12-11-1997

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June, 1997

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Abstract

We calculate the next–to–leading order contribution to the masses of the heavy baryons in the bound state approach for baryons containing a heavy quark. These $1/N_C$ corrections arise when states of good spin and isospin are generated from the background soliton of the light meson fields. Our study is motivated by the previously established result that light vector meson fields are required for this soliton in order to reasonably describe the spectrum of both the light and the heavy baryons. We note that the inclusion of light vector mesons significantly improves the agreement of the predicted hyperfine splitting with experiment. A number of aspects of this somewhat complicated calculation are discussed in detail.

PACS numbers: 12.39.Dc, 12.39.Fe, 12.39.Hg

Keywords: Heavy spin symmetry, $1/M$ expansion, Skyrmions, heavy quark solitons, collective quantization, hyperfine splitting

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1. *Introduction*

The development of the heavy quark or Isgur-Wise symmetry [1] has stimulated a great deal of interest in studying [2]–[7] properties of heavy baryons (i.e., those with the quark structure qqQ) in the bound state approach. In this picture the heavy baryon is treated as a heavy spin multiplet of mesons (with structure $Q\bar{q}$) bound in the field of the nucleon (qqq) which itself emerges as a soliton configuration of light meson fields. This treatment is suggested by the $1/N_{\rm C}$ expansion [8] of QCD. Recent reviews of the soliton approach to the light baryons are given in refs [9]–[12] while the bound state treatment of the "light" hyperons is discussed in refs [13, 14].

A compelling feature of this approach is that it permits, in principle, an exact expansion of the heavy baryon properties in simultaneous powers of $1/M$, $1/N_{\rm C}$ and, since it is based on a chiral Lagrangian, number of derivatives acting on the light components of the heavy system. In practice there are obstacles related to the large number of unknown parameters which must be introduced. Rather than treating the light soliton in a model with many derivatives of the light pseudoscalar fields it turns out to be much more efficient to use the light vector mesons. Based on a model [15] of the light vector interactions with the heavy multiplet, the leading order (in the $1/N_{\rm C}$ and $1/M$ expansions) heavy baryon mass splittings have been discussed [16], obtaining satisfactory agreement with experiment. Actually the need for light vector mesons is not surprising since, in the soliton approach, they are necessary to explain, for example, the neutron–proton mass difference [17] and the nucleon axial singlet matrix element [18].

In the present paper we focus our attention on the hyperfine splitting, which is of subleading order both in $1/M$ and $1/N_{\rm C}$. This is a more complicated calculation and also involves using a cranking procedure [19] to obtain physical states which carry good spin and isospin quantum numbers. The first calculation of the heavy baryon hyperfine splitting in the perturbative bound state framework was carried out by Jenkins and Manohar [2] who got the formula

$$m(\Sigma_Q^*) - m(\Sigma_Q) = \frac{(m(\Delta) - m(N))\,(M^* - M)}{4d\,F'(0)}\ , \tag{1.1}$$

where $M^* - M$ is the heavy vector–heavy pseudoscalar mass difference, d is the light pseudoscalar–heavy meson coupling constant and $F'(0)$ is the slope of the Skyrme "profile function" at the origin. This formula is obtained (see also section 5) by using the leading order in number of derivatives (zero) and leading order in $1/M$ heavy spin violation term. Therefore it is expected to provide the dominant contribution. Unfortunately, on evaluation, it is found to provide only a small portion of the experimental Σ_c^*–Σ_c masss difference. This naturally suggests the need for including additional higher order in derivative heavy spin violation terms. However, there are many possible terms with unknown coefficients so that the systematic perturbative approach is not very predictive.

To overcome this problem we employ a relativistic Lagrangian model [15] which uses ordinary heavy pseudoscalar and vector fields rather than the heavy "fluctuation" field multiplet [1]. This model reduces to the heavy multiplet approach in leading order and does not contain any new parameters. In a recent note [20] we showed that such a model (considered, for simplicity, to contain only light pseudoscalars; *i.e.*, the light part is the original Skyrme model [21]) yielded a "hidden" heavy spin violation which is not manifest from the form of the Lagrangian itself. This hidden part involves two derivatives and is actually more important numerically than the zero derivative "manifest" piece which leads to eq (1.1). However this new result is still not sufficient to bring the predicted Σ_c^*–Σ_c mass difference into agreement with experiment. The prediction for this difference is actually correlated to those for Σ_c–Λ_c and Δ–N, the Δ - nucleon mass difference by [13]:

$$m\left(\Sigma_c^*\right) - m\left(\Sigma_c\right) = m\left(\Delta\right) - m\left(N\right) - \frac{3}{2}\left[m\left(\Sigma_c\right) - m\left(\Lambda\right)\right] \ . \tag{1.2}$$

This formula depends only on the collective quantization procedure being used rather than the detailed structure of the model. If $m\left(\Sigma_c\right) - m\left(\Lambda\right)$ and $m\left(\Delta\right) - m\left(N\right)$ are taken to agree with experiment, eq (1.2) predicts 41 MeV rather than the experimental value of 66 MeV. This means that it is impossible to exactly predict, in models of the present type, all three mass differences which appear in eq (1.2). The goodness of the overall fit must be judged by comparing all three quantities with experiment. Our focus, of course, is the left hand side of eq (1.2) which is of order $1/M$ while the right hand side involves the difference of two order M^0 quantities. A similar calculation in the model with only light pseudoscalars was carried out by Oh and Park [22]. However, they did not make a $1/M$ expansion in order to reveal the hidden violation terms. They also introduced a one–derivative "manifest" heavy spin violation term with a new relatively large unknown constant in order to improve the agreement with experiment.

In the present paper we show that it is not necessary to introduce any new violation terms to agree with experiment if a chiral Lagrangian including light vectors is employed. Typical results are summarized, compared with experiment and compared with the Skyrme model for the light sector in Table 1.1. A much more detailed discussion is given later in the text. We notice from the last row, that the model with light vectors gives a very satisfactory account of the Σ_c^*–Σ_c hyperfine splitting in contrast to the model without light vectors. There are also noticeable effects when the use of the heavy meson reduced mass is taken as a simple

Table 1.1: Typical results for the present model (including light vectors) compared with model with light pseudoscalars only ("Skyrme" column) and compared with experiment. No "manifest" heavy spin violation effects other than $M^* \neq M$ have been included. The column "present model + CM" simply takes into account recoil corrections by replacing the heavy meson mass by the reduced mass. Λ_c' denotes a negative parity, spin 1/2 state. The quantity α in eqs (2.6) and (2.7) was taken to be zero. All masses in MeV.

mass difference	expt.	present model	present model + CM	Skyrme
$\Lambda_c - N$	1345	1257	1356	1553
$\Lambda_b - \Lambda_c$	3356±50	3164	3285	3215
$\Lambda_c' - \Lambda_c$	308	249	342	208
$\Sigma_c - \Lambda_c$	168	172	158	185
$\Sigma_c^* - \Sigma_c$	66	42	63	16

approximation for kinematical corrections. Similarly, the first four rows of Table 1.1 show that the other predictions of the model with light vectors agree well with experiment. Note that (see section 4) the predictions for mass differences are considered more reliable than those for the masses themselves.

The present article is organized as follows. In section 2 we will review the classical, leading in $1/N_C$, part of this calculation. This discussion includes both the light and heavy meson pieces of the Lagrangian. The emergence of bound state solutions is also explained in section 2. In section 3 we will describe the collective quantization in the framework of the cranking procedure for the bound states. Section 4 contains a detailed discussions of the numerical results. In section 5 we will discuss some new manifest contributions to the hyperfine splittings and the extension to different channels in the framework of the perturbation approach. We will conclude in section 6. The explicit expressions for the couplings between the bound heavy meson and the collective coordinates are listed in the appendices.

2. The Model Lagrangian

In this section we review the classical, *i.e.* leading order part in the $1/N_C$ expansion of the bound state description for the heavy baryons in the soliton picture.

2.1. Light Mesons

For the sector of the model describing the light pseudoscalar and vector mesons we adopt the chirally invariant Lagrangian discussed in detail in the literature [23, 24]. This Lagrangian can be decomposed into a regular parity part

$$\mathcal{L}_S = f_\pi^2 \mathrm{tr}\left[p_\mu p^\mu\right] + \frac{m_\pi^2 f_\pi^2}{2}\mathrm{tr}\left[U + U^\dagger - 2\right] - \frac{1}{2}\mathrm{tr}\left[F_{\mu\nu}\left(\rho\right) F^{\mu\nu}\left(\rho\right)\right] + m_V^2 \mathrm{tr}\left[R_\mu R^\mu\right] \tag{2.1}$$

and a part which contains the Levi-Civita tensor, $\epsilon_{\mu\nu\alpha\beta}$. The action for the latter is most conveniently displayed with the help of differential forms $p = p_\mu dx^\mu$, etc.

$$\begin{aligned}\Gamma_{\rm an} \quad &= \frac{2N_c}{15\pi^2}\int Tr(p^5) \\ &+ \int Tr\left[\frac{4i}{3}(\gamma_1 + \frac{3}{2}\gamma_2)Rp^3 - \frac{g}{2}\gamma_2 F(\rho)(pR - Rp) - 2ig^2(\gamma_2 + 2\gamma_3)R^3 p\right].\end{aligned} \tag{2.2}$$

In eqs (2.1) and (2.2) we have introduced the abbreviations

$$p_\mu = \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right), \quad v_\mu = \frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \quad \text{and} \quad R_\mu = \rho_\mu - \frac{1}{g}v_\mu . \tag{2.3}$$

Here ξ refers to a square root of the chiral field, *i.e.* $U = \xi^2$. Furthermore $F_{\mu\nu}\left(\rho\right) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - ig\left[\rho_\mu, \rho_\nu\right]$ denotes the field tensor associated with the vector mesons ρ and ω, which are combined in $\rho_\mu = \left(\omega_\mu \mathbb{1} + \rho_\mu^a \tau^a\right)/2$ when the reduction to two light flavors is made. The parameters g, γ_1, etc. can be determined (or at least constrained) from the study of decays of the light vector mesons such as $\rho \to 2\pi$ or $\omega \to 3\pi$ [24].

The action for the light degrees of freedom ($\int \mathcal{L}_S + \Gamma_{\rm an}$) contains static soliton solutions. The appropriate *ansätze* are

$$\xi(\boldsymbol{r}) = \exp\left(\frac{i}{2}\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}F(r)\right), \quad \omega_0(\boldsymbol{r}) = \frac{\omega(r)}{g} \quad \text{and} \quad \rho_{i,a}(\boldsymbol{r}) = \frac{G(r)}{gr}\epsilon_{ija}\hat{r}_j \tag{2.4}$$

while all other field components vanish. The resulting non–linear Euler–Lagrange equations for the radial functions $F(r), \omega(r)$ and $G(r)$ are solved numerically subject to the boundary conditions $F(0) = -\pi$, $\omega'(0) = 0$ and $G(0) = -2$ while all fields vanish at radial infinity [24]. These boundary conditions are needed to obtain a consistent baryon number one configuration.

