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ABSTRACT

We review the Skyrme model approach which treats baryons as solitons of an effective meson theory. We start out with a historical introduction and a concise discussion of the original two flavor Skyrme model and its interpretation. Then we develop the theme, motivated by the large N_C approximation of QCD, that the *effective* Lagrangian of QCD is in fact one which contains just mesons of all spins. When this Lagrangian is (at least approximately) determined from the meson sector it should then yield a zero parameter description of the baryons. We next discuss the concept of chiral symmetry and the technology involved in handling the three flavor extension of the model at the collective level. This material is used to discuss properties of the light baryons based on three flavor meson Lagrangians containing just pseudoscalars and also pseudoscalars plus vectors. The improvements obtained by including vectors are exemplified in the treatment of the *proton spin puzzle*.

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1. Historical background and motivation

The Skyrme model was born around 1960 in a series of increasingly more detailed papers [1]. At that time the prevailing dynamical model of nuclear forces was that of Yukawa which had been formulated in the 1930's. Still to come was the concept of fractionally charged quarks and much further in the future was the recognition that the correct theory of strong interactions binds these quarks together with non–Abelian (color) gauge fields.

In the Yukawa theory, of course, the nucleons are introduced as fundamental fermion fields while the spin zero pion fields are postulated to provide the "glue" which binds protons and neutrons into nuclei. This model is acknowledged to work reasonably well as a description of the long range interactions of nucleons and the prediction of the existence of pions has been amply confirmed.

Skyrme's innovation was to provide a model in which the fundamental fields consisted of just the pions. The nucleon was then obtained, in the initial approximation, as a certain classical configuration of the pion fields. The seeming contradiction of making fermi fields out of bose fields was avoided by arranging the classical field configuration to possess a non–zero "winding number". In modern language this "Skyrmion" is an example of a topological soliton. Such objects are solutions to the classical field equations with localized energy density [2]. They play an important role nowadays in many areas of physics and the papers of Skyrme are justifiably recognized as pioneering milestones in this development.

The years following this original idea saw the particle physics community actively investigating the approaches of quark models, flavor symmetry, current algebra, chiral dynamics, dual resonance models and finally color gauge theory to the problem of strong dynamics. Evidently Skyrme's model was lost in the rush. However the novelty of the model did stimulate a few interesting papers [3, 4, 5, 6] before the more recent wave of activity in the area.

At first glance, it might appear that a Lagrangian model built out of only pion fields could not be more different as a description of the nucleons from the current picture of three "valence" quarks containing a trivalent "color" index and bound together through their interaction with $SU(3)$ gauge fields. Remarkably, it has turned out that the Skyrme model is in fact a plausible approximation to this QCD picture. This may be understood as follows.

In QCD the gauge coupling constant has an effective strength which decreases for interactions at high energy scales (asymptotic freedom) but which increases at the low energy scales which are relevant when one considers the binding of quarks into nucleons and other hadrons. Thus the application of standard perturbation theory techniques to the problem of low energy interactions is not expected to be reliable and in fact has not produced definitive results. A natural alternative approach which retains the possibility of using perturbation theory is to imagine that the strong underlying gauge couplings bind the quarks into particles which may possibly interact with each other relatively weakly. At low energies these particles should evidently comprise the pseudoscalar meson fields (pions when restricted to two "flavors"). Then it is necessary to formulate some *effective* Lagrangian for the pions. Certainly the Lagrangian should be restricted by the correct symmetries of the underlying gauge theory. These must include an (approximate) $SU(N_f)$ flavor symmetry [7], where N_f is the number of light flavors.

But there is another symmetry which plays a crucial role. At about the same time that Skyrme was contemplating the model under discussion the correct formulation [8] of the structure of the effective weak (beta–decay etc.) interaction was discovered. It was noted that this interaction treated the left and right handed components of fermions on a completely separate basis. If this distinction is maintained at the level of the strong interactions one should impose a "chiral" left handed $SU(3)$ flavor $\times$ right handed $SU(3)$ flavor symmetry on the effective low energy Lagrangian of mesons. A consequence of this larger symmetry is that the meson multiplets must contain scalar as well as the desired low–lying pseudoscalar particles. This seemed a bit of an embarrassment in that the pseudoscalars are very light while the scalars were not well established and presumably heavy. Hence the possibility of a degenerate symmetry multiplet is implausible. Nevertheless it was realized [9] that the situation was likely to be similar to that met in the BCS theory of superconductivity in which the vacuum (ground state) is energetically favored to exist in a non–symmetric state. This "spontaneous breakdown" picture predicts, in the absence of a needed small explicit symmetry breaker, exactly zero mass for the pseudoscalars at the same time that the scalars are massive. In fact it may be formulated using a so called "non–linear realization of chiral symmetry" in such a way that the scalars do not appear at all [10]. The prototype Lagrangian density for this picture is

$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{tr}\,\left(\partial_\mu U \partial^\mu U^\dagger\right) + \ldots\,, \tag{1.1}$$

where $U = \exp(\sqrt{2}i\phi/f_\pi)$, ϕ being the 3×3 matrix of the ordinary pseudoscalar mesons and $f_\pi = 93\mathrm{MeV}$ the "pion decay constant". U is a unitary matrix which transforms "linearly" under the chiral transformations. Possible higher derivative and symmetry breaking terms have not been explicitly written here. It was demonstrated a long time ago that just this term compactly summarizes the low energy scattering of pseudoscalar mesons [11]. Improvements to this term form the basis of the "chiral perturbation theory approach" [12]. Now it is believed that a picture like this is expected from fundamental QCD. However the same Lagrangian was earlier written by Skyrme (in the two rather than three flavor case) in order to explain the nucleons [1] before the present justifications for it were known.

Even in the framework of the chiral Lagrangian given above it would seem that there is no special *a priori* reason not to explicitly add baryons in a chiral symmetric manner rather than to build them out of the mesons. Indeed there have been many papers over very many years which do just this with reasonable phenomenological results [13]. Nevertheless there is an indication from fundamental QCD that the soliton treatment of the baryon is more natural. This arises from an attempt [14] to consider $1/N_C$, the inverse of the number of colors in the gauge theory, as a possible expansion parameter for QCD which might be meaningful even at low energies. In this approach the product $g'^2 = g^2 N_C$, where g is the gauge theory coupling constant, is held constant. 't Hooft [14] showed that for large N_C QCD may be considered as a theory of mesons weakly interacting in the sense that scattering amplitudes or quadrilinear coupling constants are of order $g_{\rm eff} = 1/N_C$. Since the baryon mass must start out proportional to N_C (noting that the baryon in the N_C model is made of N_C quarks) it means that the predicted expressions for baryon masses should start out as the inverse of this coupling constant. In the framework of the (non–relativistic) mean field treatment Witten [15] not only pointed out that the baryon masses indeed grow linearly with N_C but also that baryon radii and meson–baryon scattering amplitudes are of the order N_C^0 while baryon–baryon scattering is of $\mathcal{O}(N_C)$. He in particular recognized that this inverse behavior with $g_{\rm eff}$ is just the usual signal that the baryon state in question is a soliton of the effective meson theory.

Naturally, one wonders how these "modern" justifications for the Skyrme approach relate to Skyrme's original motivations. We are fortunate in having available a reconstructed talk on just this topic by Skyrme [16]. He mentioned three motivations: 1) The idea of unifying bosons and fermions in a common framework. 2) The feeling that point particles are inconsistent in the sense that their quantum field theory formulation introduces infinities which are only "swept under the rug" by the renormalization process. 3) The desire to eliminate fermions from a fundamental formulation since fermions have no simple classical analog. What seems more fascinating is his awareness that there were probably some "hidden" influences pushing him toward the soliton picture. Directly, these came from his fascination with Kelvin's idea that the various atoms should correspond to vortices of different connectivities in some underlying liquid. In turn, his interest in Kelvin was sparked at an early age by the presence of a tide prediction machine, designed by Kelvin and constructed by his great–grandfather, still occupying space in his great–grandfather's house. An interesting account of this aspect is given in a paper of Dalitz [17].

Thus it seems that Skyrme's motivations were not those currently used to justify his model. In particular it appears that he did not choose his Lagrangian model to describe spontaneous breakdown of chiral symmetry. Rather the non–linear form was chosen to insure that the pions were "angular" variables which would give multi–valued functions; the crossing of different sheets of these functions might then correspond to singularities which would realize the baryons. The evident "moral" of this historical discussion is just that interesting ideas have an uncanny way of turning out to be useful and true. In this spirit, we would like to continue with the application of Skyrme's ideas to current research on baryon physics, making use of current motivations but trying to avoid getting enslaved by them.

2. The Skyrme model for two flavors

In this section we will present the basic technology of the Skyrme model for baryons. The starting point for the construction of a soliton solution is the non–linear sigma model Lagrangian (1.1) already introduced in the previous section. As we require a finite energy density the chiral field U must approach a constant value at spatial infinity. We are free to choose this to be unity, *i.e.*,

$$U(\boldsymbol{r},t) \underset{|\boldsymbol{r}|\to\infty}{\longrightarrow} 1 \ . \tag{2.1}$$

This can be considered a mapping from compactified coordinate space, a three–sphere S^3, to the space which is described by the unitary, unimodular matrix U, namely $SU(N_f)$, where N_f denotes the number of flavors. In the case of two flavors the target space is isomorphic to S^3. The mappings $S^3 \to S^3$ fall into distinct equivalence classes. This signals the existence of soliton configurations because members of different classes cannot be continuously transformed into one–another. The equivalence classes are characterized by the winding number. This number counts the coverings of the target space and is the charge $\int d^3x B_0$ associated with the topological current

$$B_\mu = \frac{1}{24\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}\left[\left(U^\dagger\partial^\nu U\right)\left(U^\dagger\partial^\rho U\right)\left(U^\dagger\partial^\sigma U\right)\right] \ . \tag{2.2}$$

When later discussing the three flavor case we will see that this topological current indeed equals the baryon number current.

Although these topological considerations allow the existence of soliton solutions it turns out that the dynamics of (1.1) do not lead to static stable classical solutions. This can be deduced from a simple consideration, known as Derrick's theorem [18]. Assume $U_0(\boldsymbol{r})$ to be such a solution. The static energy of $U_0(\lambda\boldsymbol{r})$, obtained from the Hamiltonian of (1.1), would then be

$$E_{\rm cl}^{({\rm nl}\sigma)}[U_0(\lambda\boldsymbol{r})] = \frac{1}{\lambda}E_{\rm cl}^{({\rm nl}\sigma)}[U_0(\boldsymbol{r})] \tag{2.3}$$

which does not have a minimum at $\lambda = 1$, in contradiction to the assumption. In order to obtain stable solitons Skyrme added a term to the Lagrangian which is of fourth order in the derivatives,

$$\mathcal{L}^{({\rm Sk})} = \frac{1}{32e^2}\,{\rm tr}\left(\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]\left[U^\dagger\partial^\mu U, U^\dagger\partial^\nu U\right]\right)\,. \tag{2.4}$$

Here e is the dimensionless "Skyrme constant". Although this term is quartic in the derivatives it was arranged to be at most quadratic in the time–derivatives. This makes the quantization feasible. It is now apparent that a scaled configuration may well lead to a minimum of the energy functional

$$E_{\rm cl}^{({\rm tot})}[U_0(\lambda\boldsymbol{r})] = \frac{1}{\lambda}E_{\rm cl}^{({\rm nl}\sigma)}[U_0(\boldsymbol{r})] + \lambda E_{\rm cl}^{({\rm Sk})}[U_0(\boldsymbol{r})] \tag{2.5}$$

at $\lambda = 1$ provided the configuration $U_0(\boldsymbol{r})$ satisfies $E_{\rm cl}^{({\rm nl}\sigma)}[U_0(\boldsymbol{r})] = E_{\rm cl}^{({\rm Sk})}[U_0(\boldsymbol{r})]$.

A priori the Euler–Lagrange equations of motion for the chiral field $U_0(\boldsymbol{r})$ are highly non–linear partial differential equations. To simplify these equations Skyrme adopted the famous hedgehog *ansatz*

$$U_0(\boldsymbol{r}) = \exp\left(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)\right), \tag{2.6}$$

where $\boldsymbol{\tau}$ represents the Pauli matrices. This form may actually be traced back to the old "strong coupling" theory [19]. Upon substitution of this *ansatz* the energy functional turns into a simple integral involving only the radial function $F(r)$,

$$E[F] = \frac{2\pi f_\pi}{e}\int_0^\infty dx\left\{\left(x^2F'^2 + 2{\rm sin}^2F\right) + \sin^2F\left(2F'^2 + \frac{\sin^2F}{x^2}\right)\right\}. \tag{2.7}$$

Henceforth this radial function will be called the chiral angle. In eq (2.7) a prime indicates a derivative with respect to the dimensionless coordinate $x = ef_\pi r$. In this manner we have completely extracted the dependence on the model parameters. Imposing $F(\infty)$=0 and noting that $\int d^3rB_0 = (F(0) - F(\infty))/\pi$ leads to the boundary condition $F(0) = \pi$ for a unit baryon number configuration. The profile function depicted in Fig. 2.1 minimizes (2.7) and is obtained numerically. The energy obtained by substituting this solution into (2.7) is found to be $E = 23.2\pi f_\pi/e$ [6].

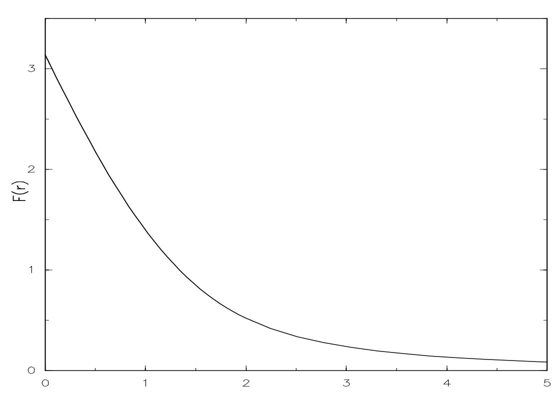

Figure 2.1: Chiral angle

As the *ansatz* (2.6) is not invariant under separate spatial or flavor rotations this field configuration does not yet describe states with good spin and flavor quantum numbers. As the first step towards generating such states, time–dependent collective coordinates $A(t)$ are introduced which describe the spin and flavor orientation of the hedgehog,

$$U(\boldsymbol{r},t) = A(t)U_0(\boldsymbol{r}\,)A^\dagger(t), \qquad A(t) \in SU(N_f)\,. \tag{2.8}$$

Note that the hedgehog structure causes rotations in coordinate and flavor space to be equivalent. For generality we assumed an arbitrary number of flavors. In the two flavor case this configuration yields the Lagrange function

$$L(A,\dot{A}) = \frac{1}{2}\alpha^2\boldsymbol{\Omega}^2 - E_{\rm cl} \tag{2.9}$$

where the quantity $\boldsymbol{\Omega}$ measures the time dependence of the collective coordinates

$$A^\dagger(t)\frac{d}{dt}A(t) = \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\Omega}\,. \tag{2.10}$$

The constant of proportionality, α^2 is computed as a spatial integral over the chiral angle to be $\alpha^2 = 53.3/(e^3 f_\pi)$. Computing the spin as the Noether charge of spatial rotations yields $\boldsymbol{J} = \alpha^2\boldsymbol{\Omega}$. Apparently the system displays all the features of a rigid top. In that language $\boldsymbol{\Omega}$ and α^2 are denoted as the angular velocity and the moment of inertia, respectively.

