

Syracuse University

**SURFACE**

---

Physics

College of Arts and Sciences

---

8-23-2000

## Exploring the Structure of a Possible Light Scalar Nonet

Joseph Schechter

*Department of Physics, Syracuse University, Syracuse, NY*

Deirdre Black

*Syracuse University*

Amir H. Fariborz

*Syracuse University*

Follow this and additional works at: <https://surface.syr.edu/phy>



Part of the [Physics Commons](#)

---

### Recommended Citation

Schechter, Joseph; Black, Deirdre; and Fariborz, Amir H., "Exploring the Structure of a Possible Light Scalar Nonet" (2000). *Physics*. 288.

<https://surface.syr.edu/phy/288>

This Article is brought to you for free and open access by the College of Arts and Sciences at SURFACE. It has been accepted for inclusion in Physics by an authorized administrator of SURFACE. For more information, please contact [surface@syr.edu](mailto:surface@syr.edu).

## Exploring the structure of a possible light scalar nonet

Deirdre BLACK, Amir H. FARIBORZ and Joseph SCHECHTER

*Department of Physics , Syracuse University  
Syracuse, NY 13244-1130, USA*

We first review the work of the Syracuse group, which uses an effective chiral Lagrangian approach, on meson-meson scattering. An illustration providing evidence for the existence of a strange scalar resonance of mass around 900 MeV is given. An attempt to fit this  $\kappa(900)$  together with a similarly obtained  $\sigma(560)$  and the well known  $a_0(980)$  and  $f_0(980)$  into a nonet pattern suggests that the underlying structure is closer to a dual quark-dual antiquark than to a quark-antiquark. A possible mechanism to explain a next higher-in mass scalar meson nonet is also discussed. This involves mixing between  $q\bar{q}$  and  $qq\bar{q}\bar{q}$  states.

### §1. Introduction

The possible existence of light scalar mesons (with masses less than about 1 GeV) has been a controversial subject for roughly forty years. The last few years have seen a revival of interest in this area. We will discuss two related aspects. The first involves the determination of the existence of light scalar mesons and their properties by comparing models for meson-meson scattering with experiment. Other speakers<sup>1)</sup> at this workshop have presented various approaches to this problem. The work of the Syracuse group<sup>2), 3), 4)</sup>, discussed in the talk of M. Harada for the case of  $\pi\pi$  scattering, is based on an effective non-linear chiral Lagrangian containing pseudoscalar, vector and scalar particles. It is well known that  $\pi\pi$  scattering very near threshold can be accurately treated with a chiral Lagrangian of only pseudoscalars, which is systematically expanded to include all terms with a given number of derivatives (chiral perturbation theory). However this essentially polynomial expansion can not be used to explain the shape of the scalar partial wave amplitude up to the 1 GeV region without using a prohibitively large number of derivatives. The inclusion of scalar resonances directly, provides a much more economical description, already at tree level, over this extended range. The tree level scattering amplitude obtained from the chiral Lagrangian is crossing symmetric but has physical divergences at the direct channel poles. These are regularized according to the prescription (for a light, broad resonance like the  $\sigma$  or  $\kappa$ ):

$$\frac{MG}{M^2 - s} \rightarrow \frac{MG}{M^2 - s - iMG'}, \quad (1.1)$$

where  $G'$ , which is not required to equal  $G$ , is taken as a fitting parameter.  $G$  and  $M$  are parameters from the chiral Lagrangian. Fitting the resulting amplitude to experiment, of course, restores unitarity. In this way both unitarity and crossing symmetry are approximately satisfied. For the  $\pi\pi$  case the amplitude up to 1 GeV has four parts: i. "current algebra" contact term, ii. vector meson exchange terms, iii.  $\sigma(560)$  exchange terms, iv.  $f_0(980)$  exchange terms including the appropriate

background (Ramsauer Townsend effect). A similar pattern seems to hold for  $\pi K$  scattering as we will briefly describe in section 2.

