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A DYNAMIC THERMAL NETWORK MODEL APPLIED TO VENTILATED ATTICS

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ABSTRACT

The need for tools to analyze heat and moisture behavior in wooden constructions is increasing with the increasing awareness of the moisture problems. An example of a construction with a history of mould problems is cold attics. Most heat models of attics use a lumped node technique for energy balance and ignore the variation in heat transfer coefficients. A model based on so called dynamic thermal networks has been developed that takes into account radiation between the interior surfaces and the different boundary conditions at the outside and inside surfaces. The first step is to develop analytical solutions for the step responses for the whole attic including a composite roof. The second step is to create a dynamic thermal network based on these solutions. With the thermal network it is possible to do hourly (or any given time step) calculations for several years in a very short computer time which makes it possible to easily test different parameters: ventilation rate, insulation thickness, solar insolation etc. The analytical response solutions also provide good insight into the physics of the thermal problem. A simple parameter study of exterior roofing insulation is presented as an example. The work is part of a large project in Sweden investigating wooden constructions.

KEYWORDS

Heat transfer, Analytical solution, Attics, Transient.

INTRODUCTION

The points of interest for mould problems in a cold attic are e.g. the attic air and the interior attic roof surface. This is because one reason for the moisture problems is the cooling of ambient air when entering the attic. There are many possible ways to implement a thermal model of an attic. Using a one-dimensional model where the ventilated air space is modeled as a space between two parallel plates is especially common when the model includes moisture transport, Vahid (2012). Since even the thermal problem is quite complex given that the attic has many layers the methods used to solve the problem are typically numerical. The paper presented here deals with an analytical solution of the transient heat transfer problem of an attic based on a thermal network. With this solution it is possible to do fast parameter studies with hourly climate data. This includes the effect of shortwave absorbation coefficients, insulation thickness, ventilation rate etc.

THERMAL MODELING OF AN ATTIC

The model consists of an attic floor, roof and a ventilated space in between. The heat transfer in the floor and roof is one dimensional but the radiation exchange between the surfaces takes into account the two dimensional view factors between the floor and roof. The floor consists of insulation. The composite roof consists of wood, insulation, a non-ventilated air gap and roof tiles. This is described in Figure 1. The heat transfer process is a linear one, which therefore includes linearization of the convective and radiative heat transfer at the surfaces.

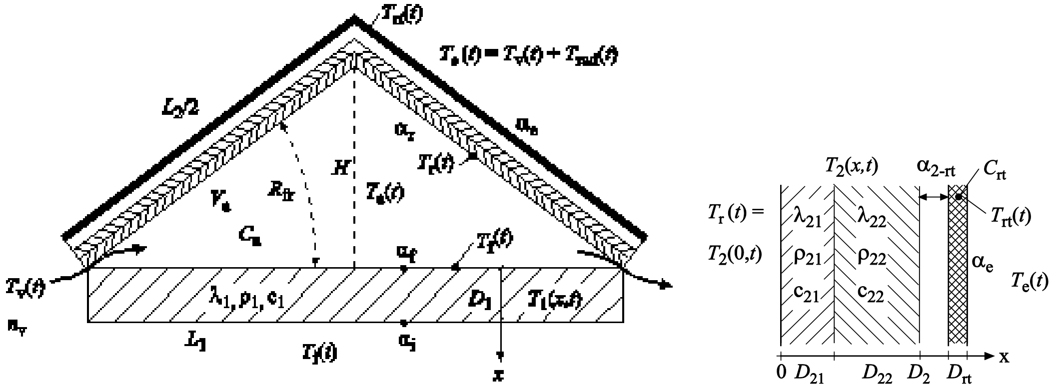

Figure 1. Left: Notations for an attic with radiation heat exchange between floor and roof. Right: Details of the composite roof involving two material layers and an air gap below roofing tiles.

