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A Panel Data Model with Generalized Higher-Order Network Effects

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Abstract

Many data situations require the consideration of network effects among the cross-sectional units of observation. In this paper, we present a generalized panel model which accounts for two features: (i) three types of network effects on the right-hand side of the model, namely through weighted dependent variable, weighted exogenous variables, as well as weighted error components, and (ii) higher-order network effects due to ex-ante unknown network-decay functions or the presence of multiplex (or multi-layer) networks among all of those. We outline the model, the basic assumptions, and present simulation results.

JEL No.: C23, C33, C34

Keywords: Spatial and Network Interdependence, Panel Data, Higher-Order Network Effects

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1 Introduction¹

In the past two decades, we have seen a big surge in the development of models for the analysis of data which are characterized by some form of spatial or network interdependence between the cross-sectional units of observation both for cross-sectional and panel data (for examples of the latter see Baltagi and Pesaran, 2007; and Bai, Baltagi, and Pesaran, 2016). While this development roots in a more narrow interest on mere geographical or spatial connections between units (see for instance Pinkse, Slade, and Brett, 2002; Egger, Pfaffermayr, and Winner, 2005; Baltagi, Egger and Pfaffermayr, 2007; Ertur and Koch, 2007; Baltagi, Egger, and Kesina, 2017), more recent research widened its scope to cover other forms of interdependence through networks, covering inter alia the following: social interactions between individuals; trade or affiliate networks between firms; migration, commuting, and transport links between regions; to mention a few.

One discomfort with associated work had been that network links were parameterized in a way such that only a single unknown scalar would parameterize the intensity of network links apart from an otherwise fully known structure of the decay or intensity of those links (see Badinger and Egger, 2011, 2013, 2015; Gupta and Robinson, 2015).² An obvious way of parting with this admittedly restrictive approach is to allow for a richer parametrization of the network links through so-called higher-order network links. Suppose a network is characterized by the mere geography and spatial location of the cross-sectional units. Rather than assuming that the decay in space is fully known up to a multiplicative scalar (the so-called spatial or network lag parameter which is commonly estimated), one could estimate a separate parameter for different rings of neighbors or units in a specific quantile of the distance distribution. More specifically, consider tax competition

¹This paper is written in honor of M. Hashem Pesaran for his many contributions to econometrics including panel data and cross-section dependence; see, e.g., the recent survey by Chudik and Pesaran (2015) in the Oxford Handbook of Panel Data. We would like to thank an anonymous referee and Alex Chudik for their useful comments. Egger gratefully acknowledges funding from SNF under grant no. 100018-169537.

²The mentioned papers consider various forms of higher-order processes. In the papers of Badinger and Egger, the number of network layers is finite, whereas in Gupta and Robinson it may grow asymptotically. Most of the mentioned work considers panel data with a random effects structure. However, what is different between this paper and the mentioned work is that the dimensions (number of layers) and the parameters governing the network structure in the time-invariant error component differ from the ones in the time-variant component. In doing so, the present work is a blend feature of processes studied, e.g., in Badinger and Egger, and the network-effects model in Baltagi, Egger, and Pfaffermayr (2013), which considers a single network layer with one parameter each in the time-invariant and time-variant error components.

between municipalities. One can allow for the estimation of a separate parameter measuring the effect of competitive pressure from the taxes set by immediate neighbors, by second-order (or indirect) neighbors, third-order neighbors, etc. This could be implemented by estimating a separate parameter measuring the effect of the competitive pressure from the taxes set by municipalities, say within a distance of 10 kilometers; between 10 and 20 kilometers; and so on. In this way, the data estimates the relatively flexible decay function, and much less is assumed about the decay structure of network links *ex ante*. Moreover, one can allow for different network channels or types of network links – in a so-called multiplex network. For example, geographical, cultural, and historical ties between countries could be considered simultaneously, and their relative importance could be estimated.

The present paper considers the estimation of a random-effects panel-data model where four types of higher-order network links can be present simultaneously: (i) one involving the dependent variable; (ii) another involving the exogenous explanatory variables; (iii) a third involving the cross-sectional (time-invariant) error component; and (iv) a fourth involving the remainder (time-variant) error component. Overall, we could dub this approach a high-order network-lag Durbin model with heterogeneous network processes regarding the error components.

The remainder of the paper is organized as follows. We introduce the notation and outline the econometric model as well as the key assumptions in the subsequent section. Section 3 presents a design and the associated results of a Monte Carlo simulation, which focuses on the bias, root-mean-squared-error, and likelihood-ratio-test findings in cases with relatively small data sets. The last section concludes with a brief summary.

2 Model

2.1 Notation and outline

Formally, the model can be introduced as

$$y = (I_T \otimes \tilde{L})y + X\beta + u \tag{1}$$

$$u = Z_\mu u_1 + u_2 \tag{2}$$

$$u_1 = \tilde{A}u_1 + \mu, \tag{3}$$

$$u_2 = (I_T \otimes \tilde{B})u_2 + \nu \tag{4}$$

where $y' = (y_{11}, \dots, y_{1N}, \dots, y_{T1}, \dots, y_{TN})$ ordered such that the slow index is $t = 1, \dots, T$ and the fast index is $i = 1, \dots, N$. The size of the data-set is $n = NT$. With this notation, the model given in (1) to (4) has the following attributes: (i) y , u , u_2 , and ν are $n \times 1$ vectors; (ii) u_1 and μ are $N \times 1$ vectors; (iii) \tilde{L} , \tilde{A} , and \tilde{B} are $N \times N$ matrices of higher-order network effects (they could involve the dependent variable, the exogenous regressors, the time-invariant error component, and the time-variant residual). These $N \times N$ matrices will be defined below; (iv) I_T is an identity matrix of size T , and Z_μ is a selector or assignment matrix which has the form $\iota_T \otimes I_N$, where ι_T is a column vector of ones of size T ; (v) X is an $n \times K$ matrix of exogenous regressors;³ and (vi) β is a vector of unknown parameters of dimension $K \times 1$.

Towards defining the matrices \tilde{L} , \tilde{A} , and \tilde{B} , it is useful to introduce the following notation; First, let W_o^d be the o th-order (or -layer) weights matrix pertaining to the network effects which are relevant for $d = \{L, A, B\}$ and $o \in \mathfrak{D}_d$, where \mathfrak{D}_d is a set of O_d fixed and finite integer values. This notation allows the network matrices to be conceptually different and have potentially different order across \tilde{L} , \tilde{A} , and \tilde{B} . Second, For the unknown network parameters ρ_o^d with $o \in \mathfrak{D}_d$. We can generally specify

$$\tilde{d} = \sum_{o \in \mathfrak{D}_d} \rho_o^d W_o^d.$$

For instance, this notation implies that for $d = L$, $\tilde{L} = \sum_{o \in \mathfrak{D}_L} \rho_o^L W_o^L$. Similarly for $d = A$ and B .

Note that \mathfrak{D}_d governs the degree or the order of network effects with regard to concept d . W_o^d parameterizes the (normalized) decay structure of network effects within order o , and ρ_o^d scales this decay. It may be useful to think of a specific example with regard to W_o^d . Suppose that network effects emerge from some geographical or spatial neighborliness, and that the researcher is looking at data on firms which are all situated within 100 kilometers from each other. Then, considering rings of neighbors in 25-kilometer bands, $O^d = 4$, so that $o = 1, \dots, 4$ would denote the four 25-kilometer rings. Moreover, $\rho_o^d W_o^d$ would measure the network effects among all pairs of firms within ring o . With the above notation, we generally allow the relevant network concepts to differ between L , A , and B . For instance, network effects could emerge through cultural similarity (e.g., different degrees of language similarity between countries or communities), through technological proximity (the similarity between technology parameters prevailing in different countries, regions,

³Note that X is assumed to be doubly exogenous, i.e., with respect to both error components. This assumption is not as strong as it sounds, because X may contain averages of time-variant regressors (as in Mundlak (1978)), as well as higher-order network-weighted time-variant and time-invariant regressors.

or even firms), through endowment proximity (the similarity of factor endowments between different countries or regions), etc.

For $d = \{L, A, B\}$, let

$$d = I_N - \tilde{d}, \quad (5)$$

so that equations (3) and (4) can be written as

$$u_1 = A^{-1}\mu, \quad u_2 = (I_T \otimes B^{-1})\nu, \quad (6)$$

and the reduced form of the above model can be written as

$$y = (I_T \otimes L^{-1})(X\beta + u). \quad (7)$$

Assuming $\mu \sim IID(0, \sigma_\mu^2 I_N)$ and $\nu \sim IID(0, \sigma_\nu^2 I_n)$, where μ and ν are independent of each other and themselves. The variance-covariance matrix of u with the above notation is

$$\begin{aligned} \Omega_u &= E(uu') = Z_\mu E(u_1 u_1') Z_\mu' + E(u_2 u_2') \\ &= (\iota_T \otimes I_N) [\sigma_\mu^2 (A'A)^{-1}] (\iota_T' \otimes I_N) + \sigma_\nu^2 [I_T \otimes (B'B)^{-1}] \\ &= \bar{J}_T \otimes (T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}) + \sigma_\nu^2 (E_T \otimes (B'B)^{-1}), \end{aligned}$$

The second equality uses the fact that $E(u_1 u_1') = \sigma_\mu^2 (A'A)^{-1}$ and $\hat{E}(u_2 u_2') = \sigma_\nu^2 [I_T \otimes (B'B)^{-1}]$. The third equality replaces the summing matrix over time $\iota_T \iota_T' = J_T$ by its idempotent counterpart $T\bar{J}_T$, with $\bar{J}_T = T^{-1} \iota_T \iota_T'$ denoting the averaging matrix over time. Also, I_T is replaced by $E_T + \bar{J}_T$, where $E_T = I_T - \bar{J}_T$ is the deviations-from mean matrix, see Wansbeek and Kapteyn (1982) and Baltagi, Egger, and Pfaffermayr (2013).⁴

Under the normality assumption, the likelihood function of this model is given by

$$\begin{aligned} L(\beta, \zeta) &= -\frac{n}{2} \ln 2\pi - \frac{1}{2} \ln |(I_T \otimes L^{-1}) \Omega_u (I_T \otimes L^{-1})'| \\ &\quad - \frac{1}{2} (y - (I_T \otimes L^{-1}) X\beta)' ((I_T \otimes L^{-1}) \Omega_u (I_T \otimes L^{-1})')^{-1} (y - (I_T \otimes L^{-1}) X\beta) \\ &= -\frac{n}{2} \ln 2\pi - \frac{1}{2} \ln |T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}| - \frac{T-1}{2} \ln |\sigma_\nu^2 (B'B)^{-1}| + T \ln |L| \\ &\quad - \frac{1}{2} ((I_T \otimes L)y - X\beta)' \Omega_u^{-1} ((I_T \otimes L)y - X\beta), \end{aligned}$$

⁴Note that the latter authors only considered first-order network processes and no network lag of the dependent variable. This model was generalized to include among other things serial correlation of the autoregressive moving average type by Lee and Yu (2011).

where $\zeta = (\sigma_\nu^2, \sigma_\mu^2, \rho_1^L, \dots, \rho_O^L, \rho_1^A, \dots, \rho_O^A, \rho_1^B, \dots, \rho_O^B)$. There are a number of LR tests of interest for testing hypotheses regarding higher-order network effects. For instance, one could use LR tests to test against the absence of individual network effects, i.e., $\rho_o^d = 0$. Similarly, one could specify LR tests regarding joint hypotheses such as $\rho_1^d = \dots = \rho_O^d = 0$ or $\rho_1^L = \dots = \rho_O^L = \rho_1^A = \dots = \rho_O^A = 0$. Moreover, such LR tests can be used to test the relevant order of the network effects. All of these tests could be done by using the difference in the maximized log-likelihood values between the restricted and unrestricted models. This LR test is asymptotically distributed as χ^2 with degrees of freedom equal to the number of restrictions.

With this setting, we generally think of a situation, where the number of time periods T is fixed (small), whereas the number of cross-sectional units N grows asymptotically (is large). E.g., this is the case with most data-sets on firms or regions but also on individuals, where network effects are often studied.

2.2 Key assumptions

We make the following standard assumptions regarding the error components, the network weights matrices, and the parameters governing the network effects.

