

Syracuse University

## SURFACE at Syracuse University

---

Center for Policy Research

Maxwell School of Citizenship and Public  
Affairs

---

3-1996

### Ruthless Prepayment? Evidence From Multifamily Mortgages

James R. Follain  
*Syracuse University*

Jan Ondrich  
*Syracuse University*

Gyan P. Sinha  
*C.S. First Boston*

Follow this and additional works at: <https://surface.syr.edu/cpr>



Part of the [Economic Policy Commons](#), [Economics Commons](#), and the [Public Policy Commons](#)

---

#### Recommended Citation

Follain, James R.; Ondrich, Jan; and Sinha, Gyan P., "Ruthless Prepayment? Evidence From Multifamily Mortgages" (1996). *Center for Policy Research*. 456.

<https://surface.syr.edu/cpr/456>

This Working Paper is brought to you for free and open access by the Maxwell School of Citizenship and Public Affairs at SURFACE at Syracuse University. It has been accepted for inclusion in Center for Policy Research by an authorized administrator of SURFACE at Syracuse University. For more information, please contact [surface@syr.edu](mailto:surface@syr.edu).

**Metropolitan Studies Program Series  
Occasional Paper No. 177**

**RUTHLESS PREPAYMENT? EVIDENCE FROM  
MULTIFAMILY MORTGAGES**

**James R. Follain, Jan Ondrich  
and Gyan P. Sinha**

**Center for Policy Research  
Maxwell School of Citizenship and Public Affairs  
Syracuse University  
Syracuse, New York 13244-1090**

**March 1996**

**\$5.00**

We would like to thank Robert Van Order, Frank Nothaft and Bill Schaumann, all of Freddie Mac, for providing the data used in the study, and for considerable assistance in getting it into a form suitable for estimation. An earlier version of this paper was presented at the University of British Columbia, the University of Wisconsin-Madison, and the 1992 Meetings of the American Real Estate and Urban Economics Association. All errors are, of course, our responsibility. James R. Follain and Jan Ondrich are, respectively, Professor and Associate Professor of Economics, and Senior Research Associates, Center for Policy Research, the Maxwell School, Syracuse University. Gyan P. Sinha is Vice President, Fixed Income Research at C.S. First Boston, New York.

# CENTER FOR POLICY RESEARCH - SPRING 1996

**Timothy M. Smeeding**  
**Professor of Economics & Public Administration**

**Director**

Richard V. Burkhauser  
Professor of Economics

Margaret M. Austin

John Yinger  
Professor of Public Administration  
& Economics

Associate Director,  
Aging Studies Program

Associate Director,  
Budget and Administration

Associate Director,  
Metropolitan Studies Program

## SENIOR RESEARCH ASSOCIATES

Amy Crews . . . . . Economics	Bernard Jump . . . . . Public Administration
William Duncombe . . . . . Public Administration	Duke Kao . . . . . Economics
Thomas Dunn . . . . . Economics	Marcia Meyers . . . . . Public Administration
James Follain . . . . . Economics	Jerry Miner . . . . . Economics
Vernon Greene . . . . . Public Administration	Jan Ondrich . . . . . Economics
David Greytak . . . . . Economics	John Palmer . . . . . Public Administration
Barbara Grosh . . . . . Public Administration	Michael Wasylenko . . . . . Economics
Christine Himes . . . . . Sociology	Douglas Wolf . . . . . Public Administration
Douglas Holtz-Eakin . . . . . Economics	Assata Zerai . . . . . Sociology

## RESEARCH ASSOCIATES

Debra Dwyer . . . . . Economics
James McNally . . . . . Demography

## GRADUATE ASSOCIATES

Hakan Aykan . . . . . Public Administration	Suzanne McCoskey . . . . . Economics
Debra Bailey . . . . . Economics	Michael McLeod . . . . . Public Administration
Lloyd Blanchard . . . . . Public Administration	Meeae Park . . . . . Social Science
Meg Bower . . . . . Public Administration	John Phillips . . . . . Economics
Don Bruce . . . . . Economics	John Poupore . . . . . Economics
Shannon Felt . . . . . Economics	Mark Robbins . . . . . Public Administration
Tess Heintze . . . . . Public Administration	Katherin Ross . . . . . Social Science
Chin Hu . . . . . Sociology	Alex Striker . . . . . Economics
Christopher Ingram . . . . . Public Administration	Robert Weathers . . . . . Economics
Carol Jenkins . . . . . Public Administration	David Wittenburg . . . . . Economics
Rick Joy . . . . . Economics	Wilson Wong . . . . . Public Administration
Anna Lukemeyer . . . . . Public Administration	

## STAFF

Martha W. Bonney . . . . . Publications and Events Coordinator	Alex Mintskovsky . . . . . Programmer
Barbara Butrica . . . . . Data Manager	Inge O'Connor . . . . . Administrative Assistant, Luxembourg Income Study
Karin D'Agostino . . . . . Computer Consultant	Mary Santy . . . . . Receptionist
Julie DeVincenzo . . . . . Librarian/Office Coordinator	Ann Wicks . . . . . Administrative Secretary
Esther Gray . . . . . Administrative Secretary	Jodi Woodson . . . . . Secretary
Gina Husak . . . . . Secretary to the Director	Sheng Zhu . . . . . Manager - Computing Services
Detlef Jurkat . . . . . Translator	

**Abstract**

Estimates of a prepayment function for multifamily mortgages are reported in this paper. These are among the first attempts to estimate such a function; most previous work along these lines focuses on single family mortgages. A further distinguishing aspect of the paper is its attempt to incorporate the impact of unobservable factors on the mortgage refinancing decision. A variant of the maximum likelihood procedure first developed by Meyer (1987) is employed. The results indicate an overall positive duration dependence for the conditional prepayment rate. The estimated response of prepayments to a change in the market rate of interest is significant with the expected sign; it is also larger once the effect of unobserved heterogeneity is taken into account. Nonetheless, the magnitude of the response is substantially less than that predicted by the ruthless option pricing model.

# Ruthless Prepayment? Evidence from Multifamily Mortgages

## Introduction

The large growth in the market for mortgage-backed securities in the past decade or so has led to substantial research designed to value or price these securities. One theoretical approach uses the option pricing model (OPM) developed in the field of financial economics. In this model, mortgage borrowers have an option to prepay their mortgages today or at any time in the future and prior to the maturity of the mortgage. This option has value to the borrower as long as there is some possibility that future interest rates may decline below their contract rates. If this occurs, the borrowers may reduce their mortgage payments by refinancing their mortgages at the new and lower market mortgage interest rates. Measuring the value of the prepayment or call option owned by the borrower is critical to the valuation of mortgage-backed securities; the larger is the value of the option, the lower is the value of the mortgage-backed security.

The option pricing model has yielded numerous insights regarding the pricing of mortgage-backed securities. However, the theoretical predictions of the option pricing model do not appear to hold empirically. The most important shortcoming among data on single family mortgages seems to be that the interest rate sensitivity of the conditional prepayment rate is less than that predicted by the pure option pricing model. Quigley and Van Order (1990), Follain, Scott, and Yang (1992) and others present evidence along these lines and suggest that most homeowners do not “ruthlessly” make their refinancing decision; Vandell (1995) discusses the same issue regarding mortgage default.

Several explanations can be offered to explain why the OPM may not apply exactly to single family mortgages. First, owner-occupants may not be as financially sophisticated as the pure OPM implies. Alternatively stated, these households may face substantial transactions costs

in their refinancing decisions because it requires much time and energy to make the correct decision given their lack of financial sophistication. Second, prepayment by homeowners is influenced by many other decisions that make it difficult to identify clearly the effects of the OPM. For example, households often prepay because the locations of their jobs change or because of major changes in family composition such as a divorce. Third, prepayment may occur as part of an overall desire to adjust the composition of one's portfolio. For example, a person may choose to refinance in order to increase his or her loan to value ratio and use the proceeds of the refinancing to make other investments. Fourth, the data available to estimate prepayment studies may constitute part of the problem. In particular, the interest rate patterns of the past 15 years may not contain enough volatility to measure with much precision their effects on prepayment.

This paper continues the investigation of the applicability of the OPM to mortgages. Attention is focused on the prepayment of multifamily mortgages. Little econometric work is available about these mortgages, yet the market for multifamily MBSs has grown considerably in recent years and has the potential for much more growth, e.g., DiPasquale and Cummings (1992), Follain and Szymanoski (1995), and Godner and Rosen (1989). Multifamily mortgages also offer an opportunity to study the OPM in an environment in which its assumptions are more suitable. Presumably, holders of multifamily mortgage are likely to be more financially sophisticated than those who do not have the wealth needed to make such investments. This is probably particularly true for the sample examined in this paper—1,083 Freddie Mac Plan A multifamily mortgages originated between 1975 and 1986—because the mortgages are, on average, quite large. In addition, investors in these properties are less likely to prepay for non-financial reasons. For example, a job change or a family change does not force a landlord to dispose of his or her rental property.

