

Syracuse University

SURFACE at Syracuse University

Center for Policy Research

Maxwell School of Citizenship and Public
Affairs

11-1996

Where Do the Rents Go? Land Ownership in an Urban Model

Stephen Ross

University of Connecticut

John Yinger

Syracuse University

Follow this and additional works at: <https://surface.syr.edu/cpr>



Part of the [Economic Policy Commons](#), [Economics Commons](#), and the [Public Policy Commons](#)

Recommended Citation

Ross, Stephen and Yinger, John, "Where Do the Rents Go? Land Ownership in an Urban Model" (1996).

Center for Policy Research. 451.

<https://surface.syr.edu/cpr/451>

This Working Paper is brought to you for free and open access by the Maxwell School of Citizenship and Public Affairs at SURFACE at Syracuse University. It has been accepted for inclusion in Center for Policy Research by an authorized administrator of SURFACE at Syracuse University. For more information, please contact surface@syr.edu.

**Metropolitan Studies Program Series
Occasional Paper No. 182**

**WHERE DO THE RENTS GO?
LAND OWNERSHIP IN AN URBAN MODEL**

Stephen Ross and John Yinger

**Center for Policy Research
Maxwell School of Citizenship and Public Affairs
Syracuse University
Syracuse, New York 13244-1090**

November 1996

\$5.00

Stephen Ross is an Assistant Professor of Economics, University of Connecticut, and John Yinger is a Professor of Economics and Public Administration, and Senior Research Associate, Center for Policy Research, The Maxwell School, Syracuse University.

CENTER FOR POLICY RESEARCH - SPRING 1995

Timothy M. Smeeding
Professor of Economics & Public Administration

Director

Richard V. Burkhauser
Professor of Economics

Margaret M. Austin

John Yinger
Professor of Public Administration
& Economics

Associate Director,
Aging Studies Program

Associate Director,
Budget and Administration

Associate Director,
Metropolitan Studies Program

SENIOR RESEARCH ASSOCIATES

Amy Crews Economics
William Duncombe Public Administration
Thomas Dunn Economics
James Follain Economics
Vernon Greene Public Administration
David Greytak Economics
Barbara Grosh Public Administration
Douglas Holtz-Eakin Economics
Bernard Jump Public Administration

Duke Kao Economics
Marcia Meyers Public Administration
Jerry Miner Economics
Jan Ondrich Economics
John Palmer Public Administration
Michael Wasylenko Economics
Douglas Wolf Public Administration
Assata Zerai Sociology

RESEARCH ASSOCIATES

Kenneth Couch Economics
Mary Daly Economics
James McNally Economics

GRADUATE ASSOCIATES

Stacey Amorosi Economics
Debra Bailey Economics
Peter Bine Public Administration
Lloyd Blanchard Public Administration
Barbara Butrica Economics
Carla Craddock Public Administration
Robert Dunskey Economics
Carol Jenkins Public Administration
Sarah Laditka Public Administration
Anna Lukemeyer Public Administration
Michael McLeod Public Administration
Cole McMahon Public Administration

Inge O'Connor Economics
Teri Pace Public Administration
John Phillips Economics
John Poupore Economics
Stephen Rhody Economics
John Ruggiero Economics
Alex Striker Economics
Ann Volpel Public Administration
Robert Weathers Economics
David Wittenburg Economics
Wilson Wong Public Administration

STAFF

Barbara Butrica Data Manager
Martha W. Bonney Publications and
Events Coordinator
Karin D'Agostino Computer Consultant
Jennifer Gantt Librarian/Office Coordinator
Esther Gray Administrative Secretary
Gina Husak Secretary to the Director
Detlef Jurkat Translator
Nicole Ladewig Administrative Assistant,
Luxembourg Income Study
Mary Santy Receptionist
Ann Wicks Administrative Secretary
Jodi Woodson Secretary
Sheng Zhu Manager - Computing Services

Abstract

Despite the predominance of homeownership, general equilibrium urban models assume that rents flow to absentee landlords or are redistributed to residents. This paper explores open urban models with more realistic forms of land ownership, namely ownership of shares in a land corporation and individual home ownership. These models, which produce capital gains and losses for residents, can yield different comparative static results than previous open models, with post-shock bid functions that are flatter at the periphery or steeper at the center. Moreover, shocks to these models cannot in general be analyzed without knowing the characteristics of all the urban areas in the system.

WHERE DO THE RENTS GO? LAND OWNERSHIP IN AN URBAN MODEL

Perhaps the most extreme, and certainly the least explored, assumption in urban models is that all land is owned by absentee landlords. Like general equilibrium models of other types, models of urban spatial structure must make simplifying assumptions. In fact, however, almost two-thirds of American households own their own homes and, in some cities, many renters live in owner-occupied buildings. A few scholars have explored urban models with an alternative, but equally unrealistic, assumption about land ownership, namely that rents are shared equally by all households. This paper provides a more general treatment of land ownership. In particular, it catalogs possible assumptions about land ownership and solves urban models with more realistic assumptions. These assumptions introduce the possibility that shocks to an urban equilibrium can create capital gains and losses for residents.

Existing Assumptions about Land Ownership

Following the assumptions in the pioneering work of Mills (1967) and Muth (1969), most urban models assume that land and housing rents are paid to absentee landlords and disappear from the urban system. This assumption is employed both for closed models, which have a fixed urban population, and open models, which allow unrestricted migration between urban areas. Wheaton (1974) and Brueckner (1986) provide comparative static analysis of both closed and open models with general functional forms and the absentee-landlord assumption.

This standard approach to land ownership has been criticized by several scholars because part of the system income escapes to agents who have no impact on the model's solution. As an alternative, Oron Pines and Sheshinski (1973) suggest and Solow (1973) and Robson and Scheffman (1979) implement closed urban models in which the total rents of an urban area are redistributed equally to all

residents. Wheaton (1977), Pines and Sadka (1986), and Sasaki (1987) solve closed models with redistributed rents and general functional forms. Fujita (1989) reviews these models and also introduces an open model with rents redistributed. In the process, he interprets rent redistribution as public ownership; a public enterprise contracts for land at the agricultural rental rate and redistributes excess rents to current residents.

A catalog of land ownership assumptions is needed to keep track of the possibilities. The first such catalog was provided by Pines and Sadka (1986), who identify open, semi-closed, and fully closed models. Their open model allows both people and rental income to leave the system, their semi-closed model allows income but not people to leave the system, and their fully closed model assumes redistributed rents and no mobility. Alternatively, Fujita (1989) categorizes models by distinguishing between mobility and land ownership assumptions. His catalog can be represented by a two-by-two table, in which the columns identify the two standard mobility assumptions (open and closed) and the rows identify two land ownership assumptions, namely absentee landlords and redistributed rents.

Alternative Land Ownership Assumptions

This paper builds on Fujita's catalog by expanding the number of land ownership assumptions, still in the context of open and closed models.

National Land Ownership

As discussed above, the absentee landlord assumption has often been criticized because rents escape to unseen agents. As Ross and Yinger (1995) point out, this concern can be avoided for an open model, because the absentee-landlord assumption is equivalent to national land ownership.

Redistributed rents can be considered nonlabor income, which is exogenous because a shock to one urban area has a negligible impact on total national rents.

Ross (forthcoming) notes that this approach does not work for a closed model. If a closed model is interpreted as applying to an isolated urban area, it is unreasonable to assume that non-mobile residents have a stake in land values at the national level. Alternatively, if a closed model is interpreted as applying to a large dominant urban area or as revealing the effect of events that occur simultaneously in all the areas in a system, national land rents are affected by the shock and are endogenous.