In the second step the collective coordinates are elevated to quantum variables. Again this is completely analogous to the quantization of the rigid top and gives the quantization rule $[J_i, J_j] = i\epsilon_{ijk}J_k$. For the hedgehog *ansatz* in $SU(2)$ spin and isospin are related via the adjoint representation of the collective coordinates, *i.e.* $I_i = -D_{ij}J_j$ with $D_{ij} = (1/2){\rm tr}(\tau_i A \tau_j A^\dagger)$ due to the equivalence of the respective rotations. Hence only states which have identical spin and isospin are allowed in the spectrum. These are the nucleon ($I = J = 1/2$) and the Δ–resonance ($I = J = 3/2$). Finally the Hamiltonian for the collective coordinates is given by

$$H_{\rm coll} = E_{\rm cl} + \frac{1}{2\alpha^2}\boldsymbol{J}^{\,2} = E_{\rm cl} + \frac{1}{2\alpha^2}\boldsymbol{I}^{\,2} \tag{2.11}$$

which yields the Δ–nucleon mass difference

$$M_\Delta - M_{\rm N} = \frac{3}{2\alpha^2}\,. \tag{2.12}$$

Using the physical value for the pion decay constant, $f_\pi = 93$MeV requires us to choose $e \approx 4.75$ to reproduce the empirical mass difference of 293MeV. Substituting $e \approx 4.75$ into eq (2.7) yields the classical nucleon energy $E = 23.2\pi f_\pi/e \approx 1430$MeV. This is not in especially good agreement with the experimental value of about 939MeV. However the following points must be kept in mind:

(i) The meson Lagrangian consisting of eq (1.1) plus eq (2.4) contains only pseudoscalars. We would expect that other low mass mesons (notably the vectors) should also be included. The large N_C expansion [14, 15] requires an infinite number but common sense suggests a reasonable approximation for explaining hadronic physics up to about 1GeV would be to keep those mesons with masses up to this value. Certainly the predictions in the mesonic sector of the theory are noticeably improved by the inclusion of vector mesons. The consistency of the overall picture requires accurate predictions both in the mesonic and baryonic sectors of the effective theory.

(ii) In nature there are three rather than two "light" flavors and this aspect should be included in a realistic formulation. (This feature also makes more transparent the origin of the topological current eq (2.2).) Furthermore the effects of flavor and chiral symmetry breaking mediated by the finite values of the quark masses have not yet been taken into account.

(iii) Order of N_C^0 corrections to the nucleon mass which have the structure of the Casimir effect in field theory have also not been included. These quantum contributions to the energy have been estimated to be negative and of the order of a few hundred MeV, predicting a total nucleon mass at the order of the experimental value [20]. Nevertheless one should be cautious about these quantum corrections, after all the Skyrme model is not renormalizable, leaving a logarithmic scale dependence of the "renormalized" Casimir energy. It seems that at best the quantum corrections can be computed in a scenario compatible with the chiral expansion.

We will postpone the discussion of a variety of nucleon (and other baryons') properties until after we have treated the more general case of flavor $SU(3)$.

To end this section on the basics of the Skyrme model we would like to briefly discuss the consistency of the Skyrme model with the large N_C picture of QCD. In section 1 we have already noted that the quadrilinear coupling between mesons scales like $1/N_C$. To check this behavior it is convenient to expand the non–linear σ model Lagrangian (1.1) in powers of the pion field:

$$\frac{1}{2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} + \frac{1}{6 f_\pi^2}\left\{(\boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi})^2 - \boldsymbol{\pi}^2 \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}\right\} + \mathcal{O}\left(\boldsymbol{\pi}^6\right) . \tag{2.13}$$

Since the quadrilinear coupling constant is $1/f_\pi^2$ we deduce that $f_\pi \sim \sqrt{N_C}$. This agrees with general arguments [15]. Similarly the Skyrme term (2.4) provides a quartic pion interaction with the coupling constant $1/(e^2 f_\pi^4)$ which implies $e \sim 1/\sqrt{N_C}$. Hence the classical energy (2.7) grows linearly with the number of colors as asserted from the corresponding generalization of QCD. Moreover, without flavor symmetry breaking large N_C QCD predicts the baryons of different J to be degenerate [21]. This is perfectly consistent with the mass formula (2.11) because the moment of inertia also grows linearly with N_C as is indicated after eq (2.10), hence the second term in (2.11) behaves like $1/N_C$.

Actually, the understanding of the N_C expansion for baryons involves some subtleties. Consider the construction of large N_C baryons in the quark model. The lowest lying baryons are made of N_C (taken to be odd) quarks in a totally antisymmetric (*i.e.* singlet) color spin state with no orbital angular momentum. One expects particles of all total angular momenta from $J = 1/2$ to $J = N_C/2$ to be obtained. In agreement with the spectrum of eq (2.11) we expect $I = J$ for these particles and an infinite number of them as $N_C \to \infty$. The trouble is that there is no experimental evidence for any $I = J = 5/2, 7/2$ etc. particles. This may be interpreted as evidence that $N_C = 3$ in nature. Still, the large N_C expansion is useful if one computes a quantity which exists in the $N_C = 3$ theory as a (presumably quickly convergent) Taylor series in $1/N_C$. For the specific case of the higher excitations $J = 5/2, 7/2, \ldots$ the above treatment of the rotational modes seems inadequate because the rotational energy gets as large as the classical contribution. By including these modes in the Euler–Lagrange equations the widths of these higher excitations have been estimated to be comparable to their masses [22]. This makes a particle interpretation of these states problematic suggesting that they are artifacts of the collective quantization method employed rather than of physical relevance. Possible caveats for these calculations are the instability of these configurations against emitting pions [23] and that the results are only obtained by analytical continuation in the spin variable.

3. Chiral symmetry and its breaking

In this section we will briefly discuss the concept of chiral symmetry which represents a guiding principle for extending the Skyrme model. Attention will be limited to those aspects of this large subject which have direct relevance to the study of Skyrmions. The basic idea is to construct a model of meson fields which "mocks up" as many symmetries and properties of the fundamental QCD Lagrangian as possible.

3.1 The QCD Lagrangian

Let us first recall the matter piece of the QCD Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm QCD}^{\rm matter} &= \sum_{f=1}^{N_f} \bar{q}_f \left(i\partial\!\!\!/ + gA\!\!\!/ - m_f\right) q_f \\ &= \sum_{f=1}^{N_f} \left\{\bar{q}_{f,{\rm L}} \left(i\partial\!\!\!/ + gA\!\!\!/\right) q_{f,{\rm L}} + \bar{q}_{f,{\rm R}} \left(i\partial\!\!\!/ + gA\!\!\!/\right) q_{f,{\rm R}} - m_f \left(\bar{q}_{f,{\rm L}} q_{f,{\rm R}} + \bar{q}_{f,{\rm R}} q_{f,{\rm L}}\right)\right\} . \qquad (3.1)\end{aligned}$$

Here A_μ is the matrix representation of the gluon fields and g the quark–gluon coupling. Most notably we have introduced the chiral representation for the QCD current quarks

$$q_{\rm L,R} = \frac{1}{2}\left(1 \mp \gamma_5\right) q \qquad (3.2)$$

of each flavor.

Strictly speaking, the quark mass terms are not part of the QCD Lagrangian but arise from the Yukawa terms of the full microscopic theory of nature. A major unsolved problem is to understand the resulting pattern of quark mass parameters. The phenomenologically determined masses [24] are $m_u \approx 5{\rm MeV}, m_d \approx 9{\rm MeV}, m_s \approx 120-170{\rm MeV}, m_c \approx 1.5{\rm GeV}, m_b \approx 4.5{\rm GeV}$ and $m_t \approx 175{\rm GeV}$. This random looking perturbation of the "strong interaction" plays a crucial role in determining the nature of elementary particle physics. In the region up to about 1GeV it is not possible to produce particles containing c, b or t quarks. Then it is usually a good approximation to simply drop them from the theory. Other approximations are useful when dealing with the subspace carrying the flavor quantum number of a single "heavy" quark [25]. Furthermore in the sector of the three "light" quarks u, d, s it turns out to be fundamental to neglect the u, d, s masses as a first approximation and include their effects as a perturbation [26]. This is reasonable because the light masses are less than the quantity $\Lambda_{QCD} \approx 250{\rm MeV}$, the scale below which the QCD effective coupling gets extremely large.

In the case $m_f = 0$ the Lagrangian (3.1) specialized to the three light quarks has the global chiral symmetry

$$U_{\rm L}(3) \times U_{\rm R}(3): \quad q_{\rm L} \longrightarrow L q_{\rm L} \quad \text{and} \quad q_{\rm R} \longrightarrow R q_{\rm R} \quad \text{with} \quad q_{\rm L,R} = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}_{\rm L,R} \qquad (3.3)$$

where L and R are each 3×3 unitary matrices. Using Noether's theorem on the classical Lagrangian then yields the conservation of the eighteen vector and axial vector currents

$$j^{\mu}_{ij} = \bar{q}_j \gamma^{\mu} q_i \quad \text{and} \quad j^{\mu}_{ij,5} = \bar{q}_j \gamma^{\mu} \gamma_5 q_i, \tag{3.4}$$

where the latin indices run over u, d, s. These currents play an important role in the theory of weak interactions.

Now a major discovery of quantum field theory is that consequences of the classical field equations of motion (which can be used to verify the conservation of the Noether currents) do not necessarily hold at the quantum level. It is necessary to consider whether there exists a suitable regularization of the divergent diagrams of the theory which maintains the classical relations. In the present case, the axial singlet current[1], $j^5_\mu = \bar{q}\gamma_\mu\gamma_5(\lambda^0/2)q$ is not conserved even for massless quarks. Rather its divergence is proportional to the gluon field tensor times its dual. This is a result of the well–known Adler–Bell–Jackiw (ABJ) triangle anomaly [27] contained in the loop–diagrams shown in figure 3.1.

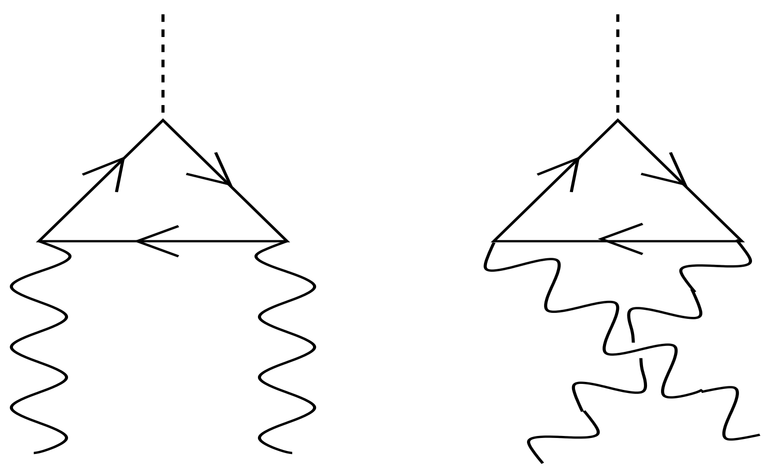

Figure 3.1: Adler–Bell–Jackiw anomaly. The arrows indicate quark lines, the curly lines refer to the gauge bosons and the dashed lines denote the coupling of the axial singlet current J^{μ}_5.

The net result is that the true global symmetry of the massless quantum theory is not $U_{\rm L}(3) \times U_{\rm R}(3)$ but $U_{\rm V}(1) \times SU_{\rm L}(3) \times SU_{\rm R}(3)$. The singlet vector symmetry, $U_{\rm V}(1)$ corresponds to baryon number conservation.

A similar situation emerges when external c–number flavor gauge fields are added to the massless QCD Lagrangian in order to further probe its structure. Then the so–called non–Abelian anomaly yields non–zero *covariant* derivatives of the $SU_{\rm L}(3) \times SU_{\rm R}(3)$ currents proportional to certain combinations of the corresponding external gauge fields [28]. The non–Abelian anomaly will be noted to have important consequences for the theory of Skyrmions.

Although the true symmetry of massless three flavor QCD is $SU_{\rm L}(3) \times SU_{\rm R}(3) \times U_{\rm V}(1)$, the resulting symmetry of the physical states of the theory is further reduced to $SU_{\rm V}(3) \times U_{\rm V}(1)$ by the "spontaneous breakdown" mechanism. In this mechanism the vacuum state is not invariant under the full symmetry group. The massless QCD vacuum is characterized by a non-vanishing "condensate" $\langle \bar{q}_u q_u + \bar{q}_d q_d + \bar{q}_s q_s \rangle \neq 0$. Under an infinitesimal chiral transformation

[1]Here λ^a, $a = 1, \ldots, N_f^2 - 1$ denote the Gell–Mann matrices of $SU(N_f)$ while λ^0 refers to the singlet generator.

$L = 1 + i\sum_{a=0}^{N_f^2-1} \epsilon_{\rm L}^a \lambda^a/2$ and $R = 1 + i\sum_{a=0}^{N_f^2-1} \epsilon_{\rm R}^a \lambda^a/2$ the variation of this quark–bilinear is found to be

$$\delta\left(\bar{q}q\right) = i\left(\epsilon_{\rm L}^a - \epsilon_{\rm R}^a\right)\left(\bar{q}_{\rm L}\frac{\lambda^a}{2}q_{\rm R} - \bar{q}_{\rm R}\frac{\lambda^a}{2}q_{\rm L}\right) = \left(\epsilon_{\rm L}^a - \epsilon_{\rm R}^a\right)\bar{q}\frac{\lambda^a}{2}i\gamma_5 q\,. \tag{3.5}$$

Clearly the condensate is invariant only for the subgroup $L = R$ in eq (3.3) which is a vector type transformation. This explains the physical $SU_{\rm V}(3)\times U_{\rm V}(1)$ invariance. Note that the right hand side of eq (3.5) represents pseudoscalar objects. These are "zero mode" fluctuations of the above vacuum configuration and the corresponding massless pseudoscalar particles are designated Nambu–Goldstone bosons. Their scalar chiral "partners" – which would be degenerate in mass were it not for the spontaneous symmetry breakdown – are *not* constrained to be massless. This splitting of low–lying pseudoscalars and scalars expected from massless QCD seems in qualitative agreement with the experimental situation.

