11-14-2007

Scalar Mesons from an Effective Lagrangian Approach

Joseph Schechter
Department of Physics, Syracuse University, Syracuse, NY

Amir H. Fariborz
State University of New York Institute of Technology

Renata Jora
Syracuse University

on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

Scalar Mesons from an Effective Lagrangian Approach

A.H. Fariborz [1]
Department of Mathematics/Physics
State University of New York Institute of Technology
P.O. Box 3050, Utica, New York 13504-3050, USA.

R. Jora [2]
Physics Department
Syracuse University, Syracuse, New York 13244-1130, USA.

J. Schechter [3]
Physics Department
Syracuse University, Syracuse, New York 13244-1130, USA.

Abstract

A brief discussion of the recent interest in light scalar mesons motivates the study of a generalized linear sigma model. In an SU(3) flavor invariant version of the model there is a prediction that the the lighter scalars have sizeable "four quark" content. It is further predicted that one of the singlet scalars should be exceptionally light. Due to the presence of scalar mesons, the model gives "controlled" corrections to the current algebra formula for threshold pion pion scattering. These corections act in the direction to improve agreement with current experiments.

1 Introduction

A linear sigma model with both fields representing quark-antiquark type composites and fields representing (in an unspecified configuration) two quarks and two antiquarks, seems useful for understanding the light scalar spectrum

[1]email:fariboa@sunyit.edu
[2]email:cjora@phy.syr.edu
[3]Speaker;email:schechte@phy.syr.edu

of QCD. To see this, note that, at present, the scalars below 1 GeV appear to fit into a nonet as:

$$\begin{aligned} I=0: m[f_0(600)] &\approx 500 \text{ MeV} \\ I=1/2: \quad m[\kappa] &\approx 800 \text{ MeV} \\ I=0: m[f_0(980)] &\approx 980 \text{ MeV} \\ I=1: m[a_0(980)] &\approx 980 \text{ MeV} \end{aligned} \tag{1}$$

This level ordering may be compared with that of the most conventional meson nonet- the low lying vector mesons. In fact it is completely flipped compared to that one; the almost degenerate I=0 and I=1 vector states lie lowest rather than highest and the other I=0 state lies highest. The ordering for vectors is conventionally understood as due to the strange quark being heavier than the up and down quarks and the vectors being quark anti-quark composites. The ordering is gotten just by counting the number of strange quarks in each state. It was pointed out a long time ago in Ref. [1], using the same reasoning, that the level order is automatically flipped when the mesons are made of two quarks and two antiquarks instead. That argument was given for a diquark- anti diquark structure but is easily seen to also hold for the meson- meson structure which was advocated for example in Ref. [2]. Thus, on empirical grounds a four quark structure for the light scalars seems very suggestive and well worth investigating.

Now one expects higher mass scalars related to p wave quark antiquark composites to also exist. It is natural to expect mixing between states with the same quantum numbers and there is some phenomenological evidence for this as noted in Refs [3] and [4]. Considering the importance of spontaneously broken chiral symmetry in low energy QCD, it seems interesting to investigate packaging the whole scheme in an effective chiral Lagrangian containing a "two quark" *chiral* nonet (with both scalars and pseudoscalars) as well as a "four quark" *chiral* nonet.

We employ the 3×3 matrix chiral nonet fields;

$$M = S + i\phi, \qquad\qquad M' = S' + i\phi'. \tag{2}$$

Here M represents scalar, S and pseudoscalar, ϕ quark-antiquark type states, while M' represents states which are made of two quarks and two antiquarks. The transformation properties under $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R} \times \mathrm{U(1)_A}$ are

$$M \rightarrow e^{2i\nu}\, U_{\mathrm{L}} M U_{\mathrm{R}}^{\dagger}, \qquad\qquad M' \rightarrow e^{-4i\nu}\, U_{\mathrm{L}} M' U_{\mathrm{R}}^{\dagger}, \tag{3}$$

where U_{L} and U_{R} are unitary unimodular matrices, and the phase ν is associated with the $\mathrm{U(1)_A}$ transformation, which also distinguishes the two quark type from the four quark type fields. However, together with our model, it does not distinguish between different types of four quark configuratons. That question is discussed in more detail in ref. [5]. The general Lagrangian density which defines our model is

