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Joseph Schechter  
*Syracuse University*

Deirdre Black  
*Jefferson Lab*

Masayasu Harada  
*Nagoya University*

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## STUDY OF SCALAR MESONS AND RELATED RADIATIVE DECAYS

D. BLACK<sup>A</sup>, M. HARADA<sup>B</sup> AND J. SCHECHTER<sup>C\*</sup>

(A) *Theory Group, Jefferson Laboratory,  
12000 Jefferson Avenue,  
Newport News, Virginia, 23606, USA  
E-mail: dblack@jlab.org*

(B) *Department of Physics,  
Nagoya University,  
Nagoya 464-8602, Japan  
E-mail: harada@eken.phys.nagoya-u.ac.jp*

(C) *Physics Department,  
Syracuse University,  
Syracuse, NY 13244-1130, USA  
E-mail: schechte@phy.syr.edu*

After a brief review of the puzzling light scalar meson sector of QCD, a brief summary will be given of a paper concerning radiative decays involving the light scalars. There, a simple vector meson dominance model is constructed in an initial attempt to relate a large number of the radiative decays involving a putative scalar nonet to each other. As an application it is illustrated why  $a_0(980)-f_0(980)$  mixing is not expected to greatly alter the  $f_0/a_0$  production ratio for radiative  $\phi$  decays.

### 1. Introduction

Why might the subject of light scalar mesons be of interest to physicists now that QCD is known to be the correct theory of Strong Interactions and the burning issue is to extend the Standard Model to higher energies? Simply put, another goal of Physics is to produce results from Theory which can be compared with Experiment. At very large energy scales, the asymptotic freedom of QCD guarantees that a controlled perturbation expansion is a practical tool, once the relevant "low energy stuff" is suitably

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\*Speaker

parameterized. At very low energy scales, for example close to the threshold of  $\pi\pi$  scattering, the running QCD coupling constant is expected to be large and perturbation theory is not expected to work. Fortunately, a controlled expansion based on an effective theory with the correct symmetry structure- Chiral Perturbation Theory<sup>1</sup>- seems to work reasonably well. The new information about Strong Interactions which this approach reveals is closely related to the spectrum and flavor "family" properties of the lowest lying pseudoscalar meson multiplet and was, in fact, essentially known before QCD.

Clearly it is important to understand how far in energy above threshold the Chiral Perturbation Theory program will take us. To get a rough estimate consider the experimental data for the real part of the  $I = J = 0$   $\pi\pi$  scattering amplitude,  $R_0^0$  displayed in Fig. 1. The chiral perturbation series should essentially give a polynomial fit to this shape, which up to about 1 GeV is crudely reminiscent of one cycle of a sine curve.

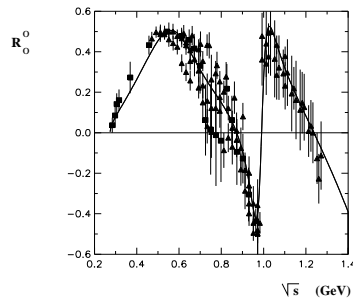


Figure 1. Illustration of the real part of the pi pi scattering amplitude extracted from experimental data.

Now consider polynomial approximations to one cycle of the sine curve with various numbers of terms. These are illustrated in Fig. 2. Note that each successive term departs from the true sine curve right after the preceding one. It is clear that something like eight terms are required for a decent fit. This would correspond to seven loop order of chiral perturbation theory and seems presently impractical.

