40)$ and $\nu_F = 0.269(7)$. The result for ν_F is in excellent agreement with the analytic continuum prediction of [3] ($\nu_F = 1/4$) but the roughness exponent is well below the analytic prediction $\zeta = 1$. Further studies are under way [6].

The most challenging problem in the physics of tubules is the incorporation of *self-avoidance*. Self-avoidance becomes relevant below the upper-critical embedding dimension $d = 11$, and here one can develop a systematic ϵ expansion [10]. This expansion can be improved and this is treated in the contribution to this proceedings by Alex Travesset [11, 12]. We are also performing Monte Carlo simulations of this system. Preliminary results indicate a flat-to-tubular transition in the physical case of three dimensions. The specific heat plot is shown in Fig. 4.

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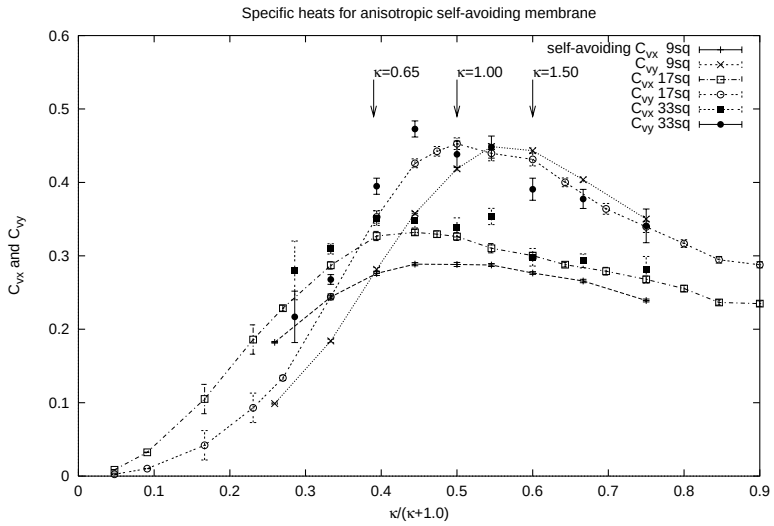

Figure 4: The specific heat C_V for self-avoiding membranes.

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