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(\partial_\mu M \partial_\mu M^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M' \partial_\mu M'^\dagger\right) - V_0\left(M, M'\right) - V_{SB}, \tag{4}$$

where $V_0(M, M')$ stands for a function made from $\mathrm{SU(3)_L}\times\mathrm{SU(3)_R}$ (but not necessarily $\mathrm{U(1)_A}$) invariants formed out of M and M'. The quantity V_{SB} stands for chiral symmetry breaking terms which transform in the same way as the quark mass terms in the fundamental QCD Lagrangian. The model was proposed in section V of ref. [6] and followed up in refs. [7], [8], [9], [10] and [11], the last two of which will be briefly described here. Related models for thermodynamic properties of QCD are discussed in refs. [13]. In [10], we focused on general properties which continued to hold when V_{SB} was set to zero while in [11] we included the SU(3) symmetric mass term:

$$V_{SB} = -2\, A\, \mathrm{Tr}(S) \tag{5}$$

where A is a real parameter.

A characteristic feature of the model is the presence of "two-quark" and "four-quark" condensates:

$$\left\langle S_a^b \right\rangle = \alpha_a \delta_a^b, \qquad \left\langle S_a'^b \right\rangle = \beta_a \delta_a^b. \tag{6}$$

We shall assume the vacuum to be $\mathrm{SU(3)_V}$ invariant, which implies

$$\alpha_1 = \alpha_2 = \alpha_3 \equiv \alpha, \qquad\qquad \beta_1 = \beta_2 = \beta_3 \equiv \beta. \tag{7}$$

The SU(3) particle content of the model consists of two pseudoscalar octets, two pseudoscalar singlets, two scalar octets and two scalar singlets. This gives us eight different masses and four mixing angles.

Note that the transformation between the diagonal fields (π^+ and π'^+) and the original pion fields is given as:

$$\begin{bmatrix} \pi^+ \\ \pi'^+ \end{bmatrix} = \begin{bmatrix} \cos\theta_\pi & -\sin\theta_\pi \\ \sin\theta_\pi & \cos\theta_\pi \end{bmatrix} \begin{bmatrix} \phi_1^2 \\ \phi_1'^2 \end{bmatrix}.$$

Thus 100 $\sin^2\theta_\pi$ represents the four quark percentage of the ordinary pion while 100 $\cos^2\theta_\pi$ represents the four quark percentage of the "heavy" pion in the model. Also of relevance is the one particle piece of the isovector axial vector current,

$$(J_\mu^{axial})_1^2 = F_\pi \partial_\mu \pi^+ + F_{\pi'} \partial_\mu \pi'^+ + \cdots, \tag{8}$$

where,

$$
\begin{aligned}
F_\pi &= 2\,\alpha\cos\theta_\pi - 2\,\beta\sin\theta_\pi, \\
F_{\pi'} &= 2\,\alpha\sin\theta_\pi + 2\,\beta\cos\theta_\pi.
\end{aligned} \tag{9}
$$

Note that $\tan\theta_\pi = -\beta/\alpha$ when the pion is massless.