Assumptions about the error components: Apart from the IID and independence assumptions mentioned above regarding μ and ν , we adopt the customary assumptions that the elements of μ and ν are bounded such that $E|\mu_i|^{4+\eta} < \infty$ and $E|\nu_{it}|^{4+\eta} < \infty$ for some $\eta > 0$ and for all i and t (see Badinger and Egger, 2015).

Assumptions about the network weights matrices: We assume that the network weights matrices W_o^d for all $d = \{L, A, B\}$ and $o \in \mathcal{D}_d$ are nonstochastic, have zero diagonal elements, and bounded off-diagonal elements which sum up to unity in each row or at most to unity for the minimum between the maximum row and column sums (see Kelejian and Prucha, 2010).

Assumptions about the unknown network-effects parameters: The matrices $d = \{L, A, B\}$ are invertible for all real parameter values ρ_o^d . We will make the sufficient assumption that $\sum_{o \in \mathcal{D}_d} |\rho_o^d|$ is an element of a compact parameter space which is a subset of $[0, 1)$.

Assumption about X : The elements of X are nonstochastic and uniformly bounded in n .

3 Monte Carlo simulations

3.1 Design

For the analysis of the small-sample performance of the proposed estimator, we consider the model given in equations (1)-(4), where $X = (t_n, x)$ with $x = \zeta_i + z_{it}$, $\zeta_i \sim IIDU[-7.5, 7.5]$, and $z_{it} \sim IIDU[-5, 5]$ as in Baltagi, Egger, and Pfaffermayr (2013), where $U[\cdot]$ denotes the uniform distribution of a given compact interval. The two processes of ζ_i and z_{it} are assumed to be independent and they are kept fixed across the Monte Carlo draws. Regarding the parameters on X , we assume $\beta = (5, 0.5)'$, where the first parameter denotes the constant.

Moreover, in the Monte Carlo simulations we consider the case where the order of the network relations is two, $O_d = 2$, for every concept $d \in \{L, A, B\}$. Moreover, we assume that the weights matrices are identical between the d so that $W_o^d = W_o$ for all d . For the sake of simplicity, W_1 and W_2 will take a very simple form on a so-called wrap-around lattice: W_1 has a normalized three-before-and-three-behind design, and W_2 has a normalized four-to-six-before and four-to-six-behind design. By this choice, the positive entries of W_1 and W_2 are $1/6$ and non-overlapping. For the purpose of illustration, we display these matrices in a condensed way here.

$$W_1 = \begin{pmatrix} 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1/6 & 1/6 \\ 1/6 & 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1/6 \\ 1/6 & 1/6 & 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & 0 \end{pmatrix},$$

$$W_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 \\ 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

The wrap-around, banded nature of the two respective lattices underlying W_1 and W_2 is apparent from the last column elements in the first three rows and the first three column elements of the last three rows of W_1 . Similarly, it is apparent from the structure of rows four to six from the top and from the bottom of W_2 . All row sums of both W_1 and W_2 sum up to unity.

For the parameters governing the strength of network relations, we postulate three configurations for each concept $d = \{L, A, B\}$ each of $\rho^d = \{(0.6, 0.2); (0.6, 0.0); (0.0, 0.0)\}$ and again use all permutations. Hence, with three parameter configurations of ρ^d and three concepts in $d = \{L, A, B\}$ there are $3^3 = 27$ permutations of configurations.

Regarding T and N , we use all permutations of the number of time periods of $T = \{5; 10\}$ and the number of cross-sectional units of $N = \{100; 200\}$.

With respect to the disturbances, we follow Baltagi, Egger, and Pfaffermayr (2013) in assuming that the individual-specific effects are drawn from a normal distribution so that $\mu_i \sim IIDN(0, 20\theta)$, and the remainder error is distributed as $\nu_{it} \sim IIDN(0, 20(1 - \theta))$, where $0 \leq \theta = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2} \leq 1$ parameterizes the proportion of heterogeneity of the individual-specific effects in the total variance of the disturbances. We set $\theta = \{0.25; 0.5\}$.

This means that there are all together 216 configurations of the design or experiments. For each experiment, we perform 1,000 replications and report the bias and root mean squared error of the results.

3.2 Results

Tables 1-8 report the bias of the parameter estimates across the 1,000 replications, while Tables 9-16 report the corresponding root mean-squared error (RMSE) across the 1,000 replications. For each table, the heading gives the corresponding N, T and θ . The block labelled *Configuration* reports the true values of σ_μ^2 and σ_ν^2 as well as ρ_o^d for $d \in \{L, A, B\}$ and $o \in \{1, 2\}$. We do not provide the true values of the parameters in β , as they are fixed at $(5, 0.5)$ for all experiments.

A comparison of Table 1 ($N = 100, T = 5$) with Table 2 ($N = 100, T = 10$), Table 3 ($N = 200, T = 5$), and Table 4 ($N = 200, T = 10$) suggests the following conclusions for a configuration where $\theta = 0.25$. First, the bias of the parameters is already quite low in Table 1, and it diminishes as T and, in particular, as N increases. The companion comparison regarding the RMSE in Tables 9-12 suggests that also the RMSE declines as T and, in particular, as N increases.

With $\theta = 0.5$ instead of $\theta = 0.25$, i.e., doubling the heterogeneity across the individual effects, the bias in ρ_o^A and ρ_o^B tends to be slightly larger in absolute value. To see this, compare the results in Tables 5-8 (where $\theta = 0.5$) with the corresponding ones in Tables 1-4 (where $\theta = 0.25$). However, this bias improves as the sample size increases. Compare the results in Tables 5 and 1, on the one hand, and those in Tables 8 and 4, respectively. A comparison of the results in Tables 9-12 with the ones in Tables 13-16 regarding the RMSE suggests the following conclusions. First, the RMSE tends to be somewhat smaller for ρ_o^d for $d \in \{L, A, B\}$ and $o \in \{1, 2\}$, where θ is higher. Second, the RMSE tends to decline as T , and particularly, as N increases, irrespective of the level of θ .

Overall, we conclude that the small-sample performance of the estimator appears to be quite good and we recommend it for use in applications with even modest sample sizes.

3.3 Likelihood-ratio tests

In the previous subsection, we documented the performance of the proposed estimator with regard to point estimates with an emphasis on bias and RMSE. In this subsection, we consider selected testing results, using likelihood-ratio (LR) tests and four alternative null hypotheses: $H_0^1 : \rho_2^L = 0$, a second one is $H_0^2 : \rho_2^A = 0$, a third one is that $H_0^3 : \rho_2^B = 0$, and the fourth one is $H_0^4 : \rho_2^L = \rho_2^A = \rho_2^B = 0$. These test that there is no second order network effect in the lagged dependent variable only, the random individual time-invariant effect only, the remainder disturbance term only, or all of them jointly.

Table 1: Bias of Parameters, $N = 100$, $T = 5$, $\theta = 0.25$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	-0.294	-0.002	-0.437	-0.475	0.010	-0.015	-0.173	-0.072	-0.057	-0.001
0.6	0.2	0.6	0.2	0.6	0	5	15	-0.190	-0.003	-0.335	-0.521	0.016	-0.020	-0.127	-0.041	-0.071	0.004
0.6	0.2	0.6	0.2	0	0	5	15	0.038	-0.002	-0.248	-0.531	-0.012	0.008	-0.043	-0.048	-0.034	-0.025
0.6	0.2	0.6	0	0.6	0.2	5	15	-0.065	-0.002	-0.480	-0.477	0.014	-0.020	-0.228	-0.057	-0.058	0.004
0.6	0.2	0.6	0	0.6	0	5	15	0.107	-0.002	-0.445	-0.497	0.021	-0.028	-0.211	-0.035	-0.070	0.013
0.6	0.2	0.6	0	0	0	5	15	0.168	-0.001	-0.263	-0.514	-0.005	-0.002	-0.090	-0.026	-0.037	-0.012
0.6	0.2	0	0	0.6	0.2	5	15	-0.186	-0.003	-0.505	-0.482	0.001	0.000	-0.158	-0.050	-0.046	-0.015
0.6	0.2	0	0	0.6	0	5	15	0.275	-0.004	-0.601	-0.477	0.008	-0.021	-0.211	-0.072	-0.056	0.012
0.6	0.2	0	0	0	0	5	15	0.289	-0.001	-0.525	-0.441	-0.014	0.002	-0.146	-0.085	-0.020	-0.012
0.6	0	0.6	0.2	0.6	0.2	5	15	-0.490	-0.002	-0.439	-0.500	-0.001	0.022	-0.175	-0.113	-0.050	-0.035
0.6	0	0.6	0.2	0.6	0	5	15	-0.324	-0.003	-0.373	-0.575	-0.001	0.008	-0.124	-0.077	-0.067	-0.025
0.6	0	0.6	0.2	0	0	5	15	-0.004	-0.003	-0.228	-0.569	-0.022	0.011	-0.042	-0.057	-0.033	-0.032
0.6	0	0.6	0	0.6	0.2	5	15	-0.241	-0.002	-0.507	-0.497	-0.001	0.010	-0.229	-0.092	-0.049	-0.025
0.6	0	0.6	0	0.6	0	5	15	0.066	-0.003	-0.473	-0.547	-0.004	-0.009	-0.193	-0.058	-0.058	-0.008
0.6	0	0.6	0	0	0	5	15	0.188	-0.003	-0.269	-0.554	-0.019	0.000	-0.084	-0.032	-0.031	-0.018
0.6	0	0	0	0.6	0.2	5	15	-0.062	-0.002	-0.511	-0.488	-0.020	0.018	-0.143	-0.071	-0.032	-0.034
0.6	0	0	0	0.6	0	5	15	0.243	-0.003	-0.586	-0.519	-0.020	-0.003	-0.191	-0.091	-0.041	-0.013
0.6	0	0	0	0	0	5	15	0.293	-0.002	-0.497	-0.494	-0.026	0.002	-0.145	-0.083	-0.014	-0.014
0	0	0.6	0.2	0.6	0.2	5	15	-0.271	-0.002	-0.438	-0.462	0.014	0.015	-0.177	-0.111	-0.048	-0.023
0	0	0.6	0.2	0.6	0	5	15	-0.239	-0.003	-0.351	-0.561	0.013	0.008	-0.127	-0.082	-0.066	-0.025
0	0	0.6	0.2	0	0	5	15	-0.026	-0.006	-0.285	-0.726	-0.023	0.005	-0.044	-0.059	-0.039	-0.034
0	0	0.6	0	0.6	0.2	5	15	-0.130	-0.002	-0.494	-0.461	0.010	0.006	-0.224	-0.093	-0.045	-0.017
0	0	0.6	0	0.6	0	5	15	0.008	-0.004	-0.466	-0.577	0.000	-0.008	-0.196	-0.077	-0.057	-0.015
0	0	0.6	0	0	0	5	15	0.176	-0.007	-0.324	-0.723	-0.029	-0.013	-0.084	-0.040	-0.032	-0.018
0	0	0	0	0.6	0.2	5	15	0.081	-0.004	-0.505	-0.540	-0.020	0.000	-0.131	-0.063	-0.029	-0.021
0	0	0	0	0.6	0	5	15	0.203	-0.006	-0.605	-0.635	-0.029	-0.012	-0.180	-0.094	-0.038	-0.016
0	0	0	0	0	0	5	15	0.334	-0.007	-0.569	-0.690	-0.048	-0.018	-0.133	-0.087	-0.009	-0.010

