6-7-2003

Radiative Decays Involving a0(980) and f0(980) in the Vector Meson Dominance Model

Joseph Schechter
Department of Physics, Syracuse University, Syracuse, NY

Deirdre Black
Jefferson Lab

Masayasu Harada
Nagoya University

DPNU-03-16

Radiative Decays involving $a_0(980)$ and $f_0(980)$ in the Vector Meson Dominance Model *)

Deirdre Black[(a)], Masayasu Harada[(b)] and Joseph Schechter[(c)]

[(a)] *Theory Group, Jefferson Lab, 12000 Jefferson Ave., Newport News, VA 23606, USA*

[(b)] *Department of Physics, Nagoya University, Nagoya, 464-8602, Japan*

[(c)] *Department of Physics, Syracuse University Syracuse, NY 13244-1130, USA*

(Received February 1, 2008)

We summarize some features of the vector meson dominance model which was recently proposed for studying radiative decays involving the scalar mesons. Using the experimental values of $\Gamma(a_0 \to \gamma\gamma)$, $\Gamma(f_0 \to \gamma\gamma)$ and $\Gamma(\phi \to a_0\gamma)$ as inputs, we show that the model predicts a large hierarchy between $\Gamma(a_0 \to \omega\gamma)$ and $\Gamma(a_0 \to \rho\gamma)$ as well as between $\Gamma(f_0 \to \omega\gamma)$ and $\Gamma(f_0 \to \rho\gamma)$.

§1. Introduction

According to the recent theoretical and experimental analysis, there is a possibility that nine light scalar mesons exist below 1 GeV, and they form a scalar nonet.[1)] In addition to the well established $f_0(980)$ and $a_0(980)$ evidence of both experimental and theoretical nature for a very broad σ ($\simeq$ 560) and a very broad κ ($\simeq$ 900) has been presented. However, the properties of the nonet members such as quark structure and interactions with other mesons are not well known. It is interesting to study the properties of these light scalar mesons, which would be of great importance for a full understanding of QCD in its nonperturbative low energy regime.

In particular, the reactions $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ have recently been observed[2)] with good accuracy and are considered as useful probes of scalar properties. The theoretical analysis was initiated by Achasov and Ivanchenko[3)] and followed up by many others.[4)] The models employed are essentially variants of the single K meson loop diagram to which a ϕ-type vector meson, a photon and two pseudoscalars or a scalar are attached.

In this paper we summarize some features of the vector meson dominance model which was recently proposed in Ref. 5) for studying radiative decays involving the scalar mesons. Our model predicts that a large hierarchy between $\Gamma(a_0 \to \omega\gamma)$ and $\Gamma(a_0 \to \rho\gamma)$ as well as between $\Gamma(f_0 \to \omega\gamma)$ and $\Gamma(f_0 \to \rho\gamma)$.

*) Talk given by M. Harada at International Symposium on Hadron Spectroscopy, Chiral Symmetry and Relativistic Description of Bound Systems, February 24-26, 2003, Nihon University, Tokyo, Japan.

§2. Vector Meson Dominance Model

Framework of the model proposed in Ref. 5) is that of a standard non-linear chiral Lagrangian containing, in addition to the pseudoscalar nonet matrix field ϕ, the vector meson nonet matrix ρ_μ and a scalar nonet matrix field denoted N. Under chiral unitary transformations of the three light quarks; $q_{\rm L,R} \to U_{\rm L,R} \cdot q_{\rm L,R}$, the chiral matrix $U = \exp(2i\phi/F_\pi)$, where $F_\pi \simeq 0.131\,{\rm GeV}$, transforms as $U \to U_{\rm L} \cdot U \cdot U_{\rm R}^\dagger$. The convenient matrix $K(U_{\rm L}, U_{\rm R}, \phi)$[6] is defined by the following transformation property of ξ ($U = \xi^2$): $\xi \to U_{\rm L} \cdot \xi \cdot K^\dagger = K \cdot \xi \cdot U_{\rm R}^\dagger$, and specifies the transformations of "constituent-type" objects. The fields we need transform as

