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# Role of Light Vector Mesons in the Heavy Particle Chiral Lagrangian

Joseph Schechter
*Department of Physics, Syracuse University, Syracuse, NY*

A. Subbaraman
*Syracuse University*

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# ROLE OF LIGHT VECTOR MESONS IN THE HEAVY PARTICLE CHIRAL LAGRANGIAN

J. Schechter and A. Subbaraman

*Department of Physics, Syracuse University,*
*Syracuse, NY 13244-1130*

## Abstract

We give the general framework for adding "light" vector particles to the heavy hadron effective chiral Lagrangian. This has strong motivations both from the phenomenological and aesthetic standpoints. An application to the already observed $D \rightarrow \overline{K}^*$ weak transition amplitude is discussed.

## 1. Introduction

Recently there has been a lot of interest in the heavy quark (or Isgur-Wise) symmetry [1] which pertains to a rigorous limit of QCD in which old fashioned quark model results may be applied. This limit corresponds to keeping the four-velocity, $V_\mu$ of the heavy quark fixed while taking its mass, $M$ to infinity. A natural application of this approach is to the chiral interactions of the heavy particles with "soft" pions and kaons. Indeed a number of interesting papers [2, 3, 4, 5, 6] have already appeared. The resulting effective Lagrangians can be used to relate amplitudes for processes with a fixed number and type of heavy quarks but with any number of soft pseudoscalars. For example the amplitude for $D^+ \to e^+\nu_e$ is related to the amplitude for $D^0 \to K^- e^+\nu_e$ in the soft $K^-$ region. Continuing, these amplitudes are related to that for $D^+ \to K^-\pi^0 e^+\nu_e$ with soft $K^-$ and $\pi^0$. It would be very interesting to compare such a relation with experiment. Unfortunately, on consulting the Review of Particle Properties we learn [7] that "it is generally agreed that the $\overline{K}\pi e^+\nu_e$ decays of the $D^+$ and $D^0$ are dominantly $\overline{K}^* e^+\nu_e$". This is not very surprising since it is known from low energy physics that two pseudoscalars often prefer to make their appearance as a vector meson. In the future it will undoubtedly be possible to disentangle the non-resonant two pseudoscalar piece. But this example provides a strong motivation for including the light vector mesons in the formulation of the heavy particle effective Lagrangian. We will begin the investigation of the heavy particle effective chiral Lagrangian with vectors in the present paper. The application to $D^+ \to \overline{K}^{*0} e^+\nu_e$ will also be discussed.

In section 2 we will discuss the derivation of the non-interacting part of the heavy meson Lagrangian in order to set down our notation and make some points which will be useful later on. Section 3 contains a brief treatment of the chiral Lagrangian of light pseudoscalars and vectors as well as the interactions of these fields with the heavy mesons. Compared to the heavy meson chiral Lagrangian with only light pseudoscalars there is now a modified chiral covariant derivative as well as a characteristic new interaction term. We will employ a phase convention for the "heavy meson fields" which is convenient for making contact with "ordinary" meson fields and verifying the CP invariance of the theory. In section 4 we will give the leading chiral covariant expression for the weak current and, apply it, in section 5, to the soft light meson regions of the $D^0 \to K^-$ and $D^+ \to \overline{K}^{*0}$ transition matrix elements.

## 2. Derivation of non-interacting Lagrangian

Let us denote the heavy mesons associated with each heavy flavor as being made out of the heavy quark (rather than anti quark); symbolically

$$\text{heavy meson field} \sim \overline{q}_{\text{light}} q_{\text{heavy}}. \tag{2.1}$$

Since there are three light flavors, (2.1) should be regarded as a three component *row* vector for each heavy flavor. In the presently known cases we thus have the experimental pseudoscalar objects $(D^0, D^+, D_s^+)$ and $(\overline{B}^-, \overline{B}^0, \overline{B}_s^0)$.

For our purpose it will be instructive to derive a heavy meson field effective Lagrangian directly from an ordinary field effective Lagrangian, in analogy ot the treatment [8] [1] of the heavy quark effective Lagrangian. Let us first consider the non-interacting terms

$$\mathcal{L}_{\text{free}}(P) = -\partial_\mu P \partial_\mu \overline{P} - M^2 P \overline{P}, \tag{2.2}$$

for the heavy pseudoscalar (row vector) field $P(x)$ of mass $M$. Note that $\overline{P} \equiv P^\dagger$ and that we are employing the "Euclidean" metric convention with $x_4 = it$. In order to implement the basic idea that deviations from straight line motion with 4-velocity $V_\mu$ of the heavy meson be small we make the change of variables

$$P = e^{iMV\cdot x} P'$$

$$\overline{P} = e^{-iMV\cdot x} \overline{P}'. \tag{2.3}$$

$V_\mu$ should be considered as fixed. Furthermore, in the free field expansion

$$P = \sum_{\underline{K}} \frac{1}{\sqrt{2E_{\underline{K}}V}} \left( a_{\underline{K}} e^{iK\cdot x} + b^\dagger_{\underline{K}} e^{-iK\cdot x} \right), \tag{2.4}$$

we are considering that the anti-particle operators $b_{\underline{K}}$ should be neglected. (In the interacting theory, the anti-particles won't be excited as $M \to \infty$). Substituting (2.3) into (2.2) yields

$$\mathcal{L}_{\rm free}(P') = -iMV_\mu P' \overleftrightarrow{\partial_\mu} \overline{P}' - \partial_\mu P' \partial_\mu \overline{P}'. \tag{2.5}$$