Table 2: Bias of Parameters, $N = 100$, $T = 10$, $\theta = 0.25$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	-0.115	-0.004	-0.322	-0.289	0.000	-0.006	-0.112	-0.057	-0.029	-0.006
0.6	0.2	0.6	0.2	0.6	0	5	15	-0.076	-0.003	-0.254	-0.278	0.006	-0.011	-0.079	-0.034	-0.037	0.002
0.6	0.2	0.6	0.2	0	0	5	15	0.095	-0.003	-0.208	-0.247	-0.007	0.000	-0.041	-0.037	-0.011	-0.005
0.6	0.2	0.6	0	0.6	0.2	5	15	0.262	-0.004	-0.385	-0.275	-0.003	-0.013	-0.150	-0.049	-0.024	0.001
0.6	0.2	0.6	0	0.6	0	5	15	0.192	-0.003	-0.319	-0.270	0.008	-0.019	-0.129	-0.033	-0.036	0.011
0.6	0.2	0.6	0	0	0	5	15	0.215	-0.003	-0.235	-0.228	-0.006	-0.003	-0.068	-0.026	-0.009	0.000
0.6	0.2	0	0	0.6	0.2	5	15	0.253	-0.004	-0.483	-0.255	-0.005	-0.008	-0.161	-0.052	-0.020	-0.004
0.6	0.2	0	0	0.6	0	5	15	0.362	-0.004	-0.531	-0.252	0.003	-0.018	-0.196	-0.070	-0.027	0.014
0.6	0.2	0	0	0	0	5	15	0.279	-0.003	-0.408	-0.203	-0.007	-0.005	-0.125	-0.063	-0.006	0.003
0.6	0	0.6	0.2	0.6	0.2	5	15	-0.272	-0.004	-0.311	-0.306	-0.002	0.015	-0.115	-0.075	-0.029	-0.024
0.6	0	0.6	0.2	0.6	0	5	15	-0.122	-0.004	-0.270	-0.357	-0.005	0.004	-0.085	-0.048	-0.037	-0.015
0.6	0	0.6	0.2	0	0	5	15	0.056	-0.004	-0.206	-0.278	-0.016	0.004	-0.039	-0.043	-0.009	-0.013
0.6	0	0.6	0	0.6	0.2	5	15	-0.078	-0.004	-0.393	-0.301	-0.006	0.009	-0.157	-0.073	-0.025	-0.019
0.6	0	0.6	0	0.6	0	5	15	0.168	-0.005	-0.367	-0.354	-0.009	-0.008	-0.132	-0.048	-0.032	-0.003
0.6	0	0.6	0	0	0	5	15	0.204	-0.004	-0.237	-0.271	-0.015	-0.003	-0.067	-0.029	-0.008	-0.003
0.6	0	0	0	0.6	0.2	5	15	0.038	-0.004	-0.483	-0.278	-0.012	0.008	-0.154	-0.066	-0.017	-0.018
0.6	0	0	0	0.6	0	5	15	0.414	-0.005	-0.552	-0.324	-0.019	-0.014	-0.185	-0.072	-0.018	0.004
0.6	0	0	0	0	0	5	15	0.278	-0.003	-0.422	-0.246	-0.015	-0.007	-0.126	-0.064	-0.002	0.004
0	0	0.6	0.2	0.6	0.2	5	15	-0.243	-0.002	-0.289	-0.208	0.026	0.014	-0.119	-0.069	-0.037	-0.011
0	0	0.6	0.2	0.6	0	5	15	-0.225	-0.003	-0.251	-0.273	0.024	0.013	-0.092	-0.056	-0.048	-0.019
0	0	0.6	0.2	0	0	5	15	0.023	-0.006	-0.243	-0.386	-0.012	0.000	-0.044	-0.043	-0.019	-0.014
0	0	0.6	0	0.6	0.2	5	15	-0.154	-0.002	-0.373	-0.221	0.020	0.008	-0.164	-0.072	-0.033	-0.008
0	0	0.6	0	0.6	0	5	15	-0.109	-0.003	-0.350	-0.293	0.014	0.006	-0.142	-0.063	-0.041	-0.015
0	0	0.6	0	0	0	5	15	0.124	-0.006	-0.293	-0.397	-0.017	-0.009	-0.070	-0.034	-0.015	-0.005
0	0	0	0	0.6	0.2	5	15	-0.045	-0.003	-0.472	-0.247	0.007	0.002	-0.156	-0.062	-0.024	-0.006
0	0	0	0	0.6	0	5	15	0.022	-0.004	-0.552	-0.309	-0.002	-0.002	-0.192	-0.088	-0.028	-0.010
0	0	0	0	0	0	5	15	0.175	-0.006	-0.465	-0.372	-0.019	-0.015	-0.127	-0.064	-0.009	0.002

Table 3: Bias of Parameters, $N = 200$, $T = 5$, $\theta = 0.25$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	-0.191	-0.004	-0.264	-0.240	0.002	-0.001	-0.081	-0.039	-0.025	-0.014
0.6	0.2	0.6	0.2	0.6	0	5	15	-0.002	-0.003	-0.199	-0.234	0.001	-0.007	-0.049	-0.015	-0.027	-0.004
0.6	0.2	0.6	0.2	0	0	5	15	0.138	-0.002	-0.149	-0.201	-0.009	0.001	-0.016	-0.018	-0.007	-0.008
0.6	0.2	0.6	0	0.6	0.2	5	15	0.064	-0.004	-0.309	-0.230	0.002	-0.008	-0.122	-0.038	-0.024	-0.006
0.6	0.2	0.6	0	0.6	0	5	15	0.187	-0.003	-0.264	-0.210	0.005	-0.015	-0.096	-0.013	-0.028	0.004
0.6	0.2	0.6	0	0	0	5	15	0.211	-0.002	-0.159	-0.186	-0.006	-0.002	-0.035	-0.009	-0.007	-0.004
0.6	0.2	0	0	0.6	0.2	5	15	0.125	-0.004	-0.392	-0.226	-0.004	-0.004	-0.125	-0.041	-0.018	-0.012
0.6	0.2	0	0	0.6	0	5	15	0.401	-0.003	-0.438	-0.217	-0.004	-0.013	-0.150	-0.053	-0.015	0.005
0.6	0.2	0	0	0	0	5	15	0.278	-0.002	-0.317	-0.156	-0.005	-0.006	-0.090	-0.039	-0.002	0.004
0.6	0	0.6	0.2	0.6	0.2	5	15	-0.288	-0.003	-0.261	-0.245	0.000	0.019	-0.089	-0.062	-0.027	-0.032
0.6	0	0.6	0.2	0.6	0	5	15	-0.103	-0.004	-0.214	-0.307	-0.006	0.008	-0.051	-0.033	-0.031	-0.023
0.6	0	0.6	0.2	0	0	5	15	0.112	-0.003	-0.154	-0.230	-0.017	0.004	-0.013	-0.026	-0.006	-0.016
0.6	0	0.6	0	0.6	0.2	5	15	-0.146	-0.003	-0.318	-0.244	-0.004	0.013	-0.130	-0.064	-0.023	-0.027
0.6	0	0.6	0	0.6	0	5	15	0.120	-0.004	-0.278	-0.287	-0.009	-0.004	-0.096	-0.030	-0.026	-0.010
0.6	0	0.6	0	0	0	5	15	0.223	-0.003	-0.175	-0.220	-0.016	-0.003	-0.032	-0.014	-0.004	-0.006
0.6	0	0	0	0.6	0.2	5	15	-0.062	-0.003	-0.392	-0.229	-0.009	0.012	-0.127	-0.059	-0.018	-0.028
0.6	0	0	0	0.6	0	5	15	0.264	-0.004	-0.468	-0.261	-0.015	-0.007	-0.157	-0.067	-0.017	-0.007
0.6	0	0	0	0	0	5	15	0.231	-0.002	-0.329	-0.185	-0.015	-0.004	-0.086	-0.045	-0.001	-0.003
0	0	0.6	0.2	0.6	0.2	5	15	-0.170	-0.002	-0.238	-0.170	0.015	0.015	-0.086	-0.050	-0.025	-0.016
0	0	0.6	0.2	0.6	0	5	15	-0.150	-0.003	-0.198	-0.247	0.012	0.015	-0.054	-0.035	-0.037	-0.024
0	0	0.6	0.2	0	0	5	15	0.041	-0.005	-0.187	-0.345	-0.016	0.004	-0.018	-0.026	-0.016	-0.021
0	0	0.6	0	0.6	0.2	5	15	-0.156	-0.002	-0.309	-0.172	0.015	0.015	-0.133	-0.064	-0.025	-0.017
0	0	0.6	0	0.6	0	5	15	-0.069	-0.003	-0.283	-0.260	0.005	0.008	-0.103	-0.043	-0.031	-0.020
0	0	0.6	0	0	0	5	15	0.126	-0.005	-0.211	-0.351	-0.021	-0.006	-0.035	-0.018	-0.010	-0.012
0	0	0	0	0.6	0.2	5	15	-0.075	-0.003	-0.390	-0.193	0.006	0.009	-0.127	-0.051	-0.021	-0.015
0	0	0	0	0.6	0	5	15	-0.029	-0.004	-0.464	-0.258	-0.001	0.006	-0.160	-0.074	-0.026	-0.019
0	0	0	0	0	0	5	15	0.133	-0.005	-0.368	-0.314	-0.019	-0.008	-0.090	-0.045	-0.008	-0.007

Table 4: Bias of Parameters, $N = 200$, $T = 10$, $\theta = 0.25$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.106	-0.001	-0.167	-0.131	-0.010	0.001	-0.037	-0.029	-0.006	-0.011
0.6	0.2	0.6	0.2	0.6	0	5	15	0.210	0.000	-0.151	-0.120	-0.008	-0.005	-0.027	-0.016	-0.007	0.001
0.6	0.2	0.6	0.2	0	0	5	15	0.156	0.000	-0.118	-0.061	-0.010	0.002	-0.013	-0.017	0.006	-0.002
0.6	0.2	0.6	0	0.6	0.2	5	15	0.438	-0.001	-0.208	-0.133	-0.014	-0.007	-0.061	-0.023	-0.002	-0.002
0.6	0.2	0.6	0	0.6	0	5	15	0.293	0.000	-0.181	-0.114	-0.006	-0.008	-0.052	-0.012	-0.008	0.004
0.6	0.2	0.6	0	0	0	5	15	0.209	0.000	-0.131	-0.056	-0.009	0.000	-0.025	-0.011	0.006	0.002
0.6	0.2	0	0	0.6	0.2	5	15	0.446	-0.001	-0.330	-0.126	-0.017	-0.004	-0.101	-0.042	0.001	-0.006
0.6	0.2	0	0	0.6	0	5	15	0.524	-0.001	-0.336	-0.100	-0.010	-0.012	-0.118	-0.047	-0.001	0.011
0.6	0.2	0	0	0	0	5	15	0.230	0.000	-0.219	-0.044	-0.009	-0.001	-0.056	-0.029	0.008	0.003
0.6	0	0.6	0.2	0.6	0.2	5	15	-0.191	0.000	-0.172	-0.129	-0.009	0.019	-0.044	-0.045	-0.009	-0.027
0.6	0	0.6	0.2	0.6	0	5	15	0.067	-0.001	-0.167	-0.181	-0.017	0.006	-0.027	-0.027	-0.009	-0.015
0.6	0	0.6	0.2	0	0	5	15	0.141	0.000	-0.124	-0.079	-0.016	0.001	-0.011	-0.020	0.006	-0.004
0.6	0	0.6	0	0.6	0.2	5	15	-0.033	-0.001	-0.218	-0.135	-0.014	0.014	-0.068	-0.046	-0.004	-0.022
0.6	0	0.6	0	0.6	0	5	15	0.292	-0.002	-0.202	-0.181	-0.022	-0.004	-0.047	-0.020	-0.003	-0.006
0.6	0	0.6	0	0	0	5	15	0.215	0.000	-0.138	-0.075	-0.015	-0.004	-0.024	-0.012	0.008	0.002
0.6	0	0	0	0.6	0.2	5	15	-0.001	-0.001	-0.331	-0.121	-0.016	0.014	-0.103	-0.060	0.000	-0.023
0.6	0	0	0	0.6	0	5	15	0.393	-0.002	-0.368	-0.158	-0.026	-0.007	-0.114	-0.054	0.004	-0.002
0.6	0	0	0	0	0	5	15	0.232	0.000	-0.230	-0.062	-0.015	-0.005	-0.055	-0.027	0.010	0.004
0	0	0.6	0.2	0.6	0.2	5	15	-0.092	0.000	-0.159	-0.099	0.001	0.009	-0.040	-0.029	-0.008	-0.009
0	0	0.6	0.2	0.6	0	5	15	-0.089	-0.001	-0.150	-0.140	0.001	0.011	-0.033	-0.026	-0.016	-0.016
0	0	0.6	0.2	0	0	5	15	0.050	-0.002	-0.150	-0.173	-0.017	0.004	-0.013	-0.023	0.001	-0.012
0	0	0.6	0	0.6	0.2	5	15	-0.065	0.000	-0.208	-0.101	0.000	0.007	-0.072	-0.037	-0.008	-0.008
0	0	0.6	0	0.6	0	5	15	-0.041	-0.001	-0.194	-0.148	-0.002	0.006	-0.058	-0.030	-0.014	-0.013
0	0	0.6	0	0	0	5	15	0.098	-0.002	-0.170	-0.177	-0.019	-0.002	-0.025	-0.017	0.003	-0.007
0	0	0	0	0.6	0.2	5	15	-0.026	-0.001	-0.334	-0.108	-0.005	0.005	-0.110	-0.052	-0.005	-0.008
0	0	0	0	0.6	0	5	15	0.016	-0.001	-0.372	-0.149	-0.009	0.003	-0.126	-0.066	-0.008	-0.011
0	0	0	0	0	0	5	15	0.106	-0.002	-0.266	-0.162	-0.018	-0.004	-0.059	-0.034	0.004	-0.003

