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ABSTRACT

In the framework of chiral soliton models we study the behavior of static nucleon properties under rescaling of the parameters describing the effective meson theory. In particular we investigate the question of whether the Brown–Rho scaling laws are general features of such models. When going beyond the simple Skyrme model we find that restrictive constraints need to be imposed on the mesonic parameters in order to maintain these scaling laws. Furthermore, in the case when vector mesons are included in the model it turns out that the isoscalar form factor no longer scales according to these laws. Finally we note that, in addition to the exact scaling laws of the model, one may construct approximate *local scaling laws*, which depend of the particular choice of Lagrangian parameters.

1. Introduction

The properties of nucleons in nuclear matter (at *finite density*) are of great interest. Recently, these properties have been intensively studied [1, 2] in the framework of a simple soliton model to the baryon. Such models (supported by the $1/N_C$ expansion in QCD) describe nucleons in terms of effective chiral Lagrangians of mesons. The central idea of these investigations is the assumption that the properties of the nucleon at finite density are to be obtained just by using different values of the parameters in the underlying meson Lagrangian. The relationship between these parameters and their zero density values can, for example, be estimated by using QCD sum rules [3]–[6] Here we will not question the central assumption but rather will study whether the scaling laws, which hold in the simple Skyrme model, remain valid when more complicated but also more realistic soliton models are considered.

The typical results of this approach are the Brown-Rho scaling laws [1, 2], which hold exactly in the semi-classical treatment of the Skyrme model with no pion mass term. These express the invariance of the combinations:

$$A = \frac{M}{f_\pi \sqrt{g_A}} = \frac{M^*}{f_\pi^* \sqrt{g_A^*}} \, ,$$
$$B = \frac{\langle r^2 \rangle_{I=0} f_\pi^{\,2}}{g_A} = \frac{\langle r^2 \rangle_{I=0}^* f_\pi^{*2}}{g_A^*} \, , \tag{1}$$

where the starred quantities correspond to evaluations at finite density. M, g_A and $\langle r^2 \rangle_{I=0}$ correspond to the nucleons' mass, axial coupling constant and isoscalar squared radius. The quantity f_π, which is present in the meson Lagrangian, is the pion decay constant.

It is natural to ask whether these equations hold in the simplest extension of the Skyrme model in which a pion mass term is added. We will show they do hold, *provided* that, in addition, m_π scales like

$$m_\pi^2 \langle r^2 \rangle_{I=0} = m_\pi^{*\,2} \langle r^2 \rangle_{I=0}^* \, . \tag{2}$$

Now, it is well known that the simple Skyrme model is unable to provide an adequate description of several nucleon properties. Examples are: the non–electromagnetic piece of the neutron proton mass difference [7], the proton matrix element of the axial singlet current [8] and the *high energy* behavior of the phase shifts in pion nucleon scattering [9]. All these short–comings can be improved in soliton models which contain explicit vector meson degrees of freedom. It is therefore natural to study the scaling behavior of static baryon properties in a more realistic vector meson model[*].

We will show that in this model the quantity B in (1) is no longer invariant under rescaling. However the combination A in (1) will be invariant provided that

$$\frac{m_\pi}{m_\rho} = \frac{m_\pi^*}{m_\rho^*} \quad \text{and} \quad \frac{m_\rho}{g f_\pi} = \frac{m_\rho^*}{g^* f_\pi^*} \, , \tag{3}$$

hold, as well as some scaling laws for coupling constants in the terms of the meson Lagrangian proportional to the Levi–Civita symbol. In (3) g is the vector meson coupling constant and m_ρ is the ρ–meson mass.

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[*]The density dependence of static properties was previously studied in such a model [10] where the variation of the meson parameters was adopted from Nambu–Jona–Lasinio model calculations.

The general strategy for obtaining these scaling laws is to first construct a universal expression for the soliton mass by absorbing as many parameters of the effective meson theory as possible into field and coordinate redefinitions. In this process not all meson fields can be reparametrized because they are subject to restrictive boundary conditions. Subsequently these redefinitions are employed to study the behavior of other nucleon properties when varying these parameters.

The scaling laws mentioned above are exact consequences of the models for any choice of parameters. It is also possible to find approximate (local) scaling laws which hold in the vicinity of a given parameter. An illustration is provided for the vector meson model.

Tests of various scaling laws should be available after the RHIC facility is completed.