The second aspect we discuss is the underlying quark structure of the light scalars which are needed in our treatment of meson-meson scattering. As examples, three models for the underlying quark structure have been discussed by many authors: i) the  $K\bar{K}$  molecule model<sup>5)</sup>, ii) the  $q\bar{q}$  model with strong meson-meson interactions (or "unitarized quark model")<sup>6)</sup>, iii) the intrinsic  $qq\bar{q}\bar{q}$  model (Jaffe type<sup>7)</sup>). These models have the common feature that four quarks are involved in some form; all are different from the "simple"  $q\bar{q}$  model. Note that in the effective Lagrangian approach, the quark substructure of the scalars is not specified. In particular a nonet field can *a priori* represent either  $q\bar{q}$  or  $qq\bar{q}\bar{q}$  (or even more complicated) states since both may have the same flavor transformation property. Information about the quark structure may however be inferred indirectly.

## §2. Pi K scattering

The  $J = 0$  partial wave amplitudes of  $\pi K$  scattering were treated<sup>3)</sup> in a similar way to those of  $\pi\pi$  scattering. In this case the low energy amplitude is taken to correspond to the sum of a current algebra contact diagram, vector  $\rho$  and  $K^*$  exchange diagrams and scalar  $\sigma(550)$ ,  $f_0(980)$  and  $\kappa(900)$  exchange diagrams. The situation in the interesting  $I = 1/2$  channel turns out to be very analogous to the  $I = 0$  channel of s-wave  $\pi\pi$  scattering. Now a  $\kappa(900)$  parametrized as in (1.1) is required to restore unitarity; it plays the role of the  $\sigma(550)$  in the  $\pi\pi$  case. Following our criterion we expect that to extend this treatment to the 1.5 GeV region, one should include the many possible exchanges of particles with masses up to about 1.5 GeV. Nevertheless we found that a satisfactory description of the 1-1.5 GeV s-wave region is obtained simply by including the well known  $K_0^*(1430)$  scalar resonance, which plays the role of the  $f_0(980)$  in the  $\pi\pi$  calculation.

It may be helpful to give a step by step pictorial approach to see how the individual components contribute to the real part of the  $I = 1/2$  scalar partial wave amplitude,  $R_0^{1/2}$  of  $\pi K$  scattering. In Fig. (1) it is seen that the "current algebra" (i.e. contact term of the non-linear chiral Lagrangian) violates the unitarity bound,  $|R_0^{1/2}| \leq 1/2$ , already not too far from threshold. The inclusion of vector meson exchanges (dashed line) improves the situation a lot but still leads to unitarity violation. The unitarity bound may be satisfied if exchanges of the scalar mesons  $\sigma(560)$ ,  $f_0(980)$  and a putative  $\kappa(900)$  are included, as shown in Fig.(2). The s-channel pole of the  $\kappa(900)$  was modified as in (1.1) and the corresponding parameters were obtained by fitting to experiment. Finally the effect of the  $K_0^*(1430)$  resonance is included as shown in Fig.(3). This inclusion was made with the background corresponding to Fig.(2). Since this approach involves fitting to experiment, unitarity is obeyed (with the appropriate elasticity assumption) rather than just the unitarity bound.

It is interesting to observe that our fit to the  $R_0^{1/2}$  amplitude has the same general structure as the one used by the experimentalists in their analysis of the data<sup>8)</sup>.

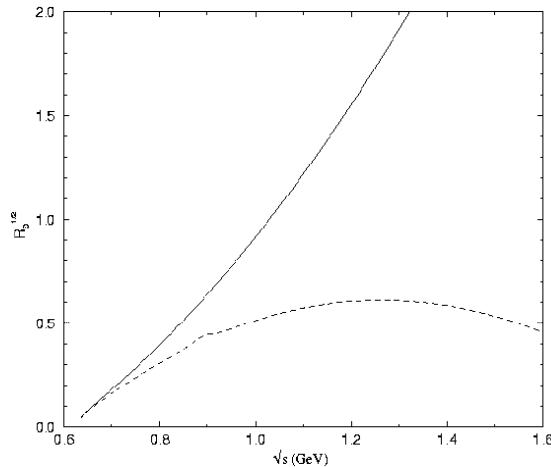


Fig. 1. Contribution of current algebra (solid line) and current algebra + vectors (dashed line) to  $R_0^{1/2}$ .