Land Corporations

Ross (forthcoming) also points out that rents can be redistributed through a private land corporation, as well as through public land ownership. This type of corporation could contract for the land and redistribute the excess rent equally to stockholders. Assuming that each resident of an urban area owns the same number of shares, this arrangement appears equivalent to redistributed rents. Indeed, a closed model with public ownership cannot be distinguished from one with stockholders because residents are not mobile.

In an open model, however, a corporation that contracts for land and redistributes rents to stockholders is not equivalent to Fujita's public enterprise. Any parameter shock to the urban equilibrium changes the value of urban land, and resident stockholders experience capital gains or losses on their stock. These gains and losses change their wealth relative to people in other urban areas who used to be identical to them. Moreover, after the shock, the stockholders may have an incentive to move out of the urban area. This open stockholder model, which has not appeared in the literature, introduces a key new element into an analysis of land ownership, namely the possibility

that different individuals have different stakes in the value of urban land, depending on when they moved in.

Home Ownership

The logical extension of the stockholding concept is to allow different individuals to have different stakes in the value of urban land that depend on where they live as well as on when they move in. Mun and Sasaki (1992) provide the only existing model of this type, a closed model of a linear city in which households own a given property for a fixed number of years.¹ At the end of this ownership period, all property is sold to a new cohort of owners. Households solve, with perfect foresight, a two-period problem in which their utility depends on land and consumption goods purchased in period one and on second-period wealth, including the sales price of their land. Mun and Sasaki anchor their model by assuming that the urban area contains a fixed number of identical individuals and that the land is originally owned by absentee landlords, who act as profit-maximizing developers with perfect foresight in deciding whether and when to sell land to households.

Mun and Sasaki use this model to examine the impacts of a transportation improvement on the urban equilibrium. They examine three possible cases. First, the resale of housing occurs prior to the announcement of the transportation improvement. In this case, the urban equilibrium is unaffected because households do not know that land values will change when the houses change hands. Second, the resale occurs after the announcement. In this case, household bids for housing are affected by the benefits of higher resale values at the end of the first period and possibly by lower transportation costs during the latter part of the first period. Third, the resale occurs after the actual implementation of the improvement. This case yields standard bids because no change in land value is expected at the time of the resale.

This model represents an important step forward but also places severe restrictions on the role of land ownership. It assumes that at the time of resale, the urban area is populated by identical households with identical utility levels. In fact, however, transportation improvements lead to capital gains that vary by location and therefore create a myriad of household classes, which differ from the class that originally lived in the area. This approach only makes sense, therefore, when households only realize capital gains at retirement (that is, at the time of resale) and then move to retirement communities where they no longer compete with new households for urban land. In effect, the Mun and Sasaki approach allows for individualized capital gains and losses but then does not allow these gains and losses to affect the urban equilibrium.

A Complete Catalog

Our catalog is created by listing all the possible assumptions about land ownership for both open and closed models. Five land ownership assumptions have been discussed here: ignored absentee landlords, rents redistributed to residents, national land ownership, ownership of shares in an urban corporation, and land ownership combined with owner-occupancy. The resulting catalog is presented in Table 1.

Standard urban models make the first assumption, namely absentee landlords, and therefore fall into the two cases in row 1 of Table 1. The rents-redistributed-to-residents assumption, row 2, has been considered extensively in the case of closed models and is solved for an open model by Fujita (1989). Because migration across urban areas is ruled out, a closed model with rents redistributed to residents is equivalent to a closed model with resident ownership of shares in an urban corporation, row 4. These two assumptions differ for open models, however, because ownership leads to capital gains and losses, not simply to changes in annual income. National land ownership, row 3, is equivalent to the first assumption in an open model (with the addition of a parameter for nonlabor income) but makes

no sense in a closed model. Finally, home ownership, row 5, in which each household retains the rights to changes in the value of its own house, has been considered in only one, very specialized case, namely the two-period, linear city model by Mun and Sasaki (1992).

The two key gaps in this catalog appear in the open-model column. In particular, urban models in which households have a stake in urban land have been implemented only for closed models. This paper investigates two types of open models in which such a stake appears. The first model treats current owners as stockholders in an urban corporation; the second allows each resident to be the unique owner of the land they occupy. These two models share a common feature, which is new to the literature, namely the possibility that unexpected shocks to the urban economy generate capital gains that vary by household—and hence produce new household classes.

These new models fill in the open-model column but leave the last row of the closed-model column only partially solved. We leave it to future research to provide a closed model that explicitly recognizes the household classes created by individualized capital gains and losses. Existing urban models with an exogenous income distribution (Beckman 1969, and Montesanto 1972) may be helpful for such an exercise.² In fact, we develop a variant of the Beckman/Montesanto approach, based on an exogenous distribution of nonlabor income, to help us analyze the last cell in the open-model column.

The conclusion from our new analysis is striking: Although the bids of new migrants are unaffected by land ownership, open models with either urban shares or home ownership can yield fundamentally different comparative static results than standard open urban models. In fact, the standard results obtain only in a special case. Moreover, we show that comparative static analysis of a shock to one urban area cannot in general be carried out without knowing the characteristics of all the other urban areas in the relevant system.

An Open, Stockholder's Model

Consider a monocentric, open urban area with agricultural rent \bar{R} , system-wide utility level \bar{V} , constant commuting costs per mile t , and urban wage rate W . Identical urban residents then solve the following optimization problem:

$$\text{Max } U \quad \text{subject to } Y \geq W + Z \geq R(u)L \geq tu, \quad (1)$$

where Z is a composite good with a price of one, L is land consumption, u is distance from a spaceless central business district (CBD), $R(u)$ is land rent, and Y is nonlabor income.

The dual of the resident's problem is to

$$\text{Max } R(u) \quad \text{subject to } \frac{Y \geq W + Z \geq tu}{L} \quad U(Z, L) \leq \bar{V}. \quad (2)$$

Now assuming that t is the sum of operating cost, t_o , and time cost, $t_w W$, the following conditions can be derived by applying the envelope theorem to (2) (Sasaki 1987):

$$\frac{\partial R}{\partial u} = -\frac{\partial t}{\partial u}, \quad \frac{\partial R}{\partial Y} = \frac{1}{L(u)}, \quad \frac{\partial R}{\partial W} = \frac{1 + t_w u}{L(u)}, \quad \frac{\partial R}{\partial \bar{V}} = -\frac{1}{U_Z L}. \quad (3)$$

The equilibrium conditions for this urban area are

$$\begin{aligned} \bar{V} &= U(Z, L) \quad \text{subject to } Y \geq W + Z \geq R(u)L \geq tu, \quad L(u) \leq \bar{L} \\ R(\bar{u} | Y \geq W + Z \geq t\bar{u}, \bar{V}) &= \bar{R} \\ N &= \int_0^{\bar{u}} \frac{2B}{L(u)} du, \end{aligned} \quad (4)$$

where \bar{u} is the outer edge of the area and N is the area's population. Wheaton (1974) presents a comparative static analysis of this model.

Stockholding Households

Initially, each household is required to purchase at market value the same number of shares in a corporation that owns and rents a large tract of land with radius A , which encompasses the entire urban area. The value of the corporation is the total discounted stream of future rents generated by the land. Under the assumption that the area is not expected to undergo any changes in the future, a household is required to make a payment equal to

$$S = \frac{2B}{N} \left(\int_0^{\bar{u}} R(u) du \& \bar{u} \bar{R} \right) \quad (5)$$

each period in order to pay for its shares in the corporation

If the area does not experience any shocks, these payments are exactly canceled by the annual payments generated by the corporation, Y , which constitute each resident's exogenous nonlabor income. In this case, ownership has no net effect on income or housing consumption, and the equilibrium conditions are identical to equation (4). The absentee landlords simply have been replaced with initial landowners who sold their land to residents through shares of a corporation. The national ownership interpretation could also be applied with Y defined as each household's exogenous share of total national land rents.