The following subscripts will be used: *a* = attic air, *f* = attic floor surface, *r* = attic roof surface, *v* = ventilation air temperature, *e* = exterior air temperature, *i* = interior air temperature, *rad* = effective outdoor air temperature due to radiation, 1 = insulation slab in attic floor, 21 = wood slab in attic roof, 22 = insulation slab in attic roof, *rt* =roofing tiles.

The model can be represented as thermal network which is described in Figure 2. It is easy to see the possible paths of the heat flow.

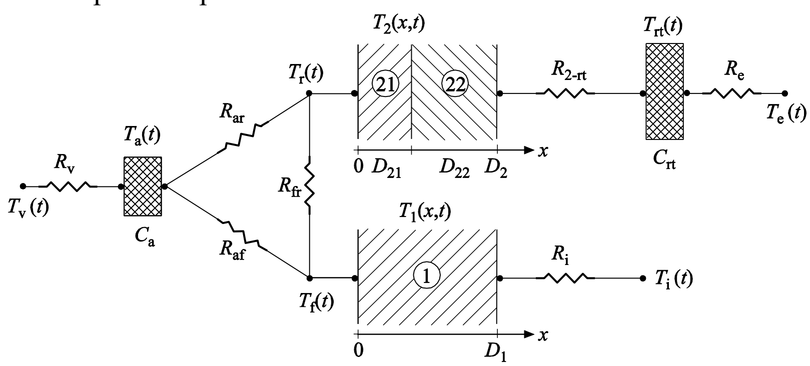

Figure 1. Network to represent the thermal interactions and equations in the attic components

The thermal process is governed by the three prescribed boundary functions for ventilation, exterior and interior temperature:

$$T_v(t), \quad T_e(t), \quad T_i(t). \tag{1}$$

These particular boundary conditions are chosen because they are simple but allow for a realistic treatment of the climate. Obvious simplifications are: constant ventilation rate, constant convective and radiative heat transfers coefficients, well mixed air in the attic space and that the influence of short and longwave radiation on the exterior surface can be simplified as an equivalent exterior temperature. The task is to calculate temperatures as functions of time at the four nodes (a, f, r, rt) and the temperature field through floor and roof:

$$T_a(t), T_f(t), T_r(t), T_{rt}(t), T_1(x,t) \ \ 0 \le x \le D_1, T_2(x,t) \ \ 0 \le x \le D_2 \tag{2}$$

The temperature of the attic air, T_a(t), and the interior surface of the attic roof, $T_r(t)= T_2(0,t)$, are of particular interest for studies of moisture problems.There is a heat balance equation at each node (*a, f, r, rt*). The equation for the air node, Ta(t), is:

$$C_a \times \frac{dT_a}{dt} = \frac{T_v(t) - T_a(t)}{R_v} + \frac{T_r(t) - T_a(t)}{R_{ar}} + \frac{T_f(t) - T_a(t)}{R_{af}}. \qquad (3)$$

The equation for the node at the surface of the attic roof, Tr(t), is, Figure 2:

$$\frac{T_a(t)-T_r(t)}{R_{ar}} + \frac{T_f(t)-T_r(t)}{R_{fr}} = q_2(0,t) = L_2 \cdot (-\lambda_{21}) \frac{\partial T_2}{\partial x} \quad {}_{x=0} \qquad (4)$$

The heat equation for the temperature T1(x,t) in the insulation slab of the attic floor reads:

$$\frac{1}{a_1} \cdot \frac{\partial T_1}{\partial t} = \frac{\partial^2 T_1}{\partial x^2}, \quad 0 \leq x \leq D_1, \; a_1 = \frac{\lambda_1}{\rho_1 c_1} \qquad (5)$$