Notice that the terms of order $M^2$ have cancelled out. The second term in (2.5) is negligible as $M \to \infty$ so we simply have in the heavy quark limit

$$\mathcal{L}_{\rm free}(P') = -2iMV_\mu P' \partial_\mu \overline{P}'. \tag{2.6}$$

This can be simplified further by redefining

$$P'' = M^{1/2} P' \tag{2.7}$$

to give a form in which the mass independence is manifest,

$$\mathcal{L}_{\rm free}(P'') = -2iV_\mu P'' \partial_\mu \overline{P}'' \tag{2.8}$$

However, $P''$ has the non-canonical dimension $\frac{3}{2}$.

Let us next consider the heavy vector field $Q_\mu$, which is relevant because it belongs [1] to the same heavy spin multiplet as $P$. The free Lagrangian in terms of ordinary spin one fields is

$$\mathcal{L}_{\rm free}(Q) = -\frac{1}{2}(\partial_\mu Q_\nu - \partial_\nu Q_\mu)(\partial_\mu \overline{Q}_\nu - \partial_\nu \overline{Q}_\mu) - M^2 Q_\mu \overline{Q}_\mu,$$

$$\overline{Q}_\mu = (-1)^{\delta\mu 4} Q^\dagger_\mu. \tag{2.9}$$

The transformation

$$Q_\mu = e^{iMV\cdot x} Q'_\mu$$

$$\overline{Q}_\mu = e^{-iMV\cdot x}\overline{Q}'_\mu \tag{2.10}$$

then yields the "small oscillation" Lagrangian,

$$\mathcal{L}_{\text{free}}(Q') = -2iMV_\nu Q'_\mu \partial_\nu \overline{Q}'_\mu, \tag{2.11}$$

in which the subsidiary condition $V_\mu Q'_\mu = 0$ was imposed and a term negligible as $M \to \infty$ was dropped.

Note that (2.6) and (2.11) both have the same structure. This is to be expected by the heavy quark symmetry and can be made [9] [1] manifest by amalgamating $P'$ and $Q'_\mu$ into a single "heavy quark" field, $H$:

$$H = \left(\frac{1 - i\gamma\cdot V}{2}\right)(\eta\gamma_5 P' + i\gamma\cdot Q'),$$
$$\overline{H} \equiv \gamma_4 H^\dagger \gamma_4 = (-\eta^*\gamma_5\overline{P}' + i\gamma\cdot\overline{Q}')\left(\frac{1 - i\gamma\cdot V}{2}\right). \tag{2.12}$$

Here $H$ is a $4\times 4$ matrix in the Dirac spinor space and the coefficients of $P'$ and $Q'_\mu$ are the kinematical operators which respectively project out the pseudoscalar and the vector combinations from $\overline{q}_{\text{light}} q_{\text{heavy}}$. $\eta$ is an arbitrary phase which we will choose as

$$\eta = i, \tag{2.13}$$

for a reason to be discussed later. In contrast, $\eta$ is chosen to be purely real in ref. 2. Using (2.12), the sum of (2.6) and (2.11) can be compactly written as:

$$\mathcal{L}_{\text{free}}(P', Q') = iMV_\mu Tr(H\partial_\mu\overline{H}), \tag{2.14}$$

where the trace refers to the $4\times 4$ Dirac space. There is also an implied summation in the light flavor space since $H$ is a row vector and $\overline{H}$ is a column vector. The use of the $H$ field evidently [1] guarantees the invariance under heavy quark spin transformations (in the Dirac space): $H \to SH, \overline{H} \to \overline{H}S^{-1}$.

Since (2.14) represents the heavy quark limit of (2.2) plus (2.9) one might think that the sum of (2.2) plus (2.9) before taking the limit should be more compactly written using an $H$ defined in terms of $P$ and $Q_\mu$ (rather than $P'$ and $Q'_\mu$) as

$$\frac{1}{2}Tr(\partial_\mu H \partial_\mu \overline{H}) + \frac{1}{2}M^2 Tr(H\overline{H}).$$

This expression is not however consistent even though it does reproduce (2.14) in the heavy quark limit after the substitutions (2.3) and (2.10) are made. The reason is that it gives a vector kinetic term $-\partial_\mu Q_\nu \partial_\mu \overline{Q}_\nu$ which (unlike (2.9)) is well-known to lead to a Hamiltonian unbounded from below. This example illustrates the danger of using $H$ outside the heavy quark regime.