However, residents in this model have a stake in the value of urban land. If an exogenous shock affects the urban area in which they reside and own stock, residents experience a capital gain or loss. Specifically,

$$)Y = \frac{2B}{N} \left[\left(\int_0^{\bar{u}} R(u) du \& \bar{u} \bar{R} \right)_{s2} \& \left(\int_0^{\bar{u}} R(u) du \& \bar{u} \bar{R} \right)_{s1} \right], \quad (6)$$

where the subscripts $s1$ and $s2$ indicate that the expression is evaluated at the pre- (1) or post- (2) shock equilibrium.

Exogenous Increase in W

First, consider the case of an increase in W to W_N . This shock results in an increase in the nonlabor income of the people residing in the urban area where this shock occurs; in other words, the “original” residents receive a capital gain on the value of their stock. A lower bound on the increase in Y can be established by assuming that the area is now inhabited entirely by people who used to live in other areas. This is a lower bound because the original residents will remain in the area after the shock only if they outbid people who have the characteristics they had before the shock. Thus from equation (6),

$$\Delta Y \geq \frac{2B}{N} \left[\int_{\bar{u}_0}^{\bar{u}_N} R(u | Y, W_N, t, u, \bar{V}) du + \int_{\bar{u}_0}^{\bar{u}} R(u | Y, W, t, u, \bar{V}) du \right] + \left(\bar{u}_N - \bar{u} \right) \bar{R} > 0 \quad (7)$$

where \bar{u}_N is the outer edge of the area after the wage shock.³ Note that N does not change, because we are evaluating the capital gain per original resident, and that, by (3) and (4), both $R(u)$ and \bar{u} increase with W .⁴ It follows that the wage shock increases redistributed rents, that is, that the change in Y is positive.

The original residents now differ from the residents of other cities, and they may no longer be indifferent between different urban areas or between different locations within an urban area. To find out where the original residents live, we must first determine what happens to the slope of their bid-rent function. According to the well-known theorem (see, for example, O’Sullivan 1993, p. 232), the initial positive shock to W flattens the bid-rent function if the income elasticity of demand for housing, η , is greater than the income elasticity of t , per-mile commuting costs. This theorem

determines what people from other areas would bid for land in the urban area where the shock to W occurs.

The bid-rent function of the original residents is affected not only by the increase in W but also by the resulting increase in Y . Differentiating the first condition in (3) by Y yields

$$\frac{M}{MY} \frac{MR}{Mu} + \frac{t}{L(u)^2} \frac{ML(u)}{MY} + \frac{t}{L(u)Y} > 0 \quad (8)$$

Thus an increase in Y always increases the slope of the bid function; after the shock, the bid function of the original residents is flatter than the bid function of other potential residents at any point where these two bid functions cross. This result differs from the standard result, because it involves nonlabor income, which, unlike labor income, has no impact on the time cost of travel and hence no impact on t .

In equilibrium, groups with flatter bid functions will live farther from the CBD. In this case, however, we cannot simply place the original residents on the outskirts of their original urban area. Unlike in a traditional open model, the original residents' new locations and welfare level can only be determined using information on the entire system of urban areas. This point is explained in Figure 1 and 2. Figure 1 shows, as solid lines, the initial bid function, the bid function after the shock in W (which is the new bid function in this area for people from other areas), and the original residents' bid functions after the boost in their nonlabor income with their utility held at the initial level, \bar{V} . This figure also shows, as dashed lines, several bid functions with the higher wage and capital gain but at higher levels of utility. These dashed lines show that some of the original residents could win the competition for land at the urban fringe at a higher utility level than \bar{V} .

Figure 2 illustrates several alternative areas to which the original residents might move. These areas all have the initial wage, W , but different values of \bar{R} . The middle value of \bar{R} , \bar{R}_2 , is

assumed to be the initial one in area that receives a shock. The original residents will move to the set of urban areas that allow them to be on the lowest possible bid function, that is, the one with the highest utility. If there exist many areas with the lowest value of \bar{R}_1 , then the original residents will move to these areas because that is where they can outbid other residents with the lowest bids. If all areas have the same value of \bar{R} as their original area, then they will distribute themselves evenly across all areas. With M areas, $1/M$ of the original residents will live in each area, including the original one. Finally, if all other areas have the highest value of \bar{R} , namely \bar{R}_3 , the original residents may not move to other areas at all.

Figure 2 illustrates, using a shock to W and variation in \bar{R} , a more general principle that applies to all the parameters of an urban model. Residents of an urban area who receive a capital gain because of their ownership in an urban land corporation will have flatter bid functions than people from other areas and will gravitate to those urban areas where they can win the competition for land at the lowest possible bids (and still have enough land to go around).

This principle can be formally derived from the indirect utility function, which is given by (4) with the demand function for land replacing $L(u)$. According to equation (8), a capital gain raises bids at all locations, holding utility constant, and, as shown above flattens the bid function. Thus the question is how much utility can rise in a given area to bring the original residents' bids at the outer edge of the urban area back down to the level before their capital gain (or, equivalently, to the level of the people who did not experience the gain). The original residents obviously will first move to the urban area(s) where this utility gain is maximized. The change in utility required to hold bids constant when households experience a capital gain can be found by totally differentiating this indirect utility function with respect to \bar{V} and Y and solving for $d\bar{V}/dY =$

$U_Z(MZ/MY) \% U_L(ML/MY)$. The original residents will move to the area(s) where this expression is

largest. Consider the Cobb-Douglas case in which $\bar{V} = k(Y + W - tu)/R(u)^\alpha$, where α is the coefficient of land in the utility function and k is a function of α . This expression implies that at \bar{u} , $d\bar{V}/dY = k/R(\bar{u})^\alpha = k/\bar{R}$ (by the middle condition in (4)). Thus the utility gain consistent with holding $R(\bar{u})$ constant when Y drops is greatest when \bar{R} is smallest and the household can buy the most land with its capital gain.

This analysis indicates where the original residents will first move, but it does not reveal their final change in utility. After all, they must compete with each other for land in the most desirable areas (identified by this analysis) and therefore will end up with a lower utility than this analysis implies, except in the extreme case in which there exists one urban area for each of the people who experience a capital gain.

Exogenous Decrease in W

This framework also allows us to analyze shocks that lead to a capital loss. Consider a decrease in W in one urban area. Following our earlier analysis, this decrease initially has an ambiguous impact on the slope of the bid function in that area, but then leads to a capital loss for the original residents, which unambiguously steepens their bid function. A steeper bid function implies that they will outbid other people near a CBD. The question is: To which areas will this new class of people go?

According to equation (3), a capital loss lowers bids at all locations, holding utility constant. Thus the question is how much utility must drop in a given area to bring the original residents' bids next to the CBD back up to the level before their capital loss (or equivalently, to the level of people who did not experience the capital loss). These original residents obviously will move to the urban area where this required utility drop is minimized.

In this case, the original residents will move to the area where $d\bar{V}/dY$ is smallest. In the Cobb-Douglas case, $R(0) = [k(Y + W)/\bar{V}]^{1/\alpha}$ and $d\bar{V}/dY = k/R(0)^\alpha = \bar{V}/(Y + W)$. Thus the utility loss required to hold $R(0)$ constant when Y drops depends inversely on $R(0)$ and hence on income, $(Y+W)$. It follows that the original residents will have the smallest utility loss if they will move to the areas with the highest income and hence the highest $R(0)$, where the real value of their capital loss is minimized. These are, of course, the largest areas. As before, if the system consists of M identical areas, $1/M$ of the original residents will move to each area—and cluster around the CBD.