2.2. The Relativistic Model for the Heavy Mesons

In this subsection we present the relativistic Lagrangian, which describes the coupling between the light and heavy mesons [15]

$$\begin{aligned}\mathcal{L}_H \quad = \quad & D_\mu P\left(D^\mu P\right)^\dagger - \frac{1}{2}Q_{\mu\nu}\left(Q^{\mu\nu}\right)^\dagger - M^2 PP^\dagger + M^{*2}Q_\mu Q^{\mu\dagger} \\ & +2iMd\left(Pp_\mu Q^{\mu\dagger} - Q_\mu p^\mu P^\dagger\right) - \frac{d}{2}\epsilon^{\alpha\beta\mu\nu}\left[Q_{\nu\alpha}p_\mu Q_\beta^\dagger + Q_\beta p_\mu\left(Q_{\nu\alpha}\right)^\dagger\right] \\ & -\frac{2\sqrt{2}icM}{m_V}\left\{2Q_\mu F^{\mu\nu}\left(\rho\right)Q_\nu^\dagger - \frac{i}{M}\epsilon^{\alpha\beta\mu\nu}\left[D_\beta P F_{\mu\nu}\left(\rho\right)Q_\alpha^\dagger + Q_\alpha F_{\mu\nu}\left(\rho\right)\left(D_\beta P\right)^\dagger\right]\right\}.\end{aligned} \tag{2.5}$$

Here we have allowed the mass M of the heavy pseudoscalar meson P to differ from the mass M^* of the heavy vector meson Q_μ. Note that the heavy meson fields are conventionally defined as *row* vectors in isospin space. The covariant derivative introduces the additional parameter α:

$$\begin{aligned} D_\mu P^\dagger &= \left(\partial_\mu - i\alpha g\rho_\mu - i\left(1-\alpha\right)v_\mu\right)P^\dagger = \left(\partial_\mu - iv_\mu - ig\alpha R_\mu\right)P^\dagger \ , \qquad (2.6) \\ D_\mu Q^\dagger_\nu &= \left(\partial_\mu - iv_\mu - ig\alpha R_\mu\right)Q^\dagger_\nu \ . \qquad (2.7) \end{aligned}$$

The covariant field tensor of the heavy vector meson is then defined as

$$(Q_{\mu\nu})^\dagger = D_\mu Q^\dagger_\nu - D_\nu Q^\dagger_\mu. \qquad (2.8)$$

The coupling constants d, c and α, which appear in the Lagrangian (2.5), have still not been very accurately determined. In particular there is no direct experimental evidence for the value of α, which would be unity if a possible definition of light vector meson dominance for the electromagnetic form factors of the heavy mesons were to be adopted [25]. We will later adjust α to the spectrum of the heavy baryons. The other parameters in (2.5) will be taken [25] to be:

$$\begin{aligned} d &= 0.53 \ , \quad c = 1.60 \ ; \\ M &= 1865\text{MeV} \ , \quad M^* = 2007\,\text{MeV} \ , \qquad \text{D}-\text{meson} \ ; \\ M &= 5279\text{MeV} \ , \quad M^* = 5325\,\text{MeV} \ , \qquad \text{B}-\text{meson}. \qquad (2.9) \end{aligned}$$

It should be stressed that the assumption of infinitely large masses for the heavy mesons has not been made in (2.5). However, a model Lagrangian which was only required to exhibit the Lorentz and chiral invariances would be more general than the relativistic Lagrangian (2.5). The additional restrictions arise from the heavy quark transformation

$$P' = \mathrm{e}^{iMV\cdot x}P \ , \qquad Q'_\mu = \mathrm{e}^{iM^*V\cdot x}Q_\mu \ , \qquad (2.10)$$

where the four–velocity V^μ characterizes the reference frame of the heavy quark. The heavy pseudoscalar and vector meson fields may then be combined in the heavy multiplet

$$H = \frac{1}{2}\left(1+\gamma_\mu V^\mu\right)\left(i\gamma_5 P' + \gamma^\nu Q'_\nu\right) \qquad \text{and} \qquad \bar{H} = \gamma_0 H^\dagger \gamma_0 \ . \qquad (2.11)$$

In the heavy quark limit, $M = M^* \to \infty$, the relativistic Lagrangian (2.5) becomes

$$\frac{1}{M}\mathcal{L}_H \to iV^\mu \mathrm{Tr}\left\{HD_\mu\bar{H}\right\} - d\mathrm{Tr}\left\{H\gamma_\mu\gamma_5 p^\mu \bar{H}\right\} - i\frac{\sqrt{2}c}{m_V}\mathrm{Tr}\left\{H\gamma_\mu\gamma_\nu F^{\mu\nu}(\rho)\bar{H}\right\} + \ldots \ . \qquad (2.12)$$

The ellipses indicate subleading pieces in $1/M$. Actually the coefficients of the various Lorentz and chirally invariant pieces in the relativistic Lagrangian (2.5) have precisely been arranged to yield the spin–flavor symmetric model (2.12) in the heavy quark limit [15].

2.3. Bound States

Here we briefly review the origin of bound states in the S– and P–wave heavy meson channels. These orbital angular momentum quantum numbers refer to those of the pseudoscalar component ($P^\dagger$) of the heavy meson multiplet ($P^\dagger, Q^\dagger_\mu$).

For the P–wave channel the appropriate *ansatz* reads

$$P^\dagger = \frac{1}{\sqrt{4\pi}}\Phi(r)\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}\rho\mathrm{e}^{i\epsilon t}\ , \qquad Q^\dagger_0 = \frac{1}{\sqrt{4\pi}}\Psi_0(r)\rho\mathrm{e}^{i\epsilon t}\ , \tag{2.13}$$

$$Q^\dagger_i = \frac{1}{\sqrt{4\pi}}\left[i\Psi_1(r)\hat{r}_i + \frac{1}{2}\Psi_2(r)\epsilon_{ijk}\hat{r}_j\tau_k\right]\rho\mathrm{e}^{i\epsilon t}\ . \tag{2.14}$$

Note that here ρ refers to a properly normalized spinor which describes the isospin of the heavy meson multiplet. Similarly the *ansatz* for the S–wave is given by

$$P^\dagger = \frac{1}{\sqrt{4\pi}}\Phi(r)\rho\mathrm{e}^{i\epsilon t}\ , \qquad Q^\dagger_0 = \frac{1}{\sqrt{4\pi}}\Psi_0(r)\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}\rho\mathrm{e}^{i\epsilon t}\ , \tag{2.15}$$

$$Q^\dagger_i = \frac{1}{\sqrt{4\pi}}\left[\Psi_1(r)\hat{r}_i\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau} + \Psi_2(r)r\boldsymbol{\tau}\cdot\partial_i\hat{\boldsymbol{r}}\right]\rho\mathrm{e}^{i\epsilon t}\ . \tag{2.16}$$

It should be remarked that the isospin matrices, which multiply the isospinor ρ, have (since there are no unmatched indices) vanishing grand spin $\boldsymbol{G}$, which is the vector sum of total spin and isospin. The above *ansätze* are substituted in the relativistic Lagrangian (2.5) and the resulting action functionals are listed in appendix A. The variation of these functionals yields the associated equations of motion. They are numerically integrated by adjusting the energy eigenvalue ϵ so that continuous normalizable configurations are obtained*. This value of ϵ directly yields the binding energy of the heavy mesons. The fact that we have $U(r=0) = -1$ at the spatial origin causes the angular barrier for the P–wave heavy meson to vanish while the S–wave acquires a finite one. As a result the P–wave heavy meson is more strongly bound.

Finally we would like to mention the connection to the heavy quark limit. In that case the multiplet H is characterized by a single radial function [26]. This implies that in the limit $M = M^* \to \infty$ the radial functions, which parametrize the bound heavy mesons, have to satisfy the linear relations

$$\Psi_1 = -\Phi, \quad \Psi_2 = -2\Phi \qquad (\mathrm{P-wave})\ , \tag{2.17}$$

*The normalizability condition is quite restrictive. For example, it was shown in ref [16] to prohibit a "pentaquark" solution which would be extracted from eq (2.12) in the heavy quark limit.

$$\Psi_1 \;=\; -\Phi, \quad \Psi_2 = \Phi \qquad\qquad (\text{S}-\text{wave})\ , \tag{2.18}$$

together with $\Psi_0 = 0$ in both cases. Indeed the numerical solutions confirm these relations as the heavy meson masses approach infinity [16]. It should be remarked that commonly more than one bound state exists in each channel. Here we will concentrate mainly on the lowest one, which is characterized by the radial functions having no nodes away from the boundaries $r = 0$ and $r \to \infty$. However, in special instances we will also discuss the first radially excited state.

2.4. Normalization

As the relativistic Lagrangian (2.5) is bilinear in the heavy meson fields the resulting equations of motion are linear. Hence the overall magnitude of the solution is not fixed by the equation of motion. Nevertheless, the equations of motion for the heavy meson fields allow us to extract a metric for a scalar product between different bound states. In particular its diagonal elements serve to properly normalize the bound state wave–functions. The Lagrange function which results from substituting the *ansätze* (2.4) and (2.13)–(2.15) may generally be written as

$$L = -M_{\rm cl}\left[F,G,\omega\right] + I_\epsilon\left[F,G,\omega;\Phi,\Psi_0,\Psi_1,\Psi_2\right]\rho^\dagger(\epsilon)\rho(\epsilon)\ . \tag{2.19}$$

Here $M_{\rm cl}$ denotes the soliton mass [24] whose minimum determines the light meson profiles F, G and ω. The explicit expressions for the functionals I_ϵ are given in appendix A. The subscript refers to the explicit dependence on the energy eigenvalues. Upon canonical quantization the Fourier amplitudes $\rho(\epsilon)$ and $\rho^\dagger(\epsilon)$ are respectively elevated to annihilation and creation operators for a heavy meson bound state with the energy eigenvalue ϵ. Demanding that each occupation of the bound state adds the amount $|\epsilon|$ to the total energy yields the normalization condition[†]

$$\Big|\frac{\partial}{\partial\epsilon} I_\epsilon\left[\Phi,\Psi_0,\Psi_1,\Psi_2\right]\Big| = 1 \tag{2.20}$$

in addition to the canonical commutation relation $[\rho_i(\epsilon), \rho_j^\dagger(\epsilon')] = \delta_{ij}\delta_{\epsilon,\epsilon'}$. Note that for bound states the energy eigenvalues are discretized. For the P–wave channel we obtain the

[†]In the case that the explicit dependence on ϵ is quadratic the proof is sketched in section 3 of ref [27]. A simple verification of eq (2.20) may be also obtained from its close connection to the Noether charge for the heavy quark number conservation. The latter is gotten by transforming the heavy fields as $P \to e^{-i\eta(x)}P$, $Q_\mu \to e^{-i\eta(x)}Q_\mu$ and computing the quantity $N = \int d^3x\, \delta\mathcal{L}(e^{-i\eta}P,\ldots)/\delta\partial_0\eta\,\big|_{\eta=\partial_0\eta=0}$. However because the present *ansatz* is of the form $P = \tilde{P}(\boldsymbol{x})e^{-i\epsilon t}$ we may equivalently compute this as $N = \int d^3x\, \delta\mathcal{L}\left(e^{-i(\eta+\epsilon t)}\tilde{P}\left(\boldsymbol{x}\right),\ldots\right)/\delta\epsilon\,\Big|_{\eta=\partial_0\eta=0}$. In the one heavy quark subspace this yields $1 = \partial I_\epsilon/\partial\epsilon$.

