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Semi-Leptonic D_s^+(1968) Decays as a Scalar Meson Probe

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Semi-leptonic $D_s^+(1968)$ decays as a scalar meson probe

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The unusual multiplet structures associated with the light spin zero mesons have recently attracted a good deal of theoretical attention. Here we discuss some aspects associated with the possibility of getting new experimental information on this topic from semi-leptonic decays of heavy charged mesons into an isosinglet scalar or pseudoscalar plus leptons.

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I. INTRODUCTION

For the first years after the quark model was accepted it was believed that the lightest scalar meson should be a quark-antiquark composite with mass value similar to those of the tensor and axial vector mesons. In particular, an occasionally discussed light, broad "sigma meson" was not expected to exist. However, more recent work has provided evidence for such a particle as well as for other similar light scalars. (Some characteristic references are [1] - [38].)

In fact there seem to be enough scalar candidates to fill up two different nonets. A model including these states, with also two nonets of low lying pseudoscalars in order to form chiral multiplets has been studied in some detail; [21], [22], [27], [30], [32]-[36], [38]. Note that chiral symmetry is the exact symmetry of QCD with massless quarks. Adding "soft" quark mass terms results in "sigma models" which give many reasonable low energy predictions. In the models just mentioned one chiral (i.e. containing both scalars and pseudoscalars) nonet is supposed to represent states with a quark-antiquark substructure while the other nonet is supposed to represent states with a two quark - two antiquark substructure. The physical states are suitable linear combinations.

On the experimental side of the subject, information on the light scalars has often been extracted from study of pion pion and other scattering processes. Another way is to search for scalar resonances explicitly in particle decay processes. Recently, the CLEO collaboration has reported [39] good evidence for the scalar $f_0(980)$ in the semi-leptonic decay of the $D_s^+(1968)$ meson. Since there is more phase space available, it may be possible to find other scalar isosinglet states in this and similar semi-leptonic decays of heavy mesons. There are also isosinglet pseudoscalar states like the η and $\eta'(980)$ which can be studied and in fact have been already reported in the decays of the $D_s^+(1968)$.

As a possibly helpful adjunct to future work in this direction we will, in the present paper, make some theoretical estimates of the semi-leptonic decay widths of the $D_s^+(1968)$ into the four scalar isosinglet states and the four pseudoscalar isosinglet states which are predicted in the chiral model mentioned above.

In section II we discuss the hadronic "weak currents" which are needed for the calculation. These are mathematically given by the so-called Noether currents of the sigma model Lagrangian being employed. We work in the approximation where renormalization of these currents from the symmetry limit are neglected. This means that there are no arbitrary parameters available to us. Nevertheless there are some subtleties. To explain these we build up the model in three stages rather than just writing the final result immediately.

In section III we give a detailed description of the calculation of the partial decay widths from the currents discussed in section II. For this purpose we also use information on the scalar and pseudoscalar meson masses and mixings obtained in [36].

A short summary and discussion is given in section IV.

In Appendix A we briefly discuss the well known $K\ell 3$ decay which has the same general structure as the semi-leptonic D_s decays.

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II. HADRONIC CURRENTS IN VARIOUS LINEAR SIGMA MODELS

These models give the usual "current algebra" results near the threshold of pion-pion scattering but also yield some additional interesting features away from threshold.

A. Chiral SU(3) model

The usual chiral nonet $M(x)$ realizing the $q\bar{q}$ structure of the pseudoscalar and scalar mesons is schematically written with chiral SU(3) indices displayed as:

$$M_a^{\dot{b}} = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}, \tag{1}$$

where a and A are respectively flavor and color indices. For clarity, on the left hand side the undotted index transforms under the left SU(3) while the dotted index transforms under the right SU(3). The decomposition in terms of scalar and pseudoscalar fields is:

$$M = S + i\phi. \tag{2}$$

Using matrix notation (e.g. $M_a^{\dot{b}} \to M_{a\dot{b}}$) the Noether vector and axial currents read (see for example Appendix A of [40]),

$$\begin{aligned} V_\mu &= i\phi \overleftrightarrow{\partial_\mu} \phi + iS \overleftrightarrow{\partial_\mu} S, \\ A_\mu &= S \overleftrightarrow{\partial_\mu} \phi - \phi \overleftrightarrow{\partial_\mu} S, \end{aligned} \tag{3}$$