The econometric investigation focuses on the estimation of the conditional probability of prepaying a mortgage in a particular quarter, which is referred to as the quarterly conditional prepayment rate (CPR) for multifamily mortgages. The CPR is an essential ingredient in modern option pricing models of mortgages. In these models, the price of the mortgage can be derived as the present value of a stream of future mortgage payments discounted at a risk-adjusted discount rate. What makes the pricing of mortgages more complex than the pricing of the standard noncallable bond is the possibility that the streams may terminate prior to the maturity of the loan because the borrower chooses to refinance. As a consequence, accurate mortgage valuation requires an understanding of the time pattern of the stream of future mortgage payments and, in particular, its sensitivity to changes in the market rate of interest. This requires knowledge of the survivor function of the mortgage or, equivalently, the hazard rate. Kau and Keenan (1995) recently reviewed the large literature on option pricing models of mortgages and discussed the role of the CPR in mortgage pricing.

In this study, a semiparametric estimation technique is used to analyze the determinants of the conditional prepayment rate or the prepayment hazard rate. The technique is semiparametric because the (log-integrated) baseline hazard is estimated completely nonparametrically and simultaneously with the covariate coefficients in a proportional hazard model. It represents the extension of the technique of Prentice and Gloeckler (1978) that was introduced into the econometrics literature by Meyer (1987). One of its principal advantages over competing semiparametric estimation techniques, for example those of Cox (1975) and Moffitt (1985), is that it easily allows the introduction of mortgage-specific unobserved heterogeneity. In essence, each borrower is assigned an error term that explicitly incorporates controls for his unobserved characteristics. This error term is then integrated out of the likelihood contribution of each borrower to leave the marginal density (or survivor function) of the time to prepayment.

Incorporation of unobserved heterogeneity among borrowers is expected to account, at least partially, for the wide variety of unobserved factors that affect prepayment; in so doing, it is expected that the responsiveness of the hazard with respect to interest rate changes will be estimated more accurately.

The key results of the analysis are:

- Prepayment is found to be sensitive to the value of the call option and, hence, the difference between the contract rate and the current market rate. Also, the responsiveness of the conditional prepayment rate to the value of the option is sensitive to whether the option is in or out of the money and the extent to which the option is in the money, as the theory suggests. However, the responsiveness of prepayment with respect to an interest rate change for an in-the-money option is still short of what is expected in the pure OPM.
- The estimates of the time-varying parameters suggest that conditional prepayment rates increase with the length of time that the mortgage has been held (positive duration dependence). Furthermore, these parameters are particularly large around 50 quarters after origination.
- Allowing for unobserved heterogeneity among the investors does improve the results substantially. Most notably, the heterogeneity variance is statistically significant and the estimates of the coefficients of the option variables are larger in the model with heterogeneity than in the model without it.

The remainder of the paper includes five additional sections. The estimation techniques are discussed, followed by an explanation of the specification of covariates and the data employed. The results are then presented. The final section discusses the implications of the analysis regarding the OPM and offers some suggestions for further work.

## **The Estimation Technique**

Each observation in our original sample describes a single mortgage securitized by Freddie Mac and originating between 1975 and 1986 and followed through the first quarter of 1989. Our goal is to find the determinants of the conditional prepayment rate (prepayment hazard rate) for



these mortgages, in other words, the rate of prepayment at a given mortgage age for all mortgages that reach that age. We do not have information on the mortgagee or lending institution that would allow us to model correlations across observations for mortgage age at prepayment, the random variable of interest. Accordingly, we consider mortgage ages at prepayment to be statistically independent in our sample.

As a starting point for our statistical analysis, let  $T_i$  be the age of mortgage  $i$  at prepayment. The prepayment hazard rate,  $\lambda_i(t)$  for mortgage  $i$  at time  $t$  is defined as:

$$\lambda_i(t) = \lim_{\Delta t \rightarrow 0^+} (\text{Prob} [t + \Delta t > T_i \geq t \mid T_i \geq t]) / \Delta t . \quad (1)$$

The prepayment hazard is assumed to take on the proportional hazard form used by Cox (1972):

$$\lambda_i(t) = \lambda_0(t) \exp \{ z_i(t)' \beta \}, \quad (2)$$

where  $\lambda_0(t)$  is the unknown baseline hazard at time  $t$ ,  $z_i(t)$  is the vector of time dependent covariates that influence the prepayment decision for mortgage  $i$  at time  $t$  and  $\beta$  is the parameter vector to be estimated.

Cox (1975) developed a partial likelihood technique to estimate the parameter vector of his proportional hazard. The partial likelihood technique suffers from at least two shortcomings. First, the correct treatment of tied failure times within the partial likelihood approach is difficult both theoretically and computationally. Second, the incorporation of individual-specific unobserved heterogeneity makes the estimation virtually intractable.

These two deficiencies are resolved in Meyer's (1987) adaptation of the Prentice and Gloeckler (1978) technique. In Meyer's approach, the parameters of the (log-integrated) baseline hazard are estimated nonparametrically and simultaneously with the parameter vector  $\beta$  of the proportional hazard. The estimation does not use the continuous quality of the variable mortgage age at prepayment; rather, it discretizes this variable into time intervals. Because a parameter

must be estimated for each such time interval, we took the intervals to be quarters. This left us with 55 parameters to estimate in the baseline hazard!

The conditional survivor function at age  $t + 1$  is defined to be the probability that mortgage age at prepayment is greater than or equal to the quarter ending at time  $t + 1$  given that the mortgage age at prepayment is greater than or equal to the quarter ending at time  $t$ :

$$\begin{aligned} P[T_i \geq t + 1 \mid T_i \geq t] &= \exp \left[ - \int_t^{t+1} \lambda_i(u) du \right] \\ &= \exp \left[ - \exp(z_i(t)'/\beta) \int_t^{t+1} \lambda_0(u) du \right], \end{aligned} \tag{3}$$

where  $z_i(t)$  is assumed constant in the quarter starting at  $t$  and ending at  $t + 1$ . Equation (3) can also be written as:

$$P [T_i \geq t + 1 \mid T_i \geq t] = \exp [ - \exp (z_i(t)'/\beta + \gamma(t)) ], \tag{4}$$

where

$$\gamma(t) = \ln \left[ \int_t^{t+1} \lambda_0(u) du \right]. \tag{5}$$

Our estimation must properly take into account the possibility that some mortgages will not have prepaid in the period in which they are observed. This can happen in one of two ways. First, the mortgage may be observed over only part of its term. Second, the mortgage may simply mature. Following Meyer (1990), we take care of the possibility of censoring by denoting the appropriate censoring time for each observation as  $C_i$  and defining  $\delta_i = 1$  if  $T_i \leq C_i$  and 0 otherwise. Furthermore, we define  $k_i = \min(\text{int}(T_i), C_i)$ .

The likelihood function for a sample of  $N$  mortgages can be written as:

$$l(\gamma, \beta) = \prod_{i=1}^N \left[ [1 - \exp\{-\exp[\gamma(k_i) + z_i(k_i)'\beta]\}]^{\delta_i} \prod_{t=0}^{k_i-1} \exp\{-\exp[\gamma(t) + z_i(t)'\beta]\} \right], \quad (6)$$

where  $\gamma = [\gamma(0), \dots, \gamma(55)]'$  and  $\beta$  are the parameters to be estimated. Note that the first term in the product in (6) is equal to 1 and so provides no information to the likelihood except when a spell ends between  $k_i$  and  $k_i + 1$ . The final product in (6) is simply the probability that the mortgage lasts at least until  $k_i$ . The log-likelihood function associated with equation (6) is:

$$L(\gamma, \beta) = \sum_{i=1}^N \left[ \delta_i \log [1 - \exp\{-\exp[\gamma(k_i) + z_i(k_i)'\beta]\}] - \sum_{t=0}^{k_i-1} \exp[\gamma(t) + z_i(t)'\beta] \right]. \quad (7)$$

We move now to the case where unobserved heterogeneity is incorporated into the analysis. Realistically, we recognize that we will never have a situation in which all relevant covariates are included in  $z_i(t)$ . We incorporate the effects of all excluded regressors that are constant over the life of the mortgage into the random variable  $\theta_i$ , which is a mortgage-specific error term. The new hazard becomes:

$$\lambda_i(t) = \theta_i \lambda_o(t) \exp\{z_i(t)'\beta\}. \quad (8)$$

The random variable  $\theta_i$  is assumed to be distributed independently of the  $z_i(t)$  and has a Gamma distribution with mean one and variance  $\sigma^2$ . Conditioning on the unobserved  $\theta_i$  and integrating out over their distribution leads to the following log-likelihood, derived by Meyer (1990):

$$L(\gamma, \beta, \sigma^2) = \sum_{i=1}^N \log \left[ \left[ 1 + \sigma^2 \sum_{t=0}^{k_i-1} \exp[\gamma(k_i) + z_i(k_i)' \beta] \right]^{-\sigma^{-2}} \right. \\ \left. - \delta_i \left[ 1 + \sigma^2 \sum_{t=0}^{k_i} \exp[\gamma(t) + z_i(t)' \beta] \right]^{-\sigma^{-2}} \right] . \quad (9)$$

Here, the variance parameter  $\sigma^2$  must be estimated together with the parameter vectors  $\beta$  and  $\gamma$ .