The heat equation for the temperature T2(x,t) of the composite roof is similar. The basic input data with values for the reference case are chosen as a relatively small attic:

$$\begin{aligned} &L_1 = 8 \text{ m}, \; H = 3 \text{ m}, \; n_v = 2/3600 \text{ s}^{-1}, \; r_a = 1.29 \text{ kg/m}^3, \; c_a = 1000 \text{ J/kg}, \; e = 0.9, \\ &T_{av} = 283\ ^0\text{C}, \; a_f = 4 \text{ W/(K,m}^2), \; D_1 = 0.4 \text{ m}, \; l_1 = 0.04 \text{ W/(K,m)}, \; r_1 = 20, \; c_1 = 800, \\ &a_i = 8, \; a_r = 4, \; D_{21} = 0.02, \; l_{21} = 0.14, \; r_{21} = 500, \; c_{21} = 1200, \; D_{22} = 0.05, \; l_{22} = 0.04, \\ &r_{22} = 50, \; c_{22} = 800, \; a_{2\text{-}rt} = 15, \; D_{rt} = 0.015 \text{ m}, \; r_{rt} = 1500, \; c_{rt} = 800, \; a_e = 15. \end{aligned} \qquad (6)$$

STEP RESPONSES FOR THE THREE BASIC CASES

The determination of the attic temperatures is based on the solutions for three basic cases, one for each boundary condition. In the first case associated with the ventilation boundary, the ventilation temperature experiences a unit temperature step from 0 to 1 at t = 0. The exterior and interior boundary temperatures are zero for all times. The temperature at the start t = 0 is zero in the whole attic. The three basic step-response problems (ventilation, exterior, interior) are defined by the following boundary conditions:

$$\begin{aligned} &\text{Ventilation:} && T_v(t) = H(t), \quad T_e(t) = 0, \quad T_i(t) = 0; \\ &\text{Exterior:} && T_e(t) = H(t), \quad T_v(t) = 0, \quad T_i(t) = 0; \qquad H(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}; \\ &\text{Interior:} && T_i(t) = H(t), \quad T_v(t) = 0, \quad T_e(t) = 0. \end{aligned} \qquad (7)$$

STEP RESPONSES FOR VENTILATION TEMPERATURE

The solution for a step in the ventilation boundary temperature involves the following temperature components (using a bold face superscript v for the ventilation step response):

$$U_a^v(t), U_f^v(t), U_r^v(t), U_{rt}^v(t), U_1^v(x,t) \;\; for\; 0 \leq x \leq D_1, U_2^v(x,t) \; for \; 0 \leq x \leq D_2 \qquad (8)$$

These functions describe the step response at all points in the thermal model. Some examples of solutions are given in Figure 3 .

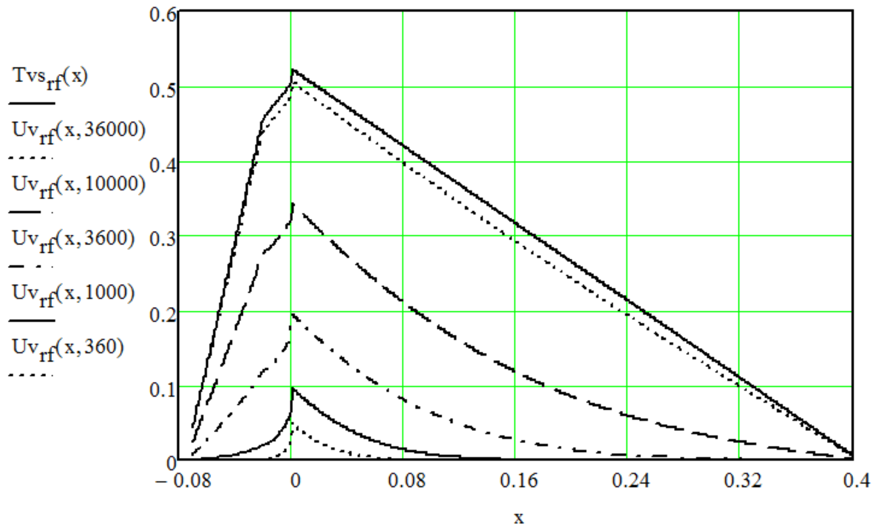

Figure 3. Temperature distribution through roof (-0.07< x <0) and floor (0< x <0.4) for the ventilation step for t = 0.1, …, 10 hours. The top curve shows the steady-state temperature.