We remark that even though (2.14) [as well as the interacting analogs to be discussed later] is very compact it is not any more difficult to use the sum of (2.2) and (2.9) for practical calculations. This is because the Feynman rules for ordinary mesons are very well known and we can just substitute for the heavy quark momentum $K_\mu$, $K_\mu = MV_\mu + K'_\mu$ at the end of the calculation. From this point of view the heavy quark symmetry just tells us to put the $P$ and $Q_\mu$ masses equal and to equate certain coefficients of the interaction Lagrangian. Certainly it is useful to keep both approaches in mind. In one respect the use of the ordinary fields might actually appear more convenient. That is the case when we want to consider the heavy limit for mesons containing heavy anti-quarks. The ordinary fields contain both quark and antiquark operators as in (2.4) so the calculation can be done using standard techniques. However, by construction, the heavy quark Lagrangian makes no reference to antiquarks. Of course this is not a big problem and one can define a heavy anti-particle Lagrangian in a similar way. We would then like to choose the heavy anti-particle fields to be related to the heavy particle fields in the same way that the ordinary fields describe both particles and anti-particles. What this amounts to is choosing a phase convention so that the right hand sides of (2.3) and (2.10) transform in the same way under charge conjugation as the left hand sides. That turns out to be the reason for our choice (2.13).

## 3. Chiral Interaction Terms

First consider the three light (current) quarks, $q$. Under a chiral transformation

$$q_L \to U_L q_L, \qquad q_R \to U_R q_R, \tag{3.1}$$

where $U_L$ and $U_R$ are $3 \times 3$ unitary matrices. The chiral matrix $U = \exp[2i\phi/F_\pi]$, where $\phi$ is the $3 \times 3$ matrix of pseudoscalars and $F_\pi \simeq 132$ MeV, is constructed [10] in such a way that the interaction term $\overline{q}_L U q_R +$ h.c. is invariant. This implies $U \to U_L U U_R^\dagger$. The interaction term is converted to a light quark "constituent" mass term by the change of variables $q_L = \xi \tilde{q}_L$, $q_R = \xi^\dagger \tilde{q}_R$ with $\xi \equiv U^{1/2}$. The transformation property of $U$ implies [11]

$$\xi \to U_L \xi K^\dagger = K \xi U_R^\dagger, \tag{3.2}$$

where the unitary matrix $K$ depends on $U_L$, $U_R$ as well as $\phi$ and is determined from (3.2). We note that the "constituent" fields transform as $\tilde{q}_L \to K\tilde{q}_L$ and $\tilde{q}_R \to K\tilde{q}_R$. The vector and pseudovector combinations:

$$\begin{aligned} v_\mu &= \frac{i}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi) \\ p_\mu &= \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi) \end{aligned} \tag{3.3}$$

are seen to transform as

$$\begin{aligned} v_\mu &\to K v_\mu K^\dagger + iK\partial_\mu K^\dagger \\ p_\mu &\to K p_\mu K^\dagger. \end{aligned} \tag{3.4}$$

Using (3.4) we can construct a covariant chiral derivative acting on "constituent" type fields:

$$D_\mu \tilde{q} = (\partial_\mu - i v_\mu)\tilde{q},$$

$$D_\mu \tilde{q} \rightarrow K D_\mu \tilde{q}. \tag{3.5}$$

These transformation properties enable us to simply construct chiral invariants.

Before adding the heavy fields into this picture let us add the "light" vector fields. There is a strong phenomenological motivation to do so, of course. But there is also a kind of aesthetic reason which is motivated by the heavy quark symmetry. This is simply that the heavy meson multiplet (2.12) involves both pseudoscalars and vectors. If we want to imagine models in which we can try to extrapolate some quark masses up and down it is necessary to include all relevant degrees of freedom.

It is straightforward to introduce *both* vector and axial vector mesons as linear combinations of fields transforming like

$$A_\mu^L \rightarrow U_L A_\mu^L U_L^{-1}, \quad A_\mu^R \rightarrow U_R A_\mu^R U_R^{-1}. \tag{3.6}$$

For reasons of economy (and because we are not including the other $\ell = 1$ $\overline{q}q$ states) we would like to "integrate out" the axials, analogously to the way one "integrates out" the scalar sigma meson in arriving at the non-linear sigma model. This can be done [12] by writing $A_\mu^L$ and $A_\mu^R$ in terms of the physical vector field $\rho_\mu$ (a $3 \times 3$ matrix):

$$\begin{aligned} A_\mu^L &= \xi \rho_\mu \xi^\dagger + \frac{i}{\tilde{g}} \xi \partial_\mu \xi^\dagger \\ A_\mu^R &= \xi^\dagger \rho_\mu \xi + \frac{i}{\tilde{g}} \xi^\dagger \partial_\mu \xi, \end{aligned} \tag{3.7}$$

where $\tilde{g}$ is a vector meson coupling constant. $\rho_\mu$ is seen to transform as

$$\rho_\mu \rightarrow K \rho_\mu K^\dagger + \frac{i}{\tilde{g}} K \partial_\mu K^\dagger \tag{3.8}$$