Conclusion

These results fundamentally alter a comparative static analysis of an open urban model. In a standard open model with absentee landlords (as in Wheaton 1974, or Brueckner 1987), a positive shock to W , for example, has well defined impacts, as illustrated by the bid function in Figure 1 that involves only a change in W . With a land corporation, however, this shock could lead to the standard result (when all the original residents move away), to an urban area with a mix of the original residents and residents who move in from other areas (with the original residents and their flatter bid function in the suburbs), or to a much flatter bid function than the standard result implies (when none of the original residents move away).⁵ Although one can conclude that the now-richer original residents will live near the urban edge wherever they end up, it is not possible to determine which of these cases applies without knowing the characteristics of other urban areas. The standard open-model result only arises if other areas have features, such as a low value of \bar{R} , that make them more attractive to the now-richer original residents than is their original area. This equivalence should not be surprising; in either case, all the rents generated by a shock flow out of the urban area.

Finally, the same ambiguity concerning comparative-static results appears when exogenous shocks lead to capital losses. In this case, the now-poorer original residents will cluster around the

CBD wherever they decide to live, but the areas they choose cannot be determined without knowing the characteristics of all areas in the system.

An Open Model with Owner-Occupants

Owner occupancy is difficult to model because it implies that a shock to an urban area creates a myriad of income classes all by itself. The change in land rent due a shock varies by distance from the CBD. As a result, urban residents living and owning land in one location may receive a different capital gain on their property than do residents living in a different location. Even residents who start out in the same income class may end up, after a shock, with different nonlabor income. To emphasize that people live on the land they own in this model, we describe them with the term “homeowner” instead of simply “landowner.”⁶

The most straightforward way to anticipate such a distribution of nonlabor income is to start with a model that already contains a continuum of income classes. Beckmann (1969) and Montesanto (1972) provide models of this type. However, their models focus on an exogenous distribution of income and do not readily extend to a shifting distribution of unearned income. Moreover, a fixed distribution of income is intractable in an open urban model, which involves equilibrium conditions across cities based on utility, not income.

Our approach to owner occupancy begins with the assumption that the system-wide utility level, \bar{V} , is a monotonically increasing function of nonlabor income, Y . This approach differs from that of Beckman and Montesanto because it allows the income distribution in an urban area to be endogenously determined by the locational and consumption choices of residents. With this new assumption, applying the envelope theorem to equation (2) yields the conditions in (3) except that now

$$\frac{\bar{M}R}{\bar{M}Y} = \frac{1}{L(u)} \quad \& \quad \frac{\bar{M}\bar{V}}{\bar{M}Y} = \frac{1}{U_z L(u)} \quad . \quad (9)$$

The income class residing at location u can be determined by maximizing $R(u)$ with respect to

Y . The relevant condition is:

$$\frac{\partial R}{\partial Y} = 0 \Leftrightarrow \frac{\partial \bar{V}}{\partial Y} = U_Z|_{L(u), Z(u)} \quad (10)$$

This condition indicates that, in equilibrium, the marginal utility of income for a resident who wins the bid for land at a given location must equal the increase in system utility that arises from increasing that resident's nonlabor income by one dollar.

To determine whether (10) maximizes $R(u)$, we examine the second-order condition:

$$\frac{\partial^2 R}{\partial Y^2} = \frac{1}{L^2} \left(1 \otimes \frac{\partial \bar{V}}{\partial Y} \frac{1}{U_Z} \right) \otimes \frac{\partial \bar{V}}{\partial Y} \frac{1}{MU_Z^2 L} \frac{MU_Z}{\partial Y} \otimes \frac{\partial^2 \bar{V}}{\partial Y^2} \frac{1}{U_Z L} \quad (11)$$

Evaluating this condition at the optimum yields

$$\frac{\partial^2 R}{\partial Y^2} = \frac{1}{U_Z L} \left(\frac{MU_Z}{\partial Y} \otimes \frac{\partial^2 \bar{V}}{\partial Y^2} \right) < 0 \quad (12)$$

The solution to (10) yields a maximum if and only if this expression is negative.

Equation (10) can be solved for a map between the nonlabor income of residents and location choice, \tilde{Y} . The manner in which residents sort by income can then be found by replacing Y with \tilde{Y} in equation (10) and differentiating the result with respect to u . The result is

$$\frac{d\tilde{Y}}{du} = \left(\frac{MU_Z}{\partial Y} \otimes \frac{\partial^2 \bar{V}}{\partial Y^2} \right)^{-1} \frac{MU_Z \partial R}{\partial Y \partial u} > 0 \quad (13)$$

The term in brackets must be negative by (12), because residents win the bid for housing where their bid is maximized. Moreover, U_Z falls with R because an increase in R causes people to substitute toward Z . Thus, higher-income residents live farther from the CBD.⁷

The minimum income in the urban area, Y^{MIN} , is found by determining \tilde{Y} at the center of the area, that is, at $u = 0$. The maximum income, Y^{MAX} , is found by evaluating \tilde{Y} at the edge of the area, \bar{u} . In addition, \bar{u} is found from the equilibrium condition:

$$\bar{R} = R(\bar{u}), \tilde{Y}(\bar{u}) = W + t\bar{u}, \bar{V}[\tilde{Y}(\bar{u})] = \bar{A} \quad (14)$$

Comparative Statics without Home Ownership

A model of this type has not appeared in the literature, even without home ownership, so we begin by deriving some basic comparative static results with a distribution of nonlabor income and absentee landlords. We focus on the impact of a change in W . To begin, a change in W might affect $R(u)$ both directly, by altering the bids of the people who live at u , and indirectly, by changing who lives at u . Applying the envelope theorem to equation (2) indicates that

$$\frac{dR}{dW} = \frac{1}{L} \left[\frac{\partial R}{\partial W} + \frac{\partial R}{\partial \tilde{Y}} \left(1 + \frac{1}{U_Z} \frac{d\tilde{Y}}{dW} \right) \right] > 0 \quad (15)$$

According to (10), the term in parentheses equals zero, so a small change in W does not have an indirect impact on $R(u)$, and dR/dW must be positive.

The impact of a change in W on \tilde{Y} can be found by differentiating (10):

Note that the partial derivatives taken with respect to Y actually are taken with respect to total

$$\frac{d\tilde{Y}}{dW} = \left(\frac{\partial \tilde{Y}}{\partial Y} + \frac{\partial \tilde{Y}}{\partial W} \right) \left(\frac{\partial Y}{\partial W} + \frac{\partial Y}{\partial \tilde{Y}} \frac{d\tilde{Y}}{dW} \right) < 0 \quad (16)$$

resident spending power, $(Y + W)$, and the total derivative of spending power with respect to W is one and is therefore omitted from the expression. Two effects can be seen in (16). The increase in R caused by an increase in W lowers U_Z at any location and lowers the income of people who live there. However, the impact of an income increase on U_Z , which reflects the income expansion path of

residents' preferences, is ambiguous in sign. Consequently, the overall impact of W on \tilde{Y} could be either positive or negative.

