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Reconstructing the Conformal Mode in Simplicial Gravity

Simon Catterall
Syracuse University

Emil Mottola
Los Alamos National Laboratory

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S. Catterall[a]and E. Mottola [b]

[a]Physics Department, Syracuse University,
Syracuse, NY 13244

[b]Theoretical Division T-8, Mail Stop B285,
Los Alamos National Laboratory, Los Alamos, NM 87545

We verify that summing 2D DT geometries correctly reproduces the Polyakov action for the conformal mode, including all ghost contributions, at large volumes. The Gaussian action is reproduced even for central charges greater than one lending strong support to the hypothesis that the space of all possible dyamical triangulations approximates well the space of physically distinct metrics independent of the precise nature of the matter coupling.

1. Quick Review of 2DQG

For 2DQG coupled to matter fields ϕ with central charge c_m the path integral is:

$$Z = \int \frac{DgD\phi}{\text{Vol}\,(\text{Diffs})} e^{-S(g,\phi)} \qquad (1)$$

It is important to include only physically distinct metrics. This can be accomplished by fixing the gauge $g = \overline{g}e^{2\sigma}$. This yields

$$Z \sim \int D\sigma e^{-S_L(\sigma)} \qquad (2)$$

The Liouville action $S_L(\sigma)$ is given by

$$S_L(\sigma) = \frac{25 - c_m}{24\pi} \int \sqrt{\overline{g}}\left(-\sigma\Box\sigma + \overline{R}\sigma\right) \qquad (3)$$

Equivalently the same result can be derived from the trace anomaly of massless fields in a curved background [1]

$$
\begin{aligned}
T &= \frac{(25 - c_m)}{24\pi}R \\
&= \frac{(25 - c_m)}{24\pi}e^{-2\sigma}\left(\overline{R} - 2\,\overline{\Box}\,\sigma\right) \\
&= \frac{1}{\sqrt{\overline{g}}}\frac{\delta}{\delta\sigma}S_L(\sigma)
\end{aligned}
$$

Thus the quantum effective action is the sum of this anomaly-induced action and the classical action.

2. Consequences of $S_L(\sigma)$

Notice that the quantum action contains non-trivial dynamics. Specifically it ensures that 2D quantum gravity has the following properties:

- Nontrivial scaling of correlation functions $\langle O_1 \ldots O_N \rangle \sim A^{p_1 + \cdots + p_2}$.

- Geometries are *fractal* eg $d_H = 4$ for pure 2D gravity.

- Baby Universe substructure ($\gamma < 0$).

3. Branched Polymers

For $c_m > 1$ we find that, although the action remains well-defined the scaling dimensions become *complex*. At this point a BKT-like argument due to Cates [2] indicates that *spike* configurations dominate where $\sigma_{\text{spike}} \sim -\log r$. Such configurations have a free energy which is sensitive to the U.V cut-off. It is presently unknown whether the dominance of these configurations indicates a complete breakdown of Liouville theory or is simply a signal that it is merely *incomplete* - perhaps the action should be augmented by more terms whose couplings must be tuned to approach a continuum limit.

4. Dynamical Triangulations

DTs furnish another approach to 2DQG - finite *simplicial* meshes are used to approximate con-

tinuum geometries. It is a fundamental postulate of this approach that summing over lattices generates the correct measure on the space of physically distinct metrics

$$Z = \sum_{T,\phi} e^{S(T,\phi)} \qquad (4)$$

where

$$S = \sum_{\langle ij \rangle \epsilon T} \phi_i \phi_j + \cdots \qquad (5)$$

The strongest evidence for this comes from the startling agreement of correlation functions computed using Liouville theory or via the DT approach [3].

5. Lattice conformal mode

Given the observed agreement between the correlaton functions we would like to demonstrate explicitly the equivalence of the two formalisms. We shall assume that every DT geometry can be thought of as approximating a continuum metric which can be conformally mapped to the round sphere with constant curvature $\overline{R}$. The conformal factor needed is given by

$$R = e^{-2\sigma} \left(\overline{R} - 2 \, \overline{\Box} \, \sigma \right) \qquad (6)$$

Since $e^{-2\sigma} \, \overline{\Box} = \Box$ this can be rewritten as a non-linear lattice equation:

$$2M_{ij}\sigma_j = \frac{2\pi}{3} (6 - q_i) - \frac{\overline{R}q_i}{3} A_\Delta e^{-2\sigma_i} \qquad (7)$$

where

$$M_{ij} = \frac{2}{\sqrt{3}} \left(q_i \delta_{ij} - C_{ij} \right) \qquad (8)$$

