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Joseph Schechter
Department of Physics, Syracuse University, Syracuse, NY

Abdou Abdel-Rehim
Syracuse University

Deirdre Black
Jefferson Laboratory

Amir H. Fariborz
State University of New York Institute of Technology

THE SCALAR SECTOR AND THE $\eta \to 3\pi$ PROBLEM

A. ABDEL-REHIM[A], D. BLACK[B], A. H. FARIBORZ[C] AND J. SCHECHTER[D*]

(A) Physics Department,
Syracuse University,
Syracuse, NY 13244-1130,USA.
E-mail: abdou@phy.syr.edu

(B) Theory Group, Jefferson Laboratory,
12000 Jefferson Avenue,
Newport News, Virginia, 23606, USA.
E-mail: dblack@jlab.org

(C) Department of Mathematics/Science,
State University of New York Institute of Technology,
Utica, NY 13504-3050, USA.
E-mail: fariboa@sunyit.edu

(D)Physics Department,
Syracuse University,
Syracuse, NY 13244-1130, USA.
E-mail: schechte@phy.syr.edu

First, recent work on light scalar mesons, which is of possible interest in connection with the strong coupling region of QCD, is briefly discussed. Then a very short highlighting of a paper concerned with an application to the $\eta \to 3\pi$ problem is presented.

1. Introduction

At very large energy scales, the asymptotic freedom of QCD guarantees that a controlled perturbation expansion is a practical tool. At very low energy scales, for example close to the threshold of $\pi\pi$ scattering. the running QCD coupling constant is expected to be large and perturbation theory is not

*Speaker

expected to work. Fortunately, a controlled expansion based on an effective theory with the correct symmetry structure- Chiral Perturbation Theory[1]- seems to work reasonably well. The new information about Strong Interactions which this approach reveals is closely related to the spectrum and flavor "family" properties of the lowest lying pseudoscalar meson multiplet and was, in fact, essentially known before QCD.

Clearly it is important to understand how far in energy above threshold the Chiral Perturbation Theory program will take us. To get a rough estimate consider the experimental data for the real part of the $I = J = 0$ $\pi\pi$ scattering amplitude, R_0^0 displayed in Fig. 1. The chiral perturbation series should essentially give a polynomial fit to this shape, which up to about 1 GeV is crudely reminiscent of one cycle of a sine curve.

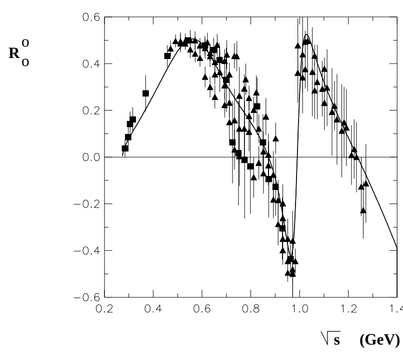

Figure 1. Illustration of the real part of the pi pi scattering amplitude extracted from experimental data.

Now consider polynomial approximations to one cycle of the sine curve with various numbers of terms. These are illustrated in Fig. 2. Note that each succesive term departs from the true sine curve right after the preceding one. It is clear that something like eight terms are required for a decent fit. This would correspond to seven loop order of chiral perturbation theory and seems presently impractical.

2. Need for light scalar mesons

Thus an alternative approach is indicated for going beyond threshold of pi pi scattering up to about 1 GeV. The data itself suggests the presence of s-wave resonances, the lowest of which is denoted the "sigma". Physically, one then expects the practical range of chiral perturbation theory to

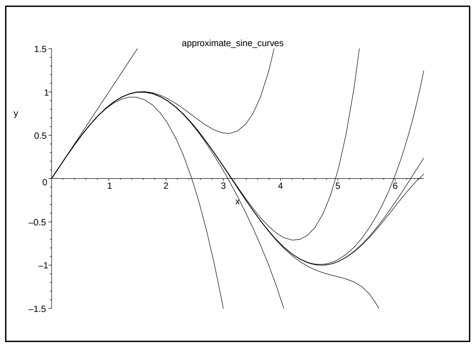

Figure 2. Polynomial approximations to one cycle of the sine curve.

