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A comparison of model order reduction methods for the simulation of wall heat transfer

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ABSTRACT
In this paper, the potential of model order reduction for simulating building performance is assessed, via a case study of modelling heat transfer through a massive masonry wall. Two model order reduction techniques – proper orthogonal decomposition and proper generalized decomposition – are investigated and compared. Moreover, to illustrate the performance of model order reduction techniques, the accuracies of the two model order reduction techniques are respectively compared with a standard finite element method. The outcomes show that both of the two model order reduction techniques are able to provide an accurate result, and the proper generalized decomposition tends to be more versatile than the proper orthogonal decomposition method.

KEYWORDS
Model order reduction, proper generalized decomposition, proper orthogonal decomposition, finite element method, heat transfer

INTRODUCTION
Today, 30% of the European building stock consists of 'historic' buildings built prior to World War II (Ribuild.eu, 2018). These buildings are typically far less energy-efficient than new buildings, and they hence account for a large share of the total energy consumption of buildings. One important measure to reduce their energy consumption is to install internal insulation. However, internal insulation is often associated with moisture damage, and much care should be taken when applying this solution. This paper is part of the EU H2020 RIBuild project, which aims at developing effective and comprehensive guidelines for internal insulation in historic buildings. Given that a multitude of scenarios and factors can be easily evaluated with numerical analyses, the use of numerical simulations for hygrothermal performance assessment tends to be the best option. However, the standard numerical methods for space and time discretization are usually very time consuming due to the high non-linearity of the equations, the multi-dimensional spatial domains and the long simulation time intervals required, in order to reduce the computation time as much as possible, an efficient solver for modelling the hygrothermal behavior of the wall is needed. Therefore, a faster surrogate model is highly desired.

Instead of using the standard numerical models, Van Gelder et al. (2014) employed statistical regression and interpolation based surrogate models (such as polynomial regression, Kriging etc.) to reduce the simulation time. However, these statistical surrogate models can only deliver static results: for the heat transfer through a wall, they may predict the yearly total heat loss, but not the temperature profile at each moment in time. Hence, to obtain the dynamic behavior with a simplifying surrogate model, model order reduction techniques as alternatives of the statistical surrogate modelling are investigated. In this paper, two model order reduction methods (proper orthogonal decomposition (POD) and proper generalized decomposition (PGD)) are investigated and compared. The first method belongs to a family of a posteriori methods - it is built based on the preliminary results of the original time-consuming model. The second method is an a priori method which can be established by a suitable iterative process. Instead of the standard finite element method (FEM), we will use both POD and PGD to simulate the building thermal performance, exemplified through a case study of modelling the heat transfer through a massive masonry wall.

Below, first a brief introduction of POD and PGD are put forward, with focus on the potential use for the modelling of wall heat transfer. Subsequently, the calculation object and the case study with its input parameters are introduced, as that forms the central application in this study. Next, the results of using POD and PGD for simulating the wall heat transfer are presented and a discussion with respect to the interpretation of their accuracies follows. Finally, conclusions on which method is considered most optimal are formulated.

POD AND PGD FOR MODELLING WALL HEAT TRANSFER

The thermal performance of a building component can be assessed by analyzing the transfer of heat through building materials. Heat transfer is mainly related to the normal flows of heat conduction, convention, radiation and advection. Thus, assessing the thermal performance of a building component requires to get numerical simulation results of the heat transport equation based on the component geometry, the boundary conditions and the material properties. The conventional thermal simulation models are mainly based on numerical simulation methods for space and time discretization, for instance, the FEM. As mentioned before, these standard numerical methods can be very time consuming due to the high number of degrees of freedom after the spatial and temporal discretization. Therefore, in this paper we investigate two model order reduction methods (POD and PGD) which reduce the degrees of freedom of the complex system and still mimic the dynamic behavior (such as time evolution of temperatures,…).

Proper orthogonal decomposition

The POD method was first proposed by Kosambi (1943), and has been successfully applied in a variety of engineering fields, such as image processing, signal analysis, data compression and recently in building physical engineering (Tallet et al., 2017). POD is also known as Karhunen - Loeve decomposition, principal component analysis, or singular value decomposition, and the connections of these three methods are provided by Liang et al. (2002). A brief tutorial of POD can be found in (Chatterjee, 2000), a detailed introduction of its theory and related application for modelling heat transfer process are respectively presented by Liang et al. (2002) and Fic et al. (2005). The basic idea of POD is approximating a high dimensional process by its 'most relevant information'. In this paper, we extract the 'most relevant information' by making use of principal component analysis (PCA). After the PCA, the POD modes are constructed by selecting the most important k components, here $k \ll m$, where m is the number of the mesh elements. As a result, these POD modes can be used to construct a reduced model for simulating different problems (for instance, variations in the boundary conditions or material properties or longer simulation period).