$$\begin{aligned}
& N \to K \cdot N \cdot K^\dagger \ , \\
& \rho_\mu \to K \cdot \rho_\mu \cdot K^\dagger + \frac{i}{\tilde{g}} K \cdot \partial_\mu K^\dagger \ , \\
& F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}\,[\rho_\mu \, , \, \rho_\nu] \to K \cdot F_{\mu\nu} \cdot K^\dagger \ ,
\end{aligned} \tag{2·1}$$

where the coupling constant $\tilde{g}$ is about 4.04. One may refer to Ref. 7) for our treatment of the pseudoscalar-vector Lagrangian and to Ref. 8) for the scalar addition. The entire Lagrangian is chiral invariant (modulo the quark mass term induced symmetry breaking pieces) and, when electromagnetism is added, gauge invariant.

In Ref. 5), the strong trilinear scalar-vector-vector terms were included into the effective Lagrangian as

$$\begin{aligned}
\mathcal{L}_{SVV} = & \beta_A \, \epsilon_{abc} \epsilon^{a'b'c'} \, [F_{\mu\nu}(\rho)]_{a'}^a \, [F_{\mu\nu}(\rho)]_{b'}^b \, N_{c'}^c \\
& + \beta_B \, {\rm Tr}\,[N] \, {\rm Tr}\,[F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)] \\
& + \beta_C \, {\rm Tr}\,[N F_{\mu\nu}(\rho)] \, {\rm Tr}\,[F_{\mu\nu}(\rho)] \\
& + \beta_D \, {\rm Tr}\,[N] \, {\rm Tr}\,[F_{\mu\nu}(\rho)] \, {\rm Tr}\,[F_{\mu\nu}(\rho)] \ .
\end{aligned} \tag{2·2}$$

Chiral invariance is evident from (2·1) and the four flavor-invariants are needed for generality. (A term $\sim {\rm Tr}(FFN)$ is linearly dependent on the four shown). Actually the β_D term will not contribute in our model so there are only three relevant parameters β_A, β_B and β_C. Equation (2·2) is analogous to the PVV interaction *) which was originally introduced as a $\pi\rho\omega$ coupling a long time ago.[10] One can now compute the amplitudes for $S \to \gamma\gamma$ and $V \to S\gamma$ according to the diagrams of Fig. 1.

The decay matrix element for $S \to \gamma\gamma$ is written as $(e^2/\tilde{g}^2) X_S \times \left(k_1 \cdot k_2 \, \epsilon_1 \cdot \epsilon_2 - k_1 \cdot \epsilon_2 \, k_2 \cdot \epsilon_1\right)$ where ϵ_μ stands for the photon polarization vector. It is related to the width by

$$\Gamma\,(S \to \gamma\gamma) = \alpha^2 \frac{\pi}{4} m_S^3 \left| \frac{X_S}{\tilde{g}^2} \right|^2 \ , \tag{2·3}$$

*) It was shown[9] that the complete vector meson dominance (VMD) is violated in the $\omega \to \pi^0\pi^+\pi^-$ decay which is expressed by PVV interactions. However, since the VMD is satisfied in other processes such $\pi^0 \to \gamma\gamma^*$ as well as in the electromagnetic form factor of pion, we here assume that it holds in the processes related to SVV interactions.

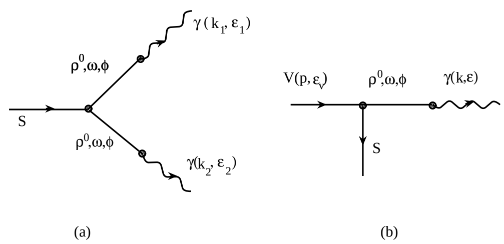

Fig. 1. Feynman diagrams for (a) $S \to \gamma\gamma$ and (b) $V \to S\gamma$.

and X_S takes on the specific forms:

$$\begin{aligned}
X_\sigma &= \frac{4}{9}\beta_A\left(\sqrt{2}s - 4c\right) + \frac{8}{3}\beta_B\left(c - \sqrt{2}s\right) \ , \\
X_{f_0} &= -\frac{4}{9}\beta_A\left(\sqrt{2}c + 4s\right) + \frac{8}{3}\beta_B\left(\sqrt{2}c + s\right) \ , \\
X_{a_0} &= \frac{4\sqrt{2}}{3}\beta_A \ .
\end{aligned} \tag{2·4}$$