The estimation of the log-likelihood in equations (7) and (9) was performed on a Sun Workstation (Release 4.1.3) and an IBM AIX Workstation (Release 3.2.5) at Syracuse University using the optimization program GQOPT, version 6.

## Specification of Covariates

The selection of the regressors in the covariate vector of the Prentice-Gloeckler-Meyer hazard analysis is motivated by option pricing theory. The following discussion highlights some of the major insights generated by the option pricing model and their implications for the specification and estimation of the econometric model.<sup>2</sup>

According to the option pricing model, the market value of an existing mortgage,  $P(t)$ , can be written as the difference of two components:  $P(t) = P_{nc}(t) - V_c(t)$ . The first term,  $P_{nc}(t)$ , is the present value of all future mortgage payments until maturity discounted at the current interest rate. The second term,  $V_c(t)$ , is the value to the mortgagee of *not* currently exercising the prepayment or call option, usually referred to simply as the value of the call option. The value of the prepayment option is nonnegative because any lender must be compensated for the uncertainty created by allowing the option to be embedded in a mortgage.

With this as background the criterion for the refinancing and the specification of our econometric model can be described precisely. A borrower will refinance if

$$P(t) - B(t) > TC(t), \quad (10)$$

where  $P(t) - B(t)$  is the difference between the market value and the book value of the outstanding loan balance.  $TC(t)$  represents transactions costs associated with refinancing. In words, a borrower refinances if the market value of its existing mortgage exceeds the book value of the debt plus incidental transactions costs. Writing  $P(t)$  in terms of the value of the call option and rearranging terms shows that prepayment is appropriate if  $P_{nc}(t) - B(t)$  exceeds the sum  $V_c(t) + TC(t)$ .

These considerations lead to the following specification of the primary covariate,  $EXOPT(t)$ , the reward or value to the mortgagee of exercising the prepayment option immediately, measured as a percentage of book value:

$$EXOPT(t) = [P(t) - B(t) - TC(t)] / B(t). \quad (11)$$

If  $EXOPT(t)$  is positive, the call option is in the money and the borrower will benefit financially by prepaying the loan prior to maturity and obtaining a new loan at the market rate of interest. If  $EXOPT(t)$  is negative, exercising the prepayment option immediately bears a cost (the call option is out of the money).

This option based view of the refinancing criterion also sheds light on the shape of the functional relationship between the criterion and the hazard. Not only is the prepayment hazard positively related to  $EXOPT(t)$ , but the relationship between the two is expected to be nonlinear. The the value of the call option is likely to be quite small when the market interest rate is well above the contract interest rate (coupon rate) because the likelihood of benefiting from refinancing in the future is small. The prepayment option is likely to rise in value substantially as the market rate approaches and falls below the coupon rate because such movements increase the likelihood that the borrower may be able to realize savings in the future by refinancing. Although the exact shape of the function depends upon the volatility of interest rates and the term to

maturity, this reasoning does suggest a highly nonlinear shape to the hazard. It is relatively flat and insensitive to changes in  $EXOPT$  when  $EXOPT$  is negative and the market rate of interest far exceeds the borrower's contract rate, i.e., the prepayment option is deeply out of the money. The hazard rises as the market rate approaches the contract rate (from above) and increases substantially when  $EXOPT$  is large and positive, i.e., the option is deeply in the money. In fact, according to the ruthless version of the option pricing model in which the prepayment is driven purely by this financial criterion, the hazard rate ought to approach unity when the option is deeply in the money.

Although the data set does not provide information about the book value of the outstanding mortgage, its computation is straightforward given knowledge of the term to maturity,  $T$ , the coupon rate,  $r_c$ , and the origination date, which are available. The book value can be written as:

$$B(t) = (M/r_c) (1 - (1 + r_c)^{-(T-t)}), \quad (12)$$

where  $M$  represents the quarterly mortgage payment.

Computation of the market value of the debt represents a more challenging problem. Ideally, the market value of the debt should be calculated using an explicit option pricing model to compute  $V_c(t)$  for each mortgage and for each quarter; this is an impractical approach in empirical work of this type. The simpler approach used in this paper relies on the contention of Richard and Roll (1989) that states that the market value of a callable bond is approximated by the present value of the payments to maturity discounted by the current callable interest rate<sup>3</sup>. As long as the current market rate used to compute the present value pertains to a new issue with a maturity close to the remaining term of the existing mortgage, this approach will provide relatively accurate estimates of market value. Thus, the expression for the value of a callable bond used here is:

$$P(t) = (M/r(t)) (1 - (1 + r(t))^{-(T-t)}), \quad (13)$$

where  $r(t)$  is the current rate on newly issued mortgages.

The final component of the primary criterion for prepayment is transactions costs,  $TC(t)$ . Transactions costs have two main components. The first is the penalty for prepayment, which is 6 months of interest within the first five years and one percent of the book balance after that. The second is the sum of the various and incidental transactions costs associated with the origination of a new mortgage. These include the cost of appraisal, application fees, and origination fees on the first mortgage; they do not include payments to buy down the interest rate on the new mortgage. One might also include less precise aspects of these costs such as the value of the time spent thinking about the issue. This second component can also be divided into an absolute dollar amount and a percentage of the book balance. We use \$1,000 as the estimate of the absolute costs of refinancing and one percent of book value as the variable component. Furthermore, our expectation is that the unobservable and idiosyncratic portion of transactions costs are captured to some extent by the model in which unobserved heterogeneity is taken into account.

The possibility of a nonlinear response of the hazard to  $EXOPT$  is examined by the use of a piecewise linear spline function. Experimentation led to the specification of four pieces: in the positive direction the first piece is for  $EXOPT$  between 0 and 0.1 and the second piece is for  $EXOPT$  greater than 0.1; in the negative direction the first piece is for  $EXOPT$  between 0 and -0.1 and the second piece is for  $EXOPT$  less than -0.1. The size of the coefficients of  $EXOPT$  are expected to increase as  $EXOPT$  increases. The introduction of unobserved heterogeneity is also expected to increase the size of the coefficients of  $EXOPT$ , since the unobservable components of transactions costs may be better taken into account.

Option pricing theory also suggests another covariate, the value of the equity that exists in the property. Holders of existing mortgages also implicitly own an option to default if the market

value of the property falls below the market value of the mortgage. Because the option to default is surrendered upon prepayment, it is an opportunity cost associated with prepayment. As such, the larger the value of the default option, the lower is the likelihood that the mortgage will be prepaid in order to refinance if the conditions for refinancing are otherwise satisfied. As with the prepayment option, it is not possible to incorporate an exact measure of the default option; instead, we use the loan to value ratio as a proxy for this option. The larger the loan to value ratio, the greater is the likelihood of default and the greater is the value of the default option.

The market value of the property is the original appraisal value increased by a state-specific index of single family housing prices; that is,

$$H(t) = H(0) (1 + \xi)^t, \quad (14)$$

where  $H(0)$  and  $H(t)$  represent original and current collateral values, respectively, and  $\xi$  is the average quarterly growth rate in housing values for the state in which the mortgage originated.<sup>4</sup>

The loan to value ratio,  $B(t)/H(t)$ , might be included in the hazard as a single variable; however, because of the likely inexactness of this measure of current collateral value, the variables  $H(t)$  and  $B(t)$  appear separately and in natural logarithms. Moreover, these variables are entered as covariates only when the prepayment is in the money, i.e.  $EXOPT > 0$ . With an exact collateral measure, the theory suggests that these coefficients will have the same magnitude and opposite signs, with the coefficient for the logarithm of  $H(t)$  positive.