The $SU_{\rm V}(3) \times U_{\rm V}(1)$ invariance (so–called "eightfold way") of massless QCD is, of course, further broken when the effects of non–zero quark mass terms are included. For later purposes it is convenient to rewrite the quark mass terms as:

$$\mathcal{L}_{\rm QCD}^{mass} = -\frac{m_u + m_d}{2}\bar{q}\mathcal{M}q \qquad \text{with} \qquad \mathcal{M} = \frac{2+x}{3}\mathbb{1} + y\lambda_3 + \frac{1-x}{\sqrt{3}}\lambda_8. \tag{3.6}$$

Here characteristic quark mass ratios are defined by

$$x = \frac{2m_s}{m_u + m_d}\,, \quad y = \frac{m_u - m_d}{m_u + m_d}. \tag{3.7}$$

In the limit $y = 0$, the theory possesses $SU_{\rm V}(2)$ or isospin invariance.

3.2 Effective Lagrangian of pseudoscalars

The simplest way to mock up low energy QCD is to employ a 3×3 matrix field M_{ij} which transforms under the chiral group in the same way as the bilinear quark combination $\bar{q}_{jR}q_{iL}$. It has the decomposition $M = S + iP$ into hermitian scalar and pseudoscalar components. A chirally invariant Lagrangian is

$$\mathcal{L} = \frac{1}{2}{\rm tr}(\partial_\mu M \partial^\mu M^\dagger) - V(M, M^\dagger), \tag{3.8}$$

where the potential V is a function of invariants like ${\rm tr}(MM^\dagger)$, ${\rm tr}(MM^\dagger MM^\dagger)$ etc. Spontaneous breakdown to $SU_{\rm V}(3)$ is implemented by choosing V to have a minimum such that $\langle M\rangle = const. \times \mathbb{1}$. Then the scalar fields S become massive and can be "integrated out" by imposing a chirally invariant constraint [10]. This represents a transition from the linear to the non–linear sigma model. Formally we may use the "polar decomposition" of the matrix M into unitary and hermitian factors $M = HU$. Setting $H \to const. \times \mathbb{1}$ then results in eq (1.1) again.

In principle, a scenario of this sort can be derived by adding a term like $\bar{q}_L M q_R + h.c.$ to the QCD Lagrangian and then integrating out the quark fields[2]. As a result one is left with a

[2] In the case of QCD this seems to be impractical. However, the simpler Nambu–Jona–Lasinio [9] model for the quark flavor dynamics nicely exemplifies how a meson functional can be constructed by integrating out quark degrees of freedom [29].

complicated action functional for M and, by further eliminating H as above, for U. Note that U inherits the chiral transformation property of the quark bilinear:

$$U \longrightarrow L U R^{\dagger}\,. \tag{3.9}$$

It is this identification of the transformation properties which provides the important link of the effective chiral theory to QCD since it in particular implies that the (Noether) currents must be identified. In turn, matrix elements of these currents will yield the static properties of hadrons.

Of course, in the chiral limit we demand the effective meson theory to be strictly invariant under the transformation (3.9). Since $U^{\dagger}U = 1$ only derivative terms can appear and the leading one is just the non–linear σ model (1.1). Clearly also the Skyrme term (2.4) is invariant under (3.9).

At this point one essential ingredient is still missing to ensure that the chiral field, $U = \exp(i\Phi)$ describes pseudoscalar fields, Φ. This requirement demands the parity transformation

$$U \stackrel{\rm parity}{\longrightarrow} U^{\dagger}\,. \tag{3.10}$$

However, it is straightforward to verify that the Skyrme model Lagrangian is invariant under (3.10) and $\boldsymbol{r} \rightarrow -\boldsymbol{r}$ separately. On the level of the equations of motion we can easily break this unwanted extra symmetry by adding a term which contains the Levi–Civita tensor. With $\alpha_{\mu} = (\partial_{\mu}U)U^{\dagger}$ we write

$$\frac{f_{\pi}^{2}}{2}\partial_{\mu}\alpha^{\mu} + \ldots + 5\lambda\epsilon_{\mu\nu\rho\sigma}\alpha^{\mu}\alpha^{\nu}\alpha^{\rho}\alpha^{\sigma} = 0\,, \tag{3.11}$$

where the ellipsis refers to the contributions from the Skyrme term (2.4) which have the same symmetries as those from the non–linear σ term (1.1). Unfortunately, the additional term cannot be easily incorporated in the effective Lagrangian since it does not correspond to the variation of a local term. Witten suggested [30] to include it at the level of the action because the variation of

$$\Gamma = \lambda \int_{\mathrm{M}_5} {\rm tr}\, \epsilon_{\mu\nu\rho\sigma\tau}\alpha^{\mu}\alpha^{\nu}\alpha^{\rho}\alpha^{\sigma}\alpha^{\tau} \tag{3.12}$$

and the use of Stoke's theorem yields the desired term in the equation of motion (3.11) provided the boundary of the five dimensional manifold, M_5 is taken to be Minkowski space, *i.e.* $\partial\mathrm{M}_5 = \mathrm{M}_4$. The choice of M_5 is not unique because its complement has the same boundary. In order to nevertheless have a unique action the constant $\lambda = \frac{-in}{240\pi^2}\,, n \in \mathbb{Z}$ must be quantized[3]. It is interesting to study the physical relevance of (3.12). Expanding in the meson fields Φ and employing again Stoke's theorem reveals that it describes processes with at least five different pseudoscalars[4] like $K^{+}K^{-} \rightarrow \pi^{+}\pi^{0}\pi^{-}$. As such processes were first discussed by Wess and Zumino [31] who essentially found a power series expression for (3.12), the term is commonly named after them[5].

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[3] The reader is referred to the literature [30] to see the analogy to Dirac's quantization of the magnetic monopole.

[4] For that reason the term (3.12) vanishes in the case of two flavors which only has four different pseudoscalar fields.

[5] Some authors refer to the Wess–Zumino term in its gauged form *i.e.* with external sources. Unless otherwise noted we will always understand the Wess–Zumino term to be (3.12).

There are further important consequences of the Wess–Zumino term (3.12) which can be read off after generalizing it so its variation with respect to external (electro–weak) gauge transformations [30, 32] yields the non–Abelian anomaly [28]. After appropriately including the corresponding gauge boson fields two striking features are observed:

(i) A contact interaction for the decay $\pi^0 \to \gamma\gamma$ is contained in the gauged Wess–Zumino action. On the quark level this process is described by the ABJ anomaly involving the diagrams of figure 3.1 with the external lines representing photons. Identifying that result with the Wess–Zumino term requires setting $n = N_C$, *i.e.* the Wess–Zumino term is proportional to the number of colors.

(ii) The linear coupling to the $U_{\rm V}(1)$ gauge boson represents the baryon number current. Indeed it turns out that this current is identical to the topological current B_μ in eq (2.2).

The mocking up of the effects of the $U_{\rm A}(1)$ anomaly in QCD involves the $SU(3)$ singlet pseudoscalar particle η' and will be discussed later when we treat the *proton spin puzzle*.

We must also take account of the effects of the finite quark mass terms (3.6). These transform according to the chiral $SU_{\rm L}(3) \times SU_{\rm R}(3)$ representation: $\mathbf{3} \times \mathbf{3}^* + \mathbf{3}^* \times \mathbf{3}$. Note that the matrix $\mathcal{M} = \mathcal{M}^\dagger$ (neglecting the possibility of strong CP violation) may be considered a "spurion" for this transformation property. Then the minimal symmetry breaking piece of the effective Lagrangian reads

$$\mathcal{L}_{\rm SB} = {\rm tr}\,\left\{\mathcal{M}\left[-\beta'\left(\partial_\mu U \partial^\mu U^\dagger U + U^\dagger \partial_\mu U \partial^\mu U^\dagger\right) + \delta'\left(U + U^\dagger - 2\right)\right]\right\}, \qquad (3.13)$$

where β' and δ' are two numerical parameters. The δ' term is required to split the pseudoscalar meson masses while the β' term is required to split the pseudoscalar "decay constants". The decay constants f_a are defined from the axial vector matrix elements $\langle 0|j^a_{5,\mu}|\phi_a, p\rangle = i f_a p_\mu$

Working in the isospin invariant limit, the parameters β', δ' and x can then be extracted from the knowledge of meson properties [33],

$$m_\pi^2 = \frac{4}{f_\pi^2}\delta', \quad m_K^2 = \frac{4}{f_K^2}\delta'(1+x) \quad \text{and} \quad \left(\frac{f_K}{f_\pi}\right)^2 = 1 + \frac{4}{f_\pi^2}\beta'(1-x). \qquad (3.14)$$

This represents the essential input when discussing the Skyrme model for three flavors. To sum up, the Lagrangian of only pseudoscalars which we shall use for discussing Skyrmions consists of the sum of (1.1), (2.4), (3.12) and (3.13).

In the chiral perturbation theory approach [12] essentially the most general chirally invariant Lagrangian is written down and ordered in powers of $\partial\partial \sim \mathcal{M}$. For example, the leading terms are eq (1.1) and the δ' term of eq (3.13). The next–to–leading terms include:

$$\begin{aligned}
&[{\rm tr}(\partial_\mu U \partial^\mu U^\dagger)]^2,\ {\rm tr}(\partial_\mu U \partial_\nu U^\dagger){\rm tr}(\partial^\mu U \partial^\nu U^\dagger),\\
&{\rm tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger),\ {\rm tr}(\partial_\mu U \partial^\mu U^\dagger){\rm tr}(\mathcal{M}(U+U^\dagger)),\ {\rm tr}(\partial_\mu U \partial^\mu U^\dagger \mathcal{M}(U+U^\dagger)),\\
&[{\rm tr}(\mathcal{M}(U+U^\dagger)]^2,\ [{\rm tr}(\mathcal{M}(U-U^\dagger)]^2,\ {\rm tr}(\mathcal{M}U^\dagger\mathcal{M}U^\dagger + \mathcal{M}U\mathcal{M}U),
\end{aligned} \qquad (3.15)$$

each with its own coupling constant. Note that the combinations of terms which can not be manipulated (by use of various matrix identities) to become a single trace are suppressed in the $1/N_C$ expansion. This procedure also entails absorbing the divergent parts of loop corrections in the coefficients of the listed terms. The result is a joint power series in energy and the quark masses which can be expected to be very accurate quite near the $\pi\pi$ threshold in the case of pion–pion scattering for example. But going higher in energy is very difficult in this scheme. Furthermore it seems that many of the coefficients mainly simulate the low energy effects of vector meson exchanges. We shall also work with a meson Lagrangian which includes the vector particles directly. This may be thought of as a start on the approach of constructing the leading (Born) term of the $1/N_C$ expansion, which should include mesons of all spins.

3.3 Effective Lagrangian of pseudoscalars and vectors

We will follow the so–called massive Yang–Mills approach [32] for introducing the vector meson nonet into the Lagrangian of pseudoscalars in a chirally invariant manner. In this approach both vector and axial vector fields are formally introduced as gauge fields (yielding invariance under *local* chiral transformations). Then globally invariant mass–type terms which break the local chiral invariance are included. Finally, as in the transition from the linear to the non–linear sigma model discussed in section 3.1 above, the (heavier) axial vector mesons are eliminated by a chirally invariant constraint.

We introduce two multiplets with spin one, $A_\mu^{\rm L}$ and $A_\mu^{\rm R}$ which we demand to transform under (3.3) as left– and right–handed fields, respectively,

$$A_\mu^{\rm L} \longrightarrow L\left(A_\mu^{\rm L}+\frac{i}{g}\partial_\mu\right)L^\dagger \quad \text{and} \quad A_\mu^{\rm R} \longrightarrow R\left(A_\mu^{\rm R}+\frac{i}{g}\partial_\mu\right)R^\dagger\,. \tag{3.16}$$

This allows us to define a covariant derivative for the chiral field and field tensors,

$$D_\mu U = \partial_\mu U - igA_\mu^{\rm L}U + igUA_\mu^{\rm R}\,, \tag{3.17}$$
$$F_{\mu\nu}^{\rm L,R} = \partial_\mu A_\nu^{\rm L,R} - \partial_\nu A_\mu^{\rm L,R} - ig\left[A_\mu^{\rm L,R}, A_\nu^{\rm L,R}\right]\,, \tag{3.18}$$

which transform homogeneously under (3.3). The chirally invariant terms with a minimal number of derivatives read

$$\text{tr}\left[(D_\mu U)^\dagger D^\mu U\right]\,, \quad \text{tr}\left[F_{\mu\nu}^{\rm L,R}F^{\rm L,R,\mu\nu}\right] \quad \text{and} \quad \text{tr}\left[F_{\mu\nu}^{\rm L}UF^{\rm R,\mu\nu}U^\dagger\right]\,. \tag{3.19}$$

In addition we can have mass–type terms for the vector mesons

$$\text{tr}\left[A_\mu^{\rm L}A^{\rm L,\mu}+A_\mu^{\rm R}A^{\rm R,\mu}\right] \quad \text{and} \quad \text{tr}\left[A_\mu^{\rm L}UA^{\rm R,\mu}U^\dagger\right]\,, \tag{3.20}$$

which are still invariant under global chiral transformations. Of course, many more terms with higher derivatives could be written down at the expense of more undetermined parameters. Now, it is our aim to construct an effective model for the vector mesons only; at present we are not interested in the axial–vector mesons. We have to find a mechanism to eliminate the latter without violating the chiral symmetry. This can be accomplished by choosing a special "gauge" for the vector fields $A_\mu^{\rm L,R}$,

$$\tilde{A}_\mu^{\rm L} = \xi\left(\rho_\mu+\frac{i}{g}\partial_\mu\right)\xi^\dagger \quad \text{and} \quad \tilde{A}_\mu^{\rm R} = \xi^\dagger\left(\rho_\mu+\frac{i}{g}\partial_\mu\right)\xi\,. \tag{3.21}$$

Here ρ_μ is a matrix field with N_f^2 components. For example, in the case of two flavors it includes both the ρ and ω mesons via $\rho_\mu = \boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau}+\omega_\mu\mathbb{1}$. In the case of three flavors, this matrix field is supplemented by the K^* and ϕ mesons. Most importantly we have introduced the "square root", ξ of the chiral field, $U = \xi\xi$ which yields the chirally invariant relation

$$\tilde{A}^{\rm L}_\mu = U\left(\tilde{A}^{\rm R}_\mu + \frac{i}{g}\partial_\mu\right)U^\dagger\,. \tag{3.22}$$

It is actually this so–called unitary constraint which eliminates the axial–vector fields in favor of the vector fields ρ without spoiling chiral symmetry.

It is interesting to study the behavior of the ρ meson under chiral transformations. To start off, we recognize that the transformation of ξ introduces the matrix K which is defined by [34]

$$\xi \longrightarrow L\xi K^\dagger \stackrel{!}{=} K\xi R^\dagger\,. \tag{3.23}$$

Clearly this leaves the transformation law of the chiral field (3.9) unchanged. Note that in general the matrix K is a position dependent quantity because of ξ. Demanding now the symmetry transformation

$$\rho_\mu \longrightarrow K\left(\rho_\mu + \frac{i}{g}\partial_\mu\right)K^\dagger \tag{3.24}$$

causes the fields $\tilde{A}^{\rm L,R}$ to transform exactly like left– and right–handed vector fields.

