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Joseph Schechter Department of Physics, Syracuse University, Syracuse, NY

Deirdre Black Syracuse University

Amir H. Fariborz Syracuse University

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Schechter, Joseph; Black, Deirdre; and Fariborz, Amir H., "Light Scalar Mesons" (1999). *Physics*. 290. https://surface.syr.edu/phy/290

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Light Scalar Mesons

Deirdre Black, Amir H. Fariborz and Joseph Schechter

Department of Physics, Syracuse University, Syracuse, New York 13244-1130, USA.

Abstract.

We review how a certain effective chiral Lagrangian approach to $\pi\pi$ scattering, πK scattering and $\eta' \rightarrow \eta\pi\pi$ decay provides evidence for the existence of light scalars $\sigma(550)$ and $\kappa(900)$ as well as describing the $f_0(980)$ and the $a_0(980)$. An attempt to fit these into a nonet suggests that their structure is closer to a dual quark–dual antiquark than to a quark–antiquark. A possible mechanism to explain the next higher mass scalar nonet is also proposed.

I INTRODUCTION

The possible existence of light scalar mesons (with masses less than about 1 GeV) has been a controversial subject for roughly forty years. There are two aspects: the extraction of the scalar properties from experiment and their underlying quark substructure. Because the J = 0 channels may contain strong competing contributions, such resonances may not necessarily dominate their amplitudes and could be hard to "observe". In such an instance their verification would be linked to the model used to describe them. The last few years have seen a revival of interest in this area. As examples, three models for the underlying quark structure have been discussed by many authors, including other contributors to this workshop [1]: i) the $K\bar{K}$ molecule model [2], ii) the $q\bar{q}$ model with strong meson-meson interactions (or "unitarized quark model") [3], iii) the intrinsic $qq\bar{q}\bar{q}$ model (Jaffe type [4]). These models have the common feature that four quarks are involved in some form; all are different from the "simple" $q\bar{q}$ model. Clearly, the elucidation of the structure of unusual low lying states can be expected to increase our understanding of non-perturbative QCD.

The present approach is based on comparing with experiment, the predictions for $\pi\pi$ scattering, πK scattering and $\eta' \to \eta\pi\pi$ decay from a phenomenological chiral Lagrangian containing particles of mass comparable to the energy regime of interest. These studies seem to require for consistency the existence of two isoscalars $\sigma(550)$ and $f_0(980)$, an isospinor $\kappa(900)$ and an isovector $a_0(980)$ with given properties. Note that in the effective Lagrangian approach, the quark substructure of the scalars is not specified. In particular a nonet field can *a priori* represent either $q\bar{q}$ or $qq\bar{q}\bar{q}$ (or even more complicated) states. From this point of view our approach is "model independent".

Section II contains a brief summary of our scattering model in the $\pi\pi$ case. The generalization to the πK case and to $\eta' \to \eta \pi \pi$ decay is even more briefly summarized in section III. Section IV deals with the "family properties" of the nonet made up from the scalars we need. The model of "ideal mixing" for meson nonets is reviewed and generalized to include "dual ideal mixing". The realistic situation is noted to be closer to dual rather than ordinary ideal mixing. Finally, in section V, a possible mechanism is proposed to explain some puzzling features of a presumably more conventional next-to-lowest-lying scalar nonet.

II PI PI SCATTERING

The most difficult partial wave amplitude to explain is just the scalar channel with I = J = 0. Our notation for the partial wave is $T_J^I(s) = R_J^I + iI_J^I$. The complicated shape of the experimentally obtained $R_0^0(s)$ shown in Figs. 2 and 3 below suggests that resonances are present. Close to threshold, the chiral perturbation theory approach, which essentially supplies a Taylor expansion of the amplitude, is very accurate. However explaining the data shown to about 1.2 GeV would appear to require a prohibitively high order of expansion in this scheme. Thus we sacrifice some accuracy near threshold and use instead an expansion of the invariant amplitude in terms of resonance exchange diagrams (including contact terms needed for chiral symmetry). This holds the possibility of achieving a fit to experiment over a larger energy regime. Some theoretical support for such an approach comes from the leading order in $1/N_c$ approximation to QCD, which features just tree diagrams.