Because the CBD boundary is fixed, the change in Y^{MIN} with respect to W equals the change in \tilde{Y} . The change in Y^{MAX} , however, depends on the change in both \tilde{Y} and \bar{u} . Differentiating equation (14) yields:

$$\frac{d\bar{u}}{dW} \cdot \left[\frac{MR}{M\bar{u}} \cdot \left(\frac{MR}{M\tilde{Y}} \cdot \frac{MR}{M\bar{V}} \cdot \frac{M\tilde{V}}{M\tilde{Y}} \right) \cdot \frac{M\tilde{Y}}{M\bar{u}} \right] \cdot \frac{MR}{M\tilde{Y}} \lessgtr 0 \quad . \quad (17)$$

As in the one-class case, absentee-landlord case, raising the wage has an ambiguous impact on \bar{u} .

Finally, differentiating Y^{MAX} with respect to \tilde{Y} and \bar{u} , we find that

$$\frac{dY^{MAX}}{dW} \cdot \frac{d\tilde{Y}}{dW} \cdot \frac{M\tilde{Y}}{M\bar{u}} \cdot \frac{d\bar{u}}{dW} \lessgtr 0 \quad . \quad (18)$$

Adding Homeowners

Now assume that each household purchases the land on which it resides. The value of its land is the present value of the future rental stream (imputed or paid) generated by its land. Under the assumption that the urban area will not change in the future, a purchasing household would contract to make payments equal to its rent each period. If the area does not experience any shocks, these obligations are exactly canceled by the stream of rents generated by the property. As a result, nonlabor income is exogenous, and each household has a nonlabor income of $\tilde{Y}(u)$, where u represents the location at which it currently resides—and owns property.

If an exogenous shock hits the urban area, however, the residents will experience a capital gain or loss. Specifically,

$$\hat{Y}(u) \cdot \tilde{Y}(u) \cdot [R(u|\mathbb{T}2) \cdot R(u|\mathbb{T}1)] \cdot L(u|\mathbb{T}1) \quad , \quad (19)$$

where τ_1 and τ_2 indicate that the expression is evaluated at the pre- (1) or post- (2) shock equilibrium, and $\hat{Y}(u)$ is the post-shock nonlabor income of a household located at u prior to the shock.

Exogenous Change in W

Now return to the case in which W increases to W_N in one urban area. This shock boosts the nonlabor income of a resident originally located at u from $\tilde{Y}(u)$ to $\hat{Y}(u)$. After this change, this resident is identical to the resident of another urban area whose nonlabor income equals $\hat{Y}(u)$ and to someone originally located at u_N in the same area so long as $\hat{Y}(u_N) = \hat{Y}(u)$. Moreover, the shock makes the original residents want to move, both because their bid function is altered by the shock to W and because they now have higher nonlabor income. These two effects correspond to the two post-shock curves in Figure 1, one with the wage effect only and one with the higher wage and the resulting capital gain. To be specific, a resident originally located at u will move to u^* defined by

$$\hat{Y}(u) = \tilde{Y}(u | W_N) \quad (20)$$

and rent or sell their property at market value to households with

$$Y = \tilde{Y}(u | W_N) \quad (21)$$

If there is not enough land available at u^* for all the original residents of u (plus any other original residents with the same value of u^*), some of these now-richer original residents will move to other areas; after all, this is an open model in which the utility of a class, defined by nonlabor income, is fixed. Similarly, if there is too much land at u^* for the original residents who want to live there after the wage shock (and resulting capital gains), members of their new income class will migrate into the shocked urban area.

These results lead to the surprising conclusion that, except at the edges of the urban area, the impact of a change in W on the urban equilibrium is the same regardless of whether land is owner

occupied or rents accrue to absentee landlords. With mobility and alternative urban areas, new residents enter or old residents exit in each income class so that $R(u)$, $L(u)$, and $\tilde{Y}(u)$ are determined by the comparative statics problem presented above. To put it another way, the analysis generally cannot distinguish between a household whose nonlabor income was exogenously set at some value and a household whose nonlabor income reaches the same value because it received a capital gain or loss.

At the edges of the income distribution (and hence at the edges of the area), however, income classes may disappear from the area or appear in the area for the first time. These are the classes created by adding capital gains to households at the top of, or giving capital losses to households at the bottom of, the pre-shock system income distribution. These new classes may not arise. Gains and losses in an area may simply shift people at the edges of an area's income distribution into classes that already existed elsewhere. In this case, home ownership does not alter the urban equilibrium. But if these new classes do arise, analysis of them is analogous to that of the original residents in the stockholder model. In some cases, therefore, the location of these classes and the shape of an area's rent function cannot be determined without knowing the characteristics of all the urban areas in the system.

This point is illustrated in Figure 3, which presents bid functions for four income classes, with Y_1 as the lowest nonlabor income and Y_4 as the highest. These bid functions are drawn with the post-shock wage, W_N . Now suppose that before a shock, the urban area contains income classes 1 to 3 and that no household in the system has an income as high as Y_4 . Suppose further that the positive shock to W lead to capital gains that shifted each household to the next higher income class. Households with original nonlabor income Y_1 find their nonlabor income boosted to Y_2 , and so on. After

the shock, therefore, the original residents of this area fall income classes 2 to 4. Under these circumstance, Figure 3 describes the post-shock equilibrium bid functions.

However, the people in class 1 will all be new residents, since none of the original residents remain in that class. The people in classes 2 through 3 will all be original residents of the area only if the space available for a class is less than or equal to the space needed to accommodate the number of original residents who were in the next lower class before the shock.⁸ Otherwise, some people from other areas will have to move in to fill up the area inhabited by that class.

None of the original residents was a member of class 4. This implies either that (a) the agricultural rental rate was so high in this area that no member of class 4 wanted to live there (as illustrated by \bar{R}_2 in Figure 3) or (b) that nobody in the system of cities had an income high enough to be in class 4. In the first case, the people who join class 4 because of their capital gain will not remain in this urban area but will spread themselves out to the other urban areas where people in class 4 currently live.⁹ In the second case, the people in the new class 4 will locate in the areas where they can achieve the highest utility. If other areas have a lower agricultural rental rate (as illustrated by \bar{R}_1 in Figure 3) then they will all move away. If other areas all have a high \bar{R} and their area has a low \bar{R} , then they will all stay. And if all areas are identical, they will spread themselves evenly across all areas and their bid function will have to be shifted to the point where they outbid other groups for the land that they require. Wherever these new classes locate, the equilibrium rent function will be flatter, and \bar{u} will be larger, than predicted by a model without home ownership.

A similar analysis applies to the case of a downward shock to W . Original residents in the highest income classes who drop down to lower classes because of capital losses are replaced by people from other areas with the same income the original residents used to have. The poorest original residents become even poorer because of their capital losses. If their new class already existed in

the system, they will go to the areas (which obviously do not include their original area) where their new class lives.¹⁰ If their new class is poorer than any other class in the system, they will locate in the areas with the highest wages, and hence the highest rents next to the CBD, which could include their original area. As in the stockholding model, locating in these areas minimizes the utility loss these classes experience and steepens the equilibrium rent function near the CBD wherever they live.

One implication of these results is that the comparative statics results with home ownership are the same as those with absentee landlords if and only if all the income classes created by a shock to an urban area already existed somewhere in the system. If some of these classes did not exist, one cannot determine the residential locations of the new classes (or the equilibrium rent functions and other features of the urban area experiencing the shock) without knowing the characteristics of all the areas in the system.¹¹ Thus comparative static results in models with home ownership can be quite different from those in models without it.

Moreover, one cannot avoid this complexity by assuming that all urban areas are alike; with identical areas, a shock to one area inevitably creates income classes that did not exist before at the top or bottom of the income distribution. In fact, no single assumption ensures that all comparative statics results are the same with absentee landlords and with homeowners. Assuming a relatively high \bar{R} in a given area, for example, implies that a wage increase in that area has the same impact regardless of which landownership assumption is employed. In contrast, the equivalence of these comparative statics results for a wage decrease requires the assumption that the area with the shock has a relatively low wage rate.