Table 5: Bias of Parameters, $N = 100$, $T = 5$, $\theta = 0.5$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	-0.419	-0.003	-0.543	-0.329	0.007	-0.013	-0.104	-0.042	-0.054	-0.003
0.6	0.2	0.6	0.2	0.6	0	10	10	-0.062	-0.003	-0.481	-0.344	0.005	-0.019	-0.082	-0.025	-0.060	0.005
0.6	0.2	0.6	0.2	0	0	10	10	-0.012	-0.002	-0.386	-0.348	-0.015	0.006	-0.040	-0.040	-0.027	-0.021
0.6	0.2	0.6	0	0.6	0.2	10	10	0.248	-0.002	-0.641	-0.325	0.005	-0.024	-0.143	-0.030	-0.049	0.007
0.6	0.2	0.6	0	0.6	0	10	10	0.897	-0.003	-0.581	-0.344	-0.003	-0.036	-0.118	-0.008	-0.051	0.020
0.6	0.2	0.6	0	0	0	10	10	0.162	-0.001	-0.416	-0.336	-0.008	0.000	-0.072	-0.027	-0.032	-0.013
0.6	0.2	0	0	0.6	0.2	10	10	0.375	-0.003	-0.866	-0.329	-0.009	-0.010	-0.145	-0.047	-0.034	-0.007
0.6	0.2	0	0	0.6	0	10	10	1.881	-0.004	-0.913	-0.331	-0.031	-0.045	-0.132	-0.033	-0.020	0.030
0.6	0.2	0	0	0	0	10	10	0.285	-0.001	-0.686	-0.302	-0.012	0.001	-0.106	-0.062	-0.020	-0.009
0.6	0	0.6	0.2	0.6	0.2	10	10	-0.581	-0.002	-0.541	-0.340	-0.004	0.020	-0.103	-0.074	-0.047	-0.033
0.6	0	0.6	0.2	0.6	0	10	10	-0.465	-0.003	-0.504	-0.387	-0.004	0.010	-0.084	-0.054	-0.062	-0.028
0.6	0	0.6	0.2	0	0	10	10	-0.060	-0.003	-0.350	-0.360	-0.028	0.014	-0.034	-0.055	-0.020	-0.031
0.6	0	0.6	0	0.6	0.2	10	10	-0.160	-0.002	-0.654	-0.329	-0.006	0.007	-0.141	-0.064	-0.043	-0.020
0.6	0	0.6	0	0.6	0	10	10	-0.002	-0.002	-0.615	-0.367	-0.004	-0.007	-0.126	-0.040	-0.058	-0.009
0.6	0	0.6	0	0	0	10	10	0.215	-0.003	-0.404	-0.354	-0.022	-0.001	-0.067	-0.032	-0.022	-0.014
0.6	0	0	0	0.6	0.2	10	10	0.093	-0.003	-0.869	-0.332	-0.024	0.012	-0.135	-0.076	-0.025	-0.027
0.6	0	0	0	0.6	0	10	10	0.314	-0.003	-0.945	-0.338	-0.022	-0.007	-0.159	-0.072	-0.034	-0.007
0.6	0	0	0	0	0	10	10	0.270	-0.002	-0.685	-0.330	-0.024	0.002	-0.103	-0.061	-0.013	-0.011
0	0	0.6	0.2	0.6	0.2	10	10	-0.397	-0.001	-0.516	-0.308	0.019	0.018	-0.110	-0.074	-0.052	-0.025
0	0	0.6	0.2	0.6	0	10	10	-0.393	-0.002	-0.442	-0.356	0.019	0.016	-0.090	-0.059	-0.067	-0.028
0	0	0.6	0.2	0	0	10	10	-0.123	-0.006	-0.439	-0.452	-0.020	0.010	-0.044	-0.052	-0.036	-0.035
0	0	0.6	0	0.6	0.2	10	10	-0.098	-0.002	-0.644	-0.328	0.006	0.004	-0.144	-0.067	-0.044	-0.016
0	0	0.6	0	0.6	0	10	10	-0.003	-0.004	-0.604	-0.385	-0.001	-0.010	-0.123	-0.051	-0.055	-0.012
0	0	0.6	0	0	0	10	10	0.146	-0.006	-0.530	-0.455	-0.029	-0.011	-0.069	-0.037	-0.028	-0.016
0	0	0	0	0.6	0.2	10	10	0.082	-0.004	-0.863	-0.357	-0.019	0.001	-0.131	-0.071	-0.026	-0.020
0	0	0	0	0.6	0	10	10	0.229	-0.006	-0.981	-0.412	-0.031	-0.016	-0.151	-0.080	-0.031	-0.012
0	0	0	0	0	0	10	10	0.333	-0.007	-0.816	-0.461	-0.047	-0.020	-0.092	-0.064	-0.008	-0.005

Table 6: Bias of Parameters, $N = 100$, $T = 10$, $\theta = 0.5$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	-0.324	-0.004	-0.427	-0.177	0.003	-0.004	-0.077	-0.039	-0.029	-0.009
0.6	0.2	0.6	0.2	0.6	0	10	10	-0.139	-0.003	-0.391	-0.174	0.001	-0.010	-0.060	-0.030	-0.030	0.001
0.6	0.2	0.6	0.2	0	0	10	10	0.018	-0.003	-0.311	-0.155	-0.006	-0.001	-0.040	-0.033	-0.010	-0.004
0.6	0.2	0.6	0	0.6	0.2	10	10	0.186	-0.004	-0.531	-0.172	-0.001	-0.013	-0.105	-0.032	-0.025	0.001
0.6	0.2	0.6	0	0.6	0	10	10	0.101	-0.003	-0.459	-0.170	0.004	-0.014	-0.091	-0.026	-0.031	0.005
0.6	0.2	0.6	0	0	0	10	10	0.224	-0.002	-0.360	-0.139	-0.005	-0.004	-0.060	-0.025	-0.006	0.003
0.6	0.2	0	0	0.6	0.2	10	10	0.327	-0.004	-0.795	-0.160	-0.007	-0.009	-0.139	-0.055	-0.016	-0.003
0.6	0.2	0	0	0.6	0	10	10	0.514	-0.004	-0.748	-0.161	-0.004	-0.016	-0.139	-0.051	-0.019	0.010
0.6	0.2	0	0	0	0	10	10	0.282	-0.002	-0.581	-0.123	-0.006	-0.005	-0.098	-0.049	-0.003	0.006
0.6	0	0.6	0.2	0.6	0.2	10	10	-0.285	-0.003	-0.404	-0.196	-0.006	0.015	-0.076	-0.059	-0.024	-0.025
0.6	0	0.6	0.2	0.6	0	10	10	-0.275	-0.004	-0.383	-0.224	-0.006	0.008	-0.066	-0.045	-0.031	-0.019
0.6	0	0.6	0.2	0	0	10	10	-0.044	-0.003	-0.297	-0.167	-0.013	0.005	-0.038	-0.039	-0.008	-0.012
0.6	0	0.6	0	0.6	0.2	10	10	-0.035	-0.004	-0.509	-0.197	-0.009	0.007	-0.105	-0.053	-0.020	-0.017
0.6	0	0.6	0	0.6	0	10	10	0.114	-0.004	-0.488	-0.230	-0.011	-0.005	-0.094	-0.037	-0.027	-0.006
0.6	0	0.6	0	0	0	10	10	0.193	-0.003	-0.379	-0.159	-0.014	-0.004	-0.058	-0.028	-0.004	-0.001
0.6	0	0	0	0.6	0.2	10	10	0.113	-0.004	-0.775	-0.181	-0.015	0.005	-0.136	-0.072	-0.012	-0.016
0.6	0	0	0	0.6	0	10	10	0.365	-0.004	-0.804	-0.207	-0.019	-0.011	-0.139	-0.061	-0.015	0.002
0.6	0	0	0	0	0	10	10	0.258	-0.003	-0.615	-0.148	-0.014	-0.006	-0.095	-0.050	0.000	0.004
0	0	0.6	0.2	0.6	0.2	10	10	-0.264	-0.002	-0.356	-0.131	0.021	0.012	-0.079	-0.047	-0.031	-0.009
0	0	0.6	0.2	0.6	0	10	10	-0.299	-0.002	-0.351	-0.168	0.021	0.015	-0.075	-0.046	-0.041	-0.020
0	0	0.6	0.2	0	0	10	10	-0.073	-0.005	-0.345	-0.225	-0.008	0.005	-0.044	-0.039	-0.016	-0.015
0	0	0.6	0	0.6	0.2	10	10	-0.120	-0.002	-0.485	-0.151	0.013	0.008	-0.110	-0.052	-0.027	-0.009
0	0	0.6	0	0.6	0	10	10	-0.082	-0.003	-0.472	-0.192	0.008	0.004	-0.100	-0.048	-0.034	-0.013
0	0	0.6	0	0	0	10	10	0.091	-0.005	-0.432	-0.231	-0.014	-0.007	-0.063	-0.031	-0.012	-0.004
0	0	0	0	0.6	0.2	10	10	-0.015	-0.003	-0.771	-0.162	0.001	0.001	-0.141	-0.068	-0.018	-0.006
0	0	0	0	0.6	0	10	10	0.078	-0.004	-0.825	-0.207	-0.009	-0.006	-0.146	-0.074	-0.021	-0.007
0	0	0	0	0	0	10	10	0.152	-0.005	-0.680	-0.230	-0.017	-0.013	-0.100	-0.052	-0.008	0.002

