Intelligent Processing in Wireless Communications Using Particle Swarm Based Methods

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Abstract

There are a lot of optimization needs in the research and design of wireless communication systems. Many of these optimization problems are Nondeterministic Polynomial (NP) hard problems and could not be solved well. Many of other non-NP-hard optimization problems are combinatorial and do not have satisfying solutions either. This dissertation presents a series of Particle Swarm Optimization (PSO) based search and optimization algorithms that solve open research and design problems in wireless communications. These problems are either avoided or solved approximately before.

PSO is a bottom-up approach for optimization problems. It imposes no conditions on the underlying problem. Its simple formulation makes it easy to implement, apply, extend and hybridize. The algorithm uses simple operators like adders, and multipliers to travel through the search space and the process requires just five simple steps. PSO is also easy to control because it has limited number of parameters and is less sensitive to parameters than other swarm intelligence algorithms. It is not dependent on initial points and converges very fast.

Four types of PSO based approaches are proposed targeting four different kinds of problems in wireless communications. First, we use binary PSO and continuous PSO together to find optimal compositions of Gaussian derivative pulses to form several UWB pulses that not only comply with the FCC spectrum mask, but also best exploit the available spectrum and power. Second, three different PSO based algorithms are developed to solve the NLOS/LOS channel differentiation, NLOS range error mitigation and multi-
lateration problems respectively. Third, a PSO based search method is proposed to find optimal orthogonal code sets to reduce the inter carrier interference effects in an frequency redundant OFDM system. Fourth, a PSO based phase optimization technique is proposed in reducing the PAPR of an frequency redundant OFDM system. The PSO based approaches are compared with other canonical solutions for these communication problems and showed superior performance in many aspects. which are confirmed by analysis and simulation results provided respectively.

Open questions and future works for the dissertation are proposed to serve as a guide for the future research efforts.
INTELLIGENT PROCESSING IN WIRELESS COMMUNICATIONS USING PARTICLE SWARM BASED METHODS

By

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Dedication

To Dr. Lisa Osadciw,
who is not only a mentor, but a friend and a family member.

To my best friend and my wife Yu Zhu
Chapter 1

General Introduction

1.1 Motivations

There are a lot of optimization needs in the research and designs in wireless communication filed. These problems are the focus of much active research in order to find efficient approaches to address them. However, the outcome of the research is still unsatisfactory.

Many of the optimization problems are Nondeterministic Polynomial (NP) hard problems and could not be solved well. Many of other non-NP-hard optimization problems are combinatorial and do not have satisfying solutions either. Evolutionary algorithms are generally more suitable to solve these difficult problems because they are population-based stochastic approaches. Thus, they can avoid being trapped in a local optimum and can often find a global optimal solution. Particle Swarm Optimization (PSO) is a population-based stochastic optimization algorithm modeled after the simulation of the social behavior of bird flocks. PSO is easy to implement and has been successfully applied to solve a wide range of optimization problems. Thus, due to its simplicity and efficiency in navigating large search spaces for optimal solutions, PSO is used in this research to develop efficient, robust and flexible algorithms to solve a selective set of
difficult problems in the field of wireless communications. One of the urgent problems to be solved is to fit suitable approaches into different application domains. Innovation and hybridization of algorithms are also necessary when facing different applications. PSO has quite a few modifications and is easy to tune to apply on different fields, which also make it a good choice when solve various problems in wireless communication research.

1.2 Objectives

The primary objectives of this thesis can be summarized as follows:

* To show that many signal processing problems in wireless communication field can be boiled down to search problems or optimization problems.

* To show that the PSO can be successfully used to solve difficult problems in wireless communications and signal processing.

* To develop a black-box tool that can aid researchers to solve similar optimization problems in wireless communications.

1.3 Methodology

Questions and problems appear in this thesis are among many particle problems the author has encountered in the research of wireless communications and optimizations. These problems either have unsatisfactory solutions or have no solutions at all. The problems of different nature are all reduced to different “non-traditional” optimization problems.
1.4 Contributions

In this thesis, we designed algorithms based PSO for four types of problems in wireless communications.

The first problem we solved is a Ultra Wide Band (UWB) pulse shape design problem. A binary PSO is used to select the different Gaussian derivative pulses that form the composite pulse. A continuous PSO is used to obtain the parameters (time duration and amplitude) for each of the selected pulses. The new composite pulse generated conforms to the FCC masks and maximizes the power. Using a bottom-up biologically inspired approach as in particle swarm optimization enables design of signals with arbitrary constraints. The speedy convergence and simplicity of the operators used by the algorithm enables its use in real time.

The second problem we engaged in is the LOS/NLOS wireless channel differentiation and Non-Line Of Sight (NLOS) range error mitigation in an indoor localization problem. Multiple metrics derived from this data are fused to identify the NLOS signals. Specifi-
cally, kurtosis, mean excess delay and root mean square delay are used as metrics for fusion. The fusion strategy is derived using PSO, considering the correlation between multiple classifiers. An NLOS error mitigation approach is also derived using PSO. The PSO based algorithm identifies the error mitigation ratio using the training data. Finally, PSO is used for multilateration to combine measurements from three nodes. We have compared our strategies to traditionally applied techniques and achieved higher performance. Performance is measured in terms of the Bayesian risk function for the NLOS identification. For locationing the performance is measured in terms of the positioning error, i.e. distance from the true position. We are able to achieve less than a meter error when PSO based strategies are used.

The third problem is to design a satisfying set of orthogonal codes for a frequency redundant OFDM system. By using this codes as spreading sequences, the inter carrier interference can be greatly reduced. Again, PSO based algorithm provide an adaptive approach to find a satisfying code for any irregular code length within an reasonable maximum.

The forth problem we resolve is the reduction of the peak to average power ratio (PAPR) in a frequency redundant OFDM system. We adopt a two stage approach. At the first stage, the frequency domain subcarriers are divided into smaller clusters, we apply random phase rotation and another specific manipulation on clusters of subcarriers. At the second stage, we propose a novel PSO based phase rotation method in reducing the PAPR. Extensive simulation tests are conducted to show the superiority of the performance of our method. Results show that the PSO method can obtain the lowest PAPR reduction with acceptable computational complexity.
1.5 Thesis Outline

This dissertation is organized as follows.

Chapter 1 (this chapter) has given an introduction to this dissertation. It includes motivation and objective of the research and explains the methodology adopted and dissertation contributions.

Chapter 2 starts with an introduction to the theory of optimization, followed by a brief review of certain existing techniques for solving optimization problems. We can not list all the existing techniques for solving optimization problems. Instead, we list some related and important techniques to help to understand PSO method.

Chapter 3 presents a detailed description of the particle swarm optimization algorithm. It also includes a presentation of the numerous key factors in PSO and a brief introduction of binary PSO algorithm which is used in later chapters.

Chapter 4 proposes a new approach in ultra wide band pulse shape design using PSO. By using the designed PSO based method, we linearly combine the Gaussian derivatives creating multiple orthogonal UWB signals. These signals comply with the jagged FCC mask. The new composite pulse best exploits the bandwidth and energy within a given spectral mask.

Chapter 5 describes a novel UWB positioning system design using particle swarm enabled learning techniques. First, a PSO based classification strategy identifies the NLOS signals from the received signal set. Then we use PSO to mitigate the distance estimation error due to the NLOS signal. Finally, the PSO is used for the multilateration problem to locate a position unknown nodes in a sensor network.

Chapter 6 use PSO as an intelligent searching algorithm to find the sub-optimal solution for a code design problem in a frequency redundant OFDM/OFDMA system. By using PSO to find an short length optimal pseudo orthogonal code, we propose an OFDM
transceiver design that employs frequency redundant subcarrier mapping to mitigate frequency selective fading and subcarrier spreading to achieve ICI self cancelation.

Chapter 7 considers the use of a two stage method to reduce the peak to average power ratio (PAPR) of a frequency redundant OFDM system. At the second stage, a PSO based method is proposed to search the suboptimal combination of phase factors for pre-divided subcarrier clusters. The phase rotation factor search of the problem can be formulated as a discrete optimization and a binary PSO method is applied on it.

Chapter 8 provides a summary of this dissertation and recommendations for future work.
Chapter 2

Optimization Problems and Solutions

Optimization, in the straightforward interpretation, is the process of finding the optimum value of a given function, the objective function, on a particular domain, possibly with a number of additional constraints. An optimum can be either a maximum or a minimum depending on the problem formulation, it is straightforward to turn a minimization problem into a maximization problem, and vice versa. In the simplest case, optimization means solving problems in which one seeks to minimize or maximize the objective function by systematically choosing the values of real or integer variables from within an allowed set. More generally, it means finding “best available” values of the objective function given a defined domain, including a variety of different types of objective functions and different types of domains.

2.1 Introduction

Optimization has its foundations dating back to the days of Newton, Lagrange, Cauchy, and Leibnitz when differential calculus methods were developed to minimize and maximize analytical functions. Substantial progress in optimization became more prominent
in the mid to late twentieth century when digital computers showed promise in offloading analytical problem solving into numerical methods through computer code for faster evaluations of designs.

The great divide in classification of optimization problems depends on the domain on which the objective function is defined. In “ordinary” optimization the domain is continuous and the objective function often have at least a first order (partial) derivative. If, on the other hand, the domain is discrete, the problem is combinatorial and combinatorial optimization problems are in general much more difficult to solve than “ordinary” optimization problems. In fact, for a large class of problems there exists no known algorithm to solve such a problem in a reasonable amount of time.

Gradient decent based approaches have been used for optimization as one of the most popular approaches to solve problems in design and engineering. To find a local minimum of a function using gradient descent, one takes steps proportional to the negative of the gradient (or of the approximate gradient) of the function at the current point. If instead one takes steps proportional to the gradient, one approaches a local maximum of that function; the procedure is then known as gradient ascent. To use gradient decent, the objective function should be differentiable. This limit the usefulness of this method. To further achieve the gradient and subsequent step size, certain restrictions and assumptions need to be made for the underlying objective function. As a result of this, the function does not represent the realistic scenario precisely any more. Another popular optimization approach is Linear Programming. Linear programming is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear equations. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. The restrictions on objective functions as well as constraints of the target problem also
greatly limit the usefulness of this kind of approaches. One example of those optimization
problems that are not well solvable by the above two approaches is localization of
an unknown node using three known nodes by multilateration. The problem is formu-
lated as a minimization problem, minimizing the squared error between the estimate of
the unknown node’s position and the known nodes’ positions. For linear programming
approach, the objective function is not linear. Linearize the objective function will bring
big errors that sacrifice the precision of the results. For gradient decent algorithm, the
process involves the inversion of a matrix. The matrix can be singular due to certain posi-
tions of the known nodes, which make intolerable computation delays and inaccuracies.
In the gradient descent approach, as the starting point is randomly chosen, the gradient
descent is likely to stuck at a local optimal because of that.

2.2 Formulation of Optimization Problems

Optimization is the mechanism by which one finds the maximum or minimum value of a
function or process. This mechanism is used in fields such as physics, chemistry, eco-
nomics, and engineering where the goal is to maximize efficiency, production, or some
other measure. Optimization can refer to either minimization or maximization; maximiza-
tion of a function \( f \) is equivalent to minimization of the opposite of this function, \(-f\).

Mathematically, a minimization task is generally defined as:

Given \( f : \mathbb{R}^n \rightarrow \mathbb{R} \)

Find \( x^* \in \mathbb{R}^n \) such that \( f(x^*) \leq f(x), \forall x \in \mathbb{R}^n \)

Similarly, a maximization task is defined as:

Given \( f : \mathbb{R}^n \rightarrow \mathbb{R} \)

Find \( x^* \in \mathbb{R}^n \) such that \( f(x^*) \geq f(x), \forall x \in \mathbb{R}^n \)

The domain \( \mathbb{R}^n \) is referred to as the search space. Each element of \( \mathbb{R}^n \) is called a
candidate solution in the search space, with $x^*$ being the optimal solution. The value $n$ denotes the number of dimensions of the search space, and thus the number of parameters involved in the optimization problem. The function $f$ is called the objective function, which maps the search space to the function space. Since a function has only one output, this function space is usually one-dimensional. The function space is then mapped to the one-dimensional fitness space, providing a single fitness value for each set of parameters. This single fitness value determines the optimality of the set of parameters for the desired task. In most cases, the function space can be directly mapped to the fitness space. However, the distinction between function space and fitness space is important in cases such as multi-objective optimization tasks, which include several objective functions drawing input from the same parameter space [1].

For a known (differentiable) function $f$, calculus can fairly easily provide us with the minima and maxima of $f$. However, in real-life optimization tasks, this objective function $f$ is often not directly known. Instead, the objective function is a “black box” to which we apply parameters (the candidate solution) and receive an output value. The result of this evaluation of a candidate solution becomes the solution’s fitness. The final goal of an optimization task is to find the parameters in the search space that maximize or minimize this fitness.

In some optimization tasks, called constrained optimization tasks, the elements in a candidate solution can be subject to certain constraints. The objective function along with some inequality constraints, and sometimes equality constraints can each be linear or non-linear functions depending upon the problem to be solved. The side constraints provide lower and upper bounds for the variables. If vector $x$ is plotted on an $n$-dimensional Cartesian coordinate system with each coordinate axis representing a variable $(x_1, x_2, x_3, ..., x_n)$, the space occupied by the coordinate system is called the search space or the design space. The objective function $f(x)$ refers to the location in the design
2.2. FORMULATION OF OPTIMIZATION PROBLEMS

space for a specific set of values assigned to the design vector \( x \). Figure 2.1 represents a simple single dimensional objective function with four minimums.

![1 D Search Space](image)

**Figure 2.1: One dimensional search space**

In this search space variable \( x \) is plotted on the X-axis and the objective function is plotted on the Y-axis. Points \( A, B \) and \( D \) are local minimums and point \( C \) is the global minimum. Figure 2.2 represents a two dimensional objective function with one minimum.

The smallest oval in the objective function contour represents the optimum and its value increases as the size of the oval increases. A design point \( A(\mathbf{x}_1, \mathbf{x}_2) \) encapsulates the variable information. For a 10 dimensional objective function point \( A \) will have variable values \((\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, ..., \mathbf{x}_{10})\).

Another class of optimization problems are known as Least-Squares problems, which are of the form

Given \( \mathbf{r}: \mathbb{R}^n \rightarrow \mathbb{R}^m, n < m \)
Find $x^* \in \mathbb{R}^n$ for which $\sum_{i=1}^m (r_i(x))^2$ is minimized.

These optimization problems present themselves when there are more non-linear requirements than there are degrees of freedom. Note that the least-squared problem can be solved using the same approach as used in the previous definition, by defining

$$f(x) = \sum_{i=1}^m (r_i(x))^2$$

and minimizing $f$. In this thesis, we will see such kind of problems.
2.2. FORMULATION OF OPTIMIZATION PROBLEMS

Optimization problems defined above mainly can be placed into two subfields: Local and Global optimization problems.

2.2.1 Local optimization

A local optimizer, $x^*_B$, of the region $B$, is defined so that

$$f(x^*_B) \leq f(x), \forall x \in B$$  \hspace{1cm} (2.2)

where $B \in S \subseteq \mathbb{R}^n$, and $S$ denotes the search space. Note that $S = \mathbb{R}^n$ when dealing with unconstrained problems. More importantly, note that $B$ is a proper subset of $S$. A given search space $S$ can contain multiple regions $B_i$ such that $B_i \cap B_j = \emptyset$ when $i \neq j$. It follows that $x^*_{B_i} \neq x^*_{B_j}$, so that the optimizer of each region $B_i$ is unique. Any of the $x^*_{B_i}$ can be considered a minimizer of $B$, although they are merely local minimizers. There is no restriction on the value that the function can assume in the optimizer, so that $f(x^*_{B_i}) = f(x^*_{B_j})$ is allowed. The value $f(x^*_{B_i})$ will be called the local minimum.

Most optimization algorithms require a starting point $z_0 \in S$. A local optimization algorithm should guarantee that it will be able to find the minimizer $x^*_B$ of the set $B$ if $z_0 \in B$. Some algorithms satisfy a slightly weaker constraint, namely that they guarantee a find a minimizer $x^*_{B_i}$ of some set $B_i$ not necessarily the one closest to $z_0$.

Many local optimization algorithms have been proposed. A distinction will be made between deterministic, analytical algorithms and the stochastic algorithms. The deterministic local optimization algorithms include simple Newton-Raphson algorithms, through Steep Descent [2] and its many variants, including the Scaled Conjugate Gradient algorithm (SCG) [3] and the quasi-Newton [4] family of algorithms. Some of the better known algorithms include Fletcher-Reeves (FR), Polar-Ribiere (PR), Davidon-Fletcher-Powell (DFP), Broyden-Fletcher-Goldfarb-Shanno (BFGS) [5]. There is even an algorithm that
2.2. FORMULATION OF OPTIMIZATION PROBLEMS

was designed specifically for solving least-squares problems, known as the Levenberg-Marquardt (LM) algorithms [2].

2.2.2 Global optimization

The *global optimizer*, or *global minimizer* in this context, $x^*$, is defined so that

$$f(x^*) \leq f(x), \forall x \in S$$

(2.3)

where $S$ is the search space. For unconstrained problems it is common to choose $S = \mathbb{R}^n$, where $n$ is the dimension of $x$. A global optimization algorithm like the local optimization algorithms describe above, also starts by choosing an initial starting position $z_0 \in S$.

Contrary to the definition above in 2.3, some texts (e.g. [4]) define a global optimization algorithm differently, namely an algorithm that is able to find a (local) minimizer of $B \subset S$, regardless of the actual position of $z_0$. These algorithms consists of two processes: “global” steps and “local” steps. Their local steps are usually the application of a local minimization algorithm, and their global steps are designed to ensure that the algorithm will move into a region $B_i$, from where the “local” step will be able to find the minimizer of $B_i$. These methods will be referred to as *globally convergent* algorithms, meaning that they are able to converge to a local minimizer regardless of their starting point $z_0$. These methods are also capable of finding the global minimizer, given that the starting position $z_0$ is chosen correctly. There is no known reliable, general way of doing this, though. A true global optimization algorithm will find $x^*$ regardless of the choice of starting position $z_0$. Dixon and Szego have edited two collections of papers on the topic of global optimization algorithms [6].
2.3 Classifications of Optimization Problems

Except the rough classification provided in previous section, optimization problems can also be classified into various categories according to different measures. Figure 2.3 partially shows the classification. Note that the word “programming” in this figure has nothing to do with a computer program or algorithm, the definition predates the modern computer [7].

Different categories may have overlaps to each other. For example, convex programming studies the case when the objective function is convex and the constraints, if any, form a convex set. This can be viewed as a particular case of nonlinear programming or as generalization of linear or convex quadratic programming. On the other hand, nonlinear programming studies the general case in which the objective function or the constraints or both contain nonlinear parts. This may or may not be a convex program. We selectively explain some of the classifications here.
2.3. CLASSIFICATIONS OF OPTIMIZATION PROBLEMS

2.3.1 Convex and Non-convex Optimization Problems

As shown in Figure 2.3, optimization problems are classified as either convex or non-convex based on whether the domain and the cost function are both convex or not. The domain is convex if a straight line between any two points $x_1$ and $x_2$ in the domain is also part of the domain, and the cost function is convex if its value at any point along the straight line between any two points $x_1$ and $x_2$ in the domain has an upper bound in the chord through $(x_1; f(x_1))$ and $(x_2; f(x_2))$. See Figure 2.4 and 2.5 for examples of the convexity criterion for domain and function, respectively.