Here $\alpha = e^2/(4\pi)$, $s = \sin\theta_S$ and $c = \cos\theta_S$ where the scalar mixing angle, θ_S is defined from[5)]

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} N_3^3 \\ (N_1^1 + N_2^2)/\sqrt{2} \end{pmatrix} \ . \tag{2·5}$$

Furthermore ideal mixing for the vectors, with $\rho^0 = (\rho_1^1 - \rho_2^2)/\sqrt{2}$, $\omega = (\rho_1^1 + \rho_2^2)/\sqrt{2}$, $\phi = \rho_3^3$, was assumed for simplicity.

Similarly, the decay matrix element for $V \to S\gamma$ is written as $(e/\tilde{g})C_V^S \times [p\cdot k\epsilon_V\cdot\epsilon - p\cdot\epsilon k\cdot\epsilon_V]$. It is related to the width by

$$\Gamma(V \to S\gamma) = \frac{\alpha}{3}\left|k_V^S\right|^3\left|\frac{C_V^S}{\tilde{g}}\right|^2 \ , \tag{2·6}$$

where $k_V^S = (m_V^2 - m_S^2)/(2m_V)$ is the photon momentum in the V rest frame. For the energetically allowed $V \to S\gamma$ processes we have

$$\begin{aligned}
C_\phi^{f_0} &= \frac{2\sqrt{2}}{3}\beta_A c - \frac{4}{3}\beta_B\left(\sqrt{2}c + s\right) + \frac{\sqrt{2}}{3}\beta_C\left(c - \sqrt{2}s\right) \ , \\
C_\phi^{\sigma} &= -\frac{2\sqrt{2}}{3}\beta_A s - \frac{4}{3}\beta_B\left(c - \sqrt{2}s\right) - \frac{2}{3}\beta_C\left(c + \frac{1}{\sqrt{2}}s\right) \ , \\
C_\phi^{a_0} &= \sqrt{2}\left(\beta_C - 2\beta_A\right) \ , \\
C_\omega^{\sigma} &= \frac{2\sqrt{2}}{3}\beta_A\left(c + \sqrt{2}s\right) + \frac{2\sqrt{2}}{3}\beta_B\left(c - \sqrt{2}s\right) - \frac{2}{3}\beta_C\left(\sqrt{2}c + s\right), \\
C_{\rho^0}^{\sigma} &= -2\sqrt{2}\beta_A c + 2\sqrt{2}\beta_B\left(c - \sqrt{2}s\right).
\end{aligned} \tag{2·7}$$

In addition, the same model predicts amplitudes for the energetically allowed $S \to V\gamma$ processes: $f_0 \to \omega\gamma$, $f_0 \to \rho^0\gamma$, $a_0^0 \to \omega\gamma$, $a_0^0 \to \rho^0\gamma$ and, if κ^0 is sufficiently heavy $\kappa^0 \to K^{*0}\gamma$. The corresponding width is

$$\Gamma(S \to V\gamma) = \alpha \left|k_S^V\right|^3 \left|\frac{D_S^V}{\tilde{g}}\right|^2 \ , \tag{2·8}$$

where $k_S^V = (m_S^2 - m_V^2)/(2m_S)$ and

$$\begin{aligned}
D_{f_0}^{\omega} &= \frac{2}{3}\beta_A\left(-2c+\sqrt{2}s\right)+\frac{2}{3}\beta_B\left(2c+\sqrt{2}s\right)+\frac{2}{3}\beta_C\left(c-\sqrt{2}s\right) \ , \\
D_{f_0}^{\rho^0} &= -2\sqrt{2}\beta_A s + 2\beta_B\left(2c+\sqrt{2}s\right) \ , \\
D_{a_0}^{\omega} &= 2\beta_C \ , \\
D_{a_0}^{\rho^0} &= \frac{4}{3}\beta_A \ , \\
D_{\kappa^0}^{K^{*0}} &= -\frac{8}{3}\beta_A \ .
\end{aligned} \tag{2·9}$$