Table 7: Bias of Parameters, $N = 200, T = 5, \theta = 0.5$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	-0.258	-0.003	-0.316	-0.166	0.001	0.000	-0.048	-0.029	-0.025	-0.015
0.6	0.2	0.6	0.2	0.6	0	10	10	-0.047	-0.003	-0.259	-0.167	-0.001	-0.005	-0.036	-0.014	-0.024	-0.005
0.6	0.2	0.6	0.2	0	0	10	10	0.127	-0.003	-0.212	-0.128	-0.009	0.001	-0.014	-0.017	-0.005	-0.006
0.6	0.2	0.6	0	0.6	0.2	10	10	0.074	-0.004	-0.379	-0.157	0.002	-0.009	-0.071	-0.021	-0.024	-0.004
0.6	0.2	0.6	0	0.6	0	10	10	0.134	-0.002	-0.321	-0.142	0.003	-0.011	-0.058	-0.009	-0.026	0.002
0.6	0.2	0.6	0	0	0	10	10	0.233	-0.002	-0.249	-0.121	-0.008	-0.002	-0.026	-0.010	-0.004	-0.001
0.6	0.2	0	0	0.6	0.2	10	10	0.148	-0.003	-0.575	-0.142	-0.002	-0.006	-0.099	-0.038	-0.018	-0.008
0.6	0.2	0	0	0.6	0	10	10	0.560	-0.003	-0.587	-0.125	-0.003	-0.020	-0.105	-0.031	-0.016	0.011
0.6	0.2	0	0	0	0	10	10	0.261	-0.002	-0.373	-0.107	-0.007	-0.003	-0.048	-0.027	-0.001	0.001
0.6	0	0.6	0.2	0.6	0.2	10	10	-0.312	-0.003	-0.334	-0.170	-0.002	0.019	-0.050	-0.048	-0.025	-0.031
0.6	0	0.6	0.2	0.6	0	10	10	-0.194	-0.004	-0.296	-0.204	-0.007	0.010	-0.036	-0.033	-0.029	-0.025
0.6	0	0.6	0.2	0	0	10	10	0.057	-0.003	-0.219	-0.147	-0.017	0.006	-0.011	-0.025	-0.004	-0.017
0.6	0	0.6	0	0.6	0.2	10	10	-0.077	-0.003	-0.398	-0.170	-0.007	0.011	-0.070	-0.044	-0.020	-0.024
0.6	0	0.6	0	0.6	0	10	10	0.098	-0.004	-0.353	-0.192	-0.010	-0.002	-0.057	-0.024	-0.024	-0.011
0.6	0	0.6	0	0	0	10	10	0.219	-0.003	-0.258	-0.135	-0.018	-0.001	-0.023	-0.015	0.001	-0.007
0.6	0	0	0	0.6	0.2	10	10	-0.012	-0.003	-0.581	-0.152	-0.011	0.011	-0.096	-0.058	-0.014	-0.025
0.6	0	0	0	0.6	0	10	10	0.240	-0.004	-0.621	-0.168	-0.014	-0.006	-0.105	-0.051	-0.015	-0.006
0.6	0	0	0	0	0	10	10	0.227	-0.002	-0.393	-0.125	-0.015	-0.003	-0.046	-0.029	0.000	-0.003
0	0	0.6	0.2	0.6	0.2	10	10	-0.152	-0.002	-0.272	-0.121	0.012	0.012	-0.047	-0.032	-0.023	-0.014
0	0	0.6	0.2	0.6	0	10	10	-0.149	-0.002	-0.260	-0.161	0.010	0.013	-0.039	-0.030	-0.032	-0.022
0	0	0.6	0.2	0	0	10	10	0.009	-0.005	-0.272	-0.205	-0.013	0.006	-0.016	-0.025	-0.013	-0.020
0	0	0.6	0	0.6	0.2	10	10	-0.072	-0.003	-0.366	-0.134	0.006	0.008	-0.069	-0.036	-0.020	-0.012
0	0	0.6	0	0.6	0	10	10	-0.023	-0.004	-0.361	-0.182	-0.001	0.004	-0.059	-0.032	-0.027	-0.017
0	0	0.6	0	0	0	10	10	0.095	-0.005	-0.324	-0.214	-0.017	-0.004	-0.029	-0.018	-0.010	-0.011
0	0	0	0	0.6	0.2	10	10	-0.045	-0.003	-0.584	-0.131	0.002	0.007	-0.100	-0.052	-0.016	-0.013
0	0	0	0	0.6	0	10	10	0.021	-0.004	-0.649	-0.174	-0.006	0.001	-0.110	-0.061	-0.020	-0.015
0	0	0	0	0	0	10	10	0.130	-0.005	-0.481	-0.205	-0.019	-0.008	-0.052	-0.032	-0.006	-0.006

Table 8: Bias of Parameters, $N = 200$, $T = 10$, $\theta = 0.5$

Configuration								Bias									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	-0.038	-0.001	-0.248	-0.082	-0.009	0.004	-0.026	-0.028	-0.007	-0.013
0.6	0.2	0.6	0.2	0.6	0	10	10	0.135	0.000	-0.237	-0.077	-0.008	-0.004	-0.021	-0.015	-0.006	0.000
0.6	0.2	0.6	0.2	0	0	10	10	0.130	0.000	-0.173	-0.038	-0.009	0.002	-0.011	-0.017	0.006	-0.002
0.6	0.2	0.6	0	0.6	0.2	10	10	0.338	-0.001	-0.290	-0.084	-0.013	-0.004	-0.036	-0.019	-0.003	-0.005
0.6	0.2	0.6	0	0.6	0	10	10	0.294	0.000	-0.266	-0.074	-0.007	-0.007	-0.036	-0.009	-0.006	0.004
0.6	0.2	0.6	0	0	0	10	10	0.196	0.000	-0.198	-0.034	-0.008	0.000	-0.021	-0.012	0.007	0.002
0.6	0.2	0	0	0.6	0.2	10	10	0.463	-0.001	-0.470	-0.080	-0.015	-0.005	-0.066	-0.033	0.001	-0.004
0.6	0.2	0	0	0.6	0	10	10	0.413	0.000	-0.448	-0.064	-0.008	-0.009	-0.069	-0.027	-0.003	0.008
0.6	0.2	0	0	0	0	10	10	0.222	0.000	-0.311	-0.029	-0.008	-0.001	-0.037	-0.022	0.008	0.004
0.6	0	0.6	0.2	0.6	0.2	10	10	-0.213	0.000	-0.236	-0.080	-0.007	0.019	-0.030	-0.040	-0.008	-0.026
0.6	0	0.6	0.2	0.6	0	10	10	0.022	-0.001	-0.250	-0.112	-0.017	0.006	-0.018	-0.028	-0.006	-0.015
0.6	0	0.6	0.2	0	0	10	10	0.089	0.000	-0.183	-0.048	-0.013	0.002	-0.010	-0.020	0.005	-0.004
0.6	0	0.6	0	0.6	0.2	10	10	-0.005	-0.001	-0.305	-0.086	-0.014	0.012	-0.041	-0.037	-0.003	-0.020
0.6	0	0.6	0	0.6	0	10	10	0.241	-0.002	-0.304	-0.113	-0.020	-0.002	-0.031	-0.019	-0.003	-0.006
0.6	0	0.6	0	0	0	10	10	0.172	0.000	-0.213	-0.045	-0.013	-0.002	-0.020	-0.013	0.007	0.001
0.6	0	0	0	0.6	0.2	10	10	0.026	-0.001	-0.481	-0.078	-0.016	0.012	-0.073	-0.054	0.001	-0.020
0.6	0	0	0	0.6	0	10	10	0.366	-0.002	-0.471	-0.101	-0.024	-0.007	-0.061	-0.033	0.003	-0.001
0.6	0	0	0	0	0	10	10	0.203	0.000	-0.323	-0.042	-0.013	-0.004	-0.036	-0.021	0.009	0.004
0	0	0.6	0.2	0.6	0.2	10	10	-0.072	0.000	-0.226	-0.070	-0.002	0.008	-0.025	-0.024	-0.006	-0.009
0	0	0.6	0.2	0.6	0	10	10	-0.070	-0.001	-0.225	-0.092	-0.002	0.009	-0.024	-0.025	-0.012	-0.014
0	0	0.6	0.2	0	0	10	10	0.026	-0.002	-0.226	-0.099	-0.015	0.005	-0.011	-0.022	0.002	-0.011
0	0	0.6	0	0.6	0.2	10	10	-0.031	-0.001	-0.280	-0.073	-0.004	0.005	-0.041	-0.026	-0.005	-0.007
0	0	0.6	0	0.6	0	10	10	-0.016	-0.001	-0.284	-0.100	-0.006	0.005	-0.038	-0.026	-0.010	-0.012
0	0	0.6	0	0	0	10	10	0.074	-0.002	-0.263	-0.102	-0.017	0.000	-0.021	-0.018	0.003	-0.006
0	0	0	0	0.6	0.2	10	10	-0.016	-0.001	-0.489	-0.072	-0.006	0.005	-0.079	-0.047	-0.004	-0.008
0	0	0	0	0.6	0	10	10	0.025	-0.001	-0.503	-0.098	-0.010	0.002	-0.078	-0.048	-0.007	-0.009
0	0	0	0	0	0	10	10	0.091	-0.001	-0.381	-0.099	-0.017	-0.003	-0.040	-0.028	0.004	-0.003

Table 9: RMSE of Parameters, $N = 100$, $T = 5$, $\theta = 0.25$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	2.360	0.045	1.266	1.217	0.162	0.136	0.399	0.280	0.197	0.153
0.6	0.2	0.6	0.2	0.6	0	5	15	2.182	0.044	1.228	1.248	0.155	0.143	0.328	0.258	0.212	0.152
0.6	0.2	0.6	0.2	0	0	5	15	1.966	0.046	1.287	1.362	0.136	0.123	0.226	0.215	0.243	0.152
0.6	0.2	0.6	0	0.6	0.2	5	15	2.367	0.045	1.287	1.216	0.159	0.139	0.450	0.284	0.194	0.156
0.6	0.2	0.6	0	0.6	0	5	15	2.194	0.045	1.263	1.234	0.151	0.144	0.414	0.271	0.206	0.156
0.6	0.2	0.6	0	0	0	5	15	1.769	0.047	1.250	1.359	0.132	0.120	0.256	0.209	0.238	0.151
0.6	0.2	0	0	0.6	0.2	5	15	2.317	0.045	1.362	1.219	0.154	0.141	0.460	0.336	0.184	0.157
0.6	0.2	0	0	0.6	0	5	15	2.108	0.046	1.375	1.222	0.149	0.146	0.470	0.321	0.194	0.155
0.6	0.2	0	0	0	0	5	15	1.636	0.047	1.260	1.295	0.129	0.120	0.387	0.295	0.224	0.149
0.6	0	0.6	0.2	0.6	0.2	5	15	2.356	0.046	1.259	1.238	0.182	0.132	0.408	0.292	0.197	0.155
0.6	0	0.6	0.2	0.6	0	5	15	2.196	0.045	1.238	1.267	0.181	0.138	0.350	0.266	0.218	0.159
0.6	0	0.6	0.2	0	0	5	15	1.935	0.047	1.262	1.361	0.152	0.134	0.225	0.216	0.252	0.185
0.6	0	0.6	0	0.6	0.2	5	15	2.248	0.046	1.292	1.230	0.179	0.129	0.459	0.286	0.194	0.152
0.6	0	0.6	0	0.6	0	5	15	2.194	0.045	1.265	1.245	0.176	0.137	0.409	0.269	0.210	0.156
0.6	0	0.6	0	0	0	5	15	1.849	0.047	1.243	1.358	0.148	0.130	0.258	0.209	0.248	0.181
0.6	0	0	0	0.6	0.2	5	15	2.238	0.046	1.347	1.221	0.179	0.138	0.469	0.337	0.183	0.154
0.6	0	0	0	0.6	0	5	15	2.155	0.046	1.383	1.228	0.171	0.140	0.466	0.325	0.194	0.156
0.6	0	0	0	0	0	5	15	1.702	0.047	1.256	1.304	0.144	0.128	0.391	0.296	0.230	0.175
0	0	0.6	0.2	0.6	0.2	5	15	2.207	0.049	1.272	1.486	0.294	0.157	0.420	0.297	0.201	0.145
0	0	0.6	0.2	0.6	0	5	15	2.137	0.049	1.255	1.497	0.295	0.180	0.355	0.270	0.225	0.156
0	0	0.6	0.2	0	0	5	15	1.883	0.048	1.284	1.360	0.251	0.186	0.233	0.220	0.271	0.208
0	0	0.6	0	0.6	0.2	5	15	2.005	0.049	1.300	1.469	0.286	0.155	0.459	0.290	0.194	0.145
0	0	0.6	0	0.6	0	5	15	2.025	0.049	1.265	1.492	0.293	0.177	0.425	0.273	0.219	0.154
0	0	0.6	0	0	0	5	15	1.784	0.048	1.259	1.355	0.253	0.188	0.267	0.215	0.268	0.207
0	0	0	0	0.6	0.2	5	15	1.944	0.049	1.344	1.497	0.281	0.154	0.474	0.337	0.182	0.142
0	0	0	0	0.6	0	5	15	1.939	0.049	1.362	1.505	0.285	0.177	0.470	0.327	0.201	0.149
0	0	0	0	0	0	5	15	1.680	0.048	1.246	1.319	0.244	0.188	0.399	0.303	0.249	0.202