![Convex and Non-convex Domains](image)

Figure 2.4: Examples of a convex (left) and a non-convex domain (right)

More precisely stated, the criteria for convexity are: given two points $x_1, x_2 \in S$, $\lambda \in [0, 1]$, the domain $S$ is convex if and only if

$$\lambda x_1 + (1 - \lambda) x_2 \in S, \quad (2.4)$$

and the objective function is convex if and only if

$$f(\lambda x_1 + (1 - \lambda) x_2) \leq \lambda f(x_1) + (1 - \lambda) f(x_2). \quad (2.5)$$
2.3. CLASSIFICATIONS OF OPTIMIZATION PROBLEMS

Figure 2.5: Examples of a non-convex function (left) and a convex function (right). The solid line, \( f \), is the function \( f(\lambda x_1 + (1 - \lambda)x_2) \) and the dashed line is the convexity criteria \( \lambda f(x_1) + (1 - \lambda)f(x_2) \).

If both the domain and the function are convex the problem is said to be convex and in this case a local minimum is also a global minimum. This is a nice property which means that the problem can be solved by powerful standard methods.

2.3.2 Continuous and Combinatorial Optimization Problems

In “ordinary” optimization the domain is continuous and the objective function often have at least a first order (partial) derivative. If, on the other hand, the domain is discrete, the problem is combinatorial. Combinatorial optimization problems are in general much more difficult to solve than “ordinary” optimization problems. In fact, for a large class of problems there exists no known algorithm to solve such a problem in a reasonable amount of time.

Continuous optimization requires the domain to be continuous, \( S \in \mathbb{R}^n \), and the objective function, \( f \) to be defined on the whole of \( S \). Furthermore, the partial first derivatives
2.3. CLASSIFICATIONS OF OPTIMIZATION PROBLEMS

\[ \frac{\partial f(x)}{\partial x_i} \] of \( f \), and thus the gradient vector

\[
\nabla f(x) = \begin{pmatrix}
\frac{\partial f}{\partial x_1}(x) \\
\vdots \\
\frac{\partial f}{\partial x_n}(x)
\end{pmatrix}
\] (2.6)

are assumed to exist. This kind of problems are usually solved by standard differential optimization methods.

If the domain \( S \) is discrete rather than continuous, the problem instance \( (S; f) \) is a combinatorial problem. Given \( S \in \mathbb{Z}^n \) one finds that much of the discussion from the previous section still holds, but with the considerable difference that no derivatives of \( f \) exists. The definition of global optimum 2.3 is still valid and by defining a neighborhood structure \( B \) the definition of local minima in 2.2 holds without change. Given a point \( x_i \in S \), the neighborhood structure \( B(x_i) \) defines a set \( S_i \subset S \) of points that in some sense is close to \( x_i \). A reasonable restriction on the neighborhood is to assume that \( x_j \in S_i \iff x_i \in S_j \).

Finally, \( x^* \) is used to denote the set of globally minimal solutions, since there may be several points in \( S \) that have the same (minimal) cost.

The classification of combinatorial optimization problems, which is completely different from that of continuous problems, is based on the computational time needed to solve a problem as the size grows. A distinction is made between problems which has a solution time, with respect to the best known algorithm, that is a polynomial function of the problem size and those which require a super-polynomial, e.g. any function that grows faster than a polynomial, execution time in terms of their size. The term exponential is often used to describe the growth rate rather than super-polynomial but it is somewhat misleading since the concept includes growth rates such as \( a^n \) where \( a > 1 \),
2.4. SOLUTIONS OF OPTIMIZATION PROBLEMS

\( n^n \) and \( n! \). The two kinds are referred to as belonging to the class of polynomial (P) and
non-polynomial (NP) problems, respectively [8].

It is important to understand the implication of a non-polynomial problem in terms of
solution time. Such a problem dwarfs Moore’s Law [9], and the optimistic view that in a
couple of years we will have computers “fast enough” falls flat to the floor. Not even the
popular concept of computing clusters have much impact on an NP problem.

Except the major classifications of optimization problems explained above and list in
Figure 2.3, there are other categories too. For example, if some or all of the variables in
the vector are restricted to take on only integer (or discrete) values, the problem is called
an integer-programming problem. Real-valued programming problems are those where
the variables are permitted to take any real value. In addition to these classifications,
an objective function can be unimodal or multimodal. Unimodal objective functions are
those that contain a single optimum while multimodal objective functions contain multiple
optimums. A real design situation more often encompasses more than one of the above
features into the design objective(s). For example, an aircraft wing design could have two
objectives (multi-objective problem), one being simple linear and the other being highly
non-linear, multimodal and multidimensional. Such problems are more difficult to solve
than single unimodal objective problems.

2.4 Solutions of Optimization Problems

2.4.1 Analytical Methods

Methods to solve design optimization problems in various categories require different
approaches and techniques [10][11] [12]. Analytical methods use classical differential
calculus theory and calculus of variations where the extremes of a function \( f(x) \) are
obtained by finding the values of \( x \) that cause the derivatives of \( f(x) \) to vanish. These methods can be used to find unconstrained maximums and minimums of an objective function with several variables, with the assumption that the search space is continuous and functions are twice differentiable.

For a problem with both convex domain and a convex function there are gradient searching algorithms, utilizing the fact that any local optimum is also the global optimum, which traverses the domain in the direction of steepest descent, \(-\nabla f(x)\). The search is complicated by the fact stated before, that even though \(-\nabla f(x) = 0\) is a necessary condition it is not a sufficient condition and more elaborate schemes are necessary. Gradient search algorithms are common. The search is often based on Newton’s method that uses a search direction given by \( H^{-1}(x)\nabla f(x) \), where \( H \) is the Hessian matrix with elements which are the second partial derivatives of \( f(x) \) [13].

A large class of convex problems, linear programs (LP), stated in canonical form as

\[
\min_x c^T x \tag{2.7}
\]

subject to

\[
Ax = bx \geq 0 \tag{2.8}
\]

can be solved by e.g. the simplex method [14]. For other problems with quadratic cost functions, non-linear constraints etc., there are a number of solvers available, examples of which include Interior Point methods [15], Quadratic and Cubic interpolation methods [16], Augmented Lagrange Multiplier method, Method of Feasible Directions, Modified Method of Feasible Directions and the Generalized Reduced Gradient Method [17].
2.4. SOLUTIONS OF OPTIMIZATION PROBLEMS

2.4.2 Numerical Methods

If the problem is non-convex, the difficulty is severely increased and one can not expect to find a certain local minimum, in particular the global minimum.

For example, combinatorial optimization problems are inherently much harder than continuous problems with a worst case corresponding to a full enumeration (exhaustive search) of all possible solutions. A full search can quickly become intractable no matter how much computing force is used, if the problem grows super-polynomially with size. It is quite reasonable to give up the requirement of an optimal solution in exchange for an algorithm that presents a good approximative solution in polynomial time. The algorithms that are true optimization algorithms include exhaustive search and branch and bound whereas the approximation algorithms include meta-heuristic methods and heuristic methods, particularly evolutionary algorithms. Generally, all evolutionary algorithms work as follows: a population of individuals is initialized where each individual represents a potential solution to the problem at hand. The quality of each solution is evaluated using a fitness function. A selection process is applied during each iteration of an evolutionary algorithm in order to form a new population. The selection process is biased toward the fitter individuals to ensure that they will be part of the new population. Individuals are altered using unary transformation (mutation) and higher order transformation (crossover). This procedure is repeated until convergence is reached. The best solution found is expected to be a near-optimum solution. Examples of evolutionary algorithms are Genetic Algorithms (GA), Simulated Annealing (SA), Ant Colony Optimization (ACO), and Particle Swarm Optimization (PSO)[1][18]. These evolutionary methods have some natural advantages over traditional methods:

* PAPR They can handle mixed continuous-discrete variables, and discontinuous and nonconvex design spaces. Use of numerical methods can either be computationally
very expensive or return incorrect values (i.e. get trapped in local minimums).

* PAPR They can easily be adjusted to the problem at hand. Almost any aspect of the algorithm may be changed and customized

* Traditional methods of optimization are not robust to dynamic changes in problem the environment and often require a complete restart in order to provide a solution (e.g., dynamic programming). In contrast, evolutionary algorithms can be used to adapt solutions to changing circumstance.

* They can also be combined with more traditional optimization techniques. This may be as simple as the use of a gradient minimization used after primary search with an evolutionary algorithm, or it may involve simultaneous application of other algorithms.

* They have the ability to address problems for which there are no human experts. Although human expertise should be used when it is available, it often proves less than adequate for automating problem-solving routines.

One thing we need to emphasize here is that evolutionary algorithms rely on the communication scheme among the population members to reach the global optimum. It is like normal numerical methods execute with multiple initial points in the search space simultaneously. However, without communication frame and learning ability, parallel execution of numerical methods still can not compete with evolutionary algorithms. To certain extent, we can say evolutionary algorithms are more efficient for reaching the global optimum than numerical methods.
2.5 Summary

This chapter provided a short overview of optimization and certain optimization methods. We explain where PSO belongs to and what kind of optimization problems that PSO might be good for in this chapter. Starting from next chapter, we will shift our attention to the real focus of this dissertation, the PSO algorithm and its various applications in wireless communications and signal processing.
Chapter 3

Particle Swarm Optimization

Alternative approaches need to be find to achieve satisfying solutions to the problems that afore-mentioned approaches can not achieve. One such approach is Particle Swarm Optimization (PSO) [1]. PSO is a method for doing numerical optimization without explicit knowledge of the gradient of the problem to be optimized. It was originally intended for simulating social behavior, but the algorithm was simplified and it was realized that the particles were actually performing optimization.

PSO optimizes a problem by maintaining a population of candidate solutions called particles and moving these particles around in the search-space according to simple formulae. The movements of the particles are guided by the best found positions in the search-space, which are continually being updated as better positions are found by the particles. The main advantages of the algorithm are summarized as follows:

1. The bottom-up approach, impose no conditions on the underlying problem.

2. Simple formulation making it easy to implement, apply, extend and hybridize. There are few parameters to adjust. The operations are just adding and multiplication.

3. It uses a relatively small population, usually the particle number is less than 50.
3.1. SWARM INTELLIGENCE

4. Perfectly fit for decentralized processing, each particle is independent. Swarms can run in parallel.

These advantages have given it increasing popularity in the field of numeric optimization since it was created in 1995. It can be applied, in the areas of system design, multi-objective optimization, classification, pattern recognition, system modeling, scheduling, planning, signal processing, robotic applications, decision making, simulation and identification. Thus far it has been applied to such difficult problems as feature selection, neural network training, reactive power and voltage control, end milling and ingredient mix optimization.

3.1 Swarm Intelligence

Since PSO belong to a bigger family of swarm intelligence approaches. In this section, we briefly introduce swarm intelligence.

The concept of Swarm Intelligence (SI) was first used in the field of cellular robotic systems by Beni, Hackwood and Wan [19][20]. In this context, simple agents occupied one- or two-dimensional grid environments and self organized through closest neighbor interactions.

SI could be defined as any attempt to design algorithms or distributed problem-solving devices whose behavior emerges from the social interaction between local neighbors. The word swarm loosely describes a collection of interactive individuals. The classical example of a swarm is bees swarming around their hive; nevertheless the metaphor can easily be extended to any other system with a similar architecture. As ant colonies can be thought of as a swarm whose individuals are ants, so can a flock of birds. The concept of swarm can be extended to an even more general one: that of any type of
collective behavior. Thus, a swarm might occur in high-dimensional cognitive spaces, where collision is no longer a concern and could simply mean agreement.

In 1999, Bonabeau, Dorigo and Theraulaz noted that the term “swarm intelligence” and extends that definition [21]. SI includes the study of collective behaviors in nature, such as nest building, foraging, and item sorting in insect societies, and swarming, flocking, herding, and schooling behaviors in vertebrates. On the other hand, from an engineering point of view, it refers to the bottom-up design of distributed systems that display forms of useful and/or interesting behavior at the global level as a result of the actions of a number of units interacting with one another and with their environment at the local level.

In recent years, SI design has been applied to a wide variety of problems in combinatorial and continuous optimization, telecommunications, robotics, etc., often with excellent results. Two of the most popular and successful examples of the SI approach are Ant Colony Optimization (ACO) [22] and Particle Swarm Optimization (PSO) [1] [18]. ACO takes inspiration from the pheromone-mediated ability of ant colonies to find shortest paths between their nest and sources of food to define a metaheuristic for combinatorial optimization based on the use of ant-like agents and stigmergic communication of artificial pheromone information. PSO translates the flocking behavior of birds into a framework based on information-sharing particle-like agents to find extremal points in optimization problems.
3.2 Particle Swarm Optimization

3.2.1 Basic Particle Swarm Optimization

This section presents a brief introduction to basic PSO algorithms. The first PSO program was a graphical simulation of a bird flock [23][24]. In this simulation, a point on the screen was defined as food, called the “cornfield vector ” [1]; the idea was for birds to find food through social learning, by observing the behavior of nearby birds, who seemed near the food source. The optimization potential was realized in the initial experiments and the algorithm was modified to incorporate topological rather than Euclidean neighborhoods and multidimensional search was attempted successfully [25][26].

In PSO a number of simple entities - the particles - are placed in the search space of some problem or function, and each evaluates the objective function at its current location. Each particle then determines its movement through the search space by combining some aspect of the history of its own current and best (best-fitness) locations with those of one or more members of the swarm, with some random perturbations. The next iteration takes place after all particles have been moved. Eventually the swarm as a whole, like a flock of birds collectively foraging for food, is likely to move close to an optimum of the fitness function.

Each individual in the particle swarm is composed of three $D$ - dimensional vectors, where $D$ is the dimensionality of the search space. For the $i^{th}$ particle, these are the current position $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$, the previous best position $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$, and the velocity $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$.

The current position $X_i$ can be considered as a set of coordinates describing a point in space. On each iteration of the algorithm, the current position is evaluated as a problem solution. If that position is better than any that has been found so far, then the coordinates
are stored in the second vector, $P_i$. The value of the best function result so far is stored in a variable that can be called *pbest* (for “previous best”), for comparison on later iterations. The objective, of course, is to keep finding better positions and updating $P_i$ and *pbest*. New points are chosen by adding $V_i$ coordinates to $X_i$, and the algorithm operates by adjusting $V_i$, which can effectively be seen as a step size.

The particle swarm is more than just a collection of particles. A particle by itself has almost no power to solve any problem; progress occurs only when the particles interact.

Problem solving is a population-wide phenomenon, emerging from the individual behaviors of the particles through their interactions. In any case, populations are organized according to some sort of communication structure or topology, often thought of as a social network. The topology typically consists of bidirectional edges connecting pairs of particles, so that if $j$ is in $i$'s neighborhood, $i$ is also in $j$'s. Each particle communicates with some other particles and is affected by the best point found by any member of its topological neighborhood. This is just the vector $P_i$ for that best neighbor, which we will denote with $P_g$. The potential kinds of population “social networks” are hugely varied, but in practice certain types have been used more frequently.

In the particle swarm optimization process, the velocity of each particle is iteratively adjusted so that the particle stochastically oscillates around $P_i$ and $P_g$ locations.

The original process for implementing PSO is as follows:

1. Initialize a population of particles with random positions and velocities. Initialize $P_i$ and $pbest_i$ respectively to the starting position and fitness.

2. For each particle, evaluate the fitness function of the position $X_i$

3. Compare the particle’s fitness with $pbest_i$. If the current value is better, copy it to $pbest_i$ and set $P_i$ equal to the current position $X_i$.
4. Identify the most successful particle in the neighborhood and assign its index to the variable \( g \).

5. Change the velocity and position of the particle according to equation 3.1 and 3.2, respectively:

\[
V_i^{(t+1)} = \omega \times V_i^{(t)} + \psi_1 \times (P_i - X_i) + \psi_2 \times (P_g - X_i) \tag{3.1}
\]

\[
X_{id}^{(t+1)} = X_{id}^{(t)} + V_{id}^{(t+1)} \tag{3.2}
\]

6. Loop to step 2 until a criterion is met, usually a sufficiently good fitness or a maximum number of iterations.

The acceleration constants \( \psi_1 \) and \( \psi_2 \) in equation 3.1 represent the weighting of the stochastic acceleration terms that pull each particle around \( P_i \) and \( P_g \). Adjustment of these variables alters the relative effect of the particle’s previous best and the neighborhood one. Higher values result in abrupt movement toward or past target regions.

Figure 3.1 shows each component of the velocity term \( V_i \) in vector form, and the resulting position, \( X_i \) (updated), for the \( i^{th} \) particle. Note that the inertia coefficient \( \omega \) is used to scale the previous velocity term, normally to reduce the “momentum” of the particle.

The pseudo-code for implementing PSO is shown in Algorithm 1.

<table>
<thead>
<tr>
<th>Term</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>fitness</td>
<td>A number representing goodness of a given solution</td>
</tr>
<tr>
<td>swarm</td>
<td>The entire collection of agents or particles</td>
</tr>
<tr>
<td>position</td>
<td>Particle’s coordinates which represent solution to the problem</td>
</tr>
<tr>
<td>pbest</td>
<td>Best fitness returned for a specific particle</td>
</tr>
<tr>
<td>gbest</td>
<td>Best fitness returned for the entire system</td>
</tr>
</tbody>
</table>
3.2. PARTICLE SWARM OPTIMIZATION

Figure 3.1: Visualization of PSO, components as vectors

3.2.2 Key Factors of Particle Swarm Optimization

Swarm Size

The swarm size selected is problem dependent. Swarm sizes ranging from 20 to 50 are the most common. It was learned early on that PSO needed smaller populations than other evolutionary algorithms to reach high quality solutions.

Velocity Bound

To prevent explosion, in the original version, the particles’ velocities were clamped to a maximum velocity $V_{\text{max}}$. If the velocity exceeded $V_{\text{max}}$ in any coordinate, it was truncated to that value. $V_{\text{max}}$ was therefore an important parameter. If it was too high, particles
3.2. PARTICLE SWARM OPTIMIZATION

Algorithm 1 Algorithm PSO

for \( t = 1 \) to the maximum number of particle generations do
for \( i = 1 \) to the population size do
for \( d = 1 \) to the dimensionality of the fitness function do

// Apply the velocity update equation:
\[
V_{id}^{(t+1)} = \omega \times V_{id}^{(t)} + \psi_1 \times (P_{id} - X_{id}) + \psi_2 \times (P_{gd} - X_{id})
\]
// where \( P_i \) is the best position visited so far by \( X_i \)
// and \( P_g \) is the best position visited so far by any particle;
// Update Position:
\[
X_{id}^{(t+1)} = X_{id}^{(t)} + V_{id}^{(t+1)}
\]
end for
// Compute fitness of \( X_i^{(t+1)} \)
// If needed, update historical information regarding \( P_i \) and \( P_g \)
end for
// Terminate if \( P_g \) meets problem requirements
end for

could fly past good solutions. If \( V_{max} \) was too small, particles explored too slowly, and good solutions could not be found. Therefore, particles could become trapped in local optima. Early experience with the acceleration constants \( \psi_1 \) and \( \psi_2 \) concluded it was possible to set them to 2.0 for almost all applications. \( V_{max} \) was the only parameter that needed to be adjusted. It was however noted that the optimal setting of this parameter was dependent on the problem.

Bound Resetting

Most of the research conducted in PSO searched for the optimal value of the fitness function in a certain hypercube; it is therefore necessary to somehow enforce the exploration to remain inside that valid hyperspace. This is usually handled by resetting the particles within valid bounds whenever necessary.

However, in some situations, bound resetting does more harm than good. The best way to solve this predicament may be not to use bound checking at all. By modifying the fitness function as to assign the \( +\infty \) value (assuming the optimization problem is
3.2. PARTICLE SWARM OPTIMIZATION

a minimization one) to non-valid solutions may be a better approach. In this way, the particle will soon enter valid space.

