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Chiral Phase Transition for $SU(N)$ Gauge Theories via an Effective Lagrangian Approach.

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Abstract

We study the chiral phase transition for vector-like $SU(N)$ gauge theories as a function of the number of quark flavors N_f by making use of an anomaly-induced effective potential. We modify an effective potential of a previous work, suggested for $N_f < N$, and apply it to larger values of N_f where the phase transition is expected to occur. The new effective potential depends explicitly on the full β -function and the anomalous dimension γ of the quark mass operator. By using this potential we argue that chiral symmetry is restored for $\gamma < 1$. A perturbative computation of γ then leads to an estimate of the critical value N_f^c for the transition.

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I. INTRODUCTION

The phase structure of strongly coupled gauge field theories as a function of the number of matter fields N_f is a problem of general interest. Much has been learned about the phases of supersymmetric theories in recent years [1–5]. An equally interesting problem is the phase structure of a non-supersymmetric $SU(N)$ theory as a function of the number of fermion fields N_f . At low enough values of N_f , the chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ is expected to break to the diagonal subgroup. At some value of N_f less than $11N/2$ (where asymptotic freedom is lost), there will be a phase transition to a chirally symmetric phase. Whether the transition takes place at a relatively small value of N_f [6] or a larger value remains unknown. The larger value ($N_f/N \approx 4$) is suggested by studies of the renormalization group improved gap equation [7] and is associated with the existence of an infrared fixed point. A recent analysis [8] indicates that instanton effects could also trigger chiral symmetry breaking at comparably large value of N_f/N . Besides being of theoretical interest, the physics of a chiral transition could have consequences for electroweak symmetry breaking [9], since near-critical gauge theories provide a natural framework for walking technicolor theories [10].

If a phase transition is second order, a useful approach is to find a tractable model in the same universality class. For chiral symmetry, a natural order parameter is the $N_f \times N_f$ complex matrix field M describing mesonic degrees of freedom. If the meson degrees of freedom are the only ones that develop large correlation lengths at the phase transition, then the transition may be studied using an effective Landau-Ginzburg theory. This time-honored approach has been used, for example, to study the QCD finite temperature transition for $N_f = 2$ [11].

For the zero-temperature transition as a function of N_f , a similar approach might also be tried. It was suggested in Ref. [7], however, that while the order parameter vanishes continuously as $N_f \rightarrow N_f^c$, the transition is not second order. With the gap equation dominated by an infrared fixed point of the gauge theory, the transition was argued to be continuous but infinite order. It has also been noted [12] that because of the associated long range conformal symmetry, the masses of all the physical states, not just the scalar mesons are expected to scale to zero with the order parameter.

In this paper we nevertheless suggest that an effective potential using only the low lying mesonic degrees of freedom might be employed to model at least some aspects of the zero-temperature chiral phase transition. The key ingredient is the presence of a new non-analytic potential term that emerges naturally once the anomaly structure of the theory is considered. The anomalies also provide a link between this effective potential term and the underlying

gauge theory.

To deduce the anomaly induced effective potential we modify an effective potential [13–15] developed for $N_f < N$, and apply it to the range $N_f > N$. The effective potential of Refs. [13–15] was suggested by starting with the effective Lagrangian for super-QCD and considering how the gluinos and squarks decouple below a supersymmetric breaking scale m_s . In Reference [14], it was noted that this potential can also be constructed, once the trace and axial anomaly constraints are saturated at one loop, by assuming holomorphicity.

In Section II, we set the stage by providing a brief review of the SUSY QCD effective potential for $N_f < N$, and comparing it to the one-loop, anomaly-induced effective QCD potential of Refs. [13–15]. In Section III an effective potential valid to all orders in the loop expansion, and appropriate for the range $N_f > N$, is proposed. It utilizes only mesonic variables to capture the low energy dynamics. In Section IV we use this potential to discuss the zero-temperature phases of an $SU(N)$ gauge theory as a function of N_f . We use the singular behavior of the curvature of the effective potential at the origin as a signal for chiral restoration. Assuming that the transition is governed by an infrared fixed point of the theory, we deduce that chiral symmetry is restored, together with long-range conformal symmetry, when $\gamma < 1$, where γ is the anomalous dimension of the mass operator. Finally we note that by using the perturbative expansion of γ , chiral symmetry is predicted to be restored above $N_f^c \approx 4N$, in agreement with a gap equation analysis. In Section V we summarize our results and provide some discussion. In Appendix A we examine some higher loop effects in the effective potential.