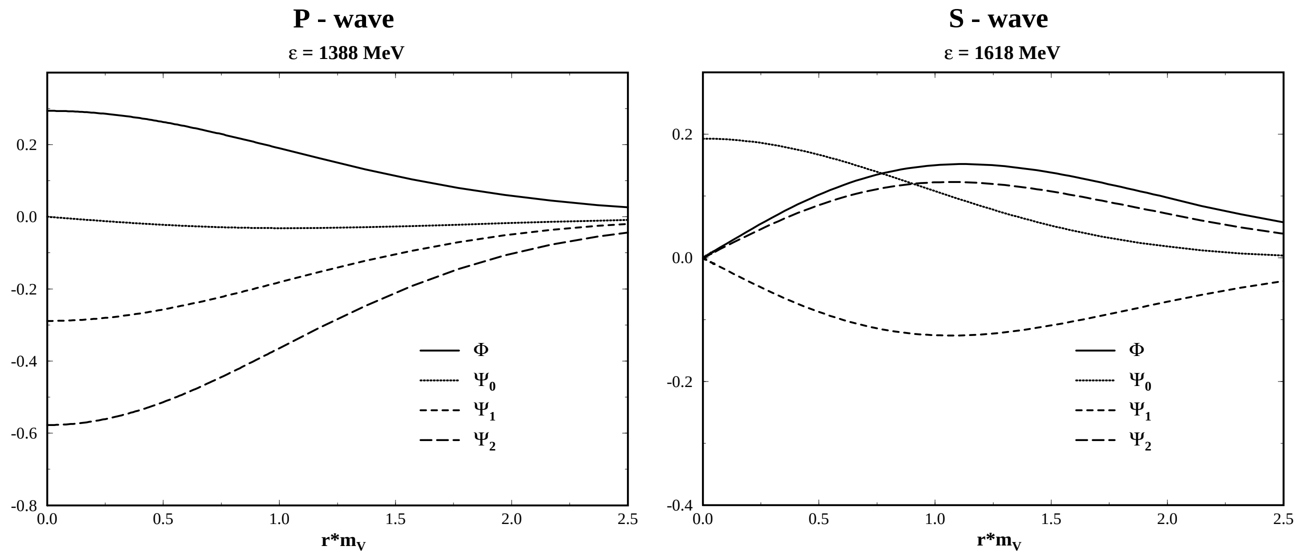

Figure 2.1: The profile functions for the bound state wave–functions in the P–wave (left panel) and S–wave (right panel) channels. These functions are measured in units of $m_V = 773\mathrm{MeV}$. See text for the specification of the remaining parameters.

normalization condition

$$\Bigg| \int dr r^2 \Bigg\{ 2\left(\epsilon - \frac{\alpha}{2}\omega\right)\Phi^2 - 2\left[\Psi_0' - \left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_1\right]\Psi_1 + \left[\frac{1}{r}R_\alpha \Psi_0 + \left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_2\right]\Psi_2$$
$$-d\left[\frac{2}{r}\mathrm{sin}F\Psi_1 - \frac{1}{2}F'\Psi_2\right]\Psi_2 + \frac{4\sqrt{2}c}{g m_V}\frac{1}{r^2}\left[G\left(G+2\right)\Psi_1 - G' r \Psi_2\right]\Phi \Bigg\}\Bigg| = 1 \qquad (2.21)$$

from eq (2.20). For convenience we have employed the abbreviation $R_\alpha = \mathrm{cos}F - 1 + \alpha\left(1 + G - \mathrm{cos}F\right)$. Similarly for the S–wave channel the condition (2.20) yields

$$\Bigg| \int dr r^2 \Bigg\{ 2\left(\epsilon - \frac{\alpha}{2}\omega\right)\Phi^2 - 2\left[\Psi_0' - \left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_1\right]\Psi_1$$
$$+4\left[\left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_2 - \frac{R_\alpha + 2}{2r}\Psi_0\right]\Psi_2 - 2d\left[\frac{2}{r}\mathrm{sin}F\Psi_1 + F'\Psi_2\right]\Psi_2$$
$$+\frac{4\sqrt{2}c}{g m_V}\frac{1}{r^2}\left[G\left(G+2\right)\Psi_1 + 2G' r \Psi_2\right]\Phi \Bigg\}\Bigg| = 1 \ . \qquad (2.22)$$

The radial profiles associated with these normalizations are displayed in figure 2.1 for the choice $\alpha = -0.3$. The parameters in the relativistic Lagrangian (2.5) have been set to the charm sector (2.9). The parameters entering the light meson Lagrangian (2.1,2.2) are given in eq (4.1).

3. Cranking the Bound Heavy Meson State

It can easily be verified that the field configurations for both the light mesons (2.4) and the heavy mesons (2.13)–(2.16) are neither eigenfunctions of the spin– nor the isospin generators. Hence these configurations do not possess the correct quantum numbers. In order to generate states which correspond to physical baryons a cranking procedure is employed. In the first step collective coordinates, which parametrize the (iso–) spin orientation of the meson configuration, are introduced.

3.1. Collective Coordinates

Time–dependent solutions to the equations of motion are required to obtain non–vanishing spin and isospin as the corresponding Noether charges. Unfortunately these solutions are unknown. Taking, however, into account that static rotations in coordinate and isospin spaces do not change the potential part of the Lagrange function, the assumption that these rotations are time–dependent seems to be a reasonable approximation. We therefore extend the soliton *ansatz* (2.4) by

$$\xi \longrightarrow A(t)\xi A^{\dagger}(t) \quad \text{and} \quad \rho_\mu \longrightarrow A(t)\rho_\mu A^{\dagger}(t)\ . \tag{3.1}$$

Note that ρ_μ contains both isoscalar and isovector pieces. It should also be remarked that introducing only an isospin rotation as in eq (3.1) is sufficient because the hedgehog structure of the classical configuration (2.4) allows one to express a spatial rotation as one in isospin space. The time–dependence of the collective rotations is most conveniently parametrized by introducing angular velocities $\boldsymbol{\Omega}$ via

$$A^{\dagger}(t)\frac{d}{dt}A(t) = \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\Omega}\ . \tag{3.2}$$

In the specific case that the Lagrangian contains terms which are linear in the time derivative as in eq (2.2), additional field components are induced. For the light vector mesons these are linear in the angular velocities

$$\omega_i = \frac{2}{r}\varphi(r)\epsilon_{ijk}\Omega_j\hat{r}_k \quad \text{and} \quad \rho_0^k = \xi_1(r)\Omega_k + \xi_2(r)\hat{\boldsymbol{r}}\cdot\boldsymbol{\Omega}\hat{r}_k\ . \tag{3.3}$$

Substituting the configurations (3.1) and (3.3) into the light meson Lagrangian yields a term which is quadratic in the angular velocities. The constant of proportionality defines the moment of inertia $\alpha^2\,[F,G,\omega;\xi_1,\xi_2,\varphi]$. The induced radial functions $\varphi(r)$, $\xi_1(r)$ and $\xi_2(r)$ are obtained from a variational approach to α^2 [28]*. The resulting equations of motion are coupled inhomogeneous differential equations with the classical fields F, G and ω, which are fixed from extremizing the classical mass, acting as sources. Here it is worth mentioning that α^2 is of the order N_C.

*The term α^2 refers to a frequently adopted notation for the moment of inertia and should not be confused with the coupling constant α in eq (2.6).

Since the heavy mesons are also isospinors the collective rotation has to be applied as well. In analogy to eq (3.1) we write

$$P^\dagger \longrightarrow A(t)P^\dagger \quad \text{and} \quad Q^\dagger_\mu \longrightarrow A(t)Q^\dagger_\mu \ . \tag{3.4}$$

Substituting the collectively rotating configurations into the total Lagrangian finally yields

$$L_P = -M_{\rm cl} + I_\epsilon^{(P)}\rho^\dagger\rho + \frac{1}{2}\alpha^2\mathbf{\Omega}^2 + \frac{1}{2}\chi_P\rho^\dagger\mathbf{\Omega}\cdot\boldsymbol{\tau}\rho \ . \tag{3.5}$$

For convenience we have omitted the argument of the iso–spinor ρ. The classical mass $M_{\rm cl}$, which upon minimization provides the soliton profiles (2.4), and the moment of inertia α^2 are functionals of only the light meson fields. The quantity $I_\epsilon^{(P)}$ is given in eq (A.2) and has already been employed to obtain the bound state P–wave profiles (2.13). The new quantity is the hyperfine parameter χ_P whose explicit expression is displayed in appendix B (B.1).

3.2. Quantization of the Collective Coordinates

Here we will discuss how the canonical quantization of the collective coordinates A leads to states which may be identified with physical baryons. In order to construct Noether charges we first have to consider the variation of the fields under infinitesimal symmetry transformations. For the isospin transformation we observe

$$\left[\phi, i\frac{\tau_i}{2}\right] = -D_{ij}(A)\frac{\partial\dot{\phi}}{\partial\Omega_j} + \ldots \ . \tag{3.6}$$

Here ϕ refers to any of the iso–rotating meson fields and the ellipses represent terms, which are subleading in $1/N_C$, as *e.g.* time derivatives of the angular velocities which might arise from eq (3.3). Furthermore $D_{ij}(A) = (1/2)\ {\rm tr}(\tau_i A\tau_j A^\dagger)$ denotes the adjoint representation of the collective rotations A. From eq (3.6) we conclude that the total isospin is related to the derivative of the Lagrange function with respect to the angular velocities

$$I_i = -D_{ij}(A)\frac{\partial L_P}{\partial\Omega_j} \ . \tag{3.7}$$

Next we note that the total spin operator $\boldsymbol{J}$ enters the grand spin operator $\boldsymbol{G}$ in the laboratory frame via

$$G_i = J_i + D^{-1}_{ij}(A)I_j = J_i - J_i^{\rm sol} \ . \tag{3.8}$$

For convenience we have defined the spin carried by the soliton $\boldsymbol{J}^{\rm sol} = \partial L_P/\partial\mathbf{\Omega}$. As a consequence of the relation (3.7), its absolute value is identical to that of the isospin, *i.e.* $(\boldsymbol{J}^{\rm sol})^2 = \boldsymbol{I}^2 = I(I+1)$. By construction the light meson fields do not contribute to $\boldsymbol{G}$. Even more importantly and as has been noted before, the pieces of the heavy meson wave–functions (3.4), which multiply the spinor $A\rho$, have zero grand spin too. Using the normalization condition (2.21) one therefore ends up with

$$\boldsymbol{G} = -\rho^\dagger \frac{\boldsymbol{\tau}}{2}\rho \ . \tag{3.9}$$

This relation will be helpful because it relates the operator multiplying the hyperfine parameter in the collective Lagrangian (3.5) to the spin and isospin operators. The collective piece of the Hamiltonian is obtained from the Legendre transformation

$$H_P^{\rm coll} = \boldsymbol{\Omega}\cdot\boldsymbol{J}^{\rm sol} - L_P^{\rm coll} = \frac{1}{2\alpha^2}\left[\boldsymbol{J}^{\rm sol} + \chi_P \boldsymbol{G}\right]^2 \ , \tag{3.10}$$