§3. Results

Let us summarize main points of the results obtained in Ref. 5) together with new results from our recent analysis.[11)]

We should stress again that all the different decay amplitudes are described by the parameters β_A, β_B and β_C. Below we shall first illustrate the procedure for the model of a single putative scalar nonet.[8)]

We determine the value of β_A from the $a_0 \to \gamma\gamma$ process. Substituting $\Gamma_{\rm exp}(a_0 \to \gamma\gamma) = (0.28 \pm 0.09)\,\text{keV}$ (obtained using[12)] $B(a_0 \to K\bar{K})/B(a_0 \to \eta\pi) = 0.177 \pm 0.024$) into Eqs. (2·3) and (2·4) yields

$$\beta_A = (0.72 \pm 0.12)\,\text{GeV} \ , \tag{3·1}$$

where we assumed positive in sign. By using this value, the value of β_C is determined from $\Gamma_{\rm exp}(\phi \to a_0\gamma) = (0.47 \pm 0.07)\,\text{keV}$ (obtained by assuming $\phi \to \eta\pi^0\gamma$ is dominated by $\phi \to a_0\gamma$) and Eq. (2·7) as

$$\beta_C = (7.7 \pm 0.5\,,\ -4.8 \pm 0.5)\,\text{GeV}^{-1} \ . \tag{3·2}$$

We stress that the values of β_A and β_C obtained above are independent of the mixing angle θ_S. It should be noticed that $|\beta_A|$ is almost an order of magnitude smaller than $|\beta_C|$. As we can see from Eq. (2·9), the amplitude $D_{a_0}^{\omega}$ is given by β_C while $D_{a_0}^{\rho^0}$ is given by only β_A. Then, the large hierarchy between β_C and β_A implies that there is a large hierarchy between $\Gamma(a_0 \to \omega\gamma)$ and $\Gamma(a_0 \to \rho\gamma)$. Actually, by using the values of β_A and β_C given in Eqs. (3·1) and (3·2), they are estimated as

$$\begin{aligned}
\Gamma(a_0 \to \omega\gamma) &= (641 \pm 87\,,\ 251 \pm 54)\ \text{keV} \ , \\
\Gamma(a_0 \to \rho\gamma) &= 3.0 \pm 1.0\,\text{keV} \ .
\end{aligned} \tag{3·3}$$

This implies that there is a large hierarchy between $\Gamma(a_0 \to \omega\gamma)$ and $\Gamma(a_0 \to \rho\gamma)$ which is caused by an order of magnitude difference between $|\beta_C|$ and $|\beta_A|$.

We next determine the value of β_B from the $f_0 \to \gamma\gamma$ process. X_{f_0} in Eq. (2·4) depends on β_B as well as on β_A and the mixing angle θ_S. Here we take the mixing angle as

$$\theta_S \simeq -20^\circ \ , \tag{3·4}$$

which is characteristic of $qq\bar{q}\bar{q}$ type scalars.[8)] By using this and the value of β_A in Eq. (3·1), $\varGamma_{\rm exp}(f_0 \to \gamma\gamma) = 0.39 \pm 0.13\,\mathrm{keV}$ yields

$$\beta_B = (0.61 \pm 0.10\,,\ -0.62 \pm 0.10)\,\mathrm{GeV}^{-1} \ . \tag{3·5}$$

This implies that $|\beta_B|$ is on the order of $|\beta_A|$, and almost an order of magnitude smaller than $|\beta_C|$. Equation (2·9) shows that $D^{\omega}_{f_0}$ includes β_C while $D^{\rho}_{f_0}$ does not. Thus, we have a large hierarchy between decay widths of $f_0 \to \omega\gamma$ and $f_0 \to \rho\gamma$: The typical predictions are given by

$$\begin{aligned} \varGamma(f_0 \to \omega\gamma) &= (88 \pm 17)\,\mathrm{keV} \ , \\ \varGamma(f_0 \to \rho\gamma) &= (3.3 \pm 2.0)\,\mathrm{keV} \ . \end{aligned} \tag{3·6}$$

This implies that there is a large hierarchy between $\varGamma(f_0 \to \omega\gamma)$ and $\varGamma(f_0 \to \rho\gamma)$ which is caused by the fact that $|\beta_C|$ is an order of magnitude larger than $|\beta_A|$ and $|\beta_B|$.