In detail, we calculate the tree diagrams of our model from an effective chiral Lagrangian which contains resonances but has interactions with a minimal (for simplicity) number of derivatives. Hence the initial computed amplitude will be (as in the $1/N_c$ expansion) purely real. This suggests that it is most sensible in the present approach to compare with the real part of the experimental amplitude. Of course we still must find a way to "regularize" the infinities which arise at the direct channel poles. We interpret the "regularization" procedure as equivalent to enforcing unitarity in the vicinity of the direct channel pole. In the case of a narrow isolated resonance we adopt the usual Breit–Wigner procedure in which the offending term in the amplitude is replaced as

$$\frac{MG}{M^2 - s} \to \frac{MG}{M^2 - s - iMG}.$$
(1)

In the case of a very broad resonance we instead replace

$$\frac{MG}{M^2 - s} \to \frac{MG}{M^2 - s - iMG'},\tag{2}$$



FIGURE 1. The solid line which shows the current algebra $+\rho$ result is much closer to the unitary bound of 0.5 than the dashed line which shows the current algebra result alone.

where G', which is not required to equal G, is taken as a fitting parameter. Finally, in the case of a "narrow" resonance in a non trivial background (characterized by a phase shift δ in the resonance partial wave) we replace,

$$\frac{MG}{M^2 - s} \to e^{2i\delta} \frac{MG}{M^2 - s - iMG}.$$
(3)

This method can be trivially modified to give a crossing symmetric invariant amplitude but unitarity may easily be violated in general. We thus choose the parameters for a putative σ meson represented by (2) to fit experiment. We then end up with an amplitude [5] which approximately satisfies unitarity and crossing symmetry. This is illustrated in a step by step manner in Figs. 1, 2 and 3.

The real part of the partial wave amplitude R_0^0 is obtained by projecting out the real part of the s-wave I = 0 component of the chiral invariant and crossing symmetric invariant amplitude. We see in Fig. 1 that the "current algebra" piece starts violating the unitarity bound, $|R_0^0| \leq 1/2$ at about 0.5 GeV and then runs away. However the inclusion of the ρ meson exchange diagrams turns the curve in the right direction and improves, but does not completely cure, the unitarity violation. This feature, which does not involve any unknown parameters, gives encouragment to our hope that the cooperative interplay of various pieces at tree level can explain the low energy scattering. In order to fix up Fig. 1 we note that the real part of a resonance contribution vanishes at the pole, is positive before the pole and *negative* above the pole. Thus a scalar resonance with a pole about 0.5 GeV (where R_0^0 in Fig. 1 needs a negative contribution to stay below 1/2) should do the job. The result of including such a σ meson, parametrized as in (2), is shown in Fig. 2. Now note that the predicted $R_0^0(s)$ in Fig. 2 vanishes around 1 GeV.



FIGURE 2. The sum of current algebra $+\rho + \sigma$ contributions compared to data.



FIGURE 3. The sum of current algebra $+\rho + \sigma + f_0(980)$ contributions compared to data.

Thus the phase δ at 1 GeV (assumed to keep rising) is about 90° there. Considering this as a background phase for the known $f_0(980)$, Eq(3) shows us that the real part of the $f_0(980)$ contribution will get reversed in sign (Ramsauer-Townsend effect). This is the missing piece in the jig-saw puzzle as Fig. 3 shows. Up to about 1.2 GeV, the amplitude R_0^0 can be explained as the sum of current algebra, ρ exchange, $\sigma(550)$ exchange and $f_0(980)$ exchange pieces.