The axial symmetry breaking is measured by the vacuum value of S:

$$S = \tilde{S} + < S >, \qquad < S_a^b >= \alpha_a \delta_a^b, \tag{4}$$

where the normalization is $\alpha_1 + \alpha_2 = F_\pi \approx 130.4$ MeV and $\alpha_1 + \alpha_3 = F_K \approx 156.1$ MeV. Note that the overall normalization constant for V_μ gives the correct value for the ordinary electromagnetic current. This determines the normalization for the weak currents in the $\mathrm{SU(3)}_L \times \mathrm{SU(3)}_R$ symmetry limit. For the vector currents this amounts to an implementation of the "conserved vector current hypothesis" introduced for beta decay many years ago [41]. Such an approximation is well known not to be as good for the axial current case, but may at least furnish an order of magnitude estimate. In detail, with the usual SU(3) tensor indices, the currents read:

$$\begin{aligned} V_{\mu a}^b &= i\phi_a^c \overleftrightarrow{\partial_\mu} \phi_c^b + i\tilde{S}_a^c \overleftrightarrow{\partial_\mu} \tilde{S}_c^b + i(\alpha_a - \alpha_b)\partial_\mu \tilde{S}_a^b, \\ A_{\mu a}^b &= \tilde{S}_a^c \overleftrightarrow{\partial_\mu} \phi_c^b - \phi_a^c \overleftrightarrow{\partial_\mu} \tilde{S}_c^b + (\alpha_a + \alpha_b)\partial_\mu \phi_a^b, \end{aligned} \tag{5}$$

For example, the relevant hadronic current needed to describe the semi-leptonic decay $K^+ \to \pi^0 + e^+ + \nu_e$ is

$$V_{\mu 1}^3 = \phi_1^c \overleftrightarrow{\partial_\mu} \phi_c^3 + i(\alpha_1 - \alpha_3)\partial_\mu \tilde{S}_1^3. \tag{6}$$

A relevant application of this formula is given in Appendix A.

B. SU(3) $M - M'$ model

For this model we first introduce another chiral field, $M^{(2)}$ constructed out of two quarks and two anti-quarks as:

$$M_a^{(2)\dot{b}} = \epsilon_{acd}\epsilon^{\dot{b}\dot{e}\dot{f}} \left(M^\dagger\right)_{\dot{e}}^c \left(M^\dagger\right)_{\dot{f}}^d. \tag{7}$$

This object has the form of a "molecule" made of two M's. Alternatively one can schematically make two quark-two antiquark states denoted by $M^{(3)}$ and $M^{(4)}$, from a diquark combined with an anti-diquark in two different ways [21]. One might as well consider the most general linear combination of $M^{(2)}$, $M^{(3)}$ and $M^{(4)}$ as a field representing an object, M' made from two quarks plus two antiquarks. M' has the decomposition,

$$M' = S' + i\phi'. \tag{8}$$

Then the Noether currents involve the sum of pieces constructed from the unprimed fields and from the primed fields. The latter take the form,

$$\begin{aligned} V'^b_{\mu a} &= i\phi'^c_a \overleftrightarrow{\partial_\mu} \phi'^b_c + i\tilde{S}'^c_a \overleftrightarrow{\partial_\mu} \tilde{S}'^b_c + i(\beta_a - \beta_b)\partial_\mu \tilde{S}'^b_a, \\ A'^b_{\mu a} &= S'^c_a \overleftrightarrow{\partial_\mu} \phi'^b_c - \phi'^c_a \overleftrightarrow{\partial_\mu} \tilde{S}'^b_c + (\beta_a + \beta_b)\partial_\mu \phi'^b_a, \end{aligned} \tag{9}$$

wherein,

$$S' = \tilde{S}' + < S' >, \qquad < {S'}^b_a >= \beta_a \delta^b_a. \tag{10}$$

The total currents are denoted as:

$$\begin{aligned} V^b_{\mu a}(total) &= V^b_{\mu a} + V'^b_{\mu a}, \\ A^b_{\mu a}(total) &= A^b_{\mu a} + A'^b_{\mu a}. \end{aligned} \tag{11}$$