Here $L_P^{\rm coll}$ refers to the $\boldsymbol{\Omega}$ dependent terms in eq (3.5). Finally the mass formula for an even parity baryon with a single heavy quark becomes

$$M_P = M_{\rm cl} + |\epsilon_P| + \frac{1}{2\alpha^2}\left[\chi_P J(J+1) + (1-\chi_P)I(I+1)\right] \ , \tag{3.11}$$

where contributions of $\mathcal{O}(\chi_P^2)$, which apparently are quartic in the heavy meson wave–function, have been omitted for consistency because terms of that order have been excluded from the very beginning. Also the omitted terms are subleading in the $1/M$ expansion since χ_P goes as $1/M$, *cf.* figure 4.1 and Appendix B. In addition, the operator contained in the omitted term, $\left(\rho^\dagger\boldsymbol{\tau}\rho\right)\cdot\left(\rho^\dagger\boldsymbol{\tau}\rho\right)$, does not contribute to the hyperfine splitting. The reason is that from canonical commutation relations for the components of the isospinor ρ, $\left[\rho_i,\rho_j^\dagger\right] = \delta_{ij}$, this operator is shown to be $N_Q\left(N_Q+2\right)$, where N_Q is the occupation number for the heavy meson bound state [13]. Hence this term contains neither the spin nor the isospin quantum numbers.

From eq (3.11) we recognize that the spin degeneracy between baryons containing a heavy quark vanishes in the heavy quark limit because χ_P approaches zero. Of course, this result is a direct consequence of the spin–flavor symmetry and would not have come out in case the various Lorentz and chirally invariant terms in eq (2.5) had been chosen arbitrarily.

3.3 The Odd Parity State

The S– and P–wave heavy channels decouple because they have opposite parity. Therefore the quantization of the S–wave bound state may be considered independently from the P–wave case, which has been discussed in the preceding section. The calculation, which proceeds along the lines of the one discussed in subsection 3.1, yields

$$L = -M_{\rm cl} + I_\epsilon^{(S)}\rho^\dagger\rho + \frac{1}{2}\alpha^2\boldsymbol{\Omega}^2 + \frac{1}{2}\chi_S\rho^\dagger\boldsymbol{\Omega}\cdot\boldsymbol{\tau}\rho \ . \tag{3.12}$$

Here, of course, the spinor ρ corresponds to the one of the bound heavy meson in the S-wave channel. The explicit expression for the corresponding hyperfine parameter χ_S is given in eq (B.3) in Appendix B.

We may apply the same quantization procedure as for the P–wave. This results in the mass formula

$$M_S = M_{\rm cl} + |\epsilon_S| + \frac{1}{2\alpha^2}\left[\chi_S J(J+1) + (1-\chi_S) I(I+1)\right] \tag{3.13}$$

for baryons constructed as a light baryon and a single occupation of the bound state for the heavy meson being in the S–wave channel.

Let us add a brief comment on the $1/N_C$ dependences in eqs (3.11) and (3.13). The classical mass is $\mathcal{O}(N_C)$ while in leading order the binding energies are $\mathcal{O}(N_C^0)$. As already noted above, the moment of inertia is $\mathcal{O}(N_C)$ while $\chi \sim \mathcal{O}(N_C^0)$. Therefore the hyperfine splitting is not only subleading in the heavy quark limit but also in the $1/N_C$ expansion.

4. Numerical Results

In this section we will discuss the numerical results obtained for the masses of the heavy baryons in the model discussed above. In particular we will concentrate on the spin and isospin splitting in the realistic case of finite heavy meson masses (2.9). It should be noted that sizable quantum corrections occur for the classical soliton mass $M_{\rm cl}$ [29]. It seems that these corrections are (approximately) equal for all baryons. Hence we will only consider mass differences between various baryons. In that case the absolute value of the classical mass $M_{\rm cl}$ is redundant. The parameters in the light sector cannot completely be determined from properties of the corresponding mesons. The remaining (limited) parameter space is, however, more than fully constrained by a best fit to the mass differences of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in the light sector. This yields:

$$\begin{aligned} g = 5.57, \qquad & m_V = 773\,\mathrm{MeV} \\ \gamma_1 = 0.3, \quad \gamma_2 \; = \; & 1.8, \quad \gamma_3 = 1.2\ . \end{aligned} \tag{4.1}$$

The resulting mass differences for the light baryons all agree within about 10% [30]. The corresponding moment of inertia is $\alpha^2 = 5.00\mathrm{GeV}^{-1}$. In eqs (3.11) and (3.13) $1/\alpha^2$ enters as the coefficient of those terms which determine the hyperfine splitting. Hence a fine–tuning of the parameters (4.1) to *e.g.* $\alpha^2 = 5.11\mathrm{GeV}^{-1}$, which exactly reproduces the Δ–nucleon mass difference, has only negligible effects on the predicted hyperfine splittings.

Before discussing the implications for the physical parameter results (2.9) we would like to comment on the heavy limit behavior of the hyperfine splitting parameters χ_P and χ_S. For

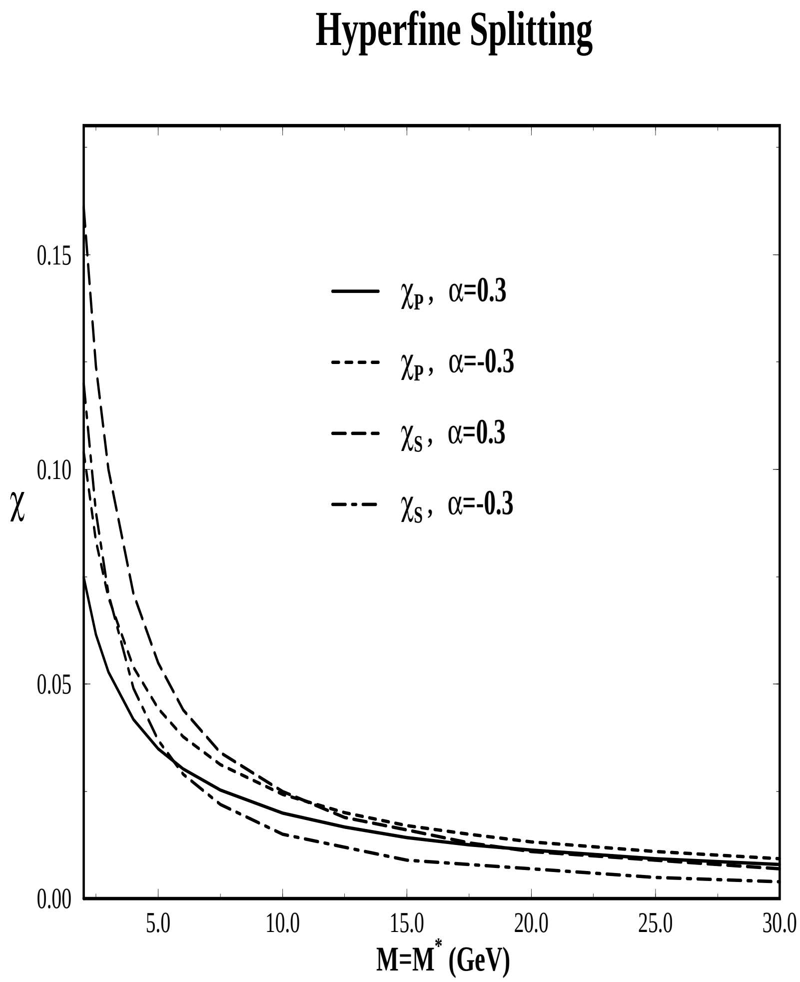

Figure 4.1: The hyperfine splitting parameters χ_P and χ_S a functions of identical heavy meson masses for different values of the coupling constant α.

this purpose we have plotted these quantities as functions of $M = M^*$ in figure 4.1. We see that for both channels the splitting parameters decrease when the heavy limit is approached. In the appendix we show that the leading order term in the heavy quark expansion indeed is proportional to $1/M$. Clearly the hyperfine parameter in the S–wave channel decreases somewhat more quickly with the heavy meson mass than in the P–wave channel.

For fixed M the hyperfine parameters in the two channels behave oppositely with regard to the undetermined coupling constant α: While χ_P decreases when α becomes larger, χ_S increases. In ref [20] we have shown that a major fraction of the P–wave hyperfine constant is due to terms in the relativistic Lagrangian (2.5) which do not manifestly break the heavy spin symmetry rather than to terms, which explicitly break this symmetry; as for example $M \neq M^*$. For a quantitative discussion of this hidden contribution we perform the calculation using identical masses from the charm sector *i.e.* $M = M^* = 1.865\text{GeV}$ and furthermore $\alpha = 0.3$. We take all other parameters as in eq (2.9). This results in $\chi_P = 0.080$.

From table 4.1 we recognize that this is about 80% of the value obtained using the physical masses $M = 1.865$GeV and $M^* = 2.007$GeV. In the case of the S–wave the hidden piece is even more dominant. For the symmetric choice of the mass parameters one finds $\chi_S = 0.175$ which is more than 90% of the value displayed in table 4.1 for $\alpha = 0.3$. It is also interesting to compare figure 4.1 with the corresponding curve in figure 1 of ref [20], pertaining to the model without light vectors. This makes it clear that the light vector model predicts a substantially larger χ_P.

In section 5, some more discussion of the "hidden" hyperfine splitting terms is given. In addition, three more "manifest" heavy spin symmetry violating terms associated with the relativistic Lagrangian (2.5) are treated in the perturbative expansion. Since the manifest $(M^* - M)$ contribution is relatively small it is reasonable to expect that the others will be small too.