so that $F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu]$ transforms as

$$F_{\mu\nu}(\rho) \rightarrow K F_{\mu\nu}(\rho) K^\dagger. \tag{3.9}$$

It is easy to construct chiral invariants using (3.6) and (3.9). The "minimal" chiral Lagrangian of light pseudoscalars and vectors is then simply [12] [13]

$$\mathcal{L}_{\text{light}} = -\frac{1}{4} Tr[F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)]$$

$$-\frac{m_v^2}{8K}(1+K)Tr(A_\mu^L A_\mu^L + A_\mu^R A_\mu^R) + \frac{m_v^2}{4K}(1-K)Tr(A_\mu^L U A_\mu^R U^\dagger),$$

$$K = (m_v/F_\pi \tilde{g})^2, \tag{3.10}$$

where $m_v$ is the light vector meson mass. Note that (3.10) also contains the kinetic and interaction terms of the pseudoscalar mesons. Chiral symmetry breaking terms as well as terms proportional to $\epsilon_{\mu\nu\alpha\beta}$ are given elsewhere. [12] [14] The coupling constant $\tilde{g}$ is related [14] to the width $\Gamma(\rho \to 2\pi)$; a suitable value is

$$\tilde{g} \simeq 3.93. \tag{3.11}$$

Now consider the "ordinary" heavy meson fields $P$ and $Q_\mu$ discussed in section 2. Under the chiral transformations only the light "constituent" degrees of freedom transform so (see (2.1)) we have

$$P \to PK^\dagger, \qquad Q_\mu \to Q_\mu K^\dagger. \tag{3.12}$$

We can upgrade (2.2) and (2.9) to chiral invariants involving interactions with light pseudoscalars and vectors merely by replacing the derivative operators appearing there by suitable covariant derivatives. At this point, however, there is an interesting choice. As can be seen from (3.5) and (3.8) both the vector combination of pseudoscalars, $v_\mu$ and the vector particles, $\rho_\mu$ transform in the same way. Let us therefore define a generalized covariant derivative,

$$\mathcal{D}_\mu \overline{P} = [\partial_\mu - i\alpha\tilde{g}\rho_\mu - i(1-\alpha)v_\mu]\overline{P},$$

$$\mathcal{D}_\mu P = P[\overleftarrow{\partial}_\mu + i\alpha\tilde{g}\rho_\mu + i(1-\alpha)v_\mu]. \tag{3.13}$$

(The same definitions hold for $\overline{Q}_\nu$ and $Q_\nu$.) The dimensionless parameter $\alpha$ specifies the extent to which two emitted pseudoscalars in a relative p-wave like to arise from an intermediate vector state; $\alpha = 1$ would correspond to "vector meson dominance". Our prejudice is that $\alpha$ should be close to unity. However, $\alpha$ should eventually be found by comparing calculations in this model to experiment. Now, making the substitutions (2.3) and (2.10) in the "covariantized" (2.2) and (2.9) yields for the small oscillation fields

$$\mathcal{L}^{(1)}_{\text{heavy}} = iMV_\mu Tr[H(\partial_\mu - i\alpha\tilde{g}\rho_\mu - i(1-\alpha)v_\mu)\overline{H}], \tag{3.14}$$

in which terms $\mathcal{O}(1)$ in $M$ were neglected and (2.12) was used. Note that the "Tr" symbol pertains to the Dirac space while the light flavor space summation is implicit.

Before discussing other chiral invariant interaction terms let us give the hermiticity and CP transformation properties for the quantities involved. Under hermiticity

$$\begin{gathered} P \leftrightarrow \overline{P}, \quad Q_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}\overline{Q}_\mu, \\ \rho_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}p_\mu, \quad p_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}p_\mu, \\ \partial_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}\partial_\mu, \end{gathered} \tag{3.15}$$

while the *usual* phase conventions would give the CP properties:

$$\begin{gathered} P \leftrightarrow -\overline{P}^T, \quad Q_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}\overline{Q}^T_\mu, \\ \rho_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}\rho^T_\mu, \quad p_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}p^T_\mu, \\ \partial_\mu \leftrightarrow -(-1)^{\delta_{\mu 4}}\partial_\mu. \end{gathered} \tag{3.16}$$

If we want to implement an effective CP operation for the heavy primed fields so that the same Lagrangian describes both the heavy quark and heavy antiquark sectors we should demand that under CP:

$$\begin{gathered} P' \leftrightarrow -\overline{P}'^T, \quad Q'_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}\overline{Q}'^T_\mu. \\ V_\mu \leftrightarrow (-1)^{\delta_{\mu 4}}V_\mu \end{gathered} \tag{3.17}$$