Finally, the distinction drawn here between pre- and post-shock income classes in the system of areas depends on the standard open model assumptions, which are quite strong with a continuum of income classes. Complete openness requires that the total population in any income class be large

relative to the population of that class in a single area. Unless all areas are the same or can be divided into a small number of types, it is possible that some income classes might be concentrated in just a few areas. If so, a shock to one of these areas could have a significant effect on the total number of people within a single income class, thereby altering both the utility of that class and the equilibrium outcomes in any area where that class is located. Under these circumstances, the total effect of a shock to one area cannot be found without specifying the entire system, even if no new income classes are created in the system.

Conclusions

With continuous utility classes and open model assumptions, the urban equilibrium for a utility class that contains some of the original residents both before and after a shock cannot be affected by the gains and losses associated with that shock. For people in this class, therefore, comparative statics results (for rents, populations, locations, and so on) are the same regardless of whether the model assumes absentee landlords or home ownership. The original residents may change utility classes, but the comparative statics results for a given class are the same. Moreover, comparative statics results are not affected by gains and losses for any utility class represented among the original residents before a shock, even if none of the original residents remain in that class after the shock, or for any class that existed elsewhere before the shock, even if some of the original residents enter that class only because of shock-induced gains or losses.

Nevertheless, comparative statics results for open urban models with home ownership are fundamentally different from those with absentee landlords whenever they generate new income classes at the top or bottom of the income distribution for the system of cities. Indeed, the location and utility level for these new classes cannot be determined without information on the characteristics of all the areas in the system. Moreover, similar complexities in a comparative statics analysis arise

whenever the change in an income class in an area that experiences a shock is large relative to the number of people in that class in the entire system. Thus, open models with home ownership can produce comparative statics results that are very different from those of traditional open models.

Conclusions

This paper tries to bring the most hidden assumption in urban models out into the open. The vast majority of general equilibrium urban models rely on the unrealistic assumption that all land is owned by absentee landlords so that the capital gains and losses associated with changes in rent disappear from the analysis. The only alternative assumptions in the literature, namely redistributed rents or national land ownership, are equally unrealistic. This paper reviews the one existing closed model with a special case of individual land ownership (Mun and Sasaki 1992) and develops open models both with ownership of shares in a local land corporation and with home ownership. We hope that future research will complete our catalog by developing a general closed model with home ownership or mixed models in which some urban residents are renters and others own the land.

Introducing a private land corporation or home ownership fundamentally alters the nature of an open urban model. Comparative statics results are not affected by this change in one special case, namely when no new income classes are created by a shock. However, urban shocks inevitably create a new highest or lowest income class in a stockholder model that begins with a single class, and the capital gains and losses generated by such shocks are likely to create new highest- or lowest-income classes in a model with a nonlabor income distribution for homeowners. Neither the post-shock locations of these new classes nor the characteristics of the urban areas where they live, which may include the area experiencing the shock, can be analyzed by previous models. In fact, the post-shock

equilibrium outcomes for these new classes and their new locations depend on the characteristics of all the urban areas in the system.

This result undermines a basic features of open models as they have been presented in previous literature, namely their ability to collapse the influence of all urban areas other than the one being analyzed down to a single parameter: the system utility level. Moreover, our analysis reveals that in a multi-class setting, the system utility level for a single income class, even one that existed before a shock, cannot summarize the influence of other urban areas without the strong assumption that the number of people placed into that class due to a shock in one area is small relative to the number of people in that class elsewhere.

Given that almost two-thirds of households in the United States own their own home and many landlords are themselves urban residents, it is ironic that most general equilibrium models of urban residential structure depend on the assumption that all residents are renters and all landlords live in the countryside (or at least in a different urban area). This article is offered as a step toward more realistic assumptions about land ownership.

Endnotes

1. Capozza and Helsley (1989) provide another model of land ownership, but they assume that residential lot size is exogenous and identical regardless of location.
2. Solving a closed model with homeownership appears to be a formidable task. The post-shock rents at a given location depend on the capital gains of the people who win the competition there, but everyone's capital gain depends on the increase in the rents at their original location.
3. If the land corporation only owned urban land, then the upper limit of the first integral should not have a prime. This lowers the capital gain for the original residents, but does not change any of our results.
4. People will not commute unless income net of commuting costs is positive, or $W - tu > 0$. Dividing by W yields $1 - t_w u > t_{ou}/W > 0$. Hence by (3), $mR/MW > 0$. This result can be derived for the general case of $t = t(W)$, so long as average cost, t/W , exceeds marginal cost, Mt/MW .
5. New residents may move into the center of the area after the wage shock, thereby creating a two-class area even if none of the original residents move away.
6. The term "homeowner" is appropriate in another sense; Brueckner 1987, shows that comparative static results from a land-only model carry over to a model with housing.
7. This result is unambiguous here because commuting costs are assumed not to depend on nonlabor income, Y .
8. If the space is less than the amount needed, then some of the original residents obviously will migrate to other areas.
9. An exception to this rule arises if the upward shock to W makes the area where the shock occurs desirable to class 5 even though it was not desirable before the shock. This outcome is possible because the shock raises the class-5 bid function—and could raise it so much that the region where it is above other bid functions moves from below \bar{R} to above \bar{R} . In this case some or all of the new members of class 5 may remain in the area, even though no members of class 5 lived there before the shock.
10. Since a drop in W to below the level in some other urban areas can, in principle, drive the lowest income class away in models without home ownership, it is theoretically possible that nonresident members of a given low-income class that existed in an area before a negative wage shock will not want to move in after the wage shock has taken place. This case can be treated using bid-functions like those in Figure 3, which are evaluated at W_N , not W . To consider this case, the beginning of the preceding sentence in the text should be amended to begin: "If their new class already existed in the system but is not attracted to their original area at the new wage rate,..."

11. If a relatively large number of people are in classes that did not exist before anywhere in the urban area, they might even crowd into the areas inhabited by groups that did exist before and, because of this competition, alter the utility levels of these groups.

References

- Brueckner, J. 1987. "The Structure of Urban Equilibria: A Unified Treatment of the Muth Mills Model." In E.S. Mills (ed.), *Handbook of Regional and Urban Economics, Vol 2, Urban Economics*. Amsterdam: North Holland, pp. 821-846.
- Capozza, D. and R. Helsley. 1989. "The Fundamentals of Land Prices and Urban Growth," *Journal of Urban Economics*, 26: 295-306.
- Fujita, M. 1989. *Urban Economic Theory*. Cambridge, England: Cambridge University Press.
- Mills, E.S. 1967. "An Aggregative Model of Resource Allocation in a Metropolitan Area," *American Economic Review*, 57: 197-210.
- Mun, S. and K. Sasaki. 1992. "Effect of Urban Transportation System Change on Land Prices in the Setting of Owner-Occupied Residence," *Journal of Urban Economics*, 32: 351-366.
- Muth, R.F. 1969. *Cities and Housing*. Chicago: University of Chicago Press.
- Oron, Y., D. Pines, and E. Sheshinski. 1973. "Optimum Versus Equilibrium Land Use Pattern and Congestion Toll," *Bell Journal of Economics and Management Science*, 4: 619-636.
- O'Sullivan, Arthur. 1993. *Urban Economics*. Homewood, IL: Irwin.
- Pines, D. and E. Sadka. 1986. "Comparative Static Analysis of a Fully Closed City," *Journal of Urban Economics*, 20: 1-20.
- Robson, A. and D. Scheffman. 1979. "Urban Structure, Gas Prices, and the Demand for Transportation," *Journal of Urban Economics*, 6: 148-155.
- Ross, S. Forthcoming. "Economic Development Policy and Resident Welfare: An Urban Economic Perspective," *Journal of Urban Economics*.
- Ross, S. and J. Yinger. Forthcoming. "Comparative Static Analysis of Open Urban Models with a Full Labor Market and Suburban Employment," *Regional Science and Urban Economics*.
- Sasaki, K. 1987. "A Comparative Static Analysis of Urban Structure in the Setting of Endogenous Income," *Journal of Urban Economics*, 22: 53-72.
- Solow, R. 1973. "Congestion Costs and the Use of Land for Streets," *Bell Journal of Economics and Management Science*, 4: 602-618.
- Wheaton, W. 1974. "Residential Decentralization, Land Rents, and the Benefit of Urban Transportation Investment," *American Economic Review*, 67: 138-143.