Let us next discuss the spectrum of the baryons containing a single heavy quark. For this case we assume the realistic masses as in eq (2.9). In table 4.1 the numerical results for the lowest S– and P–wave bound states in the charm sector are displayed. As already noted in ref [16] the binding energy

$$\omega = M - |\epsilon| \tag{4.2}$$

decreases with growing coupling constant α. This is the case for both the P– and S–wave channels. For $M \to \infty$ the heavy limit [26]

$$\omega \longrightarrow \frac{3}{2} dF'(0) + \frac{3\sqrt{2}c}{gm_V} G''(0) - \frac{\alpha}{2}\omega(0) \tag{4.3}$$

will be attained*. As in the discussion of figure 4.1 we see that the hyperfine parameters in these two channels behave oppositely as functions of α. Here we have chosen to measure the mass differences with respect to the lightest charmed baryon, Λ_c. Hence the mass differences with respect to Σ_c and Σ_c^* directly reflect the α–dependence of hyperfine parameter χ_P while the corresponding dependence of the binding energy ω_P can be extracted from the splitting relative to the nucleon. In addition the splitting with respect to the negative parity charmed baryons reflects the α–dependence of the S–wave channel binding energy ω_S. Finally the mass difference to Λ_b contains the energy eigenvalues and hyperfine parameters computed with the B and B^* meson masses in eq (2.9).

While the mass difference to the nucleon is improved with a positive value for α, the agreement for the mass differences between the heavy baryons slightly deteriorates when in-

*Note that the conventions in ref [26] differ from the present ones as explained in Appendix B of ref [16].

Table 4.1: Parameters for heavy baryons and mass differences with respect to Λ_c. Primes indicate negative parity baryons, *i.e.* S–wave bound states. All energies are in MeV.

α	-0.1	0.0	0.1	0.2	0.3	Expt.	Skyrme
ω_P	564	544	522	500	478		243
χ_P	0.147	0.140	0.131	0.123	0.114		0.053
ω_S	316	298	281	264	247		57
χ_S	0.172	0.181	0.189	0.197	0.205		0.346
Σ_c	171	172	174	175	177	168	185
Σ_c^*	215	214	213	212	211	233	201
Λ_c'	250	249	245	242	238	308	208
Σ_c'	415	413	408	402	397	?	335
$\Sigma_c'^*$	468	467	464	461	458	?	437
N	-1237	-1257	-1278	-1299	-1321	-1345	-1553
Λ_b	3160	3164	3167	3170	3173	3356 ± 50	3215

creasing this parameter. Nevertheless, fair agreement with the experimental data is achieved for quite a range of α.

Table 4.1 also contains the model predictions when the background soliton is taken from the basic Skyrme model [21, 19] which does not include the light vector mesons. Here we have adjusted the only free parameter ($e_{\text{Skyrme}} = 4.25$) to reproduce the Δ–nucleon mass difference. From the Λ_c–nucleon mass difference we observe that in comparison with the nucleon the masses of the heavy baryons are predicted about 200MeV too large. This confirms the above statement that the spectra of both the light and the heavy baryons can only be reasonably reproduced when light vector mesons are included. This conclusion can already be drawn from the too small binding energies [16]. The hyperfine corrections make only minor changes in the Λ_b–Λ_c splitting.

In table 4.2 we display the analogous predictions for the bottom sector. According to the heavy spin symmetry the binding energies of the P– and S–wave channels approach each other. Hence the mass differences between the even and odd parity baryons containing a bottom quark correspondingly decrease. As was already inferred from figure 4.1 we confirm upon comparison with table 4.1 that χ_P decreases less quickly with the heavy meson mass than χ_S. Except for Λ_b no empirical data for the masses of these baryons are known at present. These results for the mass differences among the bottom baryons are predictions

Table 4.2: Parameters for heavy baryons and mass differences with respect to Λ_b. Primes indicate negative parity baryons, *i.e.* S–wave bound states. All energies are in MeV. The empirical value for the relative position of the nucleon is 4701 ± 50MeV [31].

α	-0.1	0.0	0.1	0.2	0.3
ω_P	811	786	762	737	713
χ_P	0.055	0.053	0.050	0.048	0.045
ω_S	639	617	595	573	552
χ_S	0.043	0.046	0.049	0.052	0.055
Σ_b	189	189	190	190	191
Σ_b^*	206	205	205	205	205
Λ_b'	171	168	167	164	161
Σ_b'	363	359	358	354	351
$\Sigma_b'^*$	375	373	371	369	367
N	-4397	-4422	-4446	-4471	-4494

of the model which can, in the future, be compared with experiment. As could have been inferred from the next to last row in table 4.1 the absolute position of the bottom multiplet is about 200 ± 50MeV too low. On the absolute scale this apparently is only a 5% deviation from the data. Certainly a larger value $\alpha \approx 1$, which corresponds to a model for light vector resonance dominance of the heavy meson form factor [25], would yield an excellent agreement for the mass difference between Λ_b and the nucleon. On the other hand such a choice would slightly spoil the nice picture for the charm multiplet.

In table 4.3 we list the numerical results for baryons constructed from the first radially excited P–wave bound state in the charm sector. The particle data group (PDG) lists an excited $\Lambda_c(2625)$, about 340MeV above the Λ_c [31], although this state is more likely, according to the PDG, to have $J^P = \frac{3}{2}^-$. In the bound state picture this would require a D–wave ($G = 3/2$) heavy meson bound to the soliton. Here we have not discussed that channel. A preliminary discussion in the perturbative approach is given in the next section.

The preceding calculations are based on the $N_C \to \infty$ limit in which the nucleon is infinitely heavy. From a common sense point of view this is peculiar since the nucleon is actually lighter than the heavy mesons being bound to it in the model. Hence, for comparison with experiment it is desirable to estimate kinematic effects associated with the nucleon's motion. These are expected [26] to lower the binding energy of the heavy baryons which

Table 4.3: Parameters for radially excited heavy baryons and mass differences with respect to Λ_c. A tilde refers to a radially excited P–wave baryon. All energies are in MeV.

α	-0.1	0.0	0.1	0.2	0.3
ω_P	125	113	101	90	79
χ_P	0.020	0.014	0.007	0.000	-0.007
$\tilde{\Lambda}_c$	429	419	409	398	388
$\tilde{\Sigma}_c$	625	618	607	598	589

have up to now come out too high (see Λ_c–N mass difference in Table 4.1, for example.). In order to estimate these kinematical effects in the bound state approach we have substituted the reduced masses

$$\frac{1}{M} \longrightarrow \frac{1}{M_{\rm cl}} + \frac{1}{M} \quad \text{and} \quad \frac{1}{M^*} \longrightarrow \frac{1}{M_{\rm cl}} + \frac{1}{M^*} \tag{4.4}$$

into the bound state equations. In a non–relativistic treatment this corresponds to the elimination of the center of mass motion [26]. The results for the spectrum of the heavy baryons obtained from the replacement (4.4) are in displayed in table 4.4. Again we consider α as a free parameter. We notice that there is a remarkable improvement in the prediction for the Λ_b mass, which was previously the worst one. The changes in some of the mass parameters can approximately be compensated by a suitable re–adjustment of α. For $\alpha \approx 0.0$ to -0.4 the agreement with the existing data is quite reasonable. When using the reduced meson masses the Σ_c baryon is always predicted a bit too light while it is too heavy when the physical masses are substituted in the bound state equation. For Λ_c' the situation is opposite. While the use of the physical meson masses gives too small a mass, the substitution of the reduced masses gives too large a prediction for the mass of this baryon. These results indicate that kinematical corrections are indeed important. It should, however, be remarked that with this replacement the heavy quark limit $\chi \to 0$ cannot be attained because the mass parameters in the bound state equations will remain finite and hence the linear relations (2.17) and (2.18) will not be satisfied. However, these relations are essential to verify the limit $\chi \to 0$ for $M = M^* \to \infty$. Nevertheless we think that the replacement (4.4) provides sensible insight in the relevance of kinematical corrections.

It is interesting to see how far the heavy quark approach can be pushed to lighter quarks. To answer this question we have considered the strange quark. In the corresponding kaon sector the P–wave is only very loosely bound when the physical masses are substituted. On the other hand sizable binding energies are obtained when the reduced masses are used [16].

Table 4.4: Parameters for heavy baryons and mass differences with respect to Λ_c. Primes indicate negative parity states, *i.e.* S–wave bound states. All energies are in MeV. In this calculation the reduced masses (4.4) enter the bound state equations from which the binding energies are extracted. The physical meson masses 1865MeV and 5279MeV are used when computing the mass differences to the nucleon and the Λ_b from these binding energies. Radially excited states are omitted because they are only very loosely bound, if at all. The empirical data are taken from the PDG [31], see also [32].

α	0.0	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	Expt.
ω_P	450	469	488	508	527	546	566	585	
χ_P	0.212	0.232	0.246	0.260	0.273	0.286	0.299	0.312	
ω_S	123	134	146	158	171	184	197	210	
χ_S	0.410	0.399	0.387	0.374	0.361	0.346	0.331	0.315	
Σ_c	158	154	151	148	145	143	140	138	168
Σ_c^*	221	223	225	226	227	229	230	231	233
Λ_c'	342	346	353	359	363	367	371	375	308
Σ_c'	460	468	475	484	490	497	505	512	?
$\Sigma_c'^*$	583	587	591	596	599	601	605	607	?
N	-1356	-1338	-1320	-1302	-1283	-1265	-1246	-1228	-1345
Λ_b	3285	3282	3280	3278	3275	3272	3271	3269	3356 ± 50

Table 4.5: Same as table 4.4 for even parity baryons in the kaon sector.

α	0.0	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	Expt.
ω_P	80	94	109	124	140	155	171	188	
χ_P	0.346	0.371	0.394	0.417	0.439	0.460	0.479	0.498	
Σ	131	126	121	117	112	108	104	100	77
Σ^*	235	237	239	242	244	246	248	250	269
N	-366	-354	-341	-327	-313	-300	-285	-269	-177

This behavior is somewhat different from the charm and bottom sector and can be understood by noting that the difference $M^* - M$ is considerably reduced when using (4.4). In the heavy sectors (charm and bottom) this difference is small in any event. The resulting spectrum for the strange baryons is shown in table 4.5. The comparison with the experimental data shows that even the use of the reduced masses does not provide sufficient binding. In the S–wave channel the situation is worse, even when the reduced masses are substituted bound states are not detected unless $\alpha \leq -1.0$. The failure of the present approach in the strange sector strongly suggests that for these baryons a chirally invariant set–up [12] is more appropriate.

5. *The Perturbative Approach*

The perturbative approach can illuminate several aspects of the hyperfine splitting problem. This is due to the heavy quark symmetry which is naturally exploited by making an expansion in powers of $1/M$ using the heavy field formalism. Our starting Lagrangian (2.5) has been set up in such a way as to yield a heavy quark symmetric result as $M \to \infty$ when $M = M^*$ is assumed, *cf.* eq (2.12). The perturbative $1/M$ expansion is more general (presumably exact) but less predictive. Thus the $1/M$ expansion provides a useful calibration in the large M limit. Since it deals with perturbation matrix elements it provides us with a convenient classification of the various sources of hyperfine splitting. The method is also advantageous in that it can be extended, without too much algebraic work, to different channels of interest. On the other hand, once the particular channels of interest are settled on, it is clearly more convenient to employ the exact numerical solution, which efficiently sums up a class of $1/M$ corrections.