Note especially that the behavior of $V_\mu$ follows from requiring, for example, that $V_\mu P' \rightarrow -(-1)^{\delta_{\mu 4}}V_\mu\overline{P}'^T$ to match, using (2.3), the result for ordinary fields that $\partial_\mu P \rightarrow (-1)^{\delta_{\mu 4}}\partial_\mu\overline{P}^T$. It was shown in ref. 2 that an interaction term

$$\mathcal{L}^{(2)}_{\text{heavy}} = iMdTr(H\gamma_\mu\gamma_5 p_\mu\overline{H}) \tag{3.18}$$

plays an important role. $d$ is a real dimensionless constant. Let us expand this, using (2.12), to find

$$\mathcal{L}^{(2)}_{\text{heavy}} = 2Md[\eta P' p_\mu \overline{Q}'_\mu + \eta^* Q'_\mu p_\mu \overline{P} + \epsilon_{\beta\alpha\rho\mu} V_\rho Q'_\alpha p_\mu \overline{Q}'_\beta]. \tag{3.19}$$

As it stands (3.18) is hermitean. But, using the mnemonics in (3.17) and (3.16) we see that the first term goes to $-\eta Q'_\mu p_\mu \overline{P}'$ under CP. In order to agree with the second term we require $\eta = -\eta^*$, which is satisfied by choosing $\eta = i$. It may be verified that the third term is also CP invariant. We note that (3.19) is descended from the ordinary field Lagrangian

$$\mathcal{L}^{(2)}_{\text{heavy}}(P,Q) = 2iMd[P p_\mu \overline{Q}_\mu - Q_\mu p_\mu \overline{P} - \frac{1}{2M}\epsilon_{\beta\alpha\rho\mu}(\mathcal{D}_\rho Q_\alpha p_\mu \overline{Q}_\beta - Q_\alpha p_\mu \mathcal{D}_\rho \overline{Q}_\beta)]. \tag{3.20}$$

The heavy quark symmetry has related the coefficients of the two different pieces in (3.20).

Another important chiral invariant interaction may be written in the heavy symmetry limit

$$\mathcal{L}^{(3)}_{\text{heavy}} = \frac{icM}{m_v} Tr(H\gamma_\mu\gamma_\nu F_{\mu\nu}(\rho)\overline{H}) = \frac{-2cM}{m_v}[2iQ'_\mu F_{\mu\nu}(\rho)\overline{Q}'_\nu + \epsilon_{\mu\nu\alpha\beta}V_\beta(P' F_{\mu\nu}(\rho)\overline{Q}'_\alpha - Q'_\alpha F_{\mu\nu}(\rho)\overline{P}')], \tag{3.21}$$

where $c$ is a dimensionless constant and the light vector mass $m_\nu$ appears just for dimensional reasons. Equation (3.21) is the limit of the ordinary field Lagrangian,

$$\mathcal{L}^{(3)}_{\text{heavy}}(P,Q) = \frac{2icM}{m_v}\Big[-2Q_\mu F_{\mu\nu}(\rho)\overline{Q}_\nu + \frac{1}{M}\epsilon_{\mu\nu\alpha\beta}(\mathcal{D}_\beta P F_{\mu\nu}(\rho)\overline{Q}_\alpha + Q_\alpha F_{\mu\nu}(\rho)\mathcal{D}_\beta \overline{P})\Big]. \tag{3.22}$$

Again, CP invariance may be verified and heavy quark symmetry is seen to have related the coefficients of the two pieces. We can also construct a term similar to (3.21) or (3.22) in which $F_{\mu\nu}(\rho)$ is replaced by $F_{\mu\nu}(v) = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$:

$$\mathcal{L}^{(3)\prime}_{\text{heavy}} = \frac{ic' M}{F_\pi} Tr(H\gamma_\mu\gamma_\nu F_{\mu\nu}(v)\overline{H}). \tag{3.23}$$

In the spirit of (light) vector meson dominance we would expect this term to be less important. However it would play a role in a model where vectors are neglected.

To sum up, the leading terms of the chiral invariant heavy meson Lagrangian written in terms of the doublet $H$ field are

$$\mathcal{L}_{\rm heavy} \;=\; (3.14) \;+\; (3.18) \;+\; (3.21). \tag{3.24}$$

These involve the new coupling constants $c$ and $d$ as well as the parameter $\alpha$ which would be unity in the vector meson dominance approximation. We will not explicitly write the chiral symmetry breaking terms here. Strictly speaking, (3.24) is defined only for mesons with a heavy *quark*. A continuation of (3.24) to "ordinary" heavy fields (containing also meson states with heavy *antiquarks*) is provided by

$$\mathcal{L}(P,Q) = [(2.2) \;+\; (2.9)] \mathrm{with} \partial_\mu \rightarrow \mathcal{D}_\mu \;+\; (3.20) \;+\; (3.22). \tag{3.25}$$

The mutual consistency of (3.24) and (3.25) led to the determination of the phase $\eta$ in the definition of $H$, (2.12). These Lagrangians should be used for large $M$ and small momenta (more precisely, small $p \cdot V$) of the light pseudoscalars and vectors.