2. The Pseudoscalar Model

Although the scaling laws for dense matter were motivated [1] from an effective meson Lagrangian [11], which includes the QCD trace anomaly, these laws essentially follow from the simple Skyrme model [12, 13]. In order to present this model it is convenient to consider the root, ξ of the non–linear realization, $\xi^2 = U = \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi)$, of the pseudoscalar fields. Using the vector and pseudovector objects constructed from ξ

$$v_\mu = \partial_\mu\xi\xi^\dagger - \xi^\dagger\partial_\mu\xi \qquad \text{and} \qquad p_\mu = \partial_\mu\xi\xi^\dagger + \xi^\dagger\partial_\mu\xi \ . \tag{4}$$

the Skyrme Lagrangian assumes the simple form

$$\mathcal{L}_{\mathrm{Sk}} = -\frac{f_\pi^2}{4}\mathrm{tr}\,(p_\mu p^\mu) + \frac{1}{32e^2}\mathrm{tr}\,([p_\mu, p_\nu]\,[p^\mu, p^\nu]) \ . \tag{5}$$

Substituting the hedgehog configuration $\xi(\boldsymbol{r}) = \exp(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)/2)$ yields the classical energy functional

$$E_{\mathrm{cl}}[F] \quad = \quad 2\pi\frac{f_\pi}{e}\int_0^\infty d\zeta\left\{\zeta^2 F'^2 + 2\sin^2 F + \sin^2 F\left(2F'^2 + \frac{\sin^2 F}{\zeta^2}\right)\right\} , \tag{6}$$

where the dimensionless variable $\zeta = ef_\pi r$ has been introduced. This functional is the building block for discussing the variation of static nucleon properties with the parameters f_π and e. The chiral angle, $F(\zeta)$ is obtained by minimizing $E_{\mathrm{cl}}[F]$ subject to the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$ for topological baryon number one. Within this treatment one derives scaling formulas for the classical mass, the isoscalar radius, $\langle r^2\rangle_{I=0}$, and the axial vector charge, g_A [13]

$$E_{\mathrm{cl}} = 72.9\frac{f_\pi}{e} \ , \quad \langle r^2\rangle_{I=0} = \frac{1.12}{f_\pi^2 e^2} \quad \text{and} \quad g_A = \frac{18.0}{e^2} \ . \tag{7}$$

It should be noted that in the simple Skyrme model the isoscalar radius receives its sole contribution from the topological baryon density. This current can be interpreted as the $U_V(1)$ Noether current from the Wess–Zumino term [14]

$$\Gamma_{\mathrm{WZ}} = \frac{iN_C}{240\pi^2}\int_{M_5}\mathrm{tr}(p^5) \ , \quad p = p_\mu dx^\mu \ , \tag{8}$$

where the differential forms notation has been adopted. Furthermore M_5 refers to a manifold which has Minkowski space, M_4, as boundary. We are assuming the three flavor case in writing (8).

In leading order of the $1/N_C$ expansion the nucleon mass, M, is identical to E_{cl}. Hence by eliminating the Skyrme parameter, e, from the expressions (7) one easily obtains two quantities which only contain physical observables

$$A := \frac{M}{f_\pi\sqrt{g_A}} = 17.2 \quad \text{and} \quad B := \frac{\langle r^2\rangle_{I=0}f_\pi^2}{g_A} = 0.062 \ , \tag{9}$$

where the data refer to the Skyrme model predictions (7). In particular, these combinations are universal in the Skyrme model, in the sense that A and B remain unchanged when scaling the fundamental parameters of the Lagrangian (5) [1, 15]

$$A^* = A \quad \text{and} \quad B^* = B \ . \tag{10}$$

Here the asterisk denotes quantities, which are computed using[†] $f_\pi^* \neq f_\pi$ and $e^* \neq e$.

It is an easy exercise to verify that A and B are universal quantities independent of which higher order (in derivatives) stabilizing term is adopted. The only requirement is the restriction to a single stabilizing term.