Several other dummy variables are included to control for possible seasonal effects. Analysts of single family prepayments have identified seasonal effects in their analysis. Fabozzi and Modigliani (1992) find that conditional prepayment rates are lowest in the winter months and begin to rise in the spring, reaching a peak in the summer months or early fall. This pattern probably reflects the greater propensity for homeowners to sell their homes during the spring and



summer and move during the summer months. We test whether similar effects for multifamily investors exist.

The last covariate is also a dummy variable and pertains to the Tax Reform Act of 1986 (TRA). Several provisions of the TRA provided incentives for many owners of existing multifamily real estate projects to sell before the end of 1986. In particular, the TRA called for a higher capital gains tax rate and much longer depreciation periods than previous law. It is possible that these provisions triggered above average property sales and, hence, above average conditional prepayment rates, during the last quarter of 1986. This effect is captured by entering a dummy variable for that quarter.

## **Data Description**

This section provides a description of the data set comprised of non-assumable, fully amortizing (plan A) loans provided by Freddie Mac. Sample inclusion required that, on the date of sample collection (April 27, 1989), the loan was active, prepaid, or foreclosed, delinquent, or real-estate owned (REO). Loans which had implausible loan to value ratios, missing values for principal amount, appraisal value, contract rate at origination, or maturity date, or were recorded as paid off but had no payoff date were also deleted. Active loans are considered to be censored in the estimation in that they were still current at the date of sample collection. Loans that were foreclosed, delinquent, or REO are treated as being censored in the final quarter that they were active. Thus, instead of being dropped entirely from the estimation sample, each mortgage-quarter for these loans during the period in which they remain current provides a contribution to the likelihood function.

Since each quarter in the sample has a baseline hazard parameter associated with it, for tractability in the estimation procedure the loan sample was further restricted to loans originating

after 1974 and before 1987, which marks the beginning of the post-TRA regime. Of the 1,083 sample mortgages, there were 451 that eventually prepaid in the sample period, and 632 that were censored, usually because the mortgage had not yet reached maturity at the end of the sample period. Less than 20 were censored due to mortgage default.

There appears to have been a sharp acceleration in originations in the mid-1970s. More than half the sample, 607 loans to be precise, originated in the two-year period 1975-76. A breakdown of mortgage characteristics by origination year is presented in Table 1. There was an increase in loan size and the value of the property being financed after 1982. Interest rates at origination have ranged from a low of 9.43 percent in 1976 to a high of 14.87 percent in 1982. Originations were lower in the 1980s than the 1970s because of the high rates in the early 1980s and because Freddie Mac moved to another multifamily program during the 1980s<sup>5</sup>.

A calculation of the empirical (Kaplan-Meier) quarterly hazard for the sample of loans shows that the hazard does not reach 1 percent until the 10th quarter of the loan and first reaches 3 percent in the 41st quarter. Thereafter, it does not fall below 2 percent. An examination of the sample prepayment pattern by calendar time shows no prepayments before the first quarter of 1984, when the average mortgage rate on newly issued multifamily loans purchased and securitized by Freddie Mac stood at 13.28 percent, down from a high of 17.77 percent in the first quarter of 1982. The maximal prepayment rate of 4.67 percent occurred in the first quarter of 1987, when the average mortgage rate bottomed out at 9.97 percent. In the first quarter of 1989, the final sample calendar quarter, the prepayment rate was 2.81 percent and the average mortgage rate was 11.06 percent.

The housing value appreciation index is calculated from time-series and cross-sectional observations on median sales prices of a single family home, constructed by WEFA/DRI. A price appreciation factor for each state and quarter is computed which in turn is used to calculate the

contemporaneous value of the property by multiplying the appraisal value by this appreciation factor.

## Results

The discussion of the results is divided into two parts. The first subsection focuses on the process by which we arrived at the final specification of covariates for two models, with and without heterogeneity. The parameter estimates and predictive ability of the models are also compared. The second subsection examines the sensitivity of prepayment rates to changes in *EXOPT* implied by the model estimates and, in particular, whether the results are consistent with the “ruthless” option pricing model.

### Arriving at the Two Final Models

Maximum likelihood parameter estimates and standard errors for five different covariate specifications are presented in Tables 2-6 for models with and without unobserved heterogeneity. For each of the estimations there are estimates for 56 (log-integrated) baseline hazard parameters,  $\gamma(0)$  through  $\gamma(55)$ , where  $\gamma(t-1)$  is the parameter for period  $t$ . Because there were no prepayments of a mortgage in quarter 25,  $\gamma(24)$  could not be estimated independently and we imposed the restriction:

$$\gamma(24) = (\gamma(23) + \gamma(25))/2. \tag{15}$$

Thus, 55 independent baseline hazard parameter estimates were generated in each estimation. (We present estimates only for quarters 1, 19, 37, and 56.) All of the estimations also contain three seasonal indicators for spring, summer, and fall; the indicator for winter is subsumed in the  $\gamma$ 's. They also contain an indicator for the fourth quarter of 1986, in which conditional prepayment rates were hypothesized to be higher as a result of the TRA of 1986. The (evenly

numbered) models controlling for unobserved heterogeneity also generate an estimate for  $\sigma^2$ , the variance of the Gamma-distributed random effect.

The five specifications differ only in the set of financial variables that are included. In the specification of Table 2, the only financial variable included is  $EXOPT$ , the value of exercising the prepayment option currently. In the Table 3 specification, only an indicator for whether  $EXOPT$  is positive (in the money) is included. For Table 4, four variables representing the spline discussed in Section III are included. For contrast with Table 4, Table 5 includes indicator variables representing three of the four pieces of the spline; the indicator for  $EXOPT < -1$  is subsumed in the  $\gamma$ 's. Finally, in the preferred specification of Table 6, the financial variables consist of those in the four-piece spline and the logarithms of housing value and book value.

The results in Table 2-6 for the models without heterogeneity indicate an overall increasing pattern for the  $\gamma(t)$ 's, so that duration dependence is generally positive. The  $\gamma(t)$ 's increase even faster in the corresponding models with heterogeneity. Such a downward bias of duration dependence in the absence of heterogeneity controls was first noted by Lancaster (1976) in his work on unemployment durations. It is demonstrated in Figure 1, which plots quarterly hazard rates for representative values of the covariates for the models with and without heterogeneity in the preferred specification.<sup>6</sup> For, both models, we calculate the quarterly hazard rate as

$$h_i(t) = 1 - \exp \{ - \exp [ \gamma(t) + z_i(t)' \beta ] \} . \quad (16)$$

The interpretation is straightforward in the model without heterogeneity; in the model with heterogeneity it represents the quarterly hazard of a mortgage for which the heterogeneity component equals the mean value of unity.

Moving now to an examination of the covariates, we see first that the seasonal effects for spring and fall are always insignificantly different than the effect for winter. However, except for

Models III and IV in Table 3, the effect of summer is to significantly lower the hazard from winter quarters (at the 5 percent level). These results contrast with those found by Fabozzi and Modigliani (1992) for single family mortgages. A less striking seasonal pattern for multifamily prepayments is not too surprising, because prepayments due to borrower relocation are probably much less important in the multifamily case.

The coefficient of the dummy variable for the last quarter of 1986 is positive in all models, but significant at the 5 percent level in only three of the ten models. In each model in which it was significant, the prepayment option variable(s) were indicators. In all specifications the effect of this quarter diminished in the model with heterogeneity. The results suggest that prepayments were not higher than normal during this period— investors did not seek to sell their properties to those who wished to benefit from the more generous depreciation of benefits of the law prior to TRA.

Perhaps the covariates of most interest in our model are those associated with the exercise of the prepayment option. In all models, the associated coefficient estimates have the expected sign and virtually all are significantly different from zero at the 1 percent level. The two exceptions are the coefficient for the spline when *EXOPT* is between -0.1 and 0.0 in the models without heterogeneity in Tables 4 and 6. Of these two, the estimate is significant at the 5 percent level only in Table 4. Comparing models without heterogeneity to models with heterogeneity, the estimated effect of a covariate relating to *EXOPT* is higher in the corresponding heterogeneity model for all estimates but one, the spline estimate for *EXOPT* between 0.0 and 0.1 in Table 6. We suspect that this is because the heterogeneity model is able to control for an unobserved component of transactions costs. It is also interesting to note, by a simple comparison of log-likelihoods, that over the four preliminary specifications, the specifications that used exact quantitative information on *EXOPT* (Tables 2 and 4) performed better than the corresponding

specifications which used only qualitative information on *EXOPT* (Tables 3 and 5 respectively). Finally, in Tables 4 and 6, it is interesting to note the heightened effect of an increase in *EXOPT* when it is just in the money compared to when it is just out of the money. This point is examined in more detail below.