2 Specific Lagrangian

As discussed in ref. [10] one may obtain certain general results from tree level Ward identities without restrictions on the form of the potential V_0. For a complete description however, a specific form must be furnished. The leading choice of terms corresponding to eight or fewer quark plus antiquark lines at each effective underlying vertex reads [10]:

$$
\begin{aligned}
V_0 = \; & - \; c_2\,\mathrm{Tr}(MM^\dagger) + c_4^a\,\mathrm{Tr}(MM^\dagger MM^\dagger) \\
& + \; d_2\,\mathrm{Tr}(M'M'^\dagger) + e_3^a(\epsilon_{abc}\epsilon^{def}M_d^aM_e^bM_f'^c + h.c.) \\
& + \; c_3\left[\gamma_1\ln(\frac{\det M}{\det M^\dagger}) + (1-\gamma_1)\frac{\mathrm{Tr}(MM'^\dagger)}{\mathrm{Tr}(M'M^\dagger)}\right]^2 .
\end{aligned} \tag{10}
$$

All the terms except the last two have been chosen to also possess the $\mathrm{U}(1)_\mathrm{A}$ invariance. Further details of the $\mathrm{U}(1)_\mathrm{A}$ aspect are given in ref. [10].

As the corresponding experimental inputs [12] we take the non-strange quantities:

$$
\begin{aligned}
m(0^+\mathrm{octet}) &= m[a_0(980)] = 984.7 \pm 1.2\,\mathrm{MeV} \\
m(0^+\mathrm{octet}') &= m[a_0(1450)] = 1474 \pm 19\,\mathrm{MeV} \\
m(0^-\mathrm{octet}') &= m[\pi(1300)] = 1300 \pm 100\,\mathrm{MeV} \\
m(0^-\mathrm{octet}) &= m_\pi = 137\,\mathrm{MeV} \\
F_\pi &= 131\,\mathrm{MeV}
\end{aligned} \tag{11}
$$

Clearly m[π(1300)] has a large uncertainty and will essentially be regarded as a free parameter.

Predictions for the two unspecified 0^+ masses are shown in Fig.1. The lighter of these clearly invites us to identify it as the sigma. Predictions for the four quark contents of the lightest four SU(3) multiplets are shown in Fig.2. Clearly the lighter 0^- octet is primarily two quark and the lighter 0^- singlet has a primarily two quark solution. On the other hand, both the lighter 0^+ octet and singlet have large four quark content!

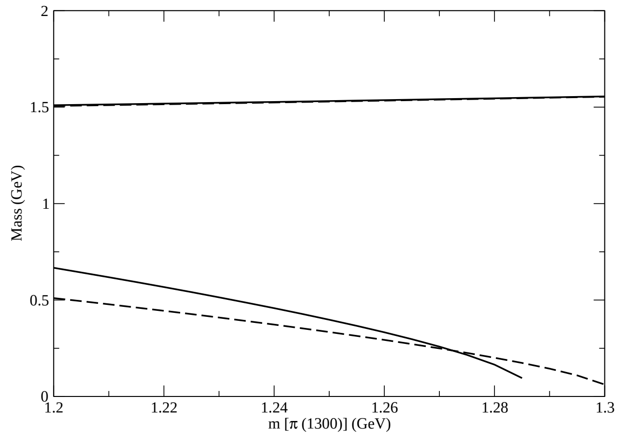


Figure 1: The predictions for the masses of the two SU(3) singlet scalars vs. $m[\pi(1300)]$. The solid lines correspond to the massive pion case while the dashed lines correspond to the massless pion case.

3 Pion pion scattering

The tree level form for the conventional Mandelstam amplitude in the present model is:

$$A(s,t,u) = -\frac{g}{2} + \sum_i \frac{g_i^2}{m_i^2 - s}, \tag{12}$$

where g is the four pion coupling constant, the four g_i's are the three-point coupling constants of two pions with each of the four scalar isosinglets and the m_i represent the masses of the four scalar isosinglets. To really understand what is happening we should expand in powers of $(s - m_\pi^2)$:

$$\begin{aligned} A(s,t,u) &= -\frac{g}{2} + \sum_i \frac{g_i^2}{m_i^2 - m_\pi^2}\left[1 + \frac{s-m_\pi^2}{m_i^2 - m_\pi^2} + (\frac{s-m_\pi^2}{m_i^2 - m_\pi^2})^2 + \cdots\right] \\ &\approx (s - m_\pi^2)\left[\frac{2}{F_\pi^2} + (s - m_\pi^2)\sum_i \frac{g_i^2}{(m_i^2 - m_\pi^2)^3} + \cdots\right]. \end{aligned} \tag{13}$$