Table 10: RMSE of Parameters, $N = 100$, $T = 10$, $\theta = 0.25$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	2.302	0.034	0.988	0.814	0.135	0.116	0.286	0.226	0.151	0.124
0.6	0.2	0.6	0.2	0.6	0	5	15	2.053	0.034	0.960	0.817	0.128	0.115	0.234	0.201	0.157	0.116
0.6	0.2	0.6	0.2	0	0	5	15	1.680	0.035	0.978	0.884	0.106	0.094	0.185	0.178	0.179	0.112
0.6	0.2	0.6	0	0.6	0.2	5	15	2.283	0.035	1.019	0.804	0.134	0.118	0.329	0.231	0.147	0.126
0.6	0.2	0.6	0	0.6	0	5	15	2.027	0.035	0.993	0.801	0.127	0.119	0.294	0.213	0.155	0.123
0.6	0.2	0.6	0	0	0	5	15	1.478	0.035	0.971	0.886	0.104	0.092	0.205	0.174	0.178	0.110
0.6	0.2	0	0	0.6	0.2	5	15	2.230	0.035	1.131	0.787	0.124	0.115	0.422	0.282	0.135	0.124
0.6	0.2	0	0	0.6	0	5	15	1.969	0.035	1.144	0.790	0.120	0.116	0.419	0.275	0.141	0.121
0.6	0.2	0	0	0	0	5	15	1.407	0.036	0.998	0.870	0.101	0.090	0.328	0.244	0.170	0.110
0.6	0	0.6	0.2	0.6	0.2	5	15	2.224	0.034	0.977	0.817	0.151	0.115	0.284	0.231	0.150	0.126
0.6	0	0.6	0.2	0.6	0	5	15	2.099	0.034	0.961	0.833	0.150	0.122	0.239	0.211	0.166	0.130
0.6	0	0.6	0.2	0	0	5	15	1.717	0.035	0.982	0.893	0.120	0.102	0.186	0.183	0.187	0.135
0.6	0	0.6	0	0.6	0.2	5	15	2.047	0.034	1.011	0.813	0.148	0.116	0.338	0.238	0.146	0.126
0.6	0	0.6	0	0.6	0	5	15	2.042	0.034	0.992	0.823	0.151	0.122	0.302	0.223	0.163	0.129
0.6	0	0.6	0	0	0	5	15	1.571	0.035	0.976	0.890	0.117	0.101	0.206	0.181	0.184	0.136
0.6	0	0	0	0.6	0.2	5	15	1.938	0.035	1.126	0.792	0.141	0.113	0.424	0.285	0.133	0.123
0.6	0	0	0	0.6	0	5	15	1.981	0.035	1.143	0.798	0.141	0.121	0.423	0.279	0.145	0.127
0.6	0	0	0	0	0	5	15	1.466	0.036	1.007	0.863	0.112	0.099	0.332	0.252	0.174	0.135
0	0	0.6	0.2	0.6	0.2	5	15	1.927	0.037	0.994	0.963	0.229	0.133	0.289	0.228	0.148	0.117
0	0	0.6	0.2	0.6	0	5	15	1.951	0.037	0.976	0.968	0.232	0.154	0.245	0.210	0.166	0.124
0	0	0.6	0.2	0	0	5	15	1.679	0.035	0.995	0.838	0.187	0.154	0.188	0.186	0.193	0.161
0	0	0.6	0	0.6	0.2	5	15	1.654	0.037	1.023	0.966	0.227	0.129	0.346	0.239	0.145	0.114
0	0	0.6	0	0.6	0	5	15	1.724	0.037	0.997	0.985	0.231	0.152	0.312	0.227	0.161	0.122
0	0	0.6	0	0	0	5	15	1.461	0.036	1.001	0.848	0.191	0.159	0.212	0.188	0.192	0.163
0	0	0	0	0.6	0.2	5	15	1.488	0.037	1.115	0.975	0.214	0.124	0.423	0.287	0.133	0.111
0	0	0	0	0.6	0	5	15	1.532	0.036	1.135	0.985	0.215	0.146	0.421	0.284	0.144	0.117
0	0	0	0	0	0	5	15	1.309	0.035	1.015	0.826	0.181	0.155	0.336	0.259	0.181	0.159

Table 11: RMSE of Parameters, $N = 200$, $T = 5$, $\theta = 0.25$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	2.118	0.031	0.891	0.817	0.119	0.102	0.241	0.200	0.133	0.114
0.6	0.2	0.6	0.2	0.6	0	5	15	1.876	0.032	0.887	0.824	0.114	0.101	0.198	0.185	0.138	0.109
0.6	0.2	0.6	0.2	0	0	5	15	1.521	0.032	0.923	0.914	0.096	0.087	0.158	0.156	0.168	0.114
0.6	0.2	0.6	0	0.6	0.2	5	15	2.114	0.031	0.921	0.814	0.116	0.105	0.292	0.211	0.131	0.117
0.6	0.2	0.6	0	0.6	0	5	15	1.890	0.032	0.894	0.810	0.110	0.104	0.244	0.198	0.135	0.113
0.6	0.2	0.6	0	0	0	5	15	1.386	0.032	0.913	0.905	0.096	0.086	0.166	0.150	0.166	0.112
0.6	0.2	0	0	0.6	0.2	5	15	2.152	0.031	1.057	0.809	0.108	0.103	0.408	0.281	0.121	0.114
0.6	0.2	0	0	0.6	0	5	15	1.892	0.032	1.067	0.820	0.106	0.102	0.392	0.269	0.125	0.110
0.6	0.2	0	0	0	0	5	15	1.310	0.032	0.923	0.878	0.089	0.081	0.291	0.223	0.156	0.108
0.6	0	0.6	0.2	0.6	0.2	5	15	2.122	0.031	0.902	0.830	0.138	0.106	0.254	0.209	0.135	0.121
0.6	0	0.6	0.2	0.6	0	5	15	1.987	0.031	0.881	0.847	0.137	0.112	0.208	0.187	0.150	0.125
0.6	0	0.6	0.2	0	0	5	15	1.579	0.032	0.911	0.912	0.112	0.094	0.160	0.157	0.177	0.135
0.6	0	0.6	0	0.6	0.2	5	15	1.996	0.031	0.932	0.829	0.137	0.106	0.306	0.223	0.133	0.119
0.6	0	0.6	0	0.6	0	5	15	1.920	0.031	0.889	0.833	0.135	0.113	0.256	0.201	0.145	0.123
0.6	0	0.6	0	0	0	5	15	1.530	0.032	0.906	0.909	0.109	0.097	0.169	0.155	0.175	0.138
0.6	0	0	0	0.6	0.2	5	15	1.930	0.031	1.059	0.824	0.129	0.105	0.407	0.287	0.124	0.116
0.6	0	0	0	0.6	0	5	15	1.860	0.031	1.082	0.820	0.123	0.113	0.404	0.274	0.132	0.119
0.6	0	0	0	0	0	5	15	1.355	0.032	0.919	0.874	0.100	0.090	0.290	0.228	0.162	0.130
0	0	0.6	0.2	0.6	0.2	5	15	1.702	0.033	0.911	0.999	0.211	0.123	0.253	0.202	0.130	0.105
0	0	0.6	0.2	0.6	0	5	15	1.765	0.034	0.900	1.008	0.219	0.145	0.203	0.187	0.149	0.117
0	0	0.6	0.2	0	0	5	15	1.514	0.032	0.932	0.862	0.183	0.143	0.161	0.159	0.184	0.158
0	0	0.6	0	0.6	0.2	5	15	1.568	0.034	0.954	1.001	0.210	0.126	0.313	0.226	0.130	0.106
0	0	0.6	0	0.6	0	5	15	1.610	0.034	0.919	1.018	0.217	0.145	0.266	0.207	0.145	0.115
0	0	0.6	0	0	0	5	15	1.381	0.033	0.928	0.865	0.184	0.149	0.171	0.159	0.182	0.160
0	0	0	0	0.6	0.2	5	15	1.470	0.033	1.057	0.995	0.199	0.123	0.412	0.291	0.122	0.102
0	0	0	0	0.6	0	5	15	1.474	0.033	1.081	1.007	0.203	0.142	0.401	0.284	0.134	0.111
0	0	0	0	0	0	5	15	1.224	0.032	0.927	0.848	0.168	0.145	0.295	0.244	0.169	0.153

Table 12: RMSE of Parameters, $N = 200$, $T = 10$, $\theta = 0.25$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	5	15	1.991	0.024	0.690	0.558	0.101	0.087	0.165	0.146	0.104	0.092
0.6	0.2	0.6	0.2	0.6	0	5	15	1.771	0.024	0.685	0.558	0.093	0.082	0.153	0.138	0.105	0.083
0.6	0.2	0.6	0.2	0	0	5	15	1.288	0.025	0.705	0.621	0.075	0.064	0.128	0.122	0.125	0.078
0.6	0.2	0.6	0	0.6	0.2	5	15	2.009	0.024	0.717	0.552	0.102	0.089	0.203	0.157	0.103	0.092
0.6	0.2	0.6	0	0.6	0	5	15	1.680	0.024	0.691	0.556	0.092	0.083	0.177	0.142	0.105	0.085
0.6	0.2	0.6	0	0	0	5	15	1.108	0.025	0.704	0.619	0.074	0.064	0.137	0.119	0.124	0.078
0.6	0.2	0	0	0.6	0.2	5	15	1.965	0.024	0.870	0.556	0.097	0.087	0.352	0.238	0.097	0.090
0.6	0.2	0	0	0.6	0	5	15	1.723	0.024	0.847	0.550	0.089	0.083	0.325	0.219	0.097	0.086
0.6	0.2	0	0	0	0	5	15	1.015	0.025	0.699	0.610	0.071	0.061	0.229	0.174	0.120	0.076
0.6	0	0.6	0.2	0.6	0.2	5	15	1.753	0.025	0.685	0.569	0.114	0.090	0.171	0.159	0.107	0.096
0.6	0	0.6	0.2	0.6	0	5	15	1.776	0.025	0.690	0.571	0.119	0.094	0.158	0.147	0.118	0.098
0.6	0	0.6	0.2	0	0	5	15	1.358	0.025	0.708	0.619	0.085	0.072	0.129	0.124	0.130	0.099
0.6	0	0.6	0	0.6	0.2	5	15	1.596	0.025	0.707	0.574	0.115	0.088	0.210	0.169	0.106	0.095
0.6	0	0.6	0	0.6	0	5	15	1.730	0.025	0.688	0.575	0.122	0.095	0.183	0.151	0.118	0.098
0.6	0	0.6	0	0	0	5	15	1.225	0.025	0.706	0.617	0.083	0.073	0.138	0.123	0.129	0.101
0.6	0	0	0	0.6	0.2	5	15	1.503	0.025	0.865	0.566	0.108	0.087	0.356	0.246	0.098	0.092
0.6	0	0	0	0.6	0	5	15	1.618	0.025	0.853	0.564	0.113	0.093	0.334	0.231	0.106	0.094
0.6	0	0	0	0	0	5	15	1.098	0.025	0.703	0.603	0.078	0.070	0.232	0.183	0.122	0.097
0	0	0.6	0.2	0.6	0.2	5	15	1.300	0.027	0.694	0.716	0.158	0.088	0.160	0.146	0.095	0.078
0	0	0.6	0.2	0.6	0	5	15	1.405	0.028	0.697	0.729	0.169	0.109	0.153	0.143	0.109	0.083
0	0	0.6	0.2	0	0	5	15	1.233	0.025	0.719	0.564	0.133	0.112	0.130	0.128	0.133	0.116
0	0	0.6	0	0.6	0.2	5	15	1.107	0.027	0.715	0.719	0.159	0.087	0.207	0.161	0.096	0.077
0	0	0.6	0	0.6	0	5	15	1.218	0.028	0.696	0.735	0.169	0.108	0.184	0.151	0.109	0.083
0	0	0.6	0	0	0	5	15	1.027	0.025	0.718	0.566	0.134	0.115	0.139	0.129	0.133	0.119
0	0	0	0	0.6	0.2	5	15	0.989	0.026	0.860	0.709	0.149	0.084	0.353	0.244	0.089	0.074
0	0	0	0	0.6	0	5	15	1.060	0.027	0.842	0.720	0.154	0.104	0.332	0.234	0.097	0.078
0	0	0	0	0	0	5	15	0.883	0.025	0.705	0.559	0.125	0.112	0.233	0.193	0.124	0.116