SOLUTION TECHNIQUES

A methodology called dynamic thermal networks to solve thermal problems involving transient heat conduction have been presented by Claesson and co-workers in a number of papers. These networks represent the relations between boundary heat fluxes and boundary temperatures. The current heat fluxes are obtained by integrals (or sums) of preceding boundary temperatures multiplied by weighting functions, Claesson (2002A, 2003). The theory is applied to composite walls in Wentzel and Claesson (2003), to a building with walls, roof and foundation in Wentzel and Claesson (2004) and Wentzel (2005). Solution techniques involving Laplace transforms and Fourier series with determination of eigenvalues are used. This whole system with all its complex interactions may be represented by a thermal network for the Laplace transform and for the eigenvalues. From these similar networks the Laplace transform and the equation for eigenvalues are readily obtained. The final Laplace solution is obtained by an integral inversion. The full thermal model for the attic requires some four pages of formulas and relations in Mathcad. The computer time for about five digits accuracy is a few minutes.

General superposition formula

Let **P** denote any considered point (node, point in floor or roof) for which the temperature is to be determined:

$$\mathbf{P}: a, f, r, rt, x = x_1 \ (0 \le x \le D_1), x = x_2 \ \ (0 \le x \le D_2) \qquad (9)$$

The temperature at **P** as function of time t depends on the three boundary temperatures taken for preceding times up to time t. General superposition gives the following *exact* formula:

$$T_{\mathbf{P}}(t) = \int_0^{\infty} \left[W_{\mathbf{P}}^{v}(\tau) \cdot T_v(t-\tau) + W_{\mathbf{P}}^{e}(\tau) \cdot T_e(t-\tau) + W_{\mathbf{P}}^{i}(\tau) \cdot T_i(t-\tau) \right] d\tau \qquad (10)$$

Here, the weighting functions are given by the time derivative of the three basic step-response solutions at the considered point **P**:

$$W_{\mathbf{P}}^{v}(\tau) = \frac{d}{d\tau}[U_{\mathbf{P}}^{v}(\tau)], \ W_{\mathbf{P}}^{e}(\tau) = \frac{d}{d\tau}[U_{\mathbf{P}}^{e}(\tau)], \quad W_{\mathbf{P}}^{i}(\tau) = \frac{d}{d\tau}\left[U_{\mathbf{P}}^{i}(\tau)\right], \qquad (11)$$

The weighting factors are positive (or zero), since the U-functions increase monotonously with time, and the derivatives tend to zero (exponentially) for large times. Equation (10) thus expresses the full analytical solution of the problem. No discretization or numerical limitations are used at this point.

Discretization

In the discrete numerical model, the boundary temperatures are by assumption piecewise constant during each time interval *n*:

$$T_v(t) = T_{v,n}, \quad T_e(t) = T_{e,n}, \quad T_i(t) = T_{i,n} \quad for\ (n-1)h < t < n \cdot h \qquad (12)$$

Here, h is the time step, which often is typically h = 1 hour. Formula (10) gives:

$$T_{\mathbf{P}}(nt) = T_{\mathbf{P},n} = \sum_{v=1}^{\infty} \int_{vh-h}^{vh} \left[W_{\mathbf{P}}^{v}(\tau) \cdot T_{v,n-v} + W_{\mathbf{P}}^{e}(\tau) \cdot T_{e,n-v} + W_{\mathbf{P}}^{i}(\tau) \cdot T_{i,n-v} \right] d\tau \qquad (13)$$

The sum of all weighting factors becomes equal to one. Equation (13) may therefore be written in the following way:

$$\sum_{v=1}^{\infty} \left[W_{\mathbf{P},v}^{v} \cdot T_{v,n-v} - T_{\mathbf{P},n} + W_{\mathbf{P},v}^{e} \cdot T_{e,n-v} - T_{\mathbf{P},n} + W_{\mathbf{P},v}^{i} \cdot T_{i,n-v} - T_{\mathbf{P},n} \right] = 0 \qquad (14)$$

This relation may be represented graphically as a kind of dynamic thermal network, Figure 4.