Let us check the dependence of the above results on the choice of the scalar mixing angle θ_S. In Ref. 8), the value of $\theta_S \simeq -90^\circ$ was obtained as another solution to reproduce the masses of the lightest scalar nonet, although the predicted value of f_0-π-π coupling is much larger than the value obtained in Ref. 13) by fitting to the $\pi\pi$ scattering amplitude

As we stressed above, the values of β_A and β_C are independent of the scalar mixing angle θ_S. The value of β_B determined from $\varGamma(f_0 \to \gamma\gamma)$ becomes

$$\beta_B = (1.1 \pm 0.1\,,\ 0.12 \pm 0.13)\,\mathrm{GeV}^{-1} \ . \tag{3·7}$$

Then the typical predictions for $\varGamma(f_0 \to \omega\gamma)$ and $\varGamma(f_0 \to \rho\gamma)$ are given by

$$\begin{aligned} \varGamma(f_0 \to \omega\gamma) &= (86 \pm 16)\,\mathrm{keV} \ , \\ \varGamma(f_0 \to \rho\gamma) &= (3.4 \pm 3.2)\,\mathrm{keV} \ . \end{aligned} \tag{3·8}$$

These predictions are very close to the ones in Eq. (3·6). This can be understood by the following consideration: From the expression of $D^{\omega}_{f_0}$ in Eq. (2·9), we can see that it is dominated by the term including β_C which is proportional to $(\cos\theta_S - \sqrt{2}\sin\theta_S)$. Then, the approximate relation

$$\cos(-20^\circ) - \sqrt{2}\,\sin(-20^\circ) \simeq \cos(-90^\circ) - \sqrt{2}\,\sin(-90^\circ) \simeq 1.4 \tag{3·9}$$

implies that the value of $D^{\omega}_{f_0}$ for $\theta_S = -90^\circ$ is close to that for $\theta_S = -20^\circ$, and thus $\varGamma(f_0 \to \omega\gamma)$ for $\theta_S = -90^\circ$ to that for $\theta_S = -20^\circ$. As for $\varGamma(f_0 \to \rho\gamma)$ we note that the following relation is satisfied for X_{f_0} in Eq. (2·4) and $D^{\rho^0}_{f_0}$ in Eq. (2·9):

$$3X_{f_0} - 2\sqrt{2}D^{\rho^0}_{f_0} = -\frac{4}{3}\sqrt{2}\beta_A(c - \sqrt{2}s) \ . \tag{3·10}$$

Since we use the experimental value of $\Gamma(f_0 \to \gamma\gamma)$, i.e., X_{f_0} as an input, this relation implies that the predicted value of $\Gamma(f_0 \to \rho\gamma)$ for $\theta_S = -90^\circ$ is roughly equal to that for $\theta_S = -20^\circ$. From the above consideration we conclude that there is a large hierarchy between $\Gamma(f_0 \to \omega\gamma)$ and $\Gamma(f_0 \to \rho\gamma)$ for $\theta_S = -20^\circ$ and $\theta_S = -90^\circ$:

$$\Gamma(f_0 \to \omega\gamma) \gg \Gamma(f_0 \to \rho\gamma) \quad \text{for } \theta_S = -20^\circ \text{ and } \theta_S = -90^\circ \ . \tag{3·11}$$

§4. Discussion

In this paper we showed the predictions of our model only for $\Gamma(a_0 \to \omega\gamma)$, $\Gamma(a_0 \to \rho\gamma)$, $\Gamma(f_0 \to \omega\gamma)$ and $\Gamma(f_0 \to \rho\gamma)$. Predictions on several other processes such as $\Gamma(\sigma \to \gamma\gamma)$ and $\Gamma(\phi \to \sigma\gamma)$ can be seen in Ref. 5).