As the Skyrme model represents the simplest version of an effective meson theory which supports soliton solutions, it is certainly incomplete. Hence the natural question arises whether the scaling laws (10) hold in more realistic models. The first extension, which comes into mind in this context, is the inclusion of a pion mass term. This neither changes the analytical form of $\langle r^2\rangle_{I=0}$ nor of g_A; however it contributes to the soliton mass:

$$(4\pi f_\pi/e)(m_\pi/ef_\pi)^2 \int_0^\infty d\zeta\,\zeta^2(1-\cos F) \ . \tag{11}$$

Hence a universal expression for the nucleon mass can only be obtained by demanding that m_π/ef_π does not scale. This implies

$$m_\pi^{*2}\langle r^2\rangle_{I=0}^* \equiv m_\pi^2\langle r^2\rangle_{I=0} \ . \tag{12}$$

We thus see that the scaling laws (10) do not simply follow from a chirally symmetric theory; rather their implementation yields relations among the scaling behaviors of various hadron observables. Note that this relation goes beyond dimensional analysis since the model contains two dimensional parameters: f_π and m_π. From Nambu–Jona–Lasinio model studies on the density dependence of m_π [16] one might conclude that $\langle r^2\rangle_{I=0}^*$ does not vary until three times nucleon matter density is reached. In any event, the role of the pion mass is special as it arises from the explicit breaking of chiral symmetry. In the following sections we will therefore discuss such constraints for coefficients of chirally symmetric expressions in more realistic models which include vector mesons.

3. The Vector Meson Model

The chirally symmetric action for a realistic vector meson model has been worked out previously [17, 18]

$$\mathcal{A} = \int d^4x \mathcal{L}_{\text{nan}} + \Gamma_{\text{an}} + \Gamma_{\text{WZ}} \tag{13}$$

[†]Although we do not specifically have in mind to restrict the discussion to the density behavior of the nucleon properties we follow the conventions of dense matter literature and denote quantities which are associated with modified meson parameters by an asterisk.

$$\mathcal{L}_{\mathrm{nan}} = \mathrm{tr}\left[-\frac{f_\pi^2}{4}p_\mu p^\mu + \frac{m_\pi^2 f_\pi^2}{4}\left(U + U^\dagger - 2\right) - \frac{1}{2}F_{\mu\nu}(\rho)F^{\mu\nu}(\rho) + m_\rho^2 R_\mu R^\mu\right]$$

$$\Gamma_{\mathrm{an}} = \int \mathrm{tr}\left(\frac{1}{6}\left[\gamma_1 + \frac{3}{2}\gamma_2\right]Rp^3 - \frac{i}{4}g\gamma_2 F(\rho)\left[pR - Rp\right] - g^2\left[\gamma_2 + 2\gamma_3\right]R^3 p\right) .$$

For convenience the differential forms notation has again been used to simplify the presentation of the $\epsilon_{\mu\nu\rho\sigma}$ terms. The vector mesons ω and ρ are contained in $R_\mu = \rho_\mu - (i/2g)v_\mu$ with $\rho_\mu = (\omega_\mu + \rho_\mu^a \tau_a)/2$. The additional parameters g and γ_i are related to the decay widths of the vector mesons. Using a slightly different notation they are found to be [18]

$$g \approx 5.6 , \quad \tilde{h} = -\frac{2\sqrt{2}}{3}\gamma_1 \approx 0.4 , \quad \tilde{g}_{VV\phi} = g\gamma_2 \approx 1.9 , \quad \kappa = \frac{\gamma_3}{\gamma_2} \approx 1 . \tag{14}$$

The value for κ was obtained from a fit to static nucleon properties.

In addition to the hedgehog *ansatz* for the pseudoscalar fields we have two more radial functions parametrizing the vector meson fields

$$\omega_0 = \frac{\omega(r)}{gm_V} , \quad \rho_i^a = \frac{G(r)}{gr}\epsilon_{ija}\hat{r}_j , \tag{15}$$

where $m_V = \sqrt{2}gf_\pi$. Introducing a scaled radial coordinate $\zeta = m_V r$ yields the classical energy functional [18]

$$\begin{aligned}
E_{\mathrm{cl}} = \ & \frac{4\pi f_\pi^2}{m_V}\int_0^\infty d\zeta\Bigg\{\frac{1}{2}\left(F'^2\zeta^2 + 2\sin^2 F\right) + \frac{1}{4}\mu_\pi^2\zeta^2\left(1 - \cos F\right) - \left(\omega'^2 + \mu_\rho^2\omega^2\right)\zeta^2 \\
& + G'^2 + \frac{G^2}{2\zeta^2}\left(G + 2\right)^2 + \mu_\rho^2\left(1 + G - \cos F\right)^2 + 2g\gamma_1 F'\omega\sin^2 F - 4g\gamma_2 G'\omega\sin F \\
& + 2g\gamma_3 F'\omega G\left(G + 2\right) + 2g\left(\gamma_2 + \gamma_3\right)F'\omega\left(1 - 2(G+1)\cos F + \cos^2 F\right)\Bigg\}. \tag{16}
\end{aligned}$$