## 2. Need for light scalar mesons

Thus an alternative approach is indicated for going beyond threshold of pi pi scattering up to about 1 GeV. The data itself suggests the presence

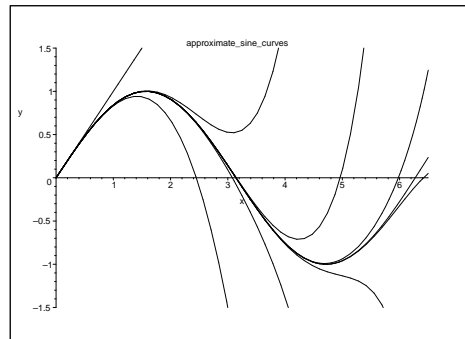


Figure 2. Polynomial approximations to one cycle of the sine curve.

of s-wave resonances, the lowest of which is denoted the "sigma". Physically, one then expects the practical range of chiral perturbation theory to be up to about 450-500 MeV, just before the location of this lowest resonance. In the last few years there have been studies<sup>2</sup> by many authors which advance this picture. All of them are "model dependent" but this is probably inevitable for the strongly coupled regime of QCD. For example<sup>3</sup>, in a framework where the amplitude is computed from a non linear chiral Lagrangian containing explicit scalars as well as vectors and pseudoscalars, the fit shown in Fig. 1 emerges as a sum of four pieces: i. the current algebra "contact" term, ii. the  $\rho$  exchange diagram iii. a non Breit Wigner  $\sigma(560)$  pole diagram and exchange, iv. an  $f_0(980)$  pole in the background produced by the other three. It is not just a simple sum of Born graphs but includes the approximate unitarization features of the non Breit Wigner shape of the sigma and a Ramsauer Townsend mechanism which reverses the sign of the  $f_0(980)$ . Also note that i. and ii. provide very substantial background to the sigma pole, partially explaining why the sigma does not "jump right out" of various experimental studies. Qualitative agreement with this approach is obtained by K-matrix unitarization of the two flavor linear sigma model<sup>4</sup> and three flavor linear sigma model<sup>5</sup> amplitudes.

Workers on scalar mesons entertain the hope that, after the revelations about the vacuum structure of QCD confirmed by the broken chiral symmetric treatment of the pseudoscalars, an understanding of the next layer of the "strong interaction onion" will be provided by studying the light scalars. An initial question is whether the light scalars belong to a flavor  $SU(3)$  multiplet as the underlying quark structure might suggest. Apart from the  $\sigma(560)$ , the  $f_0(980)$  and the isovector  $a_0(980)$  are fairly well es-

tablished. This leaves a gap concerning the four strange- so called kappa-states. This question is more controversial than that of the sigma state . In the unitarized non linear chiral Lagrangian framework one must thus consider  $pi - K$  scattering. In this case the low energy amplitude is taken<sup>6</sup> to correspond to the sum of a current algebra contact diagram, vector  $\rho$  and  $K^*$  exchange diagrams and scalar  $\sigma(560)$ ,  $f_0(980)$  and  $\kappa(900)$  exchange diagrams. The situation in the interesting  $I = 1/2$  s-wave channel turns out to be very analogous to the  $I = 0$  channel of s-wave  $\pi\pi$  scattering. Now a non Breit Wigner  $\kappa$  is required to restore unitarity; it plays the role of the  $\sigma(560)$  in the  $\pi\pi$  case. It was found that a satisfactory description of the 1-1.5 GeV s-wave region is also obtained by including the well known  $K_0^*(1430)$  scalar resonance, which plays the role of the  $f_0(980)$  in the  $\pi\pi$  calculation. As in the case of the sigma, the light kappa seems hidden by background and does not jump right out of the initial analysis of the experimental data.