The exact first equation contains, for each m_i, a geometrical expansion in the quantity $(s - m_\pi^2)/(m_i^2 - m_\pi^2)$. Thus the radius of convergence in s for this expression is the squared mass of the lightest scalar isosinglet. To apply this expression in the resonance region we must, of course, unitarize the formula in some way. Here we will look at the threshold region. In going from the first

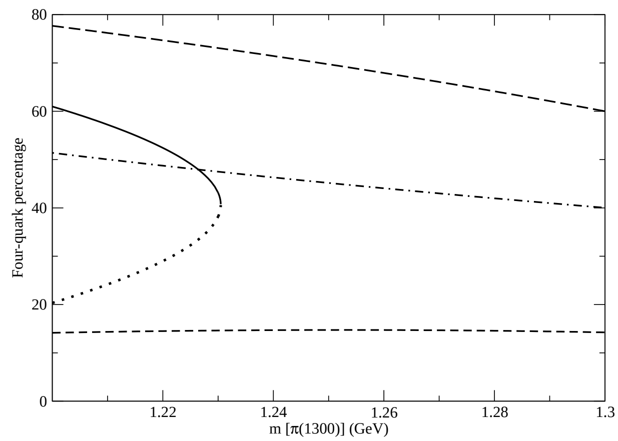


Figure 2: Four quark percentages of the pion (dashed line), the $a_0(980)$ (top long-dashed line), the very light 0^+singlet (dotted-dashed line) and the $\eta(958)$ in the scenario where the higher state is identified as the $\eta(1475)$ (curve containing both solid and dotted pieces) as functions of the undetermined input parameter, $m[\pi(1300)]$. Note that there are two solutions for the $\eta(958)$: the dotted curve choice gives it a predominant two quark structure and the solid curve choice, a larger four quark content.

to the second equation of Eq.(13) we used the facts established in ref. [10] for the $m_\pi = 0$ case and in ref. [11] (in the very good approximation where $F_{\pi'} = 0$) for the massive pion case that: 1) the sum of the first two terms of the first equation vanishes and 2) the third term of the first equation simplifies to becomes the first, current algebra, term of the second equation. These results hold for any chiral symmetric choice of the potential. The third term of the second equation represents the model dependent leading correction to the usual current algebra formula. It depends on the masses of the scalar mesons and would vanish in a hypothetical limit (often used) in which the scalar meson masses are taken to infinity. At low energies the third term is seen to be suppressed by order $(m_\pi/m_i)^2$ compared to the current algebra term.

For comparison, we give the usual current algebra results [14] for the two s-wave scattering lengths:

$$\begin{aligned} m_\pi a_0^0 &= \frac{7m_\pi^2}{16\pi F_\pi^2} \approx 0.15, \\ m_\pi a_0^2 &= \frac{-2m_\pi^2}{16\pi F_\pi^2} \approx -0.04. \end{aligned} \tag{14}$$

Recent experimental data on the s-wave scattering lengths a_0^0 and a_0^2 include the following,

NA48/2 collaboration [15]:

$$m_{\pi^+}(a_0^0 - a_0^2) = 0.264 \pm 0.015 \tag{15}$$

$$m_{\pi^+} a_0^0 = 0.256 \pm 0.011 \tag{16}$$

E865 Collaboration [16]:

$$m_{\pi^+} a_0^0 = 0.216 \pm 0.015 \tag{17}$$

DIRAC Collaboration [17]

$$m_{\pi^+} a_0^0 = 0.264^{+0.038}_{-0.020} \tag{18}$$

Clearly, the current algebra value of a_0^0 is lower than experiment. Including the corrections from non-infinite scalar meson masses with the choice of V_0 in Eq.(10), yields the results shown in Fig.3. The corresponding lightest scalar mass for each value of m[$\pi(1300)$] can be read from Fig. 1. The value of a_0^2 for the non resonant channel is not altered much but it can be seen that the prediction for a_0^0 is definitely improved.

