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Abstract

We use order statistics to analytically derive demand functions when consumers choose from among the varieties of two brands—such as Coke and Pepsi—and an outside good. Soft-drinks have no price variability across varieties within a brand, so traditional demand systems (e.g., mixed logit) are not identified. In contrast, our demand system is identified and can be estimated using a nonlinear instrumental variable estimator. Our demand functions are higher-order polynomials, where the polynomial order is increasing in variety. Because these demand curves have convex and concave sections around an inflection point, firms are more likely to respond and make large price adjustments to increases in cost than to comparable decreases in costs. We compare the profit-maximizing number of varieties within a grocery store to the socially optimal number and find that consumer surplus and welfare would increase with more variety.

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Key Words: Varieties, Product Line, Consumer Surplus, Welfare, Demand, Order Statistics

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Effects of Increased Variety on Demand, Pricing, and Welfare

How do Coke and Pepsi's product lengths—the number of varieties that each sells—affect consumers' brand choice? How does the shape of a brand's demand curve change when a retailer carries more varieties? How does variety affect optimal price adjustment? Are there too few or too many varieties? To address these and other questions, we use order statistics to develop a new theory of consumer choice across brands that have many varieties. The theory implies demand equations that are functions of brand preference parameters, which we estimate for Coke and Pepsi. Our estimated model does well in characterizing actual consumer behavior over brands. We use the estimates to determine the effects of changes in the number of varieties on demand curves and consumer welfare and to address welfare questions.

After picking the best variety within each brand (an order statistic), the consumer selects the best choice across the brands (an order statistic over order statistics) and compares that best choice to an outside good. Given that consumers' tastes are distributed uniformly, we derive a complete set of analytic results. Brand demand functions are higher-order polynomials, where polynomial order is increasing in the number of varieties. We analytically solve for the own- and cross-partial adjustments to consumers' demands from changes in the number of varieties.

The demand curves have both convex and concave sections around an inflection point. This shape leads to asymmetric price response to cost shocks: Firms are more likely to respond to and to adjust price more to a cost increase than to a comparable cost decrease.

According to food and beverage manufacturing executives, brands are maintained through product differentiation (e.g., Nijssen and Van Trijp, 1998). Firms constantly innovate to keep up with changing consumer tastes.[1] Products that are not accepted by consumers are quickly dropped. One might think of this approach of constantly providing new products as a flagpole strategy: "Let's run it up the flagpole and see who salutes it."[2] Firms differentiate by changing flavors or other aspects of the product (e.g., by altering the size or shape of the container).

We examine a market in which two firms (brands) produce many varieties of a good that sell for the same price within a brand. This is true for some clothing manufacturers, sports goods, yogurt, movie theaters, books, ice cream, and many beverages including teas, juices, and soft drinks (see, e.g., McMillan 2005, Draganska and Jain, 2005, Orbach and Einav 2007, and Heidhues and Koszegi 2008). For example, sporting good firms produce a variety of balls, gloves and shoes that differ only slightly in terms of which athlete endorses them or other aesthetic bells and whistles. Yogurts vary by flavor, whether the fruit is on the bottom, and in other ways. Ice creams vary by flavor and fat content. If there is no price variability across varieties within a brand, then traditional demand systems cannot identify cross-price elasticities over varieties.

Coca-Cola and Pepsi offer many varieties that vary by flavor, amount of caffeine, and number of calories. Coca-Cola sells many variations of its flagship product, including Coca-Cola, Cherry Coke, Diet Coke, Caffeine-Free Coke, Caffeine-Free Diet Coke, and Black Cherry Vanilla Coke. It also sells or has sold a wide variety of other soft drinks, including Tab, Sprite, Fresca, Fanta, Barq's Root Beer, Mello Yello, and Pibb Xtra, and, according to its website, it produces 700 low-calorie or no-calorie drinks throughout the world. Pepsi has a similarly large

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[1] In 2000 Snapple introduced, Diet Orange Carrot Fruit Drink ("Fruit Beverages Scope," *Beverage World*, February, 2000, p. 26), presumably reasoning that if they can sell that flavor, they can sell any flavor.

[2] We talked to representatives of Coca-Cola and Pepsi who stated categorically that they have firm policies of always pricing all varieties the same.

number of varieties. Retailers differ as to how many of these varieties they carry, but no retailer carries all of them. For simplicity in the following, we refer to the Coke brand and the Pepsi brand, whereas each company actually has several brands with many varieties in each.

Most existing theoretical works on product differentiation or product length focus on firms' behavior rather than on consumers' choices. Much of the theoretical literature abstracts from how consumers choose and assumes that there are only a small number of varieties (e.g., Brander and Eaton 1984, Gilbert and Matutes 1993, and Villas-Boas 2004). Other theoretical papers model demand using specific functional forms that depend on the total number of varieties. By employing explicit utility functions, they analyze the welfare effects of greater variety.

Four classic papers on product differentiation—Dixit and Stiglitz (1977), Spence (1976), Salop (1979), and Deneckere and Rothschild (1992)—assumed that each monopolistically competitive firm produces a single product and then asked if there are too many or too few products. The Chamberlin-representative-consumer competition papers of Dixit and Stiglitz (1977) and Spence (1976) used a constant elasticity of substitution (CES) utility function for a representative consumer that depends explicitly on the number of products. In the Hotelling-competition model of Salop (1979), consumers' tastes are uniformly distributed around a circle and products are evenly spaced around the circle (adjusting as new firms enter). Deneckere and Rothschild (1986) nested what they called a Chamberlin model (Perloff and Salop 1985) and the Hotelling-circle model (Salop 1979).

Some recent papers modified these models so that a firm has a product lines rather than only one good, but they maintain these functional assumptions. For example, Raubitschek (1987) allows brands to have varieties in the Spence-CES model, and Klemperer (1992) modified the circle model to allow for an endogenous determination of spacing of varieties.

Our work is closest to the Perloff and Salop (1985) and Anderson et al. (1992) models that show the effects of greater product diversity on prices, market share, and welfare. In those models and in the current model, consumers place a value on each product and then choose the variety with the largest net surplus: the consumer’s value minus the price. The values are drawn independently from a distribution. Each consumer buys one unit of one of these varieties if the net surplus from the consumer’s favorite exceeds that of an outside good.

There are two main differences between our model and those of Perloff-Salop and Anderson et al. First, those models assumed that each manufacturing firm produced a single product, whereas our model has two manufacturing firms (brands) each of which produces multiple varieties. Second, rather than focus on the decision of manufacturers, we examine the decision of the retailer (though we return to the manufacturers’ decision making at the end of the paper). The reason for this later difference is that, at least in grocery markets, retailers determine the number of varieties to carry rather than manufacturers, who produce a much larger number of varieties than any one retailer carries.

Our approach has several advantages over existing models. First, we can examine the effects of increases in the numbers of varieties of existing firms rather than by making the strong assumption that variety can only increase if the number of firms increases. Second, rather than make an ad hoc assumption about how the shape of the utility function or the demand curve varies with product length, we derive the shape of the demand curve from a model of consumer choice. Third, our model allows the effects of an increase in one firm’s varieties to be highly nonlinear and to vary interactively with its own price and the prices and varieties of rival products. Fourth, because our model produces closed-form, analytically tractable demand functions, it can be used to derive comparative statics results about firm behavior and welfare.

Finally, our model implies a nonlinear instrumental variable estimator that is identified when there is no price variability across varieties within a brand, which is likely to occur in soft-drink (and other) markets. Estimating traditional systems of demand functions for each variety, using mixed logit or other functional forms, is not feasible without price variation across varieties.

We start by using order statistics to derive a general model of how consumer choice varies with brands' product lengths and derive a number of analytic, comparative statics properties. We use a nonlinear instrumental variable approach to estimate a demand system for Coke, Pepsi, and an outside good. We examine the properties of the estimated demand system, and discuss the implications of the shapes of these demand curves for price adjustments. Finally we examine the welfare properties with respect to variety and price.

Varieties and Consumer Choice

In our model, each consumer buys one unit of a good by choosing among all the available varieties offered by both brands taking account of varieties' prices and the consumer's valuation of each variety. We assume that each brand charges the same price for all of its varieties, but the brand prices may differ. This phenomenon of setting identical prices for all varieties of a specific brand occurs in soft drinks as well as a number of other industries including clothing, yogurt, movie theaters, books, and many teas and juices (see McMillan 2005, Draganska and Jain, 2005, Orbach and Einav 2007, and Heidhues and Koszegi 2008).

The value that a consumer places on each variety is drawn independently from uniform distributions that may differ across brands. Each consumer picks the variety with the largest net surplus if that net surplus is greater than the net surplus provided by the outside good. Before plunging into our order-statistics model, we illustrate the basic idea with two simpler examples.

One Brand

Initially, suppose a monopoly grocery store carries varieties of only a single brand. Consumers choose their favorite variety, and the net surplus from the outside good is zero. A typical consumer places a value on each of the n varieties that is drawn independently from a uniform [0, 1] distribution. The price, p, lies within the range (0, 1) by appropriate scaling. The probability is $1 - p$ that a consumer will place a higher value on a given variety than its price. The probability that at least one variety is more valuable to the consumer than its price is 1 minus the probability that no variety has a value greater than the price: $1 - (1 - [1 - p])^n = 1 - p^n$.

The aggregate demand curve is the number of consumers, Z, multiplied by this probability: $[1 - p^n]Z$. (For simplicity, we henceforth normalize Z to equal one.) The slope of the demand curve with respect to price is $-np^{n-1} < 0$. If the number of varieties increases from n to $n + 1$, then the quantity purchased increases by $(1 - p)p^n$, which is positive because $p \in (0, 1)$. As n gets large, virtually everyone buys a variety from this brand.

Two Brands

Now, suppose that the grocery store carries two brands. The consumer might use a three-step procedure to pick which variety if any to buy. The consumer first picks the highest net-surplus variety within each brand, then the consumer selects between the two top choices for each brand, finally the consumer compares the best overall variety choice across the brands with the net surplus the consumer places on an outside product. If the best variety across brands is more attractive than the outside good, the consumer buys that variety.

We use a table (below) to illustrate the effects of adding one more variety for one brand. The numbers in the table are the net surplus a consumer obtains from each variety. The net surplus from the outside good is .5. Initially, the store carries two varieties of Brand 1 and only two variety of Brand 2 (the first two columns of the Brand 2 section of the table). Consumer A (row one) receives a net surplus of .9 from the first variety of Brand 1 and .6 from the second variety, so the consumer prefers the first variety. Similarly that consumer prefers the first Brand 2 variety (.3) to the second one (.1). This consumer buys one unit of Brand 1 because the net surplus from the preferred variety of Brand 1 (.9) exceeds the net surplus from the best Brand 2 option (.3) and the net surplus from the outside good (.5). In the table, the consumer's overall choice is indicated by expressing the relevant surplus in italics (ignore the bold type).

	Outside	**Brand 1**		**Brand 2**		
Consumer		*First*	*Second*	*First*	*Second*	*Third*
A	.5	***.9***	.6	.3	.1	.4
B	.5	.7	.6	.6	***.8***	.7
C	.5	.4	*.8*	.3	.6	**.9**
D	.5	.6	.3	***.7***	.4	.1
E	*.5*	.1	.2	.3	.2	**.6**

By similar reasoning, Consumer B buys the second variety of Brand 2, Consumer C buys the second variety of Brand 1, Consumer D chooses the first variety of Brand 2, and Consumer E opts for the outside good. In this market, 40% (2 out of 5) choose Brand 1, 40% choose Brand 2, and 20% consume the outside good.