Wheaton, W. 1977. "A Comparative Static Analysis of Urban Spatial Structure," *Journal of Economic Theory*, 9: 223-237.

Table 1. Alternative Assumptions About Land Ownership

	Open Model	Closed Model
Ignored Absentee Landlords	O1: Standard	C1: Standard
Redistribution to Residents	O2: Considered in the Literature	C2: Considered in the literature
National Land Ownership	O3: O1 with nonlabor income ^a	C3: Does not make sense ^b
Ownership of City Shares	O4: New, time-related income classes	C4: Same as C2 ^c
True Home Ownership	O5: New space/time related income classes	C5: Special case in the literature ^d

^aThe only difference between O1 and O3 is that O3 has an additional parameter, namely nonlabor income.

^bChanges that occur in one large urban area or in all areas alter the level of nonlabor income, contradicting the exogeneity assumption.

^cWith no mobility, residents and stockholders are indistinguishable.

^dTwo-period, linear-city case solved by Mun and Sasaki (1992).

Figure 1

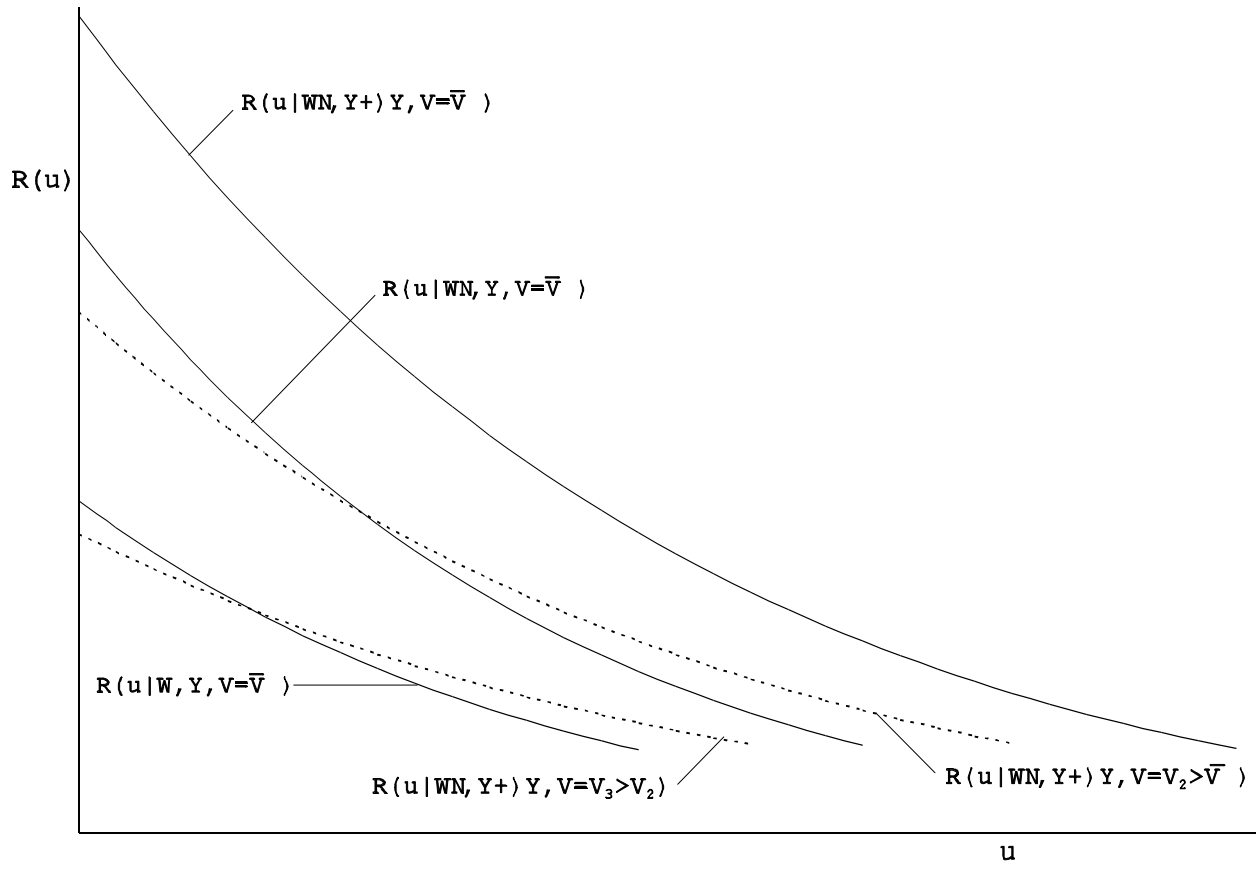


Figure 2

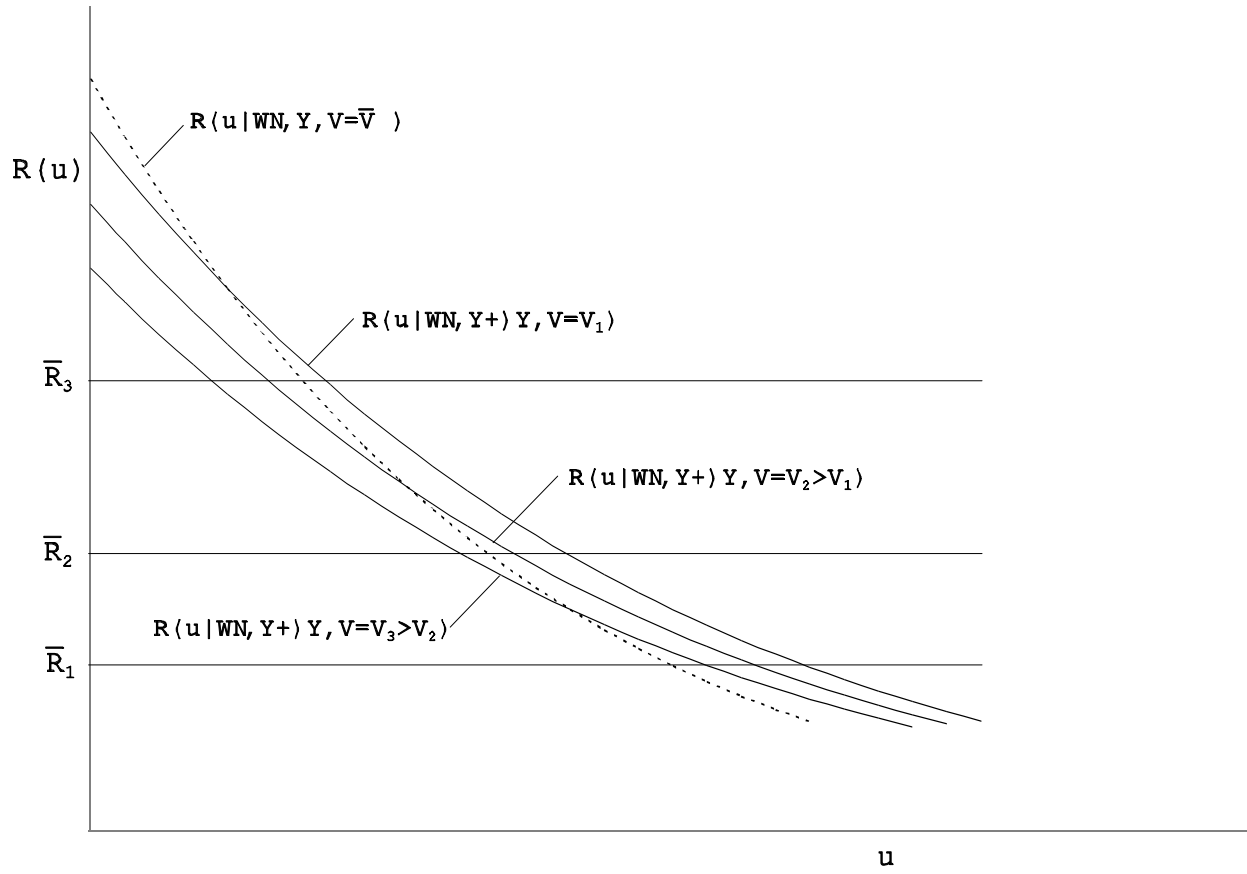
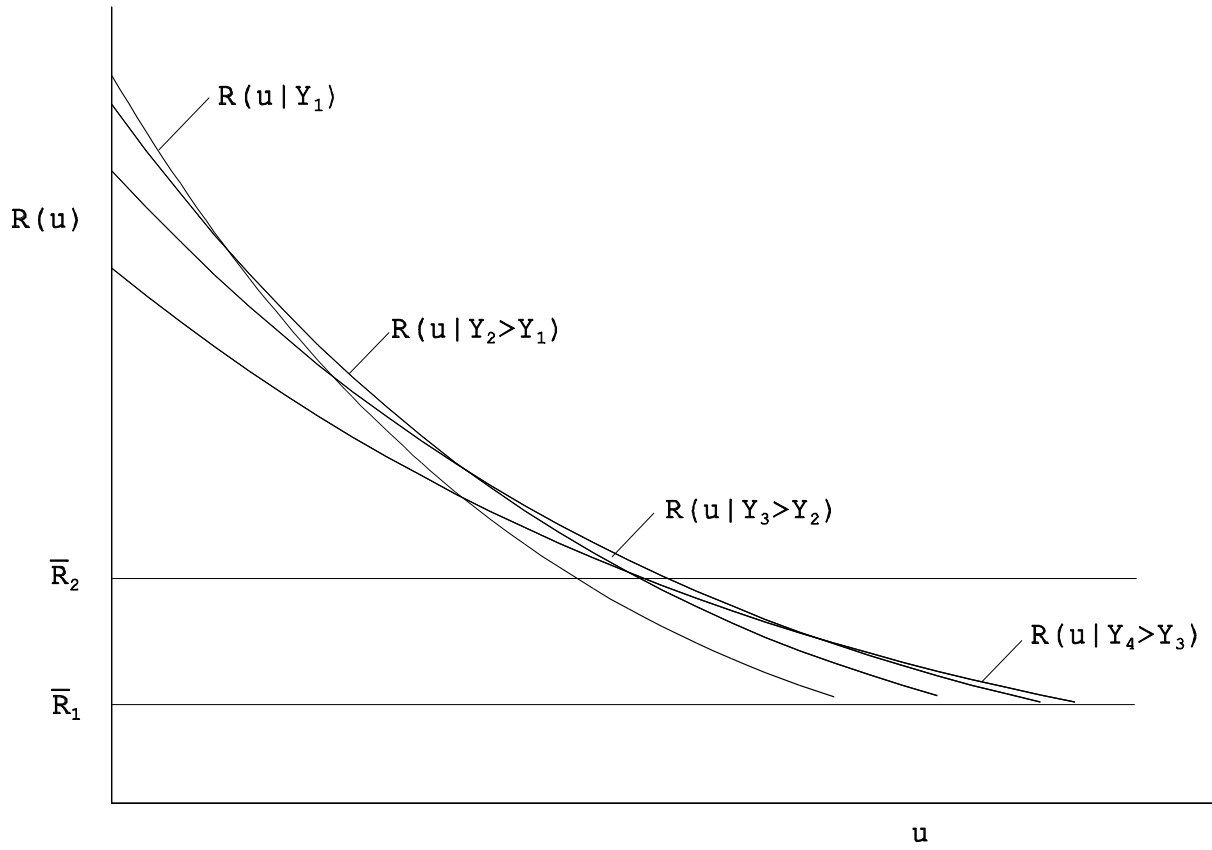


Figure 3



METROPOLITAN STUDIES PROGRAM OCCASIONAL PAPER SERIES

(\$5.00 each)

(complete list available upon request)

-
- 171.**
Robert M. Dunsky, James R. Follain, and Jan Ondrich
Using Recurrence Probabilities to Estimate the Volume of Multifamily Mortgage Originations, November 1995, 21 pp.
- 172.**
Amy D. Crews, Robert M. Dunsky and James R. Follain
Estimating the Volume of Multifamily Mortgage Originations by Commercial Banks Using the Survey of Mortgage Lending Activity and the Home Mortgage Disclosure Act Data, December 1995, 34 pp.
- 173.**
Amy D. Crews, Robert M. Dunsky and James R. Follain
