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We study the dependence on the number of colors of the leading $\pi\pi$ scattering amplitude in chiral dynamics. We demonstrate the existence of a critical number of colors for and above which the low energy $\pi\pi$ scattering amplitude computed from the simple sum of the current algebra and vector meson terms is crossing symmetric and unitary at leading order in a truncated and regularized $1/N_c$ expansion. The critical number of colors turns out to be $N_c = 6$ and is insensitive to the explicit breaking of chiral symmetry. Below this critical value, an additional state is needed to enforce the unitarity bound; it is a broad one, most likely of "four quark" nature.

INTRODUCTION

Strong interactions are hard to tackle and different tools or limits in the parameter space of a given asymptotically free theory have been proposed. The large number of colors (N_c) limit is a relevant example [1, 2]. But how large should N_c be for the counting scheme to be not only qualitatively but also quantitatively interesting for QCD ? In this note we try to answer this question in terms of the nature of the low energy hadronic spectrum.

The physical situation where the large N_c approximation seems closest to direct experimental check is perhaps low energy $\pi\pi$ (or meson meson) scattering. Large N_c predicts the low energy amplitude to be a sum of tree diagrams – contact terms and resonance pole diagrams. With the two reasonable additions: truncation of number of resonances and some kind of regularization to give the resonances finite widths, this corresponds to the way in which the analyses of experimentally derived partial wave amplitudes are usually performed. We shall proceed in this manner with the additional requirement that all pre-regularized tree amplitudes be obtained from a chiral invariant (modulo usual symmetry breaking terms) Lagrangian.

The most crucial partial wave for our considerations turns out to be the scalar, isotopic spin zero channel. Recently there has been renewed interest [3]- [30] in the low energy behavior of this channel. Especially, many physicists now believe in the existence of the light, broad $I = J = 0$ resonance, sigma in the 500-600 MeV region. The sigma is also very likely the "tip of an iceberg" which may consist of a nonet of light four quark-type scalars [31] mixing [32] with another heavier $q\bar{q}$ - type scalar nonet as well as a glueball. However there has been some controversy as to the underlying nature of these low lying scalar objects. In the present context, it was argued [2] that a four quark state in the leading large N_c limit would choose to appear as two non-interacting $q\bar{q}$ states. The necessity of such a state to explain the scattering then suggests a critical value of N_c, below which a simple analysis of $\pi\pi$ scattering up to a given energy (taken as 1 GeV for definiteness) and based on the large N_c approximation is not accurate.

To obtain this critical value, we make a reanalysis of meson-meson scattering as a function of the number of colors. The N_c dependences can be easily incorporated in the parameters (such as the pion decay constant or the vector meson ρ coupling to two pions) of the theory and of the states whose large N_c counting is known. The criterion for an accurate description will be whether the unitarity bounds are satisfied up to the energy of at least 1 GeV.

A variety of different approaches to meson-meson scattering have been developed in the literature and all are able to provide a reasonable fit to the data. An approach to describing $\pi\pi$, $I = J = 0$ scattering based on a conventional non-linear chiral Lagrangian of pseudoscalar and vector fields augmented by scalar fields introduced in a chiral invariant manner was discussed in [9, 15] and will be adoped here. This will have an advantage, for our present purpose, of not forcing the amplitudes to be unitary. Thus unitarity can be considered as a criterion for an accurate theory. In the physical $N_c = 3$ case, the experimental data from threshold to a bit beyond 1 GeV can be fit by computing the tree amplitude from this Lagrangian and choosing parameters to yield an approximate unitarization. The following ingredients are present: i) the "current algebra" contact term, ii) the vector meson exchange terms, iii) the unitarized $\sigma(560)$ exchange terms and iv) the unitarized $f_0(980)$ exchange terms. Although the $\rho(770)$ vector meson certainly is a crucial feature of low energy physics, a fit can be made [16] in which the contribution ii) is absent. This results in a somewhat lighter and broader sigma meson, in agreement with other approaches which neglect the effect of the ρ meson. We note that the $\sigma(560)$ contribution is needed to keep the $I = J = 0$ scattering amplitude within the unitarity bounds [9]. In [15] it was noted that the $\sigma(560)$ contribution is also needed to provide the correct background which flips the sign of the $f_0(980)$ resonance (the Ramsauer-Townsend effect).