Now suppose that the store starts carrying a third variety of Brand 2. In the table, the net surplus corresponding to the consumer's choice is in bold type. The extra variety affects the decisions of only Consumers C and E. Consumer C switches from buying the second variety of Brand 1 to the third variety of Brand 2. Consumer E changes from the outside good to the third variety of Brand 2. Brand 2's market share rises with the addition of another variety. The market shares are now 80% for Brand 2 and 20% for Brand 1.

Consumers A, B, and D are unaffected by the additional variety, while Consumers C and E are better off, so total consumer surplus must rise. Initially, the total surplus was .9 + .8 + .8 +. 7 + .5 = 3.7. After the third variety of Brand 2 is introduced, total surplus is .9 + .8 + .9 +. 7 + .6 = 3.9. This example illustrates how consumers benefit—have higher consumer surplus—from more choice holding price constant.

Order-Statistics Model

We now turn to a formal analysis of our model. We develop the model in four steps. First we discuss how a consumer would compare two sets of varieties if prices were zero and there is no outside good. Second, we introduce non-zero prices. Third, we allow the value distribution for each brand to have a different support, so that consumers might prefer one brand to another on average. Fourth, we introduce an outside good with a non-negative net surplus.

Distribution of the Difference of Independent Maxima

Each consumer's valuation of any variety of Brand 1 or Brand 2 is drawn independently from uniform distributions on [0, θ] with independent random sample of sizes n_1 and n_2 respectively, where n_1 is the number of varieties offered by Brand 1 and n_2 is the number of varieties offered by Brand 2. Let L_1 and L_2 be the maximal observations of a consumer's valuation of varieties for Brand 1 and Brand 2. That is, $L_1 = \max(L_{1,1},\dots,L_{1,n_1})$ and $L_2 = \max(L_{2,1},\dots,L_{2,n_2})$, where the valuations $L_{1,j}$ and $L_{2,j}$ are distributed independently uniform on [0, θ].

The distribution of the maximal valuation difference, $L_1 - L_2$, is the probability that a consumer selects Brand 1 or Brand 2, or the market shares (relative demand): $s_1 \in [0,\ 1]$ and $s_2 \in [0,\ 1]$. For now, everyone buys one unit of either Brand 1 or Brand 2—there is no outside good, so that $s_1 + s_2 = 1$. Also for now, we ignore prices. For example, suppose that a firm provides a free soft-drink at lunch, so that its employees simply have to decide which variety of which brand to choose independent of price. Let ${}_aC_b = a!/(a-b)!/b!$, then we derive

Proposition 1. The probabilities that the consumer chooses Brand 1 and Brand 2 are:

$$s_1(n_1,n_2) = \Pr(L_1 > L_2, n_1, n_2) = \sum_{j=1}^{n_1} (-1)^{j-1} \frac{{}_{n_1}C_j}{{}_{n_2+j}C_j},$$
$$s_2(n_1,n_2) = \Pr(L_1 \le L_2, n_1, n_2) = \sum_{j=1}^{n_2} (-1)^{j-1} \frac{{}_{n_2}C_j}{{}_{n_1+j}C_j}. \blacksquare$$

The proof is in the appendix. The share functions are finite hypergeometric sums, which can be reformulated in terms of the hypergeometric function (Gauss, 1813). In economic terms, cumulations in Proposition 1 are the probabilities of choosing Brand 1 ($L_1 > L_2$) or Brand 2 ($L_1 < L_2$), where the consumer's objective is to pick the Brand 1 or Brand 2 variety with the highest value (given that there is no outside good or price).

Using the Chu-Vandermonde identity for hypergeometric functions (Andrews et al., 2001, p.67), we can simplify the expressions for the shares:

Proposition 2. The shares in Proposition 1 can be reformulated as:[3]

$$s_1(n_1,n_2) = \frac{n_1}{n_1+n_2},$$
$$s_2(n_1,n_2) = \frac{n_2}{n_1+n_2}. \blacksquare$$

That is, the quantity share equals the share of varieties for each firm. If preferences are identical across consumers and the number of consumers is known, then multiplying the shares in

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[3] Murty (1955) derived a similar result when he considered the distribution of L_1/L_2 and calculated the probabilities: $\Pr(L_1/L_2 > 1) = n_1/(n_1 + n_2)$ and $\Pr(L_1/L_2 \le 1) = n_2/(n_1 + n_2)$. In the following sections, we compare the differences between L_1 and L_2 rather than their ratio because relocation of a random variate by a constant (e.g., price) is generally simpler than rescaling.

Proposition 2 by the number of consumers produces demand curves for Brand 1 and Brand 2, respectively. Therefore, as written these probabilities are relative demands or market shares.

Relocation of the Distribution of the Difference by Prices

We now introduce prices, where $p_1 > 0$ is the price for all varieties of Brand 1 and $p_2 > 0$ is the price for all varieties of Brand 2. The maximal net surplus of the choices are $\ell_1 = L_1 - p_1$ for Brand 1 and $\ell_2 = L_2 - p_2$ for Brand 2. The shares for each good are:

Proposition 3. The brand shares are a function of price differential $\pi = p_1 - p_2$:

$$\tilde{s}_1(n_1, n_2, p_1, p_2) = \sum_{j=1}^{n_1} (-1)^{j-1} \frac{{}_{n_1}C_j}{{}_{n_2+j}C_j} \left(1 - \frac{\pi}{\theta}\right)^{n_2+j} \text{ for } p_1 > p_2,$$

$$\tilde{s}_2(n_1, n_2, p_1, p_2) = \sum_{j=1}^{n_2} (-1)^{j-1} \frac{{}_{n_2}C_j}{{}_{n_1+j}C_j} \left(1 + \frac{\pi}{\theta}\right)^{n_1+j} \text{ for } p_1 < p_2. \blacksquare$$

The proof is in the appendix. These shares are functions of the price difference, π, and are higher-order polynomials in the number of varieties, where the polynomial order is increasing in variety. If $p_1 > p_2$, then $\tilde{s}_1$ is the share of Brand 1 purchased given the prices of Brand 1 and Brand 2, and $1-\tilde{s}_1$ is the corresponding share of Brand 2 purchased. If $p_1 < p_2$, then $1-\tilde{s}_2$ is the share of Brand 1, and $\tilde{s}_2$ is the share of Brand 2.

When $p_1 = p_2$, the shares in Proposition 3 are the same as those in Proposition 1: $\tilde{s}_1 = s_1$ = $\tilde{s}_2 = s_2$. Constraining π/θ to the unit circle—which is equivalent to constraining prices such that $p_1, p_2 \in (0,\ \theta)$ so that $-1 < \pi/\theta < 1$—then $(1-\pi/\theta)^{n_2+j}$ and $(1+\pi/\theta)^{n_1+j}$ are on the unit interval,

$$\tilde{s}_1 < s_1 \text{ and } \tilde{s}_2 > s_2 \text{ for } p_1 > p_2,$$

$$\tilde{s}_2 < s_2 \text{ and } \tilde{s}_1 > s_1 \text{ for } p_1 < p_2.$$

If Brand 1's price rises above Brand 2's price, then the share of Brand 1 falls ($\tilde{s}_1 < s_1$) and the share of Brand 2 rises ($\tilde{s}_2 > s_2$). We can derive the comparative statics properties of shares with respect to prices and varieties:

Proposition 4. The brands' shares in Proposition 3 have the following properties:

(a) Share $\tilde{s}_i$ is decreasing in p_i and is first convex to the origin then concave in p_i with an inflection point at $p_i = p_j$, $i \neq j$.

(b) $\frac{\Delta \tilde{s}_1}{\Delta n_1} > 0$, $\frac{\Delta^2 \tilde{s}_1}{\Delta n_1^2} < 0$, $\frac{\Delta \tilde{s}_1}{\Delta n_2} < 0$, $\frac{\Delta^2 \tilde{s}_1}{\Delta n_2^2} < 0$, and similarly for $\tilde{s}_2$.

(c) The "cross partial" with respect to the number of varieties is

$\frac{\Delta \tilde{s}_1}{\Delta n_1 \Delta n_2} = \frac{n_2+1}{n_1+n_2+2}\left[\left(1-\frac{\pi}{\theta}\right)\tilde{s}_1(n_1+1) - \tilde{s}_1(n_1)\right] + \frac{n_1+1}{n_1+n_2+2}\frac{\Delta \tilde{s}_1}{\Delta n_2}$ for $p_1 > p_2$. When prices are equal, $\frac{\Delta s_1}{\Delta n_1 \Delta n_2} = \frac{(n_2+1)}{(n_1+n_2+2)}\frac{\Delta s_1}{\Delta n_1} + \frac{(n_1+1)}{(n_1+n_2+2)}\frac{\Delta s_1}{\Delta n_2}$. If both prices are zero,

$\frac{\Delta s_1}{\Delta n_1 \Delta n_2} = \frac{n_2 - n_1}{(n_1+n_2+2)(n_2+n_1)}$, which is positive for $n_2 > n_1$ and negative for $n_1 > n_2$. ■

See the appendix for the proof.

Different Supports

So far, we have treated the two brands symmetrically. However, consumers may prefer one brand to another in the sense that overall consumers will buy more of one brand than the other if prices are equal. One way to capture the difference in how much consumers like one brand relative to another is to allow the uniform distributions for each brand to have different supports: L_1 $L_1 \in [0, \theta_1]$ and $L_2 \in [0, \theta_2]$. The difference in the upper bounds of the supports represent the extent to which some consumers prefer one brand over the other given they can choose their most preferred variety from each. That is, θ_1 and θ_2 are preference parameters.

Proposition 5. If brands' supports have different upper bounds:

$$\hat{s}_1(\theta_1,\theta_2,n_1,n_2,p_1,p_2)=\left(\frac{\theta_1}{\theta_2}\right)^{n_2}\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta_1}\right)^{n_2+j}\text{, for } p_1-\theta_1>p_2-\theta_2\text{,}$$

$$\hat{s}_2(\theta_1,\theta_2,n_1,n_2,p_1,p_2)=\left(\frac{\theta_2}{\theta_1}\right)^{n_1}\sum_{j=1}^{n_2}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1+j}C_j}\left(1+\frac{\pi}{\theta_2}\right)^{n_1+j}\text{, for } p_1-\theta_1\le p_2-\theta_2\text{. ■}$$

The proof is similar to that in the symmetric case discussed in the appendix. If $p_1 > p_2$, then $\hat{s}_1$ is the share of Brand 1 purchased given the prices of Brand 1 and Brand 2, and $1-\hat{s}_1$ is the corresponding share of Brand 2 purchased. If $p_1 < p_2$, then $1-\hat{s}_2$ is the share of Brand 1, and $\hat{s}_2$ is the share of Brand 2. The inflection point occurring at $p_1-\theta_1=p_2-\theta_2$.

If prices are zero or equal, these shares are:

$$\hat{s}_1(\theta_1,\theta_2,n_1,n_2,p_1,p_2)=\left(\frac{\theta_1}{\theta_2}\right)^{n_2}\frac{n_1}{n_1+n_2}\text{, for } \theta_2>\theta_1,\ p_1=p_2$$

$$\hat{s}_2(\theta_1,\theta_2,n_1,n_2,p_1,p_2)=\left(\frac{\theta_2}{\theta_1}\right)^{n_1}\frac{n_2}{n_1+n_2}\text{, for } \theta_2\le\theta_1,\ p_1=p_2\text{.}$$

Thus with equal prices, each firm's share is its share in Proposition 2, $n_1/(n_1 + n_2)$, adjusted by a multiplicative term that depends on the ratio of its preference parameters raised to a power equal to the number of varieties of the "more popular" brand. Each brand's share is increasing in its preference parameter and decreasing in its rival's.

