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A Two-Dimensional Lattice Model with Exact Supersymmetry

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A Two-Dimensional Lattice Model with Exact Supersymmetry

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Starting from a simple discrete model which exhibits a supersymmetric invariance we construct a local, interacting, two-dimensional Euclidean lattice theory which also admits an exact supersymmetry. This model is shown to correspond to the Wess-Zumino model with extended $N = 2$ supersymmetry in the continuum. We have performed dynamical fermion simulations to check the spectrum and supersymmetric Ward identities and find good agreement with theory.

1. Introduction

Supersymmetry is thought to be a crucial ingredient in any theory which attempts to unify the separate interactions contained in the standard model of particle physics. On rather general grounds we expect that supersymmetry must be broken non-perturbatively in any such theory. This has led to attempts to formulate such theories on lattices [2].

However, most lattice models break supersymmetry explicitly and lead generically to the appearance of relevant, SUSY violating interactions in the lattice effective action. The continuum limit will not then correspond to a supersymmetric theory without fine tuning.

In this talk we show that models exist which may be discretized in such a way as to preserve a subset of the continuum SUSY transformations exactly. Such models may evade this fine tuning problem.

2. Simple Model

Consider a set of discrete, real, commuting fields x_i and real, anticommuting fields $\psi_i, \overline{\psi}_i$ where $i = 1 \ldots K$ with action

$$S = \frac{1}{2} N_i^2(x) + \overline{\psi}_i \frac{\partial N_i}{\partial x_j} \psi_j$$

which admits a 'supersymmetry'

$$\delta x_i \quad = \quad \psi_i \xi$$

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$$\delta \overline{\psi}_i \quad = \quad N_i \xi$$
$$\delta \psi_i \quad = \quad 0$$

Corresponding to this symmetry the quantum theory exhibits Ward identities eg.

$$\langle \psi_i \overline{\psi}_j \rangle + \langle \overline{N}_i x_j \rangle = 0$$

If we choose the fields $x, \psi, \overline{\psi}$ to lie on a spatial lattice equipped with periodic boundary conditions and take the fermion matrix $M_{ij} = \frac{\partial N_i}{\partial x_j}$ to be of the form

$$M_{ij} = D_{ij}^S + P_{ij}''(x)$$

we recover a Euclidean lattice version of SUSY QM

$$S = \frac{1}{2}(D_{ij}^S x_j + P_i')^2 + \overline{\psi}_i(D_{ij}^S + P_{ij}'')\psi_j$$

Notice that if we include a Wilson term in P'' we can eliminate double modes in both fermionic and bosonic sectors. Notice also that the lattice action contains a term which behaves as a total derivative in the naive continuum limit. However, on the lattice it is non-zero for an interacting theory and its inclusion is necessary if the theory is to possess an exact SUSY invariance.

3. Mean Action

For $P' = mx + gx^Q$ we can show using a simple scaling argument and a supersymmetric Ward identity that the mean action $< S >= K$, the number of degrees of freedom, which we recognize as a Euclidean analog of the vanishing of the

vacuum energy $E_{\text{vac}} = 0$ in a supersymmetric theory. Notice also that S is approximately invariant under 2nd SUSY

$$\begin{aligned} \delta' x_i &= \overline{\psi}_i \xi \\ \delta' \psi_i &= (D_{ij} x_j - P'_i) \xi \\ \delta' \overline{\psi}_i &= 0 \end{aligned}$$

where

$$\delta' S = \delta' \left(2 D_{ij}^S x_j P'_i \right)$$

Since QM is a finite theory we hence expect the continuum model to have the full $N = 2$ SUSY. Fig 1 shows a plot of the massgap in the model where $P' = mx + gx^3$ for $m = 10$ and $g = 100$, as a function of the lattice spacing which clearly exhibits the degeneracy between bosonic and fermionic degrees of freedom at finite lattice spacing.