In contrast to the chiral SU(3) model above, all the primed and corresponding unprimed fields mix to give physical fields of definite mass. As a simple example, the transformation between the physical π^+ and π'^+ fields and the original fields (say ϕ^+ and ϕ'^+) is [32]:

$$\begin{bmatrix} \pi^+ \\ \pi'^+ \end{bmatrix} = R_\pi^{-1} \begin{bmatrix} \phi^2_1 \\ {\phi'}^2_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_\pi & -\sin\theta_\pi \\ \sin\theta_\pi & \cos\theta_\pi \end{bmatrix} \begin{bmatrix} \phi^2_1 \\ {\phi'}^2_1 \end{bmatrix}, \tag{12}$$

which also defines the transformation matrix, R_π.

The pion decay constant as well as (formally) the decay constant for the much heavier $\pi(1300)$ particle are defined by the part of the axial current linear in the fields:

$$\begin{aligned} A^2_{\mu 1}(total) &= F_\pi \partial_\mu \pi^+ + F_{\pi'} \partial_\mu \pi'^+ + \cdots, \\ F_\pi &= (\alpha_1 + \alpha_2)\cos\theta_\pi - (\beta_1 + \beta_2)\sin\theta_\pi, \\ F_{\pi'} &= (\alpha_1 + \alpha_2)\sin\theta_\pi + (\beta_1 + \beta_2)\cos\theta_\pi. \end{aligned} \tag{13}$$

The angle θ_π depends on the detailed dynamics. [32]

In what follows it will be useful for us to specify the mixing matrix for the four isoscalar scalar mesons in this model. A basis for these states is given in terms of the four component vector $f = (f_a, f_b, f_c, f_d)$ where,

$$\begin{aligned} f_a &= \frac{S^1_1 + S^2_2}{\sqrt{2}} \qquad n\bar{n}, \\ f_b &= S^3_3 \qquad s\bar{s}, \\ f_c &= \frac{S'^1_1 + S'^2_2}{\sqrt{2}} \qquad ns\bar{n}\bar{s}, \\ f_d &= S'^3_3 \qquad nn\bar{n}\bar{n}. \end{aligned} \tag{14}$$

In the above, the quark content is indicated on the right for convenience. Note that s stands for a strange quark while n stands for a non-strange quark. However these basis states are not mass eigenstates. Again, the detailed dynamics of the model is required to specify this. For typical values of the model's input parameters (see [36]) the mass eigenstates make up a four vector, $F = L_0^{-1} f$ with,

$$
(L_o^{-1}) = \begin{bmatrix} 0.601 & 0.199 & 0.600 & 0.489 \\ -0.107 & 0.189 & 0.643 & -0.735 \\ 0.790 & -0.050 & -0.391 & -0.470 \\ 0.062 & -0.960 & 0.272 & -0.019 \end{bmatrix} \quad (15)
$$

The physical states are identified, with nominal mass values, as

$$
F = \begin{bmatrix} f_0(600) \\ f_0(980) \\ f_0(1370) \\ f_0(1800) \end{bmatrix} \quad (16)
$$

It will also be interesting for us to give the typical result of the model for the mixing of the four isoscalar pseudoscalars. The analogous basis states are:

$$
\begin{aligned}
\eta_a &= \frac{\phi_1^1 + \phi_2^2}{\sqrt{2}} \qquad n\bar{n}, \\
\eta_b &= \phi_3^3 \qquad s\bar{s}, \\
\eta_c &= \frac{\phi_1'^1 + \phi_2'^2}{\sqrt{2}} \qquad ns\bar{n}\bar{s}, \\
\eta_d &= \phi_3'^3 \qquad nn\bar{n}\bar{n}.
\end{aligned} \quad (17)
$$