The leading order Lagrangian (2.12) can be supplemented by terms which manifestly break the heavy quark symmetry to leading order (M^0 with the present normalization) as follows:

$$\frac{1}{M}\mathcal{L}'_H = \frac{M-M^*}{8}\mathrm{Tr}\left[H\sigma_{\mu\nu}\overline{H}\sigma^{\mu\nu}\right] + \frac{(d-d')}{2}\mathrm{Tr}\left[Hp_\mu\overline{H}\gamma^\mu\gamma_5\right]$$

$$-i\frac{\sqrt{2}(c-c')}{m_V}\mathrm{Tr}\left[\gamma_\mu\gamma_\nu H F^{\mu\nu}(\rho)\overline{H}\right]+\tilde{\alpha}V^\beta\mathrm{Tr}\left[H\sigma_{\mu\nu}\left(\tilde{g}\rho_\beta-v_\beta\right)\overline{H}\sigma^{\mu\nu}\right] \quad (5.1)$$

Here the $(M-M^*)$ term measures the heavy spin violation due to the heavy pseudoscalar – heavy vector mass difference. The $(d-d')$ term measures the heavy spin violation induced by choosing different coefficients for the fifth and sixth terms in eq (2.5) (or see eq (2) in ref [20]) while the $(c-c')$ term corresponds to choosing different coefficients for the last and next–to last terms in eq (2.5). Finally the $\tilde{\alpha}$ term corresponds to the leading term obtained by using different values of α in eqs (2.6) and (2.7). Note that $(M-M^*)$, $(d-d')$, $(c-c')$ and $\tilde{\alpha}$ all behave as $1/M$.

In addition to the terms in eq (5.1), which manifestly break the heavy quark symmetry, there are, in fact, "hidden" violation terms contained in eq (2.5). The explicit expression for the hidden terms in the model without light vectors is given in eq (11) of ref [20]. These were shown to exist (for the model without light vectors) in ref [20] and arise from performing a detailed $1/M$ expansion of the relativistic Lagrangian. In the sense of the chiral expansion these terms carry two derivatives, but nevertheless turn out to be very important numerically for the case considered. In the previous section the numerical study has confirmed that this is also true when light vector mesons are included. It was shown (*cf.* fig 2 of ref [20]) that the dependence on d of the hyperfine splitting computed from these hidden terms using the perturbative approach generally matched the exact numerical calculation. Hence we shall not explicitly isolate the extra hidden terms due to the addition of the light vectors but shall content ourselves with the numerical treatment given in the preceding section.

In the perturbative approach the collective Lagrangian involving the variable $A(t)$ is obtained by substituting

$$\overline{H}(\boldsymbol{x},t)=A(t)\overline{H_c}(\boldsymbol{x}) \ , \quad (5.2)$$

where $\overline{H_c}(\boldsymbol{x})$ is the heavy meson bound–state wave function, into the heavy field Lagrangian. Clearly this is the analog of the replacement (3.4). (The treatment of the chiral Lagrangian of the light pseudoscalars and vectors is the same as in section 3.) The bound–state wave–function is conveniently presented in the rest frame, $V_\mu=(1,\mathbf{0})$, where

$$\overline{H_c}\rightarrow\begin{pmatrix}0 & 0\\ \overline{h}^a_{lh} & 0\end{pmatrix} \ , \quad (5.3)$$

with a, l, h representing respectively the isospin, light spin and heavy spin bivalent indices. Due to the hedgehog structure of the soliton profiles, the calculation is simplified if we deal with a reduced wave–function obtained after removing the factor $\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}$:

$$\overline{h}^a_{lh}=\frac{u(r)}{\sqrt{M}}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}\right)_{ad}\psi_{dl,h} \ , \quad (5.4)$$

where* $u(r)$ is a radial wave function, assumed to be very sharply peaked near $r = 0$ for large M. In the leading order of $1/M$ there is no violation of the heavy quark symmetry and we may perform a partial wave analysis of $\psi_{dl,h}$

$$\psi_{dl,h}(g, g_3, r, k) = \sum_{r_3,k_3} C^{r,k;g}_{r_3,k_3;g_3} Y^{r_3}_r \xi_{dl}(k, k_3)\chi_h \ . \tag{5.5}$$

Here $Y^{r_3}_r$ stands for the standard spherical harmonics representing orbital angular momentum r and C denotes ordinary Clebsch–Gordan coefficients. The heavy spinor χ_h is trivially factored in this expression as a manifestation of the heavy quark symmetry. Furthermore $\xi_{dl}(k, k_3)$ represents a wave–function in which the light spin and isospin are added vectorially to give $\boldsymbol{K} = \boldsymbol{I}_{\rm light} + \boldsymbol{S}_{\rm light}$ with eigenvalues $\boldsymbol{K}^2 = k(k + 1)$. The total "light grand spin"

$$\boldsymbol{g} = \boldsymbol{r} + \boldsymbol{K} \tag{5.6}$$

is the significant quantity in the heavy limit. The dynamics of the model dictates that the bound-states occur for $k = 0$, in which case $\xi_{dl}(0, 0) = \epsilon_{dl}/\sqrt{2}$. The bound-state wave–function simply is

$$\psi_{dl,h}(0, 0, 0, 0) = \frac{1}{\sqrt{8\pi}}\epsilon_{dl}\chi_h \ . \tag{5.7}$$

The $k = 1$ unbound wave–function with no orbital excitation ($r = 0$) is

$$\psi_{dl,h}(1, g_3, 0, 1) = \frac{1}{\sqrt{4\pi}}\xi_{dl}(1, g_3)\chi_h \ . \tag{5.8}$$

When violations of the heavy quark symmetry are included, g is no longer a good quantum number. We define the grand spin, which is a good quantum number, as,

$$\boldsymbol{G'} = \boldsymbol{g} + \boldsymbol{S}_{\rm heavy} \ . \tag{5.9}$$

In the notation of eqs (5.7) and (5.8) we have the grand spin eigenstates

$$\psi^{(1)}_{dl,h}(G' = G'_3 = 1/2) = \frac{1}{\sqrt{8\pi}}\epsilon_{dl}\delta_{2h} \ , \tag{5.10}$$

$$\psi^{(2)}_{dl,h}(G' = G'_3 = 1/2) = \frac{1}{\sqrt{4\pi}}\left[\sqrt{\frac{2}{3}}\delta_{d1}\delta_{l1}\delta_{h1} + \frac{1}{\sqrt{6}}\left(\delta_{d2}\delta_{l1} + \delta_{d1}\delta_{l2}\right)\delta_{h2}\right] \ . \tag{5.11}$$

Note that in eq (5.10) the $G'_3 = +1/2$ wave function is δ_{2h} since the index 2 corresponds to $+1/2$ for the anti–quark wave–function. The two states (5.10) and (5.11) differ with respect to their $\boldsymbol{g}$ and $\boldsymbol{K}$ labels.

*We have removed a factor of $1/\sqrt{4\pi}$ compared to ref [20], since it is now carried by the spherical harmonic in eq (5.5).

Now let us consider the potential for the bound-state wave–function in the presence of the first heavy quark symmetry violating term in eq (5.1). Substituting the G'–eigenstates $\psi^{(1)}$ and $\psi^{(2)}$ from eqs (5.10) and (5.11) into eq (2.12) and the first term of eq (5.1) yields, after a spatial integration, the potential matrix in the $\psi^{(1)}$–$\psi^{(2)}$ space:

$$V = -\frac{d\,F'(0)}{2}\begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix} + \frac{M-M^*}{4}\begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & 2 \end{pmatrix} \ , \tag{5.12}$$

where $F(r)$ is defined in eq (2.4) and $F' = dF/dr$. The first matrix shows that $\psi^{(1)}$ is bound while $\psi^{(2)}$ is unbound in the heavy spin limit. Since the second matrix gives mixing between $\psi^{(1)}$ and $\psi^{(2)}$ the latter must be included in the presence of effects which break the heavy quark symmetry. The diagonalized bound wave function is seen to be

$$\psi^{(1)} - \frac{\sqrt{3}}{8}\frac{M-M^*}{d\,F'(0)}\psi^{(2)} \ . \tag{5.13}$$

This is the proper wave–function to be "cranked" in order to generate the heavy spin violation. Using it in eq (5.2), which is then substituted into the $\alpha = 0$ limit of the first term of eq (2.12), contributes a term in the collective Lagrangian

$$\frac{\chi}{2}\Omega_3 \qquad \text{where} \qquad \chi = \frac{M^*-M}{4d\,F'(0)} \ . \tag{5.14}$$

By using the Wigner–Eckart theorem we may express this for states of either G'_3 as the matrix element of the operator $\chi\boldsymbol{\Omega}\cdot\boldsymbol{G'}$. For convenience we have chosen to consider our wave–function as representing the conjugate particle in eq (5.4). Hence the matrix element of $\boldsymbol{G'}$ in this section differs by a minus sign from that of $\boldsymbol{G}$ defined in eq (3.8). The latter is the appropriate one when we form the total heavy baryon spin $\boldsymbol{J} = \boldsymbol{G} + \boldsymbol{J}^{\rm sol}$, with $J_i^{\rm sol} \equiv (\partial L/\partial\Omega_i)$. Then the collective Lagrangian, $L_{\rm coll}$ may be written (see section 3)

$$L_{\rm coll} = \frac{1}{2}\alpha^2\boldsymbol{\Omega}^2 - \chi\boldsymbol{\Omega}\cdot\boldsymbol{G} \tag{5.15}$$

which again leads to the Hamiltonian (3.10). Substituting $\alpha^2 = (3/2)\,[m(\Delta) - m(N)]$ in eq (3.10) we get the well known formula, *cf.* eq (1.1)

$$m(\Sigma_Q^*) - m(\Sigma_Q) = [m(\Delta) - m(N)]\ \chi \ . \tag{5.16}$$

The purpose in deriving this again was to explain the perturbative method and our notation.

Next we shall give some new perturbative "manifest" contributions to χ from eq (5.1). When all these terms are included the potential V in eq (5.12) is modified so that the properly diagonalized wave–function which replaces eq (5.13) becomes

$$\psi^{(1)} + \epsilon\psi^{(2)} \ , \tag{5.17}$$

with

$$\epsilon = \frac{-\frac{\sqrt{3}}{4}(M-M^*) + \frac{\sqrt{3}}{4}(d-d')\ F'(0) + \sqrt{3}\tilde{\alpha}\omega(0) - \sqrt{6}\frac{c-c'}{m_V}\frac{G''(0)}{g}}{2d\,F'(0) + \frac{2\sqrt{2}cG''(0)}{gm_V}} \ . \tag{5.18}$$

There are two types of contribution to χ. The first type is analogous to eq (5.14) and arises when eq (5.17) is cranked and substituted into eq (2.12). The second type is obtained by substituting the leading order wave function $\psi^{(1)}$ into the $(c-c')$ and $\tilde{\alpha}$ terms in eq (5.1). The complete expression for χ resulting from the "manifest" heavy spin violation is

$$\chi = \epsilon\left[\frac{2}{\sqrt{3}}\left(1-\frac{4}{3}\alpha\right) + \frac{2}{3\sqrt{3}}\alpha g\left(\xi_1(0)-\xi_2(0)\right) - 8\sqrt{\frac{2}{3}}\frac{c}{m_V}\varphi''(0)\right]$$
$$+\frac{\tilde{\alpha}}{3}\left[8-2g\left(\xi_1(0)-\xi_2(0)\right)\right] - 4\sqrt{2}\frac{c-c'}{m_V}\varphi''(0) \ . \tag{5.19}$$

The quantities $\xi_1(0)$, $\xi_2(0)$ and $\varphi''(0)$ are defined in eq (3.3). This formula may be useful for quickly estimating the effects of heavy spin violation in the coupling constants, which were not explicitly given in the previous discussion. Unfortunately there is no determination of the magnitude of these effects from the mesonic sector at present. In ref [20] the discussion of the "hidden" heavy contributions to χ was given for the Lagrangian with only light pseudoscalars.