Figure 1. Massgaps vs a at $m = 10$, $g = 100$
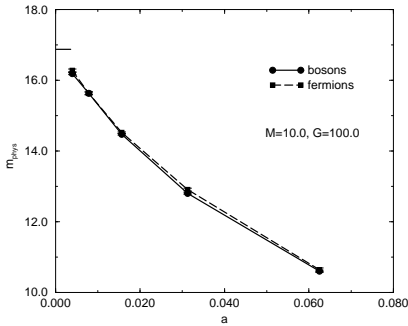
4. Wess Zumino Model

Imagine promoting the indices $i \to (i, \alpha)$ where i labels the spacetime point and $\alpha = 1, 2$ a spinor component for a theory in two dimensions. Notice immediately that the target theory will contain two real scalars. We take a fermion matrix of the form

$$M_{ij}^{\alpha\beta} = \gamma_{\alpha\beta}^{\mu} D_{ij}^{\mu} + A_{ij} \delta_{\alpha\beta} + B_{ij} i \gamma_{\alpha\beta}^5$$

We find that this matrix can only be obtained by differentiating some vector field if the bosons possess a complex structure $\phi_i = x_i^1 + i x_i^2$ and

$$S = \frac{1}{2} \overline{\eta}^{(1)} \eta^{(1)} + \overline{\psi} M \psi$$

where

$$\eta_i^{(1)} = D_z \phi_i + W'_i(\phi)$$

and $W(\phi)$ is an arbitrary analytic function.

We can also write down three other complex fields which yield the same continuum bosonic action

$$\begin{aligned} \eta^{(2)} &= D_z \overline{\phi} - W'(\phi) \\ \eta^{(3)} &= D_z \phi - W'(\overline{\phi}) \\ \eta^{(4)} &= D_z \phi + W'(\overline{\phi}) \end{aligned}$$

All these lead to same value for $\det M$ and generate approximate symmetries on the lattice $\delta S \sim g a^2$ eg.

$$\begin{aligned} \delta_2 x_i &= i \gamma_5^{\alpha\beta} \psi_i^{\beta} \xi \\ \delta_2 \overline{\psi}_i^{\alpha} &= i \gamma_5^{\alpha\beta} \overline{N}_i^{\beta} \xi \\ \delta_2 \psi_i^{\alpha} &= 0 \end{aligned}$$

with

$$\eta_i^{(2)} = (\overline{N}_i^1 + i \overline{N}_i^2)$$

We can also derive Ward identities corresponding to all these exact and approximate supersymmetries.

5. Simulations

We have checked these conclusions by explicit dynamical fermion simulations for the case $W = m\phi + g\phi^2$ with $m = 10$ and $g = 0, 3$. We employed a HMC algorithm [3] in which the fermions were replaced by (real) pseudofermions χ with action

$$S_F = \chi (M^T M)^{-1} \chi$$

We obtained substantial improvement by use of Fourier acceleration techniques described in [4,5]. Using these ideas we amassed data for lattices from $L = 4$ through $L = 32$ (1 million and 2×10^4 trajectories respectively) both for $g = 0$ and $g = 3$ [6]. Table 1 illustrates the mean action obtained

Table 1
Mean action vs L

$< S >$	L
4	31.93(6)
8	127.97(7)
16	512.0(3)
32	2046(3)

at $g = 3$ as a function of lattice size By fitting the zero-momentum correlation functions we obtained the following massgaps as a function of lattice spacing $a = 1/L$. Fig 2 shows a plot show-

Figure 2. 1st Ward identity

Table 2
Boson and fermion massgaps vs L

Lattice size	m_B	m_F
4	5.09(2)	4.95(8)
8	6.52(2)	6.44(5)
16	7.76(4)	7.75(6)
32	8.29(19)	8.33(30)

ing the bosonic and fermionic contributions to the Ward identity in eqn.1. If the Ward identity is satisfied these two curves should sum to zero as the figure confirms.
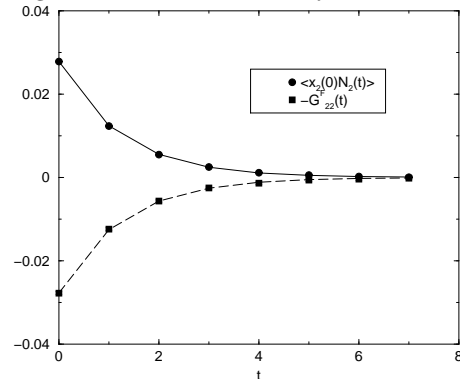
6. Conclusion and Discussion

We have shown that it is possible to write down a local lattice action for the 2D Wess Zumino model with extended ($N = 2$) supersymmetry which admits an exact, local supersymmetry. The presence of an exact SUSY-like symmetry together with the finiteness of the theory then guarantees that the full $N = 2$ supersymmetry will be regained in the continuum limit without fine tuning. We have tested these ideas by explicit dynamical fermion simulations.

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