be up to about 450-500 MeV, just before the location of this lowest resonance. In the last few years there have been studies [2] by many authors which advance this picture. All of them are "model dependent" but this is probably inevitable for the strongly coupled regime of QCD. For example [3], in a framework where the amplitude is computed from a non linear chiral Lagrangian containing explicit scalars as well as vectors and pseudoscalars, the fit shown in Fig. 1 emerges as a sum of four pieces: i. the current algebra "contact" term, ii. the ρ exchange diagram iii. a non Breit Wigner $\sigma(560)$ pole diagram and exchange, iv. an $f_0(980)$ pole in the background produced by the other three. It is not just a simple sum of Born graphs but includes the approximate unitarization features of the non Breit Wigner shape of the sigma and a Ramsauer Townsend mechanism which reverses the sign of the $f_0(980)$. Also note that i. and ii. provide very substantial background to the sigma pole, partially explaining why the sigma does not "jump right out" of various experimental studies. Qualitative agreement with this approach is obtained by K-matrix unitarization of the two flavor linear sigma model [4] and three flavor linear sigma model [5] amplitudes.

Workers on scalar mesons entertain the hope that, after the revelations about the vacuum structure of QCD confirmed by the broken chiral symmetric treatment of the pseudoscalars, an understanding of the next layer of the "strong interaction onion" will be provided by studying the light scalars. An initial question is whether the light scalars belong to a flavor $SU(3)$ multiplet as the underlying quark structure might suggest. Apart from the $\sigma(560)$, the $f_0(980)$ and the isovector $a_0(980)$ are fairly well established. This leaves a gap concerning the four strange- so called kappa-states. This question is more controversial than that of the sigma state .

In the unitarized non linear chiral Lagrangian framework one must thus consider $pi - K$ scattering. In this case the low energy amplitude is taken[6] to correspond to the sum of a current algebra contact diagram, vector ρ and K^* exchange diagrams and scalar $\sigma(560)$, $f_0(980)$ and $\kappa(900)$ exchange diagrams. The situation in the interesting $I = 1/2$ s-wave channel turns out to be very analogous to the $I = 0$ channel of s-wave $\pi\pi$ scattering. Now a non Breit Wigner κ is required to restore unitarity; it plays the role of the $\sigma(560)$ in the $\pi\pi$ case. It was found that a satisfactory description of the 1-1.5 GeV s-wave region is also obtained by including the well known $K_0^*(1430)$ scalar resonance, which plays the role of the $f_0(980)$ in the $\pi\pi$ calculation. As in the case of the sigma, the light kappa seems hidden by background and does not jump right out of the initial analysis of the experimental data.

Thus the nine states associated with the $\sigma(560)$, $\kappa(900)$, $f_0(980)$ and $a_0(980)$ seem to be required in order to fit experiment in this chiral framework. What would their masses and coupling constants suggest about their quark substructure if they were assumed to comprise an SU(3) nonet [7]? Clearly the mass ordering of the various states is inverted compared to the "ideal mixing"[8] scenario which approximately holds for most meson nonets. This means that a quark structure for the putative scalar nonet of the form $N_a^b \sim q_a \bar{q}^b$ is unlikely since the mass ordering just corresponds to counting the number of heavier strange quarks. Then the nearly degenerate $f_0(980)$ and $a_0(980)$ which must have the structure $N_1^1 \pm N_2^2$ would be lightest rather than heaviest. However the inverted ordering will agree with this counting if we assume that the scalar mesons are schematically constructed as $N_a^b \sim T_a \bar{T}^b$ where $T_a \sim \epsilon_{acd} \bar{q}^c \bar{q}^d$ is a "dual" quark (or anti diquark). This interpretation is strengthened by consideration [7] of the scalars' coupling constants to two pseudoscalars. Those couplings depend on the value of a mixing angle, θ_s between N_3^3 and $(N_1^1 - N_2^2)/\sqrt{2})$. Fitting the coupling constants to the treatments of $\pi\pi$ and $K\pi$ scattering gives a mixing angle such that $\sigma \sim N_3^3 +$ "small"; $\sigma(560)$ is thus a predominantly non-strange particle in this picture. Furthermore the states $N_1^1 \pm N_2^2$ now would each predominantly contain two extra strange quarks and would be expected to be heaviest. Four quark pictures of various types have been sugggested as arising from spin-spin interactions in the MIT bag model[9], unitarized quark models[10] and meson-meson interaction models[11].