Specifically, they write the amplitude as the sum of an effective range background piece and a  $K_0^*(1430)$  piece modified by this background. In our model their background corresponds to the sum of "current algebra",  $\rho$ ,  $\sigma(560)$  exchange,  $f_0(980)$  exchange and  $\kappa(900)$  exchange pieces. Certainly the effective range description is more economical. However pieces corresponding to current algebra, vector meson and at least  $f_0(980)$  seem to definitely exist in nature. Our evidence for the need of a  $\kappa(900)$  is in a model in which these other contributions are included. If one does not include these known other contributions, the statistical evidence for a  $\kappa(900)$  would be weaker<sup>9)</sup>. Our conclusion agrees with that of Ishida et al<sup>10)</sup>.

### §3. Scalar nonet "family" properties

The nine states associated with the  $\sigma(550)$ ,  $\kappa(900)$ ,  $f_0(980)$  and  $a_0(980)$  are required in order to fit experiment in our model. What do their masses and coupling constants suggest about their quark substructure? (See<sup>11)</sup> for more details.) Suppose we first try to assign them to a conventional  $q\bar{q}$  nonet:

$$\sigma(550) \sim \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad \kappa^+(900) \sim u\bar{s}, \quad a_0^+(980) \sim u\bar{d}, \quad f_0(980) \sim s\bar{s}. \quad (3.1)$$

Then there are two puzzles. i) Why aren't the  $a_0(980)$  and the  $\sigma(550)$ , which have the same number of non-strange quarks, degenerate? ii) Why aren't these particles, being p-wave states, in the same 1+ GeV energy region as the other p-wave states?

To study this, first note that most meson multiplets can be nicely understood

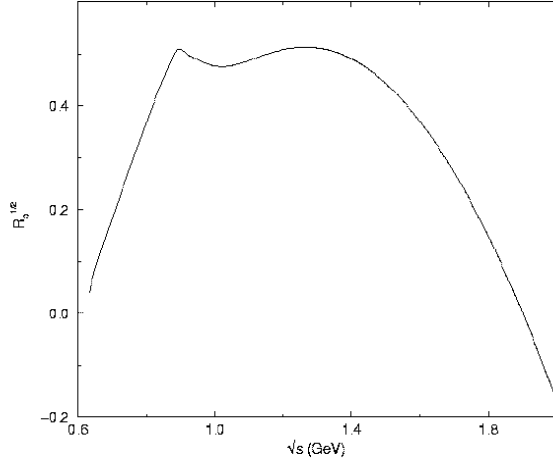


Fig. 2. Contribution of current algebra + vectors +  $\sigma$  +  $f_0(980)$  +  $\kappa$  to  $R_0^{1/2}$ .

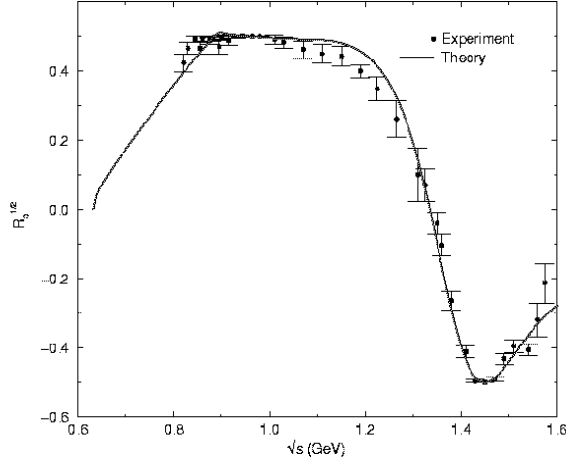


Fig. 3. Comparison of the theoretical prediction of  $R_0^{1/2}$  with the experimental data.

