6-22-2004

Study of Leptonic CP Violation

Joseph Schechter
Syracuse University

Salah Nasri
University of Maryland - College Park

Sherif Moussa
Ain Shams University

STUDY OF LEPTONIC CP VIOLATION

S. NASRI[A], J. SCHECHTER[B]*AND S. MOUSSA[C]

(A) Department of Physics,
University of Maryland,
College Park, MD 20742-4111, USA.
E-mail: snasri@physics.umd.edu

(B) Physics Department,
Syracuse University,
Syracuse, NY 13244-1130, USA.
E-mail: schechte@phy.syr.edu

(C) Department of Mathematics,
Faculty of Science,
Ain Shams University, Egypt.
E-mail: sherif@asunet.shams.eun.eg

The "complementary" Ansatz, Tr(M_ν)=0, where M_ν is the prediagonal neutrino mass matrix, seems a plausible approximation for capturing in a self contained way some of the content of Grand Unification. We study its consequences in the form of relations between the neutrino masses and CP violation phases.

1. Introduction

A favorite topic of discussion at the MRST meetings over many years has been the prediction of quark and lepton masses and mixings. Usually, an attempt is made to predict everything at once based on a suitable guess (Ansatz). Here we discuss a "complementary" Ansatz which adds just a little information to the system. This is [1,2,3] , for the symmetric prediagonal neutrino mass matrix, M_ν:

$$Tr(M_\nu) = 0 \tag{1}$$

*Speaker

If, at first, leptonic CP violation is neglected so that M_ν is diagonalized by a real orthogonal matrix, Eq. (1) yields simply

$$m_1 + m_2 + m_3 = 0, \tag{2}$$

where the neutrino masses, m_i can be taken here to be either positive or negative. Now a very recent analysis [4] of solar, atmospheric, reactor and accelerator neutrino oscillation data (but neglecting LSND) gives the best fit:

$$\begin{aligned} m_2^2 - m_1^2 &= 6.9 \times 10^{-5} eV^2, \\ |m_3^2 - m_2^2| &= 2.6 \times 10^{-3} eV^2. \end{aligned} \tag{3}$$

Together, Eqs. (2) and (3) provide three equations for three unknowns. There are two essentially different types of solutions. Type I is characterized by $|m_3|$ being largest:

$$m_1 = 0.0291 \text{ eV}, \quad m_2 = 0.0302 \text{ eV}, \quad m_3 = -0.0593 \text{ eV}, \tag{4}$$

while type II has $|m_3|$ smallest:

$$m_1 = 0.0503 \text{ eV}, \quad m_2 = -0.0510 \text{ eV}, \quad m_3 = 0.00068 \text{ eV}. \tag{5}$$

Here we will, following ref.[5] (see this for further references), discuss the situation when CP violation effects are included and give an application to leptogenesis.

2. Plausibility argument for Ansatz

SO(10) grand unified theories have the nice feature that they contain a complete fermion generation in a single irreducible representation of the group. There are three Higgs irreducible representations which can directly contribute to tree level fermion masses via the Yukawa sector: the **10**, the **120** and the **126**. In principle any number of each is allowed. For every Higgs field there is a 3 ×3 matrix of unknown coupling constants. The fermion mass matrices are linear combinations of these matrices. Clearly a very large number of different models can be envisioned.

We start from the "kinematical" relation:

$$Tr(M_{-1/3} - rM_{-1}) \propto Tr(M_{0,LIGHT}), \tag{6}$$

which holds when any number of **10**'s and **120**'s are present but only a single **126**. Here the subscript on the mass matrix indicates the electric charge of the fermion. The quantity $r \approx 3$ takes account of running masses from the GUT scale to about 1 GeV. Under the same conditions one also has:

$$M_{0,HEAVY} \propto M_{0,LIGHT}. \tag{7}$$

Note that the physical light neutrino mass matrix, M_ν is given by the well known formula:

$$M_\nu \approx M_{0,LIGHT} - M_{DIRAC}^T M_{0,HEAVY}^{-1} M_{DIRAC}. \tag{8}$$

The initial assumption we shall make is that the second, "see-saw" term in Eq. (8) is small compared to $M_{0,LIGHT}$.