For typical values of the model's input parameters (see [36]) the mass eigenstates make up a four component vector, $P = R_0^{-1}\eta$ with,

$$
P = \begin{bmatrix} \eta(547) \\ \eta(958) \\ \eta(1295) \\ \eta(1760) \end{bmatrix} \quad (18)
$$

(These identifications correspond to the favored scenario discussed in section V of [36]). The dynamically determined mixing matrix is then:

$$
(R_o^{-1}) = \begin{bmatrix} -0.675 & 0.661 & -0.205 & 0.255 \\ 0.722 & 0.512 & -0.363 & 0.291 \\ -0.134 & -0.546 & -0.519 & 0.644 \\ 0.073 & 0.051 & 0.746 & 0.660 \end{bmatrix} \quad (19)
$$

C. Hybrid M -M' model with a heavy flavor

As recently discussed in [38], the case of three flavors is special in the sense that it is the only one in which a two quark-two antiquark field has the correct chiral transformation property to mix (in the chiral limit) with M. In order to respect this property when a heavy meson is included in the Lagrangian, we should demand that "heavy" spin zero mesons be made of just one quark and one antiquark. In a linear sigma model the kinetic term would then be written as:

$$
\mathcal{L} = -\frac{1}{2}Tr^4(\partial_\mu M \partial_\mu M^\dagger) - \frac{1}{2}Tr^3(\partial_\mu M' \partial_\mu M'^\dagger), \quad (20)
$$

where the meaning of the superscript on the trace symbol is that the first term should be summed over the heavy quark index as well as the three light indices. This stands in contrast to the second term which is just summed over the three light quark indices pertaining to the two quark - two antiquark field M'. Since the Noether currents are sensitive only to these kinetic terms in the model, the vector and axial vector currents with flavor indices 1 through

3 in this model are just the same as in Eq.(11) above. However if either or both flavor indices take on the value 4 (referring to the heavy flavor) the current will only have contributions from the field M. This should be clarified by the following example,

$$\begin{aligned} V^a_{\mu 4}(total) &= V^a_{\mu 4} = i\phi^c_4 \overset{\leftrightarrow}{\partial_\mu} \phi^a_c + iS^c_4 \overset{\leftrightarrow}{\partial_\mu} S^a_c, \\ A^a_{\mu 4}(total) &= A^a_{\mu 4} = S^c_4 \overset{\leftrightarrow}{\partial_\mu} \phi^a_c - \phi^c_4 \overset{\leftrightarrow}{\partial_\mu} S^a_c. \end{aligned} \tag{21}$$

Here the unspecified indices can run from 1 to 4. This equation is correct by construction but does not tell the whole story since the connection between the fields above and the physical states involves, as in the preceding cases, the details of the non-derivative ("potential") terms of the effective Lagrangian.

III. DIFFERENT SEMI-LEPTONIC DECAY MODES OF THE D_s^+(1968)

The initial motivation for this work was the recent experimental discovery [39] of the semileptonic decay mode,

$$D_s^+(1968) \to f_0(980)e^+\nu_e, \tag{22}$$

in which the $f_0(980)$ was identified from its two pion decay mode.

A relevant generalization is to consider other scalar isosinglet candidates than the $f_0(980)$. For example the SU(3) M - M' model contains four different isoscalar scalars, F_i. In addition, there are four different isoscalar pseudoscalars in that model, P_i. Here we shall calculate the predictions of that model for all eight of these decays in the simplest approximation. This should provide some useful orientation. In fact there are no parameters which have not already been determined in the previous treatment [36] of the model.