using the concept of “ideal mixing”. In Okubo’s formulation<sup>12)</sup>, originally applied to the vector meson multiplet, the meson fields are grouped into a nonet matrix,

$$N_a^b = \begin{bmatrix} N_1^1 & a_0^+ & \kappa^+ \\ a_0^- & N_2^2 & \kappa^0 \\ \bar{\kappa}^+ & \bar{\kappa}^0 & N_3^3 \end{bmatrix}, \quad (3.2)$$

where the particle names have been chosen to fit the scalar mesons. The two  $I = 0$  states are the SU(3) singlet,  $(N_1^1 + N_2^2 + N_3^3)/\sqrt{3}$  and the SU(3) octet member,  $(N_1^1 + N_2^2 - 2N_3^3)/\sqrt{6}$ . Okubo's ansatz for the mass terms was,

$$\mathcal{L}_{mass} = -a\text{Tr}(NN) - b\text{Tr}(NN\mathcal{M}), \quad (3.3)$$

where  $a > 0$  and  $b$  are real constants and  $\mathcal{M} = \text{diag}(1, 1, x)$  (with  $x = m_s/m_u$ ) is the ‘‘spurion’’ matrix which breaks flavor SU(3) invariance. With (3.2) and (3.3) the SU(3) singlet and SU(3) octet isoscalar states mix in such a way (ideal mixing) that the physical mass eigenstates emerge as  $(N_1^1 + N_2^2)/\sqrt{2}$  and  $N_3^3$ . Furthermore there are two mass relations

$$m^2(a_0) = m^2\left(\frac{N_1^1 + N_2^2}{\sqrt{2}}\right), \quad m^2(a_0) - m^2(\kappa) = m^2(\kappa) - m^2(N_3^3). \quad (3.4)$$

Note that there are two different solutions depending on the sign of  $b$ . If  $b > 0$  we get Okubo's original case where [with the identifications  $a_0 \rightarrow \rho$ ,  $\kappa \rightarrow K^*$ ,  $(N_1^1 + N_2^2)/\sqrt{2} \rightarrow \omega$  and  $N_3^3 \rightarrow \phi$ ] there is the conventional ordering

$$m^2(\phi) > m^2(K^*) > m^2(\rho) = m^2(\omega). \quad (3.5)$$

This agrees with counting the number of (heavier) strange quarks when we identify  $N_a^b \sim q_a \bar{q}^b$ .

On the other hand if  $b < 0$  and we identify  $N_3^3 \rightarrow \sigma$  and  $(N_1^1 + N_2^2)/\sqrt{2} \rightarrow f_0$ , the resulting ordering would be

$$m^2(f_0) = m^2(a_0) > m^2(\kappa) > m^2(\sigma), \quad (3.6)$$

which is in nice agreement with the present ‘‘observed’’ scalar spectrum. But this clearly does not agree with counting the number of strange quarks while assuming that the scalar mesons are simple quark anti-quark composites. This unusual ordering will agree with counting the number of strange quarks if we assume instead that the scalar mesons are schematically constructed as  $N_a^b \sim T_a \bar{T}^b$  where  $T_a \sim \epsilon_{acd} \bar{q}^c \bar{q}^d$  is a ‘‘dual’’ quark. Specifically

$$N_a^b \sim T_a \bar{T}^b \sim \begin{bmatrix} \bar{s}\bar{d}ds & \bar{s}\bar{d}us & \bar{s}\bar{d}ud \\ \bar{s}\bar{u}ds & \bar{s}\bar{u}us & \bar{s}\bar{u}ud \\ \bar{u}\bar{d}ds & \bar{u}\bar{d}us & \bar{u}\bar{d}ud \end{bmatrix} \quad (3.7)$$

Note in particular that the light  $\sigma \sim N_3^3$  contains no strange quarks. While this picture seems unusual, precisely the configuration (3.7) was found by Jaffe<sup>7)</sup> in the framework of the MIT bag model. The key dynamical point is that the states in (3.7) receive (due to the spin and color spin recoupling coefficients) exceptionally large binding energy from the ‘‘hyperfine’’ piece of the gluon exchange interchange:

$$H_{hf} = -\Delta \sum_{i,j} (\mathbf{S}_i \cdot \mathbf{S}_j)(\mathbf{F}_i \cdot \mathbf{F}_j), \quad (3.8)$$

wherein the sum goes over all pairs  $i, j$  while  $\mathbf{S}_i$  and  $\mathbf{F}_i$  are respectively the spin and color generators acting on the  $i^{\text{th}}$  quark or antiquark.