Thus the nine states associated with the  $\sigma(560)$ ,  $\kappa(900)$ ,  $f_0(980)$  and  $a_0(980)$  seem to be required in order to fit experiment in this chiral framework. What would their masses and coupling constants suggest about their quark substructure if they were assumed to comprise an SU(3) nonet <sup>7</sup>? Clearly the mass ordering of the various states is inverted compared to the "ideal mixing"<sup>8</sup> scenario which approximately holds for most meson nonets. This means that a quark structure for the putative scalar nonet of the form  $N_a^b \sim q_a \bar{q}^b$  is unlikely since the mass ordering just corresponds to counting the number of heavier strange quarks. Then the nearly degenerate  $f_0(980)$  and  $a_0(980)$  which must have the structure  $N_1^1 \pm N_2^2$  would be lightest rather than heaviest. However the inverted ordering will agree with this counting if we assume that the scalar mesons are schematically constructed as  $N_a^b \sim T_a \bar{T}^b$  where  $T_a \sim \epsilon_{acd} \bar{q}^c \bar{q}^d$  is a "dual" quark (or anti diquark). This interpretation is strengthened by consideration <sup>7</sup> of the scalars' coupling constants to two pseudoscalars. Those couplings depend on the value of a mixing angle,  $\theta_s$  between  $N_3^3$  and  $(N_1^1 - N_2^2)/\sqrt{2}$ . Fitting the coupling constants to the treatments of  $\pi\pi$  and  $K\pi$  scattering gives a mixing angle such that  $\sigma \sim N_3^3 +$  "small";  $\sigma(560)$  is thus a predominantly non-strange particle in this picture. Furthermore the states  $N_1^1 \pm N_2^2$  now would each predominantly contain two extra strange quarks and would be expected to be heaviest. Four quark pictures of various types have been suggested as arising from spin-spin interactions in the MIT bag model<sup>9</sup>, unitarized quark models<sup>10</sup> and meson-meson interaction models<sup>11</sup>.

There seems to be another interesting twist to the story of the light

scalars. The success of the phenomenological quark model suggests that there exists, in addition, a nonet of “conventional” p-wave  $q\bar{q}$  scalars in the energy region above 1 GeV. The experimental candidates for these states are  $a_0(1450)(I = 1)$ ,  $K_0^*(1430)(I = 1/2)$  and for  $I = 0$ ,  $f_0(1370)$ ,  $f_0(1500)$  and  $f_0(1710)$ . These are enough for a full nonet plus a glueball. However it is puzzling that the strange  $K_0^*(1430)$  isn’t noticeably heavier than the non strange  $a_0(1450)$  and that they are not lighter than the corresponding spin 2 states. These and another puzzle may be solved in a natural way<sup>12</sup> if the heavier p-wave scalar nonet mixes with a lighter  $qq\bar{q}\bar{q}$  nonet of the type mentioned above. The mixing mechanism makes essential use of the “bare” lighter nonet having an inverted mass ordering while the heavier “bare” nonet has the normal ordering. A rather rich structure involving the light scalars seems to be emerging. At lower energies one may consider as a first approximation, “integrating out” the heavier nonet and retaining just the lighter one.

### 3. Radiative decays involving light scalars

In the last few years, a lot of experimental activity<sup>13</sup> at the  $e^+e^-$  machines (Novosibirsk, DAΦNE and Jefferson Lab) has resulted in definitive measurements of the interesting reactions:

$$\phi(1020) \rightarrow f_0(980) + \gamma \rightarrow \pi^0\pi^0 + \gamma, \quad (1)$$

$$\phi(1020) \rightarrow a_0(980) + \gamma \rightarrow \pi^0\eta + \gamma. \quad (2)$$

These measurements have been awaited by theorists for a number of years as proposed tests<sup>14</sup> of the nature of the  $f_0(980)$  and  $a_0(980)$  scalars. The theoretical models used for these tests were based on the observation that the vector meson,  $\phi(1020)$  mainly decays into  $K + \bar{K}$  so a virtual  $K$  loop diagram can reasonably be expected to dominate the decay mechanism. In this framework Achasov<sup>15</sup> has argued that the data are most consistent with a compact four quark structure for the  $f_0$  and  $a_0$  (as opposed to a two quark structure or a loosely bound meson meson “molecule” structure).

This situation makes it interesting to study in detail the extension of the picture to a full nonet (or two?) of scalar mesons as well as to further solidify the technical analysis of the K- loop class of diagrams. In addition, there is perhaps (depending on the exact masses and widths of the  $a_0$  and  $f_0$  mesons) a problem in that the experimentally derived ratio  $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma)$  is in the range 3-4 while theoretical estimates are

mostly clustered around unity. We are presently working on K-loop type models but decided to start for ourselves with a much simpler preliminary picture. The goal of this model<sup>16</sup> is to try to correlate many different radiative processes involving the members of a full scalar nonet by using flavor symmetry. The model has the following features: 1. It is based on a chiral symmetric Lagrangian containing complete nonets of pseudoscalar, vector as well as (the putative) scalar fields. 2. Vector meson dominance for photon vertices is automatic in the formulation. 3. An effective flavor invariant SVV (scalar-vector-vector) vertex is postulated which has three relevant parameters. These are treated as the only *a priori* unfixed parameters of the model.

