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Investigating the Light Scalar Mesons

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Abstract.

We first briefly review a treatment of the scalars in meson meson scattering based on a non-linear chiral Lagrangian, with unitarity implemented by a "local" modification of the scalar propagators. It is shown that the main results are confirmed by a treatment in the SU(3) linear sigma model in which unitarity is implemented "globally". Some remarks are made on the speculative subject of the scalars' quark structure.

I INTRODUCTION

The "hydrogen atom" problem of meson spectroscopy is the study of the pion in terms of its fundamental constituents. Typically, this difficult problem is finessed by using an effective Lagrangian treatment of the composite field which includes the important feature of (almost) spontaneously broken chiral symmetry. Then one explains the presumed next highest mass meson– the rho– as a $q\bar{q}$ bound state and continues up the spectrum. But nowadays there is increasing support for the existence of the old "sigma" resonance which may be lighter than the rho. If this is true it certainly seems worthwhile to pause and examine the issue in detail. It is also a difficult problem because the sigma is in an energy range just above where one expects chiral perturbation theory to be practical but well below where asymptotic freedom permits a systematic perturbative QCD expansion.

In this talk a recent paper [1] on the subject will be discussed. Other work is referenced in that paper and in other contributions [2] to this conference. First, a brief review of our previous results based on the non linear chiral Lagrangian will be given. Then we try to check the form of these results by using the linear sigma model. This model, while less general, provides the usual physical intuition about the problem as it contains a scalar nonet linked to the pseudoscalars.

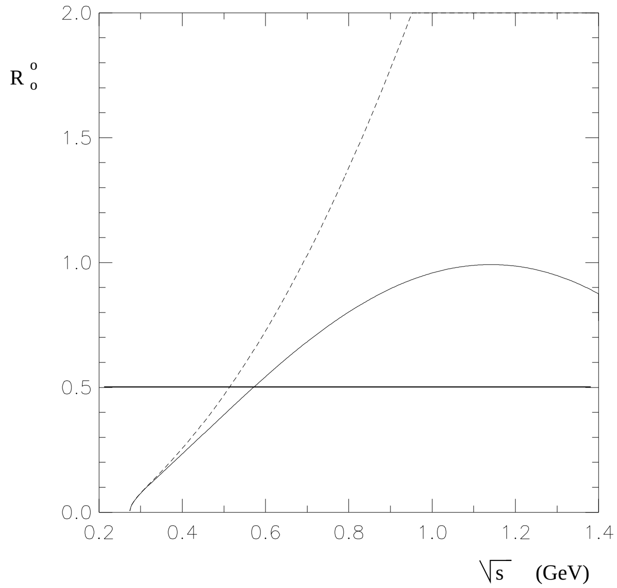

FIGURE 1. The solid line which shows the current algebra $+\rho$ result is much closer to the unitary bound of 0.5 than the dashed line which shows the current algebra result alone.

II BRIEF REVIEW OF OUR PREVIOUS WORK USING THE NONLINEAR CHIRAL LAGRANGIAN

Pi pi scattering [3]. It was noted that the $I = J = 0$ partial wave amplitude up to about 1 GeV could be simply explained as a sum of four pieces: i. the current algebra "contact" term, ii. the ρ exchange diagram iii. a non Breit Wigner σ pole diagram and exchange, iv. an $f_0(980)$ pole in the background produced by the other three. This is illustrated in a step by step manner for the real part R_0^0 in Figs. 1, 2 and 3.