What We Know about Multifamily Mortgage Originations and Why We Care, December 1995, 36 pp.
- 174.**
William Duncombe, John Ruggiero, and John Yinger
Alternative Approaches to Measuring the Cost of Education, January 1996, 46 pp.
- 175.**
William Duncombe and John Yinger
School Finance Reform: Aid Formulas and Equity Objectives, January 1996, 53 pp.
- 176.**
John Yinger
The Incidence of Development Fees and Special Assessments, February 1996, 26 pp.
- 177.**
James R. Follain, Jan Ondrich, and Gyan P. Sinha
Ruthless Prepayment? Evidence from Multifamily Mortgages, March 1996, 35 pp.
- 178.**
Amy D. Crews
Do Housing Programs for Low Income Households Improve Their Housing? April 1996, 53 pp.
- 179.**
Robert M. Dunsky and James R. Follain
The Demand for Home Mortgage Debt and the Income Tax. April 1996, 60 pp.
- 180.**
Amy S. Bogdon and James R. Follain
An Examination of Three Sets of Indicators of Financial Risk among Multifamily Properties, May 1996, 43 pp.
- 181.**
Thomas Dunn and Douglas Holtz-Eakin
Financial Capital, Human Capital, and the Transition to Self-Employment: Evidence from Intergenerational Links June 1996, 30 pp.
- 182.**
Stephen Ross and John Yinger
Where do the Rents Go? Land Ownership in an Urban Model, November 1996, 33 pp.
-

Publications Officer
Center for Policy Research
The Maxwell School - Syracuse University
426 Eggers Hall
Syracuse, New York 13244-1090

Please send me _____ copies of Occasional Paper No. _____ at \$5.00 each.

NAME _____

ADDRESS _____

CITY, STATE, ZIP CODE _____

Postage is included. For air mail delivery to points outside the United States or Canada add \$5.00 per publication. Make checks payable to Center for Policy Research. Payment **MUST** accompany orders.