PI K SCATTERING AND $\eta' \rightarrow \eta \pi \pi$ III

A similar treatment was carried out for the J = 0 partial wave amplitudes of πK scattering [6]. In this case the low energy amplitude is taken to correspond to the sum of a current algebra contact diagram, vector ρ and K^* exchange diagrams and scalar $\sigma(550)$, $f_0(980)$ and $\kappa(900)$ exchange diagrams. The situation in the interesting I = 1/2 channel turns out to be very analogous to the I = 0 channel of s-wave $\pi\pi$ scattering. Now a $\kappa(900)$ parametrized as in (2) is required to restore unitarity; it plays the role of the $\sigma(550)$ in the $\pi\pi$ case. Following our criterion we expect that to extend this treatment to the 1.5 GeV region, one should include the many possible exchanges of particles with masses up to about 1.5 GeV. Nevertheless we found that a satisfactory description of the 1-1.5 GeV s-wave region is obtained simply by including the well known $K_0^*(1430)$ scalar resonance, which plays the role of the $f_0(980)$ in the $\pi\pi$ calculation.

The $\eta' \to \eta \pi \pi$ process is a strong decay which yields information about the properties of the scalar $a_0(980)$ isovector resonance. The tree diagrams, which are similar to those of $\pi\eta$ scattering in our model [7], include $a_0(980)$, $\sigma(550)$ and $f_0(980)$ exchanges. Compared to the $\pi\pi$ and πK scatterings there is a simplification in that G-parity invariance prevents vector meson exchange diagrams from contributing. The associated "current algebra" contact diagrams also vanish. It was found that fitting the model to the experimental Dalitz plot and the rate gave $a_0(980)$ properties consistent with the recent experimental ones.

SCALAR NONET "FAMILY" PROPERTIES IV

The nine states associated with the $\sigma(550)$, $\kappa(900)$, $f_0(980)$ and $a_0(980)$ are required in order to fit experiment in our model. What do their masses and coupling constants suggest about their quark substructure? (See [8] for more details.) Suppose we first try to assign them to a conventional $q\bar{q}$ nonet:

ŀ

$$\sigma(550) \sim \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}),$$

$$\kappa^{+}(900) \sim u\bar{s},$$

$$a_{0}^{+}(980) \sim u\bar{d},$$

$$f_{0}(980) \sim s\bar{s}.$$
(4)

Then there are two puzzles. i) Why aren't the $a_0(980)$ and the $\sigma(550)$, which have the same number of non-strange quarks, degenerate? ii) Why aren't these particles, being p-wave states, in the same 1+ GeV energy region as the other p-wave states?

To study this, first note that most meson multiplets can be nicely understood using the concept of "ideal mixing". In Okubo's formulation [9], originally applied to the vector meson multiplet, the meson fields are grouped into a nonet matrix,

$$N_a^b = \begin{bmatrix} N_1^1 & a_0^+ & \kappa^+ \\ a_0^- & N_2^2 & \kappa^0 \\ \bar{\kappa}^+ & \bar{\kappa}^0 & N_3^3 \end{bmatrix},$$
 (5)

where the particle names have been chosen to fit the scalar mesons. The two I = 0 states are the SU(3) singlet, $(N_1^1 + N_2^2 + N_3^3)/\sqrt{3}$ and the SU(3) octet member, $(N_1^1 + N_2^2 - 2N_3^3)/\sqrt{6}$. Okubo's ansatz for the mass terms was,

$$\mathcal{L}_{mass} = -a \operatorname{Tr}(NN) - b \operatorname{Tr}(NN\mathcal{M}), \qquad (6)$$

where a > 0 and b are real constants and $\mathcal{M} = diag(1, 1, x)$ (with $x = m_s/m_u$) is the "spurion" matrix which breaks flavor SU(3) invariance. With (5) and (6) the SU(3) singlet and SU(3) octet isoscalar states mix in such a way (ideal mixing) that the physical mass eigenstates emerge as $(N_1^1 + N_2^2)/\sqrt{2}$ and N_3^3 . Furthermore there are two mass relations

$$m^{2}(a_{0}) = m^{2} \left(\frac{N_{1}^{1} + N_{2}^{2}}{\sqrt{2}}\right),$$

$$m^{2}(a_{0}) - m^{2}(\kappa) = m^{2}(\kappa) - m^{2}(N_{3}^{3}).$$
(7)