Our framework is that of a standard non-linear chiral Lagrangian containing, in addition to the pseudoscalar nonet matrix field  $\phi$ , the vector meson nonet matrix  $\rho_\mu$  and a scalar nonet matrix field denoted  $N$ . Under chiral unitary transformations of the three light quarks;  $q_{L,R} \rightarrow U_{L,R} q_{L,R}$ , the chiral matrix  $U = \exp(2i\phi/F_\pi)$ , where  $F_\pi \simeq 0.131$  GeV, transforms as  $U \rightarrow U_L U U_R^\dagger$ . The convenient matrix  $K(U_L, U_R, \phi)$  is defined by the following transformation property of  $\xi$  ( $U = \xi^2$ ):  $\xi \rightarrow U_L \xi K^\dagger = K \xi U_R^\dagger$ , and specifies the transformations of “constituent-type” objects. The fields we need transform as

$$\begin{aligned} N &\rightarrow K N K^\dagger, \\ \rho_\mu &\rightarrow K \rho_\mu K^\dagger + \frac{i}{\tilde{g}} K \partial_\mu K^\dagger, \\ F_{\mu\nu}(\rho) &= \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g} [\rho_\mu, \rho_\nu] \rightarrow K F_{\mu\nu} K^\dagger, \end{aligned} \quad (3)$$

where the coupling constant  $\tilde{g}$  is about 4.04. The strong trilinear scalar-vector-vector terms in the effective Lagrangian are:

$$\begin{aligned} L_{SVV} &= \beta_A \epsilon_{abc} \epsilon^{a'b'c'} [F_{\mu\nu}(\rho)]_{a'}^a [F_{\mu\nu}(\rho)]_{b'}^b N_{c'}^c \\ &+ \beta_B \text{Tr} [N] \text{Tr} [F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)] \\ &+ \beta_C \text{Tr} [N F_{\mu\nu}(\rho)] \text{Tr} [F_{\mu\nu}(\rho)] \\ &+ \beta_D \text{Tr} [N] \text{Tr} [F_{\mu\nu}(\rho)] \text{Tr} [F_{\mu\nu}(\rho)] . \end{aligned} \quad (4)$$

Chiral invariance is evident from (3) and the four flavor-invariants are needed for generality. (A term  $\sim \text{Tr}(FFN)$  is linearly dependent on the four shown). Actually the  $\beta_D$  term will not contribute in our model so there are only three relevant parameters  $\beta_A$ ,  $\beta_B$  and  $\beta_C$ . Equation (4) is analogous to the *PVV* interaction which was originally introduced as a  $\pi\rho\omega$  coupling a long time ago<sup>17</sup>. It is intended to be the simplest description of

the production mechanism which contains the full symmetries of the problem. Elsewhere we will discuss modifications due to the effect of K-loops. One can now compute the amplitudes for  $S \rightarrow \gamma\gamma$  and  $V \rightarrow S\gamma$  according to the diagrams of Fig. 3.

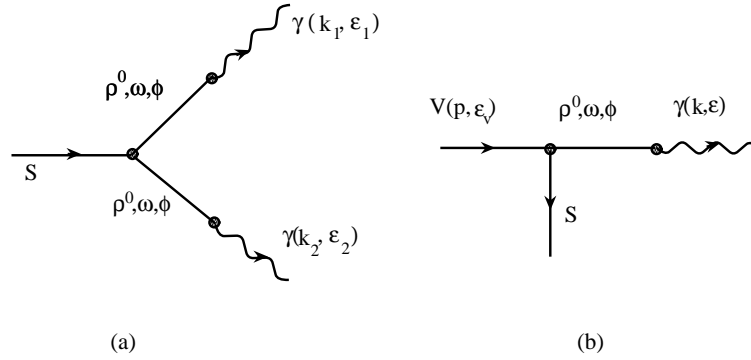


Figure 3. Feynman diagrams for (a)  $S \rightarrow \gamma\gamma$  and (b)  $V \rightarrow S\gamma$ .