The hyperfine splitting just discussed is for the ground state or P–wave heavy baryons. It is of some interest to briefly consider the negative parity heavy baryons with one unit of orbital excitation. In the heavy spin limit these bound states correspond to the $r=1$ and $k=0$ choice in eq (5.5):

$$\psi_{dl,h}(1,g_3,1,0) = \frac{\epsilon_{dl}}{\sqrt{2}}Y_1^{g_3}\chi_h \ . \tag{5.20}$$

The spin, $\boldsymbol{J}_{\rm light}$ of the "light cloud" part of the heavy baryon is gotten by adding this $g=1$ piece to the soliton spin $\boldsymbol{J}^{\rm sol}$. For the $I=0$ (which implies $J^{\rm sol}=0$) heavy baryons one finds $J_{\rm light}=1$ and the degenerate multiplet

$$\left\{\Lambda'_Q(1/2)\,,\ \Lambda'_Q(3/2)\right\} \ . \tag{5.21}$$

For the $I=J^{\rm sol}=1$ heavy baryons, $J_{\rm light}$ can be either 0, 1 or 2 and we find the degenerate heavy multiplets

$$\Sigma'_Q(1/2) \ ,$$
$$\left\{\Sigma'_Q(1/2)\,,\ \Sigma'_Q(3/2)\right\} \ ,$$
$$\left\{\Sigma'_Q(3/2)\,,\ \Sigma'_Q(5/2)\right\} \ . \tag{5.22}$$

In general, the situation is even more complicated and further discussion will be given elsewhere. At present there are experimental candidates[31] for a negative parity spin 1/2 baryon Λ_c' at 2593.6 ± 1.0 MeV and a negative parity spin 3/2 baryon Λ_c' at 2626.4 ± 0.9 MeV.

Since experimental information is available, it is especially interesting to consider the splitting between the two Λ_Q' states in eq (5.21). This splitting stems from the violation of the heavy quark symmetry. For the Λ_Q type states the total spin coincides with the grand spin $\boldsymbol{G}$ so that eq (5.21) may be alternatively considered a $G = 1/2$, $G = 3/2$ multiplet. Since the good quantum number is G, we may in general expect the hyperfine parameter χ to depend on G. The collective Hamiltonian takes the form

$$H_{\text{coll}} = \frac{\left(\boldsymbol{J}^{\text{sol}} + \chi_G \boldsymbol{G}\right)^2}{2\alpha^2} \ . \tag{5.23}$$

On general grounds we see that for the case of the Λ_Q''s the collective Hamiltonian contribution to the hyperfine splitting will be suppressed. Setting $\boldsymbol{J}^{\text{sol}} = 0$ in eq (5.23) shows that the hyperfine splitting is of order (χ^2) or equivalently of order $(1/M^2)$. Unlike the ground state which involves only the $G = 1/2$ P–wave channel, there is another possibility for hyperfine splitting here. It is allowed for the $G = 1/2$ and $G = 3/2$ bound state energies to differ from each other. In the Lagrangian with only light pseudoscalars this does not happen and the $\Lambda_Q'(1/2) - \Lambda_Q'(3/2)$ splitting is of order $1/M^2$. However when light vectors are added, there are "hidden" $1/M$ terms, which violate the heavy quark symmetry as *e.g.*

$$i \ \text{Tr} \left[\sigma_{\alpha\mu} H \gamma_\nu F^{\mu\nu}(\rho) D^\alpha \overline{H}\right] \ + \ \text{h.c.} \ . \tag{5.24}$$

This term is likely to generate splitting for the multiplet (5.21) to order $1/M$ by giving different binding energies to the $G = 1/2$ and $G = 3/2$ channels. It would be very interesting to investigate this in more detail.

Finally, we add a remark concerning an amusing conceptual feature in the computation of hyperfine splitting among the five Σ_Q''s in eq (5.22). The total angular momentum of each state is given by

$$\boldsymbol{J} = \underbrace{\boldsymbol{J}^{\text{sol}} + \overbrace{\boldsymbol{g}}^{}}_{\boldsymbol{J}_{\text{light}}}\!\!\!\!\overbrace{\phantom{\boldsymbol{g}} + \boldsymbol{S}_{\text{heavy}}}^{\boldsymbol{G}} \ , \tag{5.25}$$

where we are now considering each operator to be acting on the wave–function rather than its complex conjugate. We have illustrated two different intermediate angular momenta which can alternatively be used to label the final state. If $\boldsymbol{J}_{\text{light}}$ is used, we get the heavy-spin multiplets in eq (5.22). On the other hand, when the hyperfine splitting is turned on, the choice $\boldsymbol{G}$ is convenient, because it remains a good quantum number. According to the laws of quantum mechanics, we cannot simultaneously use both to specify the states, since the commutator

$$\left[\boldsymbol{J}^2_{\rm light}\ ,\ \boldsymbol{G}^2\right] = 4i\boldsymbol{J}_{\rm light}\cdot(\boldsymbol{S}_{\rm heavy}\times\boldsymbol{g}) \tag{5.26}$$

is generally non–vanishing. This means that we cannot uniquely trace the splitting of, say, the $\left\{\Sigma'_Q(1/2), \Sigma'_Q(3/2)\right\}$ heavy multiplet in eq (5.22), as hyperfine splitting interactions are turned on. Physically, this causes a mixing between the Σ'_Q's of the same spin. Rather, we must look at the whole pattern of the five masses. On the other hand, the problem simplifies for the computation of the ground state hyperfine splitting in eq (5.16). In that case the bound state wave function is characterized by $\boldsymbol{g} = 0$. Thus the commutator in eq (5.26) vanishes, and it is "trivially" possible to track the hyperfine splitting as a mass difference.

6. Discussion and Conclusions

In the framework of the bound state approach we have studied the hyperfine splitting for baryons containing a heavy quark. In this approach a heavy baryon is constructed from a heavy meson configuration bound in the background of a (chiral) soliton. Here we have limited ourselves to heavy mesons in the S– and P– wave channels, which exhibit the strongest binding. The study has been motivated by the earlier observation that light vector meson fields are required in the soliton configuration in order to reasonably describe the spectra of both the light and the heavy baryons when all available information on coupling constants of the elementary mesons is incorporated. The inclusion of light vector mesons causes some technical difficulties because field components which vanish classically are induced when time–dependent collective coordinates are introduced in order to generate states with good spin and isospin from the soliton. One might argue that the better agreement in the vector meson model is due to an additional parameter α; however, we have observed that the agreement is achieved for quite a wide range of this parameter. In fact the binding energies vary by only about 100MeV in the range $-0.1 \le \alpha \le 0.3$. On the other hand the discrepancy between the empirical data and the predictions obtained from the Skyrme model soliton is about twice as large. In addition the vector meson model reasonably reproduces the relative (to the nucleon) masses for both the charm and the bottom sector. Furthermore the mass difference within a given heavy multiplet, *i.e.* the hyperfine splitting, has turned out not to be very sensitive to that parameter either.

We also have estimated kinematical corrections by substituting the reduced masses. The comparison with the empirical data has certainly indicated that these corrections are important. This simple non-relativistic substitution fails, however, to satisfy the heavy quark limit result, which states that the hyperfine splitting should vanish for infinitely large quark masses. It thus seems interesting to further explore the kinematical corrections.

As an extension of earlier work [20] we have illuminated the systematics of the $1/M$ expansion of the hyperfine splitting. The main conclusion is that the dominant contribution stems from terms in the relativistic Lagrangian which do not manifestly break the heavy quark symmetry.

We have furthermore observed that the heavy quark approach does not seem to be suitable for the strange sector. The binding energies simply turned out too low for reasonable predictions of the mass differences between the heavy baryons and the nucleon.

On the other hand an interesting path to pursue would be the extension of the light sector to flavor $SU(3)$. This would make possible the description of baryons like Ξ_c or Ξ_b^*. This would in particular be interesting for the issue of flavor symmetry breaking [5]. Unfortunately the vector meson model for three flavors requires the introduction of additional induced components for the strange degrees of freedom [30].

Acknowledgements

This work was supported in part by the US DOE contract number DE–FG–02–85ER 40231 and by the Deutsche Forschungsgemeinschaft (DFG) under contract Re 856/2–2.

Appendix A: Bound State Lagrangian

In this appendix we present the Lagrangian for the *ansätze* (2.13)–(2.16) of the bound heavy mesons. These expressions have already been presented in appendix A of ref [16]. Unfortunately some typographical errors have occurred in the formulas reported there. It is therefore appropriate to list the corrected expressions. The present notation corresponds to eq (2.19).

Substituting (2.15) and (2.16) in (2.5) gives for the S–wave channel

$$
\begin{aligned}
I_\epsilon^{(S)} = \int dr r^2 \Bigg(& \Phi'^2 + \left[M^2 - \left(\epsilon - \frac{\alpha}{2}\omega\right)^2 + \frac{R_\alpha^2}{2r^2}\right]\Phi^2 + M^{*2}\left[\Psi_1^2 + 2\Psi_2^2 - \Psi_0^2\right] \\
& + \frac{2}{r^2}\left[r\Psi_2' + \Psi_2 - \frac{R_\alpha + 2}{2}\Psi_1\right]^2 + \frac{R_\alpha^2}{r^2}\Psi_2^2 - \left[\Psi_0' - \left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_1\right]^2 \\
& - 2\left[\left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_2 - \frac{R_\alpha + 2}{2r}\Psi_0\right]^2 + 2Md\left[F'\Psi_1 + \frac{2}{r}\sin F\Psi_2\right]\Phi \\
& + 2d\Bigg\{F'\left[\frac{1}{r}\left(1 + \cos F\right)\Psi_0\Psi_2 - \left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_2^2\right] \\
& \qquad + \frac{2}{r}\sin F\left[\Psi_2\Psi_0' - \left(\epsilon - \frac{\alpha}{2}\omega\right)\Psi_1\Psi_2 + \frac{R_\alpha + 2}{2r}\Psi_0\Psi_1\right]\Bigg\} \\
& + \frac{4\sqrt{2}cM}{gm_V}\left[\omega'\Psi_0\Psi_1 + \frac{2G'}{r}\Psi_1\Psi_2 + \frac{G}{r^2}\left(G + 2\right)\Psi_2^2\right]
\end{aligned}
$$

$$
-\frac{4\sqrt{2}c}{gm_V}\left\{\frac{\omega'}{r}R_\alpha\Phi\Psi_2+\frac{G}{r^2}\left(G+2\right)\Psi_0\Phi'+\frac{G'}{r}R_\alpha\Phi\Psi_0\right.
$$
$$
\left.\left.+\left(\epsilon-\frac{\alpha}{2}\omega\right)\left[\frac{2G'}{r}\Psi_2+\frac{G}{r^2}\left(G+2\right)\Psi_1\right]\Phi\right\}\right). \tag{A.1}
$$

Here a prime indicates a derivative with respect to the radial coordinate r. Furthermore the abbreviation $R_\alpha = \cos F - 1 + \alpha\left(1 + G - \cos F\right)$ has again been used.