There seems to be another interesting twist to the story of the light scalars. The success of the phenomenological quark model suggests that there exists, in addition, a nonet of "conventional" p-wave $q\bar{q}$ scalars in the energy region above 1 GeV. The experimental candidates for these states are $a_0(1450)(I=1)$, $K_0^*(1430)(I=1/2)$ and for $I=0$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. These are enough for a full nonet plus a glueball. However it is puzzling that the strange $K_0^*(1430)$ isn't noticeably heavier than the non strange $a_0(1450)$ and that they are not lighter than the corresponding spin 2 states. These and another puzzle may be solved in a natural way[12] if the heavier p-wave scalar nonet mixes with a lighter $qq\bar{q}\bar{q}$ nonet of the type mentioned above. The mixing mechanism makes essential use of the "bare" lighter nonet having an inverted mass ordering while the heavier "bare" nonet has the normal ordering. A rather rich structure involving the light scalars seems to be emerging. At lower energies one may consider as a first approximation, "integrating out" the heavier nonet and retaining just the lighter one.

3. Effect of light scalars in $\eta \to 3\pi$

Historically, this isospin violating process has been important as a relatively clean test of the effective chiral Lagrangian approach and as a source of information on the quark mass difference $m_d - m_u$. As for the experimental status, the shape of the Dalitz plot for the $\pi^+\pi^-\pi^0$ mode agrees with chiral models. The experimental width for this mode [13] is 267 ± 25 eV. On the other hand, the tree level chiral result (which might be expected to be accurate to within 25 per cent or so) is 106 eV while the one loop theoretical number [14] is 160 ± 50 eV. A correction due to final state interactions, but outside the chiral perturbation expansion, yields [15] 209 ± 20 eV,

In ref.[16] we studied the effects of light scalars at tree level for this still interesting reaction. The calculation used a chiral Lagrangian of pseudoscalars, vectors and scalars, with the scalar masses and coupling constants taken from ref.[7] mentioned above and from[17]. We included non minimal derivative terms of pseudoscalars but will only describe here the results for the minimal pseudoscalar model with scalars and neglect of vectors. The 16 Feynman diagrams are shown in Fig. 3 and Fig. 4. The "driving mechanism" for this decay is the non zero value of $m_d - m_u$ which results in isospin violating bilinear pseudoscalar terms $\eta, \eta' - \pi^0$, bilinear scalar terms $f_0.f_0' - a_0^0$ as well as the quadrilinear term in (a) of Fig.3.

The pseudoscalar diagrams in Fig. 3 (with the minimal chiral Lagrangian) predict a width of 106 eV for the $\pi^+\pi^-\pi^0$ mode. A priori one might expect important contributions from the sigma exchange diagrams in (a),(b) and (c) of Fig. 4, since the $\sigma(560)$ propagators will not be too

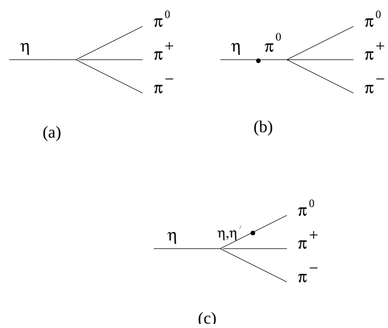

Figure 3. Feynman diagrams representing the pseudoscalar meson contribution to the decay $\eta \rightarrow \pi^+\pi^-\pi^0$.

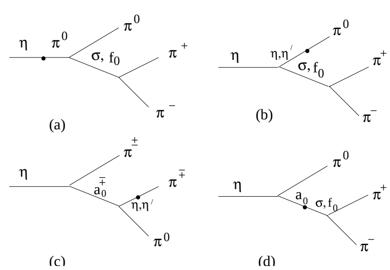

Figure 4. Feynman diagrams representing the scalar meson contributions to the decay $\eta \rightarrow \pi^+\pi^-\pi^0$.

far from their mass shells. However the signs are such that these three leading scalar contributions almost cancel each other. The net result is a total increase of the tree width by about 13 per cent to about 120 eV. This is based on the fits we obtained for the coupling constants which appear. As discussed in section IV of [16] there may be some room to increase this, owing to the delicate partial cancellations. The result is in the right direction but does illustrate that some care is needed in treating this simple looking reaction.

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