Non-negatively Valued Outside Good

In Proposition 3 (where we incorporated prices), we implicitly ignored a possible problem that a consumer would choose a Brand 1or Brand 2 variety even though the net surplus for that good was negative. We could avoid that problem by starting the uniform distribution at a high enough level that a negative net surplus is impossible.[4] More realistically, we assume that consumers will not buy either Brand or 2, if an outside good gives them greater net surplus. The outside good has non-negative, non-random net surplus, $\omega \in [0,\ \min(\theta_1 - p_1,\ \theta_2 - p_2)]$, which, for simplicity, we assume is the same for all consumers. We partition the domain of the joint distribution of $\ell_1 \in [-p_1,\ \theta_1 - p_1]$ and $\ell_2 \in [-p_2,\ \theta_2 - p_2]$ into four regions and outcomes:

Proposition 6. The general shares for Brand 1 and Brand 2 are s_1^* and s_2^*:

$$s_1^* = \hat{s}_1 + \left[1 - \left(\frac{p_1+\omega}{\theta_1}\right)^{n_1}\right]\left(\frac{p_2+\omega}{\theta_2}\right)^{n_2} - \left(\frac{\theta_1}{\theta_2}\right)^{n_2}\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(\frac{p_2+\omega}{\theta_1}\right)^{n_2+j}$$
$$-\frac{n_2}{n_1+1}\left(\frac{\theta_1}{\theta_2}\right)^{n_2}\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1+1}C_j}{{}_{n_2-1+j}C_j}\left(\frac{p_2+\omega}{\theta_1}\right)^{n_2-1+j}\left[1-\left(\frac{p_1+\omega}{\theta_1}\right)^{n_1+1+j}\right],\quad p_1-\theta_1 > p_2-\theta_2,$$

$$s_1^* = \hat{s}_1 - \left(\frac{p_1+\omega}{\theta_1}\right)^{n_1} + \left(\frac{\theta_2}{\theta_1}\right)^{n_1}\sum_{j=1}^{n_2}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1+j}C_j}\left(\frac{p_1+\omega}{\theta_2}\right)^{n_1+j}$$
$$+\frac{n_1}{n_2+1}\left(\frac{\theta_2}{\theta_1}\right)^{n_1}\sum_{j=1}^{n_2}(-1)^{j-1}\frac{{}_{n_2+1}C_j}{{}_{n_1-1+j}C_j}\left(\frac{p_1+\omega}{\theta_2}\right)^{n_1-1+j}\left[1-\left(\frac{p_2+\omega}{\theta_2}\right)^{n_2+1+j}\right],\quad p_1-\theta_1 \le p_2-\theta_2,$$

$$s_2^* = \hat{s}_2 + \left[1 - \left(\frac{p_1+\omega}{\theta_1}\right)^{n_1}\right]\left(\frac{p_2+\omega}{\theta_2}\right)^{n_2} - \left(\frac{\theta_2}{\theta_1}\right)^{n_1}\sum_{j=1}^{n_2}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1+j}C_j}\left(\frac{p_1+\omega}{\theta_2}\right)^{n_1+j}$$
$$-\frac{n_1}{n_2+1}\left(\frac{\theta_2}{\theta_1}\right)^{n_1}\sum_{j=1}^{n_2}(-1)^{j-1}\frac{{}_{n_2+1}C_j}{{}_{n_1-1+j}C_j}\left(\frac{p_1+\omega}{\theta_2}\right)^{n_1-1+j}\left[1-\left(\frac{p_2+\omega}{\theta_2}\right)^{n_2+1+j}\right],\quad p_1-\theta_1 \le p_2-\theta_2.$$

$$s_2^* = \hat{s}_2 - \left(\frac{p_2+\omega}{\theta_2}\right)^{n_2} + \left(\frac{\theta_1}{\theta_2}\right)^{n_2}\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(\frac{p_2+\omega}{\theta_1}\right)^{n_2+j}$$
$$+\frac{n_2}{n_1+1}\left(\frac{\theta_1}{\theta_2}\right)^{n_2}\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1+1}C_j}{{}_{n_2-1+j}C_j}\left(\frac{p_2+\omega}{\theta_1}\right)^{n_2-1+j}\left[1-\left(\frac{p_1+\omega}{\theta_1}\right)^{n_1+1+j}\right],\quad p_1-\theta_1 > p_2-\theta_2. \qquad \blacksquare$$

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[4] For example, if the support for Brand 1 is $[a, \theta_1]$ and that for Brand 2 is $[a, \theta_2]$ with $\theta_1 > \theta_2$ and $p_1 = p_2$, and a larger than either price, then Brand 2's share is $s_2 = n_2(n_1+n_2)^{-1}(\theta_2 - a)^{n_1}(\theta_1 - a)^{-n_1}$.

See the appendix for a proof. These are Hicksian (income compensated) demand functions. The demand for the outside good (not buying Brand 1 or Brand 2) is the residual $s_{\omega}^{*} = 1 - s_1^{*} - s_2^{*}$.

Despite the relative complexity of the general share equations in Proposition 6, all of the qualitative comparative statics results of Proposition 4 still apply but with the inflection point occurring at $p_1 - \theta_1 = p_2 - \theta_2$:

Proposition 7. Share s_1^{*} is decreasing in p_1 and is first convex to the origin then concave in p_1 with an inflection point at $p_1 = p_2 - \theta_2 + \theta_1$. Share s_2^{*} is decreasing in p_2 and is first convex then concave in p_2 with an inflection point at $p_2 = p_1 - \theta_1 + \theta_2$.[5] ■

When the price of Brand 1 is above its demand curve's inflection point, the price of Brand 2 must be below its demand curve's inflection point, and vice versa. The inflection point is not a function of the numbers of varieties, n_1 and n_2, although increases in the number of varieties affect the curvature of the demand curve on either side of the inflection point.

Estimation

Our analytical results show that increasing the number of varieties of one brand can have complex effects on the demand curves and consumer welfare measures for both brands. To illustrate the role of variety on demand and on welfare, we start by estimating the simplest possible version of our model for Coke and Pepsi and an outside good in U.S. grocery stores.

Our simplest order-statistic model has three parameters: The maximum value a consumer receives from a Coke variety, θ_1; the maximum value for a Pepsi variety, θ_2; and the surplus (value net of price) of the outside good, ω. We then estimate other, more complex versions of

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[5] The proof is identical to the proof of Proposition 4a but with all results rescaled by $(\theta_1 / \theta_2)^{n_1}$ or $(\theta_1 / \theta_2)^{n_2}$ and with s_i^{*} replacing $\tilde{s}_i$, everywhere.

this model where these parameters are functions of consumers' demographic characteristics or vary across stores. However, these generalizations produce very similar results.

Coke and Pepsi are dominant oligopolistic firms that collectively accounted for three-quarters of the U.S. carbonated beverage market in 1999, the sample period for our empirical analysis (according to *Beverage Digest*). We are aware of 27 previous studies that estimate the demand elasticity of soft drinks. Most plausible studies that estimate the demand for Coke and Pepsi (e.g., Gasmi et al. 1992, Golan et al. 2000, Dhar et al. 2005, and Chan 2006) use data that are aggregated across varieties and consequently ignore the role of variety.

Because the prices of the various varieties of a brand are identical, it is not possible to identify the price coefficients in a traditional system of demand equations for all the varieties such as mixed logit or an Almost Ideal Demand System (AIDS). Nonetheless, a number of studies estimate such models and produce implausible cross-elasticity estimates.

Data

We use Information Resources Incorporated's (IRI) InfoScan® store-level scanner data for 1998 and 1999 to obtain 5,114 weekly observations for prices and quantities at 50 randomly chosen traditional grocery stores for each soft-drink variety (as determined by Universal Product Codes, UPCs).[6] The number of varieties that stores carry and the prices they charge vary across stores and over time within a store. These traditional grocery stores belong to 32 grocery chains. Some of the grocery chains are national giants such as Kroger, Albertsons, and Safeway, while others are relatively small, regional chains such as City Markets and Piggly Wiggly.

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[6] We have less than two full years of data for a few of the grocery stores that dropped out of the sample shortly before the end of the period. We also experimented with a panel of 100 stores for a single year (5,523 observations) and the results are very similar.

We restrict our analysis to 12-packs of 12 ounce cans, the best-selling package with 46% of the total observations in the canned soft drink category in our data. Each variety has an unique UPCs for the relevant package. Varieties differ by flavor, whether diet or regular, whether caffeinated or not, as well as how the products are packaged. Across all the stores in our sample, Coca-Cola has 27 varieties and Pepsi has 36 varieties.[7] Across all stores, the average annual number of varieties within each store is 10.86 (with a standard deviation of 2.76) for Coke and 9.08 (2.52) for Pepsi, ranging from 5 to 16 for Coke and 3 to 16 for Pepsi.

Each brand's store/week price is a quantity-weighted average obtained by dividing the total revenue in cents from all the products of the two firms by total volume in ounces. The price across varieties for a given brand is identical, but because of sales, the prices fluctuate over time (including sometimes within a week). The average price across stores for all Coke varieties is 2.347¢ per ounce and that for Pepsi is 2.363¢ per ounce, with standard deviations of 0.514 and 0.473 respectively, so that Coke and Pepsi prices are equal on average.[8]

For the outside good, we use soda products of the same package and size products manufactured by firms other than Coca-Cola Co. and PepsiCo. We estimate a constant outside good net surplus ω.

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[7] Coke varieties include Coke, Diet Coke, Coke Classic, Caffeine Free Coke Classic, Caffeine-Free Diet Coke, Citra, Diet Cherry Coke, Diet Sprite, Fresca, Mello Yellow, Minute Maid, Diet Minute Maid Orange, Minute Maid Strawberry, Minute Maid Grape, Minute Maid Fruit Punch, Mr. Pibb, Sprite, Surge, and Tab. Pepsi varieties include Pepsi, Diet Pepsi, Caffeine-Free Pepsi, Caffeine-Free Diet Pepsi, Diet Minute Maid, Diet Mountain Dew, Mountain Dew Caffeine Free, Mountain Dew Citrus, Mug Root Beer, Mug Cream Soda, Josta, Diet Wild Cherry Pepsi, Pepsi One, Diet Pepsi Lemon Lime, Slice Strawberry, Slice Grape, Slice Mandarin Orange, Slice Lemon Lime, Slice Red, and Wild Cherry Pepsi.
[8] The average price ratio of Pepsi to Coke across the stores is 1.02 with a standard deviation of 0.09. The correlation coefficient between Coke and Pepsi's price is 0.89.

Share Estimates

We have observations over week t for store i. Our three-equation system is obtained by adding a random error to the Coke and Pepsi share equations from Proposition 6:

$$s_{1it}^{*} = s_{1}^{*}(n_{1it}, n_{2it}, p_{1it}, p_{2it}, \theta_1, \theta_2, \omega) + u_{1it},$$
$$s_{2it}^{*} = s_{2}^{*}(n_{1it}, n_{2it}, p_{1it}, p_{2it}, \theta_1, \theta_2, \omega) + u_{2it},$$
$$s_{1it}^{*} + s_{2it}^{*} = 1 - \left(\frac{p_{1it}+\omega}{\theta_1}\right)^{n_{1it}} \left(\frac{p_{2it}+\omega}{\theta_2}\right)^{n_{2it}},$$

where the shares are measured by volume. (The revenue shares are virtually identical as there is very little variation in the average absolute and relative prices of Coke and Pepsi.) We estimate this three-equation system using a simultaneous nonlinear instrumental variables method described by Davidson and MacKinnon (1993, p. 664, eqn. 18.82).[9]

In our system of brand-share equations, we allow for the possibility that the prices and varieties are endogenously determined by using instrumental variables. Our instruments include cost shifters at the national level for the soft drink industry, the national share of each chain, and milk sales within each store (IRI). The cost shifters are the producer price index (PPI, from the Bureau of Labor Statistics) for high-fructose corn syrup, which is the main sweetener used in most soft drink; the PPI for aluminum, which is used to make cans; and the PPI for industrial electricity, and the national PPI for gasoline price interacted with city dummies, which is a proxy for variations in transportation costs across cities. We assume that national cost shifters are correlated with prices but are not correlated with underlying consumer preferences that vary from store to store or over time. The national shares of the chains are used as proxies for possible monopsony power. The within-store milk sales variable is a proxy for the size of the store.