4 Further discussion

1. A criticism of linear sigma models is that they obtain the current algebra result as an almost complete cancellation of large quantities. This may be seen in the left panel of Fig. 4 which shows the five different terms in Eq.(12) appearing to almost cancel in a haphazard pattern. On the other hand, once we make the Taylor expansion in Eq.(13), the corrections to the current algebra result are seen in the right panel to be completely dominated by the lightest scalar; this contribution is suppressed, as noted above, by $(m_\pi/m_\sigma)^2$.

2. We plan to increase the accuracy of our model by including SU(3) breaking effects and also by unitarizing the model so that it would be suitable also in the energy region around the scalar resonances. The simplest

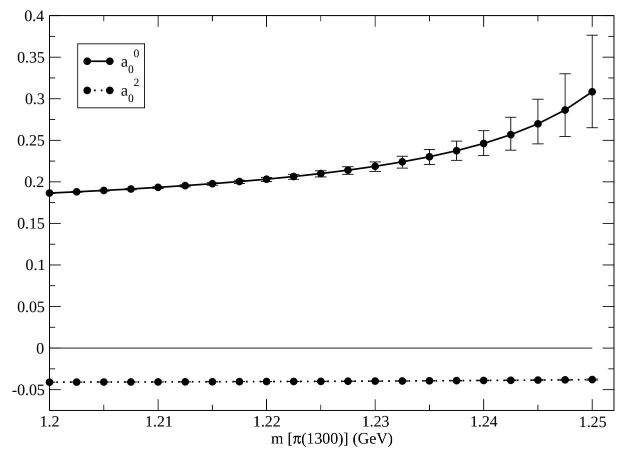
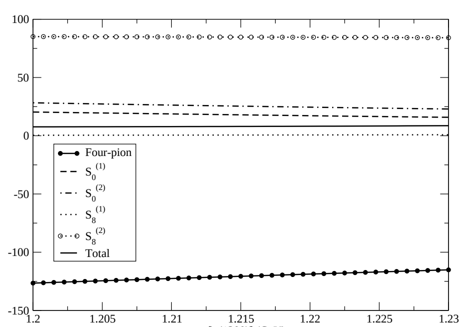


Figure 3: Top curve: $I = J = 0$ scattering length, $m_\pi a_0^0$ vs. $m[\pi(1300)]$. Bottom curve: $I = 2$, $J = 0$ scattering length, $m_\pi a_0^2$ vs. $m[\pi(1300)]$. The error bars reflect the uncertainty of $m[a_0(1450)]$.



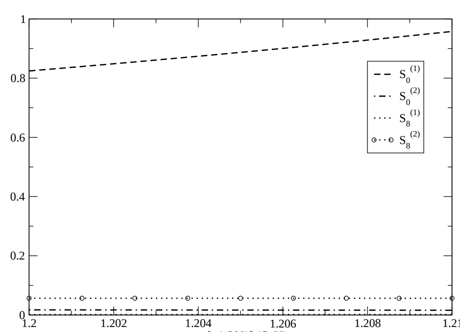


Figure 4: Left: Individual contributions in Eq.(12) to $A(s,t,u)$ at threshold. Right: Individual contributions to the second of Eq.(13) to $A(s,t,u)$ at threshold.

unitarization is the K-matrix type. It was caried out for the two flavor linear sigma model in ref. [18]. We expect the results for the present model to be generally similar to the ones obtained in ref. [6] for the single M three flavor linear model.