Here a prime denotes the derivative with respect to ζ and $\mu_i = m_i/m_V$ are dimensionless mass parameters. In order to obtain a universal mass functional we demand that none of the coefficients in eq (16) vary when f_π and/or g are scaled. This implies

$$\frac{m_\pi^*}{g^* f_\pi^*} = \frac{m_\pi}{g f_\pi} , \quad \frac{m_\rho^*}{g^* f_\pi^*} = \frac{m_\rho}{g f_\pi} , \quad g^*\tilde{h}^* = g\tilde{h} , \quad \tilde{g}_{VV\phi}^* = \tilde{g}_{VV\phi} \quad \text{and} \quad \kappa^* = \kappa . \tag{17}$$

The second equation states that for a universal mass functional the KSRF [19] relation has to be taken invariant. For $g^* \neq g$ the frequently adopted scaling relation [1, 20, 21, 22] $m_\rho^*/m_\rho = f_\pi^*/f_\pi$ is apparently violated.

Static properties are computed as the appropriate matrix elements of the symmetry currents. In order to obtain these currents one first introduces external left and right gauge fields (B_L^μ and B_R^μ) such that the action (13) is invariant under local chiral transformations (up to the anomaly). This introduces an additional term, which does not contribute to the classical energy functional, in the anomalous sector [18]

$$d_1 \int \mathrm{tr}\left(\xi F(B_R)\xi^\dagger + \xi^\dagger F(B_L)\xi\right)\left(Rp - pR\right) . \tag{18}$$

This part of the gauged action together with the γ_2–term contributes to the decay width of the process $\omega \to \pi^0 \gamma$. This determines $|2d_1 - \tilde{g}_{VV\phi}/2g^2| \approx 0.038$ [23]. The currents are extracted from the expression linear in these gauge fields. Substituting the static field configurations into the so–obtained currents and taking matrix elements provides the static nucleon properties. For the axial charge one finds [23]

$$
\begin{aligned}
g_A &= \frac{4\pi}{9g^2} \int_0^\infty d\zeta \, (2\zeta a_1 + a_2) \tag{19}\\
a_1 &= \sin F \cos F + 2\mu_\rho^2 \sin F \, (1 + G + \cos F) + g \, (2\gamma_1 + 3\gamma_2) \, \omega F' \sin F \cos F \\
&\quad - g\gamma_2 \left[\cos F \, (\omega G' - \omega'(1 + G - \cos F)) - \omega' \sin^2 F \right] \\
&\quad + g \, (\gamma_2 + 2\gamma_3) \, F' \omega \sin F (1 + G - \cos F) + 4g^2 d_1 \left(\omega' \sin^2 F + \omega F' \sin 2F \right) \\
a_2 &= \zeta^2 F' + g \, (2\gamma_1 + 3\gamma_2) \, \omega \sin^2 F - g\gamma_2 \omega G(G + 2) \\
&\quad + g \, (\gamma_2 + 2\gamma_3) \, \omega (1 + G - \cos F)^2 + 8g^2 d_1 \omega \sin^2 F \; .
\end{aligned}
$$

Although the term proportional to d_1 is a total derivative and hence does not contribute to g_A, it is clear that demanding $g^{*2} d_1^* = g^2 d_1$ in addition to the scaling laws (17) enables the higher moments of the axial current to scale universally. Note that the first Brown–Rho scaling condition in (10) $A^* = A$ is satisfied. Starting from the expression for the pion–nucleon coupling constant [23]

$$
g_{\pi NN} = \frac{8\pi}{9} M f_\pi m_\pi^2 \int_0^\infty dr \; r^3 \sin F(r) \tag{20}
$$

it is straightforward to verify that $g_{\pi NN}^* f_\pi^* / M^* g_A^* = g_{\pi NN} f_\pi / M g_A$ when (17) is imposed, *i.e.* the Goldberger Treiman relation is scale independent. From (19) it can be seen that the Wess–Zumino term (8) does not contribute to g_A. This is in contrast to the isoscalar radius [23]