However, common sense indicates and experience validates the hypothesis that valuable time is lost while the particle is wandering through non-valid space. One possible approach suggested by Vesterstrøm and Riget is to evaluate particles outside bounds as if they were at the nearest boundary point [27]. This idea can reduce convergence time in certain situations.

**Inertia Weight Parameter**

Since its introduction, PSO has seen many improvements and applications. Most modifications to the basic PSO are directed towards improving convergence of the PSO and increasing the diversity of the swarm [28].

The concept of an inertia weight was developed to better control exploration and exploitation. The aim of inertia weight was to be able to control the exploration and exploitation mechanism and eliminate the need for Vmax. The inclusion of an inertia weight in the PSO algorithm was first published in 1998 [29]. Originally, it was linearly decreased between 0.9 and 0.4 during a run, providing a balance between exploration (larger steps in the beginning) and exploitation (smaller advancements, resulting in fine tuning). Its use resulted in fewer iterations on average to attain a suitably good solution.

Experience showed that when using inertia weight the maximum velocity factor $V_{max}$ could simply be set to the value of the dynamic range of each variable (on each dimension). This limitation is sometimes necessary to keep the particle from oscillating too fast around a promising region without adequately exploring it. In this way, there is no longer the need of a strategy for setting $V_{max}$. 

32
3.2.3 Binary Particle Swarm Optimization

The basic PSO algorithms and many derived PSO algorithms are designed in continuous or real-number domains. However, there are applications where the variables are in discrete space, leading to the use of a discrete binary PSO algorithm. Kennedy and Eberhart described a very simple alteration of the canonical algorithm that operates on bit-strings rather than real numbers [30]. In their version, the velocity \( v_{id}, v_{id} \in [-V_{max}, V_{max}] \) is used as a probability threshold to determine whether \( x_{id} \) the \( d^{th} \) component of \( X_i \) should be evaluated as a zero or a one. They squashed \( v_{id} \) in a logistic function

\[
sig(v_{id}) = \frac{1}{1 + \exp(-v_{id})}
\]  

(3.3)

then generated a random number \( \rho_{id} \) from a uniform distribution between \( U[0, 1] \) for each bit-string site and compared it to \( \sig(v_{id}) \). If \( \rho_{id} \) was less that the threshold, then \( x_{id} \) was interpreted as 1, otherwise as 0.

For binary PSO, each particle position vector \( X_i \) has a dimension equal to the total number of features. Each dimension \( j \) has a binary state, where a value of 1 indicates the selection of the \( j^{th} \) feature and a value of 0 means the exclusion of the \( j^{th} \) feature. At any iteration step, the number of features that are selected by the binary PSO corresponds to the number of 1 s in the particle position vector. The number of selected features can be any value between one and the total number of features in the pool.

Kennedy and Spears compared this binary particle swarm to several kinds of GAs, using Spears’ multimodal random problem generator [31]. This paradigm allows the creation of random binary problems with some specified characteristics, e.g., number of local optima, dimension, etc. In that study, the binary particle swarm was the only algorithm that found the global optimum on every single trial, regardless of problem features. It also progressed faster than GAs with crossover, mutation, or both, on all problems ex-
cept the very simplest ones, with low dimension and a small number of local optima; the mutation-only GA was slightly faster in those cases.

### 3.3 Summary

In this chapter, the basic PSO and binary PSO are presented. These canonical methods are the base of the proposed derived PSO algorithms applied to various applications in wireless communication in later chapters. Particle swarm optimization can be and has been used across a wide range of applications. Areas where PSO has shown particular promise include multimodal problems and problems for which there is no specialized method available or all specialized methods give unsatisfactory results.

Despite its apparent simplicity, the PSO presents formidable challenges to those interested in understanding swarm intelligence through theoretical analysis. So, to date a fully comprehensive mathematical model of particle swarm optimization is still not available. In the future, we want to develop a probabilistic framework to analyze the PSO based algorithms leading to efficient design of the algorithms.
Ultra wide-band technology has been proposed as a promising solution for next generation short range high speed wireless communication systems. Instead of using carrier frequencies, UWB systems transmit information using trains of short time duration pulses that spread the energy from the near direct current to a few gigahertz. Due to its ultra wide bandwidth, UWB devices can cause interferences with other existing narrow band communication systems. In order to solve this problem, the Federal Communications Commission (FCC) released the regulations in February 2002 that set the power emission limits for all kinds of UWB devices. Since UWB is a carrier-free technique, how to design a UWB pulse shape with its power spectral density (PSD) fulfilling the mandated spectral masks become a key issue in UWB research. The choice of pulse shape can also directly affect other aspects of UWB systems, such as the performance in multi-user interference environments.

Various pulse shapes have been studied in the literature. Gaussian $2^{nd}$ derivative pulse is widely adopted in the investigation of UWB applications [32], but they are not flexible enough to conform to the FCC spectral masks and must be modified and filtered. Pulse set based on the modified Hermite polynomials (HP) are orthogonal to one an-
other [33], but frequency shifting is necessary for the original and 1st order HP pulses to meet the FCC spectral masks. Higher order HP pulses are susceptible to timing jitter and noise, and they also need bandpass filters to fit their PSD into the FCC masks [34]. Pulse designed utilizing the ideas of prolate spheroidal (PS) functions can satisfy the FCC masks [35], but they do not effectively exploit the allowable bandwidth and power.

The orthogonality of both HP pulses and PS pulses is only preserved when each user is assigned a unique pulse in a perfectly synchronous, multiuser, UWB system without considering the channel distorting effect and antenna response characterization, rendering it impractical. Pulses based on a linear combination of a set of base waveforms obtained by differentiation of the Gaussian pulse was introduced in [36][37]. However, the strategies for selecting the combination and coefficients is random, and the algorithm designing the solutions is based on the huge number of trial and error iterations. This makes the results neither determined nor reproducible. In addition, pulses generated 1st to 15th Gaussian derivative pulses are combined. Hardware complexity increases as the order of the derivative increases.

To accomplish the pulse design, we propose a novel design method using Particle Swarm Optimization (PSO) algorithm. A binary PSO is used to select the pulses that form the composite pulse. A continuous PSO is used to obtain the parameters (time duration and amplitude) for each of the selected pulses. We compare the results achieved by the PSO to an exhaustive approach employed before. The new composite pulse generated conforms to the FCC indoor mask and maximizes the power. Multiple orthogonal pulses are derived from this PSO generated composite signal. This enables us to manage our network according to different system requirements by switching between signals (pulses) that sensors use.
4.1 UWB system models and Gaussian derivative pulses

4.1.1 UWB system models and signal’s PSD

The way of forming UWB signal consists of radiating a train of pulses that are very short in time (typically a few nanoseconds). The pulses are modulated by the information data symbols using different modulation schemes which can shape the spectrum of the generated signal. Further more, in the case of multi-user communications, different pseudo-noise (PN) codes are assigned to different users. We will use TH-BPSK and TH-PPM UWB system to evaluate the performance of different pulse shapes. A brief introduction about the system models is given here.

We consider UWB systems consisting of $N_u$ active users, transmitting UWB signals simultaneously through an additive white Gaussian noise (AWGN) channel, where each user employs time-hopping spread spectrum techniques. For a binary TH-BPSK UWB system, the signal from the $i^{th}$ bit of $k^{th}$ user can be represented as:

$$s_{tr}^{(k)}(t, i) = \sum_{j=iN_s}^{(i+1)N_s-1} d_j^{(k)} p(t - jT_f - c_j^{(k)}T_c)$$

(4.1)

and a typical TH-PPM UWB signal can be written as:

$$s_{tr}^{(k)}(t, i) = \sum_{j=iN_s}^{(i+1)N_s-1} p(t - jT_f - c_j^{(k)}T_c - \epsilon d_i^{(k)})$$

(4.2)

where $p(t)$ represents the transmitted pulse that we get at the output of transmitter antenna, $N_s$ is the number of pulses to represent one data bit, $T_f$ and $T_c$ are the frame and the chip duration, respectively. The bit duration $T_b = N_sT_f$. $\{c_j^{(k)}\}$ is the distinct TH PN codes sequence of the $k^{th}$ signal, and $c_j^{(k)} \in [1, N_h]$. $\{d_i^{(k)}\}$ is the binary data sequence. In antipodal TH-BPSK UWB systems, $d_i^{(k)} \in \{1, -1\}$. In TH-PPM UWB systems, $d_i^{(k)} \in \{1, -1\}$. 

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4.1. UWB SYSTEM MODELS AND GAUSSIAN DERIVATIVE PULSES

\{0, 1\}. \(\epsilon\) is the time offset of binary PPM. Given \(T_p\) the pulse width, the limits \(T_p + \epsilon < T_c\) and \(N_h T_e \leq T_f\) are assumed. Note that a frame is divided into many chips, and the pulse generated by the \(k^{th}\) user occupies only one of those.

In order to get the PSD of a TH-PPM signal, we need to relax the hypothesis of an inconsequential effect of the time shift \(\epsilon\), because the PSD of a PPM signal is hard to evaluate due to the non-linear nature of PPM modulation. Let us define a new signal \(v(t)\) as:

\[
v(t) = \sum_{j=1}^{N_s} p(t - jT_f - \theta_j).
\]

The power spectrum of \(v(t)\) is:

\[
P_v(f) = P(f) \sum_{m=1}^{N_s} e^{-j(2\pi f(mT_s + \theta_m))},
\]

where \(P(f)\) is the Fourier transform of \(p(t)\). Then the original UWB TH-PPM signal turns out to be:

\[
s_{tr}^{(k)}(t, i) = \sum_{j=1}^{(i+1)N_s-1} v(t - jT_b - \epsilon d_i^{(k)}).
\]

We assume that \(d_i^{(k)}\) is equiprobable to be 0 and 1, the transmitted signal PSD [38] is:

\[
P_{(PPM)}(f) = \frac{|P_v(f)|^2}{T_b} \left[ 1 - |W(f)|^2 + \frac{|W(f)|^2}{T_b} \sum_{n=1}^{N_s} \delta(f - \frac{n}{T_b}) \right],
\]

where

\[
|W(f)|^2 = \frac{1}{2} \left(1 + \cos(2\pi f \epsilon)\right).
\]
\[ P_{(BPSK)}(f) = \frac{\sigma_d^2}{T_b} |P(f)|^2 + \frac{\mu_d^2}{T_b^2} \sum_{n=1}^{N_b} |P\left(\frac{n}{T_b}\right)|^2 \delta(f - \frac{n}{T_b}), \] (4.8)

where \( \sigma_d^2 \) and \( \mu_d^2 \) are the variance and the mean of \( d_i^{(k)} \) sequences, respectively.

It is clear that the PSD of TH-PPM and TH-BPSK signals both have continuous and discrete components. However, the discrete spectral spikes will vanish if the information sequence has a zero mean. We assume this is true. Because the time shift factor \( \epsilon \) in TH-PPM signal not distinctively affects the interference from UWB signal to co-existing systems. Then, the PSD of both TH-PPM and TH-BPSK signals are to some extent linearly proportional to the PSD of the individual pulse.

### 4.1.2 Gaussian derivative pulses and their PSD

Gaussian pulse has been widely adopted by UWB radar and communication systems [40]. However, Gaussian pulse has direct current offset, so it can not radiate effectively via antennas. To avoid this problem, the second derivative of the Gaussian pulse, proposed by Win and Scholtz in [32], has been widely adopted in the investigation of UWB applications. The Gaussian pulse in our design has the form:

\[ p(t) = \frac{A}{\sqrt{2\pi} \sigma} e^{-\frac{t^2}{2\sigma^2}} = \frac{\sqrt{2} A}{\alpha} e^{-\frac{2\pi t^2}{\alpha^2}}, \] (4.9)

where \( A \) is the amplitude of the pulse and \( \alpha = \sqrt{4\pi\sigma^2} \) represents a bandwidth scaling factor. In general, it is accepted that the antenna has the general effect of differentiating the time waveform presented to it [41]. Since the pulse will pass through the transmitter and receiver antennas, it is differentiated twice when it comes out of the receiver antenna. The received signal is given by:
4.1. UWB SYSTEM MODELS AND GAUSSIAN DERIVATIVE PULSES

\[ p''(t) = A \left( \frac{t^2}{\sqrt{2\pi\sigma^3}} - \frac{1}{\sqrt{2\pi\sigma^3}} \right) e^{-\frac{t^2}{2\sigma^2}}. \]  

(4.10)

Similar to the method in [36], we will use Gaussian 1st to 16th derivative pulses as the base waveforms. The PSD of a waveform can be expressed as:

\[ PSD(f) = |P(f)|^2 = \left| \int_{-\frac{1}{2}T_p}^{+\frac{1}{2}T_p} p(t)e^{-j2\pi ft} df \right|^2. \]  

(4.11)

\[ power = \int |P(f)|^2 df \]  

(4.12)

Figure 4.1 shows the amplitude normalized Gaussian 1st to 16th order derivative waveforms.

In Figure 4.2 and 4.3, we have generated and depicted the Gaussian second derivative and its PSD respectively.

Figure 4.1: Amplitude normalized Gaussian 1st to 16th derivative pulses
4.1. UWB SYSTEM MODELS AND GAUSSIAN DERIVATIVE PULSES

Figure 4.2: Amplitude normalized Gaussian second-derivative pulse

Figure 4.3: Power spectral density of amplitude normalized Gaussian second-derivative pulse
Since $\sigma$ is a bandwidth scaling factor, the pulse PSD is closely related to it. The bandwidth of the pulse is approximately $1/(pulse\ duration)$, which is the same as the bandwidth of pulse PSD. Figure 4.4 and 4.5 demonstrate how $\sigma$ affects the pulse duration in the time domain and their PSD in the frequency domain for a Gaussian second derivative pulse. From the two figures, we can see that the bigger $\sigma$ value, the wider the pulse and the narrower the PSD. If the pulse is too wide in the time domain, the PSD of the pulse will be very narrow in the frequency domain, which makes the pulse not a good choice for fitting in the FCC mask. On the other hand, if the pulse is very narrow in the time domain, a very high precision pulse generator and high speed D/A and A/D converters are required. This trade off must be properly considered in the pulse design.

Our design method is to combine several Gaussian derivatives to form a single pulse, whose PSD will effectively exploit the allowable bandwidth and power. Here, we show the PSD of different Gaussian derivatives. Figure ?? is similar to Fig. 7 in [36] with a
In [42], it has shown that as the order of the derivative pulse increases, the energy is moving to higher frequencies. By choosing the order of the derivative and a suitable pulse width, a pulse that satisfies the FCC mask can be found. In our design, we try to identify a suitable waveform by combining several Gaussian derivative pulses not in a random fashion as in [36], but in a systematic way to form one pulse that can make best utilization of the FCC indoor spectral mask.

Table 4.1 shows the mathematical expressions of Gaussian \(1^{st}-6^{th}\) derivatives. Figure 4.7 shows the time and frequency domain representation of Gaussian \(1^{st}-6^{th}\) order derivative pulses. The pulses are optimized individually by exhaustively searching for optimal \((A, \alpha)\). The corresponding power values for these pulses are given in Table 4.2. The best performing pulse is the 5th derivative pulse with a total power value of 243.77
Table 4.1: Generation of Gaussian $1^{st}$ - $6^{th}$ Derivative

<table>
<thead>
<tr>
<th>Gaussian Pulses</th>
<th>Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{st}$ Derivative</td>
<td>$4\pi te^{-\frac{2\pi t^2}{\alpha^2}}/\alpha^2$</td>
</tr>
<tr>
<td>$2^{nd}$ Derivative</td>
<td>$(-4\pi(-\alpha^2 + 4\pi t^2)e^{-\frac{2\pi t^2}{\alpha^2}})/\alpha^4$</td>
</tr>
<tr>
<td>$3^{rd}$ Derivative</td>
<td>$(16\pi^2t(-3\alpha^2 + 4\pi t^2)e^{-\frac{2\pi t^2}{\alpha^2}})/\alpha^6$</td>
</tr>
<tr>
<td>$4^{th}$ Derivative</td>
<td>$(-16\pi^2(3\alpha^4 - 24\pi t^2\alpha^2 + 16\pi^2t^4)e^{-\frac{2\pi t^2}{\alpha^2}})/\alpha^8$</td>
</tr>
<tr>
<td>$5^{th}$ Derivative</td>
<td>$(64\pi^3t(15\alpha^4 - 40\pi t^2\alpha^2 + 16\pi^2t^4)e^{-\frac{2\pi t^2}{\alpha^2}})/\alpha^{10}$</td>
</tr>
<tr>
<td>$6^{th}$ Derivative</td>
<td>$(64\pi^3(-15\alpha^6 + 180\pi t^2\alpha^4 - 240\pi^2t^4\alpha^2 + 64\pi^3t^6)e^{-\frac{2\pi t^2}{\alpha^2}})/\alpha^{12}$</td>
</tr>
</tbody>
</table>

Figure 4.6: PSD of the Different Derivatives of the Gaussian Pulse

μW. Optimal linear combination of pulses, however, require search through these parameters for all pulses jointly. The problem gains complexity even for the case of 2 pulses due to continuous nature of A and α. Decision making involves selecting the order of the derivative for the combination as well. A PSO algorithm is used in this paper to solve this problem. The algorithm arrives at the optimal order of derivatives of the pulses and the parameters for each of the derivative.
4.2 Pulse Shape Design Using Particle Swarm Optimization

The UWB pulse shape design involves selection of pulses from $1^{st}$ to $6^{th}$ derivative of the Gaussian pulse. Let us define a 6 bit binary vector given by

$$
\{ b_1, b_2, \ldots, b_6 \},
$$

where $b_i = 1$ implies that $i^{th}$ derivative is used. There are two parameters that need to be designed for each selected pulse. To simultaneously achieve the pulse selection and

<table>
<thead>
<tr>
<th>Pulse</th>
<th>$\alpha$ (ns)</th>
<th>$A$ (v)</th>
<th>Power $\mu W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{st}$ Derivative</td>
<td>$1.995 \cdot 10^{-9}$</td>
<td>0.832</td>
<td>24.87</td>
</tr>
<tr>
<td>$2^{nd}$ Derivative</td>
<td>$1 \cdot 10^{-10}$</td>
<td>4.216</td>
<td>18.718</td>
</tr>
<tr>
<td>$3^{rd}$ Derivative</td>
<td>$1.375 \cdot 10^{-10}$</td>
<td>6.16</td>
<td>63.938</td>
</tr>
<tr>
<td>$4^{th}$ Derivative</td>
<td>$1.625 \cdot 10^{-10}$</td>
<td>11.52</td>
<td>209.79</td>
</tr>
<tr>
<td>$5^{th}$ Derivative</td>
<td>$2.125 \cdot 10^{-10}$</td>
<td>9.88</td>
<td>243.77</td>
</tr>
<tr>
<td>$6^{th}$ Derivative</td>
<td>$2.5 \cdot 10^{-10}$</td>
<td>8</td>
<td>156.37</td>
</tr>
</tbody>
</table>

Figure 4.7: Individually Optimized $1^{st}$ to $6^{th}$ Derivatives of Gaussian Pulse
4.2. PULSE SHAPE DESIGN USING PARTICLE SWARM OPTIMIZATION

the parameter values for the selected pulses, we define a vector of continuous values of
length 12. The vector is decoded as follows. The first 6 give the $\alpha$ values, the next 6 give
the $A$ values for the six pulses. The continuous vector is denoted by

\[ c = \{\alpha_1, \alpha_2, \cdots, \alpha_6, a_1, a_2, \cdots, a_6\}, \quad (4.14) \]

where $[\alpha_i, a_i]$ give the time duration and the amplitude for the $i^{th}$ derivative. The final
pulse from this is derived using

\[ p(t) = \sum_{i=1}^{6} b_i \times p_i(t, c_i, c_i+6) \quad (4.15) \]

The objective of the algorithm is to maximize 4.12 without violating the FCC mask.
The PSO formulae define each particle as a potential solution to a problem in a $D$-
dimensional space. Hence the $i^{th}$ particle represented as

\[ X_q = \{x_{q1}, x_{q2}, \cdots, x_{qD}\}, \quad (4.16) \]

where $D$ is the dimension number. There are total of $D = 18$ dimensions in this problem based on 4.13 and 4.14 where $x$s in the particle replace the $b$s and $c$s respectively. Each particle also maintains a memory ($p_{best}$) of its previous best position, $P_{i}d = (p_{i1}, p_{i2}, \cdots, p_{iD})$ and a velocity along each dimension represented as $V_i = (v_{i1}, v_{i2}, \cdots, v_{iD})$ The global best ($g_{best}$) particle is denoted by $P_{g}d = (p_{g1}, p_{g2}, \cdots, p_{gD})$ [43]. Given the particle the signal is constructed using 4.15. The PSD of the signal is calculated using 4.11. The performance of this particle is evaluated using the following conditional expression:

\[
F = \begin{cases} 
  \text{power} & \text{if } P(f) < \text{Mask}(f) \\
  0 & \text{otherwise}
\end{cases} 
\quad (4.17)
\]
where power is given by 4.12. 5.10 is one of the penalty functions available to handle the constraints on an optimization problem [44]. The step nature of the above objective function induces discontinuity limiting the ability to design a gradient based approach for optimization. In each generation, the $p_{best}$ vector of the particle with the best fitness in the local neighborhood, designated as $g_{best}$, and the $p_{best}$ vector of the current particle are combined to adjust the velocity along each dimension given by

$$V_{id}^{(t+1)} = \omega \times V_{id}^{(t)} + \psi_1 \times (P_{id} - X_{id}) + \psi_2 \times (P_{gd} - X_{id}). \quad (4.18)$$

The portion of the adjustment to the velocity influenced by the individual’s own $p_{best}$ position is considered as the cognition component, and the portion influenced by $g_{best}$ is the social component. Constants $\psi_1$ and $\psi_2$ determine the relative influence of the social and the cognition components, and are often both set to the same value to give each component (the cognition and the social learning rates) equal weight.