Within the unitary gauge the various terms listed above in (3.19) and (3.20) are no longer independent. Introducing the homogeneously transforming combinations

$$p_\mu = \partial_\mu\xi\xi^\dagger + \xi^\dagger\partial_\mu\xi \quad \text{and} \quad R_\mu = \rho_\mu + \frac{i}{2g}\left(\partial_\mu\xi\xi^\dagger - \xi^\dagger\partial_\mu\xi\right) \tag{3.25}$$

the terms up to two derivatives can be combined to a chirally invariant Lagrangian of vectors (and pseudoscalars)

$$\mathcal{L}_{\rm VM} \;=\; {\rm tr}\left[-\frac{1}{4}f_\pi^2 p_\mu p^\mu - \frac{1}{2}F_{\mu\nu}(\rho)F^{\mu\nu}(\rho) + m_\rho^2 R_\mu R^\mu\right]\,, \tag{3.26}$$

where we have used the fact that the coefficient of the term quadratic in the ρ meson field is related to the vector meson mass $m_\rho = 770{\rm MeV}$. Upon expanding the square–root field ξ in powers of the pseudoscalar field, one finds that the Lagrangian (3.26) contains the $\rho\pi\pi$ coupling,

$$\mathcal{L}_{\rho\pi\pi} = \frac{m_\rho^2}{2gf_\pi^2}\boldsymbol{\rho}_\mu\cdot\left(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}\right)\,, \tag{3.27}$$

which can be utilized to fix the coupling constant $g \approx 5.6$ from the known decay–width of the process $\rho \longrightarrow \pi\pi$.

Terms which involve the Levi–Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ are also of great interest for the Skyrme model. For their presentation it is most useful to introduce the notation of differential forms: $A^R = A^R_\mu dx^\mu$, $d = \partial_\mu dx^\mu$, etc. . Since the left– and right–handed "gauge fields" are related via the unitary constraint (3.22) the number of linearly independent terms, which transform properly under chiral transformation as well as parity and charge conjugation, is quite limited [32]

$$A^L\alpha^3 \ , \quad dA^L\alpha A^L - A^L\alpha dA^L + A^L\alpha A^L\alpha \ , \quad 2\left(A^L\right)^3\alpha + \frac{i}{g}A^L\alpha A^L\alpha \ . \tag{3.28}$$

For convenience we have again made use of $\alpha_\mu = (\partial_\mu U)U^\dagger = \xi p \xi^\dagger$. Of course, including these terms in the model Lagrangian will introduce three more parameters: γ_1, γ_2 and γ_3. A suitable presentation of this part of the action is given in terms of p and R (employing again the notation of differential forms)

$$\Gamma_\epsilon \ = \ \Gamma_{\rm WZ} + \int_{M_4} {\rm tr}\left(\frac{1}{6}\left[\gamma_1 + \frac{3}{2}\gamma_2\right]Rp^3 - \frac{i}{4}g\gamma_2 F(\rho)\left[pR - Rp\right] - g^2\left[\gamma_2 + 2\gamma_3\right]R^3p\right) \ , \tag{3.29}$$

where $\Gamma_{\rm WZ}$ is given in (3.12). In ref [35] two of the three unknown constants, $\gamma_{1,2,3}$ were determined from purely strong interaction processes like $\omega \to 3\pi$. Defining $\tilde{h} = -2\sqrt{2}\gamma_1/3$, $\tilde{g}_{VV\phi} = g\gamma_2$ and $\kappa = \gamma_3/\gamma_2$ the central values $\tilde{h} = \pm 0.4$ and $\tilde{g}_{VV\phi} = \pm 1.9$ were found. Within experimental uncertainties (stemming from the errors in the $\omega - \phi$ mixing angle) these may vary in the range $\tilde{h} = -0.15, \ldots, 0.7$ and $\tilde{g}_{VV\phi} = 1.3, \ldots, 2.2$ subject to the condition $|\tilde{g}_{VV\phi} - \tilde{h}| \approx 1.5$. The third parameter, κ could not be fixed in the meson sector. From studies [36] of nucleon properties in the two flavor model it was argued that $\kappa \approx 1$ represents a reasonable choice.

The sum of the space integral of (3.26) and (3.29) comprise the chirally invariant part of the effective action we shall use for discussing the soliton in the vector meson model. Note that the second piece of (3.29) can be gauged with external fields [35] so as to make no contribution to the non–Abelian anomaly. The first piece $\Gamma_{\rm WZ}$ then correctly supplies the non–Abelian anomaly. Furthermore the second piece of (3.29) stabilizes the soliton without the need for including the Skyrme term (2.4).

We must still include the effects of symmetry breaking due to finite quark masses in the vector meson system. To leading order in the symmetry breaking, an appropriate term which behaves properly under chiral transformations, can be constructed by analogy to the last expression in (3.20)

$$-\,\alpha'{\rm tr}\left[\mathcal{M}\left(A^L_\mu U A^{R\mu} + A^R_\mu U^\dagger A^{L\mu}\right)\right] . \tag{3.30}$$

This leading contribution not only distinguishes between the ρ and K^* masses but also contributes to the different decay constants of the pseudoscalar mesons via the unitary gauge (3.21). The reader may consult ref [33] for recent discussion of higher order symmetry breaking terms.

We would like to end this section on including vector mesons by noting that the same Lagrangian is obtained within the so–called hidden gauge approach [37], once the same symmetries are required. This shows that these two approaches are in fact identical.

3.4 Other aspects

We expect that baryons should appear as solitons of the large N_C effective meson Lagrangian for any number of flavors N_f. In the case of three (or more) light flavors the Wess–Zumino term guarantees, as discussed in section 3.1, that the baryon number (2.2) is obtained in a self–contained manner from the Lagrangian. This can be used to check that the soliton indeed has the correct baryon number.

Now in the two flavor case, the same kind of soliton solution exists. However the Wess–Zumino term vanishes identically so we cannot similarly check its baryon number in a self–contained way. The situation is even more peculiar for $N_f = 1$. There the Skyrme model represents a mapping $S^3 \rightarrow S^1$ which does not contain topologically stable configurations. However, we are not forced to use an effective Lagrangian of the same form. In this case it is probably more realistic to construct the Lagrangian by including isoscalars like the spin–0 σ–field and the spin–1 ω–field. Such a Lagrangian might have a soliton solution (not necessarily topological) but a check of its baryon number may also not be available in a self–contained way. These examples seem to indicate that the form of the relevant effective Lagrangian may have a non–trivial N_f dependence (at least for small N_f).

Another interesting question, related in the sense of understanding whether physical features of the solitons can be traced to particular pieces of the effective Lagrangian, concerns the *stabilization* of the soliton. In section 2 we noted that the Skyrme term (2.4) was introduced precisely for this purpose. There is an often mentioned "derivation" of this term from the piece of the vector meson Lagrangian (3.26) above which goes as follows. In a large mass expansion, $m_\rho \rightarrow \infty$ the equation of motion for the vector meson field simply becomes $R_\mu = 0$. Substituting this into the remainder of the vector meson Lagrangian (3.26)

$$F_{\mu\nu}(\rho) \longrightarrow F_{\mu\nu}\left(\frac{-i}{2g}[\partial_\mu \xi \xi^\dagger - \xi^\dagger \partial_\mu \xi]\right)$$

yields exactly the Skyrme term (2.4) with the identification $g = e$. Although the numbers 5.6 and 4.75 are in reasonable agreement there is one caveat to this appealing derivation of the Skyrme term. While the Skyrme model does yield stable solitons, however, for arbitrary large but finite m_ρ the model (3.26) does not contain stable soliton solutions. Thus one seems to have achieved stabilization merely by approximating a model in which stabilization does not exist. Clearly we have not obtained a "physical origin" for the stabilization mechanism. As mentioned in the previous section, the second piece of (3.29) stabilizes the soliton in the vector meson Lagrangian without a need for the Skyrme term. It is also possible that, as in the case of the s–wave ground state of hydrogen, stability is achieved at the quantum, rather than at the classical, level. Several investigations of this possibility have been made [38] based on just the non–linear sigma model term (1.1), although an assumption on the allowed chiral profiles seems to be required.

4. The Skyrme model with three flavors

It is well established [7] that the neutron (n) and proton (p) belong to a multiplet with six other members (the iso–singlet Λ, the iso–doublet Ξ and the iso–triplet Σ). To try to understand n and p alone is to look at only a small piece of a large picture. Thus we must consider the three flavor generalization of the treatment in section 2. First (in the present section) we shall consider the Lagrangian of pseudoscalars alone, discussed in section 3.2. The new features arise from the inclusion of flavor $SU(3)$ symmetry breaking terms (see (3.13) together with (3.6) and (3.7)) as well as the Wess–Zumino term (3.12). Both of these features involve non–trivial extensions of the formalism and interesting "physics".

The first step towards including the strangeness degrees of freedom is to actually take the chiral field to be a $U(1) \otimes SU(3)$ matrix. To be precise, the three flavor chiral field is defined as

$$U(x) = \exp\left(i\frac{\sqrt{2}}{\sqrt{3}f_\pi}\eta_0\right)\exp\left(i\Phi\right). \tag{4.1}$$

While the singlet field η_0 is separated the matrix field Φ now not only contains the pion degrees of freedom but also the kaons and the non–singlet component of the η fields,

$$\Phi = \sum_{a=1}^{8}\frac{\sqrt{2}}{f_\pi}\phi^a\lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \tag{4.2}$$

Here λ^a denote the Gell–Mann matrices. Note that in the presence of derivative–type symmetry breakers (*e.g.* the β' term in (3.13)) the normalization of the fields gets shifted; the "physical" fields are gotten by multiplying the fields above by some constants as $Z_\pi\pi^+$, $Z_K K^+$, etc.; similarly the physical decay constants are $Z_\pi f_\pi = 93\text{MeV}$, $Z_K f_K \approx 113\text{MeV}$, etc. For the Z's we have

$$Z_\pi = \left(1 - \frac{8}{f_\pi^2}\beta'\right)^{\frac{1}{2}}, \quad Z_K = \left(1 - \frac{4}{f_\pi^2}(1+x)\beta'\right)^{\frac{1}{2}} \quad \text{etc.} \tag{4.3}$$

We clearly need a suitable generalization of the Skyrme *ansatz* (2.6). It turns out that it is correct to just embed the $SU(2)$ hedgehog in the $SU(3)$ matrix. Flavor symmetry breaking implies that field configurations which have non–zero strangeness possess a classical energy which (at least in the unit baryon number sector) is larger than that of a zero strangeness configuration. Thus we choose the embedding:

$$U_0(\boldsymbol{r}) = \left(\begin{array}{cc|c} \multicolumn{2}{c|}{\exp\left(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)\right)} & \begin{matrix}0\\0\end{matrix} \\ \hline 0 & 0 & 1 \end{array}\right). \tag{4.4}$$

Hence the classical energy will not be modified and the soliton profile, $F(r)$ is that in figure 2.1. The effects of the strange degrees of freedom are hence visible when states with baryon quantum numbers are generated via the collective coordinate approach.

The collective coordinate matrix $A(t)$ defined in eq (2.8) is now taken from $SU(3)$ and in analogy to eq (2.10), now leads to eight angular velocities,

$$A^\dagger(t)\frac{d}{dt}A(t) = \frac{i}{2}\sum_{a=1}^{8}\lambda^a\Omega_a\,. \tag{4.5}$$

In addition to the angular velocities Ω_a the adjoint representation

$$D_{ab} = \frac{1}{2}\,\mathrm{tr}\left(\lambda_a A\lambda_b A^\dagger\right) \tag{4.6}$$

of the collective rotations, $A(t)$ will be important, in particular in the context of flavor symmetry breaking.

Substituting $U = A(t)U_0(\boldsymbol{r})A^\dagger(t)$ into the pseudoscalar Lagrangian of section 3.2 without the symmetry breaker gives rise after a spatial integration to the collective Lagrangian

$$L_{\rm Skyrme}(A,\dot{A}) + L_{\rm WZ}(A,\dot{A}) = \frac{1}{2}\alpha^2\sum_{i=1}^{3}\Omega_i^2 + \frac{1}{2}\beta^2\sum_{\alpha=4}^{7}\Omega_\alpha^2 - \frac{N_C B}{2\sqrt{3}}\,\Omega_8 - E_{\rm cl}\,. \tag{4.7}$$

The $SU(2)$ moment of inertia α^2 remains unchanged while the moment of inertia β^2 for rotations into the strange directions is a new functional of the pseudoscalar fields. The fact that the eighth component of the angular velocity vector does not appear quadratically in (4.7) is a consequence of $[U_0, \lambda_8] = 0$. The term proportional to $B\Omega_8$, where B is the baryon number arises from $\Gamma_{\rm WZ}$. In order to obtain it we make use of the separation [39]

$$\Gamma_{\rm WZ}[U] = \Gamma_{\rm WZ}[U_0] - \frac{iN_C}{48\pi^2}\int d^4x\, {\rm tr}\left\{\left[(U_0^\dagger dU_0)^3 + (U_0 dU_0^\dagger)^3\right](A^\dagger dA)\right\}\,, \tag{4.8}$$

where, again, Stoke's theorem has been employed. As U_0 is static we have $\Gamma_{\rm WZ}[U_0] = 0$ and the remainder becomes a local object which is straightforwardly evaluated.

4.1 Quantization of the three flavor collective Lagrangian

In order to quantize the three flavor Lagrangian (4.7) we require the operators for spin and flavor as Noether charges. As a consequence of the hedgehog structure, the infinitesimal change under spatial rotations can be written as a derivative with respect to $\boldsymbol{\Omega}$

$$[\boldsymbol{r}\times\boldsymbol{\partial}, U(\boldsymbol{r},t)] = \frac{\partial\dot{U}(\boldsymbol{r},t)}{\partial\boldsymbol{\Omega}}\ . \tag{4.9}$$

By the Noether construction this leads to the spin operator $\boldsymbol{J} = \partial L(A,\Omega_a)/\partial\boldsymbol{\Omega}$. The quantization of the "$SU(3)$ rigid top" proceeds by generalizing this result to the so–called right generators

$$R_a = -\frac{\partial}{\partial\Omega_a}\left(L_{\rm Skyrme} + L_{\rm WZ}\right) = \begin{cases} -\alpha^2\Omega_a = -J_a, & \text{a=1,2,3} \\ -\beta^2\Omega_a, & \text{a=4,..,7} \\ \frac{N_C B}{2\sqrt{3}}, & \text{a=8} \end{cases}\ . \tag{4.10}$$

The quantization prescription then demands the commutation relation $[R_a, R_b] = -if_{abc}R_c$ with f_{abc} being the antisymmetric structure constants of $SU(3)$. Explicit expressions for these generators in terms of an "Euler–angle" parameterization of A are presented in ref [40]. The so–called left generators, which are defined by the rotation $L_a = D_{ab}R_b$, satisfy the commutation relations $[L_a, L_b] = if_{abc}L_c$. They provide the isospin, $I_i = L_i$ $(i = 1,2,3)$ and hypercharge, $Y = 2L_8/\sqrt{3}$ operators.