$$
\begin{aligned}
\langle r^2 \rangle_{I=0} &= \frac{8\pi}{m_V^2 N_C} \int_0^\infty d\zeta \zeta^2 R_{I=0} \tag{21}\\
R_{I=0} &= \frac{\mu_\rho}{g^2} \zeta^2 \omega + \left[\frac{N_C}{4\pi^2} - \frac{1}{g} \left(\gamma_1 + \frac{3}{2} \gamma_2 \right) \right] F' \sin^2 F + \frac{\gamma_2}{2g} \left[F' G(G+2) - 2G' \sin F \right] \\
&\quad - \frac{\gamma_2 + 2\gamma_3}{2g} F'(1 + G - \cos F)^2 + 4d_1 \frac{d}{d\zeta} \left[\sin F(1 + G - \cos F) \right] \; ,
\end{aligned}
$$

where the term which involves N_C stems from Γ_{WZ}. This term scales differently from all the others and causes $\langle r^2 \rangle_{I=0}$ <u>not</u> to scale universally. If the Wess–Zumino term had yielded the sole contribution the isoscalar radius would scale like $1/m_V^* \sim 1/g^* f_\pi^*$. As a consequence the quantity B, which is defined in eq (9), would be scale independent and one would recover the simple Skyrme model result. Also the relation (12) would remain valid. On the other hand, if the other terms, which represent the vector contributions, were the only ones present the scale independent quantity would instead be

$$
\tilde{B} := \frac{\langle r^2 \rangle_{I=0} f_\pi^2}{g_A^2} \; . \tag{22}
$$

In figure 1 we display the numerical comparison of the scale dependences of B and $\tilde{B}$. For this study the variations of the meson parameters are described by the *ansätze*

$$
f_\pi^* = f_\pi (1 - x) \quad \text{and} \quad g^* = g(1 - cx) \; . \tag{23}
$$

6

According to eq (17) this implies $m_\rho^* \approx m_\rho(1 - (1 + c)x)$ for small x. With $c = -0.34$ we obtain $m_\rho^*/m_\rho \approx 0.78$ and $g_A^*/g_A \approx 0.80$ at $x = x_{\mathrm{E}} = 0.35$ as suggested for nuclear density by QCD sum rules [3, 4] and studies of the neutron beta–decay in heavy nuclei [24], respectively. Note that $c \neq 0$ is necessary for g_A to vary. The numerical calculation yields $M^*/M \approx 0.60$
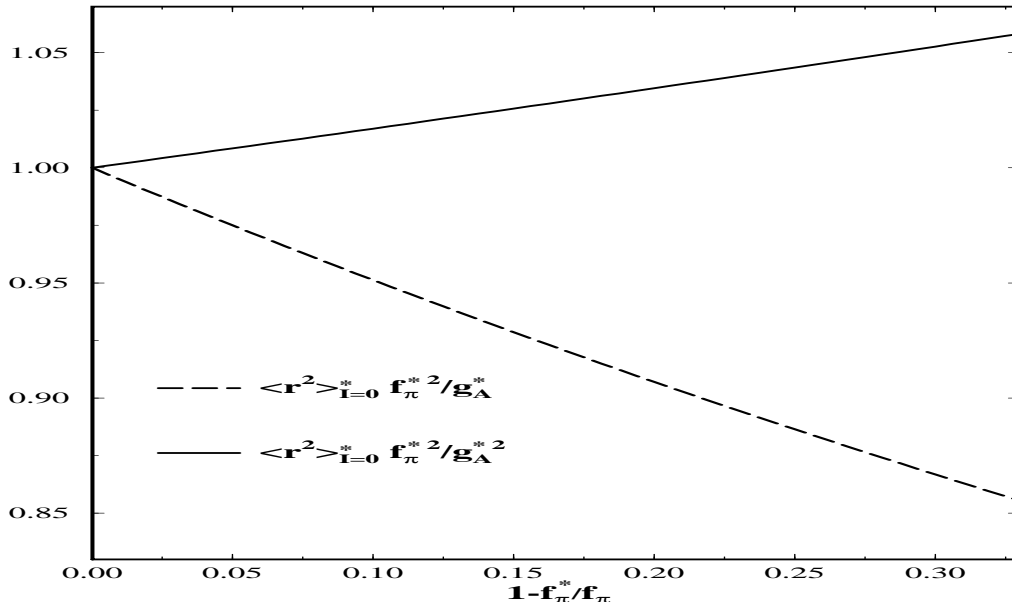

Figure 1: Comparison of the two scaling behaviors for the isoscalar radius in the vector meson model. These quantities are normalized to their unscaled values. The variation of the soliton properties corresponds to $c = -0.34$ in eq (23).

at $x = x_{\mathrm{E}}$, which is slightly smaller than the QCD sum rules estimate of 0.67 ± 0.05 [3, 4].