We see in Fig. 1 that the "current algebra" piece starts violating the unitarity bound, $|R_0^0| \leq 1/2$ at about 0.5 GeV and then runs away. However the inclusion of the ρ meson exchange diagrams turns the curve in the right direction and improves, but does not completely cure, the unitarity violation. These pieces, which do not involve any unknown parameters, give encouragment to our hope that the cooperative interplay of various pieces can explain the low energy scattering. In order to fix up Fig. 1 we note that the real part of a resonance contribution vanishes at the pole, is positive before the pole and *negative* above the pole. Thus a scalar resonance with a pole roughly about 0.5 GeV (above which R_0^0 in Fig. 1 needs a negative contribution to stay below 1/2) should do the job. The result of including such a σ pole , with three parameters, is shown in Fig. 2. Now note that the predicted $R_0^0(s)$ in Fig. 2 vanishes around 1 GeV. Thus the phase δ at 1 GeV (assumed to keep rising) is about 90^o. Considering this as a background phase for the known $f_0(980)$, the real part of the $f_0(980)$ contribution will get reversed in sign (Ramsauer–Townsend effect). As Fig. 3 shows this is the missing piece needed to give a simple explanation of the $J = I = 0$ $\pi\pi$ scattering up to about 1 GeV.

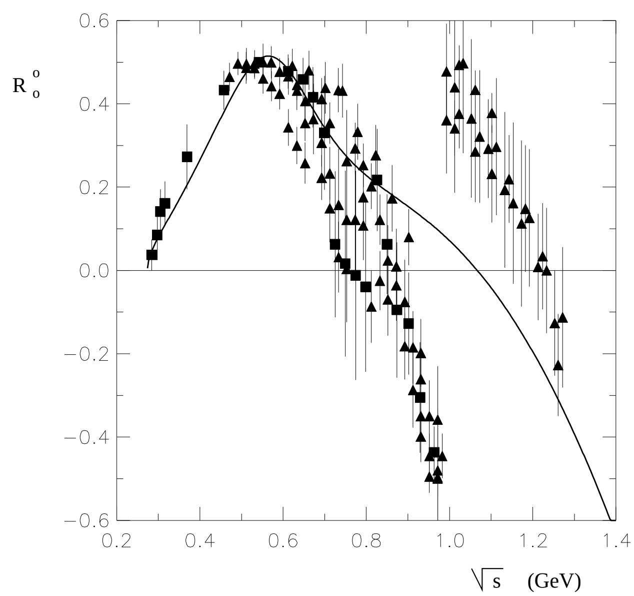

FIGURE 2. The sum of current algebra $+\rho+\sigma$ contributions compared to data.

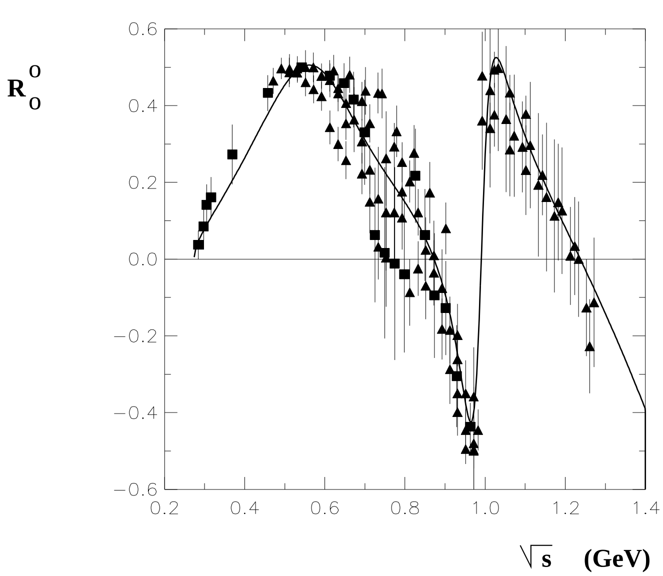

FIGURE 3. The sum of current algebra $+\rho+\sigma+f_0(980)$ contributions compared to data.

Pi K scattering [4]. In this case the low energy amplitude is taken to correspond to the sum of a current algebra contact diagram, vector ρ and K^* exchange diagrams and scalar $\sigma(560)$, $f_0(980)$ and $\kappa(900)$ exchange diagrams. The situation in the interesting $I = 1/2$ s-wave channel turns out to be very analogous to the $I = 0$ channel of s-wave $\pi\pi$ scattering. Now a non Breit Wigner κ is required to restore unitarity; it plays the role of the $\sigma(560)$ in the $\pi\pi$ case. We found that a satisfactory description of the 1-1.5 GeV s-wave region is also obtained by including the well known $K_0^*(1430)$ scalar resonance, which plays the role of the $f_0(980)$ in the $\pi\pi$ calculation.