The velocity is then used to compute a new position for the particle. The position update for the continuous part of the particle is given by

$$X_{id}^{(t+1)} = X_{id}^{(t)} + V_{id}^{(t+1)}. \quad (4.19)$$

For position update of the binary component of the particle, first the velocity is transformed into a $[0, 1]$ interval using the sigmoid function given by

$$\text{sig}_{id}(V_{id}) = \frac{1}{1 + e^{-V_{id}}}. \quad (4.20)$$

where $V_{id}$ is the velocity of the $i^{th}$ particle’s $d^{th}$ dimension. A random number is generated using a uniform distribution, which is compared to the value generated from the sigmoid
function, and a decision is made about the $X_{id}$ from

$$X_{id} = u(sig_{id} - U[0,1]) \quad (4.21)$$

where $u$ is a unit function. The decision regarding $X_{id}$ is probabilistic.

## 4.3 Results and discussions

We ran the PSO algorithm with 30 particles and for 1000 iterations. Multiple runs of algorithm converged to two competent designs. In the first two rows of Table 4.3, the two composite pulses that occupy the full band, achieved using PSO are presented. The two pulses are also shown in Figure 4.8 and Figure 4.9, respectively. The power values achieved by the two pulses are 362.89 $\mu W$ and 369.44 $\mu W$. The percentage of the achieved power to total power available within the spectrum (550 $\mu W$) is 65.9% and 67.1% respectively. Figure 4.12 shows the convergence of the PSO to a solution. The PSO algorithm converges to a solution within 700-800 iterations. The algorithm uses simple add and multiply operators to search through the space and arrive at solutions as presented in 4.18 and 4.19.

### 4.3.1 Bit Error Rate Performance of PSO designed signal

According to Shannons theory, the channel capacity of an Additive White Gaussian Noise (AWGN) channel is:

$$C = B \log_2(1 + SNR) \quad (4.22)$$

The beauty of the new designed pulses is that, in theory, given a specific bandwidth $B$ for the UWB signal, our designed pulse can provide more power than other pulses,
Table 4.3: Power/Parameter Values for Pulses Achieved Using Particle Swarm Optimization

<table>
<thead>
<tr>
<th>Signal</th>
<th>Derivatives Used</th>
<th>( A ) (v)</th>
<th>( \alpha ) (ns)</th>
<th>Power ( \mu \text{W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, 5, 6</td>
<td>[0.80904, 11.3402, 4.350]</td>
<td>[1.969, 0.1809, 0.3070]</td>
<td>362.89</td>
</tr>
<tr>
<td>B</td>
<td>1, 3, 4, 6</td>
<td>[0.80904, 5.3128, 14.2475, 6.6191]</td>
<td>[1.969, 0.1, 0.1635, 0.1]</td>
<td>369.44</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>[0.80904]</td>
<td>[1.969]</td>
<td>25.34</td>
</tr>
<tr>
<td>D</td>
<td>3, 4, 6</td>
<td>[5.3128, 14.2475, 6.6191]</td>
<td>[0.1, 0.1635, 0.1]</td>
<td>344.10</td>
</tr>
</tbody>
</table>

which means we will have a better SNR value at the receiver, thus we can provide either higher data rate “C” or at the same “C” but with less BER at a fixed noise level. This is demonstrated in the bit error rate plot in Figure 4.13 as we compare the average BER performance for the PSO generated pulse and the optimized 5th derivative pulse presented in Table 4.3. The simulations are carried out using a TH-PPM modulation scheme presented in previous section. The parameters used for simulation are given in Table 4.4. Since the two pulses designed used different power we normalize the signal to noise ratio (SNR) for fair comparisons. The average BER is plotted against normalized SNR in Figure 4.13. As we can see the composite pulse designed using PSO achieves lower average BER. The enhanced performance in terms of BER lets us do the trade-offs as per the Shannons theory.

The most important benefit of the proposed system lies in the ability to generate orthogonal pulses within the FCC mask. Using a bottom-up biologically inspired approach as in particle swarm optimization enables design of signals with arbitrary constraints. The speedy convergence and simplicity of the operators used by the algorithm enables
4.3. RESULTS AND DISCUSSIONS

Figure 4.8: Pulse ‘A’ and its PSD Achieved using Particle Swarm Optimization Algorithm

Table 4.4: UWB System Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chip Width (ns)</td>
<td>$T_c$</td>
<td>2</td>
</tr>
<tr>
<td>Frame Width (ns)</td>
<td>$T_f$</td>
<td>32</td>
</tr>
<tr>
<td>Number of Chips Per Frame</td>
<td>$N_c$</td>
<td>16</td>
</tr>
<tr>
<td>Repetition Code Length</td>
<td>$N_s$</td>
<td>4</td>
</tr>
</tbody>
</table>

its use in real time. An interesting feature of the design method is that since the pulses are linearly combined by different Gaussian derivative pulses, by using only a part of the pulses in the combination we can also design pulses that can be considered orthogonal due to their frequency separation. Figure 4.8 and Figure 4.9 show two alternative designs that can be derived from the composite pulse design achieved by the PSO. Table 4.5 gives the power values of this pulse in the two frequency bands. The signal ‘C’ uses only the first derivative pulse from the composite pulse designed by the PSO. The pulse optimally occupies the 0-960 MHz band. The total power in this signal as given in Table 4.3 is 25.34 out of which 25.3324 is contained in the 0-960 MHz band. The signal ‘D’ only uses the 3, 4, 6 out of the second composite pulse presented in Table 4.3. The pulse has most of the power concentrated in the second band i.e., 960 MHz - 10.6 GHz and has

50
4.4 Summary

In this chapter we presented a PSO algorithm to linearly combine the Gaussian derivatives creating multiple orthogonal UWB signals. These signals comply with the jagged FCC mask. The new composite pulse exploits the bandwidth and energy within a given spectral mask. The pulses generated by the PSO algorithm achieve 65% spectrum efficiency. In future work we intend to incorporate the number of orthogonal pulses as an objective function for evaluating the composite pulse in addition to the objective function

Table 4.5: Orthogonality of Signals Generated by PSO in Frequency Domain

<table>
<thead>
<tr>
<th>Signal</th>
<th>Power in 0-960 MHz (( \mu W ))</th>
<th>Power in 960 MHz - 10.6 GHz (( \mu W ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25.3324</td>
<td>0.0076</td>
</tr>
<tr>
<td>B</td>
<td>0.000065073</td>
<td>344.10</td>
</tr>
<tr>
<td>C</td>
<td>25.3324</td>
<td>337.5576</td>
</tr>
<tr>
<td>D</td>
<td>25.3324</td>
<td>344.1076</td>
</tr>
</tbody>
</table>

only $6.5 \times 10^{-5} \mu W$ which is hardly detectable and would not cause any interference with the first signal in case both are used simultaneously.
4.4. SUMMARY

Figure 4.10: Pulse ‘C’ and its PSD Achieved using Particle Swarm Optimization Algorithm

5.10. The algorithm will also be utilized to generate signals for submasks created with the FCC mask.
4.4. SUMMARY

Figure 4.11: Pulse ‘D’ and its PSD Achieved using Particle Swarm Optimization Algorithm

Figure 4.12: Convergence of the PSO
Figure 4.13: Bit error probability comparisons for optimized 5 derivative pulse and the PSO generated pulse ‘B’
Chapter 5

PSO Based Localization System

In this chapter, an adaptive learning algorithm is developed using particle swarm to identify and mitigate the non-line of sight (NLOS) signals in ranging measurements. Training data is generated using the IEEE 802.15.4a UWB channel model for different conditions. Multiple metrics derived from this data are fused to identify the NLOS signals. Specifically, kurtosis, mean excess delay and root mean square delay are used as metrics for fusion. The fusion strategy is derived using PSO, considering the correlation between multiple classifiers. We compare the fusion methodology achieved by PSO for the correlated data set to the likelihood ratio based fusion methodology assuming independence. This work also presents an NLOS mitigation approach derived using PSO. A scalar called “error mitigation ratio (EMR)” is defined. The EMR transforms a NLOS measurement into an equivalent LOS measurement. The PSO identifies the EMR using the training data. Application of PSO generated EMR enhances the positioning accuracy and is demonstrated in this chapter for indoor wireless channel. This mitigation approach enables us to arrive at a position for the unknown node even when one of the measurements is identified as NLOS. Finally, PSO is used for multilateration to combine measurements from three nodes. Comparisons are done with the linearized least square method.
5.1. UWB LOS/NLOS IDENTIFICATION CLASSIFIERS

The market demand for location-aware sensor network is continuously growing. A wide range of applications such as object tracking, environmental monitoring, warehouse inventory, vehicle network, building security and health care systems all require accurate locationing systems. Many of these applications are at indoor or underground environments where GPS systems can not work. The design of such location-aware networks typically requires the capability of peer-to-peer range or distance measurements. Ultra-wideband (UWB) technology is a promising solution for precision ranging due to its fine time resolution to resolve multipath fading. The IEEE 802.15.4a Task Group has developed a UWB based physical layer standard for short-range networks with a precision ranging capability [32],[3],[45].

5.1 UWB LOS/NLOS Identification Classifiers

5.1.1 UWB Channel Model

The impulse response of IEEE 802.15.4a channel model can be represented as the sum of the contributions of the different multipath components (MPC):

$$h(t, \tau) = \sum_{i=1}^{N} a_i(t) \delta(\tau - \tau_i) = a_{DP}(t) \delta(\tau - \tau_{DP}) + \sum_{i=1}^{N} a_i(t) \delta(\tau - \tau_i)$$ (5.1)

where $a_{DP} = a_1(t)$, $\tau_{DP} = \tau_1$ represent the amplitude and delay of the direct path MPC, $N$ is the total number of the MPCs. $h(t, \tau)$ can usually be simplified as $h(t)$.

In this work, we first collect 100,000 samples of CIR $h(t)$ as training data by running IEEE 802.15.4a channel model. Then we extract different metrics from the received MPCs forming the training data. These metrics enable us to classify the signals as NLOS and LOS signals. We choose kurtosis, mean excess delay and RMS delay as
our LOS/NLOS identification classifiers.

### 5.1.2 Multiple Identification Classifiers

The first classifier is the kurtosis of CIR. The kurtosis is defined as the ratio of the fourth order moment of the data to the square of the second order moment (i.e., the variance) of the data [46]. The kurtosis of a CIR $h(t)$ can be written as:

$$
\kappa = \frac{E[(|h(t) - \mu_h(t)|^4)]}{E[(|h(t) - \mu_h(t)|^2)^2]}
$$

(5.2)

Since the kurtosis is the degree of peakness of a distribution and CIR of LOS channel tends to be less dispersive than NLOS channel, we can use the kurtosis as one classifier. NLOS signals tend to have much higher values of delay-spread statistics [47]. The second and the third classifiers are the mean excess delay and the RMS delay spread. The mean excess delay of $h(t)$ is defined as:

$$
\tau_m = \frac{\int_{-\infty}^{\infty} t|h(t)|^2 dt}{\int_{-\infty}^{\infty} |h(t)|^2 dt},
$$

(5.3)

and the RMS delay spread is defined as:

$$
\tau_{rms} = \frac{\int_{-\infty}^{\infty} (t - \tau_m)^2|h(t)|^2 dt}{\int_{-\infty}^{\infty} |h(t)|^2 dt}
$$

(5.4)

In [46] the authors showed that probability density functions of all the three statistics for the sample realizations of the different IEEE 802.15.4a channels (Residential, Indoor, Outdoor, Industrial). These are modeled by log-normal PDFs. Figure 5.1 and Figure 5.2 show all the 3 classifiers for different IEEE 802.15.4a CIRs, we compare the LOS and NLOS channel statistics in the same scenario in these figures, respectively.
5.1. UWB LOS/NLOS IDENTIFICATION CLASSIFIERS

5.1.3 LOS/NLOS Identification Using PSO Based Fusion of Classifiers

The LOS/NLOS identification problem can be considered as a binary hypothesis-testing problem, by acquiring the kurtosis, mean excess delay and RMS delay as statistics of the CIRs. The two hypothesis are:
There are two errors in this hypothesis testing problem, known as probability of false LOS ($P_{FL}$) and probability of false NLOS ($P_{FN}$). The probability of false LOS is the detection of the LOS when actually NLOS is present. Similarly, probability of false NLOS is the identification of an LOS signal as an NLOS. In order to formulate the problem, it is assumed that the prior probabilities of encountering a LOS channel or a NLOS channel are the same. Also, we define the cost of false LOS detection $C_{FL}$ and the cost of false NLOS detection $C_{FN}$. These are incorporated into a performance function for evaluating the fusion methodology. The Bayesian cost (error), which we intends to minimize, is

$$ R = C_{FL} \times P(H_0) \times P_{FL} + C_{FN} \times P(H_1) \times P_{FN}, $$

(5.7)
5.1. UWB LOS/NLOS IDENTIFICATION CLASSIFIERS

where $C_{FL} + C_{FN} = c$ and $c$ is a constant. Here we assume $c = 2$.

5.1.4 Likelihood Ratio Test (LRT) Assuming Independence

A likelihood ratio test can be applied to each metric determining the presence of LOS/NLOS. This method works the best if the likelihood probability density models are available for each of the models. In absence of such models, the models can be estimated using the training data. Authors of [46] estimate the conditional density models for each of the metric to the lognormal distribution. We employ the same procedure and estimate the parameters for the lognormal distribution for each of the channels data:

The parameters for the lognormal distribution for each channel are not given here for the sake of brevity. The joint likelihood ratio test, given values $\kappa, \tau_{rms}, \tau_m$ for a particular observation of CIR, can be applied as in

$$
\frac{P(\kappa, \tau_{rms}, \tau_m | H_1)}{P(\kappa, \tau_{rms}, \tau_m | H_0)} \overset{NLOS}{\geq} \frac{C_{FN}}{C_{FL}} \times \frac{P_{H_1}}{P_{H_0}}
$$

Assuming independence for the three observations and equal possibility of NLOS channel and LOS channel, the LRT can be simplified, as in

$$
\frac{P(\kappa | H_1)}{P(\kappa | H_0)} \times \frac{P(\tau_{rms} | H_1)}{P(\tau_{rms} | H_0)} \times \frac{P(\tau_m | H_1)}{P(\tau_m | H_0)} \overset{NLOS}{\geq} \frac{C_{FN}}{C_{FL}} \times \frac{c - C_{FN}}{c}
$$

Note that the three metrics are highly correlated. The pearson correlation factors is given in Table 5.1.

5.1.5 Correlated Decision Level Fusion Using PSO

In decision level fusion, each classifier applies a threshold on its metric and provides the ensuing decision to the fusion engine. Different classifiers have different thresholds. If a
Table 5.1: Correlation Between Different Metrics for Different Channels

<table>
<thead>
<tr>
<th>Channel Type</th>
<th>( \kappa, \tau_{rms} )</th>
<th>( \tau_{rms}, \tau_m )</th>
<th>( \kappa, \tau_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM 1 (Residential LOS)</td>
<td>-0.0859</td>
<td>0.6763</td>
<td>-0.446</td>
</tr>
<tr>
<td>CM 2 (Residential NLOS)</td>
<td>-0.20954</td>
<td>0.63955</td>
<td>-0.5281</td>
</tr>
<tr>
<td>CM 3 (Indoor LOS)</td>
<td>-0.0839</td>
<td>0.71932</td>
<td>-0.33084</td>
</tr>
<tr>
<td>CM 4 (Indoor NLOS)</td>
<td>0.06739</td>
<td>0.513283</td>
<td>-0.34728</td>
</tr>
<tr>
<td>CM 5 (Outdoor LOS)</td>
<td>-0.02282</td>
<td>0.5923</td>
<td>-0.37269</td>
</tr>
<tr>
<td>CM 6 (Outdoor NLOS)</td>
<td>0.0341</td>
<td>0.60434</td>
<td>-0.28427</td>
</tr>
</tbody>
</table>

If the metric exceeds this threshold, the LOS hypothesis is rejected. If the metric falls below the threshold, the LOS hypothesis is accepted. This decision process using the threshold, \( \lambda_i \), for classifier \( i \) can be summarized as,

\[
    u_i = \begin{cases} 
    1 & x_i \geq \lambda_i \\ 
    0 & x_i < \lambda_i 
    \end{cases} 
\]  

(5.10)

Let \( [U] = [u_\kappa, u_{\tau_{rms}}, u_{\tau_m}] \) be the binary vector of decisions generated by multiple classifiers based on decision thresholds \( [\lambda_\kappa, \lambda_{\tau_{rms}}, \lambda_{\tau_m}] \). These decisions can then be combined using a fusion rule of the form . For more detailed description of the formation of the fusion rule, reader is referred to [48].

There two errors in this hypothesis testing problem, known as probability of false LOS (\( P_{FL} \)) and probability of false NLOS (\( P_{FN} \)) and are denoted as,

\[
P_{FL} = P(u_f = 1|H_0),
\]

(5.11)

\[
P_{FN} = P(u_f = 0|H_1).
\]

(5.12)

where \( u_f \) is the final decision rendered by the fusion engine based on the decisions output by the individual classifiers. The goal is to minimize these errors. The error probabilities (\( P_{FL}, P_{FN} \)) of the fused system are
5.1. UWB LOS/NLOS IDENTIFICATION CLASSIFIERS

\[ P_{FN} = \sum_{i=0}^{l-1} d_i \times P(u_1, u_2, \ldots, u_n | H_0) \]  
(5.13)

and

\[ P_{FL} = \sum_{i=0}^{l-1} (1 - d_i) \times P(u_1, u_2, \ldots, u_n | H_0) \]  
(5.14)

Let us assume that the classifiers decision thresholds are fixed at \([\lambda_\infty, \lambda_{\text{rms}}, \lambda_{\text{rms}}] \). Hence, 5.13 and 5.14 require the calculation of joint probabilities. The number of combinations of individual classifier decisions \(n\) determines the number of joint probabilities \(l\) that need to be estimated:

\[ l = 2^n \]  
(5.15)

The performance of decision level fusion depends on design of the thresholds and fusion rule. The optimal design of fusion rule and thresholds for each individual classifier is considered in [46]. A particle swarm optimization based (PSO) algorithm is designed in [46] to optimize the thresholds and fusion rule given the training data. A hybrid of binary and continuous PSO is used to achieve the fusion strategy. We employ this algorithm on the training data to achieve the thresholds and the fusion rule. The resultant configuration is applied to testing data.