Table 13: RMSE of Parameters, $N = 100$, $T = 5$, $\theta = 0.5$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	2.442	0.043	1.789	0.819	0.162	0.139	0.283	0.211	0.193	0.156
0.6	0.2	0.6	0.2	0.6	0	10	10	2.983	0.043	1.767	0.831	0.157	0.144	0.245	0.208	0.202	0.153
0.6	0.2	0.6	0.2	0	0	10	10	2.121	0.046	1.811	0.885	0.134	0.124	0.199	0.189	0.233	0.155
0.6	0.2	0.6	0	0.6	0.2	10	10	2.859	0.043	1.843	0.819	0.162	0.138	0.331	0.217	0.188	0.156
0.6	0.2	0.6	0	0.6	0	10	10	3.967	0.044	1.806	0.842	0.171	0.148	0.297	0.207	0.203	0.156
0.6	0.2	0.6	0	0	0	10	10	1.785	0.046	1.790	0.883	0.127	0.117	0.218	0.179	0.228	0.149
0.6	0.2	0	0	0.6	0.2	10	10	2.904	0.044	2.101	0.821	0.161	0.139	0.406	0.289	0.178	0.154
0.6	0.2	0	0	0.6	0	10	10	4.928	0.045	2.091	0.836	0.181	0.155	0.404	0.281	0.185	0.156
0.6	0.2	0	0	0	0	10	10	1.633	0.046	1.868	0.860	0.126	0.119	0.323	0.241	0.217	0.150
0.6	0	0.6	0.2	0.6	0.2	10	10	2.424	0.045	1.810	0.823	0.186	0.134	0.287	0.223	0.195	0.156
0.6	0	0.6	0.2	0.6	0	10	10	2.278	0.044	1.792	0.841	0.180	0.141	0.250	0.215	0.210	0.160
0.6	0	0.6	0.2	0	0	10	10	2.077	0.045	1.806	0.884	0.152	0.134	0.197	0.193	0.240	0.182
0.6	0	0.6	0	0.6	0.2	10	10	2.236	0.045	1.845	0.814	0.179	0.129	0.333	0.223	0.186	0.152
0.6	0	0.6	0	0.6	0	10	10	2.174	0.044	1.804	0.830	0.172	0.138	0.300	0.213	0.204	0.158
0.6	0	0.6	0	0	0	10	10	1.868	0.045	1.778	0.888	0.141	0.130	0.216	0.184	0.234	0.182
0.6	0	0	0	0.6	0.2	10	10	2.212	0.045	2.079	0.815	0.174	0.136	0.404	0.291	0.172	0.152
0.6	0	0	0	0.6	0	10	10	2.116	0.045	2.112	0.817	0.165	0.140	0.403	0.278	0.183	0.154
0.6	0	0	0	0	0	10	10	1.638	0.046	1.838	0.864	0.138	0.131	0.324	0.252	0.221	0.178
0	0	0.6	0.2	0.6	0.2	10	10	2.338	0.047	1.837	0.988	0.295	0.161	0.299	0.223	0.202	0.148
0	0	0.6	0.2	0.6	0	10	10	2.226	0.047	1.834	0.977	0.286	0.178	0.260	0.215	0.220	0.154
0	0	0.6	0.2	0	0	10	10	2.021	0.046	1.847	0.883	0.233	0.179	0.203	0.197	0.251	0.200
0	0	0.6	0	0.6	0.2	10	10	2.067	0.048	1.866	0.989	0.289	0.157	0.342	0.225	0.193	0.145
0	0	0.6	0	0.6	0	10	10	2.037	0.047	1.817	0.989	0.289	0.177	0.307	0.214	0.214	0.153
0	0	0.6	0	0	0	10	10	1.746	0.046	1.845	0.880	0.237	0.183	0.227	0.192	0.250	0.201
0	0	0	0	0.6	0.2	10	10	1.884	0.048	2.047	1.001	0.276	0.157	0.409	0.291	0.173	0.144
0	0	0	0	0.6	0	10	10	1.875	0.047	2.051	0.999	0.275	0.175	0.404	0.277	0.188	0.147
0	0	0	0	0	0	10	10	1.660	0.046	1.837	0.877	0.236	0.190	0.335	0.263	0.240	0.203

Table 14: RMSE of Parameters, $N = 100$, $T = 10$, $\theta = 0.5$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	2.318	0.032	1.581	0.533	0.127	0.111	0.215	0.182	0.144	0.120
0.6	0.2	0.6	0.2	0.6	0	10	10	2.164	0.032	1.596	0.533	0.122	0.111	0.198	0.178	0.146	0.113
0.6	0.2	0.6	0.2	0	0	10	10	1.891	0.033	1.583	0.572	0.097	0.088	0.166	0.162	0.164	0.108
0.6	0.2	0.6	0	0.6	0.2	10	10	2.237	0.032	1.634	0.531	0.127	0.111	0.252	0.188	0.140	0.120
0.6	0.2	0.6	0	0.6	0	10	10	1.938	0.032	1.617	0.532	0.120	0.112	0.230	0.182	0.146	0.115
0.6	0.2	0.6	0	0	0	10	10	1.513	0.033	1.589	0.572	0.095	0.086	0.180	0.159	0.163	0.108
0.6	0.2	0	0	0.6	0.2	10	10	2.193	0.032	1.876	0.521	0.118	0.110	0.365	0.251	0.129	0.118
0.6	0.2	0	0	0.6	0	10	10	2.445	0.032	1.816	0.530	0.118	0.108	0.345	0.237	0.135	0.112
0.6	0.2	0	0	0	0	10	10	1.346	0.033	1.633	0.568	0.093	0.084	0.278	0.214	0.159	0.107
0.6	0	0.6	0.2	0.6	0.2	10	10	2.357	0.032	1.577	0.541	0.148	0.111	0.217	0.194	0.143	0.123
0.6	0	0.6	0.2	0.6	0	10	10	2.162	0.032	1.588	0.550	0.143	0.119	0.203	0.188	0.154	0.128
0.6	0	0.6	0.2	0	0	10	10	1.872	0.033	1.601	0.575	0.107	0.095	0.166	0.168	0.169	0.129
0.6	0	0.6	0	0.6	0.2	10	10	2.039	0.032	1.610	0.538	0.143	0.109	0.255	0.197	0.139	0.120
0.6	0	0.6	0	0.6	0	10	10	2.027	0.032	1.596	0.551	0.144	0.119	0.237	0.189	0.154	0.126
0.6	0	0.6	0	0	0	10	10	1.563	0.033	1.594	0.579	0.106	0.095	0.180	0.165	0.168	0.131
0.6	0	0	0	0.6	0.2	10	10	1.883	0.032	1.859	0.528	0.135	0.108	0.366	0.254	0.126	0.117
0.6	0	0	0	0.6	0	10	10	1.911	0.032	1.829	0.534	0.134	0.117	0.355	0.248	0.138	0.123
0.6	0	0	0	0	0	10	10	1.376	0.033	1.641	0.569	0.104	0.094	0.280	0.222	0.164	0.129
0	0	0.6	0.2	0.6	0.2	10	10	2.074	0.035	1.606	0.627	0.214	0.127	0.213	0.182	0.136	0.109
0	0	0.6	0.2	0.6	0	10	10	2.069	0.034	1.610	0.633	0.215	0.148	0.203	0.183	0.152	0.119
0	0	0.6	0.2	0	0	10	10	1.858	0.033	1.622	0.542	0.163	0.144	0.166	0.172	0.171	0.152
0	0	0.6	0	0.6	0.2	10	10	1.684	0.035	1.636	0.637	0.215	0.127	0.257	0.195	0.136	0.109
0	0	0.6	0	0.6	0	10	10	1.724	0.035	1.613	0.645	0.217	0.147	0.239	0.190	0.149	0.117
0	0	0.6	0	0	0	10	10	1.426	0.033	1.632	0.541	0.168	0.148	0.184	0.172	0.172	0.154
0	0	0	0	0.6	0.2	10	10	1.406	0.034	1.824	0.635	0.202	0.118	0.367	0.252	0.124	0.104
0	0	0	0	0.6	0	10	10	1.495	0.034	1.798	0.652	0.207	0.139	0.353	0.249	0.135	0.110
0	0	0	0	0	0	10	10	1.233	0.033	1.650	0.540	0.166	0.148	0.285	0.233	0.168	0.154

Table 15: RMSE of Parameters, $N = 200$, $T = 5$, $\theta = 0.5$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	2.185	0.030	1.279	0.548	0.119	0.103	0.177	0.157	0.132	0.114
0.6	0.2	0.6	0.2	0.6	0	10	10	1.930	0.031	1.291	0.550	0.112	0.101	0.164	0.153	0.134	0.108
0.6	0.2	0.6	0.2	0	0	10	10	1.647	0.031	1.299	0.597	0.093	0.086	0.137	0.136	0.160	0.113
0.6	0.2	0.6	0	0.6	0.2	10	10	2.122	0.031	1.306	0.544	0.117	0.107	0.200	0.162	0.130	0.117
0.6	0.2	0.6	0	0.6	0	10	10	1.840	0.031	1.294	0.539	0.111	0.105	0.182	0.155	0.133	0.112
0.6	0.2	0.6	0	0	0	10	10	1.407	0.031	1.302	0.594	0.092	0.085	0.143	0.131	0.158	0.112
0.6	0.2	0	0	0.6	0.2	10	10	2.079	0.031	1.549	0.540	0.107	0.102	0.321	0.227	0.118	0.112
0.6	0.2	0	0	0.6	0	10	10	2.351	0.031	1.526	0.540	0.106	0.104	0.307	0.218	0.121	0.111
0.6	0.2	0	0	0	0	10	10	1.282	0.031	1.312	0.584	0.088	0.081	0.223	0.173	0.152	0.110
0.6	0	0.6	0.2	0.6	0.2	10	10	2.151	0.030	1.284	0.554	0.139	0.107	0.184	0.164	0.133	0.121
0.6	0	0.6	0.2	0.6	0	10	10	1.997	0.031	1.271	0.563	0.137	0.111	0.170	0.159	0.144	0.123
0.6	0	0.6	0.2	0	0	10	10	1.628	0.031	1.298	0.592	0.106	0.093	0.137	0.138	0.165	0.130
0.6	0	0.6	0	0.6	0.2	10	10	1.995	0.030	1.303	0.553	0.137	0.105	0.209	0.169	0.132	0.118
0.6	0	0.6	0	0.6	0	10	10	1.910	0.031	1.264	0.553	0.134	0.114	0.191	0.162	0.142	0.123
0.6	0	0.6	0	0	0	10	10	1.488	0.031	1.294	0.594	0.107	0.095	0.146	0.136	0.166	0.134
0.6	0	0	0	0.6	0.2	10	10	1.874	0.030	1.569	0.550	0.128	0.104	0.327	0.238	0.120	0.115
0.6	0	0	0	0.6	0	10	10	1.784	0.030	1.541	0.546	0.120	0.111	0.312	0.228	0.125	0.118
0.6	0	0	0	0	0	10	10	1.317	0.031	1.313	0.581	0.099	0.091	0.226	0.184	0.157	0.130
0	0	0.6	0.2	0.6	0.2	10	10	1.819	0.033	1.306	0.665	0.209	0.120	0.176	0.152	0.126	0.102
0	0	0.6	0.2	0.6	0	10	10	1.838	0.033	1.314	0.662	0.211	0.138	0.164	0.152	0.141	0.112
0	0	0.6	0.2	0	0	10	10	1.582	0.031	1.333	0.559	0.164	0.133	0.138	0.141	0.167	0.147
0	0	0.6	0	0.6	0.2	10	10	1.574	0.033	1.334	0.671	0.209	0.121	0.207	0.162	0.125	0.102
0	0	0.6	0	0.6	0	10	10	1.626	0.033	1.324	0.676	0.214	0.142	0.190	0.159	0.140	0.112
0	0	0.6	0	0	0	10	10	1.313	0.031	1.343	0.563	0.167	0.141	0.147	0.144	0.168	0.152
0	0	0	0	0.6	0.2	10	10	1.395	0.032	1.551	0.664	0.195	0.118	0.331	0.237	0.116	0.098
0	0	0	0	0.6	0	10	10	1.427	0.032	1.526	0.669	0.197	0.138	0.312	0.232	0.127	0.107
0	0	0	0	0	0	10	10	1.173	0.031	1.313	0.563	0.161	0.141	0.232	0.203	0.161	0.150