Putative light scalar nonet [5]. The nine states associated with the $\sigma(560)$, $\kappa(900)$, $f_0(980)$ and $a_0(980)$ are required in order to fit experiment in our model. What do their masses and coupling constants suggest about their quark substructure? Clearly the mass ordering of the various states is inverted compared to the "ideal mixing" [6] scenario which approximately holds for most meson nonets. This means that a quark structure for the putative scalar nonet $N_a^b \sim q_a\bar{q}^b$ is unlikely since the mass ordering just corresponds to counting the number of heavier strange quarks. Then the degenerate $f_0(980)$ and $a_0(980)$ which must have the structure $N_1^1 \pm N_2^2$ would be lightest rather than heaviest. However the inverted ordering will agree with this counting if we assume that the scalar mesons are schematically constructed as $N_a^b \sim T_a\bar{T}^b$ where $T_a \sim \epsilon_{acd}\bar{q}^c\bar{q}^d$ is a "dual" quark (or anti diquark). This interpretation is strengthened by consideration [5] of the scalars' coupling constants to two pseudoscalars. That shows $\sigma \sim N_3^3 +$ "small", so it is a predominantly non-strange particle in this picture. Furthermore the states $N_1^1 \pm N_2^2$ now would each have two strange quarks and would be expected to be heaviest. The four quark picture was first suggested a long time ago [7] on dynamical grounds.

Mechanism for next heavier scalar nonet [8]. Of course, the success of the phenomenological quark model suggests that there exists, in addition, a nonet of "conventional" p-wave $q\bar{q}$ scalars in the 1+ GeV range. The experimental candidates for these states are $a_0(1450)(I = 1)$, $K_0^*(1430)(I = 1/2)$ and for $I = 0$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. These are enough for a full nonet plus a glueball. However it is puzzling that the strange $K_0^*(1430)$ isn't noticeably heavier than the non strange $a_0(1450)$ and that they are not lighter than the corresponding spin 2 states. These and another puzzle may be solved in a natural way [8] if the heavier p-wave scalar nonet mixes with a lighter $qq\bar{q}\bar{q}$ nonet of the type mentioned above. The mixing mechanism makes essential use of the "bare" lighter nonet having an inverted mass ordering while the heavier "bare" nonet has the normal ordering. A rather rich structure involving the light scalars seems to be emerging. At lower energies one may consider as a first approximation, "integrating out" the heavier nonet and retaining just the lighter one.

III THE PICTURE IN A 3 FLAVOR LINEAR SIGMA MODEL

In [1] we employed the conventional chiral field $M = S + i\phi$, where S is a hermitian matrix of nine scalars and ϕ is a hermitian matrix of nine pseudoscalars. The Lagrangian density is,

$$\mathcal{L} = -\frac{1}{2}Tr(\partial_\mu M \partial_\mu M^\dagger) - V_0 + \sum A_a(M_{aa} + M^\dagger_{aa}). \tag{1}$$

The three A_a's are numbers proportional to the (current) quark masses. V_0 may be considered to be an arbitrary function of the chiral $SU(3) \times SU(3)$ invariants constructed from M and $M^\dagger$. Note that most consequences at tree level follow just from chiral symmetry, irrespective of the form of V_0.