While the picture above seems close to our expectations it is not quite right in detail. For example the masses do not exactly obey (3.4). Furthermore the simplest model for decay would give that  $f_0 \rightarrow \pi\pi$  vanishes, in contradiction to experiment. Hence we add the extra mass terms

$$\mathcal{L}_{mass} = \text{Eq.}(3.3) - c\text{Tr}(N)\text{Tr}(N) - d\text{Tr}(N)\text{Tr}(N\mathcal{M}). \quad (3.9)$$

The  $c$  and  $d$  terms give  $f_0 - \sigma$  mixing. Now we solve for  $(a, b, c, d)$  in terms of the four masses  $m_\sigma = 550$  MeV,  $m_\kappa = 900$  MeV,  $m_{a_0} = 983.5$  MeV and  $m_{f_0} = 980$  MeV. The solution boils down to a quadratic equation for (say)  $d$ . This gives two possible values for the mixing angle  $\theta_s$  defined by,

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} N_3^3 \\ \frac{N_1^1 + N_2^2}{\sqrt{2}} \end{pmatrix}. \quad (3.10)$$

The solution  $\theta_s \approx -90^\circ$ , giving  $\sigma \approx (N_1^1 + N_2^2)/\sqrt{2}$  seems to correspond to restoring the  $q\bar{q}$  model (3.1) for the scalars once more. The other solution  $\theta_s \approx -20^\circ$  corresponds to  $\sigma$  being mainly  $N_3^3$  which was just noted to be a characteristic signature of the  $qq\bar{q}\bar{q}$  model (3.7). The very existence of these two different solutions highlights the fact that by just assuming a flavor transformation property for the scalars we are not forcing a particular identification of their underlying quark structure. Different substructures are naturally associated with different values of the parameters in the same effective Lagrangian. In any event, the extra terms in (3.9) have restored the ambiguity about the scalars' structure. We need more information to decide the issue. For this purpose we look at the trilinear couplings.

Using SU(3) invariance we write

$$\begin{aligned} \mathcal{L}_{N\phi\phi} = & A \epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^e \partial_\mu \phi_c^f + B \text{Tr}(N) \text{Tr}(\partial_\mu \phi \partial_\mu \phi) \\ & + C \text{Tr}(N \partial_\mu \phi) \text{Tr}(\partial_\mu \phi) + D \text{Tr}(N) \text{Tr}(\partial_\mu \phi) \text{Tr}(\partial_\mu \phi), \end{aligned} \quad (3.11)$$

where  $A, B, C, D$  are four real constants and  $\phi$  represents the usual pseudoscalar nonet matrix. The derivatives stem from the requirement that (3.11) be the leading part of a chiral invariant object. If desired, we can rewrite the  $A$  term as a linear combination of the usual  $\text{Tr}(N \partial_\mu \phi \partial_\mu \phi)$  and the three other terms. The motivation for the form given is that by itself the  $A$  term yields zero for  $f_0 \rightarrow \pi\pi$  and  $\sigma \rightarrow K\bar{K}$ , both of which should vanish in a dominant "quark-line rule" picture of a  $T\bar{T}$  scalar decaying into two pseudoscalars. Note that all the coupling constants which enter into our treatment of  $\pi\pi$  and  $\pi K$  scattering depend on just  $A$  and  $B$ ;  $C$  and  $D$  contribute only to the decays containing  $\eta$  or  $\eta'$  in the final state. For examples of couplings:

$$\begin{aligned} \gamma_{\kappa K\pi} = \gamma_{a_0 K K} &= -2A, \\ \gamma_{\sigma\pi\pi} &= 2B \sin\theta_s - \sqrt{2}(B - A) \cos\theta_s, \text{ etc.} \end{aligned} \quad (3.12)$$