6. Gaussian distribution

For each DT triangulation generated in the Monte Carlo simulation we solve this equation. This yields a distribution of the lattice conformal mode. To check against Liouville we then decompose the lattice field on eigenmodes of $\overline{\Box}$. On the lattice we have

$$L_{ij} = \frac{3}{A_\Delta} \frac{e^{\sigma_i + \sigma_j}}{\sqrt{q_i q_j}} M_{ij} \qquad (9)$$

where

$$L_{ij} u_j^\ell = \lambda^\ell u_i^\ell \qquad (10)$$

The amplitude of each mode $\sigma^\ell = \sum_i \frac{q_i}{3} e^{-2\sigma_i} u_i^\ell \sigma_i$ should then be distributed according to

$$\exp - \frac{(25 - c_m)}{24\pi} \sum_l \lambda^\ell \left(\sigma^\ell \right)^2 \qquad (11)$$
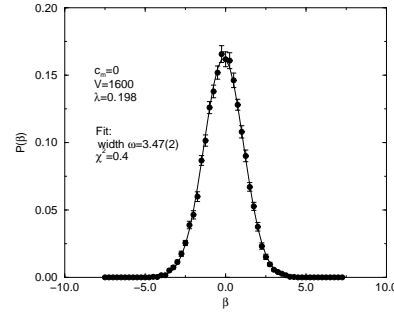
7. Results

Figure 1. Distribution of $l = 10$ mode with gaussian fit for $c_m = 1$ and $V = 1600$

First we rescale amplitudes $\sigma^\ell \rightarrow \sigma^\ell \sqrt{\lambda^\ell}$. Each mode ℓ should now be distributed with equal width depending only on the central charge. Gaussian fits are then performed to extract these widths ω_ℓ. Results for $c_m = 1$ and $c_m = 10$ are shown in figures 1 and 2. The widths of the fits agree well with Liouville theory. The mode dependence of the width is shown in fig.3 and fig.4 as a function of lattice volume V. The lattice eigenvalue $A_\Delta \lambda_\ell$ is plotted along the x-axis. In the continuum limit $A_\Delta \rightarrow 0$ and $V \rightarrow \infty$ we see good agreement with the Liouville prediction for all c_m (for a more complete discussion see [4])

Furthermore, the same arguments can be used to analyze the zero mode distribution

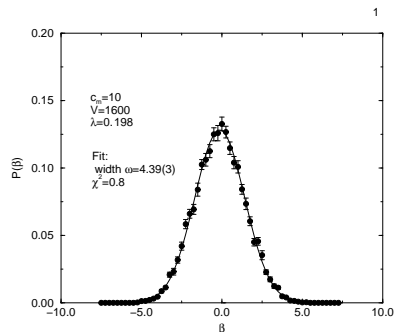$$P \left(\sigma^0 \right) \sim \exp - \frac{(25 - c_m)}{12\pi V} \sigma^0 \qquad (12)$$

Figure 2. Distribution of $l = 8$ mode with gaussian fit for $c_m = 10$ and $V = 1600$

Figure 4. Width versus lattice eigenvalue for $c_m = 10$

We have observed that the distribution of the zero mode follows this theoretical prediction for large argument (where the fixed area constraint plays no role) and allows for an independent measurement of the central charge. As for the nonzero modes this yields values that are statistically consistent with the continuum value.
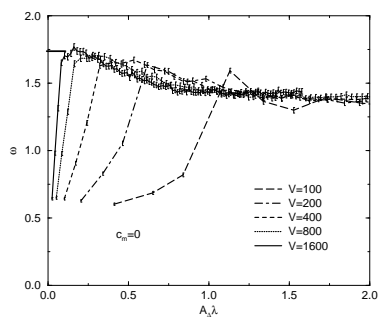

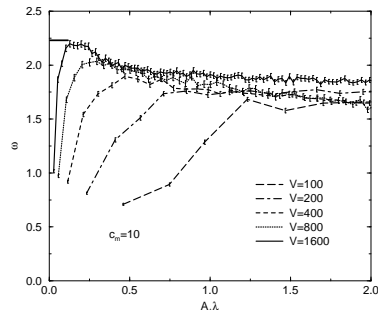Figure 3. Width versus lattice eigenvalue for $c_m = 1$

8. Conclusions

It is possible to recover the conformal mode in 2D simplicial QG. It is distributed according to Polyakov-Liouville action with the correct central charge including all ghost contributions. For $c_m < 1$ this amounts to anothing convincing test of the equivalance of DT approach to continuum aproaches to 2D quantum gravity. Our results for $c_m > 1$ would further support the conclusion that the DT measure is an appropriate measure for sampling the physical space of geometries even when there are truly propagating matter degrees of freedom.

REFERENCES

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2. M. Cates, Euro. Phys. Lett.8 (1988) 719.
3. J. Ambjorn, Proceedings of Les Houches 1994, hep-lat/9412006.
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