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[9] We estimated the model using the unconstrained nonlinear estimation routine in MatLab. At each step in the algorithm, the theoretical constraints, $\omega \in [0, \min(\theta_1 - p_1, \theta_2 - p_2)]$ were checked, revealing no violations. Were there violations, we would have used a constrained estimation algorithm. Davidson and MacKinnon (1993) discuss identification in this simultaneous nonlinear instrumental variables model.

Presumably larger stores carry more varieties due to lower shelf-space costs; however, store size should not be explicitly correlated with the error terms of our share equations. We also include squared terms of these instruments as additional instruments because prices and varieties enter the share equations nonlinearly (Davidson and MacKinnon, 1993).

Our order-statistics model estimates are very precise: $\theta_1 = 25.62$ (with an asymptotic standard error of 0.01), $\theta_2 = 25.21$ (0.01), and $\omega = 21.02$ (0.01). These results indicate that, all else equal, consumers slightly prefer Coke's soft drinks to Pepsi's: $\theta_1 = 25.62 > \theta_2 = 25.21$.

This terse model fits the data well. Coke's average share in the data is 0.49, our model's average predicted share is 0.49, and the correlation coefficient between the actual and predicted share is 0.54. Similarly for Pepsi, the share is 0.26, the average predicted share is 0.27, and the correlation is 0.31. For the outside good, the corresponding numbers are 0.25, 0.24, and 0.16.

Robustness Checks

We estimated three variations of our basic model with an outside good to check for the robustness of our results.[10] First, we eliminated the main variety for each brand (Coke and Pepsi), which have very large market shares, and re-estimated the model. The estimated coefficients and the fit are virtually unaffected.

Second, we generalized our model so that the three coefficients were linear functions of the average household income and the average number of family members in a household at a store. Doing so reduced the correlation coefficients (the fit of the model), and the coefficients on the

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[10] As an experiment, we also tried to estimate a mixed-logit model where the shares of the goods are a function of the prices, the number of varieties, a Coca-Cola dummy, and store dummies. The dependent variables are the logarithm share of Coca-Cola minus the log outside share and the log Pepsi share minus the log outside share. We used the same instruments as for our model. We randomly drew errors from a normal distribution with zero mean. Although the mixed logit fit well—about the same as our model—the mixed-logit price coefficients were not statistically significantly different from zero because the prices of the varieties tend to move in lock-step within a brand within a store.

demographic variables were not statistically different from zero. However, the estimated model was very close to our three-parameter version.

Third, using a sample of 106 stores over a single year (5,523 observations), we estimated a model that allowed the three parameters to vary over stores. That is, $\theta_{1i} = \theta_1 + \delta_{1i}$; $\theta_{2i} = \theta_2 + \delta_{2i}$; and $\omega_i = \omega + \delta_{\omega i}$, where the δ's are estimated coefficients for each store. Presumably, the store dummies capture differences in all (average) demographic and other variables that vary across stores. The results of this flexible model were similar to the three-parameter model. The basic parameter estimates at the means were similar: $\theta_1 = 25.57$, $\theta_2 = 25.39$, and $\omega = 21.46$, which are close to those from our simpler, three-parameter specification: 25.62, 25.21, and 21.02, respectively. Also, the estimated δ's were relatively small in magnitude (the average is 0.13 with a standard deviation of 0.26). Moreover, 98% of the estimated θ_{1i} and θ_{2i} were between 25 and 26, with the three exceptions being for a few stores with very high Pepsi prices and low Pepsi varieties. In all cases, Coke was preferred to Pepsi at the store level ($\theta_{1i} > \theta_{2i}$). Thus, we use our simple three-parameter model in the following simulations as it simplifies the analysis and does not differ substantially from the more general estimation models.

Properties of Cola Demand Curves

Using our estimated values for θ_1, θ_2, and ω, we can calculate demand curves and welfare measures and examine the comparative statics properties, which illustrate our analytic results. Unless otherwise stated, our results are calculated at sample average prices of $p_1 = 2.35$¢ per ounce for Coke and $p_2 = 2.36$¢ per ounce for Pepsi, and the sample average number of varieties of $n_1 = 11$ for Coke and $n_2 = 9$ for Pepsi.

Demand Curves

At the sample average prices, the point estimate of Pepsi's own share elasticity with respect to price is –1.5. That is, a 1% increase in the price of Pepsi, holding the price of Coke and the number of varieties fixed, lowers Pepsi's share by 1.5%. The corresponding point estimate of Coke's elasticity is –0.9. These elasticities show how share or the quantity demanded changes as the price of *all* the brand's varieties change at once. These estimated elasticities lie within the range of elasticities (–0.14, –2.59) estimated in the 10 traditional demand studies that use aggregate data and are, hence, comparable.

The demand curves in Figure 1 panels a and b, illustrate the results in Proposition 2 that the demand curves are first convex then concave in own price. The Pepsi demand curves in panel have an inflection point at $p_2 = p_1 - \theta_1 + \theta_2$, and the Coke demand curves in panel b have an inflection point at $p_1 = p_2 - \theta_2 + \theta_1$.

By varying the number of varieties, we can show how the demand curves shift, thereby illustrating our analytic partial and cross-partial derivative results. As the effects of a change in the number of varieties is qualitatively the same for both Coke and Pepsi demand curves, in Figure 1 panels a and b, we show how the Pepsi demand curve and the Coke demand curve change as we increase the number of Pepsi varieties, holding the number of Coke varieties and the rival's price fixed.

In Figure 1 panel a, we fix the price and varieties of Coke at their sample averages, p_1 = 2.35¢ per ounce and n_1 = 11. This figure shows the effect of an own variety change. Moving from the demand curve on the left to the one on the right, we increase the number of Pepsi varieties in increments of two from n_2 = 7 to 9 (the average in the sample) and then to 11. The figure shows that, as the number of varieties of Pepsi increases, Pepsi's demand curve rotates around the price-axis intercept, becoming flatter.

We illustrate the effect of the increase in the number of Pepsi varieties on the Coke demand curve in panel b of Figure 1. As the number of Pepsi varieties increases from n_2 = 7 to 9 to 11, the Coke demand curve rotates in around the price-axis intercept. Thus, an increase in the number of Pepsi varieties causes the Coke demand curve to become steeper.

As expected, these graphs show that a change in the number of Pepsi varieties has a larger (own) effect on Pepsi's demand curve than its (cross-partial) effect on Coke's demand curve. Similarly, changes in the number of Coke varieties have a larger effect on its demand curve than on Pepsi's. At the sample mean prices, increasing the number of Pepsi varieties from 9 to 10 increases Pepsi's share by 0.02, reduces Coke's by 0.01, and reduces the share of the outside good by 0.01 (= 0.02 – 0.01).

Panel c of Figure 1 illustrates cross-partial effects for Pepsi's demand curve. The central demand curve is evaluated at the sample averages where n_1 = 11 and n_2 = 9. The one to its left has two fewer varieties of each brand, n_1 = 9 and n_2 = 7, while the one to its right has two additional varieties, n_1 = 13 and n_2 = 11. When the numbers of varieties of both brands rise, fewer consumers buy the outside good, so the demand curves for both brands shift to the right. The effect of increasing the numbers of varieties for both brand equally is larger for Pepsi than for Coke because there are initially more Coke varieties.

As the outside good's net surplus rises, consumers who receive a relatively low net surplus from their brand switch to the outside good. Figure 2 shows the degree to which Pepsi's demand curve shifts to the left as the net surplus on the outside good increases by 5 percent increments.

Consumer Surplus

The appendix derives the formulas for calculating consumer surplus. Panel a of Figure 3 shows that consumer surplus is increasing at a decreasing rate in the number of varieties of both goods. The figure is slightly asymmetric because consumer prefer Coke to Pepsi ($\theta_1 > \theta_2$).

Panel b of Figure 3 shows the corresponding "iso-welfare" curves, where each curve holds consumer surplus constant and the number of varieties of each brand vary (treating the number of varieties as a continuous variable). These are curves are horizontal slices of the three-dimensional surface in panel a. These curves are virtually straight lines with slope less than –1. That is, consumers are willing to trade slightly more than one Pepsi variety for a single Coke variety to keep consumer surplus constant. This slight deviation from –1 is the result of a slight preference for Coke: $\theta_1 - p_1 > \theta_2 - p_2$. Indeed, the slope becomes slightly more negative as the number of varieties increase, so that the iso-welfare lines are not parallel. That is as varieties increase, it takes a greater increase in Pepsi varieties to offset the loss of one Coke variety. This same effect appears in panel a. Total surplus is increasing at a decreasing rate in variety, but the rate decreases more slowly for Coke than for Pepsi. The cross-partial analysis is consistent with these figures: If we increase by one the number of varieties of both goods, Coke's share rises by 0.02, which is more than Pepsi's share increases, 0.01.

Consumer surplus varies with the number of varieties and price, as Figure 4 demonstrates. In panel a, as the number of varieties of Coke increases holding the number of varieties of Pepsi and prices fixed, the consumer surplus of Coke rises, while that of Pepsi and the outside good fall.[11] Consequently, total consumer surplus rises, but only slightly as the gain to Coke barely exceeds the combined losses from Pepsi and the outside good.

As the price of Coke increases, holding the price of Pepsi constant, consumer surplus from Coke falls, while the consumer surplus from Pepsi and that from the outside good rise, as panel b of Figure 4 illustrates. As the price gets very large, total consumer surplus levels off and there is little decrease in total surplus because virtually all consumers have switched to the other goods.

[11] The curves are interpolated to smooth the discrete changes in the varieties of Coke.

Pricing Implications of the Shape of the Demand Curves

The convex-concave shape of the demand curves has implications for price setting behavior by retailers and manufacturers (whose derived demand curves must have this shape). For example, Cowan (2009) shows that the degree of price discrimination and the welfare effect from third-degree price discrimination depend on concavity or convexity of demand.

More importantly, the shape affects the rate of adjustment and the probability of a price adjustment. To illustrate the importance of the shape of the demand curve for the price response to a shift in marginal cost, we assume that the product length is arbitrarily fixed and consider the monopoly's pricing problem (the same type of argument holds for oligopolies). If the monopoly faces a constant marginal and average cost of m, the monopoly's profit is $[p(Q) - m]Q$, so its first-order condition is $Qp'(Q) + p(Q) - m = 0$. Totally differentiating the first-order condition, we learn that $\mathrm{d}Q/\mathrm{d}m = 1/[2p'(Q) + Qp''(Q)]$. By the chain rule, the change in the price in response to a change in the marginal cost is $\mathrm{d}p/\mathrm{d}m = p'(Q)\mathrm{d}Q/\mathrm{d}m = p'(Q)/[2p'(Q) + Qp''(Q)]$. The numerator is the slope of the inverse demand curve and the denominator is the slope of the marginal revenue curve. Dividing both the numerator and the denominator by $p'(Q)$, we can rewrite this expression as $\mathrm{d}p/\mathrm{d}m = 1/[2 + x]$, where $x = Qp''(Q)/p'(Q)$. Thus, if the demand curve is linear, $p''(Q) = 0$ so $x = 0$ and $\mathrm{d}p/\mathrm{d}c = ½$. If $p''(Q) < 0$, then $x > 0$ and $\mathrm{d}p/\mathrm{d}c < ½$. Finally, if $p''(Q) > 0$, then $x < 0$ and $\mathrm{d}p/\mathrm{d}c > ½$.