The scaling behavior shown in figure 1 indicates that $\tilde{B}$ is closer to an invariant than B; the variation of the former is only about 5% compared to a 17% decrease of B at x_E. The physical interpretation of this result is that the vector mesons provide the major contribution to the isoscalar form factor. This in turn alters the scaling law obtained in the simple Skyrme model which does not contain any vector meson degrees of freedom. On the other hand the isovector radius $\langle r^2 \rangle_{I=1}$ has no direct contribution from the Wess–Zumino term. Using formulae (3.5a) and (B2) of ref [23] one verifies that $\langle r^2 \rangle_{I=1} f_\pi^2/g_A$ does not change when (17) is imposed. This behavior is also obtained in the pure pseudoscalar Skyrme model.

For completeness we should note that a similar behavior, *i.e.* the non–existence of a universal scaling law is also observed for the isoscalar magnetic moment. Hence the above discussion applies to the whole isoscalar current.

These studies raise the question whether it is possible to construct a vector meson model without the Wess–Zumino term. Noting that this term is crucial not only from a conceptual point of view (chiral anomaly) but also for the proper normalization of the nucleon charge this question can immediately be answered in the negative. It turns out that the normalization of the isoscalar charge does not completely fix the scaling law for the associated radius because the spatial integral over those parts of the isoscalar density which do not arise from the Wess–Zumino term vanishes identically [23].

4. Local Scaling Laws

The scaling laws (17) for the mesonic parameters, which were imposed to obtain a universal mass functional, are clearly very restrictive and one wonders whether or not other relations can be found. While the scaling behavior of the parameters entering $\mathcal{L}_{\mathrm{nan}}$ acquires some justification from QCD sum rules there is no a priori analysis which prevents one from choosing scaling laws different from (17) for the parameters describing Γ_{an}. In turn a different choice might yield scaling laws for the soliton properties which significantly deviate from those obtained in the simple Skyrme model. In contrast to the general relations given in eq (17) such alternative scaling laws will depend on the particular values the mesonic parameters take on at zero scaling. We therefore refer to these relations as *local scaling* laws. As this subject is potentially vast we will consider only one example:

$$\tilde{h}^* = \tilde{h}(1 - cx) \quad \text{and} \quad g_{VV\phi}^* = g_{VV\phi}(1 - cx)^2 \ , \tag{24}$$

while f_π^* and g^* are taken as given in eq (23). The unscaled parameters are given in eq (14). The associated Brown–Rho scaling as well as a modified Brown–Rho scaling, $\tilde{A}^* = M^*/f_\pi^* g_A^*$, are shown in figure 2. Although the x–dependences of both quantities are very moderate,
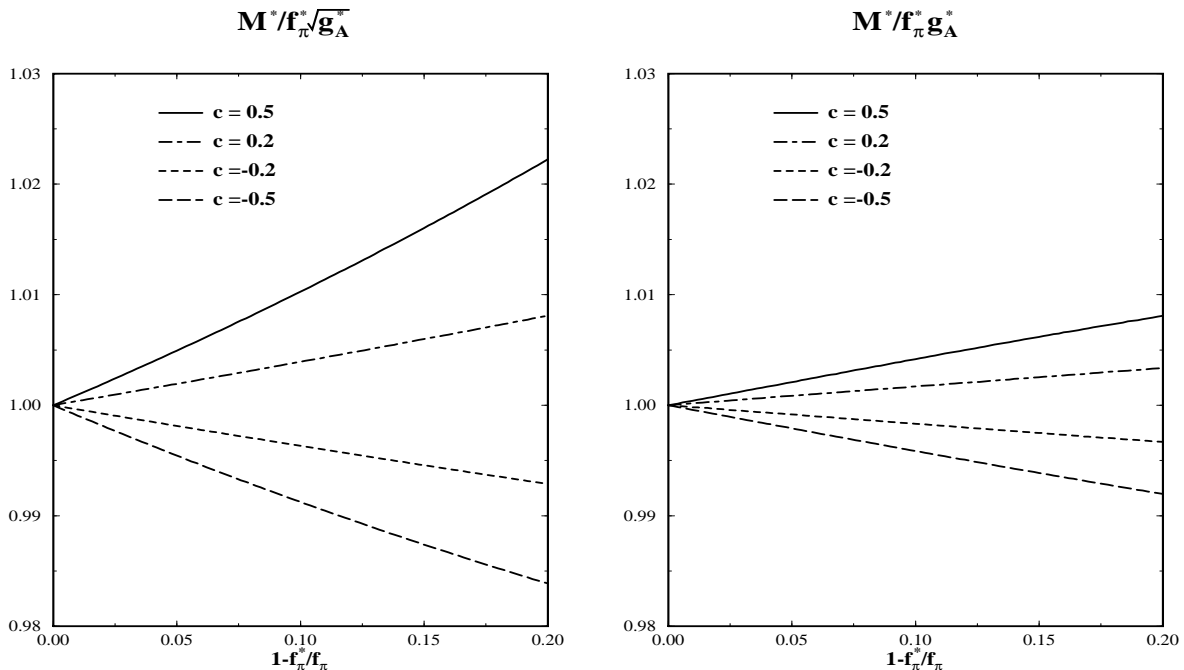