The value of $\Gamma(\phi \to f_0\gamma)$ predicted in Ref. 5) is considerably smaller than the experimental value of $\Gamma(\phi \to f_0\gamma)$. We may need to include the effect of K-loop which would give a large enhancement as shown in Ref. 3). We leave the analysis with K-loop corrections to future publications.[11)]

Acknowledgements

We are happy to thank N. N. Achasov for a suggested correction to the first version of Ref. 5) and with A. Abdel-Rahiem and A. H. Fariborz for very helpful discussions. D.B. wishes to acknowledge support from the Thomas Jefferson National Accelerator Facility operated by the Southeastern Universities Research Association (SURA) under DOE contract number DE-AC05-84ER40150. The work of J.S. is supported in part by DOE contract DE-FG-02-85ER40231.

References

1) See, e.g., the confereproceedings, S. Ishida et al, "Possible existence of the sigma moson and its implication to hadron physics", KEK Proceedings 2004, Soryushiron Kenkyu 102, No. 5, 2001; D. Amelin and A.M. Zaitsev, "Hadron spectroscopy", Ninth international conference on hadron spectroscopy, Protvino, Russia (2001).
2) M. N. Achasov *et al.* [SND Collaboration], Phys. Lett. B **479**, 53 (2000); R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **462**, 380 (1999); A. Aloisio *et al.* [KLOE Collaboration], arXiv:hep-ex/0107024.
3) N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315**, 465 (1989).
4) F. E. Close, N. Isgur and S. Kumano, Nucl. Phys. B **389**, 513 (1993); N. N. Achasov and V. V. Gubin, Phys. Rev. D **56**, 4084 (1997); Phys. Rev. D **57**, 1987 (1998); N. N. Achasov, V. V. Gubin and V. I. Shevchenko, Phys. Rev. D **56**, 203 (1997); J. L. Lucio Martinez and M. Napsuciale, Phys. Lett. B **454**, 365 (1999); arXiv:hep-ph/0001136; A. Bramon, R. Escribano, J. L. Lucio M., M. Napsuciale and G. Pancheri, Phys. Lett. B **494**, 221 (2000); F. E. Close and A. Kirk, Phys. Lett. B **515**, 13 (2001); N. N. Achasov, S. A. Devyanin and G. N. Shestakov, Phys. Lett. B **88**, 367 (1979); N. N. Achasov and A. V. Kiselev, Phys. Lett. B **534**, 83 (2002).
5) D. Black, M. Harada and J. Schechter, Phys. Rev. Lett. **88**, 181603 (2002); D. Black, M. Harada and J. Schechter, arXiv:hep-ph/0303223.
6) C. G. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2247 (1969).
7) M. Harada and J. Schechter, Phys. Rev. D **54**, 3394 (1996).
8) D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. D **59**, 074026 (1999).
9) See, e.g., M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. **164**, 217 (1988); M. Harada and K. Yamawaki, to appear in Phys. Rept., arXiv:hep-ph/0302103.
10) M. Gell-Mann, D. Sharp and W. G. Wagner, Phys. Rev. Lett. **8**, 261 (1962). A general

formulation of the PVV interaction in a gauge invariant manner was given in: T. Fujiwara, T. Kugo, H. Terao, S. Uehara and K. Yamawaki, Prog. Theor. Phys. **73**, 926 (1985); O. Kaymakcalan, S. Rajeev and J. Schechter, Phys. Rev. D **30**, 594 (1984); O. Kaymakcalan and J. Schechter, Phys. Rev. D **31**, 1109 (1985); P. Jain, R. Johnson, U. G. Meissner, N. W. Park and J. Schechter, Phys. Rev. D **37**, 3252 (1988).

11) D. Black, M. Harada and J. Schechter, in preparation.
12) D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15**, 1 (2000) and 2001 partial update for edition 2002 (URL: http://pdg.lbl.gov).
13) M. Harada, F. Sannino and J. Schechter, Phys. Rev. D **54**, 1991 (1996).