II. REVIEW OF THE ANOMALY-INDUCED EFFECTIVE POTENTIAL

We start by recalling the role of the effective potential in super QCD theories. For $N_f < N$, the effective low energy superpotential takes the Affleck-Dine-Seiberg (ADS) [16] form

$$W_{ADS}(T) = -C_s(N, N_f) \left[\frac{\Lambda_S^{3N-N_f}}{\det T} \right]^{\frac{1}{N-N_f}}, \quad (2.1)$$

where the composite meson superfield T has the same quantum numbers as $Q\tilde{Q}$, with Q and \tilde{Q} being the quark superfields, and Λ_S is the intrinsic scale of super QCD (SQCD). In this instanton-generated super potential, the exponent of Λ_S is the coefficient of the lowest order term in the supersymmetric β function. Through a suitable decoupling procedure, one can show [1] that the function $C_s(N, N_f)$ takes the form $C_s \propto (N - N_f) K^{1/(N-N_f)}$ where K is an

arbitrary constant independent of the number of colors and flavors. By an explicit instanton calculation one finds $K = 1$. In supersymmetric theories the axial anomaly together with the superconformal anomaly can be cast in the same chiral supermultiplet. This fact together with the holomorphic constraint has led to the idea that the ADS superpotential can be constructed by using only the information contained at one-loop in the underlying theory.

According to the ADS potential there is no stable vacuum in the massless theory for any $N_f < N$. Furthermore, the ADS superpotential is singular for $N_f = N$. Seiberg argued that the superpotential should be modified for $N_f = N$ and that the singularity signals the occurrence of new, massless degrees of freedom. In the case $N_f = N$, these massless degrees of freedom are identified with the superfield baryon $B \propto \epsilon^{c_1, \dots, c_N} \epsilon_{i_1, \dots, i_{N_f}} Q_{c_1}^{i_1} \cdots Q_{c_N}^{i_{N_f}}$ (a similar construction holds for the \tilde{B} field). Depending on the choice of the vacuum, chiral symmetry can be either broken or unbroken. For $N_f > N$, a variety of phases is possible depending on N_f [1].

In Refs. [13,14], an attempt was made to construct a potential in the same spirit as the ADS superpotential for a (non-SUSY) $SU(N)$ gauge theory by saturating at one-loop the energy-momentum-trace and axial anomalies and imposing holomorphicity. The result, for $N_f < N$, was

$$V = -C(N, N_f) \left[\frac{\Lambda^{\frac{11}{3}N - \frac{2}{3}N_f}}{\det M} \right]^{\frac{12}{11(N - N_f)}} + \text{h.c.}, \quad (2.2)$$

where M_i^j is the $N_f \times N_f$ complex matrix field possessing the same quantum numbers as $q_i \tilde{q}^j$. Upon quantization, it would describe mesonic degrees of freedom. $C(N, N_f)$ is a coefficient which, after defining a suitable one loop decoupling procedure, turns out to be proportional to $(N - N_f) D(N)^{1/(N - N_f)}$ where $D(N)$ is an unknown function of N . Λ was taken to be the confinement scale of the theory and its exponent in Eq. (2.2) is the first coefficient in the perturbative expansion of the β function. For $N_f < N$ this potential displays a fall to the origin.

In Ref. [13] it was noted that the fall to the origin can be cured by introducing non holomorphic terms that implement spontaneous chiral symmetry breaking. The non-holomorphic piece is constructed so that it does not contribute to either the $U(1)_A$ anomaly or the trace anomaly. The holomorphic piece was shown to play a special role in that it alone describes the η' self interactions including an η' mass term. For $N_f < N$, [14]

$$M_{\eta'}^2 \propto \frac{N_f}{N - N_f} \Lambda^2. \quad (2.3)$$

The large N behavior was anticipated in Ref. [17,18]. The mass squared of the η' -field is seen to diverge as $N_f \rightarrow N$, a singularity also present in V (Eq. (2.2)). In the analogous

supersymmetric case, the corresponding singularity in Eq. (2.1) is overcome by the appearance of additional baryonic light degrees of freedom. However, there is no indication that this is the case for a QCD-like theory so the singularity as $N_f \rightarrow N$ is very likely an artifact of the perturbative approximations leading to Eq. (2.2).