Figure 2: Comparison of the two local scaling laws in the vector meson model.

$\tilde{A}^*$ apparently varies less than A^*, *cf.* eq (9). Certainly a fine–tuning of either the scaling rules (24) or the modified Brown–Rho law could be imposed to yield $\tilde{A}^*$ invariant. This simple example of a local scaling law indicates that other scaling behaviors for the baryon properties could be justified by a different choice for the variation of the parameters entering Γ_{an}. Hence the confirmation of specific scaling laws for static nucleon properties from a realistic soliton model seems to be difficult as long as the *a priori* scaling behavior of the anomalous terms is as poorly known as at present. When axial vector mesons are included [25] the situation is most likely even more arbitrary.

5. Conclusions

In the framework of chiral soliton models we have studied the dependence of static nucleon properties on rescaling of the parameters describing the effective meson theory. The first step in deriving scaling relations between nucleon properties and the mesonic parameters consists of constructing a universal energy functional for the soliton. In the case of the simple Skyrme model this immediately leads to parameter dependences of the axial charge and the isosinglet radius which obey the Brown–Rho scaling laws. When extending the Skyrme model by adding the pion mass term and/or higher order stabilizing terms, the scaling laws no longer follow automatically. However, it is possible to impose conditions on the additional parameters such that the scaling laws are recovered. In the pseudoscalar Skyrme model the isoscalar form factor completely arises from the Wess–Zumino term, which has no effect on the classical mass functional. Therefore the conditions on the mesonic parameters, which are derived from the mass functional, have no direct consequences for the isoscalar form factor. The situation changes drastically when explicit vector meson are contained in the soliton model. Then the isoscalar form factor receives contributions not only from the anomalous Wess–Zumino term but also from the direct coupling of the photon to the ω meson and its source terms which are contained in the chirally symmetric ϵ–terms. Since these quantities also appear in the mass functional there is no unique scaling law for the isoscalar form factor. Numerical calculations indicate that the vector terms actually dominate the isoscalar form factor, making necessary a modification of the Brown–Rho scaling law for the isoscalar radius. In the context of the vector meson model we have also seen that a universal mass functional requires the KSRF relation to be scale invariant rather than just the dimensionless ratio of the vector meson mass over the pion decay constant. In addition to this ratio the KSRF relation involves the vector meson coupling constant. This coupling constant has to be taken scale dependent in order to allow the axial charge to vary.

These studies were motivated by the derivation of the Brown–Rho scaling for the description of dense matter from the simple Skyrme model. In general this approach is based on the assumption that the density dependence of nucleon properties can be obtained by a suitable scaling of the parameters contained in the mesonic Lagrangian from which the soliton is constructed. Unfortunately only little is known about the density dependence of the coefficients of the ϵ terms. As these play an important role for stabilizing the soliton it is not surprising that by choosing an arbitrary behavior of these parameters almost any scaling law for the nucleon mass can be obtained. We have illustrated this for the case that $M/f_\pi g_A$ can be made less sensitive to parameter variations than $M/f_\pi \sqrt{g_A}$. When applying these results to investigate the density dependence of nucleon properties one should also bear in mind that at finite density additional terms may be needed in the effective meson theory as, for example, suggested by heavy baryon chiral perturbation theory [26].

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