The results are presented for the PSO based strategy for the three different scenarios in Table 5.2. The LRT based approach is presented in Table 5.3. Highest performance benefits are achieved in the residential scenario. From Figure 5.1 one can see that the overlap between the two channels is higher in the residential scenario. A better detection performance in this scenario is hard to achieve. Error is calculated using 5.7. We achieve close to 13% performance benefit by using a PSO based fusion strategy. Similar performance benefits are observed in other scenarios as well.
Table 5.2: LOS/NLOS Identification Rate Using Correlated Decision Fusion Strategy

<table>
<thead>
<tr>
<th>Channel Type</th>
<th>LRT</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM 1 (Residential LOS)</td>
<td>87.176</td>
<td>0.14793</td>
</tr>
<tr>
<td>CM 2 (Residential NLOS)</td>
<td>83.238</td>
<td>0.14793</td>
</tr>
<tr>
<td>CM 3 (Indoor LOS)</td>
<td>99.962</td>
<td></td>
</tr>
<tr>
<td>CM 4 (Indoor NLOS)</td>
<td>99.892</td>
<td>0.00073</td>
</tr>
<tr>
<td>CM 5 (Outdoor LOS)</td>
<td>97.177</td>
<td></td>
</tr>
<tr>
<td>CM 6 (Outdoor NLOS)</td>
<td>93.256</td>
<td>0.04783</td>
</tr>
</tbody>
</table>

Table 5.3: LOS/NLOS Identification Rate Using Data Level Likelihood Ratio Test Assuming Independence

<table>
<thead>
<tr>
<th>Channel Type</th>
<th>LRT</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM 1 (Residential LOS)</td>
<td>79.23</td>
<td>0.1735</td>
</tr>
<tr>
<td>CM 2 (Residential NLOS)</td>
<td>86.07</td>
<td>0.1735</td>
</tr>
<tr>
<td>CM 3 (Indoor LOS)</td>
<td>99.99</td>
<td></td>
</tr>
<tr>
<td>CM 4 (Indoor NLOS)</td>
<td>99.5</td>
<td>0.003</td>
</tr>
<tr>
<td>CM 5 (Outdoor LOS)</td>
<td>96.36</td>
<td></td>
</tr>
<tr>
<td>CM 6 (Outdoor NLOS)</td>
<td>93.51</td>
<td>0.05066</td>
</tr>
</tbody>
</table>

5.2 PSO Based NLOS Error Mitigation and Multilateration

UWB sensor networks are a promising solution for ranging in short-range environments, such as underground and indoor where GPS does not work. Due to severe multipath conditions in these environments, estimation of time of arrivals) of UWB signals results in random and sometimes large errors. The signals that go through LOS channels cause small random errors and the signals that go through NLOS channels will have large positive errors. We build an UWB ranging simulation system using Matlab to collect the ranging information to serve for our NLOS mitigation technique and different positioning algorithms between transmitter Tx and receiver Rx [49], [50]. Naturally one expects an increase in the distance measurement error with the increase of distance between Tx
5.2. PSO BASED NLOS ERROR MITIGATION AND MULTILATERATION

(transmitter) and Rx (Receiver), so we modify the IEEE model according to empirical measurement results from [51].

5.2.1 UWB NLOS Range Error Modeling

Following the impulse response of IEEE 802.15.4a channel model in 5.1, the TOA of the received signal is given by:

\[
\begin{align*}
\tau_{TOF} &= \tau_1 + \tau_n \quad \text{LOS} \\
\tau_{TOF} &= \tau_1 + \tau_n + \tau_c \quad \text{NLOS}
\end{align*}
\]  

(5.16)

where \(\tau_n\) is the measurement error caused by noise. Its value can be either positive or negative, modeled as Additive White Gaussian Noise (AWGN). \(\tau_c\) is the extra time that the first MPC takes to go through the NLOS channel, which is modeled in the IEEE 802.15.4a channel model report [49] as:

\[
\tau_c = \left( \frac{1}{\sqrt{2\Lambda}} \times r_{n1} \right)^2 + \left( \frac{1}{\sqrt{2\Lambda}} \times r_{n2} \right)^2
\]  

(5.17)

where \(\Lambda\) is the cluster arrival rate. The report lists 4 types of NLOS channels, each has a unique value for \(\Lambda\). \(r_{n1}\) and \(r_{n2}\) are random numbers between 0 and 1. Since \(\tau_c\) does not employ the measurement errors dependence on Tx and Rx distance, we modify it according to [51] as below

\[
\tau_c = \left[ \left( \frac{1}{\sqrt{2\Lambda}} \times r_{n1} \right)^2 + \left( \frac{1}{\sqrt{2\Lambda}} \times r_{n2} \right)^2 \right] \log(1 + c\tau_1),
\]  

(5.18)

where \(c\) is speed of light, \(d = c\tau_1\) is the actual distance between TX and Rx. can be define as the NLOS ranging error, its value is always positive. After the modification to the channel model, the range estimates are more close to the real empirical measurement
results. The NLOS ranges then can be describe as:

\[ d_{NLOS} = d + \varepsilon_{NL} + \varepsilon_n \]  

(5.19)

where \( \varepsilon_n \) is the ranging error caused by noise.

\subsection*{5.2.2 PSO enabled NLOS Error Mitigation Technique}

There are a wide variety of algorithms that can used to calculate the position of an unknown location node from the range measurements between the target node and some known neighbor nodes. However, they all severely suffer from the NLOS range errors we mentioned above. After we identify the NLOS signals, there are two ways of using the NLOS ranges. One is to discard them. The other is to mitigate the NLOS errors first and use them in positioning algorithms. In absence of enough LOS ranges it becomes paramount to use the NLOS ranges as well.

The problem now deduces in mitigating the NLOS range \( d_{NLOS} \) and use it to estimate the real distance \( d \). Specifically the NLOS range measurement can be transformed into its equivalent LOS measurement. In absence of a closed form solution for this transformation, we learn this transformation from training data. We define an “error mitigation ratio” (EMR) \( r \) between the NLOS ranges \( d_{NLOS} \) and real distance as below:

\[ r_i = d_i / d_{NLOS_i} \]  

(5.20)

\[ r = \arg \min F(r_i) \]  

(5.21)
Thus NLOS mitigation problem becomes a minimization problem. We use PSO to find the EMR $r$ for different $d_{\text{NLOS}}$ values. The results in Figure 5.3 show that at different $d_{\text{NLOS}}$ range, the optimized ratio (found by PSO) value $r$ is different. We also show the mean ratio value which average the $r$ value across all the NLOS distances and PSO find sub-optimal $r$ value for 4-16 meter NLOS distance. The different empirical $r$ values we get from our training data enable us to find the find the error mitigated $d_m$ value for the $d_{\text{NLOS}}$ results given by

\begin{equation}
    d_m = r \times d_{\text{NLOS}}.
\end{equation}

In our positioning algorithm, we assume we have only 2 LOS ranges and one NLOS range. $d_m$ has to be used along with the 2 LOS ranges to locate the nodes in a 2-D space.

### 5.2.3 PSO Based Multilateration Positioning Algorithm

We use PSO based multilateration in our positioning procedure. Multilateration is a simple positioning technique, but the specific mathematics of its implementation vary widely. The idea of multilateration algorithm is to use the Cartesian positions of $n$ known nodes $b_i = (x_i, y_i), i = 1, 2, \cdots, n$ and their measured ranges to the target node $d_i$ to determine the unknown position of the target node $s = (x, y)$.

If those ranges are absolutely precise, the true position of the target node should be at the intersection of all the spheres whose centers are the coordinates of the beacons and radius are the ranges between the beacons and the target node (as shown in Figure
Figure 5.3: Empirical $r$ values vs. measured NLOS distance

5.4 ‘A’, ‘B’ and ‘C’ are nodes whose positions are known). However, since the accuracy of range estimation is affected by noise and the NLOS transmission channel, the spheres will not always give a conclusive single intersection point. The basic multilateration algorithm finds the best estimate of the true position $s(x,y)$ by minimizing the sum of the squared errors between the measured ranges and the predicted distance as:

$$ s = \arg \min_s E(s), $$ (5.24)
5.2. PSO BASED NLOS ERROR MITIGATION AND MULTILATERATION

$$E(s) = \sum_{i=1}^{n} (\|s - b_i\| - d_i)^2.$$  

(5.25)

Particle swarm solves this problem efficiently. The fitness function for PSO is $E(s)$, in another form:

$$f(x, y) = \sum_{i=1}^{n} (\sqrt{(x_i - x)^2 + (y_i - y)^2} - d_i)^2$$

(5.26)

Figure 5.4: NLOS error mitigation in multilateration positioning

The swarm search space is the two dimensional position of the unknown node $s = (x, y)$ which has the least error $f(x, y)$. The $d_i$'s are the range measurements from each
of the known nodes. When an measurement is identified as a signal coming from a NLOS path using the algorithm presented in section 5.2.1, the range measurement is transformed using the error mitigation ratio. This process is illustrated in the Figure 5.4. It is the zoomed version of Figure 5.5 and shows that when NLOS error mitigated range (shown as “modified NLOS range”) in the figure is used in 5.26, the “location error” is greatly reduced.

In order to test the performance of the NLOS error mitigation technique and the PSO based multilateration method, we consider a room (Indoor scenario) of size 40m x 40m. The center of the room is assumed to be the origin and the target node is assumed to be
at this position, i.e., at \([x, y] = (0, 0)\) m. The nodes whose positions are known are randomly scattered around the room, at distances of 4 m to 10 m from the target. The resultant measured NLOS ranges are distributed from 4 m to 16 m. We compare four different approaches. First, we use the EMR value found by PSO and the PSO based multilateration approach (PSO MTLR, PSO-ratio). In the second approach a simple mean ratio value (calculated in each range bin) in the NLOS error mitigation procedure is adopted along with the PSO based multilateration (PSO MTLR, mean-ratio). The third approach and fourth approach employ linearized least square method with the two different ratios.

![Figure 5.6: The average positioning error comparison between PSO based multilateration and linear least square multilateration algorithm with mean r value and PSO find sub-optimal r value](image-url)
5.3. SUMMARY

We ran a Monte Carlo simulation for 10,000 cases. The results presented in Figure 5.6 show that using \( r \) for NLOS error mitigation found by PSO, combined with our PSO based multilateration algorithm, the average location error drops significantly to a value less than 1 meter. Traditionally it is difficult to estimate the position of the unknown node with one NLOS and 2 LOS measurements. Usually on encountering an NLOS measurement, more measurements are collected causing delays and/or higher energy utilization. With the swarm based learning approaches presented in this work we are able to achieve a accuracy of 1m by transforming the measurement. The linearized least square approach is highly sensitive to the ratio used. The ratio found by the PSO gives better results than the mean ratio in this case. This demonstrates the efficacy of the PSO algorithm.

Greater advantages are found for the ratio if LLS approach is used. A single step LLS is used to solve the multilateration problem. This is computationally less expensive than PSO based multilateration (which is iterative). The use of PSO defined ratio brings the performance of LLS closer to the PSO based multilateration approaches. Thus the PSO based ratio lets us use the computationally simple LLS technique.

5.3 Summary

In this Chapter, we presented a novel UWB positioning system design using particle swarm enabled learning techniques. First a classification strategy is developed using the particle swarm optimization technique. The classification strategy identifies the NLOS signals from the received signal set. Then a PSO based strategy is developed to mitigate the error due to the NLOS signal. Specifically a ratio is achieved using PSO that transforms the NLOS measurement into its equivalent LOS measurement. Finally, the PSO is used for the multilateration problem, which combines the measurements from three
different nodes. We have compared our strategies to traditionally applied techniques and achieved higher performance. Performance is measured in terms of the Bayesian risk function for the NLOS identification. For locationing the performance is measured in terms of the positioning error, i.e. distance from the true position. We are able to achieve less than a meter error when PSO based strategies are used.
Chapter 6

PSO Based Signal Processing for OFDM Systems

PSO can be used in a wide series of applications. In this chapter, we use PSO as an intelligent searching algorithm to find the optimal or sub-optimal solution for a code design problem in a frequency redundant OFDM/OFDMA system. Two of the major challenges facing OFDM/OFDMA systems are their sensitivity to frequency selective fading and Inter Carrier Interference (ICI) due to Carrier Frequency Offset (CFO) or Doppler shift, especially when the subcarrier spacing becomes smaller. By using PSO to find an optimal code for our design, we propose an OFDM transceiver design that employs frequency redundant subcarrier mapping to mitigate frequency selective fading and subcarrier spreading to achieve ICI self cancelation. Both our theoretical analysis and simulation show that such a code-spread-interleaved-redundant OFDM system design offers significant (over 10 dB) improvement in Carrier to Interference Ratio (CIR) and robust Bit Error Rate (BER) performance in different channel conditions.


6.1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) and Orthogonal Frequency Division Multiple Access (OFDMA) are widely adopted in current communication systems for its high spectrum efficiency and easy implementation [52][53]. One of the recent advancements in OFDM/OFDMA system design is the increasing subcarriers density (reduce the subcarrier spacing) in order to minimize the cyclic prefix (CP) overhead.

Reduced sub-carrier spacing not only increases the transceiver complexity, more importantly, it makes an OFDM system more susceptible to frequency selective fading and ICI, which is caused by CFO or Doppler effect.

Adding frequency diversity in an OFDM design is an effective way of mitigating the effect of frequency-selective fading. This is generally achieved by subcarrier redundancy, or channel coding. Most popular coding schemes are convolutional codes, Turbo codes and low density parity check (LDPC) codes.

To reduce ICI, we can minimize CFO by using accurate and stable local reference clocks, or implementing phase-lock-loops (PLLs) or frequency tracking between two communicating nodes. Unfortunately, neither of these solutions are viable in many systems due to the cost, power, complexity constraints, or upper layer protocols which do not support continuous transmission.

Mitigating ICI in digital domain is desirable for many reasons and there have been research work published in recent years. [54] gives a comprehensive overview of the commonly used ICI mitigation techniques. Generally all these techniques fall into three categories: i) frequency domain equalization (FDE); ii) time domain windowing and iii) subcarrier self-cancelation. In FDE, the CFO is first estimated using training symbols and then equalized in frequency domain (after FFT) at the receiver side [55][56]. FDE minimizes ICI by compensating for CFO and thus requires an accurate CFO estimation,
which is difficult to achieve when the received signal to noise ration (SNR) is low. Also the computation complexity is high in generating the correction matrix. Time domain windowing refers to techniques which use Nyquist windows other than rectangular window (e.g., Hanning window) and reduce energy leakage between subcarriers in the transmitted symbols. These windowing methods have poor performance with respect to additive channel noise [57, 54]. It also reduces the effective length of the CP and thus results in increased inter-symbol-interference (ISI). The third type is the ICI self-cancelation. Zhao et al. proposed in [58, 59] to map the same data onto an adjacent pair of subcarriers with opposite polarities and as a result, interference to other subcarriers from these subcarrier pair cancel each other.

Here we propose an OFDM/OFDMA design which offers not only ICI self-cancelation but frequency diversity as well. The key features we proposed are the interleaved redundant subcarriers mapping and subcarriers spreading with orthogonal codes. We use PSO to find an set of optimal codes for this application. We provide both theoretical analysis and numerical results on the system performance. In our simulations, the proposed designs demonstrate robust performance in both additive white Gaussian noise (AWGN) channels and dispersive channels. The spreading scheme improves Carrier-to-Interference-Ratio (CIR) by over 10 dB, and significantly lowers the bit error rate (BER).

6.2 OFDM System Model

Figure 6.1 shows the block diagram of the proposed transmitter and receiver structure. Compared to a conventional OFDM transmitter and receiver, the key components of our design are the redundant subcarrier mapping, spreading and de-spreading blocks.

In a conventional transmitter, data to be transmitted are read in blocks. Each data block can be represented by a size-$M$ vector, $A \in \mathbb{C}^m$, $A = [a_1, a_2, \cdots, a_m]$, where $a_m$ is
6.2. OFDM SYSTEM MODEL

Figure 6.1: Block Diagram of the Proposed OFDM System

A complex number representing a modulation alphabet based on a particular modulation scheme for the \( m \)th subcarrier (e.g., QPSK, QAM and etc.). A mapping function, \( \mathcal{P}(\cdot) \), maps input data symbols in \( A \) to a size \( N \) vector, \( S = [S_1, S_2, \cdots, S_N] \).

\[
S = \mathcal{P}(A),
\]

\( N \) is the number of subcarriers in an OFDM symbol. \( \mathcal{P}(\cdot) \) can be 1-to-1 mapping, which maximize spectrum efficiency, or 1-to-many. In practice, however, the mapping block also carries out pilot insertion as well as null-tone insertion in the guard band and at DC.

\( S \) is also referred to as the frequency domain symbol block. It is then transformed to
a time domain sequence, \( s(t) \), via the inverse fast Fourier transform (IFFT) given by

\[
s(t) = \mathcal{F}^{-1}(S) = \sum_{k=0}^{N-1} S_k e^{j \frac{2 \pi k t}{T}},
\]

(6.1)

where \( S_k \) is the frequency-domain symbol for the \( k \)th subcarrier, and \( \mathcal{F}(\cdot) \) denotes the FFT operation. Cyclic prefix (CP) is added to each output time domain sequence before it is transmitted.

The received time-domain signal, \( r(t) \), is given by

\[
r(t) = h(t) \otimes s(t) e^{-2 \pi j \Delta f(t) t} + \nu(t),
\]

(6.2)

where \( h(t) \) is the channel impulse response, \( \otimes \) denotes linear convolution and \( \nu(t) \) is the additive noise. The equivalent expression in frequency domain is

\[
R = H \cdot (W \times S) + \eta,
\]

(6.3)

where \( H \), the channel frequency response matrix, is a diagonal matrix and its diagonal element, \( H_{k,k} \), represents the response of the \( k \)th subcarrier, where \( |H_{k,k}| \) is the gain and \( \angle H_{k,k} \) the phase delay. \( \eta \) is noise spectrum power, \( W \in \mathbb{C}^{N \times N} \) is the ICI coefficient matrix. More details will be given in 6.2.1.