3. A possible general question about the present model is that it introduces both states made of a quark and an antiquark as well as states with two quarks and two antiquarks. According to the usual 't Hooft large N_c extrapolation [19] of QCD the "four quark" states are expected to be suppressed. However, it was recently pointed out [20] that the alternative, mathematically allowed, Corrigan Ramond [21] extrapolation does not suppress the multiquark states. This kind of extrapolation may be relevant for understanding the physics of the light scalar mesons.

Acknowledgments

We are happy to thank A. Abdel-Rehim, D. Black, M. Harada, S. Moussa, S. Nasri and F. Sannino for many helpful related discussions. The work of A.H.F. has been partially supported by the NSF Award 0652853. The work of R.J. and J.S. is supported in part by the U. S. DOE under Contract no. DE-FG-02-85ER 40231.

References

[1] R.L. Jaffe, Phys. Rev. D **15**, 267 (1977).

[2] J.D. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982).

[3] D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. D **61** 074001 (2000).

[4] T. Teshima, I. Kitamura and N. Morisita, J. Phys. G **28**, 1391 (2002); *ibid* **30**, 663 (2004); F. Close and N. Tornqvist, *ibid.* **28**, R249 (2002); A.H. Fariborz, Int. J. Mod. Phys. A **19**, 2095 (2004); 5417 (2004); Phys. Rev. D **74**, 054030 (2006); F. Giacosa, Th. Gutsche, V.E. Lyubovitskij and A. Faessler, Phys. Lett. B **622**, 277 (2005); J. Vijande, A. Valcarce, F. Fernandez and B. Silvestre-Brac, Phys. Rev. D **72**, 034025 (2005); S. Narison, Phys. Rev. D **73**, 114024 (2006); L. Maiani, F. Piccinini, A.D. Polosa and V. Riquer, hep-ph/0604018; J.R. Pelaez, Phys. Rev. Lett. **92**, 102001 (2004); J.R. Pelaez and G. Rios, Phys. Rev. Lett. **97**, 242002 (2006).

[5] Y. S. Kalashnikova, A. E. Kudryavtsev, A. V. Nefediev, C. Hanhart and J. Haidenbauer, Eur. Phys. J. A **24** (2005) 437.

[6] D. Black, A.H. Fariborz, S. Moussa, S. Nasri and J. Schechter, Phys. Rev. D **64**, 014031 (2001).

[7] M. Napsuciale and S. Rodriguez, Phys. Rev. D **70**, 094043 (2004).

[8] A.H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D **72**, 034001 (2005).

[9] A.H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D **76**, 014011 (2007).

[10] A.H. Fariborz, R. Jora and J. Schechter, arXiv:0707.0843 [hep-ph].

[11] A.H. Fariborz, R. Jora and J. Schechter, arXiv:0708.3402 [hep-ph].

[12] W-M Yao et al, J. Phys. G: Nucl. Part. Phys. **33**, 1 (2006).

[13] N. Yamamoto, M. Tachibana, T. Hatsuda and G. Baym, Phys. Rev. D **76**, 074001 (2007), arXiv:0704.2654 [hep-ph]; A.A. Andrianov and D. Espriu, arXiv:0709.0049 [hep-ph].

[14] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).

[15] J.R. Betley et al [NA48/2 Collaboration], Phys. Lett. B **633**, 173 (2006) [arXiv:hep-ex/0511056].

[16] S. Pislak et al [E865 Collaboration], Phys. Rev. D **67**, 072004 (2003) [arXiv: hep-ex/0301040].

[17] B. Adeva et al [DIRAC Collaboration], Phys. Lett. B **619**, 50 (2005) [arXiv:hep-ex/0504044].

[18] N.N. Achasov and G.N. Shestakov, Phys. Rev. D **49**, 5779 (1994).

[19] G.'t Hooft, Nucl. Phys. B **72**, 461 (1974).

[20] F. Sannino and J. Schechter, Phys. Rev. D **76**, 014014 (2007), arXiv:0704.0602 [hep-ph].

[21] E. Corrigan and P. Ramond, Phys. Lett. B **87**, 73 (1979).