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Simon Catterall
Syracuse University

Emil Mottola
Los Alamos National Laboratory

T. Bhattacharya
Los Alamos National Laboratory

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S. Catterall[a], E. Mottola and T. Bhattacharya [b]

[a]Physics Department, Syracuse University,
Syracuse, NY 13244

[b]Theoretical Division T-8, Mail Stop B285,
Los Alamos National Laboratory, Los Alamos, NM 87545

We review the derivation of the Liouville action in 2DQG via the trace anomaly and emphasize how a similar approach can be used to derive an effective action describing the long wavelength dynamics of the conformal factor in 4D. In 2D we describe how to make an explicit connection between dynamical triangulations and this continuum theory, and present results which confirm the equivalance of the two approaches. By reconstructing a lattice conformal mode from DT simulations it should be possible to test this equivalence in 4D also.

1. Quick Review of 2DQG

The usual FPI approach to 2DQG starts from the partition function

$$Z=\int\frac{Dg}{\mathrm{Vol(Diffs)}}e^{-S(g)} \tag{1}$$

The measure is constructed to sum only over physically inequivalent metrics. Choosing the conformal gauge $g=\overline{g}e^{\phi}$ and implementing the usual FP procedure leads to the following form for Z

$$Z\sim\int D\phi e^{-S_L(\phi)}\times(\mathrm{ghosts}) \tag{2}$$

where the celebrated Liouville action $S_L(\phi)$ is given by

$$S_L(\phi)=\frac{(25-c_M)}{96\pi}\int\sqrt{\overline{g}}\left(\overline{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+2\overline{R}\phi\right) \tag{3}$$

The parameter c_M is the central charge of massless matter fields. An entirely equivalent way to derive this action proceeds via the trace anomaly of the energy momentum tensor of massless fields in a curved background [1]. The form of this is

$$\begin{aligned} T &= \frac{(25-c_M)}{24\pi}R && (4)\\ &= \frac{(25-c_M)}{24\pi}e^{-\phi}\left(\overline{R}-\overline{\Delta}\phi\right) && (5)\end{aligned}$$

This trace can be derived from an effective action $S_{\mathrm{eff}}(\phi)$ via

$$T=\frac{2}{\sqrt{g}}\frac{\partial}{\partial\phi}S_{\mathrm{eff}}(\phi) \tag{6}$$

We then find $S_{\mathrm{eff}}(\phi)=S_L(\phi)$. The true action is then a sum of the classical and Liouville actions and contains all the information on gravitational dressing, Hausdorff dimension and baby universe structure.

The form of $S_L(\phi)$ has another consequence - it admits *spike* solutions of the form $\phi_S\sim-\ln r$. These possess an action which depends logarithmically on the IR cutoff L. They correspond to quasi 1D geometries termed branched polymers. A KT-like argument then suggests that such configurations dominate Z if $c_M>1$ [2]. In this regime Liouville theory fails to give an adequate description of the consequences of gravitational fluctuations.

2. Equivalence to DTs?

Dynamical triangulations furnish another approach to quantum gravity in which a finite simplicial mesh with invariant edge-length cutoff is used to approximate the continuum geometries. Summing over such lattices is hypothesized to generate the correct measure on the space of geometries in some suitable scaling limit. This conjecture is supported by the remarkable fact that all correlation functions computed within Liouville theory agree with those coming from the DT ensemble for arbitrary genus surfaces [3].

The question we would like to pose is whether it is possible to demonstrate this connection between LT and DTs explicitly - initially in 2D and later within the wider context of 4D models based on quantizations of the conformal factor.

To answer this question within the context of numerical simulation (the natural technique for exploring the DT models) we have devised a procedure to reconstruct a lattice conformal mode for a generic lattice drawn from a DT ensemble. We simply solve the equation

$$R = e^{-\phi}\left(\overline{R} - \overline{\Delta}\phi\right) \tag{7}$$

For $\overline{g}$ we take a round sphere with fixed radius. Precise lattice analogs exist for the Laplacian operator Δ on a DT geometry and for the curvature scalar R. This is a nonlinear matrix equation which we solve using an algorithm based on Newton iteration.

2.1. Points to note

On the basis of the Gaussian model LT predicts

$$\langle S_L\left(\phi\right)\rangle = \frac{48\pi}{\left(25 - c_M\right)} \tag{8}$$

We have checked this for pure gravity $c_M = 0$ and found reasonable agreement. Fig 1. shows a plot of the mean Liouville action versus lattice size V.

We have also examined the distribution of the lattice conformal mode $P\left(\phi\right)$ as a function of c_M (Figs. 2 and 3). We see that $P\left(\phi\right)$ develops a broad tail for $c_M > 1$ indicating the increasing dominance of *spikes* with increasing central charge.