The generator R_8 is linearly connected to the so–called right hypercharge $Y_R = 2R_8/\sqrt{3} = 1$ for $B = 1$ and $N_C = 3$. In analogy to the Gell–Mann Nishijima relation a right charge

$$Q_R = -J_3 + \frac{Y_R}{2} \tag{4.11}$$

may be defined. Completing the analogy we note that the eigenvalues of Q_R are $0, \pm1/3, \pm2/3, \pm1, \ldots$. Hence for $Y_R = 1$ the relation (4.11) can only be fulfilled when the eigenvalue of J_3 is half–integer. This yields the important conclusion that the $SU(3)$ model describes fermions. *A priori* this is not expected since the starting point has been an effective model of bosons. This discussion can be generalized to arbitrary N_C showing that the Skyrmion describes fermions when N_C is odd and bosons when N_C is even [30]. This, of course, is expected from considering baryons as being composed of N_C quarks. We conclude that the proper incorporation of the anomaly structure of QCD leads to the desired spin–statistics relation.

4.2 Flavor symmetry breaking and baryon spectrum

For a realistic treatment of baryon states in the space of the collective coordinates we have to supplement the collective Lagrangian by the flavor symmetry breaking pieces associated with (3.13). Substituting the flavor rotating hedgehog yields the symmetry breaking piece in the collective Lagrangian,

$$L_{\rm SB} = -\frac{1}{2}\gamma\left(1 - D_{88}\right) \tag{4.12}$$

with the coefficient, γ being linear in the symmetry breaking parameter $1 - x$, *i.e.*

$$\gamma = \frac{32\pi}{3}(x-1)\int dr\left\{\delta' r^2(1-{\rm cos}F) - \beta'{\rm cos}F\left(F'^2 r^2 + 2{\rm sin}^2 F\right)\right\}\,. \tag{4.13}$$

$D_{88}(A)$ is defined in eq (4.6). Putting pieces (4.7) and (4.12) together, the Hamiltonian for the collective coordinates is obtained as the Legendre transform $H = -\sum_{a=1}^{8} R_a\Omega_a - L$

$$H(A, R_a) \;=\; E_{\rm cl} + \frac{1}{2}\left[\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right]\boldsymbol{J}^2 + \frac{1}{2\beta^2}C_2(SU(3)) - \frac{3}{8\beta^2} + \frac{1}{2}\gamma\left(1 - D_{88}\right) \tag{4.14}$$

for $B = 1$ and $N_C = 3$. The constraint $R_8 = \frac{\sqrt{3}}{2}$, which yielded the spin–statistics relation, commutes with H permitting one to substitute this value. The term involving $\sum_{\alpha=4}^{7} R_\alpha^2$ has been re–expressed by introducing the quadratic Casimir operator of $SU(3)$, $C_2(SU(3)) = \sum_{a=1}^{8} R_a^2$. The standard $SU(3)$ representations are eigenstates of $C_2(SU(3))$ with eigenvalues μ. For example, the octet representation **8** has $\mu_{\mathbf{8}} = 3$ while $\mu_{\mathbf{10}} = \mu_{\overline{\mathbf{10}}} = 6$ and $\mu_{\mathbf{27}} = 8$. These representations diagonalize[1] the collective Hamiltonian in the absence of symmetry breaking, $\gamma = 0$.

Now consider the full collective Hamiltonian including the symmetry breaking. It seems reasonable to assume these $\gamma = 0$ eigenstates as a basis to diagonalize the full Hamiltonian. In a perturbative treatment up to third order in γ for the $\frac{1}{2}^+$ baryons only the representations $\mathbf{8}, \overline{\mathbf{10}}$ and $\mathbf{27}$ contribute [42]. For that reason the perturbative treatment is still simple, although one must go beyond leading order. In particular this implies that the nucleon is no longer a pure octet state but rather contains sizable admixture of the nucleon type states in higher dimensional representations,

$$|N\rangle = |N, \mathbf{8}\rangle + 0.0745\gamma\beta^2|N, \overline{\mathbf{10}}\rangle + 0.0490\gamma\beta^2|N, \mathbf{27}\rangle + \ldots\ , \tag{4.15}$$

where the coefficients of the effective symmetry breaker $\gamma\beta^2$ are computed from $SU(3)$ Clebsch–Gordon coefficients [43]. The nucleon is seen to have a roughly 25% amplitude to contain the $\overline{\mathbf{10}}$ state.

[1] The hedgehog structure of the classical configuration U_0 constrains the permissible $SU(3)$ irreducible representations to those which have at least one state with $I = J$ [41].

Although this perturbative treatment provides a physical picture of the symmetry breaking effects it actually turns out that the full Hamiltonian (4.14) can be exactly diagonalized numerically. The important ingredient is that within a suitable "Euler–angle" representation of the rotations A, the symmetry breaker $1 - D_{88}$ depends only on one of these eight angles. In each isospin channel the eigenvalue equation

$$\Big[C_2(SU(3)) + \beta^2\gamma\,(1 - D_{88})\Big]\,\Psi = \epsilon_{\rm SB}\Psi \tag{4.16}$$

then reduces to a set of coupled ordinary differential equations which can be integrated numerically. Here we do not wish to discuss this approach in full detail; rather we refer the reader to the original work by Yabu and Ando [44] and exhaustive applications of this method involving the present authors [45, 46, 40]. Having obtained the eigenvalue $\epsilon_{\rm SB}$ the baryon masses are straightforwardly computed from

$$M_B = E + \frac{1}{2}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right) J(J+1) - \frac{3}{8\beta^2} + \frac{1}{2\beta^2}\epsilon_{\rm SB}\ . \tag{4.17}$$

As already mentioned this diagonalization procedure is equivalent to the perturbation expansion. For small enough symmetry breaking $\beta^2\gamma$ even first order is sufficient. In that (unjustified) case the famous Gell–Mann–Okubo mass formulae [47, 7] holds exactly:

$$2\,(M_N + M_\Xi) \;=\; M_\Sigma + 3M_\Lambda \tag{4.18}$$

$$M_\Omega - M_{\Xi^*} \;=\; M_{\Xi^*} - M_{\Sigma^*} = M_{\Sigma^*} - M_\Delta\,. \tag{4.19}$$

Additional corrections [45] arise when we allow for non–zero classical K–meson fields to get induced by "rotations" Ω_α into the strange directions. These are energetically favorable since they maximize the strange moment of inertia β^2. With a parameterization

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = W(r)\,\hat{\boldsymbol r}\cdot\boldsymbol\tau\,\begin{pmatrix} \Omega_4 - i\Omega_5 \\ \Omega_6 - i\Omega_7 \end{pmatrix}\,, \tag{4.20}$$

the radial function $W(r)$ is determined from applying a variational principle to β^2. In principle one must enforce that the *ansatz* (4.20) has no overlap with any global rotation of the classical solution (4.4).

We adjust the only free parameter, $e \approx 4$ to the mass differences of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The resulting baryon spectrum is shown in table 4.1. Apparently the three flavor Skyrme model reasonably accounts for the empirical mass differences. The original studies [48, 49, 50, 44] yielded far too low mass splittings between baryons of different strangeness for physically motivated parameters of the effective Lagrangian[2]. A major reason for the improvement is the fact that γ is significantly enlarged by including the effects associated with $f_K \neq f_\pi$ [45]. It is also apparent from table 4.1 that enforcing the zero overlap condition for the induced kaon components can be compensated by a small variation of the Skyrme parameter, e. This indicates that possible double counting effects play only a minor role. It is interesting to remark that the mass differences for the $\frac{1}{2}^+$ baryons deviate strongly from the predictions in leading order of the flavor symmetry breaking. This can easily be observed from the ratios

$$(M_\Lambda - M_N) : (M_\Sigma - M_\Lambda) : (M_\Xi - M_\Sigma) = 1 : 0.52 : 0.85 \tag{4.21}$$

[2]Many of these authors considered f_π as a free parameter fitted to the absolute values of the baryon masses. Without the β' term this yielded f_π as low as 25MeV [49].

Table 4.1: The mass differences, which are obtained by exact diagonalization of the collective Hamiltonian (4.14), of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in the pseudoscalar model for e=4.0 are compared to the experimental data. The values in parentheses are obtained by enforcing the zero overlap condition mentioned after (4.20) [40]. In that case the Skyrme parameter has slightly been readjusted to e=3.9. All data are in MeV.

Baryons	Model	Expt.
$\Lambda - N$	154 (163)	177
$\Sigma - N$	242 (264)	254
$\Xi - N$	366 (388)	379
$\Delta - N$	278 (268)	293
$\Sigma^* - N$	410 (406)	446
$\Xi^* - N$	544 (545)	591
$\Omega - N$	677 (680)	733

which are in much better agreement with the experimental data (1:0.43:0.69) than the leading order result (1:1:0.5). Obviously the higher order contributions are important. This also indicates that the baryon wave–functions contain sizable admixture of higher dimensional $SU(3)$ representations, *cf.* eq (4.15). Nevertheless the deviation from the Gell–Mann–Okubo relations (4.18) is only moderate, in particular the equal spacing among the $\frac{3}{2}^+$ baryons is well reproduced. Finally we note that, as discussed in point (iii) of section 2, the absolute mass of the nucleon is also too high in the three flavor case. Again we must rely on the N_C^0 corrections mentioned.

4.3 Electromagnetic properties of $\frac{1}{2}^+$ baryons

The value for the Skyrme parameter $e = 4.0$ obtained from this best fit to the baryon mass differences is next employed to evaluate static properties of baryons within this model. In order to do so one first constructs the Noether currents associated with the symmetry transformation (3.9). A convenient method is to extend these global symmetries to local ones by introducing external gauge fields (*e.g.* the gauge fields of the electroweak interactions) into the total action *i.e.* (1.1), (2.4), (3.12) and (3.13). The Noether currents are then read off as the expressions which couple linearly to these gauge fields. This procedure is especially appropriate for the Wess–Zumino term (3.12) because this non–local term can only be made gauge invariant by a lengthy iterative procedure [30, 32]. The final form of the nonet ($a = 0, \ldots, 8$) vector (V_μ^a) and axial–vector (A_μ^a) currents reads [46] (for $N_C = 3$)

$$\begin{aligned} V_\mu^a(A_\mu^a) \;=\; & -\frac{i}{2} f_\pi^2 \, \mathrm{tr}\left\{ \left(\xi Q^a \xi^\dagger \mp \xi^\dagger Q^a \xi\right) p_\mu \right\} - \frac{i}{8e^2} \mathrm{tr}\left\{ \left(\xi Q^a \xi^\dagger \mp \xi^\dagger Q^a \xi\right) [p_\nu, [p_\mu, p_\nu]] \right\} \\ & - \frac{1}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left\{ \left(\xi Q^a \xi^\dagger \pm \xi^\dagger Q^a \xi\right) p_\nu p_\rho p_\sigma \right\} \\ & - i\beta' \mathrm{tr}\left\{ Q^a \left(\{U\mathcal{M} + \mathcal{M}U^\dagger, \alpha_\mu\} \mp \{\mathcal{M}U + U^\dagger \mathcal{M}, \beta_\mu\} \right) \right\}, \end{aligned} \tag{4.22}$$

where $Q^a = (\frac{1}{3}, \frac{\lambda^1}{2}, \ldots, \frac{\lambda^8}{2})$ denote the Hermitian nonet generators. The combination

$$Q^{\mathrm{e.m.}} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) = Q^3 + \frac{1}{\sqrt{3}} Q^8 \tag{4.23}$$

is of special interest because it enters the computation of the electromagnetic properties. The associated form factors of the $\frac{1}{2}^+$ baryons (B) are defined by

$$\langle B(\boldsymbol{p}')|V_\mu^{\rm e.m.}|B(\boldsymbol{p})\rangle = \overline{u}(\boldsymbol{p}')\left[\gamma_\mu F_1^B(q^2) + \frac{\sigma_{\mu\nu}q^\nu}{2M_B}F_2^B(q^2)\right]u(\boldsymbol{p}) \ , \qquad q_\mu = p_\mu - p'_\mu \ . \qquad (4.24)$$

Frequently it is convenient to introduce "electric" and "magnetic" form factors

$$G_E^B(q^2) = F_1^B(q^2) - \frac{q^2}{4M_B^2}F_2^B(q^2) \ , \qquad G_M^B(q^2) = F_1^B(q^2) + F_2^B(q^2) \ . \qquad (4.25)$$

Substituting the rotating hedgehog configuration into the defining equation of the currents (4.22) yields for the spatial components of the vector current[3]

$$\begin{aligned} V_i^a &= V_1(r)\epsilon_{ijk}x_jD_{ak} + \frac{\sqrt{3}}{2}B(r)\epsilon_{ijk}\Omega_jx_kD_{a8} + V_2(r)\epsilon_{ijk}x_jd_{d\alpha\beta}D_{a\alpha}\Omega_\beta \\ &\quad + V_3(r)\epsilon_{ijk}x_jD_{88}D_{ak} + V_4(r)\epsilon_{ijk}x_jd_{d\alpha\beta}D_{8\alpha}D_{a\beta} + \ldots \ , \end{aligned} \qquad (4.26)$$

where

$$B(r) = \frac{-1}{2\pi^2}F'\frac{\sin^2 F}{r^2} \qquad (4.27)$$

is the baryon number density (2.2). The explicit form of the radial functions $V_1(r), \ldots, V_4(r)$ is given in appendix B of ref [46]. According to the quantization prescription (4.10) the angular velocities Ω_a are replaced by their expressions in terms of the right generators R_a of $SU(3)$. Taking the Fourier transform of the resulting matrix elements allows one to identify the magnetic form factor in the Breit frame [51, 52]

$$\begin{aligned} G_M^B(\boldsymbol{q}^2) &= -8\pi M_B\int_0^\infty r^2dr\frac{r}{|\boldsymbol{q}|}j_1(r|\boldsymbol{q}|)\Big\{V_1(r)\langle D_{e3}\rangle_B - \frac{1}{2\alpha^2}B(r)\langle D_{e8}R_8\rangle_B \qquad (4.28) \\ &\quad -\frac{1}{\beta^2}V_2(r)\langle d_{3\alpha\beta}D_{e\alpha}R_\beta\rangle_B + V_3(r)\langle D_{88}D_{e3}\rangle_B + V_4(r)\langle d_{3\alpha\beta}D_{e\alpha}D_{8\beta}\rangle_B\Big\} \ . \end{aligned}$$

Here the flavor index e refers to the "electromagnetic" direction (4.23). The magnetic moment corresponds to the magnetic form factor at zero momentum transfer $\mu_B = G_M^B(0)$. Similarly the electric form factor is given by Fourier transforming the time component of the electromagnetic current

$$G_E^B = 4\pi\int_0^\infty r^2drj_0(r|\boldsymbol{q}|)\left\{\frac{\sqrt{3}}{2}B(r)\langle D_{e3}\rangle_B + \frac{1}{\alpha^2}V_7(r)\langle D_{ei}R_i\rangle_B + \frac{1}{\beta^2}V_8(r)\langle D_{e\alpha}R_\alpha\rangle_B\right\}. \qquad (4.29)$$

The two new radial functions $V_7(r)$ and $V_8(r)$ are listed in appendix B of ref [46] as well. Integrating V_7 and V_8 yields the moments of inertia, α^2 and β^2, respectively. Hence the electric charges are properly normalized. It should be remarked that the baryon matrix elements in the space of the collective coordinates are computed using the exact eigenstates of (4.16) and adopting the Euler–angle representations for the $SU(3)$ generators [40]. The results for the magnetic moments and the radii

$$r_M^2 = -\frac{6}{\mu_B}\frac{dG_M^B(\boldsymbol{q}^2)}{d\boldsymbol{q}^2}\bigg|_{\boldsymbol{q}^2=0} \quad \text{and} \quad r_E^2 = -6\frac{dG_E^B(\boldsymbol{q}^2)}{d\boldsymbol{q}^2}\bigg|_{\boldsymbol{q}^2=0} \qquad (4.30)$$

Table 4.2: The electromagnetic properties of the baryons compared to the experimental data. The predictions of the Skyrme model are taken from ref [46].