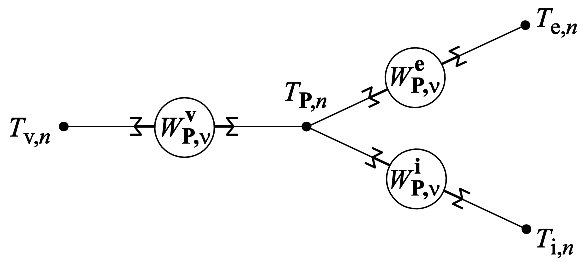

Figure 4. Dynamic thermal network to represent the weighting formulas (14).

Data from the reference case (6) gives that sum above may be truncated after 15 time steps of one hour each.

PARAMETER STUDY. AN EXAMPLE

With the model and solution described above it is easy to investigate the importance of the parameters in the model for any given boundary conditions. As an example has the importance of the exterior insulation thickness been studied using a climate in Lund for one year (1990). When investigating moisture related issues it is of interest to calculate the risk of high relative humidity on the attic roof surface which is directly dependent on the difference between the outdoor (ventilation) temperature and the roof surface *Diff*=T_r-T_v. The risk for mould problems increases with *Diff*. Figure 5 shows *Diff* for a year in Lund 1990 presented in cumulative histogram form. The x axis is *Diff* and the y axis is the number of hours the value is below *Diff*. The calculations show that 1cm of exterior insulation does not reduce the number of hours with cooling of the ventilated air. There are about 4500 hours with cooling. With 5 cm insulation, as in Figure 10 however, does the number of hours with cooling of ventilates air go down to about 2500 hours.

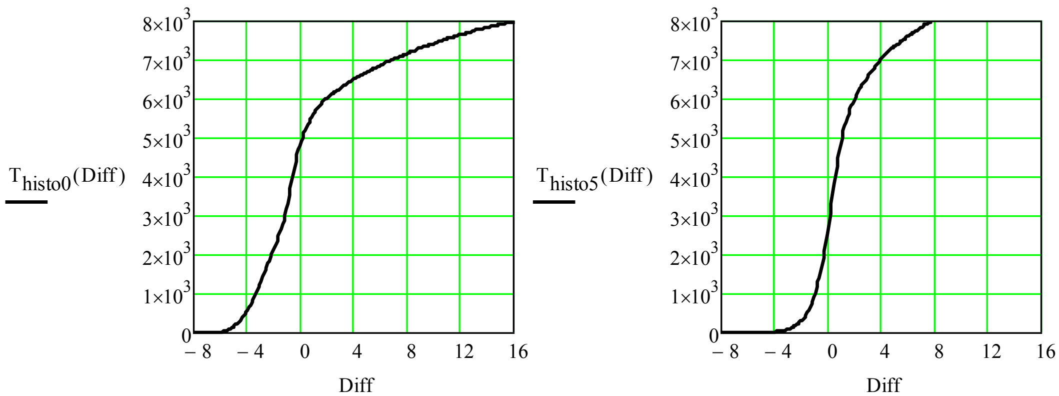

Figure 5 Cumulative histogram of *Diff* from hourly simulation of year 1990 in Lund, Sweden. Left: Exterior insulation thickness 0cm. Right. Exterior insulation thickness 5cm.

CONCLUSION

The paper presents an analytical solution method based on dynamic thermal networks for the heat transfer problem in an attic with a pitched roof. Although there exists a number of numerical approaches to solve this problem (Energy+, IDA-ICE etc), no analytical solution has as far as the authors know been presented before. The solution shows the physical behavior of the different components in a very clear way. It is also easy to investigate the effect of the included parameters. The temperature at any point can be calculated using about 15 preceding boundary temperatures. A few examples of possible parameter studies are presented.

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