Thus, given our demand curves have alternating curvature around the inflection point at the initial symmetric equilibrium, the firm's price adjustment is asymmetric with respect to an increase or a decrease in cost. Holding the number of varieties constant, a cost shock has a larger price and smaller quantity effect for a positive cost shock than for a negative cost shock.

The shape also has implications for whether a firm adjusts its price if it incurs an adjustment or menu cost when it changes its price. It pays for a firm to adjust its price if the increase in gross profit from adjusting the price (ignoring adjustment costs) exceeds the menu cost. Thus, given that the shape of the demand curve causes asymmetric price adjustments, a firm is more likely to adjust its prices at all when faced with a positive cost shock rather than to a comparable size negative shock.

Optimal Varieties

We can use our estimated model to investigate whether grocery stores carry the socially optimal number of varieties. Our welfare analysis considers the welfare of consumers and grocery stores only, ignoring manufacturers' welfare. All else the same, consumers benefit from greater variety, but grocery stores must consider the cost of carrying extra varieties.

A grocery stores' profit is

$$\Pi = (p_1 - m)s_1^* Z + (p_2 - m)s_2^* Z + (p_\omega - m_\omega)s_\omega^* Z - (cn_1 + kn_1^2) - (cn_2 + kn_2^2)$$

where m is the common wholesale price for Coke and Pepsi; m_ω is the wholesale price of other soft drinks; Z = total ounces of all soft-drinks including the outside good; s_1^* and s_2^* are functions of all the prices, varieties, marginal costs, and estimated parameters θ_1, θ_2, and ω; and the share of the outside good is $s_\omega^* = 1 - s_1^* - s_2^*$.

In our simulations, we assume that grocery stores are price takers.[12] Consequently, their only control variables are the numbers of varieties of Coke and Pepsi that they carry.[13] Presumably

[12] As many types of grocery, drug store, warehouse stores, restaurants, and other retailers sell soft drinks, grocery stores are probably price takers. The average coefficient of variation of prices across stores is small despite frequent sales: 0.13 for Coke and 0.11 for Pepsi, and 0.18 for the ratio of the Coke to Pepsi prices (with a mean of 1.01). The average Coke and Pepsi prices vary little across stores within a city.

[13] Using one weekly observation per store, we regressed the prices of Coke and Pepsi and of the price difference between Coke and Pepsi on the number of Coke and Pepsi varieties, where the instruments were mean household size, mean household income, local grocery chain share, the store's milk sales, and

the store's costs vary with the number of varieties they carry for a given brand due to shelf-space opportunity costs as well as storage, accounting, and other costs. The store incurs shelf-space costs, inventory costs, label costs, and other expenses from carrying an additional variety. (It is possible that some of these costs are offset by a slotting fee that a manufacturer pays when a grocery store agrees to carry one more variety.) We assume that the (net) cost with respect to varieties is quadratic: $cn_i + kn_i^2$.

The store's first-order conditions to maximize profit with respect to a discrete (partial) change in the number of varieties of each brand are:

$$\frac{\Delta\Pi}{\Delta n_1} = (p_1 - m)\frac{\Delta s_1^*}{\Delta n_1} Z + (p_2 - m)\frac{\Delta s_2^*}{\Delta n_1} Z + (p_\omega - m_\omega)\frac{\Delta s_\omega^*}{\Delta n_1} Z - c - k(2n_1 + 1) = 0$$

$$\frac{\Delta\Pi}{\Delta n_2} = (p_1 - m)\frac{\Delta s_1^*}{\Delta n_2} Z + (p_2 - m)\frac{\Delta s_2^*}{\Delta n_2} Z + (p_\omega - m_\omega)\frac{\Delta s_\omega^*}{\Delta n_2} Z - c - k(2n_2 + 1) = 0$$

That is, the firm sets the number of varieties so that the marginal profit from the last variety (the first three terms) equals the marginal cost of one more variety (the last two terms). In our simulations, we assume that these first-order conditions hold at the sample average where the typical grocery store carries 11 varieties of Coke and 9 of Pepsi.

Given our demand estimates and marginal costs, we can solve this system of two equations for the two unknowns, c and k. According to a major supermarket chain that we consulted, the marginal cost is about 30% to 40% of price (depending on the frequency of sales). We assume that m is 1¢ per ounce (about 42% of the observed price) and m_ω is 1.5¢ per ounce.[14]

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the PPI for high fructose corn syrup. The coefficients for all the variables and the asymptotic t-statistics were virtually zero. Thus, there is no systematic evidence of a relationship between a store's prices and the number of varieties it carries.

[14] We set the wholesale price of the outside good higher than that of Coke and Pepsi because otherwise we often get a corner solution. If we vary the marginal cost measures proportionately, our qualitative results still hold. According to our interviews at a major supermarket chain, it charges low prices for Coke

We can use our "estimates" of *c* and *k* to compare the optimal number of varieties under profit maximization to three possible outcomes determined by a social planner who is interested in maximizing welfare, defined as consumer surplus plus profit (for soft drinks):

(a) the *social optimum*, where a social planner maximizes social welfare by setting the Coke, Pepsi, and outside good prices and Coke and Pepsi varieties;

(b) the second-best *varieties-only optimum*, where the planner sets only the number of varieties for Coke and Pepsi;

(c) the second-best *prices-only optimum*, where the planner sets the price of Coke, Pepsi, and the outside good.

These two second-best approaches provide a means of decomposing the welfare gain from the social optimum approach, so that we can determine if most of the gain comes from controlling varieties or prices.

We solve for the social optimum by setting prices equal to their marginal cost (subsidizing fixed costs as necessary), and then choosing the numbers of varieties to maximize welfare by doing an exhaustive comparison of welfare for all plausible pairs of numbers of varieties. The planner sets the marginal cost of an extra variety equal to the marginal welfare from one more variety, where the marginal welfare is the marginal profit plus the marginal consumer surplus. That is, the planner's marginal benefit contains one more term, the marginal consumer surplus, than does the firm's marginal benefit.

The first column of Table 1 reports sample average profit-maximizing solution, which we compare to the social optimum (second column) and to the second-best varieties-only optimum, where the planner sets only varieties (third column) and the prices-only optimum (fourth

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and Pepsi to attract customers, even though they make larger profits on their generic soft-drink sales.

column). The social optimum prices are, of course, lower than in the unregulated case. When we switch from the profit-maximizing solution to the social optimum, the numbers of varieties increase from 11 to 26 for Coke, n_1, and from 9 to 11 for Pepsi, n_2. In the varieties-only optimum, n_2 increases by the same as in the social optimum, but n_1 increases more than in the social optimum to 27. We conclude that society would be better off if a typical grocery carried substantially more Coke varieties and slightly more of Pepsi varieties.

In our typical store, our weekly consumer surplus, *CS*, is $48,729 without regulation, $52,662 (8.1% higher than without regulation) in the social optimum, $49,719 (2.0% higher) in the varieties-only optimum, and $51,510 (5.7% higher) in the prices-only optimum. Thus, most of the gain in consumer surplus is due to regulating price rather than the number of varieties.

Welfare is $51,176 without regulation, $51,825 (1.3% higher than without regulation) in the social optimum, $51,805 (1.2% higher) in the varieties-only optimum, and $51,237 (0.1% higher) in the prices-only optimum. Thus, in contrast to the *CS* results, most of the welfare gain is due to regulating varieties rather than prices.

Varying the marginal costs, we find qualitatively similar effects, but the size of the price effect rises when the marginal cost falls, which would occur if manufacturers' wholesale prices were capped by a regulator. We conclude that consumers and society as a whole would benefit from more varieties of Coke and Pepsi. However, even with extreme marginal cost values, regulating varieties raises consumer surplus and welfare by only relatively small percentages.

Summary and Conclusions

We develop a new theory of consumer choice conditional on the number of varieties provided by firms. As the number of varieties of one brand—its product length—increases holding prices and number of varieties of other firms fixed, consumers are more likely to buy that brand, as more consumers will find a variety that they prefer to those of the other brand and to the outside good. Rather than imposing an explicit functional form on utility, we derive consumers' demand functions for each brand using order statistics where consumers' valuations of varieties are distributed independently uniform. Consumers rank choices based on net surplus—valuation minus price—of the varieties of two brands and an outside good.

One advantage of the new model is that it allows us to examine many questions about variety analytically. We derive analytic partial derivatives that show how changes in price and varieties affect demand. A second advantage of this model is that it allows us to estimate a system of demand for markets in which all varieties of a brand sell for the same price. Mixed logit, AIDS, and other standard models cannot be used because the prices of all varieties are identical.

We fit our model for Coke, Pepsi, and an outside good using U.S. grocery store data. We show both analytically and empirically that the brand demand curves have a convex and a concave section around an inflection point. Given that the prices of Coke and Pepsi are generally very close to each other, stores operate at or near the inflection point. As a result, a positive cost shock has a larger price effect and a smaller quantity effect than does a comparable negative cost shock. Given that grocery stores face adjustment or menu costs, stores are more likely to adjust price when faced with a positive cost shock than with a negative cost shock. Our simulations indicate that society would benefit if stores carried more varieties.

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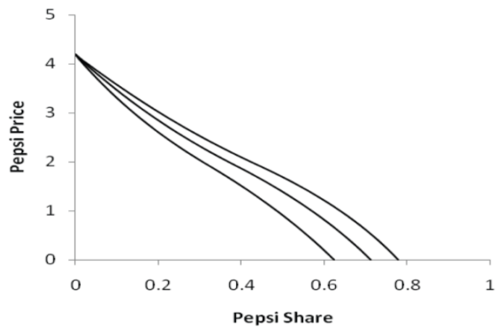
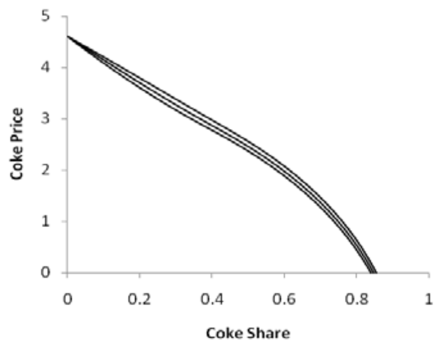
FIGURES AND TABLES

Figure 1. Effect on Demand Curves as the Number of Pepsi Varieties Change

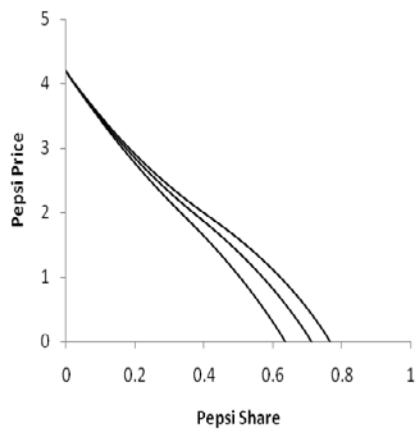
(a) Shift in Pepsi's demand curve as the number of Pepsi varieties increases from n_2 = 7 to 9 to 11 (left to right)



(b) Shift in Coke's demand curve as the number of Pepsi varieties increases from n_2 = 7 to 9 to 11 (right to left)



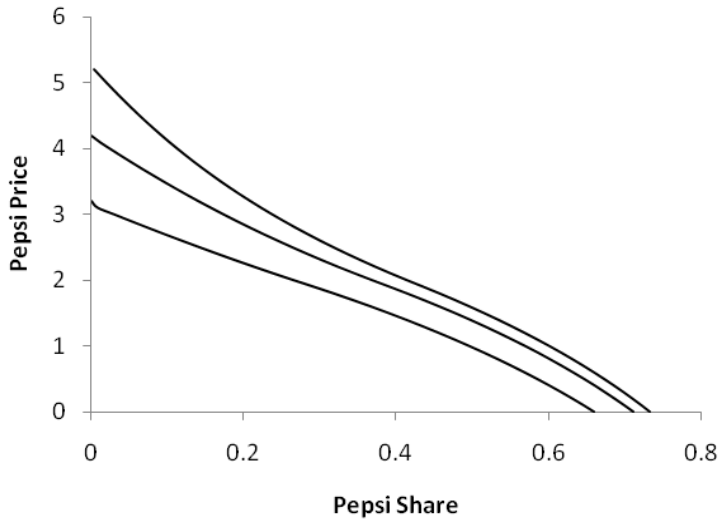
(c) Pepsi demand curves: Cross-partial effect (left: n_1 = 13, n_2 = 11; middle: n_1 = 11, n_2 = 9; right: n_1 = 9, n_2 = 7)



Note: Unless other stated, these simulations are based on n_1 = 11, n_2 = 9, p_1 = 2.347¢ per ounce, p_2 = 2.363¢ per ounce, θ_1 = 25.6181, θ_2 = 25.2096, and ω = 21.0219.