For the preferred specification (Table 6), the estimated coefficients for the logarithms of housing and book values have the expected sign in both models, but only one of these four estimates (housing value in the model with heterogeneity) is individually significant at the 5 percent level. However, a likelihood ratio test (comparing Models VI and X) suggests joint significance at the 1 percent level—the  $\chi^2$ -statistic has two degrees of freedom and a value of 11.63 (the critical value at the 1 percent level is 9.21). The fact that our analysis uses a single family housing price index instead of a multifamily housing price index is a possible explanation for our less than satisfactory results. Nonetheless, the results do indicate the potential importance of incorporating the value of the default option.

For three of the five specifications (Tables 2, 4, and 6), there are marked differences in the parameter estimates across models with and without heterogeneity. (The fact that estimates of  $\sigma^2$  are substantially smaller in Tables 3 and 5 is likely an artifact of there being no continuous variables in the specification.) Still, it is problematic, for any one specification, to conclude which of the two models is the correct one. Score tests for unobserved heterogeneity that do not assume a specific form for the heterogeneity distribution may give misleading results when a large proportion of the data are censored (see, for example, Horowitz and Neumann (1989)). If we do assume a specific distribution for heterogeneity (the Gamma in our case), simple Wald and likelihood ratio tests for whether the variance of the heterogeneity component is different from zero are not, strictly speaking, valid because zero is a boundary value for the set of possible values of the variance.

Gouriéroux, Holly and Montfort (1982) show that the appropriate critical value for a test of size  $\alpha$  is the critical value for a test of size  $2\alpha$  under standard conditions (see also Chernoff (1954)). Thus, for a test of size .01, the critical  $\chi^2(1)$  value is 5.41. For the preferred specification (Table 6), the value of the Wald  $\chi^2$ -statistic is 7.41 while for the likelihood ratio it is 14.41. Thus both tests reject the null in favor of the alternative at the 1 percent level.

### Some Implications of the Results

A central question addressed in this paper is whether the prepayment behavior of multifamily mortgages performs similarly to that predicted by the standard or ruthless option pricing model. Several predictions emerge from the theoretical option pricing model. The prepayment rate is relatively insensitive to interest rate changes as long as the value of exercising the option is negative. The predicted hazard rate should also rise relatively rapidly and nonlinearly as the value of exercising the option becomes positive and increases in value. Lastly, the probability of prepayment is expected to be near unity when the value of the call option is deeply in the money.

An examination of these predictions requires an examination of the shape of the hazard function for different values of *EXOPT*. For this examination we use the aggregate quarterly hazard rate, which is based on the marginal density and survivor function of the duration, i.e., aggregating over the effect of heterogeneity. The formula for the aggregate quarterly hazard at quarter  $t$  in the model with no heterogeneity can be obtained from equation (4) in a straightforward manner:

$$h_i(t) = 1 - \exp \{ - \exp [ \gamma(t) + z_i(t) \beta ] \} . \quad (17)$$

In the model with heterogeneity, the aggregate quarterly hazard has the form:

$$h_i(t) = 1 - \left\{ \frac{1 + \sigma^2 g_i(t)}{1 + \sigma^2 g_i(t-1)} \right\}^{-\sigma^2} \quad (18)$$

where

$$g_i(t) = \sum_{s=0}^t \exp(\gamma(t) + z_i(t)'\beta) \quad (19)$$

and  $g_i(-1)=0$ . The values assigned to the covariates are zero except for *EXOPT* which is allowed to change in value and the time period is fixed at the second quarter since origination.

Several points emerge from an examination of this particular hazard function, which is plotted in Figure 2. First, the model with heterogeneity yields systematically higher hazard rates than the model without heterogeneity. Second, the function is insensitive to reductions in the market rate of interest (increases in *EXOPT*) if the value of exercising the option is negative. Third, the function rises steeply and nonlinearly with *EXOPT* when it is positive (in the money). Fourth, and most importantly, the quarterly hazard rate never approaches unity even when the option is deeply in the money. The second and third results are consistent with the option pricing model, but the last result is not. Although this is only one of many different hazard functions that can be graphed, it is based upon representative and reasonable values of the covariates. Some experimentation with other values and other time periods generated similar results.

The upshot of these results is that multifamily borrowers do not follow the ruthless pricing model. Although prepayment rates do increase nonlinearly when the value of exercising the option is positive, they do not rise to unity even when the option is deeply in the money. This result holds even when unobserved heterogeneity is taken into account. As such, these results are consistent with the results of Quigley and Van Order (1990) regarding the applicability of the ruthless pricing model to single family mortgages.

The results also shed light on the value of a parameter often used to make simple calculations of the price of a mortgage—the expected holding period of the borrower. This simple way of computing the value of a mortgage computes the present value of remaining mortgage



payments for a typical or average expected holding period. The estimates of the baseline components of the hazard function can be used to determine this average holding period by plotting the survivor function for typical values of the covariates. This is done for the marginal survivor function (integrating out the heterogeneity component) in Figure 3 using covariate values of zero. For this particular plot, the survival rate for the model with heterogeneity equals 50 percent at about 8.5 years or 34 quarters; the median holding period is calculated to be at least four quarters longer for the model without heterogeneity. An average holding period of eight to ten years is consistent with the results recently reported by Bogdon and Follain (1995), who report an average holding period of about ten years for a recent national sample of multifamily properties.

Of course, using a rule of thumb such as this to compute the value of a mortgage is less precise than one that fully incorporates the estimated hazard rate. Although the development of such a complete model is beyond the scope of this paper, it is a fruitful area for future research. Therefore, it may be useful to summarize briefly the steps needed to produce one type of pricing model. First, an interest rate process would be used to generate a set of future interest rate paths; Chen and Yang (1995) discuss several possible ways of doing this. Second, a weighted average of future mortgage payments would be calculated using the hazard rate for each interest rate scenario; the interest rate scenario would also determine the appropriate rate at which to discount the cash flows. Finally, the simple average of the weighted average calculations for all interest rate scenarios would equal the estimate of the price of the mortgage for a given interest rate scenario. A pricing model based upon such hazard rate calculations would be a substantial improvement over the simple rule of thumb approach used above because it reflects the asymmetric response of the prepayments to interest rate changes.

## Concluding Remarks

This study reports on the determinants of the prepayment of multifamily mortgages on the borrower's decision to prepay. A special effort is made to incorporate the effects of unobservable factors on the decision to refinance. The results indicate that the effect of the unobservables on the prepayment decision is significant, even after conditioning on seasonality, the effects of the 1986 TRA, and relevant financial variables, as evidenced by the large upward revisions in parameter estimates that result from the explicit incorporation of the unobservables.

The results are relevant to an ongoing debate regarding the ruthlessness with which borrowers use the refinancing privilege to benefit from lower interest rates. Quigley and Van Order (1990) cast doubt on the pure or ruthless version of the option pricing model for single family mortgage borrowers; such borrowers appear to refinance more slowly than the ruthless version of the model predicts. Our approach hopes to shed further light on this issue by focusing on mortgages held by investors with presumably greater financial acumen and fewer reasons to move for non-financial reasons and by taking account of unobserved heterogeneity. Surprisingly, our results are consistent with the single family analysis of Quigley and Van Order. Prepayment rates among multifamily mortgages surely respond positively to declines in the market rate of interest; however, estimated and observed hazard rates are much lower than would be predicted by the ruthless option pricing model.

Given the paucity of studies on the topic, more study of the prepayment and default behavior is needed before one can conclude that investors in multifamily properties behave suboptimally. A priority for further research should be the estimation of prepayment and default behavior for other data sets and other time periods. In addition, the model of prepayment should be expanded to include the influence of factors other than debt-cost minimization. One

potentially important factor is the investor's desire to rearrange his or her portfolio; unfortunately, data are unavailable to incorporate this factor into our analysis. Furthermore, investors may churn or turn over their properties in order to take maximum advantage of tax depreciation rules (see Hendershott and Ling 1984)). Our analysis finds only modest support for a tax generated turnover impact in the fourth quarter of 1986, but more investigation may be useful.

**Table 1: Loan Origination Statistics<sup>a</sup>**

<b>Year</b>	<b>Loan Size</b>	<b>Note Rate</b>	<b>Appraised Value</b>	<b>Number</b>
1975	220099	9.45	298468	245
1976	253337	9.43	344558	362
1977	553930	9.50	792975	67
1978	341777	10.26	472680	57
1979	285502	11.64	397696	61
1980	151052	12.97	254183	30
1982	154123	14.87	240800	2
1983	832021	13.61	1383313	135
1984	1066937	14.16	2877875	8
1985	381182	10.74	772386	43
1986	1135808	11.02	2722739	73

<sup>a</sup>The columns represent within year means. The sample does not contain any loans originated in 1981.

