10-25-2008

Exploration of a Physical Picture for the QCD Scalar Channel

Joseph Schechter
Department of Physics, Syracuse University, Syracuse, NY

Amir H. Fariborz
State University of New York Institute of Technology

Renata Jora
Syracuse University

arXiv:0810.4640v1 [hep-ph] 25 Oct 2008

SU-4252-879

Exploration of a physical picture for the QCD scalar channel

Amir H. Fariboz [a ‡], Renata Jora [b †], and Joseph Schechter [c §]
[a] Department of Mathematics/Science, State University of New York Institute of Technology, Utica, NY 13504-3050, USA. and
[b,c] Department of Physics, Syracuse University, Syracuse, NY 13244-1130, USA,

(Dated: October 28, 2008)

A generalized linear sigma model is employed to study the quark structure of low lying scalar as well as pseudoscalar states. The model allows the possible mixing of quark anti-quark states with others made of two quarks and two antiquarks but no *a priori* assumption is made about the quark contents of the predicted physical states. Effects of SU(3) symmetry breaking are included. The lighter conventional pseudoscalars turn out to be primarily of two quark type whereas the lighter scalars turn out to have very large four quark admixtures.

PACS numbers:

I. INTRODUCTION

Recently the Belle collaboration [1] provided strong evidence for the Z(4430) resonance in the Ψ'- π channel. It has the quantum numbers of $c\bar{c}n\bar{n}$,where n stands for either a u or d quark, and would thus seem to be a "smoking gun" candidate for a meson containing two quarks and two antiquarks. Are there others? Many people believe that the mass ordering of the candidates for the lightest scalar nonet suggests such a picture:

$$\begin{aligned} I=0: m[f_0(600)] &\approx 500 \text{ MeV} \qquad n\bar{n}n\bar{n} \\ I=1/2: \qquad m[\kappa] &\approx 800 \text{ MeV} \qquad n\bar{n}n\bar{s} \\ I=0: m[f_0(980)] &\approx 980 \text{ MeV} \qquad n\bar{n}s\bar{s} \\ I=1: m[a_0(980)] &\approx 980 \text{ MeV} \qquad n\bar{n}s\bar{s} \end{aligned} \tag{1}$$

Here the postulated four quark content is displayed for each state. This level ordering, obtained by simply counting the number of strange (s) type quarks, is seen to be flipped [2] compared to that of the standard vector meson nonet:

$$\begin{aligned} I=1: m[\rho(776)] &\approx 776 \text{ MeV} \qquad n\bar{n} \\ I=0: m[\omega(783)] &\approx 783 \text{ MeV} \qquad n\bar{n} \\ I=1/2: m[K^*(892)] &\approx 892 \text{ MeV} \qquad n\bar{s} \\ I=0: m[\phi(1020)] &\approx 1020 \text{ MeV} \qquad s\bar{s} \end{aligned} \tag{2}$$

Note that the level inversion of four quark states would hold either for "molecular" or diquark- antidiquark pictures.

There is another side to this story. Why are the experimental candidates for a "normal" p-wave $q\bar{q}$ scalar nonet e.g.,

$$a_0(1450), K_0(1430) etc., \tag{3}$$

somewhat heavier than expected? A possible answer, based on the repulsion of"two quark" and "four quark" states which mix with each other, was proposed some time ago [3], see also [4]. This level repulsion also would explain why the lower scalars seem to be unusually light.

II. TOY MODEL TO CHECK MIXING PICTURE

Note that QCD with massless light quarks obeys SU(3)$_L$ x SU(3)$_R$ symmetry spontaneously broken to SU(3)$_V$. We wish to realize this in a Lagrangian model [5] with linearly transforming chiral nonet fields (There are necessarily

‡ Email: fariboa@sunyit.edu
† Email: cjora@physics.syr.edu
§ Speaker;Email: schechte@physics.syr.edu

two scalar nonets as well as two pseudoscalar nonets present in this approach). We input some physical particle masses and predict the two vs. four quark contents of each state. The non-inputed masses are also predicted. To check the stability of the approach we have carried out the calculations for the zero mass [6] quark case, the non-zero equal quark mass case [7] and finally the non equal quark mass case. The method is conceptually straightforward but complicated in detail.