The mixing angle solution which best fits the couplings needed to explain the  $\pi\pi$  and  $\pi K$  scattering turns out to be  $\theta_s \approx -20^\circ$ . Together with a suitable choice of

$C$  and  $D$ , the interactions involving  $\eta$  and  $\eta'$  are also consistently described<sup>4)</sup>. Thus it seems that our results point to a picture in which the light scalars are closer to dual quark- dual antiquark rather than simple quark-antiquark type. Very recently Achasov<sup>13)</sup> has argued that new experimental data from Novosibirsk on the radiative decay  $\phi(1020) \rightarrow \pi^0\eta\gamma$  are better fit with a  $qq\bar{q}\bar{q}$  type model of the  $a_0(980)$ .

#### §4. Possible mechanism for next lowest-lying scalars

Of course, the success of the phenomenological quark model suggests that there exists a nonet of “conventional”  $q\bar{q}$  scalars in the 1+ GeV range. Let us consider the experimental candidates<sup>14)</sup> for the isovector and isospinor members:

$$\begin{aligned} a_0(1450) : M &= 1474 \pm 19\text{MeV}, & \Gamma &= 265 \pm 13\text{MeV}, \\ K_0^*(1450) : M &= 1429 \pm 6\text{MeV}, & \Gamma &= 287 \pm 23\text{MeV}. \end{aligned}$$

On the way to taking these states seriously as members of an ordinary p-wave nonet we encounter three puzzles. i) The mass of the  $a_0^+(1450)$  (presumably a  $u\bar{d}$  state) is greater than that of the  $K_0^{*+}(1430)$  (presumably a  $u\bar{s}$  state). ii) The  $a_0(1450)$  and  $K_0^*(1430)$  are not less massive than the corresponding p-wave tensor mesons  $a_2(1320)$  and  $K_2^*(1430)$ , as expected from an  $L \cdot S$  interaction (e.g.  $m[\chi_{c2}(1p)] > m[\chi_{c0}(1p)]$ ). iii) Assuming the known decay modes  $K_0^*(1430) \rightarrow K\pi$  and  $a_0(1450) \rightarrow \pi\eta, K\bar{K}, \pi\eta'$  saturate the total widths, we have from SU(3) flavor invariance that  $\Gamma[a_0(1450)] = 1.51\Gamma[K_0^*(1430)]$ . However, experimentally it is  $(0.92 \pm 0.12)\Gamma[K_0(1430)]$  instead.

These puzzles can be simply resolved<sup>15)</sup> if we assume that an ideally mixed heavier  $q\bar{q}$  nonet  $N'$  in turn mixes with an ideally mixed  $T\bar{T}$  nonet  $N$  (as in (3-7)) via

$$\mathcal{L}' = -\gamma\text{Tr}(NN'). \quad (4-1)$$

This mixing term involves the product of six quark fields in our picture and is related to the instanton determinant. The mechanism is driven by the fact that  $m(a'_0) < m(K'_0)$  while  $m(a_0) > m(K_0)$ . Here the subscript zero refers to the unmixed  $N$  and  $N'$  members. The splittings are summarized in Fig. 4.

The explanations are: i) Think of a perturbation theory approach. There is a smaller “energy denominator” for  $a_0 - a'_0$  mixing than for  $K_0 - K'_0$  mixing. Thus there is more  $a_0 - a'_0$  repulsion as shown in Fig. 4. ii) Since the mixing of two levels “repels” them, both  $a_0(1450)$  and  $K_0^*(1430)$  are heavier than would be expected otherwise. Similarly the light scalars  $a_0(980)$  and  $\kappa(900)$  are lighter than they would be without the mixing (4-1). iii) The difference between the  $a_0(1450)$  and  $K_0^*(1430)$  decay coupling constants can be understood from the necessarily greater mixture of the  $qq\bar{q}\bar{q}$  component in the  $a_0(1450)$  than in the  $K_0^*(1430)$ .