III. A NONPERTURBATIVE EFFECTIVE POTENTIAL

In this section we construct an effective potential valid to all orders in the loop expansion and appropriate for the range $N_f > N$. The new ingredients are:

- i) Using the full, rather than the one loop, beta function in the trace anomaly saturation.
- ii) Taking account of the anomalous dimension of the fermion mass operator.

This anomaly-induced effective potential is based on the QCD trace and $U_A(1)$ anomalies:

$$\theta_m^m = \frac{\beta(g)}{2g} F_a^{mn} F_{mn;a} \equiv 2bH , \quad (3.1)$$

$$\delta_{U_A(1)} V_{QCD} = N_f \alpha \frac{g^2}{32\pi^2} \epsilon_{mnr s} F_a^{mn} F_a^{rs} \equiv 4N_f \alpha G , \quad (3.2)$$

where we have defined $\beta(g) \equiv -bg^3/(16\pi^2)$. We take the coupling to be defined at some low energy scale appropriate for the phase transition to be studied. Eventually, we will assume that the transition is governed by an infrared fixed point of the gauge theory and set $b = 0$. At one loop, $b = \frac{11}{3}N - \frac{2}{3}N_f$.

H and G are composite fields describing, upon quantization, scalar and pseudoscalar glueballs [19]. The general, non derivative effective potential saturating the anomalies is:

$$V = -F \sum_n c_n \ln \left(\frac{\mathcal{O}_n}{\Lambda^{d_n}} \right) + \text{h.c.} , \quad (3.3)$$

where Λ is some fixed intrinsic scale of the theory and where $F = H + i\delta G$ with δ a positive constant (a negative δ is equivalent to interchanging F with F^\dagger) to be chosen. The \mathcal{O}_n are gauge invariant fields built out of the relevant degrees of freedom with naive mass dimension d_n and axial charge q_n . The presence of the $\ln(\mathcal{O}_n/\Lambda^{d_n})$ structure insures the correct implementation of the underlying anomalies at the effective potential level.

The anomaly constraints are:

$$\sum_n c_n D_n = b , \quad \sum_n c_n q_n = \frac{2N_f}{\delta} , \quad (3.4)$$

where $D_n = d_n - \gamma_n$ is the dynamical, scaling dimension of \mathcal{O}_n , with γ_n the anomalous dimension. We remark that the derivation of V is based on making explicit scale and $U(1)_A$ transformations on the composite operators \mathcal{O}_n . Since the scaling dimension enters as a parameter in this approach it is natural to associate it with the dynamical quantity D_n .

Here we choose $\delta = 1$ [20], but we will note in Section IV that our result is independent of the specific positive value assigned to δ . The gauge degrees of freedom described by the dimension-four field F , have been introduced as an intermediate device to implement correctly at the effective potential level the underlying anomalous transformations. The anomalous dimension of F is zero. As indicated in the introduction, we will build the potential out of the $N_f \times N_f$ complex meson matrix M_i^j transforming as the operator $q_i \tilde{q}^j$. So we assign naive mass dimension 3 to M_i^j . The operator $q \tilde{q}$ acquires an anomalous dimension γ when quantum corrections are considered and the full dynamical dimension is thus $3 - \gamma$. To make our effective potential capture the low-energy quantum dynamics of the underlying theory, we take $3 - \gamma$ to be the scaling dimension of M_i^j . The anomalous dimension γ is of course a function of the coupling g , which in turn depends on the relevant scale.

We next make the simplifying assumption that the fields \mathcal{O}_n in the potential Eq. (3.3) may be restricted to a minimal set (with lowest dimension) sufficient to satisfy the anomaly constraints Eq. (3.4). Thus we include only two terms $\mathcal{O}_1 = -F$ and $\mathcal{O}_2 = \det M$. Including additional terms would introduce arbitrary parameters in the model, which seems inappropriate for an initial investigation. Retaining just the minimal set is plausible (see section VII of Ref. [21]) and would correspond to the "holonomic" structure which emerges if the potential is considered to arise ([13–15]) from broken super QCD. In the same spirit we take $\det M$ to have the scaling dimension $(3 - \gamma)N_f$.