Now, it has been known for a long time that the quark mixing matrix is of the form $diag(1,1,1) + O(\epsilon)$. Thus it was very surprising when analysis of neutrino oscillation observations showed that the lepton mixing matrix is not at all close to the unit matrix but rather has large (12) and (23) mixing elements. In a GUT framework, this suggests that a first approximation to the prediagonal mass matrices might be to take the charged fermion mass matrices to be diagonal while the neutrino mass matrix would presumably differ drastically from the diagonal form. As examples we would set $M_{-1} \approx diag(m_e, m_\mu, m_\tau)$ and $M_{-1/3} \approx diag(m_d, m_s, m_b)$. Substituting these into the left hand side of Eq. (6) shows it to be about $(m_b - 3m_\tau)$, which is about zero. Hence the right hand side should also be about zero as should $Tr(M_\nu)$ in the non-seesaw dominance case. Although this is clearly an approximation, it seems likely to be close to the physical situation in the same sense as $m_b \approx 3m_\tau$. Of course, the approximation gets better as the mixing matrices needed to bi-diagonalize the charged lepton mass matrix get closer to the unit matrix. For simplicity, in what follows we shall also approximate these matrices to equal the unit matrix.

3. Parameterized Ansatz equation

The prediagonal neutrino mass matrix may be brought to diagonal form by a transformation:

$$U^T M_\nu U = \hat{M}_\nu = diag(m_1, m_2, m_3), \tag{9}$$

where U is a unitary matrix. M_ν is a symmetric but complex matrix which has in general 12 real parameters. This equals the sum of the three parameters from m_i and the nine parameters from U. Now the observable lepton mixing matrix, K is given [1] by $K = \Omega^\dagger U$, where we have just agreed to approximate the charged lepton diagonalizing matrix factor, Ω to be essentially the unit matrix (or alternatively we could choose to work in a basis where the charged leptons are diagonal). Thus we replace U by the observable matrix K. K is parameterized in a conventional way as $K = K_{exp}\omega_0^{-1}(\tau)$, where $\omega_0(\tau) = diag(e^{i\tau_1}, e^{i\tau_2}, e^{i\tau_3})$ with $\tau_1 + \tau_2 + \tau_3 = 0$ and:

$$K_{exp} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix} \quad (10)$$

where $s_{ij} = sin\theta_{ij}\ and\ c_{ij} = cos\theta_{ij}$. As written, the matrix K is parameterized by three angles and three independent CP violating phases. To get the most general K three more phases are required; these can be inserted for example [1] by multiplying K on the left by $\omega_0(\sigma)$. However these phases can always be canceled by rephasing the (diagonal) charged lepton fields which sit to the left of K. Thus even if the phases, σ_i were included in U there would always be an allowed choice of charged lepton phases which would cancel their effect when we get restrictions (as we shall) on the physical K. Taking this into account the Ansatz reads in terms of physical quantities:

$$Tr(\hat{M}_\nu K_{exp}^{-1} K_{exp}^{*} \omega_0(2\tau)) = 0. \quad (11)$$

In more detail it is:

$$\begin{aligned} m_1 e^{2i\tau_1}\left[1 - 2i(c_{12}s_{13})^2 sin\delta e^{-i\delta}\right] + \\ m_2 e^{2i\tau_2}\left[1 - 2i(s_{12}s_{13})^2 sin\delta e^{-i\delta}\right] + \\ m_3 e^{2i\tau_3}\left[1 + 2i(s_{13})^2 sin\delta e^{i\delta}\right] = 0. \end{aligned} \quad (12)$$

In this equation we can choose the diagonal masses m_1, m_2, m_3 to be real positive. The mixing angles are known from the best fit [4],

$$s_{12}^2 = 0.30, \quad s_{23}^2 = 0.50, \quad s_{13}^2 = 0.003, \quad (13)$$

wherein the first two have about 25 per cent uncertainty while the third is just known to be small.

Together Eqs. (3) and (12) are now seen to provide 4 real equations for the 6 unknowns: $m_1, m_2, m_3, \delta, \tau_1, \tau_2$. Thus further assumptions are needed to make some predictions. We already saw that assuming all the CP phases to vanish gives three equations for three unknowns. If we assume only the "conventional" CP phase δ not to vanish there are four equations for four unknowns, with the results described in [5]. This case would become trivial in the limit where s_{13}^2 is assumed to vanish. There is no reason to expect it to vanish exactly but, considering our lack of knowledge, that seems to be also an interesting assumption to investigate. According to Eq. (12) it yields the same result as setting $\delta = 0$. Then we have 4 equations for 5 unknowns and can get a one parameter family of solutions.

4. Family of Majorana phases

In this case, Eq. (12) takes the form

$$m_1 e^{2i\tau_1} + m_2 e^{2i\tau_2} + m_3 e^{2i\tau_3} = 0, \tag{14}$$

which corresponds, as illustrated in Fig. 1, to a triangle in the complex plane.