Note that there are two different solutions depending on the sign of *b*. If b > 0 we get Okubo's original case where [with the identifications $a_0 \to \rho$, $\kappa \to K^*$, $(N_1^1 + N_2^2)/\sqrt{2} \to \omega$ and $N_3^3 \to \phi$] there is the conventional ordering

$$m^{2}(\phi) > m^{2}(K^{*}) > m^{2}(\rho) = m^{2}(\omega).$$
 (8)

This agrees with counting the number of (heavier) strange quarks when we identify $N_a^b \sim q_a \bar{q}^b$.

On the other hand if b < 0 and we identify $N_3^3 \to \sigma$ and $(N_1^1 + N_2^2)/\sqrt{2} \to f_0$, the resulting ordering would be

$$m^2(f_0) = m^2(a_0) > m^2(\kappa) > m^2(\sigma),$$
(9)

which is in nice agreement with the present "observed" scalar spectrum. But this clearly does not agree with counting the number of strange quarks while assuming that the scalar mesons are simple quark anti-quark composites. This unusual ordering will agree with counting the number of strange quarks if we assume instead that the scalar mesons are schematically constructed as $N_a^b \sim T_a \bar{T}^b$ where $T_a \sim \epsilon_{acd} \bar{q}^c \bar{q}^d$ is a "dual" quark. Specifically

$$N_a^b \sim T_a \bar{T}^b \sim \begin{bmatrix} \bar{s} \bar{d} ds & \bar{s} \bar{d} us & \bar{s} \bar{d} ud \\ \bar{s} \bar{u} ds & \bar{s} \bar{u} us & \bar{s} \bar{u} ud \\ \bar{u} \bar{d} ds & \bar{u} \bar{d} us & \bar{u} \bar{d} ud \end{bmatrix}$$
(10)

Note in particular that the light $\sigma \sim N_3^3$ contains no strange quarks. While this picture seems unusual, precisely the configuration (10) was found by Jaffe [4] in the framework of the MIT bag model. The key dynamical point is that the states in (10) receive (due to the spin and color spin recoupling coefficients) exceptionally large binding energy from the "hyperfine" piece of the gluon exchange interchange:

$$H_{hf} = -\Delta \sum_{i,j} (\mathbf{S}_i \cdot \mathbf{S}_j) (\mathbf{F}_i \cdot \mathbf{F}_j), \qquad (11)$$

wherein the sum goes over all pairs i, j while S_i and F_i are respectively the spin and color generators acting on the i^{th} quark or antiquark.

While the picture above seems close to our expectations it is not quite right in detail. For example the masses do not exactly obey (7). Furthermore the simplest model for decay would give that $f_0 \rightarrow \pi\pi$ vanishes, in contradiction to experiment. Hence we add the extra mass terms

$$\mathcal{L}_{mass} = \mathrm{Eq.}(6) - c\mathrm{Tr}(N)Tr(N) - d\mathrm{Tr}(N)\mathrm{Tr}(N\mathcal{M}).$$
(12)

The c and d terms give $f_0 - \sigma$ mixing. Now we solve for (a, b, c, d) in terms of the four masses $m_{\sigma} = 550$ MeV, $m_{\kappa} = 900$ MeV, $m_{a_0} = 983.5$ MeV and $m_{f_0} = 980$ MeV. The solution boils down to a quadratic equation for (say) d. This gives two possible values for the mixing angle θ_s defined by,