We employ the 3×3 matrix chiral nonet fields:

$$M = S + i\phi, \qquad\qquad M' = S' + i\phi'. \tag{4}$$

The matrices M and M' transform in the same way under chiral SU(3)x SU(3) transformations but may be distinguished by their different $\mathrm{U}(1)_A$ transformation properties. M desribes the "bare" quark antiquark scalar and pseudoscalar nonet fields while M' describes "bare" scalar and pseudoscalar fields containing two quarks and two antiquarks. At the symmetry level with which we are working, it is is unnecessary to further specify the four quark field configuration.

The Lagrangian density is:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(\partial_\mu M \partial_\mu M^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M' \partial_\mu M'^\dagger\right) \\ -V_0\left(M, M'\right) - V_{SB}, \tag{5}$$

where $V_0(M, M')$ stands for a function made from $\mathrm{SU(3)_L}\times\ \mathrm{SU(3)_R}$ (but not necessarily $\mathrm{U(1)_A}$) invariants formed out of M and M'. The leading choice of terms corresponding to eight or fewer quark plus antiquark lines at each effective vertex reads:

$$\begin{aligned} V_0 &= -c_2\,\mathrm{Tr}(MM^\dagger) + c_4^a\,\mathrm{Tr}(MM^\dagger MM^\dagger) \\ &+d_2\,\mathrm{Tr}(M'M'^\dagger) + e_3^a(\epsilon_{abc}\epsilon^{def}M_d^a M_e^b M_f'^c + h.c.) \\ &+c_3\left[\gamma_1 \ln(\frac{\det M}{\det M^\dagger}) + (1-\gamma_1)\frac{\mathrm{Tr}(MM'^\dagger)}{\mathrm{Tr}(M'M^\dagger)}\right]^2 . \end{aligned} \tag{6}$$

All the terms except the last two (more discussion of them is given in [8]) possess the $\mathrm{U(1)_A}$ invariance. The symmetry breaking term which models the QCD mass term takes the form:

$$V_{SB} = -2\,\mathrm{Tr}(A\,S) \tag{7}$$

where $A = diag(A_1, A_2, A_3)$ is proportional to the three light quark mass matrix, $diag(m_u, m_d, m_s)$. The model allows for two quark condensates, $\alpha_a = \langle S_a^a \rangle$ as well as four quark condensates $\beta_a = \langle {S'}_a^a \rangle$. These are assumed to obey isotopic spin symmetry:

$$\alpha_1 = \alpha_2 \neq \alpha_3, \qquad\qquad \beta_1 = \beta_2 \neq \beta_3 \tag{8}$$

We also need the "minimum" conditions,

$$\left\langle \frac{\partial V_0}{\partial S} \right\rangle + \left\langle \frac{\partial V_{SB}}{\partial S} \right\rangle = 0, \qquad \left\langle \frac{\partial V_0}{\partial S'} \right\rangle = 0. \tag{9}$$

There are twelve parameters describing the Lagrangian and the vacuum. These include the six coupling constants given in Eq.(6), the two quark mass parameters, $(A_1 = A_2, A_3)$ and the four vacuum parameters $(\alpha_1 = \alpha_2, \alpha_3, \beta_1 = \beta_2, \beta_3)$.The four minimum equations reduce the number of needed input parameters to eight.

Five of these eight are supplied by the following masses together with the pion decay constant:

$$\begin{aligned} m[a_0(980)] &= 984.7 \pm 1.2\,\mathrm{MeV} \\ m[a_0(1450)] &= 1474 \pm 19\,\mathrm{MeV} \\ m[\pi(1300)] &= 1300 \pm 100\,\mathrm{MeV} \\ m_\pi &= 137\,\mathrm{MeV} \\ F_\pi &= 131\,\mathrm{MeV} \end{aligned} \tag{10}$$

Because $m[\pi(1300)]$ has such a large uncertainty, we will, as previously, examine predictions depending on the choice of this mass within its experimental range. The sixth input will be taken as the light "quark mass ratio" A_3/A_1, which will be varied over an appropriate range. The remaining two inputs will be taken from the masses of the four (mixing) isoscalar, pseudoscalar mesons. This mixing is characterized by a 4x4 matrix M_η^2. A practically convenient choice is to consider TrM_η^2 and detM_η^2 as the inputs. Note that the presence of the last two terms in Eq.(6)- which exactly mock up the QCD U(1)$_A$ anomaly - decouples the initial treatment of the other particles from that of the complicated pseudoscalar singlet sector.