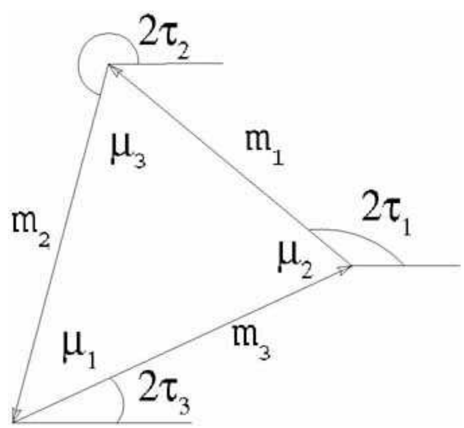

Figure 1. Vector triangle representing Eq. (14).

We proceed to obtain the family of solutions by assuming a value for m_3, getting m_1 and m_2 from Eq. (3) and finally by solving for the two interior angles μ_1 and μ_2 using trigonometry. Interesting CP violation quantities turn out to be:

$$\begin{aligned} sin[2(\tau_1 - \tau_2)] &= -sin(\mu_1 + \mu_2), \\ sin[2(\tau_1 - \tau_3)] &= sin\mu_2, \\ sin[2(\tau_2 - \tau_3)] &= -sin\mu_1, \end{aligned} \tag{15}$$

The criterion for the existence of CP violation effects is the area of the triangle being different from zero. We may express the area as:

$$Area = \frac{1}{4}\left([(m_1 + m_2)^2 - m_3^2][m_3^2 - (m_1 - m_2)^2]\right)^{1/2}. \tag{16}$$

Now we see that the vanishing of the first factor corresponds to the type I real solution while the vanishing of the second factor corresponds to the type II real solution. Furthermore, for a solution to exist, the argument of the square root should be positive. With the second factor, that establishes the minimum allowed value of m_3 while with the first factor, that establishes the minimum value of m_3 which allows a type I solution.

The table below shows a "panorama" of solutions decreasing from $m_3 =$ 0.3 eV, (which is about the highest value compatible with the cosmology bound [6] that the sum of the neutrino masses be less than about 1 eV) to the lowest value imposed by the model. In the type I solutions m_3 is the largest mass while in the type II solutions m_3 is the smallest mass. For each value of m_3, the values of the model predictions for m_1 and m_2 as well as the internal angles μ_1 and μ_2 are given. The model prediction for the neutrinoless double beta decay quantity $|m_{ee}|$ is next shown. Finally, the last column shows the estimated lepton asymmetries due to the decays of the heavy neutrinos. Note that the reversed sign of lepton asymmetry is also possible.

	m_1, m_2, m_3 in eV	μ_1, μ_2 rad.	$\|m_{ee}\|$	ϵ_1, ϵ_2, ϵ_3
I	.2955, .2956, .300	1.038, 1.039	.185	.342, .433, .017
II	.3042, .3043, .300	1.055,1.056	.187	.330, .426, -.0172
I	.0856, .0860, .100	0.946, 0.952	.058	.138, .060, .00137
II	.1119, .1122, .100	1.106, 1.111	.065	.194, .088,-.0024
I	.0305, .0316, .060	0.258, 0.268	.030	.00982, .00422, .00004
II	.0783, .0787, .060	1.172, 1.187	.043	.094, .041,-.0011
I	.0291, .0302, 0.0592715649	.000552, 0.000574	.030	1.96 $\times 10^{-6}$, .84 $\times 10^{-6}$, .71 $\times 10^{-7}$
II	.0774, .0782, 0.0592715649	1.174, 1.188	.042	.047, .020, -.0011
II	.0643, .0648, .040	1.243, 1.268	.033	.052, .023,-.000681
II	.0541, .0548, .020	1.355. 1.442	.024	.018, .0078,-.000335
II	.0506, .0512, .005	1.386, 1.658	.021	.0057, .0025,-.0000824
II	.0503, .0510, .001	0.814, 2.313	.021	.00073, .00031, -.0000122
II	0.0503, 0.0510, 0.0006996	0.051361, 3.089536	.021	.0000348, .0000150, -0.601 $\times 10^{-6}$

Notice that when m_3 is decreased to about 0.0593 eV, we get to the real type I case (no CP violation). Below this value of m_3 only the type II solutions exist. At m_3 about 7×10^{-4} eV, we get the real type II case and no solutions exist for m_3 below this value. In the m_3 regions just above the two real cases we can evidently tune the CP violation phases continuously to be as small as desired.