## 4. Weak currents

One of the main applications [1] of the heavy quark approach is to the semi-leptonic decays of heavy mesons. These are governed by the effective weak interaction

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} J_\mu^{(+)}(x) J_\mu^{(-)}(x),$$

$$J_\mu^{(-)} = i \sum_{\ell,k} \overline{u}_\ell V_{\ell k} \gamma_\mu (1+\gamma_5) d_k + i \overline{\nu}_e \gamma_\mu (1+\gamma_5) e + \cdots,$$

$$J_\mu^{(+)} = (-1)^{\delta_{\mu 4}} J_\mu^{(-)\dagger}, \tag{4.1}$$

wherein $G_F$ is the Fermi constant, $V_{\ell k}$ is the quark mixing (Kobayashi Maskawa) matrix, $u_l$ is the $\ell^{th}$ charge 2/3 quark, $d_k$ is the $k^{th}$ charge -1/3 quark and other leptonic terms (as well as possible lepton mixings) have not been included. Note that if the matrix elements of $V_{\ell k}$ were all real, $J_\mu^{(-)}$ would go to $J_\mu^{(+)}$ under CP and $\mathcal{L}_W$ would be CP invariant. The hadronic currents of immediate interest are

$$J_\mu^{(-)} = iV_{cs}\overline{c}\gamma_\mu(1+\gamma_5)s + iV_{ub}\overline{u}\gamma_\mu(1+\gamma_5)b + \cdots,$$
$$J_\mu^{(+)} = iV_{cs}^*\overline{s}\gamma_\mu(1+\gamma_5)c + iV_{ub}^*\overline{b}\gamma_\mu(1+\gamma_5)u + \cdots. \tag{4.2}$$

In this paper we will confine our attention to hadronic transitions (current matrix elements) of the form heavy meson → light mesons. We need the realization of the operator $i\overline{q}_a\gamma_\mu(1+\gamma_5)q_{\text{heavy}}$, where $q_a$ is a light quark, in terms of meson fields. The normalizations are provided by the matrix elements for the pseudoscalar → vacuum transitions:

$$i\overline{q}_a\gamma_\mu(1+\gamma_5)q_{\text{heavy}} = F\partial_\mu P_a + \cdots,$$
$$i\overline{q}_{\text{heavy}}\gamma_\mu(1+\gamma_5)q_a = F\partial_\mu \overline{P}_a + \cdots, \tag{4.3}$$

where $F$ is the decay constant for each particular heavy meson. (In this normalization convention $F_\pi \simeq 132$ MeV). Heavy quark symmetry gives the normalization for the heavy vector → vacuum transition in terms of $F$. The leading order chiral covariant generalization was already presented in ref. 2. In our notation it reads

$$i\overline{q}_a\gamma_\mu(1+\gamma_5)q_{\text{heavy}} = \frac{-iFM}{2}Tr[\gamma_\mu(1+\gamma_5)H_b]\xi_{ba}^\dagger + \cdots,$$
$$i\overline{q}_{\text{heavy}}\gamma_\mu(1+\gamma_5)q_a = \frac{-iFM}{2}\xi_{ab}Tr[\gamma_\mu(1+\gamma_5)\overline{H}_b] + \cdots, \tag{4.4}$$

wherein all fields are being evaluated at $x_\mu = 0$ (to eliminate a phase which would arise when (4.4) is derived using (2.3), (2.10) and (2.12)). The factor of $\xi$ in the second equation, for example, is required for chiral covariance: The quark current on the LHS transforms with a factor $U_L$. On the other hand $\overline{H} \rightarrow K\overline{H}$. Using (3.2) it is seen that $\xi\overline{H} \rightarrow U_L(\xi\overline{H})$. Eq. (4.4) gives the currents in the heavy quark limit. It is descended from (using (2.3), (2.10) and neglect of subleading terms in $M$) the "ordinary field" currents

$$i\overline{q}_a\gamma_\mu(1+\gamma_5)q_{\text{heavy}} = F(\mathcal{D}_\mu P_b + MQ_{b\mu})(\xi^\dagger)_{ba} + \cdots,$$

$$i\overline{q}_{\rm heavy}\gamma_\mu(1+\gamma_5)q_a = F\xi_{ab}(\mathcal{D}_\mu\overline{P}_b + M\overline{Q}_{b\mu}) + \cdots. \tag{4.5}$$

Note that the covariant derivatives include pieces proportional to $\rho_\mu$ which are formally suppressed by $\frac{1}{M}$.

Without necessarily endorsing the notions that the $c$ quark is truly heavy and the $s$ quark is truly light we give a specific example of a term in (4.2) and (4.5):

$$J_\mu^{(+)} = V_{cs}^* F_D[\partial_\mu D_b + i\alpha\tilde{g}D_c\rho_{\mu cb} + i(1-\alpha)v_{\mu cb} + M_D D_{\mu b}^*](\xi)^\dagger_{\beta 3} + \cdots, \tag{4.6}$$

where $D_b = (D^0, D^+, D_s^+)$ and $D_{\mu b}^*$ denotes the vector triplet field. We will present results for the $D \rightarrow \overline{K}$ and $D \rightarrow \overline{K}^*$ transitions based on (4.6). Essentially identical formulas will hold for the $B \rightarrow \pi$ and $B \rightarrow \rho$ transitions, etc.