3. Analog 4D theory

Consider again the conformal decomposition $g = \overline{g}e^{2\sigma}$. As a first approximation we can consider fixing $\overline{g}$ in order to study the quantum dynamics of the conformal factor σ. This is expected to be a reasonable approach in the I.R where for example the divergences encountered in the graviton propagator stem from the spin zero sector. We can treat the I.R dynamics of σ

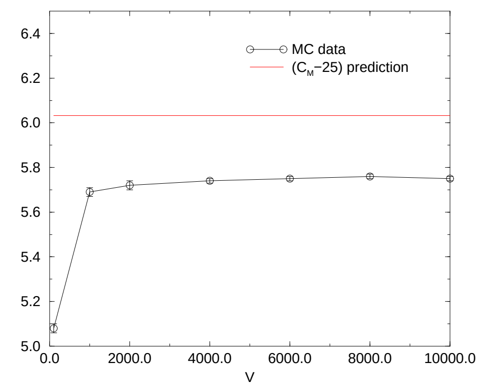

Figure 1. Mean Liouville action versus lattice size

exactly by analogy with the 2D case – from the form of the trace of the energy momentum tensor we can write down an effective action whose variation yields the anomaly [1]. In conformal coordinates this yields an effective action

$$S_{\text{eff}}\left(\sigma\right) = \frac{Q^2}{\left(4\pi\right)^2}\int\sqrt{\overline{g}}\left(\sigma\Delta_4\sigma + \frac{1}{2}\overline{M}\sigma\right) \tag{9}$$

$$\overline{M} = \left(\overline{G} - \frac{2}{3}\overline{\Delta R}\right) \tag{10}$$

Here, Δ_4 is the unique, fourth-order derivative operator which transforms covariantly under conformal transformations and the parameter Q^2 is the analog of the central charge in 2D – depending on the numbers of massless scalar, fermion and vector degrees of freedom.

$$Q^2 = \frac{1}{180}\left(N_S + \frac{11}{2}N_{WF} + 62N_V - 28\right) \tag{11}$$

Th quantity $\overline{G}$ is the Gauss-Bonnet density for the background metric. It is possible to absorb the contributions of the transverse gravitons into an additive renormalization of Q^2 - $Q^2 \rightarrow Q^2 + Q^2_{\text{graviton}}$. Again, *spike* solutions are possible and one expects two phases; a smooth Liouville-like phase and a branched polymer phase caused by a

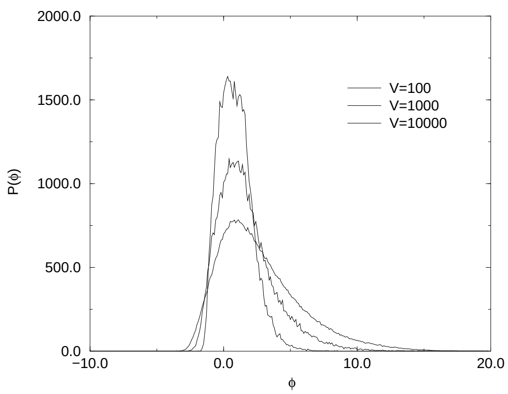

Figure 2. Distribution of ϕ for $c_M = 0$

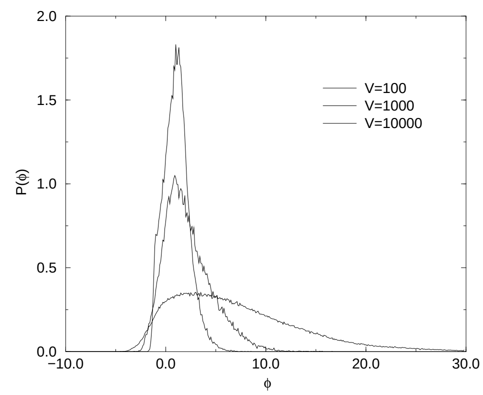

Figure 3. Distribution of ϕ for $c_M = 10$

condensation of spikes. Notice though that in 4D the addition of more matter fields *decreases* the influence of the spike-like configurations. Such branched polymer configurations have been seen in simulations of 4D DTs and have led to the speculation that these represent regularizations of this conformal factor gravity. If so, it should be possible to enter the Liouville-like phase by the addition of suitable massless fields – preliminary numerical evidence indeed hints at this [4,5]. If this were so the simulations could be used to both measure Q^2_{graviton} and to do nonperturbative studies of relevance to cosmology [6].

To investigate this further we propose to follow our 2D calculations and solve the analagous equation to eqn. 7

$$-\Delta\sigma + g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma = \frac{R}{6} - \frac{\overline{R}}{6}e^{-2\sigma} \qquad (12)$$

If this is done for an entire DT ensemble of 4-geometries we can ask whether the resulting distribution for the lattice conformal mode is again consistent with a simple gaussian action like eqn.9. This work is in progress.

REFERENCES

1. I. Antoniadis, P. Mazur and E. Mottola, Phys. Lett. B394 (1997) 49.
2. M. Cates, Euro. Phys. Lett.8 (1988) 719.
3. J. Ambjorn, Proceedings of Les Houches 1994, hep-lat/9412006.
4. S. Bilke, Z. Burda, A. Krzywicki, B. Petersson, J. Tabaczek and G. Thorleifsson, Phys. Lett. B418 (1998) 266.
5. H. Egawa, A. Fujitsu, S. Horata, N.Tsuda and T. Yukawa, hep-lat/9809028.
6. I. Antoniadis, P. Mazur and E. Mottola, hep-th/9808070.