The usual weak interaction Lagrangian is,

$$\mathcal{L} = \frac{g}{2\sqrt{2}}(J^-_\mu W^+_\mu + J^+_\mu W^-_\mu), \tag{23}$$

wherein,

$$\begin{aligned} J^-_\mu &= i\bar{U}\gamma_\mu(1+\gamma_5)VD + i\bar{\nu}_e\gamma_\mu(1+\gamma_5)e, \\ J^+_\mu &= i\bar{D}\gamma_\mu(1+\gamma_5)V^\dagger U + i\bar{e}\gamma_\mu(1+\gamma_5)\nu_e. \end{aligned} \tag{24}$$

Here the column vectors of the quark fields take the form:

$$U = \begin{bmatrix} u \\ c \\ t \end{bmatrix}, \qquad D = \begin{bmatrix} d \\ s \\ b \end{bmatrix}, \tag{25}$$

and the CKM matrix, V is explicitly,

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}. \tag{26}$$

A picture describing the relevant D_s decays is given in Fig. (1).

The corresponding semi-leptonic decay amplitudes are thus,

$$\begin{aligned} amp(D_s^+(p)) \to \left\{ \begin{array}{c} P_i(q) \\ F_i(q) \end{array} \right\} + e^+(k) + \nu_e(l)) &= -i\frac{G_F}{\sqrt{2}}V_{cs} \left\{ \begin{array}{c} < P_i(q)|V^3_{\mu 4}(total)|D_s^+(p) > \\ < F_i(q)|A^3_{\mu 4}(total)|D_s^+(p) > \end{array} \right\} \\ &\quad \times \bar{u}(\mathbf{l})\gamma_\mu(1+\gamma_5)v(\mathbf{k}), \end{aligned} \tag{27}$$

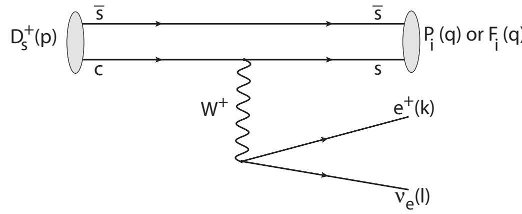

FIG. 1: D_s decay.

where the spinor $v(\mathbf{k})$ represents the outgoing e^+ and $\bar{u}(\mathbf{l})$ represents the outgoing ν_e. The relevant hadronic operators can be rewritten in terms of the mass eigenstate scalar isosinglets and the pseudoscalar isosinglets using Eqs. (16) and (18) as:

$$\begin{aligned} V^3_{\mu 4}(total) &= iD_s^+ \overleftrightarrow{\partial}_\mu \phi_3^3 + \cdots \\ &= iD_s^+ \sum_j (R_0)_{2j} \overleftrightarrow{\partial}_\mu P_j + \cdots \end{aligned} \tag{28}$$

$$\begin{aligned} A^3_{\mu 4}(total) &= -D_s^+ \overleftrightarrow{\partial}_\mu S_3^3 + \cdots \\ &= -D_s^+ \sum_j (L_0)_{2j} \overleftrightarrow{\partial}_\mu F_j + \cdots \end{aligned} \tag{29}$$

The transposed matrices L_0 and R_0 are given in Eqs. (15) and (19) respectively, based on a typical numerical solution for the model parameters [36]. Next the amplitudes are given by,

$$amp(D_s^+(p) \rightarrow \left\{ \begin{array}{c} P_i(q) \\ F_i(q) \end{array} \right\} + e^+(k) + \nu_e(l)) = \frac{G_F}{\sqrt{2}} V_{cs} \left\{ \begin{array}{c} (R_0)_{2i} \\ -i(L_0)_{2i} \end{array} \right\} (p_\mu + q_\mu)\bar{u}(\mathbf{l})\gamma_\mu(1+\gamma_5)v(\mathbf{k}), \tag{30}$$

The squared amplitudes, summed over the emitted lepton's spins, are then,

$$G_F^2 |V_{cs}|^2 \frac{1}{m_e^2} \left\{ \begin{array}{c} ((R_0)_{2i})^2 \\ ((L_0)_{2i})^2 \end{array} \right\} [2k \cdot (p+q) l \cdot (p+q) - l \cdot k (p+q)^2], \tag{31}$$

wherein m_e has been set to zero except for the overall $1/m_e^2$ factor.