Altogether there are many processes of these types. For the two photon decays one may consider the initial scalar to be any of  $\sigma(560)$ ,  $f_0(980)$  or  $a_0^0(980)$ . With an initial vector state we have, in addition to  $\phi \rightarrow f_0, a_0^0 + \gamma$ , the possibilities  $\phi \rightarrow \sigma + \gamma$ ,  $\omega \rightarrow \sigma + \gamma$  and  $\rho^0 \rightarrow \sigma + \gamma$ . Furthermore for the cases when the scalar may be heavier than the vector, the same diagram allows one to compute the five modes  $f_0, a_0^0 \rightarrow \omega, \rho^0 + \gamma$  as well as  $\kappa^0 \rightarrow K^{*0} + \gamma$ . These are not all measured yet but an initial predicted correlation, is shown in <sup>16</sup>.

This model can also be used to study a recent conjecture<sup>18</sup> which attempts to produce a large value for the ratio  $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma)$  by invoking the iso spin violating  $a_0(980) - f_0(980)$  mixing. Actually, a detailed refutation of this conjecture has already been presented<sup>19</sup>. However the calculation may illustrate our approach. One may simply introduce the mixing by a term in the effective Lagrangian:  $L_{af} = A_{af}a_0^0f_0$ . A recent calculation <sup>20</sup> for the purpose of finding the effect of the scalar mesons in the  $\eta \rightarrow 3\pi$  process obtained the value  $A_{af} = -4.66 \times 10^{-3} \text{ GeV}^2$ . It is convenient to treat this term as a perturbation. Then the amplitude factor for  $\phi \rightarrow f_0\gamma$  includes a correction term consisting of the  $\phi \rightarrow a_0^0\gamma$  amplitude factor  $C_\phi^{a_0} = \sqrt{2}(\beta_C - 2\beta_A)$  multiplied by  $A_{af}$  and by the  $a_0$  propagator. The  $\phi \rightarrow a_0^0\gamma$  amplitude factor has a similar correction. The desired ratio



is then,

$$\frac{\text{amp}(\phi \rightarrow f_0\gamma)}{\text{amp}(\phi \rightarrow a_0^0\gamma)} = \frac{C_\phi^f + A_{af}C_\phi^a/D_a(m_f^2)}{C_\phi^a + A_{af}C_\phi^f/D_f(m_a^2)}, \quad (5)$$

where  $D_a(m_f^2) = -m_f^2 + m_a^2 - im_a\Gamma_a$  and  $D_f(m_a^2) = -m_a^2 + m_f^2 - im_f\Gamma_f$ . In this approach the propagators are diagonal in the isospin basis. The numerical values of these resonance widths and masses are, according to the Review of Particle Physics<sup>21</sup>  $m_{a_0} = (984.7 \pm 1.3)$  MeV,  $\Gamma_{a_0} = 50$ – $100$  MeV,  $m_{f_0} = 980 \pm 10$  MeV and  $\Gamma_{f_0} = 40$ – $100$  MeV. For definiteness, from column 1 of Table II in Ref.<sup>3</sup> we take  $m_{f_0} = 987$  MeV and  $\Gamma_{f_0} = 65$  MeV while in Eq. (4.2) of Ref.<sup>22</sup> we take  $\Gamma_{a_0} = 70$  MeV. In fact the main conclusion does not depend on these precise values. It is easy to see that the mixing factors are approximately given by

$$\frac{A_{af}}{D_a(m_f^2)} \approx \frac{A_{af}}{D_f(m_a^2)} \approx \frac{iA_{af}}{m_a\Gamma_a} \approx -0.07i. \quad (6)$$

Noting that  $C_\phi^f/C_\phi^a \approx 0.75$  in the present model, the ratio in Eq.(5) is roughly  $(0.75 - 0.07i)/(1 - 0.05i)$ . Clearly, the correction to  $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0^0\gamma)$  due to  $a_0^0$ - $f_0$  mixing only amounts to a few per cent, nowhere near the huge effect suggested in<sup>18</sup>.