Given these inputs there are a very large number of predictions. At the level of the quadratic terms in the Lagrangian, we predict all the remaining masses and decay constants as well as the angles describing the mixing between each of (π, π'), (K, K'), (a_0, a_0'), (κ, κ') multiplets and each of the 4×4 isosinglet mixing matrices (each formally described by six angles).

III. BRIEF SUMMARY OF RESULTS

A detailed report on the results when the SU(3) flavor symmetry breaking, which yields $m_K \neq m_\pi$, is taken into account will be given elsewhere (in preparation). Here we just mention some main features.

It is comforting to first note that the appropriate predicted results do not change much as one proceeds from zero quark masses (and hence, by spontaneously broken chiral symmetry, zero masses for the lighter pseudoscalar octet) to non-zero but degenerate light quark masses (and hence, the lighter pseudoscalar masses all taking the value, m_π) and finally to the realistic case where m_π, m_K and m_η all differ from each other. The zero quark mass case is an especially important "touchstone" since it was noted in the second of ref.[5] that there are actually twenty one different allowed terms which might replace the symmetry breaker, Eq.(7). Hence, without such a check, one might worry that the somewhat surprising results obtained could be an artifact of a particular choice of symmetry breaking terms.

In the zero light mass case the only change in the inputs of Eq.(10) is to set $m_\pi = 0$. A suitable choice for the "adjustable" parameter, m[π(1300)] turned out [6] to be 1215 MeV. Then the masses of the two scalar SU(3) singlets are predicted as,

$$m_\sigma \approx 450 MeV \qquad m_{\sigma'} \approx 1500 MeV. \tag{11}$$

Clearly the lighter SU(3) singlet is the abnormally light $f_0(600)$ candidate. These two states turn out to be roughly the linear combinations,

$$\frac{1}{\sqrt{6}}[(S_1^1 + S_2^2 + S_3^3) \pm ({S'}_1^1 + {S'}_2^2 + {S'}_3^3)], \tag{12}$$

so the "sigma" appears to be 50 per cent two quark and 50 percent four quark in nature. As for the other SU(3) multiplets, the model predicts the 2 quark [2] and four quark [4] contents as roughly :

$$\begin{aligned} &lighter\, 0^+\, octet:\ 0.24[2]\ 0.76[4] \\ &lighter\, 0^-\, octet:\ 0.83[2]\ 0.17[4] \end{aligned} \tag{13}$$

The difference between the mainly two quark lighter 0^- octet and the mainly four quark lighter 0^+ octet is evident. Since, for example, the lighter and heavier 0^+ octets mix with each other, the heavier 0^+ octet would have a 24 percent four quark content and a 76 percent two quark content.

These results are not essentially changed in the case [7] when the light 0^- octet has the mass, m_π instead of mass zero.

To trace what happens in the scalar isosinglet sector when the SU(3) symmetry breaking is turned on, we note that all four such states (i.e. including the appropriate SU(3) octet members) will mix with each other. We use the convenient basis fields:

$$\frac{S_1^1 + S_2^2}{\sqrt{2}},\ S_3^3,\ \frac{{S'}_1^1 + {S'}_2^2}{\sqrt{2}},\ {S'}_3^3, \tag{14}$$

and label the four scalar isosinglets, which are linear combinations of the above, in order of increasing mass as f_1, f_2, f_3, f_4. The lightest, f_1 is identified with the "sigma" and is predicted to have a mass about 730 MeV which is qualitatively similar to that of the previous lighter scalar SU(3) singlet state. It is predicted to have percentages of the basis states in Eq.(14)

$$0.38,\ 0.06,\ 0.32,\ 0.24. \tag{15}$$

Thus we may give the 2 quark and 4 quark percentages of the "sigma" as

$$f_1:\ 0.44[2],\ 0.56[4], \tag{16}$$

which is similar to the previous 50-50 split. The predicted two quark vs. four quark percentages for some other lighter particles in this model are,

$$\begin{aligned}
&\pi: \; 0.85[2], \; 0.15[4] \\
&K: \; 0.86[2], \; 0.14[4] \\
&\kappa: \; 0.09[2], \; 0.91[4].
\end{aligned} \tag{17}$$

These are again similar to those in Eq.(13). It seems as though, large four quark content for the lighter scalar states is a stable result of the present model.