Pi pi scattering amplitude. The computed $I = J = 0$ partial wave amplitude at tree level has the form,

$$T^0_{0tree}(s) = cos^2\psi[\alpha(s) + \frac{\beta(s)}{m^2_{BARE}(\sigma) - s}] + sin^2\psi[\tilde{\alpha}(s) + \frac{\tilde{\beta}(s)}{m^2_{BARE}(\sigma') - s}], \tag{2}$$

where $\alpha(s)$,$\beta(s)$ etc. are given in connection with Eq (3.2) of [1]. ψ is a mixing angle between the two $I = 0$ scalars, denoted as σ and σ'. The subscript "BARE" on their masses means the value at tree level. If V_0 is general, the three quantities ψ, $m_{BARE}(\sigma)$ and $m_{BARE}(\sigma')$ may be chosen at will. However if V_0 is taken to be renormalizable there is only one arbitrary parameter (say $m_{BARE}(\sigma)$) in the theory when the input set (say $m_\pi, m_K, m_\eta, m_{\eta'}, F_\pi$) is fixed.

It is instructive to first go back to the widely treated two flavor case. This corresponds to choosing $\psi = 0$ in (2). Then $m_{BARE}(\sigma)$ is the only unknown parameter. Near threshold, if $m_{BARE}(\sigma)$ is not too low, the amplitude is the "current algebra" result which agrees fairly well with experiment. It is a small quantity which emerges from an almost complete cancellation of the pole and non pole terms in (2). One would like to keep this result and utilize the effect of the sigma at higher energies. Since there is a true pole in (2) it seems reasonable to regulate this in the usual way by adding a term $-im\Gamma$ in the pole denominator. However, as Achasov and Shestakov [9] pointed out, this regulation completely destroys the good current algebra result. They instead adopt the K matrix approach (whereas the usual solution is to adopt the non linear model instead since the derivative coupling of the σ there suppresses the pole contribution near threshold). In this way the tree amplitude is not only regularized but made exactly unitary. One calculates the amplitude in terms of its tree value as:

$$T = \frac{T_{tree}}{1 - iT_{tree}}. \tag{3}$$

When T_{tree} is small, $T \approx T_{tree}$ so the behavior near threshold will now not be spoiled. At the other extreme, when T_{tree} gets very large $T \to i$.

Note that the pole position, z of the unitarized T will typically correspond to mass and width (via $z = m^2 - im\Gamma$) which differ from m_{BARE} and the starting perturbative width. Which one should be chosen? Since T in (3) evidently has the structure of a "bubble sum" in field theory it seems reasonable to regard the K matrix unitarization as an approximation to including the "radiative corrections". Then, as in usual field theory, the pole found is interpreted as giving the physical mass and width while the values of m_{BARE} and Γ_{BARE} would have no special significance. For the two flavor model we verified the result of [9] that a choice of $m_{BARE}(\sigma)$ around 0.8 to 1.0 GeV would result in a physical $m(\sigma)$ around 0.45 GeV and fit the first bump in Fig. 3. The physical mass is not very sensitive to the exact choice of bare mass and also the physical width is very greatly reduced. The predicted mass in this model is a bit less than the one we found in the non linear model reviewed in the previous section, but this is readily understandable as being due [10] to the neglect of vector mesons in the present model.

Three flavor linear model amplitude. The procedure was simply to use the full two pole tree amplitude (2) in the unitarization formula (3). We were not able to fit the entire T_0^0 amplitude shown in Fig. 3 up to about 1.2 GeV in the renormalizable model (which contains only the single unknown parameter $m_{BARE}(\sigma)$). However it is easy to find a fit in the chiral model with general V_0, in which we were able to choose the three parameters $m_{BARE}(\sigma) = 0.847$ GeV, $m_{BARE}(\sigma') = 1.300$ GeV and $\psi = 48.6^o$. The physical isoscalar masses (after unitarization) turned out to be 0.457 GeV and 0.993 Gev associated with respective widths 0.632 GeV and 0.05 GeV. Again these represent large shifts from the bare values. For illustrative purposes the unitarized amplitude is reasonably approximated as

$$T_0^0 \approx const. + \frac{0.167 + 0.210i}{s - (0.209 - 0.289i)} + \frac{0.053 + 0.005i}{s - (0.986 - 0.051i)}. \quad (4)$$

Neither the first (σ) pole nor the second ($f_0(980)$) pole is precisely of Breit Wigner type. However the $f_0(980)$ pole approximates a Breit Wigner except for an overall minus sign, which corresponds to the well known "flipping" of this resonance.