Table 16: RMSE of Parameters, $N = 200$, $T = 10$, $\theta = 0.5$

Configuration								RMSE									
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2	β_0	β_1	σ_μ^2	σ_ν^2	ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B
0.6	0.2	0.6	0.2	0.6	0.2	10	10	2.024	0.022	1.157	0.370	0.097	0.084	0.140	0.127	0.100	0.091
0.6	0.2	0.6	0.2	0.6	0	10	10	1.784	0.023	1.157	0.369	0.090	0.082	0.134	0.121	0.100	0.084
0.6	0.2	0.6	0.2	0	0	10	10	1.476	0.023	1.164	0.402	0.069	0.061	0.113	0.109	0.115	0.076
0.6	0.2	0.6	0	0.6	0.2	10	10	1.965	0.022	1.168	0.368	0.098	0.085	0.157	0.130	0.100	0.089
0.6	0.2	0.6	0	0.6	0	10	10	1.684	0.023	1.157	0.368	0.088	0.082	0.148	0.122	0.099	0.085
0.6	0.2	0.6	0	0	0	10	10	1.121	0.023	1.164	0.402	0.069	0.061	0.121	0.107	0.115	0.077
0.6	0.2	0	0	0.6	0.2	10	10	1.927	0.022	1.361	0.368	0.094	0.083	0.273	0.193	0.094	0.087
0.6	0.2	0	0	0.6	0	10	10	1.635	0.023	1.303	0.366	0.084	0.080	0.250	0.176	0.093	0.084
0.6	0.2	0	0	0	0	10	10	0.981	0.024	1.170	0.399	0.067	0.060	0.194	0.148	0.113	0.076
0.6	0	0.6	0.2	0.6	0.2	10	10	1.844	0.023	1.151	0.375	0.106	0.089	0.143	0.136	0.101	0.094
0.6	0	0.6	0.2	0.6	0	10	10	1.805	0.023	1.166	0.377	0.113	0.091	0.138	0.129	0.111	0.095
0.6	0	0.6	0.2	0	0	10	10	1.462	0.023	1.165	0.399	0.076	0.068	0.113	0.111	0.117	0.093
0.6	0	0.6	0	0.6	0.2	10	10	1.595	0.023	1.165	0.380	0.110	0.086	0.163	0.139	0.101	0.092
0.6	0	0.6	0	0.6	0	10	10	1.672	0.023	1.163	0.377	0.115	0.093	0.156	0.131	0.111	0.096
0.6	0	0.6	0	0	0	10	10	1.167	0.023	1.165	0.398	0.075	0.069	0.121	0.111	0.116	0.095
0.6	0	0	0	0.6	0.2	10	10	1.437	0.023	1.361	0.377	0.104	0.085	0.281	0.204	0.094	0.090
0.6	0	0	0	0.6	0	10	10	1.571	0.023	1.301	0.375	0.108	0.091	0.259	0.189	0.102	0.092
0.6	0	0	0	0	0	10	10	1.026	0.024	1.171	0.395	0.073	0.067	0.195	0.156	0.114	0.093
0	0	0.6	0.2	0.6	0.2	10	10	1.488	0.025	1.174	0.471	0.152	0.085	0.134	0.122	0.091	0.075
0	0	0.6	0.2	0.6	0	10	10	1.559	0.026	1.180	0.477	0.159	0.103	0.132	0.123	0.102	0.080
0	0	0.6	0.2	0	0	10	10	1.388	0.023	1.184	0.368	0.117	0.103	0.114	0.115	0.118	0.109
0	0	0.6	0	0.6	0.2	10	10	1.112	0.025	1.179	0.473	0.151	0.081	0.153	0.124	0.091	0.073
0	0	0.6	0	0.6	0	10	10	1.223	0.026	1.176	0.481	0.160	0.104	0.150	0.126	0.102	0.079
0	0	0.6	0	0	0	10	10	0.995	0.023	1.186	0.369	0.117	0.106	0.122	0.115	0.118	0.111
0	0	0	0	0.6	0.2	10	10	0.939	0.024	1.340	0.467	0.144	0.081	0.275	0.196	0.086	0.071
0	0	0	0	0.6	0	10	10	1.037	0.025	1.277	0.475	0.150	0.101	0.257	0.189	0.095	0.076
0	0	0	0	0	0	10	10	0.822	0.023	1.168	0.369	0.115	0.106	0.197	0.165	0.116	0.111

As the model is relatively rich and relatively high-dimensional in terms of parameters of interest, simulating the whole response surface of tests as in Baltagi, Egger, and Pfaffermayr (2013) would be very time consuming. Therefore, we simulate selected points of that surface in the following way. We present the results of the tests for five true parameter configurations, where each one of the hypotheses is true for exactly one configuration, and there is one configuration for which none of the null hypotheses is true. Moreover, as with the bias and RMSE, we present the corresponding test results for four configurations of the sample size in the combinations $N = \{100, 200\}$ and $T = \{5, 10\}$ and each of these combinations for one of two cases of the relative importance of the cross-sectional variation in the disturbances, $\theta = \{0.25, 0.50\}$.

We present the rejection rates for the corresponding LR tests in Table 17 for $\theta = 0.25$ and in Table 18 for $\theta = 0.5$. The two tables are organized identically in terms of their horizontal and vertical structure. The vertical structure corresponds to true-parameter-sample-size-configuration blocks, and the horizontal structure reflects the corresponding null hypotheses.

In each vertical block, the first parameter configuration does not match with anyone of the four null hypotheses, so the rejection rate is the power of this test (not size adjusted). H_0^1 is true in the second row (and first column) of each vertical block, H_0^2 is true in the third row (and second column) of each vertical block, H_0^3 is true in the fourth row (and third column) of each vertical block, and H_0^4 is true in the fifth row (and fourth column) of each vertical block, so the rejection rate is the size of these tests, which is expected to be close to the nominal test size of 0.05.

Indeed, the results indicate that the rejection rates are close to the nominal test size. In Table 17, for low heterogeneity $\theta = 0.25$, the LR test is slightly undersized for $N = 100$ and $T = 5$ ranging between 0.035 for H_0^1 , 0.04 for H_0^2 , 0.041 for H_0^3 , and 0.05 for H_0^4 . But this improves as N or T increases. In Table 18, when we double the heterogeneity $\theta = 0.5$, the LR test size for $N = 100$ and $T = 5$ ranges between 0.04 for H_0^1 , 0.027 for H_0^2 , 0.047 for H_0^3 , and 0.045 for H_0^4 . So, $H_0^2 : \rho_2^A = 0$ which tests that the second order network effect in the individual time-invariant error is undersized and affected by this heterogeneity increase. Again, this improves as T doubles to 10, where the empirical size for H_0^2 becomes 0.045. It also improves when N doubles to 200, where the empirical size for H_0^2 becomes 0.044. Overall, we conclude that the proposed model can be recommended for both point estimation and testing having demonstrated this with limited Monte Carlo experiments for panel data of small to moderate size.

Table 17: LR Tests

Configuration								$H_0^1: \rho_2^L = 0$	$H_0^2: \rho_2^A = 0$	$H_0^3: \rho_2^B = 0$	$H_0^4: \rho_2^L = \rho_2^A = \rho_2^B = 0$
$N = 100, T = 5, \theta = 0.25$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.212	0.037	0.232	0.942
0.6	0	0.6	0.2	0.6	0.2	5	15	0.035	0.029	0.144	0.290
0.6	0.2	0.6	0	0.6	0.2	5	15	0.200	0.040	0.252	0.935
0.6	0.2	0.6	0.2	0.6	0	5	15	0.086	0.081	0.041	0.569
0.6	0	0.6	0	0.6	0	5	15	0.038	0.039	0.037	0.050
$N = 100, T = 10, \theta = 0.25$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.423	0.087	0.381	0.999
0.6	0	0.6	0.2	0.6	0.2	5	15	0.052	0.073	0.252	0.656
0.6	0.2	0.6	0	0.6	0.2	5	15	0.398	0.036	0.412	0.999
0.6	0.2	0.6	0.2	0.6	0	5	15	0.223	0.113	0.040	0.885
0.6	0	0.6	0	0.6	0	5	15	0.052	0.041	0.055	0.052
$N = 200, T = 5, \theta = 0.25$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.518	0.127	0.390	0.999
0.6	0	0.6	0.2	0.6	0.2	5	15	0.041	0.104	0.270	0.671
0.6	0.2	0.6	0	0.6	0.2	5	15	0.454	0.042	0.396	0.999
0.6	0.2	0.6	0.2	0.6	0	5	15	0.243	0.185	0.040	0.956
0.6	0	0.6	0	0.6	0	5	15	0.041	0.053	0.045	0.042
$N = 200, T = 10, \theta = 0.25$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.741	0.186	0.627	1.000
0.6	0	0.6	0.2	0.6	0.2	5	15	0.056	0.257	0.416	0.999
0.6	0.2	0.6	0	0.6	0.2	5	15	0.690	0.051	0.666	1.000
0.6	0.2	0.6	0.2	0.6	0	5	15	0.449	0.173	0.038	0.922
0.6	0	0.6	0	0.6	0	5	15	0.056	0.058	0.061	0.047

Table 18: LR Tests

Configuration								$H_0^1: \rho_2^L = 0$	$H_0^2: \rho_2^A = 0$	$H_0^3: \rho_2^B = 0$	$H_0^4: \rho_2^L = \rho_2^A = \rho_2^B = 0$
$N = 100, T = 5, \theta = 0.5$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.216	0.091	0.273	0.951
0.6	0	0.6	0.2	0.6	0.2	5	15	0.040	0.064	0.162	0.322
0.6	0.2	0.6	0	0.6	0.2	5	15	0.194	0.027	0.306	0.959
0.6	0.2	0.6	0.2	0.6	0	5	15	0.104	0.125	0.047	0.696
0.6	0	0.6	0	0.6	0	5	15	0.039	0.030	0.047	0.045
$N = 100, T = 10, \theta = 0.5$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.449	0.119	0.397	1.000
0.6	0	0.6	0.2	0.6	0.2	5	15	0.047	0.103	0.275	0.674
0.6	0.2	0.6	0	0.6	0.2	5	15	0.407	0.045	0.448	1.000
0.6	0.2	0.6	0.2	0.6	0	5	15	0.263	0.147	0.052	0.922
0.6	0	0.6	0	0.6	0	5	15	0.051	0.047	0.047	0.052
$N = 200, T = 5, \theta = 0.5$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.487	0.203	0.377	0.999
0.6	0	0.6	0.2	0.6	0.2	5	15	0.037	0.154	0.292	0.722
0.6	0.2	0.6	0	0.6	0.2	5	15	0.430	0.044	0.415	0.999
0.6	0.2	0.6	0.2	0.6	0	5	15	0.291	0.259	0.042	0.983
0.6	0	0.6	0	0.6	0	5	15	0.040	0.057	0.038	0.051
$N = 200, T = 10, \theta = 0.5$											
ρ_1^L	ρ_2^L	ρ_1^A	ρ_2^A	ρ_1^B	ρ_2^B	σ_μ^2	σ_ν^2				
0.6	0.2	0.6	0.2	0.6	0.2	5	15	0.763	0.241	0.650	1.000
0.6	0	0.6	0.2	0.6	0.2	5	15	0.058	0.323	0.439	0.999
0.6	0.2	0.6	0	0.6	0.2	5	15	0.711	0.044	0.694	1.000
0.6	0.2	0.6	0.2	0.6	0	5	15	0.522	0.207	0.040	0.936
0.6	0	0.6	0	0.6	0	5	15	0.056	0.054	0.061	0.057

4 Conclusions

This paper proposes a higher-order network panel-data model with a structure that permits discerning the influence of spatial and network interactions channeled through the presence of endogenous network lags of the dependent variable, exogenous network lags of the explanatory variables, and separate network lags in the time-invariant and the time-variant parts of the disturbances. All of the latter network links can be of the higher-order type and be simultaneously present in the model.

The availability of a model of this kind is desirable, because we often *ex ante* do not know either the exact decay function of network effects with an increasing network (or spatial) distance between the cross sectional units (or the nodes) in the network, nor may we know which ones of several potential channels of network links matter at all or matter more than others. Either lack of knowledge may come at the cost of bias or, at best, efficiency losses.

We found that the proposed model, which is a generalization of the one in Baltagi, Egger, and Pfaffermayr (2013), works remarkably well in terms of bias (and root-mean-squared error) already in modest sample sizes of $N = 100$ and $T = 5$. In practice, the networks typically consist of much more than just 100 cross-sectional units (or nodes). Hence, we are confident that this model can be recommended for applications with real-world data, where it may help unveiling the aforementioned desirable knowledge regarding decaying network links with greater network distance as well as the relative importance of different network channels at work.

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