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} N_3^3 \\ \frac{N_1^1 + N_2^2}{\sqrt{2}} \end{pmatrix}.$$
 (13)

The solution $\theta_s \approx -90^\circ$, giving $\sigma \approx (N_1^1 + N_2^2)/\sqrt{2}$ seems to correspond to restoring the $q\bar{q} \mod (4)$ for the scalars once more. The other solution $\theta_s \approx -20^\circ$ corresponds to σ being mainly N_3^3 which was just noted to be a characteristic signature of the $qq\bar{q}\bar{q} \mod (10)$. The very existence of these two different solutions highlights the fact that by just assuming a flavor transformation property for the scalars we are not forcing a particular identification of their underlying quark structure. Different substructures are naturally associated with different values of the parameters in the same effective Lagrangian. In any event, the extra terms in (12) have restored the ambiguity about the scalars' structure. We need more information to decide the issue. For this purpose we look at the trilinear couplings.

Using SU(3) invariance we write

$$\mathcal{L}_{N\phi\phi} = A \,\epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^e \partial_\mu \phi_c^f + B \operatorname{Tr}(N) \operatorname{Tr}(\partial_\mu \phi \partial_\mu \phi) + C \operatorname{Tr}(N \partial_\mu \phi) \operatorname{Tr}(\partial_\mu \phi) + D \operatorname{Tr}(N) \operatorname{Tr}(\partial_\mu \phi) \operatorname{Tr}(\partial_\mu \phi), \qquad (14)$$

where A, B, C, D are four real constants and ϕ represents the usual pseudoscalar nonet matrix. The derivatives stem from the requirement that (14) be the leading part of a chiral invariant object. If desired, we can rewrite the A term as a linear combination of the usual $\text{Tr}(N\partial_{\mu}\phi\partial_{\mu}\phi)$ and the three other terms. The motivation for the form given is that by itself the A term yields zero for $f_0 \to \pi\pi$ and $\sigma \to K\bar{K}$, both of which should vanish in the "fall apart" picture of a $T\bar{T}$ type scalar meson. Note that all the coupling constants which enter into our treatment of $\pi\pi$ and πK scattering depend on just A and B; C and D contribute only to the decays containing η or η' in the final state. For examples of couplings:

$$\gamma_{\kappa K\pi} = \gamma_{a_0 KK} = -2A,$$

$$\gamma_{\sigma\pi\pi} = 2B \sin\theta_s - \sqrt{2}(B-A) \cos\theta_s, etc.$$
(15)

The mixing angle solution which best fits the couplings needed to explain the $\pi\pi$ and πK scattering turns out to be $\theta_s \approx -20^\circ$. Together with a suitable choice of C and D, the interactions involving η and η' are also consistently described (as mentioned in section III). Thus it seems that our results point to a picture in which the light scalars are mainly dual quark- dual antiquark rather than quark-antiquark type. Very recently Achasov [10] has argued that new experimental data from Novosibirsk on the radiative decay $\phi(1020) \to \pi^0 \eta \gamma$ are better fit with a $qq\bar{q}\bar{q}$ type model of the $a_0(980)$.

To sum up: assuming $\sigma(550)$, $\kappa(900)$, $f_0(980)$ and $a_0(980)$ to belong to a nonet N_a^b which is fitted into a chiral Lagrangian, we have found the parameters A, B, C, Dwhich specify sixteen scalar-pseudoscalar-pseudoscalar coupling constants. These couplings and masses are used to explain $\pi\pi$ scattering (σ, f_0) , πK scattering (κ) and $\eta' \to \eta \pi \pi$ (a_0) with regularized tree amplitudes. Furthermore, a small $\sigma - f_0$ mixing angle in (13) suggests that N_a^b is describing a structure similar to a dual quark-dual antiquark. If this picture is correct there are many interesting applications and questions.