The potential in Eq. (3.3) then takes the form

$$V(F, M) = \left(\frac{\beta(g)}{g^3} 16\pi^2 + (3 - \gamma)N_f \right) \frac{F}{4} \ln \left(\frac{-F}{\Lambda^4} \right) - F \ln \left(\frac{\det M}{\Lambda^{3N_f}} \right) + AF + \text{h.c.}, \quad (3.5)$$

where A is a dimensionless constant that cannot be fixed by saturating the QCD anomalies and assuming holomorphicity. In fact, the term AF does not contribute to θ_m^m and is also a chiral singlet. The potential of Eq. (3.5) is seen to be consistent with the constraints in Eq. (3.4) when $4c_1 = b - (3 - \gamma)N_f$ and $c_2 = 1$. The coupling g and anomalous dimension γ are defined at some scale μ . For our study of a chiral phase transition governed by an infrared stable fixed point, g will be the fixed-point coupling and γ will be the associated anomalous dimension. The β function will then vanish.

To construct a potential depending only on the meson degrees of freedom we "integrate out" the gluonic degrees of freedom by imposing the field equation $\partial V / \partial F = 0$, which

provides

$$F = -e^{\frac{4A}{f(g)}-1} \Lambda^4 \left[\frac{\Lambda^{3N_f}}{\det M} \right]^{\frac{4}{f(g)}} , \quad (3.6)$$

where

$$f(g) = -\frac{\beta(g)}{g^3} 16\pi^2 - (3 - \gamma)N_f . \quad (3.7)$$

After substituting the expression for F in Eq. (3.5) we obtain:

$$V = -C \Lambda^4 \left[\frac{\Lambda^{3N_f}}{\det M} \right]^{\frac{4}{f(g)}} + \text{h.c.} , \quad (3.8)$$

where C is related to A via:

$$C = \frac{f(g)}{4e} \exp \left[\frac{4A}{f(g)} \right] . \quad (3.9)$$

Finally we integrate out the η' field, which can be isolated by setting

$$\det M = |\det M| e^{i\phi} , \quad (3.10)$$

where $\phi \propto \eta'$. This is done anticipating that the η' will be heavy with respect to the intrinsic scale of the theory and the other mesonic degrees of freedom. Now using Eq. (3.10) we derive the field equation $\phi = 0$ which leads to the final potential

$$V = -2C \Lambda^4 \left[\frac{\Lambda^{3N_f}}{|\det M|} \right]^{\frac{4}{f(g)}} . \quad (3.11)$$

The shape of this potential is determined by the function $f(g)$ Eq. (3.7). The potential of Eq.(2.2) simply used the lowest order perturbative expansion of $f(g)$ ($\gamma = 0$ and $-\frac{\beta(g)}{g^3} 16\pi^2 = \frac{11}{3}N - \frac{2}{3}N_f$).

Our interest here is in the range $N < N_f < (11/2)N$ where the chiral phase transition is expected to occur. For N_f close to $(11/2)N$, a weak infrared fixed point will occur. The β function will be negative and small at all scales and γ will also be small. Thus $f(g)$ will be negative. As N_f is reduced, the fixed point coupling increases as does γ . We will argue (in Appendix A), however, that in the range of interest, $f(g)$ will remain negative ($(3 - \gamma)N_f > -(\beta(g)/g^3)16\pi^2$). The potential in Eq. (3.11) may then be written as

$$V = +2|C| \Lambda^4 \left[\frac{|\det M|}{\Lambda^{3N_f}} \right]^{\frac{4}{g^3 16\pi^2 + (3-\gamma)N_f}} . \quad (3.12)$$

It is positive definite and vanishes with the field $|\det M|$.

IV. THE CHIRAL PHASE TRANSITION

To study the chiral phase transition, we need the combined effective potential

$$V_{tot} = V + V_I \tag{4.1}$$

where V_I is a generic potential term not associated with the anomalies. It is instructive, however, to investigate first the extremum properties of the anomaly term (Eq. (3.12)). Assuming the standard pattern for chiral symmetry breaking $SU_R(N_f) \times SU_L(N_f) \rightarrow SU_V(N_f)$, M_j^i may be taken to be the order parameter for the transition. For purposes of this discussion, we restrict attention to the vacuum value of M_j^i , which can be rotated into the form

$$M_j^i = \delta_j^i \rho, \tag{4.2}$$

where $\rho \geq 0$ is the modulus. Substituting (4.2) in the anomaly induced effective potential gives the following expression:

$$V = +2|C|\Lambda^4 \left[\frac{\rho}{\Lambda^3} \right]^{\frac{4N_f}{\frac{\beta(g)}{g^3}16\pi^2 + (3-\gamma)N_f}}. \tag{4.3}$$