**Table 2: Models with Relative Prepayment Option Value**

	<b>Model I</b>		<b>Model II</b>	
	<b>(without Heterogeneity)</b>		<b>(with Heterogeneity)</b>	
	<b>Coefficient</b>	<b>Standard Error</b>	<b>Coefficient</b>	<b>Standard Error</b>
$\gamma(0)$	-6.363	1.004	-6.156	1.007
$\gamma(18)$	-3.781	0.425	-3.136	0.478
$\gamma(36)$	-2.531	0.288	-1.831	0.364
$\gamma(55)$	-1.952	1.013	-0.765	1.115
Spring	-0.238	0.147	-0.326	0.152
Summer	-0.466	0.160	-0.587	0.167
Fall	0.225	0.143	0.183	0.145
1986 Q4	0.338	0.208	0.218	0.214
Prepayment Option	9.146	0.437	11.271	0.814
$\sigma^2$			1.411	0.494
Log Likelihood		-2039.320		-2032.043

**Table 3: Models with Positive Relative Prepayment Option Value**

	<b>Model III</b>		<b>Model IV</b>	
	<b>(without Heterogeneity)</b>		<b>( with Heterogeneity)</b>	
	<b>Coefficient</b>	<b>Standard Error</b>	<b>Coefficient</b>	<b>Standard Error</b>
$\gamma(0)$	-7.542	1.006	-7.695	1.011
$\gamma(18)$	-5.694	0.427	-5.729	0.433
$\gamma(36)$	-4.294	0.286	-4.084	0.302
$\gamma(55)$	-3.318	1.011	-2.854	1.046
Spring	-0.066	0.146	-0.088	0.148
Summer	-0.251	0.159	-0.284	0.161
Fall	0.243	0.143	0.214	0.145
1986 Q4	0.507	0.210	0.438	0.214
Positive Prepayment Option	2.811	0.125	3.201	0.208
$\sigma^2$			0.667	0.317
Log Likelihood		-2081.872		-2077.981

**Table 4: Models with Spline for Relative Prepayment Option Value**

	Model V (without Heterogeneity)		Model VI (with Heterogeneity)	
	Coefficient	Standard Error	Coefficient	Standard Error
$\gamma(0)$	-6.647	1.019	-6.368	1.027
$\gamma(18)$	-4.662	0.497	-4.098	0.568
$\gamma(36)$	-3.005	0.345	-2.380	0.427
$\gamma(55)$	-2.428	1.047	-1.274	1.156
Spring	-0.201	0.148	-0.292	0.153
Summer	-0.436	0.162	-0.552	0.168
Fall	0.200	0.144	0.173	0.145
1986 Q4	0.330	0.211	0.249	0.215
Option Spline				
Option $\geq 0.1$	5.815	1.914	9.536	2.726
Option 0.0 to 0.1	20.443	3.088	21.733	3.616
Option -0.1 to 0.0	5.609	2.776	9.005	3.335
Option $\leq -0.1$	6.269	1.336	6.916	1.510
$\sigma^2$			1.184	0.448
Log Likelihood		-2031.198		-2024.794

**Table 5: Models with Indicators for Relative Prepayment Option Value**

	Model VII (without Heterogeneity)		Model VIII (with Heterogeneity)	
	Coefficient	Standard Error	Coefficient	Standard Error
$\gamma(0)$	-7.524	1.010	-7.590	1.012
$\gamma(18)$	-6.115	0.437	-6.107	0.448
$\gamma(36)$	-4.144	0.285	-3.990	0.294
$\gamma(55)$	-3.192	1.011	-2.697	1.052
Spring	-0.193	0.148	-0.237	0.150
Summer	-0.419	0.162	-0.482	0.165
Fall	0.203	0.145	0.191	0.146
1986 Q4	0.420	0.213	0.376	0.216
Option Indicators				
Option $\geq 0.1$	3.741	0.164	4.241	0.273
Option 0.0 to 0.1	2.102	0.191	2.456	0.260
Option -0.1 to 0.0	0.409	0.133	0.483	0.144
$\sigma^2$			0.704	0.349
Log Likelihood		-2044.730		-2040.987

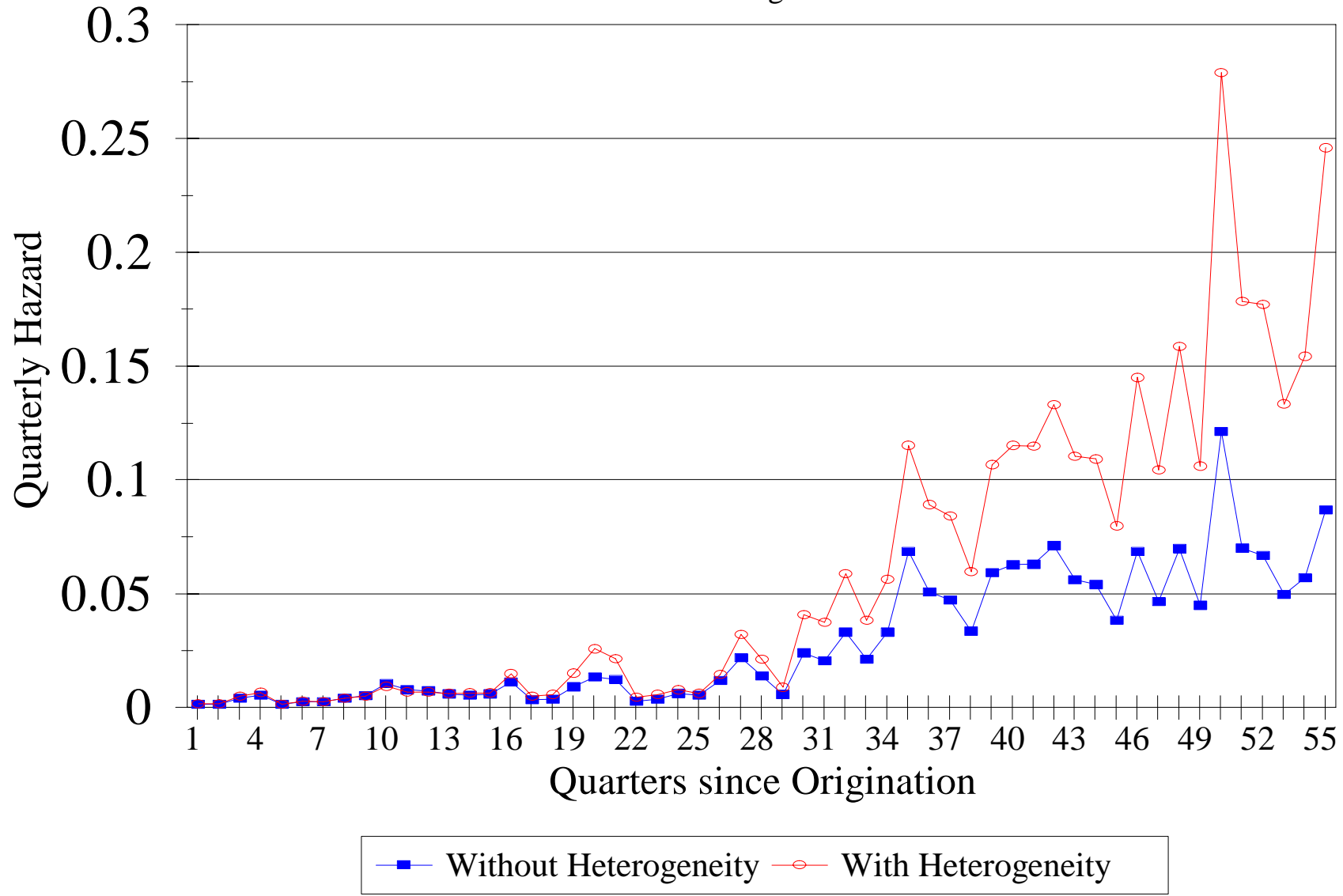


**Table 6: Models with Option Spline, Log Book Value and Housing**

	<b>Model IX (without Heterogeneity)</b>		<b>Model X (with Heterogeneity)</b>	
	<b>Coefficient</b>	<b>Standard Error</b>	<b>Coefficient</b>	<b>Standard Error</b>
$\gamma(0)$	-6.632	1.023	-6.437	1.031
$\gamma(18)$	-4.696	0.506	-4.196	0.574
$\gamma(36)$	-2.952	0.360	-2.371	0.427
$\gamma(55)$	-2.399	1.053	-1.265	1.152
Spring	-0.209	0.148	-0.296	0.152
Summer	-0.443	0.162	-0.559	0.168
Fall	0.191	0.144	0.160	0.145
1986 Q4	0.318	0.211	0.232	0.216
Option Spline				
Option $\geq 0.1$	5.188	1.947	8.747	2.789
Option 0.0 to 0.1	20.968	3.403	20.807	3.985
Option -0.1 to 0.0	5.730	2.985	8.588	3.466
Option $\leq -0.1$	6.587	1.349	7.316	1.532
Log Book Value	-0.072	0.260	-0.418	0.364
Log Housing Value	0.327	0.249	0.809	0.388
$\sigma^2$				
Log Likelihood	-2026.184		-2018.977	