The OFDM receiver reverses the processes occurred in the transmitter by performing the CP removal, and FFT on the received signal to produce received frequency domain symbols, \( R \), which is de-mapped to generate the received data symbol, \( V \). This processes can be expressed as

\[
V = \mathcal{P}^{-1}(R) = \mathcal{P}^{-1}(\mathcal{F}(r)) = \mathcal{P}^{-1}(\mathcal{F}(r)) = \mathcal{P}^{-1}(R) = \mathcal{P}^{-1}(H \cdot (W \times S) + \eta),
\]

(6.4)
6.2.1 Inter Carrier Interference

The severity of ICI is represented by the ICI coefficient matrix, $W$. $W_{m,k}$ quantifies the interference from the $k$th subcarrier to the $m$th subcarrier. In CFO free system, $\Delta f = 0$ and $W$ is an identity matrix. (6.3) can be simplified to

$$R_k = H_{k,k}S_k + \eta_k. \quad (6.5)$$

(6.5) indicates that the received signal at $k$th subcarrier is only dependent on the transmitted signal $S_k$, plus the noise $\eta_k$ and therefore is ICI free. However, the presence of CFO or Doppler effect disturbs the orthogonality between subcarriers. This is reflected in $W$. For a given CFO, $\Delta f$, $\epsilon = \frac{\Delta f}{f_s}$ is the normalized CFO wrt the subcarrier spacing, $f_s$, we have [58, 54]

$$W_{k,m} = \sin\left[\frac{\pi(m - k + \epsilon)}{\pi(m - k + \epsilon)}\right]e^{-j\pi(m-k+\epsilon)}, \quad (6.6)$$

and the received signal on $k$th subcarrier becomes

$$R_k \approx H_k \sum_{n=0}^{N-1} W_{n,k}S_n + \eta_k$$

$$= H_kW_{k,k}S_k + H_k \sum_{m=0, m \neq k}^{N-1} W_{k,m}S_m + \eta_k. \quad (6.7)$$

The first term in (6.7) is the received power from the signal subcarrier, the second term is the total interference from all other subcarriers. Clearly ICI is a function of $\epsilon$. As $\epsilon$ grows, the power from the signal tone decreases and the interference from individual tones as well as the total interference increases as shown in Figure 6.2. $W_{n,k}$ is only a function of $(n - k)$ and therefore can be simplified as $W_{n-k}$, e.g, $W_0 \equiv W_{k,k}$. ICI is
quantified with carrier to interference ratio (CIR) to quantify ICI, defined as

\[
CIR(\epsilon) = E \left[ \frac{W_0}{\sqrt{E\left[ \sum_{m=0, m \neq k}^{N-1} W_{k-m}a_m \right]}^2} \right],
\]

(6.8)

where \( E[\cdot] \) is the expectation over all subcarriers and input symbols. Figure 6.2 shows that at \( \epsilon = -0.3 \), \( CIR \) approaches 0 dB. Clearly, CFO induced ICI can be the system performance bottleneck and must be dealt with.

![ICI Coefficients vs CFO](image.png)

Figure 6.2: ICI Coefficients vs CFO
6.2. OFDM SYSTEM MODEL

6.2.2 OFDM Systems with Frequency Diversity

The coherent bandwidth in most of the wireless channels is much greater than the subcarrier-spacing and therefore each subcarrier is subject to deep fading. Frequency diversity is introduced in OFDM systems to mitigate this. An easy and convenient way of providing such frequency diversity is to map each input symbol, \( a_m \), to \( l > 1 \) subcarriers, i.e.,

\[
S_k = a_m, \quad \forall k \in \mathcal{G}_m = \{k_1, k_2, \ldots, k_L\}.
\]

\( L \) is the degree of frequency diversity, and \( \mathcal{G}_m \) is the set of subcarriers assigned to \( a_m \), referred hereafter as the \( m \)th subcarrier group. To maximize the frequency diversity, it is essential that the subcarriers assigned to the same input data are spread across the entire band. This can be achieved when an “interleaving subcarrier mapping” scheme is used. For example, assuming size-\( M \) inputs vector, a diversity degree of \( L \) and omit all non-data tones, a mapping function would look as follows:

\[
S = \mathcal{P}(A) = [a_1, a_2, \ldots, a_m, a_{M+m}, \ldots, a_{(L-1)M+m}],
\]

(6.9)

The mapping function given in (7.9) maps \( M \) inputs to \( L \times M \) subcarriers. The \( m \)th input, \( a_m \) is mapped to \( L \) subcarriers, \( \{a_m, a_{M+m}, a_{2M+m}, \ldots, a_{(L-1)M+m}\} \). Subcarriers in the same group have a minimum separation of \( M \) subcarriers spacing. We call this subcarrier mapping scheme “interleaved redundant subcarrier” since the subcarriers for different input symbols interleave with each others.

The corresponding receiver combines power from subcarriers that are mapped to the same input symbols. (6.4) is rewritten as

\[
V_m = \mathcal{P}^{-1}(R) = \sum_{k \in \mathcal{G}_m} g_k R_k,
\]

(6.10)
6.2. OFDM SYSTEM MODEL

where \( g_k \) is the combining weight defined by the combining scheme. Two most commonly used schemes are equal gain combining (EGC) and maximum ratio combining (MRC). EGC is simpler and in many cases sufficiently effective but MRC offers better performance in frequency selective channels [60]. The combination of redundant-mapping and combining processes effectively mitigate the frequency selective fading as it guarantees the combined SNR for a given input \( a_m \) remains acceptable when deep fading occurs on just one or a few subcarriers assigned to it.

With redundant subcarrier, CIR becomes the ratio of the total power of signal subcarriers for the same input symbol to the total power from the interfering subcarriers. Compared to an OFDM design without subcarrier redundancy, the “interleaved redundant subcarrier” mapping scheme described above offers no CIR improvement. The mathematic proof is relatively simple and will not be given here. Intuitively, we can see that both the total signal power and the total interference power increase proportionally to the diversity degree \( L \).

6.2.3 OFDM with Spread Redundant Subcarrier

To improve the ICI performance in an OFDM system with subcarrier redundancy, we propose a subcarrier spreading scheme which extends the “interleaved redundant subcarrier” design discussed above but offers ICI cancelation and significant CIR improvement. The block diagram is shown in Figure 6.1 and details of the design are as follows:

The transmitter first maps \( M \) input alphabets, \( A \), to \( N \) subcarriers as given in (7.9). However, instead of taking \( S \) directly as the IFFT input, a spreading operation is first carried out on \( S \). The spreading operation is defined as
6.2. OFDM SYSTEM MODEL

\[ S' = Q(S) = C \cdot S \]

\[ = \begin{bmatrix}
  c_1a_1, c_2a_2, \ldots, c_Ma_M, \ldots, & c_{LM-L+1}a_1, \ldots, c_{LM}a_M
  \\
  \text{1st set} & \text{Lth set}
\end{bmatrix}, \quad (6.11) \]

where \( C \) is a length-\( LM \) spreading sequence. We can reformat \( C \) into an \( L \times M \) matrix and each row vector in this spreading matrix, \( C^R_m = [c_m, c_{m+m}, \ldots, c_{LM-L+m}] \), is the spreading vector corresponding to a subcarrier group. If we design \( C \) such that all row vectors, \( C^R_m \), are chosen from a set of length-\( L \) orthogonal codes, \( O_L \), ICI self-cancelation can be achieved as shown in Section 6.3. Fig. shows the OFDM system with Spread Redundant Subcarrier.

The proposed spreading scheme does not specify the orthogonal codes that can or should be used. For example, Walsh codes based on Hadamard matrix can be used as spreading vectors. A length-4 Hadamard matrix is given as

\[ H_4 = \begin{bmatrix}
  1 & 1 & 1 & 1 \\
  1 & 1 & -1 & -1 \\
  1 & -1 & 1 & -1 \\
  1 & -1 & -1 & 1
\end{bmatrix} \]

and each row vector is a length-4 Walsh code, denoted as \( W_4^1 \) to \( W_4^4 \).

When the number of subcarrier groups, \( M \) is greater than the total number of length-\( L \) orthogonal vectors available (which is generally the case), it is necessary to reuse the spreading vectors, i.e., the same spreading vector is applied to multiple subcarrier groups. Two subcarrier groups are orthogonal \( (\mathcal{G}_i \perp \mathcal{G}_j) \) if their spreading vectors are orthogonal, or compatible \( (\mathcal{G}_i \parallel \mathcal{G}_j) \) if they share the same orthogonal code. To maximize
ICI cancelation, the compatible subcarrier groups should be separated far apart.

Correspondingly in the receiver, the received symbols $R$ needs to be de-spread before combined. The output after combining is given as:

$$V_m = P^{-1}(R') = \sum_{k \in \Theta_m} g_k R'_k = \sum_{k \in \Theta_m} g_k Q^{-1}(R'_k).$$

(6.12)

$Q^{-1}(\cdot)$ denotes the de-spreading function. When Walsh codes are used as spreading vectors, $Q^{-1}(\cdot) = Q(\cdot)$. 

---

**Figure 6.3:** OFDM with spread redundant subcarrier
6.2. OFDM SYSTEM MODEL

Table 6.1: Barker Codes

<table>
<thead>
<tr>
<th>Length</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>+1 -1 — +1 +1</td>
</tr>
<tr>
<td>3</td>
<td>+1 +1 -1</td>
</tr>
<tr>
<td>4</td>
<td>+1 +1 +1 — +1 +1 +1 -1</td>
</tr>
<tr>
<td>5</td>
<td>+1 +1 +1 -1 +1</td>
</tr>
<tr>
<td>7</td>
<td>+1 +1 +1 -1 -1 +1 -1</td>
</tr>
<tr>
<td>11</td>
<td>+1 +1 +1 -1 -1 -1 +1 -1 -1 +1 -1</td>
</tr>
<tr>
<td>13</td>
<td>+1 +1 +1 +1 -1 -1 +1 +1 -1 +1 -1 +1</td>
</tr>
</tbody>
</table>

6.2.4 Orthogonal Code Design Using PSO

The selection of the orthogonal codes is critical in our design since we need codes that have maximum autocorrelation and minimum cross-correlation to combat with the ICI. In this design, for code length $2^n (n = 1, 2, 3, \cdots)$, Walsh-Hadamard codes are the optimal choices thanks to their perfect autocross-correlation properties. However, for certain applications in this scheme, the orthogonal codes need to have "irregular" length (e.g. no equal to $2^n$). For finite length codes especially “irregular” short length orthogonal codes, there are no perfect candidates available. We develop a group of cyclic Baker codes that can serve for our application. A Barker code is a sequence of N binary values of +1 and -1, $a_j$ for $j=1,\ldots,N$, such that

$$\left| \sum_{j=1}^{N-v} a_j a_{j+v} \right| \leq 1$$ (6.13)

for all $1 \leq v < N$ [61].

A Barker code has a maximum autocorrelation of 1 (when codes are not aligned). There are Barker codes of lengths 2, 3, 4, 5, 7, 11, and 13, and it is conjectured that no longer Barker codes exist. A list of known Barker codes is given in Table 6.1

Considering the low autocorrelation side lobes that Barker codes have, simply by cyclically shifting the Barker code, we can get code groups of length 3, 5, 7, etc. For example, we can get a group of 7 codes by shifting length 7 Barker code one bit at a time.
Although they are not orthogonal to each other, their cross correlations are always 1 while their autocorrelations are always 7. These codes partially satisfy the requirements in our design. However, the length 6 codes are still not available. We resort to generalized Baker code for solutions. A generalized Barker code is a finite sequence $a_r$ of complex numbers having absolute value 1, and possessing a correlation function $C(\tau)$ satisfying the constraint $|C(\tau)| \leq 1, \tau \neq 0$ [62].

Walsh codes are available for $L = 2^k$. All Walsh codes have only $+1$ and $-1$ as elements. This allows very simple implementation for the spreading and de-spreading operations. It is, however, not required for the coefficients to be real. By extending coefficients to complex numbers with absolute value of 1, we can find orthogonal codes for any $L$ while preserving the transmission spectrum. For example, a set of $L$ sequences $\{F^0_L, F^1_L, \ldots, F^{L-1}_L\}$ form the orthonormal basis and can be used as spreading vectors. Each vector, $F^k_L$, is defined by a Fourier series given as $F^k_L = [1, e^{2\pi k L}, e^{4\pi k L}, \ldots e^{2\pi k (L-1) L}]$. Cyclic orthogonal sequences can also be generated that are mutually orthogonal [62]. Here, the problem boils down to given a fixed value of $L$, find the right set of $F^k_L$ (not necessarily Fourier series based) that have the lowest cross-correlation among themselves.

Note that a set of $L$ sequences $\{F^0_L, F^1_L, \ldots, F^{L-1}_L\}$ with each element having the absolute value of 1 has infinite possibilities. The searching space can be considered as continuous, but we are looking for a set of discrete numbers as the solution inside of it. Due to the prohibitive time needed to find a solution using exhaustive search, we have to concentrate our efforts on methods that provide approximate solutions and also can finish the search in reasonable time. Although this is not a real-time searching problem, as far as the author know, there is no reported tangible method to solve the problem satisfactorily. Therefore, we can design a specific PSO approach for this problem.

As we have explained before, resembling the social behavior of a swarm of bees to search the location with the most flowers in a field, the optimization procedure of
6.2. OFDM SYSTEM MODEL

![PSO algorithm flowchart](image)

Figure 6.4: PSO algorithm flowchart
PSO is based on a population of particles which fly in the solution space with velocity dynamically adjusted according to its own flying experience and the flying experience of the best among the swarm.

Figure 6.4 shows the flow chart of a PSO algorithm. During the PSO process, each potential solution is represented as a particle. In this application, a particle with a position vector $X_q$ represent a vector $\mathbb{R}^L_k$. There are total $L$ dimensions. Each particle also maintains a memory ($p$best) of its previous best position, $P_{id} = (p_{i1}, p_{i2}, \cdots, p_{iL})$ and a velocity along each dimension represented as $V_i = (v_{i1}, v_{i2}, \cdots, v_{iL})$. The global best (gbest) particle is denoted by $P_{gd} = (p_{g1}, p_{g2}, \cdots, p_{gL})$. The fitness function of this problem is the sum of cross-correlation of one vector with all the other vectors in the candidate set.

PSO algorithms run and give us an satisfied result. Table 6.2 lists a set of cyclic orthogonal codes of length-6 found by PSO.

<table>
<thead>
<tr>
<th>Code Index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{B}_0^1$</td>
<td>+1</td>
<td>$e^{j2\pi/6}$</td>
<td>-1</td>
<td>+1</td>
<td>$-e^{j2\pi/6}$</td>
<td>-1</td>
</tr>
<tr>
<td>$\mathbb{B}_0^2$</td>
<td>$e^{j2\pi/6}$</td>
<td>-1</td>
<td>+1</td>
<td>$-e^{j2\pi/6}$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$\mathbb{B}_0^3$</td>
<td>-1</td>
<td>+1</td>
<td>$-e^{j2\pi/6}$</td>
<td>-1</td>
<td>-1</td>
<td>$-e^{j2\pi/6}$</td>
</tr>
<tr>
<td>$\mathbb{B}_0^4$</td>
<td>+1</td>
<td>$-e^{j2\pi/6}$</td>
<td>-1</td>
<td>-1</td>
<td>$-e^{j2\pi/6}$</td>
<td>+1</td>
</tr>
<tr>
<td>$\mathbb{B}_0^5$</td>
<td>$-e^{j2\pi/6}$</td>
<td>-1</td>
<td>-1</td>
<td>$-e^{j2\pi/6}$</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>$\mathbb{B}_0^6$</td>
<td>-1</td>
<td>-1</td>
<td>$-e^{j2\pi/6}$</td>
<td>+1</td>
<td>-1</td>
<td>$e^{j2\pi/6}$</td>
</tr>
</tbody>
</table>

When perfect orthogonal codes are unavailable or difficult to generate, quasi-orthogonal codes (QOC) can also be used. Due to the non-zero cross-correlation sidelobe, QOC generally offers less ICI cancelation.
6.3 Analysis on Effectiveness of ICI-Cancellation

We now show the ICI cancelation in the proposed spread redundant carrier design. For clarity of the analysis, we ignore the pilot and null tones insertion and use the simple mapping defined in (7.9).

Combines (6.7), (6.11) and (6.12), the output of the combining stage at the receiver can be expressed as

\[
V_m = \sum_{k \in \mathcal{S}_m} g_k c_k R_k \\
= \sum_{k \in \mathcal{S}_m} g_k c_k (h_k \sum_{n=0}^{N-1} W_{n,k} c_n S_n + \eta_k) \\
= \sum_{k \in \mathcal{S}_m} g_k h_k W_0 c_k c_n S_k + \sum_{k \in \mathcal{S}_m} g_k c_k h_k \sum_{n=0, n \neq k}^{N-1} W_{n,k} c_n S_n \\
+ \sum_{k \in \mathcal{S}_m} g_k c_k \eta_k \\
= W_0 a_m \sum_{k \in \mathcal{S}_m} g_k h_k + \sum_{k \in \mathcal{S}_m} g_k c_k h_k \sum_{n=0, n \neq k}^{N-1} W_{n,k} c_n S_n \\
+ \sum_{k \in \mathcal{S}_m} g_k c_k \eta_k.
\]  

(6.14)

The first term in (6.14) describes the power from the transmitted signal subcarrier and the second term the interfering subcarriers. The third term is from the additive noise and its total power remain unchanged with or without de-spreading, i.e., \( E[\sum_{k \in \mathcal{S}_m} g_k \eta_k] = E[\sum_{k \in \mathcal{S}_m} g_k c_k \eta_k] \) given \( |c_k| = 1 \).

For the clarity in the following analysis, we further assume AWGN channels \( (h_k = h) \), and EGC \( (g_k = 1) \) in the receiver. The first term (signal carrier) in (6.14) is simplified to

\[
Z_m = h W_0 \sum_{k \in \mathcal{S}_m} c_k c_k S_k = h W_0 \sum_{k \in \mathcal{S}_m} S_k = h LW_0 a_m, 
\]  

(6.15)
unchanged from the original non-spreading design. The interference term, however, becomes

\[ I_m = h \sum_{k \in \mathcal{S}_m} c_k \sum_{n=0, n \neq k}^{N-1} W_{n-k} c_n S_n \]

\[ = h \sum_{k \in \mathcal{S}_m} c_k \left( \sum_{n \in \mathcal{S}_p, \mathcal{S}_p \parallel \mathcal{S}_m} W_{n-k} c_n a_p + \sum_{n \in \mathcal{S}_p, \mathcal{S}_p \perp \mathcal{S}_m} W_{n-k} c_n a_p \right) \quad (6.16) \]

\[ = h \sum_{k \in \mathcal{S}_m} c_k \sum_{n \in \mathcal{S}_p, \mathcal{S}_p \parallel \mathcal{S}_m} W_{n-k} c_n a_p + h \sum_{k \in \mathcal{S}_m} c_k \sum_{n \in \mathcal{S}_p, \mathcal{S}_p \perp \mathcal{S}_m} W_{n-k} c_n a_p \]

Rewrite the first term in (6.16) as

\[ h \sum_{k \in \mathcal{S}_m} c_k \sum_{n \in \mathcal{S}_p, \mathcal{S}_p \parallel \mathcal{S}_m} W_{n-k} c_n a_p \]

\[ = h \sum_{l=1}^{L} c_{m+l-p} W_{m-p} a_p + h \sum_{j \neq 0} \sum_{l=1}^{L} c_{m+jL-p} W_{jL+m-p} a_p \quad (6.17) \]

\[ \approx hL \sum_{p \in \mathcal{S}_p, \mathcal{S}_p \parallel \mathcal{S}_m} W_{m-p} a_p. \]

The approximation is based on \( W_{m-p} \gg W_{jL+m-p} \), i.e., interference coefficient from subcarriers far away is negligible in the overall interference. Similarly, the second term in (6.16) can be approximated to

\[ h \sum_{k \in \mathcal{S}_m} c_k \sum_{n \in \mathcal{S}_p, \mathcal{S}_p \perp \mathcal{S}_m} W_{n-k} c_n a_p \]

\[ = h \sum_{l=1}^{L} c_{m+l-p} W_{m-p} a_p + h \sum_{j \neq 0} \sum_{l=1}^{L} c_{m+jL-p} W_{jL+m-p} a_p \]

\[ \approx hL \sum_{p \in \mathcal{S}_p, \mathcal{S}_p \perp \mathcal{S}_m} W_{m-p} a_p. \]

\[ \ll hL \sum_{p \in \mathcal{S}_p, \mathcal{S}_p \parallel \mathcal{S}_m} W_{m-p} a_p. \]
6.3. ANALYSIS ON EFFECTIVENESS OF ICI-CANCELATION

Therefore we have

\[ I_m \approx hL \sum_{p \in \mathcal{P} \parallel \mathcal{G}_m} W_{m-p} a_p, \]  

(6.19)

and

\[ \text{CIR}_m = E \left| \frac{Z_m}{I_m} \right|^2 \approx \left| \frac{W_0}{E \left[ \sum_{p \in \mathcal{P} \parallel \mathcal{G}_m} W_{m-p} \right]} \right|^2. \]  

(6.20)

The above equation shows that at the \( m \)th symbol, the majority of interference comes from subcarriers in its compatible subcarrier groups. The larger the diversity degree, the higher the ICI. If all groups are mutually orthogonal, i.e., where each subcarrier group is assigned a unique orthogonal code, the overall CIR can be very high. Such a design would, however, be very spectrum inefficient and is hardly seen in practice.