Figure 2

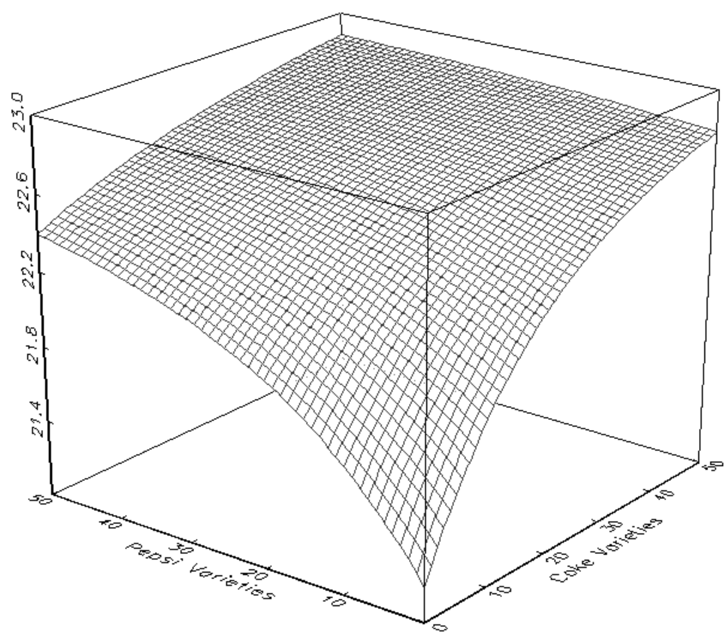
Effect on Pepsi's Demand Curve of a 5% Change in the Net Surplus of the Outside Good



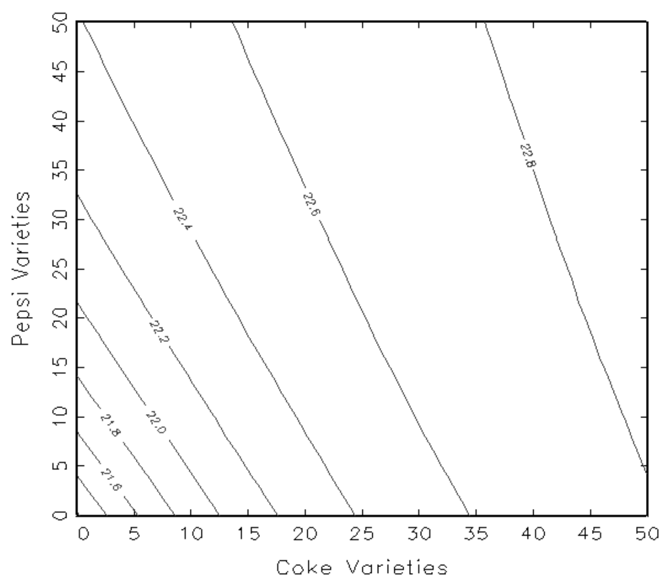
Note: In these simulations, n_1 = 11, n_2 = 9, Pepsi's price = 2.363¢ per ounce, θ_1 = 25.62, and θ_2 = 25.21. In the middle demand curve, we use the estimated ω = 21.02; while ω is 5% larger in the demand curve on the left, and ω is 5% smaller in the demand curve on the right.

Figure 3
Total Consumer Surplus as a Function of the Number of Coke and Pepsi Varieties

(a) Consumer surplus as a function of the number of Coke and Pepsi varieties from 0 to 50



(b) Iso-welfare curves

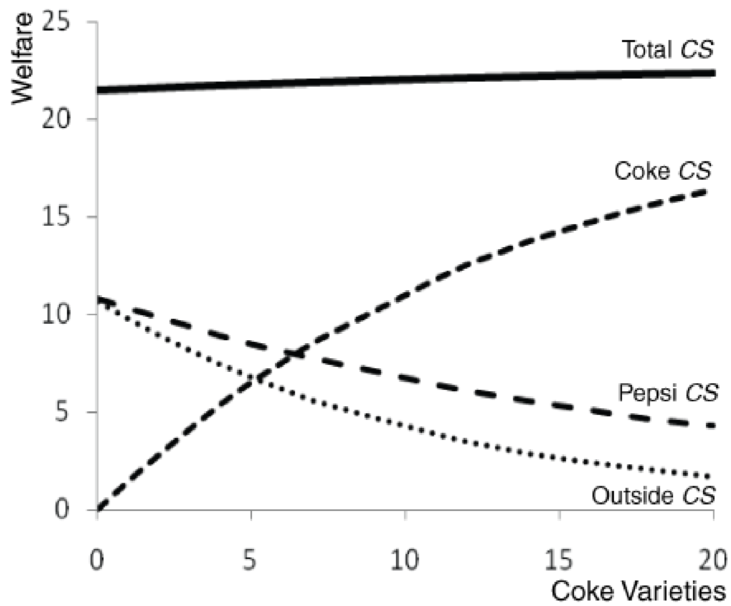


Note: These simulations assume that Coke's price = 2.347¢ per ounce, Pepsi's price = 2.363¢ per ounce, θ_1 = 25.6181, and θ_2 = 25.2096, ω = 21.0219. When Coke and Pepsi varieties are 0, the total consumer surplus equals ω = 21.0219.

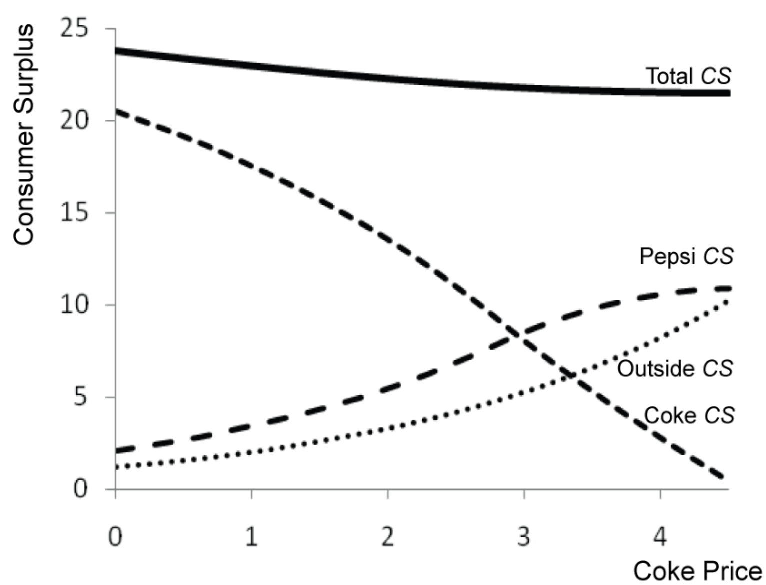
Figure 4

Variation in Consumer Surplus with the Number of Coke Varieties or Price

(a) Consumer surplus and the number of Coke's varieties



(b) Consumer surplus and Coke's price



Note: These simulations assume that $n_1 = 11$ in (b), $n_2 = 9$, Coke's price = 2.347¢ per ounce in (a), Pepsi's price = 2.363¢ per ounce, $\theta_1 = 25.6181$, $\theta_2 = 25.2096$, and $\omega = 21.0219$.

Table 1

Profit Maximization vs. Social Optima

		Planner Sets		
	Profit Max	*Prices & Varieties*	*Varieties Only*	*Prices Only*
n_1	11	26	27	11
n_2	9	11	11	9
p_1	2.35¢	1¢	2.35¢	1¢
p_2	2.36¢	1¢	2.36¢	1¢
p_ω	2.21¢	1.5¢	2.21¢	1.5¢
Profit	$2,447	-$797	$2,086	-$273
Variety Cost	$273	$797	$840	$273
CS	$48,729	$52,662	$49,719	$51,510
Welfare	$51,176	$51,825	$51,805	$51,237

Notes: Z = 220,581.7, m = 1¢, m_ω = 1.5¢, so that c = 666.89¢ and k = 69.05¢.

Appendix: Proofs and Derivations

Proof of Proposition 1.

The marginal distributions of the maxima are $n_1\theta^{-n_1}L_1^{n_1-1}dL_1$ and $n_2\theta^{-n_2}L_2^{n_2-1}dL_2$, and the cumulations below *c* of each maxima are $\theta^{-n_1}c_1^{n_1}$ and $\theta^{-n_2}c_2^{n_2}$. The joint distribution of the maximal net surplus of Brand 1 and Brand 2 is:

$$f_L(L_1,L_2)=n_1n_2\theta^{-(n_1+n_2)}L_1^{n_1-1}L_2^{n_2-1},$$

with cumulations below $(c_1,\ c_2)$ as $\theta^{-(n_1+n_2)}c_1^{n_1}c_2^{n_2}$. The distribution of the difference of the maxima, $D_0=L_1-L_2\in[-\theta,\ \theta]$, has two parts:

$$f_{D_0}(D_0)=\begin{Bmatrix}\int_0^{\theta-D_0} f_L(D_0+L_2,L_2)dL_2 & D_0\in(0,\ \theta] \\ \int_0^{\theta+D_0} f_L(L_1,L_1-D_0)dL_1 & D_0\in[-\theta,\ 0]\end{Bmatrix}.$$

The upper-bound of the first integral maps $D_0\in[0,\ \theta]$ through $L_2=\theta-D_0$ to $L_2\in[0,\ \theta]$, and the upper-bound of the second maps $D_0\in[-\theta,\ 0]$ through $L_1=\theta+D_0$ to $L_1\in[0,\ \theta]$. After repeated integration by parts:

$$f_{D_0}(D_0)=\begin{Bmatrix}\frac{n_2}{\theta^{m+n}}\sum_{j=1}^{m}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2-1+j}C_j}(L_2+D_0)^{n_1-j}(L_2)^{n_2-1+j}\Big|_0^{\theta-D_0} & D_0\in(0,\ \theta] \\ \frac{n_1}{\theta^{m+n}}\sum_{j=1}^{n}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1-1+j}C_j}(L_1)^{n_1-j}(L_1-D_0)^{n_2-1+j}\Big|_0^{\theta+D_0} & D_0\in[-\theta,\ 0]\end{Bmatrix}$$

Evaluating the limits of integration:

$$f_{D_0}(D_0)=\begin{Bmatrix}\frac{n_2}{\theta}\sum_{j=1}^{m}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2-1+j}C_j}\left(1-\frac{D_0}{\theta}\right)^{n_2-1+j}, & D_0\in(0,\ \theta] \\ \frac{n_1}{\theta}\sum_{j=1}^{n}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1-1+j}C_j}\left(1+\frac{D_0}{\theta}\right)^{n_1-1+j}, & D_0\in[-\theta,\ 0]\end{Bmatrix}.$$

Evaluating cumulations above and below zero the results of Proposition 1 follow. ■

Proof of Proposition 3.