V POSSIBLE MECHANISM FOR NEXT LOWEST-LYING SCALARS

Of course, the success of the phenomenological quark model suggests that there exists a nonet of "conventional" $q\bar{q}$ scalars in the 1+ GeV range. What are the experimental candidates for these? [11] The situation for the isoscalar candidates is presently confusing. The $f_0(1370)$ may actually correspond to two different states. The $f_0(1500)$ may be a glueball while the $f_J(1710)$ does not necessarily have spin zero. Thus we will not focus on the isoscalars now. On the other hand the Review of Particle Physics "endorses" the isovector and isospinor candidates

$$a_0(1450): M = 1474 \pm 19 \text{MeV}, \quad \Gamma = 265 \pm 13 \text{MeV},$$

 $K_0^*(1450): M = 1429 \pm 6 \text{MeV}, \quad \Gamma = 287 \pm 23 \text{MeV}.$

On the way to taking these states seriously as members of an ordinary p-wave nonet we encounter three puzzles. i) The mass of the $a_0^+(1450)$ (presumably a



FIGURE 4. Mixing of two nonets-a',K',a and K stand respectively for the "physical" states $a_0(1450), K_0^*(1430), a_0(980)$ and $\kappa(900)$. K_0 and a_0 are the unmixed isospinor and isovector $qq\bar{q}\bar{q}$ states, while K'_0 and a'_0 are the corresponding unmixed $q\bar{q}$ states.

 $u\bar{d}$ state is greater than that of the $K_0^{*+}(1430)$ (presumably a $u\bar{s}$ state). ii) The $a_0(1450)$ and $K_0^*(1430)$ are not less massive than the corresponding p-wave tensor mesons $a_2(1320)$ and $K_2^*(1430)$, as expected from an $L \cdot S$ interaction (e.g. $m[\chi_{c2}(1p)] > m[\chi_{c0}(1p)]$). iii) Assuming the known decay modes $K_0^*(1430) \to K\pi$ and $a_0(1450) \to \pi\eta, K\bar{K}, \pi\eta'$ saturate the total widths, we have from SU(3) flavor invariance that $\Gamma[a_0(1450)] = 1.51\Gamma[K_0^*(1430)]$. However, experimentally it is $(0.92 \pm 0.12)\Gamma[K_0(1430)]$ instead.

These puzzles can be simply resolved [12] if we assume that an ideally mixed heavier $q\bar{q}$ nonet N' in turn mixes with an ideally mixed $T\bar{T}$ nonet N (as in (10)) via

$$\mathcal{L}' = -\gamma \mathrm{Tr}(NN'). \tag{16}$$

Actually the assumption of exact ideal mixings is a simplification which can be relaxed. The mechanism is driven by the fact that $m(a'_0) < m(K'_0)$ while $m(a_0) > m(K_0)$. Here the subscript zero refers to the unmixed N and N' members. The splittings are summarized in Fig. 4.

The explanations are: i)Think of a perturbation theory approach. There is a smaller "energy denominator" for $a_0 - a'_0$ mixing than for $K_0 - K'_0$ mixing. Thus there is more $a_0 - a'_0$ repulsion as shown in Fig. 4. ii) Since the mixing of two levels "repels" them, both $a_0(1450)$ and $K_0^*(1430)$ are heavier than would be expected otherwise. Similarly the light scalars $a_0(980)$ and $\kappa(900)$ are lighter than they would be without the mixing (16). iii) The difference between the $a_0(1450)$ and

 $K_0^*(1430)$ decay coupling constants can be understood from the necessarily greater mixture of the $qq\bar{q}\bar{q}$ component in the $a_0(1450)$ than in the $K_0^*(1430)$.

Clearly, looking at the isoscalars will be especially interesting when the experimental situation becomes clearer.

We would like to thank Francesco Sannino and Masayasu Harada for fruitful collaboration. One of us (J.S.) would like to thank the organizers for arranging a stimulating and enjoyable conference. The work has been supported in part by the US DOE under contract DE-FG-02-85ER40231.

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