For the P–wave channel one obtains upon substitution of the *ansatz* (2.13) and (2.14)

$$
\begin{aligned}
I_\epsilon^{(P)} \;=\; & \int dr r^2\Bigg(\Phi'^2+\left[M^2-\left(\epsilon-\frac{\alpha}{2}\omega\right)^2+\frac{2}{r^2}\left(1+\frac{1}{2}R_\alpha\right)^2\right]\Phi^2+M^{*2}\left[\Psi_1^2+\frac{1}{2}\Psi_2^2-\Psi_0^2\right] \\
& +\frac{1}{2}\left[\Psi_2'-\frac{1}{r}\Psi_2\right]^2+\frac{1}{r}R_\alpha\Psi_1\Psi_2'+\frac{1}{r^2}R_\alpha\left(\Psi_1+\Psi_2\right)\Psi_2+\frac{1}{2r^2}R_\alpha^2\left(\Psi_1^2+\frac{1}{2}\Psi_2^2\right) \\
& -\left[\Psi_0'-\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_1\right]^2-\frac{1}{2}\left[\frac{R_\alpha}{r}\Psi_0+\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_2\right]^2 \\
& -d\Bigg\{\frac{2}{r}\sin F\left[\Psi_2\Psi_0'-\frac{R_\alpha}{r}\Psi_0\Psi_1-\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_1\Psi_2\right] \\
& \qquad +\frac{F'}{r}\left[\frac{r}{2}\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_2^2-\left(1-\cos F\right)\Psi_0\Psi_2\right]\Bigg\}+2Md\left[F'\Psi_1-\frac{\sin F}{r}\Psi_2\right]\Phi \\
& +\frac{2\sqrt{2}cM}{gm_V}\left[2\omega'\Psi_0\Psi_1-\frac{2G'}{r}\Psi_1\Psi_2+\frac{G}{2r^2}\left(G+2\right)\Psi_2^2\right] \\
& -\frac{4\sqrt{2}c}{gm_V}\Bigg\{\frac{1}{r^2}\left(\epsilon-\frac{\alpha}{2}\omega\right)\left[G\left(G+2\right)\Psi_1-rG'\Psi_2\right]\Phi-\frac{\omega'}{r}\left[1+\frac{R_\alpha}{2}\right]\Psi_2 \\
& \qquad +\frac{1}{r^2}\left[G\left(G+2\right)\Phi'+G'\left(2+R_\alpha\right)\Phi\right]\Psi_0\Bigg\}\Bigg).
\end{aligned} \tag{A.2}
$$

The typographical errors in ref [16] only affect the expressions involving the parameter c.

Appendix B: Hyperfine Parameters

In this appendix we give the explicit expressions for the hyperfine splitting parameters used in section 4. For convenience we employ additional abbreviations with regard to the light meson profiles defined in eqs (2.4) and (3.3)

$$
\begin{aligned}
V_1 &= \cos F - \alpha\left(\xi_1 - 1 + \cos F\right)\ , \\
V_2 &= 1 - \alpha\left(\xi_1 + \xi_2\right)\ .
\end{aligned}
$$

The explicit expression for the P–wave hyperfine parameter, which enters the mass formula for the even parity heavy baryon (3.11), reads

$$
\chi_P \;=\; \frac{2}{3}\int_0^\infty dr\ r^2\ \rho_\chi^{(P)}(r) \tag{B.1}
$$

$$
\begin{aligned}
\rho_\chi^{(P)}(r) \;=\; & \left[\left(\epsilon-\frac{\alpha}{2}\omega\right)(V_2-2V_1)-\frac{2\alpha}{r^2}\left(2+R_\alpha\right)\varphi\right]\Phi^2 \\
& +\left(2V_1+V_2\right)\left[\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_1-\Psi_0'\right]\Psi_1 \\
& -\frac{1}{2}\left(V_2\Psi_2+\frac{4\alpha}{r}\varphi\Psi_0\right)\left[\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_2+\frac{R_\alpha}{r}\Psi_0\right] \\
& +\frac{2\alpha}{r}\varphi\Psi_1\left(\Psi_2'+\frac{1}{r}\Psi_2+\frac{R_\alpha}{r}\Psi_1\right)-\frac{\alpha}{r^2}\left(2+R_\alpha\right)\varphi\Psi_2^2+4Md\,\sin F\Phi\Psi_0 \\
& -\frac{d}{r}\left\{\sin F\left[\left(2+R_\alpha+V_1\right)\Psi_1\Psi_2-\frac{4\alpha}{r}\varphi\Psi_0\Psi_1\right]+F'\left[\frac{r}{4}V_2\Psi_2^2+2\alpha\varphi\Psi_0\Psi_2\right]\right\} \\
& -\frac{4\sqrt{2}cM}{gm_\rho}\left\{\left(3\xi_1'+\xi_2'\right)\Psi_0\Psi_1+\frac{G}{r}\left(2-2\xi_1-\xi_2\right)\Psi_0\Psi_2+\frac{2}{r}\varphi'\Psi_1\Psi_2+\frac{1}{r^2}\varphi\Psi_2^2\right\} \\
& -\frac{4\sqrt{2}c}{gm_\rho}\left\{\left(\epsilon-\frac{\alpha}{2}\omega\right)\left(\frac{4}{r^2}\varphi\Psi_1+\frac{2}{r}\varphi'\Psi_2\right)\Phi+\left(V_1-\frac{V_2}{2}\right)\left[\frac{G}{r^2}\left(G+2\right)\Psi_1-\frac{G'}{r}\Psi_2\right]\Phi\right. \\
& +\frac{G}{r^2}\left(2+R_\alpha\right)\left(2\xi_1+\xi_2-2\right)\Phi\Psi_1+\frac{2}{r^2}\left[2\varphi\Phi'+\left(2+R_\alpha\right)\varphi'\Phi-\alpha G'\varphi\Phi\right]\Psi_0 \\
& \left.-\frac{1}{r}\left[\left(G+2\right)\xi_2\Phi'+\left(1+\frac{1}{2}R_\alpha\right)\left(\xi_1'+\xi_2'\right)\Phi-\alpha\omega'\varphi\Phi\right]\Psi_2\right\}
\end{aligned}
\tag{B.2}
$$

It can easily be verified that the terms involving $\left(\epsilon-\frac{\alpha}{2}\omega\right)\approx M$ cancel when the heavy limit relations for the radial functions (2.17) are substituted. Taking into account that the radial functions which parametrize the heavy meson wave–functions are normalized to $1/\sqrt{|\epsilon|}\approx 1/\sqrt{M}$ (*cf.* eq (2.21)), it is obvious that the hyperfine parameter χ_P vanishes in the heavy quark limit..

The hyperfine parameter for the odd parity baryon (*cf.* eq (3.13)) is found to be

$$
\chi_S \;=\; \frac{2}{3}\int_0^\infty dr\; r^2\;\rho_\chi^{(S)}(r) \tag{B.3}
$$

$$
\begin{aligned}
\rho_\chi^{(S)}(r) \;=\; & \left[\left(\epsilon-\frac{\alpha}{2}\omega\right)(2V_1+V_2)+\frac{2\alpha}{r^2}R_\alpha\varphi\right]\Phi^2+\left(2V_1-V_2\right)\left[\Psi_0'-\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_1\right]\Psi_1 \\
& +2\left[\frac{R_\alpha+2}{2r}\Psi_0-\left(\epsilon-\frac{\alpha}{2}\omega\right)\Psi_2\right]\left(V_2\Psi_2+2\frac{\alpha}{r}\varphi\Psi_0\right)+4\frac{\alpha}{r^2}R_\alpha\varphi\Psi_2^2 \\
& +4\frac{\alpha}{r^2}\varphi\left[r\Psi_2'+\Psi_2-\frac{R_\alpha+2}{2}\Psi_1\right]\Psi_1-4Md\,\sin F\Phi\Psi_0 \\
& -d\left\{F'\left(V_2\Psi_2+\frac{4\alpha}{r}\varphi\Psi_0\right)\Psi_2+\frac{2}{r}\sin F\left[\left(V_1+R_\alpha\right)\Psi_2+\frac{2\alpha}{r}\varphi\Psi_0\right]\Psi_1\right\} \\
& -\frac{4\sqrt{2}cM}{m_Vg}\left\{\left(\xi_2'-\xi_1'\right)\Psi_0\Psi_1+\frac{2}{r}\left(G+2\right)\xi_2\Psi_0\Psi_2+\frac{4}{r}\varphi'\Psi_1\Psi_2+\frac{4}{r^2}\varphi\Psi_2^2\right\} \\
& -\frac{4\sqrt{2}c}{m_Vg}\left\{\frac{2}{r}G\left(2\xi_1+\xi_2-2\right)\Phi'\Psi_2+\frac{R_\alpha}{r^2}\left(G+2\right)\xi_2\Phi\Psi_1\right. \\
& \qquad +\frac{1}{r}\left[R_\alpha\left(\xi_1'+\xi_2'\right)+2\alpha\omega'\varphi\right]\Phi\Psi_2+\frac{2}{r^2}\left[2\varphi\Phi'-\left(R_\alpha\varphi'-\alpha G'\varphi\right)\Phi\right]\Psi_0
\end{aligned}
$$

$$+\frac{4}{r^2}\left(\epsilon-\frac{\alpha}{2}\omega\right)(\varphi\Psi_1+r\varphi'\Psi_2)\,\Phi$$
$$-\left(V_1+\frac{1}{2}V_2\right)\left[\frac{1}{r^2}G\,(G+2)\,\Psi_1+\frac{2}{r}G'\Psi_2\right]\Phi\Bigg\}\ . \qquad \text{(B.4)}$$

Again, it can easily be verified that the terms involving $\left(\epsilon-\frac{\alpha}{2}\omega\right)\approx M$ vanish when the heavy limit relations for the radial functions (2.18) are substituted. With regard to the normalization condition (2.22) the S–wave hyperfine parameter also behaves like $\chi_S\sim 1/M$ in the heavy quark limit.

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