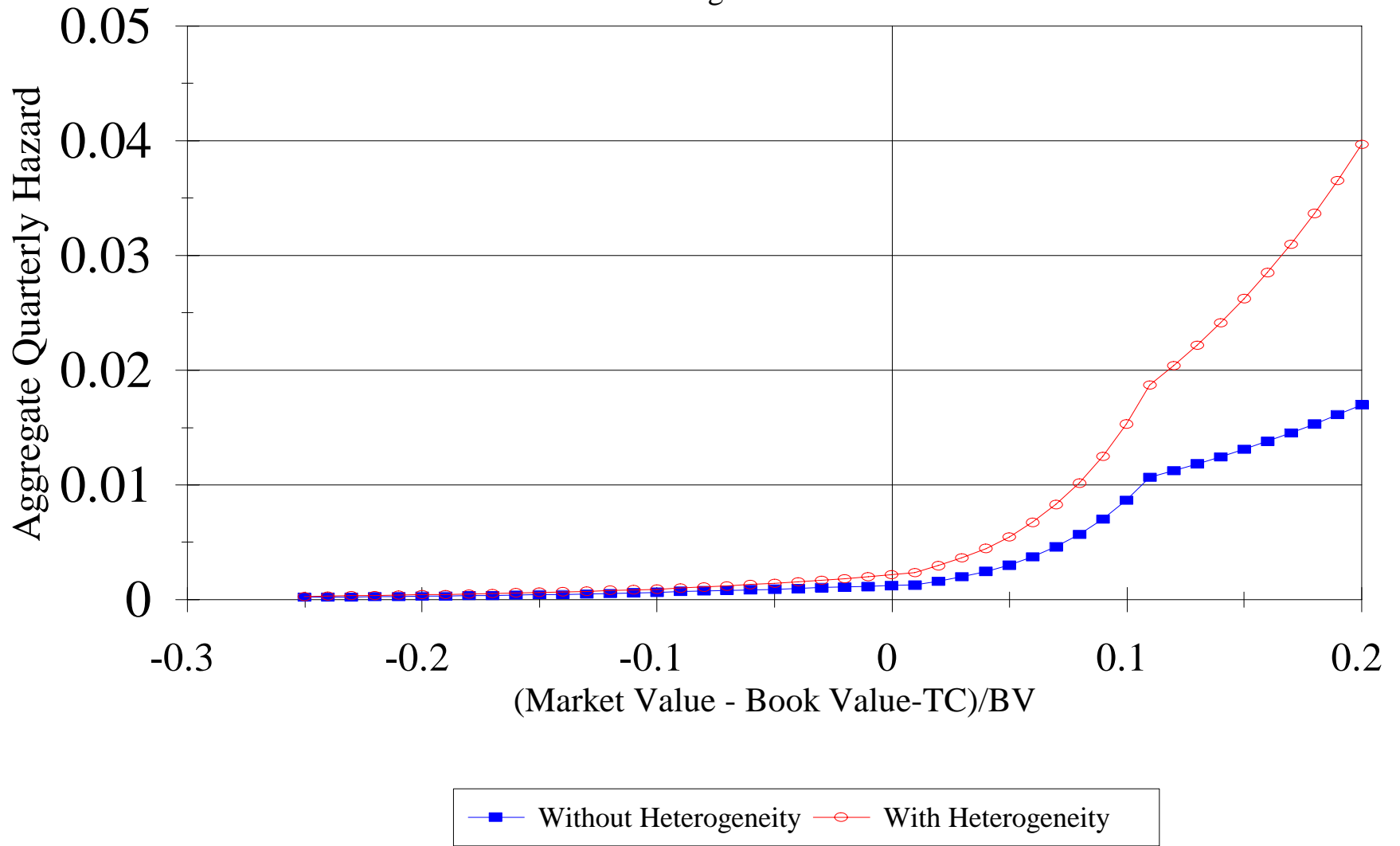
# Hazard Rates w and w/o Heterogeneity

Figure 1



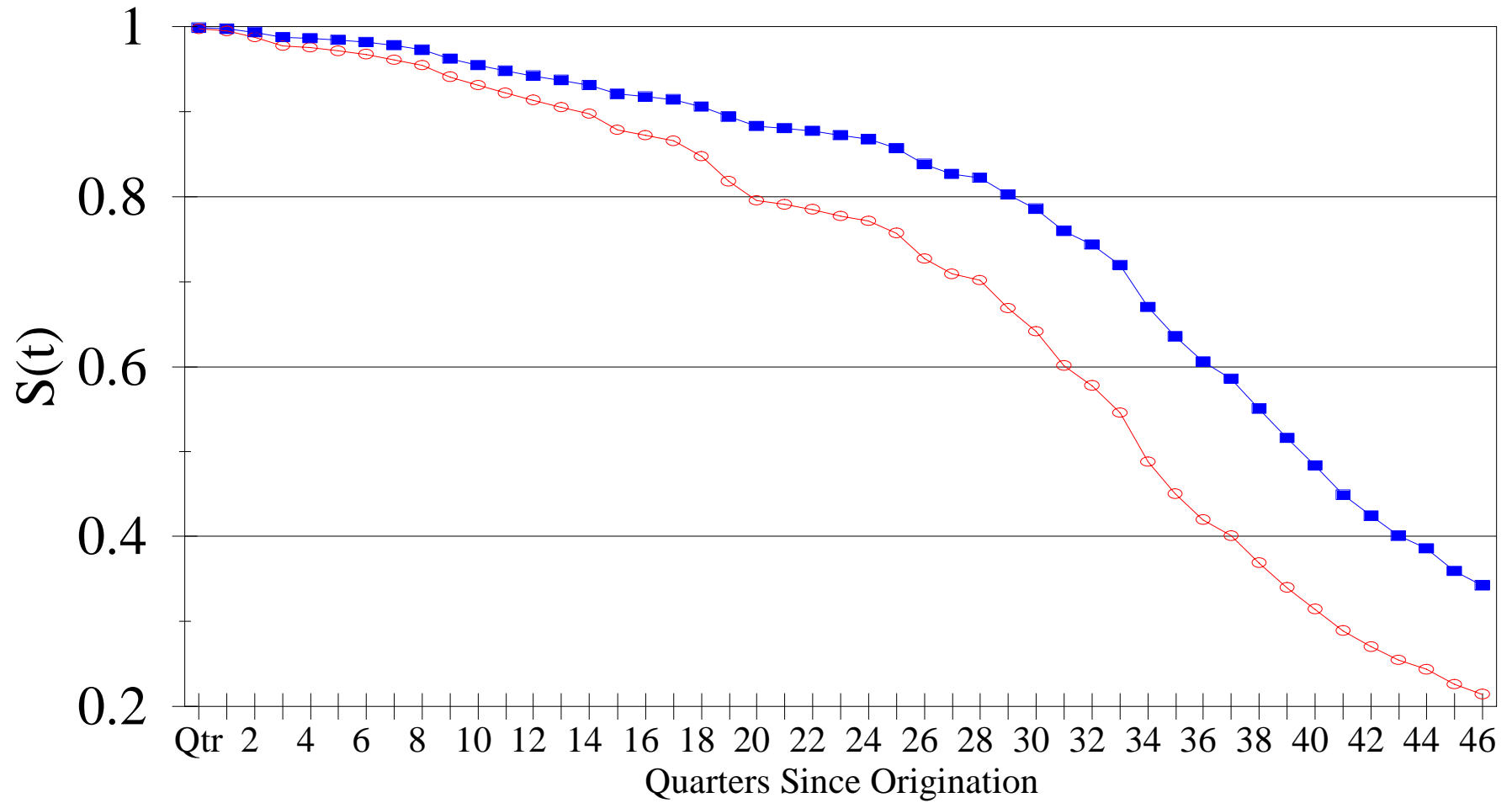
# Prepayment and Interest Rate Change

Figure 2



# Survivor Function

Figure 3



—■— Without Heterogeneity —○— With Heterogeneity

## Endnotes

1. In fact, the maximal observation period across mortgages spanned 56 quarters and under ideal conditions we would have had 56 baseline parameters to estimate. However, no mortgage in its 25th quarter was prepaid and we could not estimate a baseline parameter for this quarter independent of the other estimates. We decided on a smoothed estimate for this quarter, setting it equal to the simple mean of the 24th and 26th quarter estimates.
2. A call option on a share of stock allows its owner to purchase a share of a stock at a specified (exercise) price on or before a particular (exercise) date. The call option is said to be in the money if the market price exceeds the exercise price, because the owner of the option can earn a profit by exercising the option; a call option which is not in the money is said to be out of the money. The prepayment option can also be characterized as a call option because the borrower has the opportunity to buy his or her mortgage from the bank at its book or par value prior to maturity. Ignoring transactions costs for the moment, the prepayment option is in the money whenever the current mortgage interest rate moves below the contract rate because the borrower will realize savings by refinancing the mortgage.
3. Richard and Roll (1989) argue that interest rate volatility, an important component of option values, is impounded into the rate on newly originated mortgages. They point out that the logarithm of mortgage rates is well approximated as a linear function of the yield on a ten-year treasury bond and the logarithm of interest rate volatility. Higher levels of interest rate volatility lead to lower market values for the callable mortgage, reflecting a higher value of  $V_c(t)$ .
4. Indices for multifamily housing prices are unavailable.
5. Freddie Mac's Plan B program, which began in the early 1980s and features a yield maintenance prepayment penalty for the first five years of the mortgage. This penalty completely eliminates any benefits of exercising the prepayment option during these five years. For this reason, Plan B loans were not included in the sample.
6. The calculated hazard rates in Figure 1 are computed using the estimates from Models IX and X of Table 6. All covariates as well as  $EXOPT$  in the spline are set equal to zero.

## References

- Bogdon, A.S. and J.R. Follain. 1995. "Learning about Multifamily Housing with the 1991 Residential Finance Survey," Center for Policy Research, The Maxwell School, Syracuse University, mimeo.
- Chernoff, H. 1954. "On the Distribution of the Likelihood Ratio," *Annals of Mathematical Statistics*, 25: 573-578.
- Chen, R.R. and T.L. Yang. 1995. "The Relevance of Interest Rate Processes in Pricing Mortgage-Backed Securities," *Journal of Housing Research*, 6: 315-332.
- Chow, G. 1983. *Econometrics*. New York: McGraw-Hill.
- Cox, D.R. 1972. "Regression Models and Life Tables" (with discussion), *Journal of the Royal Statistical Society B*, 34: 187-220.
- Cox, D.R. 1975 "Partial Likelihood," *Biometrika*, 62: 269-276.
- DiPasquale, D., and J. Cummings 1992. "Financing Multifamily Rental Housing: The Changing Role of Lenders and Investors," *Housing Policy Debate*, 3: 77-116.
- Fabozzi, F.J. and F. Modigliani. 1992. *Mortgage and Mortgage-Backed Securities Markets*. Cambridge, MA: Harvard Business School Press.
- Follain, J.R. and E.J. Szymanoski. 1995. "A Framework for Evaluating Government's Evolving Role in Multifamily Mortgage Markets," *Cityscape*, 1: 151-177.
- Follain, J.R., L. Scott and T. Yang. 1992. "Microfoundations of a Mortgage Prepayment Function," *Journal of Real Estate Finance and Economics*, 5: 197-217.
- Godner, J.H. and K.T. Rosen. 1989. "Mobilizing the Multifamily Secondary Market," *Secondary Mortgage Markets*, 6: 2-5.