	μ_B(n.m.)		$r_M^2(\text{fm}^2)$		$r_E^2(\text{fm}^2)$	
B	$e=4.0$	Expt.	$e=4.0$	Expt.	$e=4.0$	Expt.
p	2.03	2.79	0.43	0.74	0.59	0.74
n	-1.58	-1.91	0.46	0.77	-0.22	-0.12
Λ	-0.71	-0.61	0.36	—	-0.08	—
Σ^+	1.99	2.42	0.45	—	0.59	—
Σ^0	0.60	—	0.36	—	-0.02	—
Σ^-	-0.79	-1.16	0.58	—	-0.63	—
Ξ^0	-1.55	-1.25	0.38	—	-0.15	—
Ξ^-	-0.64	-0.69	0.43	—	-0.49	—
$\Sigma^0 \to \Lambda$	-1.39	-1.61	0.48	—	—	—

are shown in table 4.2. As in the two flavor model [6] the isovector part of the magnetic moments is underestimated while the isoscalar part is reasonably well reproduced. Despite the fact that the flavor symmetry breaking is large for the baryon wave–functions, the predicted magnetic moments do not strongly deviate from the $SU(3)$ relations [53]

$$\begin{aligned} \mu_{\Sigma^+} &= \mu_p \ , \quad \mu_{\Sigma^0} = \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) \ , \quad \mu_{\Sigma^-} = \mu_{\Xi^-} \ , \\ 2\mu_\Lambda &= -(\mu_{\Sigma^+} + \mu_{\Sigma^-}) = -2\mu_{\Sigma^0} = \mu_n = \mu_{\Xi^0} = \frac{2}{\sqrt{3}}\mu_{\Sigma^0\Lambda} \ . \end{aligned} \tag{4.31}$$

A more elaborate treatment of the flavor symmetry breaking is necessary in order to accommodate the experimentally observed details of breaking the U–spin symmetry which *e.g.* causes the approximate identity $\mu_{\Sigma^+} \approx \mu_p$ [54]. The moderate differences between the various magnetic radii r_M^2 is a further hint that symmetry breaking effects are mitigated. The comparison with the available empirical data for the radii shows that the predictions turn out too small in magnitude (except for the neutron electric radius). This is a strong indication that essential ingredients are still missing in the model. In section 5 it will be explained that the effects, which are associated with vector meson dominance (VMD), will account for this deficiency. Nevertheless the overall picture gained for the electromagnetic properties of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ can at least be characterized as satisfactory, especially in view of the fact that the only free parameter of the model has been fixed beforehand.

4.4 Effects of symmetry breaking on baryon matrix elements and strangeness in the nucleon

Theorists typically wish for symmetry breaking effects to be negligible (the so–called "spherical cow" approximation) but Nature says otherwise. This is very apparent in the case of low energy strong interactions (QCD). The Gell–Mann–Okubo mass formulae, which amount to applications of a Wigner–Eckart theorem for first order λ_8 type symmetry breaking, furnish sum rules rather than a complete description. As we discussed in section 4.2, the Skyrme model provides a non–trivial playground for treating symmetry breaking. The Yabu–Ando equation

[3] The conventions are $i, j, k = 1, 2, 3$ and $\alpha, \beta = 4, \ldots, 7$.

(4.16) gives an exact (within the model) wave–function for each baryon state at any strength of the symmetry breaking parameter $\gamma\beta^2$ ($\propto$ underlying quark masses). The physical results vary smoothly with $\gamma\beta^2$ (although higher quantum corrections would be expected to give weak non–analytic corrections).

The physical interpretation of symmetry breaking in the model may be seen from eq (4.15). The higher $SU(3)$ representation components in the baryon wave–function can only emerge in a quark framework by having quark–antiquark pairs present in addition to the three "valence" quarks. Clearly such effects would be difficult to treat in the non–relativistic quark model approach. On the other hand, it should be recognized that the Skyrme model approach is based on a collective semi–classical treatment.

In the last few years there has been a greatly renewed interest in the study of symmetry breaking effects for ordinary nucleons. This was stimulated by new experiments on polarized lepton deep inelastic scattering off nucleons [55] which seem to indicate that the pure valence quark picture of the nucleon has serious drawbacks. The Skyrme model has the advantage of giving a simple and roughly accurate quantitative explanation of these experiments. In detail one needs the strangeness conserving proton matrix elements

$$\langle P(\boldsymbol{p}')|\overline{q}_i\gamma_\mu\gamma_5 q_i|P(\boldsymbol{p})\rangle = \overline{u}(\boldsymbol{p}')\left[\gamma_\mu H_i(q^2) + \frac{q_\mu}{2M_p}\tilde{H}_i(q^2)\right]\gamma_5 u(\boldsymbol{p}) \qquad (4.32)$$

for this discussion. Of related interest are the flavor changing matrix elements

$$\begin{aligned}\langle B'(\boldsymbol{p}')|V^\alpha_\mu|B(\boldsymbol{p})\rangle &= \overline{u}(\boldsymbol{p}')\left[\gamma_\mu g_V(q^2) + \ldots\right]u(\boldsymbol{p})\ ,\\ \langle B'(\boldsymbol{p}')|A^\alpha_\mu|B(\boldsymbol{p})\rangle &= \overline{u}(\boldsymbol{p}')\left[\gamma_\mu\gamma_5 g_A(q^2) + \ldots\right]u(\boldsymbol{p})\ .\end{aligned} \qquad (4.33)$$

between different baryons (B', B). Here we have omitted contributions proportional to the momentum transfer q_μ.

Knowledge of the $g_A(B, B')$ and $g_V(B, B')$ are crucial for the theory of baryon semi–leptonic decays. First let us consider the calculation of the axial vector matrix elements $g_A(B, B')$. Our main interest in this brief discussion will be to examine the effects of symmetry breaking. The leading order term (in $1/N_C$) of the spatial components of the axial current is straightforwardly obtained to be

$$\int d^3r A^a_i = \mathcal{C} D_{ai}(A)\ , \qquad (4.34)$$

where $A(t)$ is the collective coordinate matrix. The constant $\mathcal{C}$ denotes an integral over the chiral angle. We refer the interested reader to refs [57, 46] for the explicit expression. Then,

$$g^a_A(B', B) = \mathcal{C}\langle B'|D_{a3}|B\rangle\ . \qquad (4.35)$$

The flavor index a has to be chosen according to whether strangeness conserving ($a = 1, 2, 3, 8$) or strangeness changing ($a = 4, \ldots, 7$) processes are being considered. The corresponding result for the axial charge of the nucleon $g_A = g^{1+i2}_A(p, n)$, as measured in neutron beta–decay, is predicted too low in many soliton models. This problem is already encountered in the two flavor model and gets worse in $SU(3)$ as the Clebsch–Gordon coefficient associated with $D_{1+i2\ 3}$

Table 4.3: The matrix elements of the axial–vector current (4.35) between different baryon states in the flavor symmetric limit. Displayed are both the strangeness conserving (a) and strangeness changing (b) processes. The first column gives the relevant flavor component of the axial current.

			(a)	
	$n \to p$	$\Sigma^- \to \Lambda$	$\Sigma^- \to \Sigma^0$	$\Xi^- \to \Xi^0$
A^{π^-}	$F + D$	$\frac{2}{\sqrt{6}}D$	$\sqrt{2}F$	$D - F$
			(b)	
	$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^- \to \Lambda$	$\Xi^- \to \Sigma^0$
A^{K^-}	$\frac{1}{\sqrt{6}}(3F + D)$	$D - F$	$\frac{1}{\sqrt{6}}(3F - D)$	$\frac{1}{\sqrt{2}}(F + D)$

changes by a factor of 7/10. As symmetry breaking is increased the $SU(3)$ prediction for g_A becomes larger [42]

$$g_A(SU(3)) = \frac{7}{10}\left[1 + 0.0514\gamma\beta^2 + \ldots\right] g_A(SU(2)) \ . \tag{4.36}$$

Actually the exact treatment shows that with increasing symmetry breaking the two flavor result is approached, although only slowly. Taking everything together, including subleading terms in (4.35), finally gives $g_A = 0.98$ for $e = 4.0$ [46] which is about 4/5 of the experimental value $g_A(\text{expt.}) = 1.26$.

We may understand the tendency to approach the $SU(2)$ limit for large $\gamma\beta^2$ as follows. In the small $SU(3)$ breaking case, there is just a small extra "cost" for producing an $\bar{s}s$ pair rather then a $\bar{u}u$ or $\bar{d}d$ pair. As $\gamma\beta^2$ gets larger it is more expensive to make an $\bar{s}s$ pair and eventually $\bar{s}s$ pairs should be absent from the nucleon wave–function, recovering the $SU(2)$ picture.

Returning to the general case one should first note that flavor symmetry relates the octet axial current matrix elements between various baryons. Conventionally they are expressed using $SU(3)$ covariance in terms of two unknown constants (or reduced matrix elements) F and D. One has to use models to determine these constants. In the flavor symmetric Skyrme model one finds [53] $D/F = 9/5$ and $D + F = 7\mathcal{C}/15 = g_A$. In table 4.3 the flavor symmetric dependences of the axial matrix elements on F and D are displayed. As one departs from the flavor symmetric case the baryon wave–functions acquire admixture from higher dimensional $SU(3)$ representations making the $SU(3)$ covariant parameterization in terms of F and D inadequate. In the presence of $SU(3)$ symmetry breaking we must, without further assumptions, parameterize each decay amplitude separately. It is still reasonable to maintain the isospin invariance relations.

As an example of the perturbative corrections consider the axial $\Lambda \to p$ transition in the Cabibbo scheme [56] for semi–leptonic hyperon decays. The analog of (4.15) for the Λ hyperon is

$$|\Lambda\rangle = |\Lambda, \mathbf{8}\rangle + \frac{3}{50}\gamma\beta^2|\Lambda, \mathbf{27}\rangle + \ldots \ . \tag{4.37}$$

Noting that the D–functions mix different $SU(3)$ representations, we get

$$\langle p \uparrow |D_{K^-3}|\Lambda \uparrow\rangle = \frac{2}{5\sqrt{3}} - \frac{7\sqrt{3}}{1125}\gamma\beta^2 + \ldots \ , \qquad D_{K^-3} = \frac{1}{\sqrt{2}}\left(D_{43} - iD_{53}\right) \ . \tag{4.38}$$

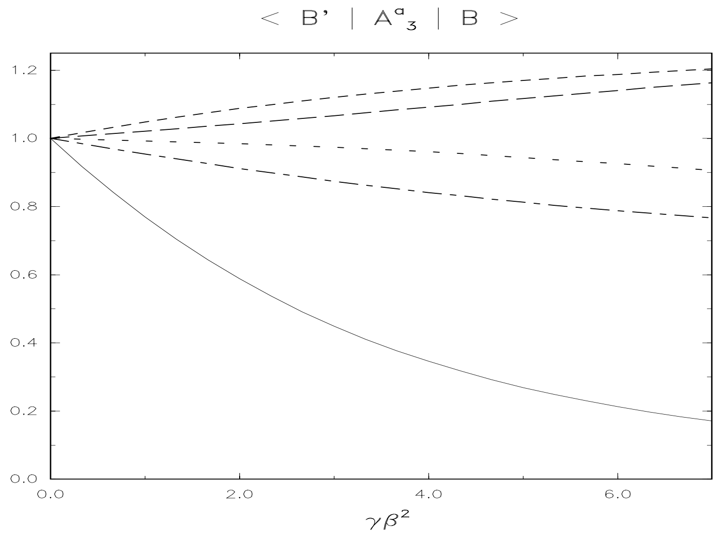

Figure 4.1: The variation of axial vector matrix elements with the effective symmetry breaking parameter $\gamma\beta^2$.
Full line: $\langle p|\overline{s}\gamma_3\gamma_5 s|p\rangle$; dashed dotted line: $\langle p|\overline{u}\gamma_3\gamma_5 s|\Lambda\rangle$; dotted line: $\langle n|\overline{u}\gamma_3\gamma_5 s|\Sigma^-\rangle$; long dashed line: $\langle \Lambda|\overline{u}\gamma_3\gamma_5 s|\Xi^-\rangle$; dashed line: $\langle p|\overline{u}\gamma_3\gamma_5 d|n\rangle$. These matrix elements, which are taken from refs [57] and [58], are normalized to the flavor symmetric values.

Of course, this expansion just provides a first approximation to the symmetry breaking dependence of the Cabibbo matrix elements. Using the exact treatment initiated by Yabu and Ando [44] this dependence can be computed numerically as shown in figure 4.1 for some processes of interest [57, 58]. Those results are normalized to the $SU(3)$ symmetric values in table 4.3 to illustrate that the matrix elements vary in different ways with symmetry breaking. In this figure also the variation of the nucleon matrix element of the flavor conserving axial current $H_3(0) = \langle N|\overline{s}\gamma_3\gamma_5 s|N\rangle$ is displayed. Obviously $H_3(0)$ decreases very rapidly with increasing symmetry breaking. This is easily visualized, as mentioned above, as a reflection of the increased cost of making extra $\bar{s}s$ pairs in the nucleon wave–function as $\gamma\beta^2$ increases. On the contrary the Cabibbo matrix elements exhibit only a moderate dependence on $\gamma\beta^2$. It is this different behavior of the matrix elements that makes the application of exact flavor symmetry to the analyses of the EMC–SLAC–SMC experiments suspicious. Stated otherwise, the strange quark contribution to the nucleon matrix element of the axial singlet current (loosely "proton spin") may be decreased significantly as a consequence of symmetry breaking without contradicting the successful Cabibbo scheme for the semi–leptonic decays of the hyperons.