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## References

1. J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142(1984); Nucl. Phys. **B250**, 465 (1985).
2. See the dedicated conference proceedings, S. Ishida et al "Possible existence of the sigma meson and its implication to hadron physics", KEK Proceedings 2000-4, Soryyushiron Kenkyu 102, No.5 2001. Additional points of view are expressed in the proceedings, D. Amelin and A. M. Zaitsev "Hadron spectroscopy", Ninth international conference on hadron spectroscopy, Protvino, Russia (2001).

3. M. Harada, F. Sannino and J. Schechter, Phys. Rev. D **54**, 1991 (1996).
4. N. N. Achasov and G. N. Shestakov, Phys. Rev. D **49**, 5779 (1984). See also S. Ishida, M. Y. Ishida, H. Takahashi, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. **95**, 745 (1966).
5. See for example D. Black, A. H. Fariborz, S. Moussa, S. Nasri and J. Schechter, Phys. Rev. D **64**, 014031 (2001). A more complete list of references is given here.
6. D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. D **58**, 054012 (1998).
7. D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. **D59**, 074026 (1999).
8. S. Okubo, Phys. Lett. **5**, 165 (1963).
9. R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).
10. N. A. Tornqvist, Z. Phys. C **68**, 647 (1995); E. van Beveren and G. Rupp, Eur. Phys. J. C **10**, 469 (1999).
11. J. D. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982).
12. D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. D **61**, 074001 (2000); D. Black et al,<sup>5</sup> above; T. Teshima, I. Kitamura and N. Morisita, J. Phys. G **28**, 1391 (2002); F. Close and N. Tornqvist, arXiv:hep-ph/0204205.
13. M. N. Achasov *et al.* [SND Collaboration], Phys. Lett. B **479**, 53 (2000); R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **462**, 380 (1999); A. Aloisio *et al.* [KLOE Collaboration], arXiv:hep-ex/0107024.
14. N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315**, 465 (1989); F. E. Close, N. Isgur and S. Kumano, Nucl. Phys. B **389**, 513 (1993); N. N. Achasov and V. V. Gubin, Phys. Rev. D **56**, 4084 (1997); Phys. Rev. D **57**, 1987 (1998); N. N. Achasov, V. V. Gubin and V. I. Shevchenko, Phys. Rev. D **56**, 203 (1997); J. L. Lucio Martinez and M. Napsuciale, Phys. Lett. B **454**, 365 (1999); arXiv:hep-ph/0001136; A. Bramon, R. Escribano, J. L. Lucio M., M. Napsuciale and G. Pancheri, Phys. Lett. B **494**, 221 (2000).
15. N. N. Achasov, page 112 of "Hadron Spectroscopy"<sup>2</sup> above.
16. D. Black, M. Harada and J. Schechter, Phys. Rev. Letts. **88**, 181603 (2002).
17. M. Gell-Mann, D. Sharp and W. G. Wagner, Phys. Rev. Lett. **8**, 261 (1962).
18. F. E. Close and A. Kirk, Phys. Lett. B **489**, 24 (2000); **515**, 13 (2001).
19. N.N. Achasov and A. V. Kiselev, Phys. Lett. B **534**, 83 (2002).
20. A. Abdel-Rehim, D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. **D67**, 054001 (2003).
21. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15**, 1 (2000) and 2001 partial update for edition 2002 (URL: <http://pdg.lbl.gov>).
22. A. H. Fariborz and J. Schechter, Phys. Rev. D **60**, 034002 (1999).