For the purpose of checking the above strong interaction calculation, the meson-meson scattering was also calculated in a general version of the *linear* SU(3) sigma model, which contains both $\sigma(560)$ and $f_0(980)$ candidates [30]. The procedure adopted was to calculate the tree amplitude and then to unitarize, without introducing any new parameters, by identifying the tree amplitude as the K-matrix amplitude. This also gave a reasonable fit to the data, including the characteristic Ramsauer-Townsend effect of the $f_0(980)$ resonance contribution. At a deeper and more realistic level of description in the linear sigma model framework, one expects two different chiral multiplets - $q\bar{q}$ as well as $qq\bar{q}\bar{q}$ - to mix with each other. A start on this model seems encouraging [33].

Although we shall adopt the non-linear realization approach [9, 15], our results are expected to be robust. We will also focus, for simplicity, on the SU(2) flavor limit of QCD. In section II we study the real parts of the $\pi\pi$ scattering amplitudes for the $I = J = 0$ and $I = J = 1$ channels as functions of the number of colors. The contributions of the current algebra and the vector meson ρ exchange pieces are both taken into account. We show that chiral symmetry, crossing symmetry and the unitarity bounds are all satisfied in the leading order in N_c for $N_c \geq 6$. However the unitarity bound will not hold for lower values of N_c. Section III is dedicated to the low energy scalar (the σ) which is needed to unitarize the R_0^0 amplitude for $N_c \leq 6$.

APPROACHING THE LARGE N_c LIMIT IN $\pi\pi$ SCATTERING

We first analyze the $I = J = 0$ pion-pion scattering amplitude as a function of the number of colors. In terms of the conventional amplitude, $A(s,t,u)$ the $I = 0$ amplitude is $3A(s,t,u) + A(t,s,u) + A(u,t,s)$. One gets the $J = 0$ state by projecting out the correct partial wave. We focus, following [9, 15], on the real part of the scattering amplitude since the imaginary part can be determined, using the unitarity relation, from the real part. Of course, unitarity requires that the real part of the elastic scattering amplitude itself must satisfy the constraint $|R_J^I| \leq 1/2$. This condition is crucial to obtain the critical number of colors in the following analysis.

The current algebra contribution to the conventional amplitude is

$$A_{ca}(s,t,u) = 2\frac{s - m_\pi^2}{F_\pi^2} \ , \tag{1}$$

where the pion decay constant, F_π depends on N_c as $F_\pi(N_c) = 131\sqrt{N_c}/\sqrt{3}$ so that $F_\pi(3) = 131$ MeV. Furthermore $m_\pi = 137$ MeV is independent of N_c. In Fig. 1 we plot the current algebra contribution to the real part of the $I = J = 0$ partial wave amplitude, R_0^0 for increasing values of the number of colors N_c. We observe that the unitarity bound is violated for a value of s_{ca}^* which increases linearly with N_c. Thus the net effect of

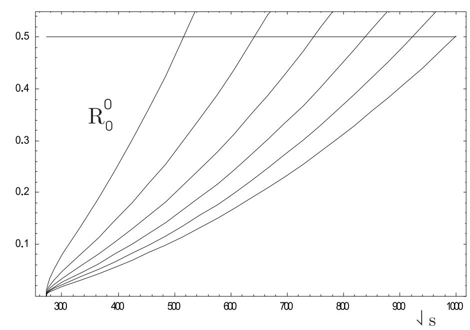

FIG. 1: Real part of the $I = J = 0$ partial wave amplitude due to the current algebra term plotted for the following increasing values of N_c (from left to right), $3, 5, 7, 9, 11, 13$.

increasing N_c is to delay the onset of the unitarity bound violation. For $N_c = 13$ the unitarity bound is satisfied up to about 1 GeV. Thus the current algebra contribution alone indicates that N_c should be of the order 13 to lead to a unitary chiral theory.