Proper generalized decomposition

Despite the POD method being able to provide a reduced basis and save the computational time when simulating similar problems, this method has an important drawback: to construct a POD, 'a priori knowledge' – the snapshots of the large original model – is needed. This disadvantage in turn leads to an extra computational cost and limits its application to 'different but similar problems'. On the contrary, Ladeveze (1985) proposed a different strategy, called 'radial approximation'. This method is based on the hypothesis that the solution of the considered problem is given by a finite sum representation:

$$u(\boldsymbol{x},t) = \sum_{i=1}^{N} \boldsymbol{X}_i(x) \cdot T_i(t) \tag{1}$$

Here, u is the solution of the target problem, $\boldsymbol{X}_i$ usually stands for the spatial parameters, T_i represents the temporal parameter. Next, injecting equation (1) into the weak formulation of the differential equation and starting from an initial point based on the related initial and boundary conditions, the solution $u(\boldsymbol{x},t)$ can be constructed by successive iterative enrichment methods. The procedure is stopped when the convergence criteria are reached. As a result, this strategy allows to approximate the solution without any 'a priori knowledge'. Inspired by this strategy, Ammar et al. (2006) generalized this method to the multidimensional situation and named it proper generalized decomposition (PGD). A detailed tutorial of PGD is proposed by Chinesta et al. (2013), and an application of PGD for simulating thermal processes is provided by Pruliere et al. (2013) . In addition, two reviews of PGD are provided by Chinesta et al. (2010) and Berger et al. (2016), with attention for general and physical engineering applications respectively.

CALCULATION OBJECT AND CASE STUDY

For investigating the performance of POD and PGD for hygrothermal simulations, a calculation object hence needs to be formulated. Since the reference situation prior to retrofit is often a massive masonry wall, and that configuration is adopted here as calculation object. In order to judge the feasibility of internal insulation in historic buildings, the hygrothermal performances of internally insulated massive walls – heat loss, mould growth, wood rot, … – need to be investigated (Vereecken et al. 2015). To simplify the calculation complexity in this study, this paper limits that performance assessment to the transmission of heat loss through the wall. Since quantifying the heat loss requires solution of the temperature profiles of the wall, both the temperature profiles and heat losses over the entire year are taken as the targeted outputs. To do so, the thermal behavior of the wall is simulated with FEM, POD and PGD, wherein the conductive heat transfer equation is solved under the relevant interior and exterior boundary conditions. The simulation result of the FEM is taken here as the reference solution: more specifically, this reference solution is calculated by the FEM with 200 mesh elements and a fixed time step of one hour.

As mentioned before, since POD is constructed for simulating different problems, in this paper, several POD models are constructed by using snapshots of different time intervals (one year, one month, one day, half day, six hours and three hours). Except for the time interval of one year, all the other scenarios are performed 12 times: once for every month. In addition, each of the one day, half day, six hours and three hours are taken at the start of each month. In order to evaluate the performance of different model order reduction methods as a function of the number of modes, both of the POD and PGD models are calculated with 1 to 15 modes.

For the comparison case study of PGD and POD, the detailed information of the input parameters is mentioned here. For the material properties, the density, thermal capacity and conductivity of the wall are $2087\ kg/m^3$, $870\ J/kgK$ and $1.07\ W/mK$. The boundary conditions are kept restricted to combined convection and radiation, governed by climate data of Gaasbeek (Belgium) at the exterior surface, and by the indoor air temperature as described in (EN 15026) at the interior surface. The related interior and exterior surface transfer coefficient are 8 W/m^2K and 25 W/m^2K, respectively. In relation to the component geometry, the thickness of the wall is 0.2 m.

RESULTS

To compare the accuracies of PGD and different POD models, the average temperature difference between the FEM solution and different model order reduction models, as a function of the number of modes, is shown in Figure 1. For getting a more direct view of the performance of POD and PGD methods, different profiles of temperature are compared at different moments, and the result is presented in Figure 2. In addition, since in practice the cumulated heat loss is usually considered as an indicator of the thermal performance of the wall, the relative deviation of heat losses between the reference solution and different model order reduction models are shown in Figure 3.

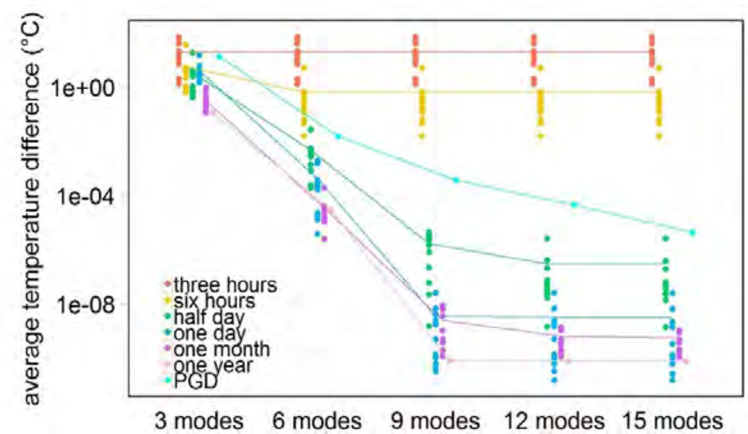

Figure 1. Average temperature differences between reference solution and different model order reduction approximations.