We similarly studied the $I = 0, J = 1/2$ scattering amplitude to find the properties of the κ resonance in the linear model. The bare mass of the κ is fixed once the input parameters are given. To allow us to vary this quantity (in a range where the $\pi\pi$ scattering is not much affected) we chose the alternative input set $(m_\pi, m_K, m_{\eta'}, F_\pi, F_K)$ and varied F_K. In this case, because the $K_0^*(1430)$ is not included in the model we can only fit the data up to about 1 GeV. It was found that the best fit corresponded to the bare κ mass about 1.3 GeV. After unitarization the physical kappa mass turned out to be about 0.800 GeV and this didn't change much as the bare value was varied from 0.9 to 1.3 GeV. Unitarization also substantially narrowed the physical kappa width. Furthermore, as for the case of the σ the κ pole is not of Breit Wigner type. An analogous calculation was carried out to study the properties of the $a_0(980)$ as observed in $\pi\eta$ scattering. A summary shown in Table 1 compares the physical widths obtained in this linear model with

	σ	f_0	κ	a_0
Present Model				
mass (MeV), width (MeV)	457, 632	993, 51	800, 260-610	890-1010, 110-240
Comparison				
mass (MeV), width (MeV)	560, 370	980±10, 40-100	900, 275	985, 50-100

TABLE 1. Predicted "physical" masses and widths in MeV of the nonet of scalar mesons contrasted with suitable (as discussed in the text) comparison values.

those obtained in the non linear model. In the cases of the $f_0(980)$ and the $a_0(980)$ the entries were taken from the Particle Data Group [11], with which the non linear model calculations agree.

Clearly, the complex pole positions and nature of the poles (non Breit Wigner for σ and κ and "Ramsauer Townsend" for $f_0(980)$) of the scalar nonet in the linear sigma model are similar to those obtained previously (putative scalar nonet) using a non linear chiral Lagrangian with a different "local" regulation. This statement makes heavy use of the unitarization of the three flavor linear model; otherwise the $f_0(980)$ and κ might be considered too high and wide to belong to a light scalar nonet. In particular, the κ clearly cannot be identified with the $K_0^*(1430)$ in this unitarized linear sigma model.

Speculation on scalar quark structure (Section V of [1]). At an intuitive level one might expect the scalar nonet, being the "chiral partner" of the light pseudoscalar nonet, to have a quark- anti quark structure. It was stressed [5] however that in the more general non linear Lagrangian approach (e.g. [12]) the scalar and pseudoscalar transformation properties are decoupled. Only the flavor SU(3) transformation property, not the chiral one, of the scalars is fixed in the effective non-linear Lagrangian treatment. Features, mentioned above, like isoscalar mixing angle and mass ordering suggest in fact the $qq\bar{q}\bar{q}$ structure for the light scalars as an initial approximation.

How might this kind of scenario play out in the linear model where the chiral properties of the scalars and the pseudoscalars are clearly linked? Even there, the quark substructure implied by the $SU(3) \times SU(3)$ transformation properties of the chiral matrix M in (1) is not unique [1] (However the $U(1)_A$ transformations do distinguish between $q\bar{q}$ and $qq\bar{q}\bar{q}$). There are three different "four quark" structures with the same transformation properties. Physically, they correspond to making the chiral mesons as a) meson meson "molecule" b) spin 0 diquark -spin 0 anti diquark and c) spin 1 diquark - spin 1 anti diquark. Actually these three are not linearly independent. Thus the molecule [13] and diquark- anti diquark [7] pictures are not clearly distinguished at the effective Lagrangian level. Presumably, large changes in the properties of the scalars due to unitarization in the effective theory must be counted as "four quark" admixtures at the underlying level.