## 5. Applications

For orientation let us first consider the hadronic matrix element for the decay $D^0 \rightarrow K^- e^+ \nu_e$, even though it is practically the same as that for $\overline{B}^0 \rightarrow \pi^+ e^- \nu_e$, already discussed. [1] [4] The invariant matrix element is parametrized by

$$\sqrt{4p_0p_0'V^2} < K^-(p')|J_\mu^+|D^0(p)> = V_{cs}^*[f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu], \tag{5.1}$$

where $q = p - p'$. There is first of all, a direct transition which is read off from (4.6) and (3.3) to be $V_{cs}^*\frac{F_D}{F_\pi}p_\mu$. In addition, there is a contribution from the $D_s^{*+}$ pole diagram. This has three factors: $V_{cs}^* M F_D$ from (4.6), $2iMdp_\nu'/F_\pi$ from (3.20) and the usual $D_s^{*+}$ propagator. Putting everything together gives:

$$f_+ + f_- = \frac{F_D}{F_\pi}\left[1 + \frac{2dM^2 p'\cdot q}{M_s^{*2}}\,\frac{1}{q^2 + M_s^{*2}}\right],$$

$$f_+ - f_- = 2dM^2\frac{F_D}{F_\pi}\,\frac{1 - p'\cdot q/M_s^{*2}}{q^2 + M_s^{*2}}. \tag{5.2}$$

These formulas are expected to be valid only for soft kaons, $p'\cdot V$ small where $V_\mu = p_\mu/M$. They may be simplified by writing $q_\mu = MV_\mu - p_\mu'$ and formally neglecting terms $\mathcal{O}(p'^2)$

to yield [1]

$$f_+ + f_- = \frac{F_D}{F_\pi}\left[1 + \frac{dV\cdot p'}{\Delta - p'\cdot V}\right],$$

$$f_+ - f_- = \frac{F_D}{F_\pi}\frac{dM}{\Delta - p'\cdot V}, \tag{5.3}$$

where $\Delta = M_s^* - M$. We may check the large $M$ scaling laws which say [1] that (5.1) should be of order $M^{1/2}$. $f_+ + f_-$ is the coefficient of $MV_\mu$ so it should go as $M^{-1/2}$ which it does because $F_D \sim M^{-1/2}$. On the other hand $f_+ - f_-$ is the coefficient of $p'_\mu$ so it should go as $M^{1/2}$. That it does so is most evident from (5.3). Note that in the extreme $M \to \infty$ limit we thus expect $f_+ + f_-$ to vanish. In this limit

$$f_+(q^2) \sim \frac{dM^2 F_D/F_\pi}{q^2 + M_s^{*2}} \tag{5.4}$$

We should stress that (5.4) is theoretically justified only near $q^2 = -M^2$; there is no reason for it to hold near $q^2 = 0$ in the present approach. Nevertheless, Anjos et.al. [15] find that such a global form fits their experiment with $f_+(0) = 0.79 \pm 0.05 \pm 0.06$. This would imply

$$d\frac{F_D}{F_\pi} \approx 1, \tag{5.5}$$

which may perhaps be safely interpreted as giving the rough order of magnitude of $d$ (Wise [1] finds $|d| < 1.7$).

Now let us turn to the matrix element

$$\sqrt{4p_0p_0'V^2} < \overline{K}^{*0}(p',\epsilon)|J_\mu^{(+)}/V_{cs}^*|D^+(p)>, \tag{5.6}$$

where $\epsilon$ is the $\overline{K}^{*0}$ polarization vector, which is relevant for $D^+ \to \overline{K}^{*0}e^+\nu_e$. One would of course expect the transition $B \to \rho$ to be a case which is better approximated in our approach. The predicted formulas would be the same but we have chosen the case in (5.6) because experimental data exist for it. According to the large $M$ scaling rules, (5.6) behaves as $M^{1/2}$, just like (5.1). [Actually, with our state normalization convention, this can be read off from the external heavy meson factor $\sqrt{p_0}$ on the LHS]. There are several contributions to (5.6) from the leading order current operator (4.6) together with the heavy Lagrangian (3.25). First there is a direct contribution from (4.6):

$$i\alpha\tilde{g}F_D\overline{\epsilon}_\mu \tag{5.7}$$

where $\overline{\epsilon}_\mu = (-1)^{\delta_{\mu 4}}\epsilon^*_\mu$. However this formally goes as $M^{-1/2}$ and must be interpreted as vanishing in the strict heavy quark limit. There is also a $D_s^+$ pole contribution which is found from (4.6) and the covariantized heavy kinetic term to be:

$$-2i\alpha\tilde{g}F_D\,\frac{p\cdot\overline{\epsilon}q_\mu}{q^2+M_s^2}, \tag{5.8}$$