The analysis for frequency selective fading channel, or MRC is more complicated and less insightful but can be carried out similarly. Due to the variable channel gain \( h_k \) at different subcarriers, the orthogonal groups are no longer perfectly orthogonal to the signal subcarrier group and therefore will have higher residual interference energy. As results, we can expect degraded CIR compared to AWGN channels.

Figure 6.5 plots the calculated CIRs based on (6.20) for several different spreading schemes in flat fading scenario, with the CFO, \( \epsilon \), ranges from 0 up to 0.3. The total number of subcarriers, \( M \), is 256. The CIR of “interleaved redundant subcarrier” with diversity degree of 4, 6 and 8 are identical and plotted as the baseline. For “spread redundant subcarrier” with diversity of 4 and 8, we use Walsh codes \( \mathbb{W}_4 \) and \( \mathbb{W}_8 \) respectively. For diversity of 6, we use truncated \( \mathbb{W}_8 \), i.e., all 8 codes are truncated to 6 bits.

All spreading-based design offers significant CIR improvement, with the highest CIR improvement, close to 30 dB, achieved with \( L = 8 \). Figure 6.5 shows that the diversity degree, \( L \), directly affect the CIR, as it determines the spacing between compatible subcarriers. Figure 6.5 shows that CIR is 10 dB higher when \( L \) increases from 4 to 8. Using
quasi-orthogonal codes \((L = 6)\) suffers little degradation.

![Figure 6.5: Theoretical Calculation of CIR for Different Schemes](image)

### 6.4 Performance Evaluation

To evaluate the effectiveness of different spreading schemes, we implement several OFDM transceivers with different redundancy and spreading schemes and simulate their BER performance. The simulation is set up as follows: the center frequency of the OFDM systems is 2.4GHz and the signal bandwidth is 20MHz. Of a total of 256 subcarriers, we allocate at least 32 subcarriers as null tones in the guard band. DC tone is nulled as well. There are also 12 pilot tones evenly distributed and modulated with random generated...
symbols. Each tone is QPSK modulated and input symbols are randomly generated. During the simulation, we sweep $\varepsilon$ from 0 to 0.3.

Our first set of simulations are carried out using flat fading channels and the results are given in Figure 6.6. To observe the effect of our proposed frequency spreading scheme on system BER performance, no additive noise is added to the channel. Our simulation shows that when spreading is not applied, system BER is insensitive to the diversity degree, as predicted by our analysis. Note that since the simulations use noise free channel, the frequency diversity gain after combining is not reflected in the BER.

For both $L = 4$ and $L = 8$, Walsh codes are used as spreading vectors. For $L = 6$,
we compare three different spreading schemes: i) truncated Walsh codes, ii) alternat-
ing orthogonal codes and iii) length-6 cyclic orthogonal codes. In the first scheme, we
truncate all $W_8$ to length-6 vectors. The truncation causes the spreading vectors to be
quasi-orthogonal instead perfectly orthogonal to each other and as consequences, re-
sults in higher residual ICI. The alternating orthogonal codes scheme uses only one pair
of length-6 orthogonal codes (e.g., $[1, 1, 1, 1, 1, 1]$ and $[1, 1, 1, -1, -1, -1]$) and apply them
on adjacent channels. Such arrangement results in half of the total subcarrier groups to
be orthogonal to the other half of the groups, but compatible to each other within the two
halves. In the third schemes, $\mathbb{B}_6$ listed in Table 6.2 are used as spreading vectors.

The simulation shows that the second scheme has the worst BER of the three, even
worse than the $L = 4$ case. This shows that even though quasi-orthogonal suffers slight
performance degradation, the CIR gain is more directly affected by the separation of the
compatible groups. Even though the second scheme uses a set of 6 QOC as spreading
vectors, it has much better performance than the first one. As expected, the third scheme,
which uses 6 truly orthogonal spreading vectors, offers the best BER performance.

Significant spreading gain can be observed in Figure 6.6. For example, to achieve
BER of $10^{-3}$ and better, the maximum CFO tolerable is $\epsilon = 0.11$ without spreading. With
spreading, the tolerable CFO goes up to 0.2 for $L = 4$ and up to 0.25 when $L = 8$.

We also compare the spreading gain in frequency selective fading channels. The
channels we use are generated from indoor, non-line-of-sight IEEE 802.15.4 channel
models. From the results shown in Figure 6.7, we can see that all scheme suffer perfor-
ance loss. The spreading gain is still significant but reduced. This is attributed to the
fact that the orthogonality is not maintained any more when the gain of each subcarrier
is different. The gain is still directly related to the diversity degree.

Both Figure 6.6 and 6.7 show high spreading gain in the low CFO regime and it slowly
reduces as $\epsilon$ grows and in both cases, approaches 0 when $\epsilon > 0.3$. 

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Figure 6.7: BER of the OFDM system vs. CFO in indoor NLOS channels
6.5 SUMMARY

In addition to OFDM systems, we simulate the performance in OFDMA systems. A total of 256 subcarriers are assigned to multiple users. Each user has an independent clock with CFO randomly distributed within $[-\frac{\epsilon}{2}, \frac{\epsilon}{2}]$. Simulation results shows similar BER improvement.

6.5 Summary

This chapter proposed an ICI cancelation scheme for OFDM/OFDMA systems, which spreads the redundant data subcarriers with orthogonal or quasi-orthogonal codes. We use PSO to find a set of orthogonal code for this design which works perfectly. We have to emphasize the benefit of using PSO here again, since this design provides us with good codes in very short time. We also present design details of both the transmitter and receiver and analysis on the spreading gain in terms of CIR improvement. Theoretical analysis and simulations are given in the chapter as well. The numerical results confirm that for a given BER requirement, designs using the proposed ICI cancelation scheme are twice or more tolerant to carrier frequency offset.
Chapter 7

PSO Based PAPR Reduction Method

This chapter considers the use of a two stage method to reduce the peak to average power ratio (PAPR) of a frequency redundant OFDM system. At the first stage, the subcarriers are divided into smaller clusters, we apply random phase rotation and another specific manipulation on clusters of subcarriers. At the second stage, we treat each cluster of subcarriers as a group and propose a PSO phase optimization technique in reducing the PAPR. The conventional selective mapping (SLM) and partial transmit sequence (PTS) technique are highly successful in PAPR reduction for OFDM signals. However, both methods need to find the optimal phase factors, which is a complex, non-linear optimization problem. The considerable computational complexity for the required search through a high-dimensional vector space is problematic for practical implementation. Moreover, the conventional PTS requires an exhaustive search from all combinations of allowed phase factors. It turns out that search complexity increases exponentially with the number of subblocks. To reduce the search complexity while still improving the PAPR statistics, stochastic search techniques have recently been proposed [63]-[64]. They can obtain the desirable PAPR reduction with low computational complexity. In our method, at the second stage, PSO is used to search for phase factors that reduce both
the PAPR statistics and the computational load.

7.1 Introduction

OFDM is an attractive technique for achieving high data rate while combating with the frequency selective fading channel. However, it is well known that uncoded OFDM does not provide any frequency diversity. Adding frequency diversity by modulating the same information bit on multiple interleaved subcarriers is an effective way to further mitigate the effect of frequency-selective fading as well as an enhancement to the system signal to noise ratio (SNR), which leads to a more robust system.

One of the major disadvantages of OFDM systems, especially for the frequency redundant design, is the high peak-to-average power ratio (PAPR) of the transmitted signals, which requires expensive high power amplifier with a large linear range. Fig. 7.1 show the concepts of how PAPR problem is created in a simple OFDM system.

Figure 7.1: A simple example of PAPR problem
In addition, large PAPR also demands AD converters with large dynamic ranges. In order to reduce the PAPR, a number of approaches have been proposed \[65\][66]. Deterministic method such as clipping the OFDM signal before amplification is the most intuitive and basic method that limits the PAPR within a given threshold. However, this method causes performance degradation and creates out-of-band radiation \[67\]. In comparison, probabilistic schemes statistically improve the characteristics of the PAPR distribution without signal distortion. Selective mapping (SLM) and partial transmit sequence (PTS) belong to this category. Conventional SLM pre-generates a number of statistically independent sequences from the same data, and chooses the one with the lowest PAPR to send out \[68\]. PTS divides the subcarriers into a set of disjoint subblocks or continuous clusters, each subblock or cluster of subcarriers is multiplied by different phase factors, the subblocks/clusters are then added to form the different OFDM symbols. The phase factor that generates the time domain OFDM signal which has the lowest PAPR is chosen for transmission \[69\]. Both SLM and PTS techniques can be considered as multiple signal representation methods, one favorable OFDM symbol is selected from a very large set of statistically independent symbols. For both techniques, a large amount of IFFT calculation and complex multiplication with phase sequences are required, in proportion to the number and length of the phase sequences used. For example, optimally PTS may require an exhaustive search over all the possible phase factor combinations, whose algorithm complexity is exponential. Then for an OFDM system that has significantly large number of subcarriers, the required computational load and hardware complexity can become prohibitively high.

In this chapter, we propose a two stage PAPR reduction method. At the first stage, we apply phase rotation on one set of the cluster of subcarriers and map it strategically on the OFDM subcarriers. At the second stage, we treat each cluster of subcarriers as a group and use a method similar to SLM to generate the favorable OFDM symbol for
transmission.

7.2 PAPR Problem and Conventional SLM and PTS

An OFDM transmitter reads in data to be transmitted in blocks. Each data block can be represented by a size-$Q$ vector, $A = [a_0, a_1, \ldots, a_{Q-1}]$, where $a_i$, $(0 \leq i \leq Q - 1)$ is a complex number representing a modulation alphabet based on a particular modulation scheme (e.g., PSK, QAM, etc.). A mapping function, $\mathcal{P}(\cdot)$, maps input data in $A$ to a size-$N$ vector, $S = [S_0, S_1, \ldots, S_{N-1}]$.

$$S = \mathcal{P}(A), \quad (7.1)$$

where $N$ is the number of subcarriers in an OFDM symbol. In a conventional OFDM system, there is no subcarrier redundancy, so we have $N = Q$, $S_i = a_i$ $(0 \leq i \leq N - 1)$. $S$ is referred as frequency domain symbol. The time domain OFDM signal $s(t)$ is obtained by the inverse fast Fourier transform (IFFT) given by:

$$s(t) = \mathcal{F}^{-1}(S) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \cdot e^{j \frac{2\pi k t}{T}}, \quad 0 \leq t \leq T, \quad (7.2)$$

where $T$ is the OFDM signal duration. In practice, a cyclic prefix (CP) is added to the signal $s(t)$ in order to avoid the inter-symbol interference (ISI) that occurs in multipath channels. Since the CP does not impact the PAPR, we ignore it [70]. Because of central limit theorem and the fact that IFFT is a linear operation, the transmitted OFDM signal $s(t)$ follows complex Gaussian distribution when the number of subcarriers $N$ is large.
7.2. PAPR PROBLEM AND CONVENTIONAL SLM AND PTS

The PAPR of OFDM is given by:

\[
PAPR(s(t)) = \max \frac{|s(t)|^2}{E\{|s(t)|^2\}}
\]  

(7.3)

where \(E\{\cdot\}\) denotes the expectation or a statistical average operator. In the literature, the complementary cumulative distribution function (CCDF) is used to evaluate the PAPR reduction performance. The CCDF of the PAPR is given in [66] as

\[
Pr(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N
\]  

(7.4)

7.2.1 Selective Mapping Scheme

SLM is a simple PAPR suppression method for OFDM signals. In the classical SLM technique, frequency domain symbol block \(S\) is multiplied element by element with \(U\) phase rotation vectors (also known as scrambling sequences) \(r^{(u)} = [e^{j\phi_0^{(u)}}, \ldots, e^{j\phi_{N-1}^{(u)}}], (u = 1, \ldots, U)\), resulting in a set of \(U\) different sequences with each entry being

\[
S_k^{(u)} = S_k e^{j\phi_k^{(u)}}, \; k = 0, 1, \ldots, N - 1.
\]  

(7.5)

All \(U\) sequences are usually oversampled by a factor \(L\) [65] and then transformed into the time domain by IFFT. The time domain sequence with the lowest PAPR is selected for transmission. Consequently, the SLM technique is required to perform \(U\) IFFT operations to generate these \(U\) candidates at the transmitter. Since the number of information bits \(k = N\), the explicit side information bits about the selected phase rotation vectors should be transmitted reliably such that the original codeword \(S\) can be recovered at the receiver.

The performance of PAPR reduction in SLM strongly depends on the number and the selection of the phase rotation vectors.
7.2. PAPR PROBLEM AND CONVENTIONAL SLM AND PTS

In the SLM implementation shown in Fig. 7.2, $U$ transmit sequences are produced by multiplying the information sequence by $U$ random sequences of length $N$. If the CCDF of the original sequence is $Pr(PAPR > PAPR_0)$, then, the CCDF of the best of the $U$ sequences will be $[Pr(PAPR > PAPR_0)]^U$. Thus, in theory, the probability of the PAPR exceeding some threshold can be made as small as possible at the expense of added complexity (i.e., additional IFFTs).

7.2.2 Partial Transmit Sequence Scheme

PTS method [69] divides the input frequency domain symbol $S$ into $M$ disjoint subblocks or clusters consisting of a contiguous set of subcarriers, $\{\bar{S}_m|m = 0, 1, \cdots, M - 1\}$. After zero padding at corresponding positions, each subblock $\bar{S}_m$ becomes a length-$N$ vector, $\bar{S}_m = [S_{m,0}, S_{m,1}, \cdots, S_{m,N-1}]$ satisfy that $S = \sum_{m=0}^{M-1} \bar{S}_m$ and $S_i \cdot S_j = 0 \ (n = 0, 1, \cdots, N - 1)$ when $i \neq j, \ (i, j \in \{0, \cdots, M - 1\})$. Through this process, the original
vector $S$ turns to a $M \times N$ matrix. Let the partial transmit sequence $s_m$ of length-$N$ be the IFFT of the subblock $S_m$, we have the time domain transmitted sequence:

$$s = \text{IFFT}(S) = \sum_{m=0}^{M-1} s_m.$$  \hfill (7.6)

Applying phase factors to subblocks/clusters allows optimization of combining partial transmit sequences. The combined sequence is:

$$s = \text{IFFT} \left( \sum_{m=0}^{M-1} b_m S_m \right) = \sum_{m=0}^{M-1} b_m s_m$$  \hfill (7.7)

where $\{b_m = e^{j\phi_m}, \ m = 0, \ldots, M - 1\}$ is the phase rotation factor, each factor is applied to one subblock/cluster. Assume $\phi_m \in \{2\pi \omega / W, \ \omega = 0, \ldots, W - 1\}$, then there will be $W^M$ possible unique sets of phase factors to choose from. One selection approach is that we exhaustively try all the possible phase rotation factors and choose the sequence gen-
7.3 FREQUENCY REDUNDANT OFDM SYSTEM

erated with the lowest PAPR, but the computational complexity of this method increase exponentially \( M \). Another much simpler approach is to randomly generate \( U \) phase rotation vectors \( b^{(u)} = [b_1^{(u)}, \ldots, b_M^{(u)}] \) \( (u = 1, \ldots, U) \) to apply on \( S_m \) and choose \( s^{(u)} \) with the lowest PAPR. A PTS transmitter is shown in Fig. 7.3.

7.3 Frequency Redundant OFDM System

The OFDM system we consider here is a frequency redundant system, which utilizes the frequency diversity across OFDM subcarriers. Since the coherent bandwidth in most of the wireless channels is much greater than the subcarrier-spacing and therefore each subcarrier is subject to deep fading. Frequency diversity is introduced in OFDM systems to mitigate this. An easy and convenient way to provide such frequency diversity is to map each input symbol, \( a_n \), to multiple subcarriers \([71]\),

\[
S_k = a_n, \quad \forall k \in \mathcal{G}_n = \{k_0, k_2, \ldots, k_{D-1}\}. \tag{7.8}
\]

\( D \) is the degree of frequency diversity, and \( \mathcal{G}_n \) is the set of subcarriers assigned to \( a_n \). To maximize the frequency diversity, it is essential that the subcarriers assigned to the same input data are spread across the entire band. This can be achieved when an interleaving subcarrier mapping scheme is used. In a generic OFDM system which has no non-data subcarriers, for a size-\( Q \) input vector, a mapping function would be as follows:

\[
S = \mathcal{P}(A) = [a_0, \ldots, a_{Q-1}, a_0, \ldots, a_{Q-1}, \ldots, a_0 \cdots a_{Q-1}] \\
= [\hat{S}_1, \hat{S}_2, \ldots, \hat{S}_D] \tag{7.9}
\]
where \( \hat{S}_i \) (\( 1 \leq i \leq D \)) stands for the \( i \)th subcarrier cluster. The mapping function given in (7.9) maps \( Q \) inputs to \( D \times Q = N \) subcarriers. The \( q \)th input \( a_{q-1} \) is mapped to \( D \) subcarriers, \( \{ k_0 = q - 1, \ldots, k_{D-1} = (D - 1)Q + q - 1 \} \). Subcarriers carrying the same data have a minimum separation of \( Q \) subcarriers spacing. The advantage of this design is that it make use of the frequency diversity to mitigate the effects from the frequency selective channel, when transmitted signals on some frequency band are affected by the channel and can not be detected, signals on other spectrum bands can still be received by the receiver correctly.

Another obvious advantage of frequency diversity for an OFDM system is the improvement of the system BER due to the signal diversity gain. With redundant subcarrier, signal to noise ratio (SNR) becomes the ratio of the total power of signal subcarriers for the same input symbol to the noise. Assuming the simplest additive white Gaussian noise channel and equal gain combining receiver, the SNR is \( D \) times of the non-frequency redundant system.

Except for the above advantages and the trade off between robustness and the data rate loss, there is a disadvantage of this frequency redundant OFDM system - its high PAPR. From the probability perspective, if there are \( D \) sets of subcarriers carrying the same data, the probability of having a high peak in time domain is much higher due to the dependency of the signal in frequency domain [65].