The joint distribution of the maxima adjusted for price is:

$$f_\ell(\ell_1,\ell_2)=n_1n_2\theta^{-(n_1+n_2)}(\ell_1+p_1)^{n_1-1}(\ell_2+p_2)^{n_2-1},\ \ell_g\in[-p_g,\theta-p_g],\ g=1,2.$$

The distribution of the difference $D=\ell_1-\ell_2=D_0-\pi$ is:

$$f_D(D)=\begin{Bmatrix} \displaystyle\int_{-p_2}^{\theta-p_1-D} f_\ell(\ell_2+D,\ell_2)d\ell_2, & D\in(-\pi,\ \theta-\pi],\ p_1>p_2 \\ \displaystyle\int_{-p_1}^{\theta-p_2+D} f_\ell(\ell_1,\ell_1-D)d\ell_1, & D\in[-\theta-\pi,\ -\pi],\ p_1\le p_2 \end{Bmatrix}.$$

The upper-bound of the first integral maps $D\in[-\pi,\ \theta-\pi]$ through $\ell_2=\theta-p_1-D$ to $\ell_2\in[-p_2,\ \theta-p_2]$, and the upper-bound of the second maps $D\in[-\theta-\pi,\ -\pi]$ through $\ell_1=\theta-p_2+D$ to $\ell_1\in[-p_1,\ \theta-p_1]$. After repeated integration by parts:

$$f_D(D)=\begin{Bmatrix} \dfrac{n_2}{\theta^{m+n}}\displaystyle\sum_{j=1}^{m}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2-1+j}C_j}(\ell_2+D+p_1)^{n_1-j}(\ell_2+p_2)^{n_2-1+j}\Big|_0^{\theta-p_1-D} & D\in(0,\ \theta-\pi] \\ \dfrac{n_1}{\theta^{m+n}}\displaystyle\sum_{j=1}^{n}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1-1+j}C_j}(\ell_1+p_1)^{n_1-j}(\ell_1-D+p_2)^{n_2-1+j}\Big|_0^{\theta-p_2+D} & D\in[-\theta-\pi,\ 0] \end{Bmatrix}.$$

Evaluating the limits of integration:

$$f_D(D)=\begin{Bmatrix} \dfrac{n_2}{\theta}\displaystyle\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2-1+j}C_j}\left(1-\frac{\pi-D}{\theta}\right)^{n_2-1+j} & D\in(-\pi,\ \theta-\pi],\ p_1>p_2 \\ \dfrac{n_1}{\theta}\displaystyle\sum_{j=1}^{n_2}(-1)^{j-1}\frac{{}_{n_2}C_j}{{}_{n_1-1+j}C_j}\left(1+\frac{\pi+D}{\theta}\right)^{n_1-1+j} & D\in[-\theta-\pi,\ -\pi],\ p_1\le p_2 \end{Bmatrix}.$$

As one might expect, this distribution is the same as the distribution of D_0 displaced by the price differential. Evaluating cumulations above and below zero the results of Proposition 3 follow. ■

Proof of Proposition 4.

(a) Applying Leibniz's rule to the probability integrals (evaluated at $D = 0$) in the proof of proposition 3, the derivatives of the shares with respect to prices are,

$$\frac{\partial \tilde{s}_1(n_1,n_2)}{\partial p_2} = -\frac{\partial \tilde{s}_2(n_1,n_2)}{\partial p_2} = \frac{n_2}{\theta}\tilde{s}_1(n_1,n_2-1) \geq 0, \quad p_1 > p_2,$$
$$\frac{\partial \tilde{s}_2(n_1,n_2)}{\partial p_1} = -\frac{\partial \tilde{s}_1(n_1,n_2)}{\partial p_1} = \frac{n_1}{\theta}\tilde{s}_2(n_1-1,n_2) \geq 0, \quad p_1 \leq p_2.$$

Interestingly, derivatives of the shares (w.r.t. price) are functions of the shares evaluated at lower numbers of varieties. Second derivatives follow directly,

$$\frac{\partial^2 \tilde{s}_2(n_1,n_2)}{\partial p_2^2} = -\frac{\partial^2 \tilde{s}_1(n_1,n_2)}{\partial p_2^2} = -\frac{n_2(n_2-1)}{\theta}\tilde{s}_1(n_1,n_2-2) \leq 0, \quad p_1 > p_2,$$
$$\frac{\partial^2 \tilde{s}_1(n_1,n_2)}{\partial p_1^2} = -\frac{\partial^2 \tilde{s}_2(n_1,n_2)}{\partial p_1^2} = -\frac{n_1(n_1-1)}{\theta}\tilde{s}_2(n_1-2,n_2) \leq 0, \quad p_1 \leq p_2.$$

Taking derivatives of the equations in Proposition 3, we have,

$$\frac{\partial \tilde{s}_1(n_1,n_2)}{\partial p_1} = -\frac{\partial \tilde{s}_1(n_1,n_2)}{\partial p_2} = -\frac{n_2}{\theta}\tilde{s}_1(n_1,n_2-1), \quad p_1 > p_2,$$
$$\frac{\partial \tilde{s}_2(n_1,n_2)}{\partial p_2} = -\frac{\partial \tilde{s}_2(n_1,n_2)}{\partial p_1} = -\frac{n_1}{\theta}\tilde{s}_2(n_1-1,n_2), \quad p_1 \leq p_2.$$

Taking derivatives again,

$$\frac{\partial^2 \tilde{s}_1(n_1,n_2)}{\partial p_1^2} = -\frac{\partial^2 \tilde{s}_1(n_1,n_2)}{\partial p_1 \partial p_2} = -\frac{n_2}{\theta}\frac{\partial \tilde{s}_1(n_1,n_2-1)}{\partial p_1}, \quad p_1 > p_2,$$
$$\frac{\partial^2 \tilde{s}_2(n_1,n_2)}{\partial p_2^2} = -\frac{\partial^2 \tilde{s}_2(n_1,n_2)}{\partial p_2 \partial p_1} = -\frac{n_1}{\theta}\frac{\partial \tilde{s}_2(n_1-1,n_2)}{\partial p_2}, \quad p_1 \leq p_2.$$

To sign these last two results, we must sign the two derivatives of the right-hand sides by again applying Leibniz's rule to the probability integrals in the proof of proposition 3, yielding,

$$\frac{\partial \tilde{s}_1(n_1,n_2-1)}{\partial p_1} = -\frac{n_1}{\theta}\left[\left(1-\frac{\pi}{\theta}\right)^{n_2} - \tilde{s}_1(n_1-1,n_2-1)\right] \leq 0, \quad p_1 > p_2.$$
$$\frac{\partial \tilde{s}_2(n_1-1,n_2)}{\partial p_2} = \frac{n_2}{\theta}\left[\left(1+\frac{\pi}{\theta}\right)^{n_1} - \tilde{s}_2(n_1-1,n_2-1)\right] \leq 0, \quad p_1 \leq p_2.$$

Hence, we have that $\partial \tilde{s}_i / \partial p_i \leq 0$ everywhere, but $\partial^2 \tilde{s}_1 / \partial p_1^2 \geq 0$ and $\partial^2 \tilde{s}_2 / \partial p_2^2 \leq 0$ for $p_1 > p_2$, and $\partial^2\tilde{s}_1/\partial p_1^2 \le 0$ and $\partial^2\tilde{s}_2/\partial p_2^2 \ge 0$ for $p_1 \le p_2$. So the main results hold.

(b) First consider the case where $p_1 > p_2$. If we increase n_1 by one in Proposition 3,

$$\tilde{s}_1(n_1+1) = \sum_{j=1}^{n_1+1}(-1)^{j-1}\frac{{}_{n_1+1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j} = \sum_{j=1}^{n_1+1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j}\left(\frac{n_1+1}{n_1+1-j}\right)$$
$$> \sum_{j=1}^{n_1+1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j} = \sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j} = \tilde{s}_1.$$

This result follows because $(n_1 + 1)/(n_1 + 1 - j) > 1$ and the $(n_1 + 1)^{th}$ term in the sum of the second to last line is zero because ${}_{n_1}C_{n_1+1} = 0$. Thus we have:

$$\frac{\Delta\tilde{s}_1}{\Delta n_1} = \sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j}\left(\frac{j}{n_1+1-j}\right) > 0.$$

Given this, it must be true that $\frac{\Delta^2\tilde{s}_1}{\Delta n_1^2} < 0$, for if it were not, then as $n_1 \to \infty$, $\tilde{s}_1 > 1$, which violates the axioms of probability. Increasing n_2 in Proposition 3,

$$\tilde{s}_1(n_2+1) = \sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+1+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+1+j}$$
$$= \sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j}\left(\frac{n_2+1}{n_2+1+j}\cdot\left(1-\frac{\pi}{\theta}\right)\right)$$
$$< \sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j} = \tilde{s}_1(n_2).$$

This result follows because $(n_2 + 1)/(n_2 + 1 + j) < 1$ and $(1 - \pi/\theta) \le 1$. Thus we have for $p_1 > p_2$:

$$\frac{\Delta\tilde{s}_1(n_2)}{\Delta n_2} = -\sum_{j=1}^{n_1}(-1)^{j-1}\frac{{}_{n_1}C_j}{{}_{n_2+j}C_j}\left(1-\frac{\pi}{\theta}\right)^{n_2+j}\left(1-\frac{n_2+1}{n_2+1+j}\cdot\left(1-\frac{\pi}{\theta}\right)\right) < 0$$

Using the difference of the differences, it is easy to show that $\frac{\Delta^2\tilde{s}_1}{\Delta n_2^2} < 0$. By showing similar results for the case $p_1 < p_2$, the proof is complete.

(c) To understand the cross-difference (cross-partial difference) with respect to n_1 and n_2 it is useful to restate the shares in terms of the Gaussian hypergeometric function (Gauss 1813 or Pachhammer 1870):

$$F(a,\ b;\ c;\ z) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j\, j!} z^j,$$

where $(a)_j = a(a+1)...(a+j-1)$ is the Pochhammer (1870) rising factorial. When the first argument is negative, the hypergeometric sum is finite. As a result, the probability in Proposition 3 can be restated as:

$$\tilde{s}_1(n_1, n_2, p_1, p_2) = \left(1 - \frac{\pi}{\theta}\right)^{n_2} \left[1 - F(-n_1,\ 1;\ n_2+1;\ 1-\pi/\theta)\right].$$

Rearranging:

$$F(-n_1,\ 1;\ n_2+1;\ 1-\pi/\theta) = 1 - \left(1 - \frac{\pi}{\theta}\right)^{-n_2} \tilde{s}_1.$$

Incrementing n_1 and n_2:

$$F(-n_1-1,\ 1;\ n_2+1;\ 1-\pi/\theta) = 1 - \left(1 - \frac{\pi}{\theta}\right)^{-n_2} \tilde{s}_1(n_1+1),$$

$$F(-n_1,\ 1;\ n_2+2;\ 1-\pi/\theta) = 1 - \left(1 - \frac{\pi}{\theta}\right)^{-n_2-1} \tilde{s}_1(n_2+1),$$

$$F(-n_1-1,\ 1;\ n_2+2;\ 1-\pi/\theta) = 1 - \left(1 - \frac{\pi}{\theta}\right)^{-n_2-1} \tilde{s}_1(n_1+1, n_2+1).$$

Substituting these into Equation 15.2.17 of Abramowitz and Stegun (1972) yields:

$$\tilde{s}_1(n_1+1, n_2+1) = \frac{n_2+1}{n_1+n_2+2}\left(1 - \frac{\pi}{\theta}\right)\tilde{s}_1(n_1+1) + \frac{n_1+1}{n_1+n_2+2}\tilde{s}_1(n_2+1).$$

Subtracting $\tilde{s}_1$ yields the result. ■

Proof of Proposition 6.