Recall that $((3 - \gamma)N_f > -(\beta(g)/g^3)16\pi^2)$ in the range of interest. The first derivative $\partial V/\partial\rho$ vanishes at $\rho = 0$ provided that

$$\frac{4N_f}{\frac{\beta(g)}{g^3}16\pi^2 + (3 - \gamma)N_f} > 1, \tag{4.4}$$

a condition that is clearly satisfied. The second derivative,

$$\frac{\partial^2 V}{\partial\rho^2} \propto \rho \left[\frac{4N_f}{\frac{\beta(g)}{g^3}16\pi^2 + (3-\gamma)N_f} - 2 \right], \tag{4.5}$$

also vanishes at $\rho = 0$ if the exponent in Eq. (4.5) is positive. The second derivative at $\rho = 0$ is a positive constant when the exponent vanishes, and it is $+\infty$ for

$$\frac{4N_f}{\frac{\beta(g)}{g^3}16\pi^2 + (3 - \gamma)N_f} - 2 < 0. \tag{4.6}$$

The curvature of V_{tot} at the origin is given by the sum of the two terms $\frac{\partial^2 V}{\partial\rho^2}$ and $\frac{\partial^2 V_I}{\partial\rho^2}$, evaluated at $\rho = 0$.

To proceed further, we assume that the phase transition is governed by an infrared stable fixed point of the gauge theory. We thus set $\beta(g) = 0$. The curvature of V at the origin is

then 0 for $\gamma > 1$, finite and positive for $\gamma = 1$, and $+\infty$ for $\gamma < 1$. The value of γ depends on the fixed point coupling, which in turn depends on N_f . As N_f is reduced from $(11/2)N$, the fixed point coupling increases from 0, as does γ . Assuming that γ remains monotonic in N_f , growing to 1 and beyond as N_f decreases, there will be some critical value N_f^c below which $\frac{\partial^2 V}{\partial \rho^2}$ vanishes at the origin. The curvature of V_{tot} will then be dominated by the curvature of V_I at the origin. For $N_f = N_f^c$, there will be a finite positive contribution to the curvature from the anomaly-induced potential. For $N_f > N_f^c$ ($\gamma < 1$), V possesses an infinite positive curvature at the origin, suggesting that chiral symmetry is necessarily restored. We will here take the condition $\gamma = 1$ to mark the boundary between the broken and symmetric phases, and explore its consequences. This condition was suggested in Ref. [22], based on other considerations. It is straightforward to see that here, this condition is independent of the value assigned to δ in Eq. (3.4). The δ parameter enters the potential multiplying $\beta(g)$ and is therefore irrelevant when $\beta(g) = 0$.

We next investigate the behavior of the theory near the transition by combining the above behavior with a simple model of the additional, non-anomalous potential V_I . We continue to focus only on the modulus ρ and take the potential to be a traditional Ginzburg-Landau mass term, with the squared mass changing from positive to negative as $\gamma - 1$ goes from negative to positive:

$$(1 - \gamma) \Lambda^{-2} \rho^2 . \quad (4.7)$$

Additional, stabilizing terms, such as a ρ^4 term, could be added but will not affect the qualitative conclusions. The full potential is then

$$V_{tot} = 2|C|\Lambda^4 \left(\frac{\rho}{\Lambda^3}\right)^{\frac{4}{3-\gamma}} - (\gamma - 1) \Lambda^{-2} \rho^2 . \quad (4.8)$$

For $\gamma > 1$ (but < 3), the first term stabilizes the potential for large ρ , and the potential is minimized at

$$\langle \rho \rangle = \Lambda^3 \left[\frac{(\gamma - 1)(3 - \gamma)}{4|C|} \right]^{\frac{1}{2} \frac{3-\gamma}{\gamma-1}} , \quad (4.9)$$

It seems to us that this form may very well represent a generic extension of the Landau-Ginzburg potential to the present case.

In the limit $\gamma \rightarrow 1$ this expression reduces to

$$\langle \rho \rangle = \Lambda^3 \left[\frac{\gamma - 1}{2|C|} \right]^{\frac{1}{\gamma-1}} \quad (4.10)$$

which describes an infinite order phase transition as $\gamma \rightarrow 1$, in qualitative agreement with the gap equation studies. This behavior would not be changed by the addition of higher power terms (ρ^4, ρ^6, \dots) to the potential.