- Gouriéroux, C., A. Holly, and A. Montfort. 1982. "Likelihood Ratio Test, Wald Test, and Kuhn-Tucker Test in Linear Models with Inequality Constraints on the Regression Parameters," *Econometrica*, 50: 63-80.
- Hendershott, P.H. and D.C. Ling. 1984. "Trading and the Tax Shelter Value of Depreciable Real Estate," *National Tax Journal*, 37: 213-234.
- Horowitz, J.L. and G.R. Neumann. 1989. "Specification Testing in Censored Regression Models: Parametric and Semiparametric Methods," *Journal of Applied Econometrics*, 4: S61-S86.

- Kau, J.B. and D. Keenan. 1995. "An Overview of the Option-Theoretic Pricing of Mortgages," *Journal of Housing Research*, 6: 217-244.
- Lancaster, T. 1979. "Econometric Methods for the Duration of Unemployment," *Econometrica*, 47: 939-956.
- Meyer, B.D. 1987. *Semiparametric Estimation of Duration Models*, Ph.D. Thesis, MIT.
- Meyer, B. D. 1990. "Unemployment Insurance and Unemployment Spells," *Econometrica*, 58: 757-782.
- Moffitt, R. 1985. "Unemployment Insurance and the Distribution of Unemployment Spells," *Journal of Econometrics*, 28: 85-101.
- Prentice, R.L. and L.A. Gloeckler. 1978. "Regression Analysis of Grouped Survival Data with Application to Breast Cancer Data," *Biometrics*, 34: 57-67.
- Quigley, J. and R. Van Order. 1990. "Efficiency in the Mortgage Market: The Borrower's Perspective," *AREUEA Journal*, 18: 237-252.
- Richard, Scott F. and R. Roll. 1989. "Prepayments of Fixed-Rate Mortgage-Backed Securities," *The Journal of Portfolio Management*, 15: 73-89.
- Vandell, K. D. 1995. "How Ruthless Is Mortgage Default? A Review and Synthesis of the Evidence," *Journal of Housing Research*, 6: 245-269.

**METROPOLITAN STUDIES PROGRAM OCCASIONAL PAPER SERIES**

**(\$5.00 each)**

**(complete list available upon request)**

---

**166.**

**James R. Follain and Orawin T. Velz**

*Incorporating the Number of Existing Home Sales into a Structural Model of the Market for Owner-Occupied Housing*, August 1994, 36 pp.

**167.**

**Jan Ondrich, Stephen L. Ross, and John Yinger**

*Measuring the Incidence of Discrimination*, December 1994, 41 pp.

**168.**

**Dixie M. Blackley and James R. Follain**

*In Search of Empirical Evidence that Links Rent and User Cost*, January 1995, 32 pp.

**169.**

**Robert Carroll, Douglas Holtz-Eakin, Mark Rider, and Harvey S. Rosen**

*Income Taxes and Entrepreneurs' Use of Labor*, September 1995, 26 pp.

**170.**

**Stephen Ross and John Yinger**

*Sorting and Voting: A Review of the Literature on Urban Public Finance*, September 1995, 78 pp.

**171.**

**Robert M. Dunskey, James R. Follain, and Jan Ondrich**

*Using Recurrence Probabilities to Estimate the Volume of Multifamily Mortgage Originations*, November 1995, 21 pp.

**172.**

**Amy D. Crews, Robert M. Dunskey and James R. Follain**

*Estimating the Volume of Multifamily Mortgage Originations by Commercial Banks Using the Survey of Mortgage Lending Activity and the Home Mortgage Disclosure Act Data*, December 1995, 34 pp.

**173.**

**Amy D. Crews, Robert M. Dunskey and James R. Follain**

*What We Know about Multifamily Mortgage Originations and Why We Care*, December 1995, 36 pp.

**174.**

**William Duncombe, John Ruggiero, and John Yinger**

*Alternative Approaches to Measuring the Cost of Education*, January 1996, 46 pp.

**175.**

**William Duncombe and John Yinger**

*School Finance Reform: Aid Formulas and Equity Objectives*, January 1996, 53 pp.

**176.**

**John Yinger**

*The Incidence of Development Fees and Special Assessments*, February 1996, 26 pp.

**177.**

**James R. Follain, Jan Ondrich, and Gyan P. Sinha**

*Ruthless Prepayment? Evidence from Multifamily Mortgages*, March 1996, 35 pp.

---

Publications Officer  
Center for Policy Research  
The Maxwell School - Syracuse University  
426 Eggers Hall  
Syracuse, New York 13244-1090

Please send me \_\_\_\_\_ copies of Occasional Paper No. \_\_\_\_\_ at \$5.00 each.

NAME \_\_\_\_\_

ADDRESS \_\_\_\_\_

CITY, STATE, ZIP CODE \_\_\_\_\_

Postage is included. For air mail delivery to points outside the United States or Canada add \$5.00 per publication. Make checks payable to Center for Policy Research. Payment **MUST** accompany orders.