In detail, the schematic structure for the matrix $M(x) = S + i\phi$ realizing a $q\bar{q}$ composite in terms of quark fields $q_{aA}(x)$ can be written

$$M_a^{(1)b} = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}, \tag{5}$$

where a and A are respectively flavor and color indices. For the "molecule" model a) the schematic quark structure with the same $SU(3)_L \times SU(3)_R$ transformation property is,

$$M_a^{(2)b} = \epsilon_{acd}\epsilon^{bef}\left(M^{(1)\dagger}\right)_e^c \left(M^{(1)\dagger}\right)_f^d. \tag{6}$$

In the spin 0 diquark - spin 0 anti diquark case the same transformation property is realized with,

$$M_g^{(3)f} = \left(L^{gA}\right)^\dagger R^{fA}, \tag{7}$$

where

$$\begin{aligned} L^{gE} &= \epsilon^{gab}\epsilon^{EAB} q_{aA}^T C^{-1} \frac{1+\gamma_5}{2} q_{bB}, \\ R^{gE} &= \epsilon^{gab}\epsilon^{EAB} q_{aA}^T C^{-1} \frac{1-\gamma_5}{2} q_{bB}, \end{aligned} \tag{8}$$

in which C is the charge conjugation matrix of the Dirac theory. Finally the spin 1 diquark - spin 1 anti diquark case c) has the schematic structure,

$$M_g^{(4)f} = \left(L_{\mu\nu,AB}^g\right)^\dagger R_{\mu\nu,AB}^f, \tag{9}$$

where

$$\begin{aligned} L_{\mu\nu,AB}^g = L_{\mu\nu,BA}^g &= \epsilon^{gab} q_{aA}^T C^{-1} \sigma_{\mu\nu} \frac{1+\gamma_5}{2} q_{bB}, \\ R_{\mu\nu,AB}^g = R_{\mu\nu,BA}^g &= \epsilon^{gab} q_{aA}^T C^{-1} \sigma_{\mu\nu} \frac{1-\gamma_5}{2} q_{bB}, \end{aligned} \tag{10}$$

and $\sigma_{\mu\nu} = \frac{1}{2i}\left[\gamma_\mu, \gamma_\nu\right]$.

Now, as discussed before, the realistic situation is likely to contain substantial mixing between scalar $q\bar{q}$ and $qq\bar{q}\bar{q}$ nonets. To explore this we formulated a linear sigma model containing both a $q\bar{q}$ chiral matrix $M = S + i\phi$ and a $qq\bar{q}\bar{q}$ chiral matrix $M' = S' + i\phi'$. This is a very complicated system so we started with a "toy model" in which all current quark masses are neglected and only a minimum number of non derivative terms are included. In addition to two minimal kinetic terms as in (1), we took the simplified potential,

$$V_0 = -c_2 \mathrm{Tr}\left(MM^\dagger\right) + c_4 \mathrm{Tr}\left(MM^\dagger MM^\dagger\right) + d_2 \mathrm{Tr}\left(M'M'^\dagger\right) + e\mathrm{Tr}\left(MM'^\dagger + M'M^\dagger\right). \tag{11}$$

Here c_2, c_4 and d_2 are positive real constants. The M matrix field is chosen to have a wrong sign mass term so that there will be spontaneous breakdown of chiral symmetry. A pseudoscalar octet will thus be massless. The mixing is controlled by the parameter e. It is amusing to note that there will then be an induced $qq\bar{q}\bar{q}$ condensate $\langle S' \rangle$ in addition to the usual $q\bar{q}$ condensate $\langle S \rangle$.

We found that it is easy to obtain a situation where the the next highest state above the predominantly $q\bar{q}$ Nambu Goldstone pseudoscalar octet is a predominantly $qq\bar{q}\bar{q}$ scalar octet. Still heavier is the predominantly $qq\bar{q}\bar{q}$ pseudoscalar while heaviest of all is the $q\bar{q}$ scalar octet. Of course, SU(3) symmetry breaking and unitarization would be expected to modify this picture. It seems very interesting to further pursue a model of this type. There is evidently a possibility of learning a lot about non perturbative QCD from the light scalar system.

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