Generally, for this frequency redundant OFDM system, the time domain baseband signal can be written as in (7.2). By sampling the above signal \( s(t) \) with sampling interval
\[ \Delta t = T_s/N, \] we get discrete time domain signal as
\[
s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \cdot e^{j2\pi kn/N} = \frac{1}{\sqrt{N}} \sum_{q=0}^{Q-1} a_q \sum_{d=0}^{D-1} e^{j2\pi n(dQ+q)/N} = \frac{1}{\sqrt{N}} \sum_{q=0}^{Q-1} a_q \cdot e^{j2\pi n_q/N} \sum_{d=0}^{D-1} e^{j2\pi ndQ/N}. \tag{7.10} \]

We can see how redundancy affects the OFDM signal’s PAPR from this equation. \( \frac{1}{\sqrt{N}} \tilde{s}(n) \) is the scaled periodic extension of the IFFT of \( \tilde{S}_i \) and \( \zeta_D \) is the IFFT of a length-\( N \) vector \([1, 0, \cdots, 0, \cdots, 1, 0, \cdots, 0]\). Fig. 7.4 shows one example of the amplitude of \( \zeta \). We can see that due to the dependency of the subcarriers, \( \zeta \) periodically raise the amplitude of \( \tilde{s}(n) \). Clearly this subcarrier dependency affects the PAPR of the OFDM signal. In the following section, we propose a two stage phase rotation method to change the probabilistic behavior of the PAPR of this design.

![Figure 7.4: Amplitude of \( \zeta \) (N=128, D=4, Q=32)](image)
7.4 A Two Stage PAPR Reduction Method

We propose a two stage PAPR reduction method. As shown in Fig. 7.5, after the modulation, we have a vector of length-$Q$. This is a set in (7.9), that carry one time of the original input data. Due to the frequency redundancy in our design, the same set of input data will be mapped on $D$ clusters of subcarriers to compose the length-$N$ OFDM symbol.

7.4.1 The First Stage

In our PAPR reduction method, we first apply the phase rotation on this subblock before mapping it on the $D$ subcarrier clusters. In this case, the chosen phase rotation vector $p^{(u)}$ only needs to have $Q$ components. The phase rotation sequence is generated using the unit-magnitude complex number. For convenience, binary ($\{\pm 1\}$) or quaternary elements ($\{\pm 1, \pm j\}$ or $\{\pm \sqrt{2}, \pm j\sqrt{2}\}$) are usually used for elements of $p^{(u)}$.

![Figure 7.5: Block diagram of the proposed design](image_url)
Different subcarrier clusters contain the same information, hence, in order to avoid accumulated components of particular phase which might produce excessive peak power signal in time domain, we use a simple alternative signal allocation method. We convert one of the adjacent clusters to the conjugate of themselves \((\hat{S}_d, \hat{S}_{d+1} = -\hat{S}_{d+1}^*)\). By doing this, the phase difference between two adjacent clusters varies with respect to the original input data symbols themselves. Thus, the dependency between the different clusters reduced.

After this stage, the input frequency symbol \(S\) in (7.9) turns to
\[
\tilde{S}^{(u)} = p^{(u)}\cdot [\hat{S}_1, -\hat{S}_2^*, \ldots, \hat{S}_{D-1}, -\hat{S}_D^*]. \tag{7.11}
\]

### 7.4.2 The Second Stage

Then we enter the second stage. At this stage, we need to treat each cluster as a group and rotate every cluster by one rotation factor. Now \(\tilde{S}^{(u)}\) can be expressed as
\[
\tilde{S}^{(u)} = p^{(u)}\cdot [\hat{S}_1 \cdot b_1^{(u)}, -\hat{S}_2^* \cdot b_2^{(u)}, \ldots, \hat{S}_{D-1} \cdot b_{D-1}^{(u)}, -\hat{S}_D^* \cdot b_D^{(u)}]. \tag{7.12}
\]

After choose the rotation factors, in order to obtain an improved approximation of the true PAPR in the discrete-time signal, we need oversample the candidate signals. An oversampling rate of \(L\) for the system can be achieved by inserting \((L - 1) \cdot N\) zeros in the middle of the encoded symbol vectors. Thus, \(\tilde{S}^{(u)}\) becomes
\[
\tilde{S}^{(u)} = p^{(u)}\cdot [\hat{S}_1 \cdot b_1^{(u)}, -\hat{S}_2^* \cdot b_2^{(u)}, \ldots, -\hat{S}_{D/2}^* \cdot b_{D/2}^{(u)}, 0, \ldots, 0, \underbrace{0, \ldots, 0}_{(L - 1) \cdot N}]
\hat{S}_{D/2+1} \cdot b_{D/2+1}^{(u)}, \ldots, \hat{S}_{D-1} \cdot b_{D-1}^{(u)}, -\hat{S}_D^* \cdot b_D^{(u)}]. \tag{7.13}
\]
7.4. A TWO STAGE PAPR REDUCTION METHOD

To generate signal representations, we get the time-domain signal by using IFFT on \( \tilde{S}(u) \)

\[
\tilde{s}^{(u)} = \text{IFFT}(\tilde{S}(u)).
\]  
(7.14)

Then the selecting can be mathematically expressed as

\[
\tilde{s} = \text{argmin}\{PAPR(\tilde{s}^{(u)})\}
\]  
(7.15)

Finally, the transmitter selects the most favorable time domain signal \( \tilde{s} \) with the lowest PAPR for transmission.

Note that during this stage, the phase rotation factors \( B^{(u)} = [b_1^{(u)}, b_2^{(u)}, \cdots, b_D^{(u)}] \) can be chosen freely within \([0, 2\pi)\). Traditionally, the selection of the phase rotation factors is limited to a set with finite number of elements to reduce the search complexity. The best phase rotation factor set among the available sets that minimizes the PAPR can be obtained from an exhaustive simulation of all possible combinations.

However, since we are choosing discrete numbers in a continuous space, in theory, there is unlimited number of possibilities to get an arbitrary point in the space. This resembles the analog to digital conversion in signal processing, the smaller the resolution is, the more candidate points we can get, thus the more possible combinations exist. In other words, \( b_i^{(u)} = e^{j\phi_i^{(u)}} \), and \( \phi_i^{(u)} = 2\pi/W, i = (0, 1, \cdots, D-1) \) and \( W \) can be an arbitrary large integer.

The optimization problem of PAPR at this stage can be considered as a combinatorial optimization problem. In other words, the objective function 7.16 is to minimize the PAPR of the transmitted OFDM signals.

To minimize

\[
f(B) = \text{argmin}\{PAPR(\tilde{s}^{(u)})\}
\]  
(7.16)
Subject to
\[ b_m = \{ e^{j\phi_m} \} \]  
(7.17)

where \( \phi_m \in \{ \frac{2\pi k}{W} | k = 0, 1, \cdots, W-1 \} \).

We propose a PSO based algorithm here to achieve better PAPR reduction with low complexity. The original PSO algorithm was designed for a problem with continuous parameters. Since the optimization parameters (rotation factors) are discrete, another way to tackle discrete parameters is to apply the PSO algorithm modified for binary parameters - Binary PSO (we already introduced in Chapter 3).

The transformation function is a sigmoid limiting function as in equation (3.2), we rewrite it here:
\[ \sigma(v_{id}) = \frac{1}{1 + \exp(-v_{id})} \]  
(7.18)

whose range is (0, 1). A random number uniformly distributed over [0, 1] is generated to be compared with \( \sigma(v_{id}) \), the location \( x_{id} \) is updated by the following formula:
\[ x_{id} = \begin{cases} 1 & \text{random} < \sigma(v_{id}) \\ 0 & \text{else} \end{cases} \]  
(7.19)

Other than above, the rest of Binary PSO is the same as the canonical PSO. Suppose the range of parameter \( |v_{id}| \) is [0, \( n \)], then the change probability of particle \( i \) is defined as
\[ \text{swap}(v_{id}) = \frac{|v_{id}|}{n} \]  
(7.20)

which is constrained to the interval [0, 1]. A random number \( \rho_{id} \) uniformly distributed over [0, 1] is generated to be compared with \( \text{swap}(v_{id}) \), if \( \rho_{id} < \text{swap}(v_{id}) \), then the value of particle \( i \) is changed with the value of global best location in corresponding position, as shown in Figure 7.6. Therefore, in the update process, the solution will
always approaches to the global optimum solution.

\[
\begin{array}{ccccccc}
\frac{v}{n} & \vdots & 1.0 & 0.1 & 0.2 & 0.8 & 0.5 & \vdots \\
\gamma_{id} & \vdots & 1 & 1 & 0 & 0 & 1 & \vdots \\
\chi_{id}^k & \vdots & 0 & 1 & 1 & 1 & 1 & \vdots \\
\chi_{id}^k & \vdots & 1 & 1 & 1 & 0 & 1 & \vdots \\
\end{array}
\]

Figure 7.6: Particle position update in Binary PSO

For our problem, during the PSO process, for a \(D\)-dimensional optimization, the position and velocity of the \(i^{th}\) particle can be represented as \(B^{(i)} = [b_1^{(i)}, b_2^{(i)}, \ldots, b_D^{(i)}]\) and \(V_i = V_{i,1}, V_{i,2}, \ldots, V_{i,D}\), respectively. Each particle has its own best position corresponding to the individual best objective value obtained so far. The global best \((g_{best})\) particle represents the best particle found so far in the entire swarm. The velocity updating follows equation (3.1) and the new position for particle \(i\) is computed according to equation (3.2). The populations of particles are then moved according to the new velocities and locations and tend to cluster together from different directions. Thus, the evaluation of each associate fitness of the new population of particles begins again. The algorithm runs through these processes iteratively until it stops. The termination criteria can be either a predefined PAPR threshold or a preset PSO iteration number.

We can see that compared with SLM and PTS, this scheme rotates the subcarriers
twice in two stages instead of only one conventionally. The increased freedom of rotation can further randomize the phase of different subcarriers. The PSO-assisted second stage exploits heuristics to search the optimal combination of phase factors with low complexity. After the two phase rotation stages, the subcarrier clusters are then mapped in cascade to form the length-$N$ OFDM symbol.

# 7.5 Simulation Results

We conduct a series of simulations to evaluate the proposed scheme on its PAPR reduction performance. The simulation system is set up as follows: the QPSK modulation is used, number of subcarriers $N$ span from 128 to 2048, the OFDM signal is oversampled by a factor of $L = 4$. In order to generate the complementary cumulative distribution function (CCDF) of the PAPR, 10000 random OFDM frames have been generated. For PSO, we use 20 particles and iteration number is 30 times.

For simplicity, the elements of the phase sequence $p^{(u)}$ in traditional SLM and $b^{(u)}$ in traditional PTS are randomly chosen from set $\{\pm 1, \pm j\}$. We ignore the cyclic prefix and pilots in the OFDM subcarriers. The frequency diversity $D$ is assigned by 4 and 8, respectively. We run the regular OFDM systems with same number of subcarriers which have no frequency redundancy to compare with our frequency redundant designs. We also compare with the so called “Optimal PTS” (OPTS) technique, but within a limited set of $\{\pm 1, \pm j, \pm \pi/2, \pm \pi/4, \pm 3\pi/4\}$. The OPTS technique requires an exhaustive search over all the combinations of the allowed phase factors. However, in our PSO-based method, we use the set of $\{\pm 1, \pm j, \pm \pi/2, \pm \pi/4, \pm 3\pi/4, \pm 5\pi/8, \pm 3\pi/8, \cdots, \pm 15\pi/16\}$ as the candidates for phase rotation factors.

Fig. 7.7 through Fig. 7.14 show the CCDF of PAPR for different methods. It can be seen that the PAPR of the original redundant OFDM signal is the largest which match our
7.5. SIMULATION RESULTS

Figure 7.7: Comparison of PAPR reduction performance, subcarrier number N = 128 redundant degree D = 4

Figure 7.8: Comparison of PAPR reduction performance, subcarrier number N = 128 redundant degree D = 8
7.5. SIMULATION RESULTS

Figure 7.9: Comparison of PAPR reduction performance, subcarrier number \( N = 256 \) redundant degree \( D = 4 \)

Figure 7.10: Comparison of PAPR reduction performance, subcarrier number \( N = 256 \) redundant degree \( D = 8 \)
7.5. SIMULATION RESULTS

Figure 7.11: Comparison of PAPR reduction performance, subcarrier number
N = 1024 redundant Degree D = 4,

Figure 7.12: Comparison of PAPR reduction performance, subcarrier number
N = 1024 redundant degree D = 8,
7.5. SIMULATION RESULTS

Figure 7.13: Comparison of PAPR reduction performance, subcarrier number
N = 2048 redundant degree D = 4

Figure 7.14: Comparison of PAPR reduction performance, subcarrier number
N = 2048 redundant degree D = 8
guess based on the theory. We also simulate regular OFDM systems with same number of subcarriers, which have no frequency redundancy to compare with our frequency redundant designs. These figures verify that redundancy does affect the PAPR greatly.

In Fig. 7.7, for example, the performance of different techniques are shown, to compare the performance of PAPR reduction with proposed technique. Note that traditional PTS and SLM use 16 time random choose, which means $U = 16$. After 16 random generations, they pick the signal which has the lowest PAPR to send out. We can see that when $Pr(PAPR > PAPR_0) = 10^{-4}$, the $PAPR_0$ of OFDM with redundant degree of 4 is 16.7dB, PTS with $M = 16$ is 12.3dB, PTS with $M = 32$ is 11.7dB and OPTS is 9.2dB with $M = 16$ and factor number 16 ($\{\pm1, \pm j, \pm \frac{\pi}{4}, \pm \frac{11\pi}{4}, \pm \frac{3\pi}{4}\}$). And PSO-based method is 8.8dB. It is evident that the PSO-based method can provide the better performance of PAPR reduction.

We know that the OPTS need $2^{16} = 65536$ times of IFFT operations which is prohibitively expensive to get a good result, meanwhile our PSO based method are using a much larger set to get the rotation factor $\{\pm1, \pm j, \pm \frac{\pi}{2}, \pm \frac{11\pi}{8}, \pm \frac{3\pi}{8}, \cdots, \pm \frac{15\pi}{16}\}$. This is almost impossible for OPTS to use since it needs $2^{32} = 4294967296$ IFFT operations to exhaustively cover the whole set. However, our propose approach still get a better result. In the following figures, we can find some trends:

* PAPR grows with subcarrier number. When $N = 2048$, the redundant OFDM (redundant degree = 8) has a PAPR of more than 20dB.

* PAPR also grows with the redundant degree, the higher the redundant degree, the bigger the PAPR which matches our previous analysis.

* In all cases, our proposed PSO-based method performs closely but better than the OPTS methods, which shows our methods advantage.
* Due to the low complexity of PSO methods, when the rotation factor set is big and number of subblock is big, when it is prohibitively expensive to do an exhaustive search, PSO-method can still render a good result.

7.6 Summary

In this chapter, a PSO based PAPR reduction method for frequency redundant OFDM system is proposed to search the suboptimal combination of phase factors to reduce the complexity. We formulate the phase rotation factor search of the problem as a discrete optimization problem with bound constraints and design a binary PSO based algorithm for it. Simulation results demonstrate that the performance of the proposed method has slightly better performance compared to that of OPTS. However, the complexity of the proposed method was remarkably lower than that of OPTS. In one word, it can achieve the good tradeoff between PAPR performance and complexity compared with the conventional PTS techniques.
Chapter 8

Conclusions and Future Work

Research is an iterative process very similar to the one modeled by the particle swarm. Researchers keep testing ideas based on their previous successes and the successes observed in other researchers in the area. The work in this dissertation is no exception.

8.1 Concluding Remarks

In this dissertation we developed particle swarm optimization based methods for problems in wireless communication area. The aim is to provide intelligent tools to the system designers to apply to real world problems. The PSO based algorithms we designed provide a comprehensive treatment to the problems encountered in wireless communications. They are adaptable and can be applied to different problem structures and problem types. Particle swarm optimization can be and has been used across a wide range of applications by many other researchers in the world. Areas where PSOs have shown particular promise include multimodal problems and problems for which there is no specialized method available or all specialized methods give unsatisfactory results. From the data of 2010, in the IEEE Xplore database there are around 4500 papers (286
of which are journal papers) can be classified as applications of PSO, although many of these also involve the customization or extension of the method to best suit the specific application of interest.

To sum up the advantages of PSO, we list its three attracting feathers here:

**Simplicity** The particle swarm optimization algorithm uses simple operators like adds, and multipliers to travel through the search space. The algorithm structure only requires just five simple steps;

**Independence from the objective function** The swarm treats the objective function as a black-box. It queries the objective function to estimate the performance of each particle in the swarm. This clear separation from the objective function, enables the swarm to be used for a variety of problems without any additional steps;

**Easy control** PSO has limited number of parameters. In the basic PSO, only the size of the population, two inertial coefficients and the bound of the velocity need to be fixed before running a PSO. Also, PSO is less sensitive to parameters. PSO is also less dependent on initial points. Contradict to some other methods, randomly picked initial positions can still lead to convergence for PSO.

PSO algorithms have been applied to optimization problems ranging from classical problems such as scheduling, the traveling salesman problem, neural network training, and task assignment, to highly specialized applications such as reactive power and voltage control, biomedical image registration, and even music composition. Our work can be categorized into highly specialized applications, in all applications we achieve provide successful solutions and get good results. Meanwhile, new problems and new methods in wireless communication and signal processing field are appearing from time to time. It seems not surprised at all that some of the results presented during the course of this
work are superseded by other results presented later. We hope the work presented in this thesis may help other researchers explore new aspects in this area that were not considered before.

8.2 Future Works

Despite its apparent simplicity, the PSO presents formidable challenges to those interested in understanding swarm intelligence through theoretical analysis. So, to date a fully comprehensive mathematical model of particle swarm optimization is still not available. We would like to explore the theoretical side of PSO algorithm and gain more insight on PSO. The research on PSO and its application in the following directions are going to be useful:

8.2.1 Mathematical Explanation of PSO Algorithm

Although PSO's application has been proved to be effective, its theoretical foundation is rather weak. Clerc and Kennedy [72] make a analysis on the convergence of the method from the point of maths. By analyzing the stability of the condition transmitting matrix, they find the limited conditions where the particle can move stably. Based on this, Bergh makes the further analysis on it. Lebesgue and Borel explore the effect of casualty on the locus of the particle, and analyze the convergence from the point of measuring space. Still, there is no mathematically proved about the convergence and the speed of the convergence. The most optimist solution of PSO can not be ensured in theory.
8.2. FUTURE WORKS

8.2.2 Hybridization of PSO with other intelligent optimization algorithm

Another important area of active research is the hybridization of PSO with other computational intelligence techniques. This is often used to solve complex real-world problems where one technique is typically used to fix the weaknesses of the other. Another popular paradigm of swarm intelligence is called ant colony optimization (ACO). The algorithm is inspired by the stigmergistic communication system employed by ants to evaluate alternative choices and take decisions in dynamic optimization problems. We are particularly interested in combining PSO and ACO to see if the hybridization can lead to much more effective algorithms. Blending PSO with ACO means combining the advantages of the PSO with the advantages of ACO to create the compound algorithm that has more practical value.

8.2.3 Expanding the application area of PSO

In wireless communication area, issues of node deployment, localization, energy-aware clustering, and data-aggregation are often formulated as optimization problems. Most analytical methods suffer from slow or lack of convergence to the final solutions. This calls for fast optimization algorithms that produce quality solutions utilizing less resources. PSO has been a popular technique used to solve optimization problems in wireless sensor networks due to its simplicity, high quality of solution, fast convergence and insignificant computational burden. Although the PSO algorithm has been used widely, it will be very meaningful to explore the developing area further. Areas in wireless sensor networks and mobile networks that PSO suits are cross-layer optimization, multi-target tracking, heterogeneous resources allocation, multi-objective optimized routing and so on.
More generally, most research on PSO aim at the coordinate system nowadays. Although in practical usage, it is used in non-coordinate system, scattered system and compound optimization system, there is less research on the PSO algorithm application in these systems.
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4. X. Ye, W. Gao, Y. Yan and L. Osadciw, “Multiple Tests for Wind Turbine Fault Detection and Score Fusion Using Two - level Multidimensional Scaling”, in proceeding


AWARDS AND HONORS

• Nunan Award (Electrical Engineering) 2008

• Travel Grant in SIS’09, 2009