We now demonstrate that this result is strongly modified by the presence of the well established $q\bar{q}$ companion of the pion – the ρ vector meson. The amplitude we consider now is obtained by adding to the current algebra contribution the following vector meson $\rho(770)$ contribution:

$$\begin{aligned} A_\rho(s,t,u) &= \frac{g_{\rho\pi\pi}^2}{2m_\rho^2}(4m_\pi^2 - 3s) \\ &- \frac{g_{\rho\pi\pi}^2}{2}\left[\frac{u-s}{(m_\rho^2 - t) - im_\rho\Gamma_\rho\theta(t - 4m_\pi^2)}\right. \\ &+ \left.\frac{t-s}{(m_\rho^2 - u) - im_\rho\Gamma_\rho\theta(u - 4m_\pi^2)}\right] \ , \end{aligned} \tag{2}$$

where $g_{\rho\pi\pi}(N_c) = 8.56\sqrt{3}/\sqrt{N_c}$ is the $\rho\pi\pi$ coupling constant. Also, $m_\rho = 771$ MeV is independent of N_c and

$$\Gamma_\rho(N_c) = \frac{g^2_{\rho\pi\pi}(N_c)}{12\pi\, m_\rho^2}\left(\frac{m_\rho^2}{4} - m_\pi^2\right)^{\frac{3}{2}} \quad . \tag{3}$$

It should be noted that the first term in Eq. (2), which implies the existence of an additional non-resonant contact interaction other than the current algebra contribution, is required by the chiral symmetry when we include the ρ vector meson contribution in a chiral invariant manner. Although the vector meson contribution to the $I = J = 0$ partial wave amplitude is due only to the crossed channel, as noted in Ref. [9], adding the $\rho(770)$ for the 3 color case greatly decreases the amount of unitarity violation. However for three colors this contribution is not sufficient to keep the scattering amplitude completely within the unitarity bounds and a broad state (the sigma) is needed [9].

Here we follow a different route and investigate the unitarity bound restoration –in the absence of the sigma– as a function of N_c. We compute analytically the relevant partial wave projections. In Fig. 2 we plot the real part of the $I = J = 0$ amplitude (i.e. R^0_0) due to current algebra plus the ρ contribution for increasing values of N_c. Since in this channel the vector meson is never on shell we suppress the contribution of the width in the vector meson propagator in Eq. (2). We observe that the unitarity bound (i.e., $|R^0_0| \leq 1/2$) is satisfied for $N_c \geq 6$ till well beyond the 1 GeV region. However unitarity is still a problem for $3, 4$ and 5 colors. At energy scales larger than the one associated with the vector meson clearly other resonances are needed [9] but we shall not be concerned about those here. It is also interesting to note that these considerations are essentially unchanged when the pion mass (i.e. explicit chiral symmetry breaking in the Lagrangian) is set to zero. We also note that the curves in Fig. 2 are multiples of each other, so all of the extrema coincide as well as the points where the curves cross zero. This is so since the N_c dependent scattering amplitude which is the sum of the current algebra and vector meson contributions obeys, by construction:

$$A(s,t,u) = \frac{3}{N_c}\widetilde{A}(s,t,u) \ . \tag{4}$$

where $\widetilde{A}(s,t,u)$ is defined replacing F_π and $g_{\rho\pi\pi}$ with the N_c independent quantities $\widetilde{F}_\pi = F_\pi\sqrt{3}/\sqrt{N_c}$ and $\widetilde{g}_{\rho\pi\pi} = g_{\rho\pi\pi}\sqrt{N_c}/\sqrt{3}$ [38].

One might be worried that the critical value, $N_c = 6$ determined by studying the $I = J = 0$ channel would not hold for other channels. This is not so since $I = J = 0$ is actually the worst channel with respect to unitarity bound violation at the low energies of present interest. Indeed channels with larger J have angular momentum suppression factors which delay the onset of such a violation. Let us consider as a relevant example the $I = J = 1$