It is also interesting to describe how the order parameter ρ approaches zero at the critical point (i.e. $\gamma = 1$) as a function of the quark mass. At the effective potential level (for $m \ll \Lambda$) the quark mass enters in the following way

$$-m\text{Tr} [M + M^\dagger] = -2N_f m \rho, \quad (4.11)$$

where m is a diagonal quark mass. This new operator when added to the potential in Eq. (4.8) yields

$$\langle \rho \rangle_{\gamma=1} = \frac{m\Lambda^2 N_f}{2|C|}. \quad (4.12)$$

The curvature of the potential Eq. (4.8) at the minimum describes a mass associated with the field ρ . To interpret this mass physically, one should construct the kinetic energy term associated with this field (at least to determine its behavior as a function of $\gamma - 1$). We hence rescale ρ to a field σ via $\rho = \sigma^{3-\gamma}\Lambda^\gamma$ with σ possessing a conventional kinetic term $-\frac{1}{2}(\partial^\mu\sigma)^2$ [‡]. This then leads to the following result for the physical mass M_σ and $\langle \sigma \rangle$

$$\langle \sigma \rangle \simeq \left[\frac{\gamma - 1}{2|C|} \right]^{\frac{1}{2(\gamma-1)}} \Lambda, \quad M_\sigma \simeq 2\sqrt{6}|C| \left[\frac{\gamma - 1}{2|C|} \right]^{\frac{1}{2(\gamma-1)}} \Lambda. \quad (4.13)$$

Likewise, in the presence of the quark mass term we have

$$\langle \sigma \rangle_{\gamma=1} \simeq \left[\frac{mN_f\Lambda}{2|C|} \right]^{\frac{1}{2}}, \quad [M_\sigma]_{\gamma=1} \simeq 2[2mN_f\Lambda]^{\frac{1}{2}}. \quad (4.14)$$

[‡]The present rescaling procedure is consistent with the respect to the construction of the energy momentum tensor at the effective potential level.

It is interesting to notice that in the variable σ the effective potential reads

$$V = 2|C|\sigma^4 - (\gamma - 1)\sigma^{2(3-\gamma)}\Lambda^{2(\gamma-1)}.$$

Clearly the first term is conformally invariant and the second term can be understood as a small deviation from conformality. We expect this effective potential to be a suitable generalization of the Ginzburg-Landau theory when a global symmetry (associated with the non vanishing vacuum expectation value of the order parameter σ) is restored together with the conformal symmetry.

Thus the order parameter σ for $\gamma = 1$ vanishes according to the power $1/2$ with the quark mass in contrast with an ordinary second order phase transition where the order parameter is expected to vanish according to the power $1/3$.

Finally we note an important distinction between our effective potential describing an infinite order transition and the Ginzburg-Landau potential describing a second order transition. The latter may be used in both the symmetric and broken phases, describing light scalar degrees of freedom as the transition is approached from either side. Our potential develops infinite curvature at the origin in the symmetric phase, indicating that no light scalar degrees of freedom are formed as the transition is approached from that side. This is in agreement with the conclusions of Ref. [23], indicating that as one crosses to the symmetric phase, mesons melt into quarks and gluons and hence the physics is described via only the underlying degrees of freedom. The present effective Lagrangian formalism for describing the chiral/conformal phase transition is close in spirit to the one developed in Ref. [24].

V. PERTURBATION THEORY AND THE DETERMINATION OF N_F^C

Our discussion of the chiral transition so far, using the anomalous dimension γ as the control parameter, has made no direct reference to N_f and has been independent of perturbation theory. The critical value N_f^c for the transition may be estimated by making use of a perturbative expansion of γ . Through two orders in perturbation theory, γ is given by

$$\gamma = a_0\alpha + a_1\alpha^2, \quad (5.1)$$

with $a_0 = \frac{1}{2\pi}3C_2(R)$ and $a_1 = \frac{1}{16\pi^2} \left[3C_2(R)^2 - \frac{10}{3}C_2(R)N_f + \frac{97}{3}C_2(R)N \right]$, where $C_2(R) = \frac{N^2 - 1}{2N}$. We evaluate γ at the fixed point value of the coupling constant, which at two loops in the beta function expansion may be expressed as:

$$\alpha^* = -\frac{b_0}{b_1} \simeq \frac{4\pi}{N} \left[\frac{11N - 2N_f}{13N_f - 34N} \right], \quad (5.2)$$

where we have used the large N_f and N expansion to simplify the expression.