As an interesting contrast to the axial matrix elements, consider the evaluation of the vector matrix elements $g_V(B, B')$ needed for the hyperon semi–leptonic decays. The dominant contribution is given by the matrix elements of the $SU(3)$ flavor generators

$$g_V^a(B', B) = \langle B'|L_a|B\rangle \ . \tag{4.39}$$

For example if we sandwich the generators L_{K^-} between the perturbative Λ and p states given in (4.37) and (4.15) and recognize that group generators can only connect states belonging to the same irreducible representation, we see that symmetry breaking corrections start out as $(\gamma\beta^2)^2$ rather than $\gamma\beta^2$. This is just a demonstration of the Ademollo–Gatto theorem [59], which "protects" the vector matrix elements against small symmetry breaking corrections. Since $\gamma\beta^2$ is large the numerical validity of this result is questionable. However, the exact Yabu–Ando scheme does confirm [57] that vector matrix elements suffer at most, 10% deviation from the symmetric values, even for large symmetry breaking, *e.g.* $\gamma\beta^2 \approx 7$.

A reduction of strangeness in the nucleon with increasing $\gamma\beta^2$ is also predicted for the scalar strange content fraction of the proton

$$X_s = \frac{\langle p|\overline{s}s|p\rangle - \langle 0|\overline{s}s|0\rangle}{\langle p|\overline{u}u + \overline{d}d + \overline{s}s|p\rangle - \langle 0|\overline{u}u + \overline{d}d + \overline{s}s|0\rangle} \ . \tag{4.40}$$

Here the state $|0\rangle$ refers to the soliton being absent. Models of quark flavor dynamics, as *e.g* the one of Nambu–Jona–Lasinio [9], indicate that matrix elements of quark bilinears $\bar{q}\lambda_a q$ may be taken as proportional to the matrix elements of tr $\left[\lambda_a \left(U + U^\dagger - 2\right)\right]$. Then we straightforwardly get

$$X_s = \frac{1}{3}\langle p|1 - D_{88}|p\rangle \approx \frac{7}{30} - \frac{43}{2250}\gamma\beta^2 + \ldots \ . \tag{4.41}$$

In this case, however, the deviation from the flavor symmetric result [60] ($X_s = 7/30$) is considerably mitigated [61] as compared to the variation of H_s defined in (4.32). The symmetry breaking has to be as large as $\gamma\beta^2 \approx 4.5$ to obtain a reduction of the order of 50%. In the case of H_s this was already achieved for $\gamma\beta^2 \approx 2.5$. In any event, the additional quark–antiquark excitations in the nucleon, which are parametrized by the admixture of higher dimensional $SU(3)$ representations (4.15), clearly tend to cancel the virtual strange quarks of the pure octet nucleon.

The three flavor Skyrme model under present consideration provides a convenient way to study the nucleon matrix elements of the vector current $\bar{s}\gamma_\mu s$. These are theoretically interesting because they would vanish in a pure valence quark model of the nucleon and so test finer details of nucleon structure. They are experimentally interesting because they can be extracted from measurements of the parity violating asymmetry in the elastic scattering of polarized electrons from the proton. The precise form factors needed are defined by

$$\langle P(\boldsymbol{p}')|\overline{q}_s\gamma_\mu q_s|P(\boldsymbol{p})\rangle = \overline{u}(\boldsymbol{p}')\left[\gamma_\mu F_s(q^2) + \frac{\sigma_{\mu\nu}q^\nu}{2M_p}\tilde{F}_s(q^2)\right]u(\boldsymbol{p}) \ . \tag{4.42}$$

These form factors are currently under intensive experimental investigation, *cf.* refs [62] and have been estimated in various models. The models range from vector–meson–pole fits [63] of dispersion relations [64] through vector meson dominance approaches [46] and kaon–loop calculations with [65] and without [66] vector meson dominance contributions to soliton model calculations [46, 67, 68]. The numerical results for the strange magnetic moment $\mu_S = \tilde{F}_s(0) \approx -0.31 \pm 0.09 \ \ldots \ 0.25$ are quite diverse. The predictions for the strange charge radius $r_S^2 = -6dF_s(q^2)/dq^2|_{q=0}$ are almost equally scattered $r_S^2 \approx -0.20 \ \ldots \ 0.14\text{fm}^2$.

In order to evaluate these form factors in the three flavor Skyrme model one requires the matrix elements of the "strange" combination

$$Q^s = \frac{1}{3}\mathbb{1} - \frac{1}{\sqrt{3}}\lambda_8 = Q^0 - \frac{2}{\sqrt{3}}Q^8 \tag{4.43}$$

rather than the electromagnetic one (4.23) between proton states. Using the same value $e = 4.0$ as used consistently for the three flavor pseudoscalar model yields the predictions

$$\mu_S = -0.13\text{n.m.} \ , \qquad r_S^2 = -0.10\text{fm}^2 \ . \tag{4.44}$$

Here n.m. stands for nuclear magnetons. It should be stressed that these results are obtained within the Yabu–Ando approach, *i.e.* the proton wave–function contains sizable admixture of higher dimensional representations. If a pure octet wave–function were employed to compute the matrix elements of the collective operators the strange magnetic moment would have been $\mu_S = -0.33$. The proper inclusion of symmetry breaking into the nucleon wave–function is again seen to reduce the effect of the strange degrees of freedom in the nucleon. We already discussed above that as the strange quarks within the nucleon become more massive (effect of symmetry breaking) their excitation becomes less likely.

In the next few years a great deal of new experimental information on the form factors F_s and $\tilde{F}_s$ should become available. This would enable more accurate comparison with (for a given effective meson Lagrangian) the essentially parameter free predictions of the soliton theory.

5. The nucleon as a vector meson soliton

According to the modern view of the Skyrme model approach we should start from the "full" effective Lagrangian which contains mesons of all spins. The practical criterion on which particles to include is to find an effective Lagrangian which does a good job of explaining the low energy experimental data in the meson sector. On these grounds it is evident that the vector mesons should be included in the effective Lagrangian. In this section we give a brief sketch of the soliton sector of the Lagrangian of pseudoscalars and vectors (see section 3.3) and note that it leads to significant improvements of many predictions. In particular it is crucial for discussing the so–called *proton spin puzzle*.

5.1 Generalized soliton ansatz and profile functions

As a first step we construct the soliton of the Lagrangian defined in section 3.3. The generalization of the hedgehog *ansatz* (2.6) to the vector meson model requires the time component of the ω field and the space components of the ρ field to be different from zero. Parity and grand spin symmetry[1] allow for three radial functions

$$\xi_\pi = \exp\left(\frac{i}{2}\hat{\boldsymbol{r}} \cdot \boldsymbol{\tau} F(r)\right) \ , \quad \omega_0 = \frac{\omega(r)}{2g} \ , \quad \rho_i^a = \frac{G(r)}{gr}\epsilon_{ija}\hat{r}_j \ . \tag{5.1}$$

Substituting these *ansätze* into the action described in section 3.3 yields the classical mass,

$$\begin{aligned} E \;=\; 4\pi \int dr \Big[& \frac{f_\pi^2}{2}(F'^2 r^2 + 2\sin^2 F) - \frac{r^2}{2g^2}(\omega'^2 + m_\rho^2 \omega^2) + \frac{1}{g^2}[G'^2 + \frac{G^2}{2r^2}(G+2)^2] \\ & + \frac{m_\rho^2}{g^2}(1 + G - \cos F)^2 + \frac{\gamma_1}{g} F' \omega \sin^2 F - \frac{2\gamma_2}{g} G' \omega \sin F \\ & + \frac{\gamma_3}{g} F' \omega G (G+2) + \frac{1}{g}(\gamma_2 + \gamma_3) F' \omega [1 - 2(G+1)\cos F + \cos^2 F] \\ & + (1 - \cos F)\Big\{ 4\delta' r^2 + 2(2\beta' - \frac{\alpha'}{g^2})(F'^2 r^2 + 2\sin^2 F) \\ & - \frac{2\alpha'}{g^2}[\omega^2 r^2 - 2(G + 1 - \cos F)^2 - 4(1 + \cos F)(1 + G - \cos F)]\Big\}\Big]. \end{aligned} \tag{5.2}$$

Application of the variational principle to this functional yields second order coupled non–linear differential equations for the radial functions $F(r)$, $\omega(r)$ and $G(r)$. The boundary conditions

[1] The grand spin $\mathbf{J} + \mathbf{I}$ is the characteristic invariance of the hedgehog ansatz.

for the chiral angle $F(r) = \pi$ and $F(\infty) = 0$, which correspond to unit baryon number, also determine the boundary conditions of the vector meson profiles via the differential equations and the requirement of finite energy. For example we find $G(0) = -2$. A typical set of resulting profile functions is shown in figure 5.1.

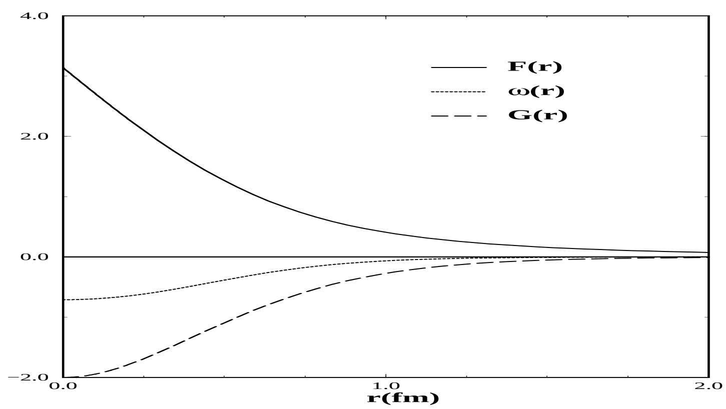

Figure 5.1: The profile functions which minimize the classical vector meson energy functional (5.2) for the parameters $g = 5.85$, $\tilde{h} = 0.4$, $\tilde{g}_{VV\Phi} = 1.9$ and $\kappa = 1.0$. $\omega(r)$ is measured in units of m_ρ.

As in the pseudoscalar model we have to generate states with good spin and isospin from this classical field configuration. To start with, one introduces collective coordinates (2.8) for all fields which have non–vanishing spin or isospin. However, an additional complication arises because there are vector meson field components which vanish classically but get excited by the collective rotation. In the two flavor case the appropriate *ansatz* for these excitations reads

$$\boldsymbol{\rho}_0 = \frac{1}{2g} A(t) \left[\xi_1(r)\boldsymbol{\Omega} + \xi_2(r)(\hat{\boldsymbol{r}} \cdot \boldsymbol{\Omega})\hat{\boldsymbol{r}}\right] A^\dagger(t) \ , \quad \omega_i = \frac{\Phi(r)}{2g}\epsilon_{ijk}\Omega_j\hat{r}_k \ , \tag{5.3}$$

where the angular velocity of the rotating soliton, Ω_i is defined in (2.10). The three radial functions ξ_1, ξ_2 and Φ are not the only ones which get excited. As these radial functions are non–zero they provide sources for the non–strange component of the iso–singlet pseudoscalar field via the ϵ–terms (3.29). In the two flavor formulation the appropriate *ansatz* which takes into account the pseudoscalar nature of the η field reads

$$U(\boldsymbol{r}, t) = \mathrm{e}^{i\eta_T(\boldsymbol{r})} A(t) U_0(\boldsymbol{r}) A^\dagger(t) \quad \text{with} \quad \eta_T(\boldsymbol{r}) = \frac{1}{f_\pi}\eta(r)\hat{\boldsymbol{r}} \cdot \boldsymbol{\Omega} \, . \tag{5.4}$$

As we will observe shortly, the excitation of this η field plays a decisive role in the context of the *proton spin puzzle*. The additional radial functions are determined from extremizing the moment of inertia for rotations in coordinate space,

$$\begin{aligned}
\alpha^2 \ = \ & \frac{8\pi}{3}\int dr \Big\{ f_\pi^2 r^2 \sin^2 F - \frac{4}{g^2}(\phi'^2 + 2\frac{\phi^2}{r^2} + m_\rho^2\phi^2) + \frac{m_\rho^2}{2g^2} r^2 \left[(\xi_1 + \xi_2)^2 + 2(\xi_1 - 1 + \cos F)^2\right] \\
& + \frac{1}{2g^2}\left[(3\xi_1'^2 + 2\xi_1'\xi_2' + \xi_2'^2)r^2 + 2(G^2 + 2G + 2)\xi_2^2 + 4G^2(\xi_1^2 + \xi_1\xi_2 - 2\xi_1 - \xi_2 + 1)\right] \\
& + \frac{4}{g}\gamma_1\phi F' \sin^2 F + \frac{4}{g}\gamma_3\phi F'\left[(G - \xi_1)(1 - \cos F) + (1 - \cos F)^2 - G\xi_1\right] \\
& + \frac{2\gamma_2}{g}\Big\{\phi' \sin F(G - \xi_1 + 2 - 2\cos F) + \phi \sin F(\xi_1' - G') \\
& + \phi F'\left[2 + 2\sin^2 F + (\xi_1 - G - 2)\cos F - 2(\xi_1 + \xi_2)\right]\Big\}
\end{aligned}$$

$$
-\frac{1}{2}\left[\eta'^2 r^2 + 2\eta^2 + m_\eta^2 r^2 \eta^2\right] + \frac{\gamma_2 g}{2f_\pi}\left[\eta(\phi\omega' - \omega\phi') - \eta'\phi\omega\right]
$$
$$
-\frac{\gamma_1}{3gf_\pi}\left[\eta'(\xi_1+\xi_2)\sin^2 F + 2\eta F'(G+\xi_1)\sin F\right] - \frac{3\gamma_3}{gf_\pi}\eta'(G+1-\cos F)^2(\xi_1+\xi_2)
$$
$$
-\frac{\gamma_2}{gf_\pi}\left\{\eta'\left[(G+\xi_1)G + (\xi_1+\xi_2)[(1-\cos F)^2 - 2G\cos F]\right] + \eta(G\xi_1' - G'\xi_1)\right\}\Bigg\}, \qquad (5.5)
$$

together with suitable boundary conditions. In eq (5.5) we have not displayed the explicit contributions from the symmetry breakers (which are in fact small). We will mostly limit the present discussion to the two flavor case. In the case of three flavor vector meson models the situation is even more complicated as also K^* type fields get excited. Also there will be additional symmetry breakers on the level of the collective Lagrangian which are of the form $\sum_{i=1}^{3} D_{8i}\Omega_i$ and stem from terms which are linear in the time derivative. They can straightforwardly be implemented in the collective quantization approach. Here we will omit details but rather refer the reader to the literature [67, 40]. The general pattern for computing baryon properties is essentially the same as that discussed for the Lagrangian of only pseudoscalars in section 4.