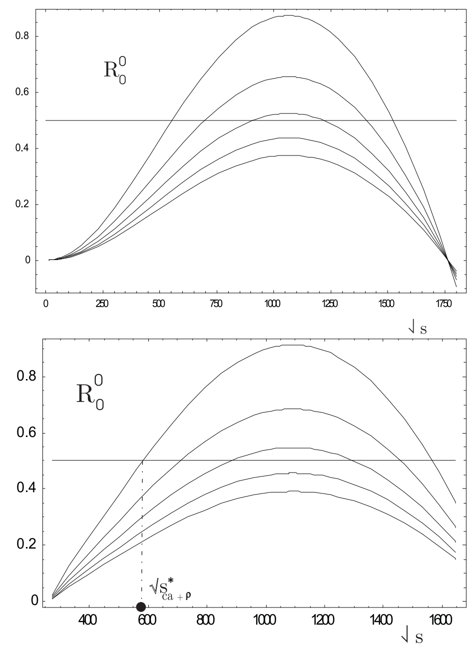

FIG. 2: Upper Panel: Real part of the $I = J = 0$ partial wave amplitude due to the current algebra $+\rho$ terms, plotted for the following increasing values of N_c (from up to down), $3, 4, 5, 6, 7$. The curve with largest violation of the unitarity bound corresponds to $N_c = 3$ while the ones within the unitarity bound are for $N_c = 6, 7$. Here we have set $m_\pi = 0$. Lower Panel: Same plot for the physical value of the pion mass.

channel. Figure 3 shows that, even including just the current algebra term, the unitarity bound is not an issue for 3 colors till energies well above the ρ mass. The com-

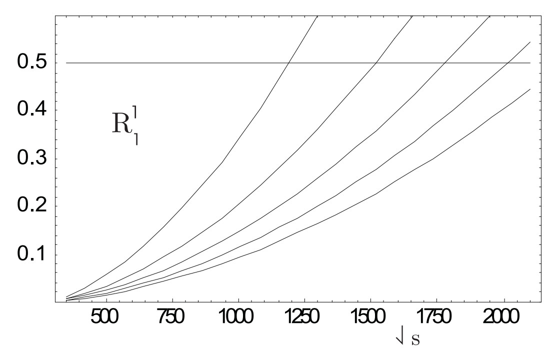

FIG. 3: Real part of the $I = J = 1$ partial wave amplitude due to current algebra plotted for the following increasing values of N_c (from left to right), $3, 5, 7, 9, 11$. The curve which first violates the unitarity bound corresponds to $N_c = 3$.

bined contributions of the vector meson ρ term and the current algebra term in this channel as a function of the number of colors is shown in Fig. 4. For this channel the non zero ρ width has been of course taken into acount. The already narrow ρ becomes more and more narrow as we increase N_c while the N_c effects are not very relevant concerning unitarity bounds. Near the vector resonance we observe practically a Breit-Wigner consistent with the well known cancellation between the current algebra contribution and part of the non resonant vector meson contribution in chiral models.

It is amusing that the vector meson plays a major role in helping satisfy the unitarity bound for large N_c not in the direct channel but in the crossed one. It is also interesting that the critical number of colors emerges

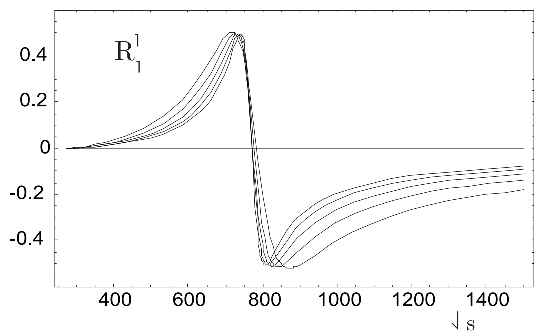

FIG. 4: Real part of the $I = J = 1$ partial wave amplitude due to the current algebra $+\rho$ terms, plotted for the following increasing values of N_c, $3, 4, 5, 6, 7$. $N_c = 3$ is the curve with the broadest ρ meson.

from the relation between $g_{\rho\pi\pi}$, F_π and m_ρ while explicit chiral symmetry breaking has been seen to play a negligible role. Notice that in the most crucial $I = J = 0$ channel the vector meson width can be deleted to good accuracy and plays no role in satisfying the unitarity bound. Thus apart from the truncation of higher mass resonant states the calculation of this partial wave is exactly that of the leading large N_c approximation. We also note that the $I = 2, J = 0$ partial wave amplitude computed in this way obeys the unitarity bound to well above 1 GeV [34].