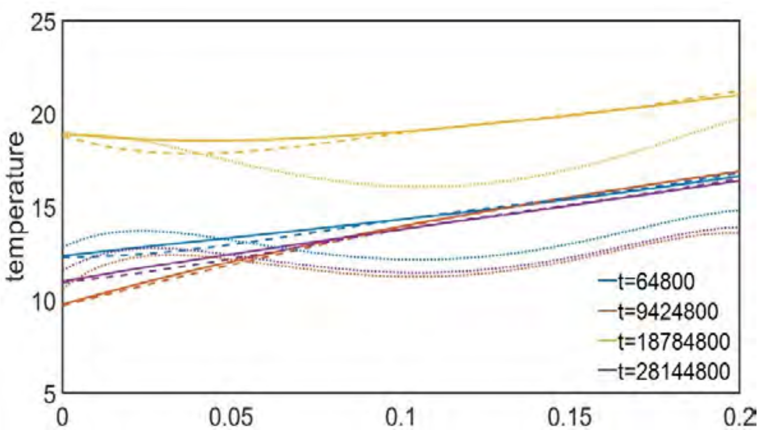

Figure 2. Temperature profiles of the reference solution and PGD solution (solid lines), POD solution constructed from 6 hours' snapshots (dashed lines) and from 3 hours' snapshots (dotted lines). All the reduced models are construed by 15 modes.

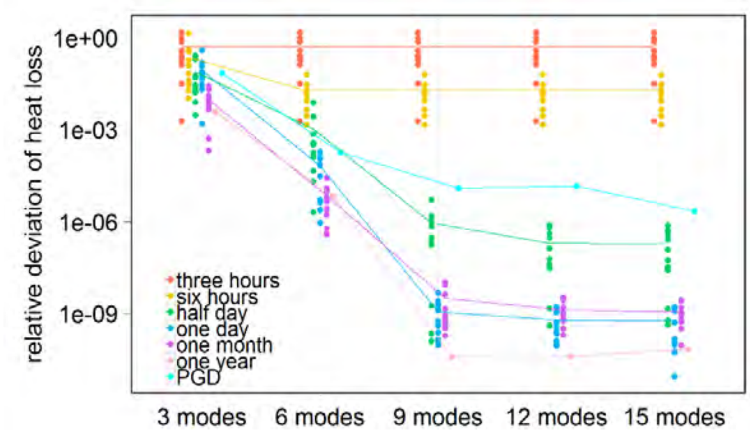

Figure 3. Relative deviation of heat loss between reference solution and different model order reduction approximations.

DISCUSSIONS

Figure 1 illustrates that, except for the POD constructed from the 3 and 6 hours' snapshots, the accuracy of the other reduced order models increases as the number of their construction modes raises. However for the POD, after 9 modes this improvement becomes negligible. In addition, one can see that the accuracy of the POD increases as the time duration of its snapshots raises. In relation to the PGD, one can see that a relatively accurate result can be reached with a sufficient number of construction modes. Figure 2 confirms the result of Figure 1, it is shown that visually there is no difference between the reference solution and PGD approximation. On the other hand, larger differences can be respectively found between the reference solution and the solution calculated by the POD with the 6 and 3 hours snapshots. Furthermore, compared with Figure 1, a very similar result can be found in Figure 3 - except for the POD constructed from the 3 and 6 hours' snapshots, the relative errors of the other reduced order models decreases as the number of their construction modes increases and these relative errors can be reached below 1% with a very limited number of modes. These findings indicate that the performance of all the model order reduction methods do not vary much for quantifying the heat loss instead of calculating the temperature profile.

In summary, combined the results of all the Figures, we can conclude that, with enough number of modes the PGD method can provide a relatively accurate result. In relation to the POD, only when the number of snapshots is really insufficient (three and six hours), an inaccurate result may be obtained. As a consequence, for using the POD method to obtain an accurate result with the smallest size of snapshot, an error estimation method is thus needed. On the other hand, since the PGD model is constructed based on a suitable iterative method, an error controller is naturally embedded in this method. Therefore, together with the advantage that the PGD model is constructed without any 'prior knowledge', this method tends to be more versatile than the POD.

CONCLUSIONS

In this paper, we investigated the performance of two model order reduction methods (POD and PGD), based on a case study of modelling heat transfer through a massive masonry wall. It is shown that, both of the two methods can provide a very accurate result. In addition, since the construction of PGD does not rely on any 'a priori information', this method tends to be more versatile than the POD and should be preferred.

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