In Ref. [25], it was noted that in lowest (ladder) order, the gap equation leads to the condition $\gamma(2 - \gamma) = 1$ for chiral symmetry breaking. To all orders in perturbation theory, this condition is gauge invariant (since γ is gauge invariant) and is equivalent to the condition $\gamma = 1$ Ref. [22]. To any finite order in perturbation theory these conditions are of course different. To leading order in the expansion of γ , the condition $\gamma(2 - \gamma) = 1$ leads, to the critical coupling

$$\alpha_c = \frac{\pi}{3C_2(R)}, \quad (5.3)$$

above which the ladder gap equation has a non-vanishing solution. Using Eq. (5.2) together with Eq. (5.1) and the condition $\gamma < 1$ leads to the conclusion that chiral symmetry is restored for

$$N_f > N_f^c \simeq 3.9N. \quad (5.4)$$

If, on the other hand, the condition $\gamma = 1$ is implemented using the lowest order expression for γ , a somewhat smaller value of N_f^c emerges.

The advantage of using the anomalous dimension γ as the control parameter to study the chiral transition is that the problem can be formulated in a way that is free of these perturbative uncertainties[§].

VI. CONCLUSIONS

We have explored the chiral phase transition for vector-like $SU(N)$ gauge theories as a function of the number of flavors N_f via an anomaly induced effective potential. The effective potential was constructed by saturating the trace and axial anomalies. It depends on the full beta function and anomalous dimension of the quark-mass operator. The mesonic degrees of freedom are the only variables included at low energies. We assumed the anomaly induced effective potential to have a holomorphic structure. We note that holomorphicity was also used by other groups [27] to constrain similar anomaly induced potentials. The present potential is a generalization of a previous potential [13–15] which was constructed by saturating the QCD anomalies at just one-loop.

We showed that the anomaly induced effective potential for $N_f > N$ is positive definite and vanishes with the field M_i^j . We then investigated the stability of the potential at the origin, and discovered that the second derivative is positive and divergent when the underlying β function and the anomalous dimension of the quark-mass operator satisfy the relation of Eq. (4.6). We took this to be the signal for chiral restoration. With conformal symmetry being restored along with chiral symmetry (due to the β function vanishing at an infrared fixed point), the criticality relation becomes a constraint on the anomalous dimension of the quark-mass operator:

[§]Recently a perturbative study of the conformal window region in QCD and supersymmetric QCD was performed in Ref. [26]

$$\gamma < 1 . \tag{6.1}$$

To convert this inequality into a condition for a critical number of flavors, we used the perturbative expansion of the anomalous dimension evaluated at the fixed point, deducing that chiral symmetry is restored for $N_f \simeq 4N$, in agreement with gap equation studies.

The core of this paper is the proposal that the chiral/conformal phase transition, suggested by gap equation studies to be continuous and infinite order, may be described by an effective potential whose form is dictated by the trace and axial anomalies of the underlying $SU(N)$ gauge theory. It will be important to explore more completely both the derivation of this potential and its application to the chiral/conformal phase transition. In particular we note that we have here used the potential only at the classical, mean-field level. The development of kinetic energy terms and the consideration of long wavelength quantum fluctuations of M_i^j could next be considered.

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APPENDIX A: HIGHER LOOP EFFECTS FOR THE EFFECTIVE POTENTIAL

The potential in Eq. (3.11), when $f(g)$ is evaluated to lowest order in perturbation theory ($\gamma = 0$ and $-\frac{\beta(g)}{g^3}16\pi^2 = \frac{11}{3}N - \frac{2}{3}N_f$), leads to Eq. (2.2). Here we note that the special location of the singularity in that potential is an artifact of lowest order perturbation theory. Let us thus investigate the behavior of $f(g)$ to next order. Thus

$$-\frac{\beta(g)}{g^3}16\pi^2 = b_0 + b_1\alpha, \quad \gamma = a_0\alpha, \tag{A1}$$

with

$$b_0 = \frac{11}{3}N - \frac{2}{3}N_f, \quad b_1 = \frac{1}{4\pi} \left(\frac{34}{3}N^2 - \frac{10}{3}NN_f - \frac{N^2 - 1}{N}N_f \right), \quad a_0 = \frac{3}{4\pi} \frac{N^2 - 1}{N}, \tag{A2}$$

which provides

$$f(g) = \frac{11}{3}(N - N_f) + \left[\frac{34}{3}N^2 - \frac{10}{3}NN_f + 2\frac{N^2 - 1}{N}N_f \right] \frac{\alpha}{4\pi}. \quad (\text{A3})$$