where $q_\mu = p_\mu - p'_\mu$. Because of the overall $q_\mu$ this term will not contribute to physical processes to the extent that the lepton current is conserved (where $m_e$ is negligible). It may, however, be someday measured in $D^+ \to \overline{K}^{*0}\mu^+\nu_\mu$. Its heavy quark limit is obtained by setting $p_\mu = MV_\mu$, $q_\mu = MV_\mu - p'_\mu$ and considering $p'_\mu$ small; the result

$$-i\alpha\tilde{g}F_D M\frac{V\cdot\overline{\epsilon}V_\mu}{(M_s-M)-p'\cdot V} \tag{5.9}$$

immediately shows that (5.8) properly scales as $M^{1/2}$. The remaining leading contribution to (5.6) is of vector rather than axial-vector type; using (4.6) and (3.22) we find that the $D_s^{*+}$ pole diagram gives

$$4icF_D(M/m_v)\epsilon_{\sigma\nu\mu\beta}\frac{p_\beta p'_\sigma\overline{\epsilon}_\nu}{q^2+M_s^{*2}}, \tag{5.10}$$

where $m_v$ is the light vector mass and $c$ is the new coupling constant introduced in (3.22). Its heavy quark limit,

$$2icF_D(M/m_v)\epsilon_{\sigma\nu\mu\beta}\frac{V_\beta p'_\sigma\overline{\epsilon}_\nu}{(M_s^*-M)-p'\cdot V} \tag{5.11}$$

is seen to scale as $M^{1/2}$ (with $c$ scaling as $M^0$).

To sum up, our result for the $D\to\overline{K}^*$ transition matrix element is given by adding (5.7), (5.8) and (5.10). It should be used only for $q^2$ near $-M^2$. In particular, there is no justification for using it near $q^2=0$.

The experimental situation, which has recently been reviewed by Pham [16], is a little complicated since three different form factors (one other proportional to $p_\mu$ in addition to those in (5.7) and (5.10)) must be inferred from the data. He shows that the coefficient of $\overline{\epsilon}_\mu$ is most crucial in establishing the overall rate; with a phenomenological $q^2$ damping the coefficient of $\overline{\epsilon}_\mu$ at $q^2 = 0$ is found to be roughly 1.4 GeV in magnitude. This might be contrasted with our (5.7) which (for $\alpha = 1$) has a magnitude of about 0.6 GeV. Note that (5.7) is only expected to be valid near $q^2 = -M^2$. Typically the form factors fall with increasing $-q^2$ so our result does not seem unreasonable. Since (5.7) vanishes in the $M \to \infty$ limit, this also provides a caution about using the infinite $M$ limit for the $D$ meson. Continuing, Pham finds that the coefficient of $p_\mu$ in (5.6) is perhaps consistent with zero. This feature would also agree with our result. Finally, the coupling constant $c$ could be eventually determined by comparison of the experimental vector-type form factor in the $q^2 \approx -M^2$ region with (5.10).

All in all, the use of the chiral symmetric expression (5.7) + (5.8) + (5.10) for small light vector momenta does not seem to be unreasonable at our present stage of experimental understanding of the process $D^+ \to \overline{K}^{*0} e^+ \nu_e$. For $D$ decays, at least, it seems better to use the expression (5.7) which follows from the "ordinary" field Lagrangian rather than its strict $M \to \infty$ limit (of zero). In the future it would be interesting to include terms non-leading in $M$ and also terms containing more derivatives of the light fields. An example of the latter which has a piece giving a direct (non-pole) heavy pseudoscalar $\to$ light vector transition is (cf (4.5)):

$$i\overline{q}_a \gamma_\mu (1 + \gamma_5) q_{\text{heavy}} = \cdots + i\beta \mathcal{D}_\nu P_c [F_{\mu\nu}(\rho)]_{cb} [\xi^\dagger]_{ba} + \cdots, \qquad (5.12)$$

where $\beta$ is a real constant. This yields the extra contribution to the matrix element (5.6):

$$i\beta[p \cdot \overline{\epsilon}(p_\mu - q_\mu) + p \cdot p' \overline{\epsilon}_\mu]. \qquad (5.13)$$

This is seen to contain a piece proportional to $p_\mu$ which was absent from our leading order result above.

## 6. Summary

We have given the general framework for adding "light" vector particles to the heavy hadron effective chiral Lagrangian. The leading order in $M$ strong Lagrangian was presented in section 3 with special attention to a convenient phase convention for including the anti-quark sector. There are many possible applications, the most immediate being to the semi-leptonic decays of heavy mesons. We discussed the process $D^+ \rightarrow \overline{K}^{*0} e^+ \nu_e$ (chosen because data exists) and found that the leading order result was reasonable at our present stage of experimental knowledge. In the future it would be instructive to compare the results with those computed for $D^+ \rightarrow K^- \pi^+ e^+ v_e$. Many other processes with (extra) "soft" light vectors and/or pseudoscalars can be considered. Decays like $B \rightarrow D \rho e \overline{\nu}_e$ are very natural to look at next. Progress along these and similar lines will be reported elsewhere.

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