An interesting application of the model is to neutrinoless double beta decay (for example, the decay $^{76}Ge \rightarrow {}^{76}Se + e^- + e^-$). The current experimental bound on the amplitude factor [7] is: $|m_{ee}| < (0.35 \rightarrow 1.30)$ eV, where

$$|m_{ee}| = \sum |m_i (K_{exp1i})^2 e^{-2i\tau_i}|. \tag{17}$$

From the table we see that the predicted values of $|m_{ee}|$ are typically about one order of magnitude below the experimental bound. Furthermore, the predicted values do not vary drastically for m_3 less than about 0.1 eV.

5. Estimate for leptogenesis

An intriguing possibility for learning more about leptonic CP violation is the study of the proposed leptogenesis mechanism for generation of the present baryon number asymmetry of the universe. According to this scheme, the lepton number violating decays of the heavy neutrinos at a high temperature (early universe) establish a lepton asymmetry which gets converted as the universe cools, through a (B+L) violating but (B-L) conserving "sphaleron" interaction to a baryon asymmetry. References and a rough estimate in the present framework are given in ref. [5]. According to Eq. (7) the heavy neutrino masses are supposed to be proportional to the light ones here and have the same diagonalizing matrix, U. The effective term for calculating the heavy neutrino decays at very high temperature is

$$L_{YUKAWA} = -\sum \bar{L}_i h_{ij} \Phi^c \hat{N}_j + H.c., \tag{18}$$

where

$$h_{ij} \approx \frac{M_{2/3i} K_{expij} e^{-i\tau_j}}{< \phi^0 > r'}. \tag{19}$$

The quantities needed for the calculation are the matrix products $(h^\dagger h)_{ij}$; it is thus seen that the effect of a diagonal matrix of phases multiplying K_{expij} on the left would cancel out. The lepton CP asymmetry ϵ_i, due to the decay of the ith heavy neutrino, is defined as the ratio of decay widths:

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow L + \Phi) - \Gamma(N_i \rightarrow \bar{L} + \bar{\Phi})}{\Gamma(N_i \rightarrow L + \Phi) + \Gamma(N_i \rightarrow \bar{L} + \bar{\Phi})}. \tag{20}$$

In this equation $L+\Phi$ stands for all lepton- Higgs pairs of the types $e_j^- + \phi^+$ and $\nu_j + \bar{\phi}^0$. This is an effect which violates C and CP conservation, in agreement with the requirement of Sakharov. The numerical values of the ϵ_i, which depend on the ratios of heavy neutrino masses rather than their absolute values, are displayed in the last column of the table. Notice that Eq. (18) represents the same term which generates the Dirac mass, M_{DIRAC} in Eq. (8). Since our motivation for the Ansatz assumes dominance of the non seesaw term, this feature requires [5] the heavy neutrino mass scale to be suitably large. This scale plays a role in the estimation of the present baryon to photon ratio, η_B of the universe which is obtained by convoluting the ϵ_i with factors obtained by solving the Boltzmann evolution equations for the (B-L) asymmetry. It turns out [5] that for typical values of the parameter, m_3 in the table, η_B is considerably larger than its experimental value [6] of about 6.5×10^{-10}. Thus agreement with experiment requires tuning close to the two real type solutions; the correct order of magnitude is obtained when either $m_3 \approx 0.059$ eV (type I) or $m_3 \approx 0.005$ eV (type II).

We thank D. Black, A. H. Fariborz, C. Macesanu , M. Trodden and D. Schechter for their help. The work of S.N is supported by National Science foundation grant No. PHY-0099544. The work of J.S. is supported in part by the U. S. DOE under Contract no. DE-FG-02-85ER 40231.

Finally, we are pleased to take this opportunity to express our wishes for Happy Birthday, Good Health and Continued Important Contributions to Physics to PAT O'DONNELL and to HARRY LAM.

References

1. D. Black, A. H. Fariborz, S. Nasri and J. Schechter, Phys. Rev. **D62**, 073015 (2000).
2. X.-G. He and A. Zee, Phys. Rev. **D68**, 037302, (2003).
3. W. Rodejohann, Phys. Lett. **B579**, 127 (2004).
4. M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, arXiv:hep-ph/0309130.
5. S. Nasri, J. Schechter and S. Moussa, arXiv:hep-ph/0402176.
6. D. N. Spergel et al, arXiv:astro-ph/0302209; S. Hannestad, arXiv:astro-ph/0303076.
7. H. V. Klapdor-Kleingrothaus et al, Eur. Phys. J. **A12**, 147 (2001).