Imposing the equation $f = 0$ we find a zero for

$$N_f^s = N \frac{1 + \frac{17\alpha}{22\pi}N}{1 + \frac{3\alpha}{22\pi} \left(\frac{5N}{3} - \frac{N^2-1}{N} \right)}, \quad (\text{A4})$$

which shows that the singularity at $N_f = N$ is shifted once higher order corrections are included.

Whether even this shifted value has any significance depends on the magnitude of α (the convergence of the expansion). As indicated in Section III, α can only be guaranteed to be small when N_f is close to, but below $11N/2$, leading to a weak infrared fixed point. This is a range (well above N_f^s) in which $f(g)$ is clearly negative since β and γ are small. We next decrease N_f and see whether $f(g)$ has a zero in the range of interest. We reduce N_f until it reaches the value N_f^c (Eq. (5.4)) where the infrared fixed point (Eq. (5.2)) has reached the critical coupling (Eq. (5.3)). Assuming only that the perturbative expansions leading to these expressions are roughly accurate (remember that α is never larger than its fixed point value), $f(g)$ will be negative throughout the N_f range from $11N/2$ down to N_f^c , the onset of chiral symmetry breaking. The quantity N_f^s is well below N_f^c , and corresponds, probably, to large values of α .

To conclude, there is no evidence that $f(g)$ will have changed sign from negative to positive when N_f is reduced to the critical value N_f^c of interest here. N_f^c appears to be safely above any possible zeros of $f(g)$.

This analysis is based on the existence of an infrared fixed point. It is instructive and reassuring ** to observe from Fig. 1 that, without requiring the existence of any infrared fixed point, for any value assumed by the quantity αN (at two loops and for large N) the critical number of flavors N_f^c (defined as the number for which the exponent in Eq. (4.5) vanishes) is always greater than N_f^s . We notice that for a wide range of values of αN , N_f^s/N is below the horizontal line $11/2$, above which asymptotic freedom is lost. It is also clear that there is a region where N_f^* (the point where the beta function vanishes) is close to N_f^c (see Fig. 1). In the large N limit we have:

$$\frac{N_f^c}{N} \simeq \frac{11 + \frac{17}{2\pi}\alpha N}{5 + \frac{1}{\pi}\alpha N}, \quad \frac{N_f^*}{N} \simeq \frac{11 + \frac{17}{2\pi}\alpha N}{2 + \frac{13}{4\pi}\alpha N}. \quad (\text{A5})$$

**Although perturbation theory is now probably less reliable.

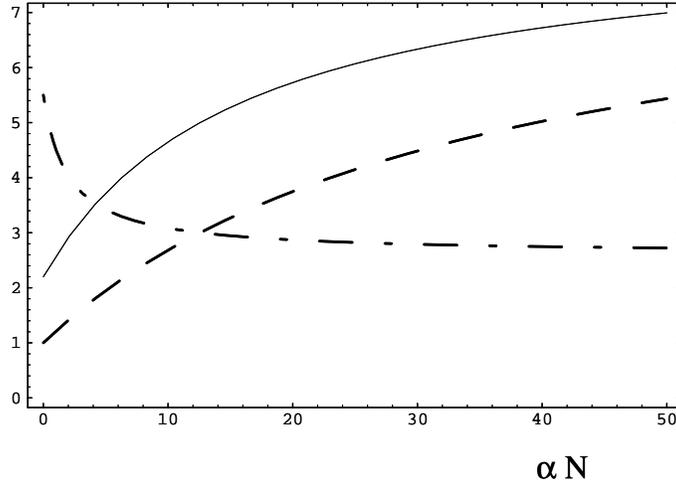


FIG. 1. N_f^c/N as a function of αN is shown as a solid line. The dashed line represents N_f^s/N , while the dot-dashed line describes the N_f^*/N function.

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