IV. PHYSICAL INTERPRETATION OF SCALAR MASSES

The masses obtained above appear as tree level quantities in the effective Lagrangian under discussion. Especially in the case of the scalars the physical states are rather broad and will appear as poles in the scattering of two pseudoscalar mesons. A simple way to estimate the scattering amplitude is to first compute the tree level scalar partial wave scattering amplitude and then unitarize it by using the K-matrix method. This is equivalent to an earlier approach [9] and amounts to replacing the tree level amplitude, T_{tree} by,

$$T = \frac{T_{tree}}{1 - iT_{tree}}. \tag{18}$$

For example, the sigma pole at 730 MeV discussed in the last section appears in the unitarized pion scattering amplitude at $z \equiv M^2 - iM\Gamma$ with

$$M = 473MeV, \; \Gamma = 473MeV. \tag{19}$$

This is of the same order as usual estimates for the sigma. Such scattering calculations should be performed to find the "actual" mass and width parameters for all the scalars in the present model.

Acknowledgments

One of us (JS) would like to thank the organizers of QCD08 for arranging such a stimulating and friendly conference. We are happy to thank A. Abdel-Rehim, D. Black, M. Harada, S. Moussa, S. Nasri and F. Sannino for many helpful related discussions. This work was supported in part by the U. S. DOE under contract no. DE-FG-02-85ER 40231.

[1] S.-K.Choi et al, arXiv:0708.1790.
[2] R.L. Jaffe, Phys. Rev. D **15**, 267 (1977);D. Black, A.H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. **D59**, 074026 (1999).
[3] D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. D **61** 074001 (2000).
[4] T. Teshima, I. Kitamura and N. Morisita, J. Phys. G **28**, 1391 (2002); *ibid* **30**, 663 (2004); F. Close and N. Tornqvist, *ibid.* **28**, R249 (2002); A.H. Fariborz, Int. J. Mod. Phys. A **19**, 2095 (2004); 5417 (2004); Phys. Rev. D **74**, 054030 (2006); M. Napsuciale and S. Rodriguez, Phys. Rev. D **70**, 094043 (2004); F. Giacosa, Th. Gutsche, V.E. Lyubovitskij and A. Faessler, Phys. Lett. B **622**, 277 (2005); J. Vijande, A. Valcarce, F. Fernandez and B. Silvestre-Brac, Phys. Rev. D **72**, 034025 (2005); S. Narison, Phys. Rev. D **73**, 114024 (2006); L. Maiani, F. Piccinini, A.D. Polosa and V. Riquer, hep-ph/0604018; J.R. Pelaez, Phys. Rev. Lett. **92**, 102001 (2004); J.R. Pelaez and G. Rios, Phys. Rev. Lett. **97**, 242002 (2006); F. Giacosa, Phys. Rev. D **75**,054007 (2007); G. 't Hooft, G. Isidori, L. Maiani, A.D. Polosa and V. Riquer, arXiv:0801.2288[hep-ph].
[5] Section V of D. Black, A.H. Fariborz, S. Moussa, S. Nasri and J. Schechter, Phys. Rev. D **64**, 014031 (2001); A.H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D **72**, 034001 (2005).
[6] A.H.Fariborz, R.Jora and J.Schechter, Phys. Rev. D **77**, 034006 (2008), arXiv:0707.0843 [hep-ph].
[7] A.H.Fariborz, R.Jora and J.Schechter, Phys. Rev. D **76**, 114001 (2007), arXiv:0708.3402 [hep-ph].
[8] A.H.Fariborz,R.Jora and J.Schechter, Phys. Rev. D **77**, 094004 (2008), arXiv:0801.2552 [hep-ph].
[9] N.N. Achasov and G.N. Shestakov, Phys. Rev. D **49**, 5779 (1994).