5.2 Axial singlet current and proton spin puzzle

Notice that in (5.5) we included by hand a mass term for the rotationally excited profile $\eta(r)$ of a pseudoscalar isosinglet field. Actually the existence of such a term has not yet been justified. Before proceeding we must do so since the term turns out to be very important.

In section 3.1 we mentioned that the QCD axial singlet current

$$
J_{5,\mu}^{(0)} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s, \qquad (5.6)
$$

is not conserved even for zero quark masses: $\partial^\mu J_{5,\mu}^{(0)} = G$, where the $U_A(1)$ anomaly G is proportional to the product of the QCD field strength tensor $F_{\mu\nu}^a$ and its dual. In order to mock up this non–conservation equation at the effective Lagrangian level [69] we may add the terms

$$
\frac{c}{2}G^2 + \frac{iG}{12}\ln\left(\frac{\det U}{\det U^\dagger}\right), \qquad (5.7)
$$

where G is now considered a composite glueball field which "dominates" the $U_A(1)$ anomaly. Here we assumed three light flavors and also that the strong CP violation parameter θ is zero. Furthermore it is necessary that, except for the terms representing quark mass symmetry breaking, all the other terms in the effective Lagrangian be invariant under $U_A(1)$. The parameter c above is determined by

$$
m_{\eta_0}^2 \approx \frac{1}{6cf_\pi^2}, \qquad (5.8)
$$

in the approximation where the quark mass terms are neglected. η_0 is the $SU(3)$ singlet pseudoscalar field as in (4.1). This equation arises after noting that G is like an auxiliary field and may be integrated out: $G = \eta_0/(\sqrt{6}cf_\pi)$. In the effective Lagrangian the realization of the axial singlet current, obtained by a Noether variation, is

$$
J_{5,\mu}^{(0)} = \sqrt{6}f_\pi\partial_\mu\eta_0 + \tilde{J}_{5,\mu}^{(0)}\,. \qquad (5.9)
$$

Here the first term is the contribution from the pseudoscalar field and the second term is due to the addition of vector fields. $\tilde{J}^{(0)}_{5,\mu}$ has a complicated structure but, in particular, contains *non–derivative* terms like $\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(\rho_\nu\rho_\alpha\rho_\beta)$. Using this decomposition we may write the equation of motion for the η_0 field as

$$(\partial^2 + m_{\eta_0^2})\eta_0 = \frac{1}{\sqrt{6}f_\pi}\partial^\mu \tilde{J}^{(0)}_{5,\mu}\,, \tag{5.10}$$

which shows that the vector meson contribution to the axial singlet current may act as a source for a non–trivial excitation associated with the η_0 field in the soliton sector.

Now the form factors for the proton matrix elements of the axial singlet current are obviously just the sums of the three separate form factors introduced in (4.32):

$$H(q^2) = H_u(q^2) + H_d(q^2) + H_s(q^2) \quad \text{and} \quad \tilde{H}(q^2) = \tilde{H}_u(q^2) + \tilde{H}_d(q^2) + \tilde{H}_s(q^2). \tag{5.11}$$

If the vector mesons are not present, eq (5.9) shows that the operator for the axial singlet current must be (even in the soliton sector) a pure derivative. This means that, regardless of the details of the calculation, the matrix element for the sum of the three terms in (4.32) must be proportional to the momentum transfer q_μ . Thus $\tilde{H}(q^2)$ is non–zero and $H(q^2) = 0$. From the theory of Dirac particles we recognize that the quantity $H(0)$ has the interpretation of twice the quark spin part of the proton's angular momentum. We see that the Skyrme model of only pseudoscalars predicts that the expectation value of the net quark spin operator vanishes; the total angular momentum (1/2) of the proton must involve, at a fundamental level, the rotational and gluonic pieces! Note that the above argument for $H(0) = 0$ with the Lagrangian of only pseudoscalars continues to hold even if symmetry breaking contributions are taken into account [58].

The situation is a little different when vector mesons are included in the effective Lagrangian. Since $\tilde{J}^{(0)}_{5,\mu}$ has pieces which are not pure derivatives it then is possible to obtain $H(0) \neq 0$. A convenient parameterization for this calculation in the effective Lagrangian model is

$$\langle P(\boldsymbol{p}')|\tilde{J}^{(0)}_{5,i}|P(\boldsymbol{p})\rangle = H(q^2)\langle 2J_i\rangle\ . \tag{5.12}$$

Once all the radial functions have been determined as before from the appropriate variational principles, it is straightforward to compute $H(0)$ from eq (5.12). One only has to recall that under the collective coordinate quantization the angular momentum operator is given by $\boldsymbol{J} = \boldsymbol{\Omega}/\alpha^2$. The numerical results for a variety of allowed parameters (*cf.* discussion after eq (3.29)) are displayed in table 5.1. Surprisingly the predictions of the vector meson model for $H(0)$ are

Table 5.1: Predictions for the matrix element of the proton axial singlet current for various allowed sets of parameters in the vector meson model. Results are taken from ref [70].

$\tilde{h}$	0.4	0.4	0.4	0.7	0.5	0.2	0.1
$\tilde{g}_{VV\Phi}$	1.9	1.9	1.9	2.2	2.0	1.7	1.5
κ	0.0	0.5	1.0	0.0	0.0	0.0	0.0
$H(0)$	0.34	0.33	0.30	0.29	0.34	0.32	0.28

very robust against possible changes of the parameters of the model.

Even though we get a non–zero value for $H(0)$ in the vector meson model it is still small compared to $H(0) = 1$, the expectation from the simple non–relativistic quark model. Qualitatively the soliton model results with and without vectors are similar. Since one has a natural prejudice that the quark model results should be roughly correct, this would at first seem to be a serious defect of the soliton approach to nucleon properties.

Of course one can only make an accurate judgment on the matter by appealing to experiment. $H(0)$ can be found from eq (5.11) if we can experimentally obtain separately $H_u(0), H_d(0)$ and $H_s(0)$. The linear combination

$$H_u(0) - H_d(0) = g_A = 1.257, \tag{5.13}$$

is reliably obtained by an isotopic spin rotation of the axial form factor describing neutron beta–decay. Similarly the estimate for the "eighth" octet component

$$H_u(0) + H_d(0) - 2H_s(0) \approx 0.575 \pm 0.016, \tag{5.14}$$

may be gotten from an $SU(3)$ flavor rotation of the data on hyperon beta–decay experiments. Clearly one more linear combination is needed in order to disentangle the individual $H_i(0)$ and that situation existed for many years. About ten years ago different experimental groups (EMC, SLAC, SMC) used polarized lepton beams to probe the structure of nucleons. The deep inelastic scattering data [55] were used to extract the combination

$$4H_u(0) + H_d(0) + H_s(0), \tag{5.15}$$

in which the axial current form factor for each quark is weighted proportionally to the square of the quark electric charge. Combining these data resulted in $H(0) \approx 0.3$ [71]. The experimental results were later on confirmed [72, 73, 74]. However, it turns out that the theoretical extraction of $H(0)$ is quite complicated as it involves a careful treatment of perturbative QCD corrections. The value [75]

$$H(0) = 0.27 \pm 0.04 \tag{5.16}$$

is nowadays considered correct. At the time this low value was considered hard to understand and the situation was called the *proton spin puzzle*. We have just seen that the soliton approach does however provide a simple explanation of such a low value.

Clearly the prediction of the vector meson treatment described in Table 5.1, yielding H(0) about 0.30, is in good agreement with the data. From this we learn two things. First, the simplest quark model does not give a good description of the spin structure of the nucleon. Second, the soliton approach based on an effective Lagrangian including vector mesons markedly improves the qualitatively reasonable predictions of the soliton treatment based on a pseudoscalars only effective Lagrangian. A physical interpretation of the latter statement is that the pseudoscalars only Lagrangian mainly probes the "pion cloud" of the nucleon while the vector Lagrangian probes a little more deeply.

For completeness we remark on a possible caveat. The estimate of (5.14) is based on the use of exact $SU(3)$ symmetry. However in Fig 4.1 of section 4.4 we showed that precisely this current matrix element is expected to exhibit stronger suppression than others due to $SU(3)$ symmetry breaking. Nevertheless it turns out that [70] the numerical evaluation of $H(0)$ is not very sensitive to this feature. This is to be contrasted with the behavior of $H_s(0)$, which decreases rapidly with symmetry breaking, *cf.* section 4.4.

5.3 Other improvements with vector mesons

The famous problem of explaining the neutron–proton mass difference is another one which requires the addition of vector mesons to the effective Lagrangian in order to obtain a satisfactory solution in the nucleon–as–soliton picture. It is known that the electromagnetic (*i.e.* one photon loop) contribution has the wrong sign. After correcting for the electromagnetic interaction the remaining "strong" part of the neutron–proton mass–difference should be $(M_n - M_p)_{\rm strong} \approx (2.0 \pm 0.3)$MeV [76]. At the quark level this arises from the down quark–up quark mass difference, controlled by the parameter y in eqs (3.6) and (3.7). Information on y can be most easily gained by analyzing the K^+–K^0 mass–difference, yielding $y \approx (-0.4 ... -0.2)$ [33]. To understand the problem it is helpful to consider the contribution of the (presumably dominating) δ'–type symmetry breaker to the neutron–proton mass–difference. Since the d–u quark mass difference clearly exists with only two flavors it is interesting to first consider the problem at this level. Then the relevant piece of the δ' term is proportional to

$$\mathrm{tr}\left[\tau_3\left(U + U^\dagger\right)\right]. \tag{5.17}$$

Using the ansatz (5.4) we see that $U = \exp(i\eta_T)[\cos(\psi) + i\boldsymbol{n}\cdot\boldsymbol{\tau}\sin(\psi)]$, where ψ is some angle. Then (5.17) is proportional to $\sin(\eta_T)$. In other words the contribution vanishes unless the field η_T gets excited due to the collective rotation (or any kind of symmetry breaking). Now (5.10) together with (5.9) shows that this will not happen if only pseudoscalars are present in the effective Lagrangian; the vector meson contribution $\tilde{J}^{(0)}_{5,\mu}$ must also be present. This is analogous to the situation concerning the *proton spin puzzle*. The contribution of the δ' term turns out to be

$$(M_n - M_p)_{\rm strong} = -\frac{2y\delta'}{3\alpha^2}\int d^3r\ \sin F(r)\eta(r) + \dots . \tag{5.18}$$

Using the full two–flavor vector meson result for $\tilde{J}^{(0)}_{5,\mu}$ which was already employed to compute $H(0)$ yields [77]

$$(M_n - M_p)_{\rm strong} \approx 1.4\,\mathrm{MeV} \tag{5.19}$$

which, not surprisingly, turns out to be about as robust against changes of the parameters as is $H(0)$. This prediction is still somewhat too small when compared to the empirical value. However, it turns out that the missing ~ 0.5MeV can be attributed to three flavor effects as matrix elements of D_{38} are non–vanishing[2].

The addition of vector mesons also plays an important role in the discussion of the "sizes" of the nucleons: the nucleon radii. As can be observed from table 4.2 the Skyrme model of pseudoscalars only seriously underestimates the empirical values for the baryon radii. The presence of the ω meson provides an increase of the isoscalar radius [52]

$$\langle r^2\rangle_{I=0} \approx \langle r^2\rangle_B + \frac{6}{m_\rho^2}\ , \tag{5.20}$$

[2]It should be remarked that $\langle p|D_{38}|p\rangle$ quickly approaches zero as $SU(3)$ symmetry breaking is increased. This decreases $SU(3)$ type contributions to (5.19).

where $\langle r^2 \rangle_B$ is the radius associated with baryon number current (2.2). The additional piece in eq (5.20) is a consequence of (approximate) vector meson dominance in this model [78], which indeed is observed when including the vector mesons in a chirally invariant manner. As can be seen from table 4.2 this increase of about 0.35fm^2 will significantly improve the predictions for the radii.

A similar interesting improvement due to vector mesons is obtained in the context of meson–baryon scattering. In these investigations one introduces small fluctuations off the classical soliton. Eventually these fluctuations are quantized to represent in– and out–going meson fields, thereby determining the scattering matrix [79]. It turns out that in the pseudoscalar Skyrme model the phase–shifts extracted from this scattering matrix rise almost linearly with the momentum of the in–going pion. This undesired feature is mostly due to the contact interaction between pions contained in the Skyrme model Lagrangian (*cf.* section 2). When introducing vector mesons this contact interaction is essentially replaced by the exchange of such a vector meson,

$$\frac{-1}{m_\rho^2} \longrightarrow \frac{1}{q^2 - m_\rho^2} \,. \tag{5.21}$$

As this interaction decreases for large momentum transfers, q^2 the resulting phase–shifts assume a constant value for large energies rather than rising linearly [80]. Clearly this effect is similar to the one observed when going from the Fermi to the standard model of electro–weak interactions.

These examples show that while the inclusion of vector meson degrees of freedom involves quite a few technical details it clearly provides a more realistic picture of the nucleon as a chiral soliton.

6. Summary and discussion

Aside from the mass spectra and current matrix elements of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons treated here the soliton approach has been extensively employed to study meson nucleon scattering [79, 80], baryons containing a heavy quark [81], nucleon–nucleon scattering [82], few nucleon systems [83] and nuclear matter [84].

In the present survey, we started out with a historical introduction (section 1) and a concise technical summary of the original two flavor Skyrme model (section 2). In these sections the physical interpretation and justification of the model were emphasized; it is hoped that they will be useful to beginners in this area of research (see also [85] and [86]).

We next attempted to develop the generalization of the original Skyrme model which is suggested by the large N_C approximation to QCD. In this approach the Skyrme Lagrangian is to be replaced by a more general effective Lagrangian containing mesons of all spins. Perhaps some day an analytic expression in this framework will be found. At present it seems necessary to obtain an approximation based on including the lowest energy resonances and constraining the model by the symmetries of the underlying QCD. The concept of chiral symmetry which plays a crucial role in this extension was explored in section 3. Furthermore the original Skyrme model of two light flavors was extended to three flavors, as it is now well established that the nucleons belong to a flavor SU(3) multiplet.

It is worthwhile to stress that once the effective Lagrangian has been determined from the meson sector, the soliton approach provides in principle a *zero parameter* description of baryon properties (In our case we introduced just one parameter which had to be fit from the baryon sector.).

In section 4 we studied the technical tools needed to treat the flavor SU(3) symmetry and its breaking at the (collective) baryon level. These were applied to the calculation of various interesting baryon matrix elements. Finally section 5 sketched the treatment of baryons based on an effective Lagrangian which also included the vector mesons. An application to the so–called *proton spin puzzle* demonstrated that the soliton approach seems to give a neat description of, otherwise hard to explain, experimental results on the quark spin structure of the nucleon. The improvements one encounters on including the vector mesons are in accord with the intuitive notion that the addition of higher mass resonances in the meson sector leads to a progressively more detailed understanding of the short distance structure of the nucleon–as–soliton.

We are happy to acknowledge the stimulating interactions we have had with many collaborators and colleagues while doing research related to the topics reviewed here.

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