Consider the following regions in two-dimensional Cartesian space.

Region	*Market Share*

$R_{\omega\omega} = \{(\ell_1,\ \ell_2):\ \ell_1 \in [-p_1,\ \omega),\ \ell_2 \in [-p_2,\ \omega)\}$	$s_1^* = s_2^* = 0$
$R_{1\omega} = \{(\ell_1,\ \ell_2):\ \ell_1 \in [\omega,\ \theta_1 - p_1],\ \ell_2 \in [-p_2,\ \omega)\}$	$s_1^* \in [0,\ 1]$; $s_2^* = 0$
$R_{\omega 2} = \{(\ell_1,\ \ell_2):\ \ell_1 \in [-p_1,\ \omega),\ \ell_2 \in [\omega,\ \theta_2 - p_2]\}$	$s_1^* = 0$; $s_2^* \in [0,\ 1]$
$R_{12} = \{(\ell_1,\ \ell_2):\ \ell_1 \in [\omega,\ \theta_1 - p_1],\ \ell_2 \in [\omega,\ \theta_2 - p_2]\}$	$s_1^* \in [0,\ 1]$; $s_2^* \in [0,\ 1]$

Given that some consumers purchase the outside good instead of a Brand 1 or Brand 2 variety, $s_1^* + s_2^* \leq 1$. Brand 1 and Brand 2 shares equal the probabilities:

$$(*) \quad s_1^*(n_1, n_2, p_1, p_2, \theta_1, \theta_2) = \begin{Bmatrix} \Pr(\ell_1 > \ell_2 \cap R_{12}) + \Pr(R_{1\omega}) & p_1 - \theta_1 > p_2 - \theta_2 \\ 1 - \Pr(\ell_1 \leq \ell_2 \cap R_{12}) - \Pr(R_{\omega 2}) - \Pr(R_{\omega\omega}) & p_1 - \theta_1 \leq p_2 - \theta_2 \end{Bmatrix},$$

$$(**) \quad s_2^*(n_1, n_2, p_1, p_2, \theta_1, \theta_2) = \begin{Bmatrix} \Pr(\ell_1 \leq \ell_2 \cap R_{12}) + \Pr(R_{\omega 2}) & p_1 - \theta_1 \leq p_2 - \theta_2 \\ 1 - \Pr(\ell_1 > \ell_2 \cap R_{12}) - \Pr(R_{1\omega}) - \Pr(R_{\omega\omega}) & p_1 - \theta_1 > p_2 - \theta_2 \end{Bmatrix}.$$

The probability masses on $R_{1\omega}$, $R_{\omega 2}$, and $R_{\omega\omega}$ are cumulations:

$$\Pr(R_{1\omega}) = \iint_{R_{1\omega}} f_\ell(\ell_1, \ell_2)\, d\ell_1 d\ell_2 = \left[1 - \left(\frac{p_1 + \omega}{\theta_1}\right)^{n_1}\right]\left(\frac{p_2 + \omega}{\theta_2}\right)^{n_2},$$

$$\Pr(R_{\omega 2}) = \iint_{R_{\omega 2}} f_\ell(\ell_1, \ell_2)\, d\ell_1 d\ell_2 = \left[1 - \left(\frac{p_2 + \omega}{\theta_2}\right)^{n_2}\right]\left(\frac{p_1 + \omega}{\theta_1}\right)^{n_1},$$

$$\Pr(R_{\omega\omega}) = \iint_{R_{\omega\omega}} f_\ell(\ell_1, \ell_2)\, d\ell_1 d\ell_2 = \left(\frac{p_1 + \omega}{\theta_1}\right)^{n_1}\left(\frac{p_2 + \omega}{\theta_2}\right)^{n_2}.$$

The last equation is the probability that the outside good is purchased, and is, hence, the share function for that good. Probabilities in R_{12} are:

$$\Pr(\ell_1 > \ell_2 \subset R_{12}) = \left(\frac{\theta_1}{\theta_2}\right)^{n_2} \sum_{j=1}^{n_1} (-1)^{j-1} \frac{{}_{n_1}C_j}{{}_{n_2+j}C_j} \left[\left(1 - \frac{\pi}{\theta_1}\right)^{n_2+j} - \left(\frac{p_2 + \omega}{\theta_1}\right)^{n_2+j}\right]$$
$$- \frac{n_2}{n_1 + 1}\left(\frac{\theta_1}{\theta_2}\right)^{n_2} \sum_{j=1}^{n_1} (-1)^{j-1} \frac{{}_{n_1+1}C_j}{{}_{n_2-1+j}C_j} \left(\frac{p_2 + \omega}{\theta_1}\right)^{n_2-1+j} \left[1 - \left(\frac{p_1 + \omega}{\theta_1}\right)^{n_1+1+j}\right],$$

for $p_1 - \theta_1 > p_2 - \theta_2$, and

$$\Pr(\ell_1 \le \ell_2 \subset R_{12}) = \left(\frac{\theta_2}{\theta_1}\right)^{n_1} \sum_{j=1}^{n_2} (-1)^{j-1} \frac{{}_{n_2}C_j}{{}_{n_1+j}C_j} \left[\left(1 + \frac{\pi}{\theta_2}\right)^{n_1+j} - \left(\frac{p_1+\omega}{\theta_2}\right)^{n_1+j} \right]$$

$$- \frac{n_1}{n_2+1} \left(\frac{\theta_2}{\theta_1}\right)^{n_1} \sum_{j=1}^{n_2} (-1)^{j-1} \frac{{}_{n_2+1}C_j}{{}_{n_1-1+j}C_j} \left(\frac{p_1+\omega}{\theta_2}\right)^{n_1-1+j} \left[1 - \left(\frac{p_2+\omega}{\theta_2}\right)^{n_2+1+j} \right],$$

for $p_1 - \theta_1 \le p_2 - \theta_2$. Substituting the probabilities into equations (*) and (**) produces the main result of Proposition 6. ■

Calculating Consumer Surplus.

The area under the demand curves s_1^* and s_2^* can be used to calculate the effect of increasing varieties (Δn_1 or Δn_2) on total consumer surplus (*CS*). For instance, if we increase the varieties of Brand 1 by one ($\Delta n_1 = 1$), demand for Brand 1 increases so that demand for Brand 2 and the outside good change, and the change (increase) in total CS can be represented as the difference of the areas under the compensated demand function for Brand 1 (and above its price) before and after the change. That is, when $p_1 - \theta_1 > p_2 - \theta_2$,

$$\frac{\Delta CS(p_1 - \theta_1 > p_2 - \theta_2)}{\Delta n_1} = \int_{p_1}^{\theta_1 - \omega} s_1^*(n_1 + 1, p_1 - \theta_1 > p_2 - \theta_2) dp_1 - \int_{p_1}^{\theta_1 - \omega} s_1^*(n_1, p_1 - \theta_1 > p_2 - \theta_2) dp_1.$$ In this equation, with a slight abuse our notation, the share $s_1^*(n_1, p_1 - \theta_1 > p_2 - \theta_2)$ corresponds to Brand 1 demand when $p_1 - \theta_1 > p_2 - \theta_2$. Because the demand functions are compensated, we need calculate only areas under the Brand 1 function. We integrate with respect to price from the given price, p_1, to the upper bound, $\theta_1 - \omega$, because demand for Brand 1 is zero if the price is higher (the outside good dominates for all consumers).

When $p_1 - \theta_1 \le p_2 - \theta_2$ the calculation is slightly different:

$$\frac{\Delta CS(p_1-\theta_1\le p_2-\theta_2)}{\Delta n_1}=\int_{p_1}^{p_2-\theta_2+\theta_1} s_1^*(n_1+1,\ p_1-\theta_1\le p_2-\theta_2)dp_1-\int_{p_1}^{p_2-\theta_2+\theta_1} s_1^*(n_1,\ p_1-\theta_1\le p_2-\theta_2)dp_1$$
$$+\int_{p_2-\theta_2+\theta_1}^{\theta_1-\omega} s_1^*(n_1+1,\ p_1-\theta_1> p_2-\theta_2)dp_1-\int_{p_2-\theta_2+\theta_1}^{\theta_1-\omega} s_1^*(n_1,\ p_1-\theta_1> p_2-\theta_2)dp_1.$$

To calculate the change in total consumer surplus, we must use both parts of the demand function, s_1^*. These CS calculations are complicated because the demand equation for a brand differs depending on relative prices.

Similarly, the equation for s_2^* can be used to calculate the increase in total consumer demand when the varieties of Brand 2 are increased by one. For example,

$$\frac{\Delta CS(p_1-\theta_1\le p_2-\theta_2)}{\Delta n_2}=\int_{p_2}^{\theta_2-\omega} s_2^*(n_2+1,\ p_1-\theta_1\le p_2-\theta_2)dp_2-\int_{p_2}^{\theta_2-\omega} s_2^*(n_2,\ p_1-\theta_1\le p_2-\theta_2)dp_2,$$

while the integrals are straight-forward to calculate, the resulting formulae are long and not presented here.

Proof of Proposition 8.

To derive the comparative statics results for the manufacturers' model, we totally differentiate the first-order conditions, $b_i n_j/(n_1 + n_2)^2 - c = 0$, and write them in matrix form:

$$A=\frac{1}{(n_1+n_2)^3}\begin{bmatrix} -2n_2b_1 & (n_1-n_2)b_1 \\ (n_2-n_1)b_2 & -2n_2b_2 \end{bmatrix}=\begin{bmatrix} \mathrm{d}c & -n_2/(n_1-n_2)^2\mathrm{d}b_1 & 0 \\ \mathrm{d}c & 0 & -n_1/(n_1-n_2)^2\mathrm{d}b_2 \end{bmatrix}.$$

Consequently, $|A| = b_1b_2[4(n_2)^2 + (n_2 - n_1)^2]/(n_1 + n_2)^3 > 0$. Then, using Cramer's rule, we find that $\mathrm{d}n_1/\mathrm{d}c = [2n_1b_2 + b_1(n_2 - n_1)]/[(n_1 + n_2)|A|]$. Thus, the sign of $\mathrm{d}n_1/\mathrm{d}c$ is the opposite of the sign of $2n_1b_2 + b_1(n_2 - n_1)$. If $b_1 = b_2$, then $n_1 = n_2$, and the sign of $\mathrm{d}n_1/\mathrm{d}c$ is negative. Unless b_1 substantially differs from b_2, the sign is negative. In addition, $\mathrm{d}n_1/\mathrm{d}b_1 = 2n_1n_2b_2/[(n_1 + n_2)^5|A|] > 0$, and $\mathrm{d}n_1/\mathrm{d}b_2 = b_1n_1(n_1 - n_2)/[(n_1 + n_2)^5|A|]$, which has the opposite sign of $n_1 - n_2$. Given symmetry, the comparative results for n_2 are similar.■