We have thus demonstrated that for $N_c \geq 6$ and two flavor QCD the low energy theory is crossing symmetric and unitary up to well beyond the one GeV region considering just the pion and the ρ vector meson. This supports the usual expectation that when N_c is large *enough* the meson-meson scattering theory is well represented by including just the relevant $q\bar{q}$ resonances at the tree level.

INCLUDING THE SIGMA

In order to explain low energy $\pi\pi$ scattering for the physical value $N_c = 3$ (and for the cases $N_c = 4, 5$) using the attractive notion of tree diagram dominance involving near by resonances, it thus seems necessary to include a scalar singlet resonance like the light sigma which is likely to be of "four quark" type and hence not to contribute in the leading large N_c approximation. As mentioned before, many models [3]-[30] have recently been proposed which successfully employ the sigma in one way or another. Some discussions of N_c dependence have also been very recently given [35].

Using the approach of Refs. [9, 15] it is easy to make a quick estimate of the mass of the needed sigma particle. A unitarization procedure is needed when the current algebra plus the ρ amplitude first starts violating the unitarity bound. Denote this point as s^*. The σ pole structure is such that the real part of its amplitude is positive for $s < M_\sigma^2$ and negative for $s > M_\sigma^2$. Identifying the squared sigma mass roughly with s^* will then give a negative contribution where the real part of the amplitude exceeds $+1/2$. In the case when only the current algebra term is included we get

$$M_\sigma^2 \approx s^*_{ca} = 4\pi\, F_\pi^2\ . \tag{5}$$

This shows that the squared mass of the sigma meson needed to restore unitarity for $N_c = 3, 4, 5$ increases roughly linearly with N_c. This estimate gets modified a bit when we include the vector meson (see Fig. 5), yielding $M_\sigma^2 \approx s^*_{ca+\rho}$, where $s^*_{ca+\rho}$ is to be obtained from Fig. 2. In the case of three colors we then get

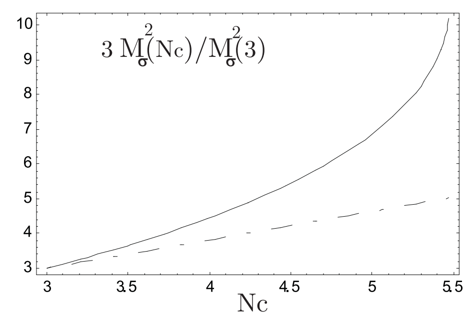

FIG. 5: The value $M^2(N_c)$ for which unitarity bounds are first violated as function of N_c and normalized to $M^2(3)/3$. The dashed line corresponds to the pure current algebra contribution while the solid line to current algebra $+\rho$ contribution.

$M_\sigma \approx 580$ MeV which is close to the best fit to the $\pi\pi$ scattering data determined in Refs. [9, 15].

Finally, we briefly discuss the possible four quark nature of the sigma. Most naively, the needed particle is rather light while a usual constituent quark $q\bar{q}$ bound state should be a somewhat heavy p-wave state. Of course we have been working in an effective Lagrangian framework which does not directly tell us anything about the inner structure of the sigma meson. Indirectly one can examine the three flavor generalization and observe that the lightest nine scalars found in the present manner have the right flavor quantum numbers to make up a usual nonet. However the characteristics of this nonet, such as usual vs. inverted mass ordering and the value of the mixing angle between the two isoscalars, turn out [36] to favor a four quark interpretation. Still the four quarks may be of $qq\bar{q}\bar{q}$ type [31], meson-meson molecule type [37] or any linear combination. For our present purpose it is enough to know that they are not of $q\bar{q}$ type.

CONCLUSIONS

We studied the dependence on number of colors of $\pi\pi$ scattering. We showed the existence of a critical number of colors for and above which, chiral theory is unitary at leading order in the $1/N_c$ expansion to beyond the one GeV range. The critical number of colors is $N_c = 6$ and is insensitive to the explicit breaking of chiral symmetry. For $N_c \geq 6$ the $q\bar{q}$ resonances are sufficient to keep the low energy theory unitary. Below this critical value the σ meson is needed to keep the amplitude within the unitarity bounds. This is a light, broad object which is unlikely to be of $q\bar{q}$ nature.

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