This treatment suggests that the light scalar mesons have an interesting and non-trivial story to tell. Clearly further work will be needed to complete the picture.

We would like to thank Masayasu Harada and Francesco Sannino for fruitful collaboration. One of us (J.S.) would like to thank the organizers for arranging a stimulating and enjoyable conference and the Japan Society for the Promotion of



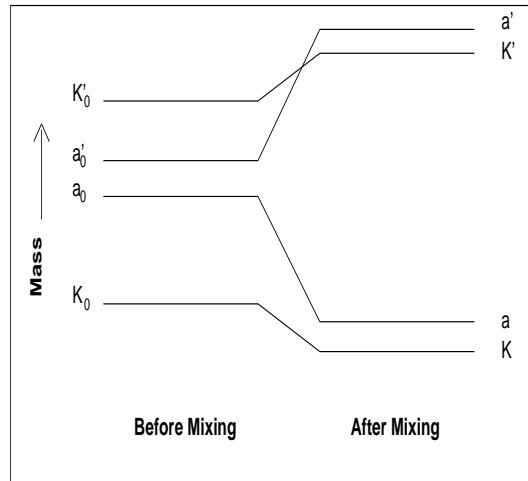


Fig. 4. Mixing of two nonets- $a', K', a$  and  $K$  stand respectively for the "physical" states  $a_0(1450)$ ,  $K_0^*(1430)$ ,  $a_0(980)$  and  $\kappa(900)$ .  $K_0$  and  $a_0$  are the unmixed isospinor and isovector  $qq\bar{q}$  states, while  $K'_0$  and  $a'_0$  are the corresponding unmixed  $q\bar{q}$  states.

Science for supporting his visit to Japan. The work has been supported in part by the US DOE under contract DE-FG-02-85ER40231.

### References

- [1] See the write-ups of M. Harada, T. Hatsuda, K. Igi, M. Ishida, S. Ishida, R. Kaminski, T. Kunihiro, S. Nishiyama, M. Ohbu, J. Oller, T. Sawada, M. Scadron, E. Shabalín, K. Takamatsu, N. Tornqvist and T. Watabe.
- [2] ( $\pi\pi$  scattering) F. Sannino and J. Schechter, Phys. Rev. **D52**, 96 (1995); M. Harada, F. Sannino and J. Schechter, Phys. Rev. **D54**, 1991 (1996); *ibid.* Phys. Rev. Lett. **78**, 1603 (1997).
- [3] ( $\pi K$  scattering) D. Black, A. H. Fariborz, F. Sannino and J. Schechter. Phys. Rev. **D58**, 054012 (1998).
- [4] ( $\eta' \rightarrow \eta\pi\pi$  and  $\pi\eta$  scattering) A. H. Fariborz and J. Schechter, Phys. Rev. **D60**, 034002 (1999); D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. **D61**, 074030 (2000).
- [5] N. Isgur and J. Weinstein, Phys. Rev. Lett. **48**, 659(1982).
- [6] N. Tornqvist, Z. Phys. **C68**, 647(1995); E. van Beveren et al, Z. Phys. **C30**, 615(1986).
- [7] R. Jaffe, Phys. Rev. **D15**, 267 (1977).
- [8] D. Aston et al, Nucl. Phys. **B296**, 493 (1988).
- [9] S. Cherry and M. Pennington hep-ph/0005208.
- [10] S. Ishida, M. Ishida, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. **98**, 621 (1997).
- [11] D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. **D59**, 074026 (1999).
- [12] S. Okubo, Phys. Lett. **5**, 165 (1963).
- [13] N. Achasov hep-ph/9904223.
- [14] Review of Particle Physics, Euro. Phys. J. **C3** (1999).
- [15] D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. **D61**, 074001 (2000).