This yields the unintegrated decay width,

$$\frac{d\Gamma}{d|\mathbf{q}|} = \frac{G_F^2 |V_{cs}|^2}{12\pi^3} \left\{ \begin{array}{c} ((R_0)_{2i})^2 \\ ((L_0)_{2i})^2 \end{array} \right\} m(D_s) \frac{|\mathbf{q}|^4}{q_0}. \tag{32}$$

For integrating this expression we need,

$$|q_{max}| = \frac{m^2(D_s) - m_i^2}{2m(D_s)}, \tag{33}$$

where m_i is the mass of the isosinglet meson F_i or P_i and also the indefinite integral formula, where $x = |\mathbf{q}|$,

$$\int \frac{x^4 dx}{\sqrt{x^2 + m_i^2}} = \frac{x^3}{4}\sqrt{x^2 + m_i^2} - \frac{3}{8} m_i^2 x \sqrt{x^2 + m_i^2} + \frac{3}{8} m_i^4 ln(x + \sqrt{x^2 + m_i^2}). \tag{34}$$

Table I summarizes the calculations of the predicted widths, for D_s^+ decays into the four pseudoscalar singlet mesons ($\eta_1 = \eta(547)$, $\eta_2 = \eta(982)$, $\eta_3 = \eta(1225)$, $\eta_4 = \eta(1794)$. Notice that the listed masses, m_i are the "predicted" ones in the present model) and leptons.

m_i (MeV)	$(R_0)_{2i}$	$(q_{max})_i$ (MeV)	Γ_i (MeV)
553	0.661	906.20	4.14×10^{-11}
982	0.512	739.00	7.16×10^{-12}
1225	-0.546	602.74	2.57×10^{-12}
1794	0.051	166.31	2.65×10^{-17}

TABLE I: pseudoscalars.

m_i (MeV)	$(L_0)_{2i}$	$(q_{max})_i$ (MeV)	Γ_i (MeV)
477	0.199	933.23	4.56×10^{-12}
1037	0.189	710.79	7.80×10^{-13}
1127	-0.050	661.30	3.62×10^{-14}
1735	-0.960	219.21	3.85×10^{-14}

TABLE II: scalars.

Table II, with the same conventions, summarizes the calculations of the predicted widths for D_s^+ decays into the four scalar singlet mesons $((f_1, f_2, \cdots) = (\sigma, f_0(980), \cdots))$ and leptons.

Experimental data exist for only three of these eight decay modes.

$$\begin{aligned}
\Gamma(D_s^+ \to \eta e^+ \nu_e) &= (3.5 \pm 0.6) \times 10^{-11} \quad \text{MeV} \\
\Gamma(D_s^+ \to \eta' e^+ \nu_e) &= (1.29 \pm 0.30) \times 10^{-11} \quad \text{MeV} \\
\Gamma(D_s^+ \to f_0 e^+ \nu_e) &= (2.6 \pm 0.4) \times 10^{-12} \quad \text{MeV}
\end{aligned} \tag{35}$$

It is encouraging that even though our calculation utilized the simplest model for the current and no arbitrary parameters were introduced, the prediction for the lightest hadronic mode, $\Gamma(D_s^+ \to \eta e^+ \nu_e)$ agrees with the measured value. In the case of the decay $D_s^+ \to \eta e^+ \nu_e$ the predicted width is about 30% less than the measured value. For the mode $D_s^+ \to f_0(980) e^+ \nu_e$ our predicted value is about one third the measured value. Conceivably, considering the large predicted width into the very broad sigma state centered at 477 MeV, some of the higher mass sigma events might have been counted as $f_0(980)$ events, which would improve the agreement. It would be very interesting to obtain experimental information about the energy regions relevant to the other five predicted isosinglet modes.

Furthermore, these width predictions are based on Eqs. (15) and (19) corresponding to particular choices for the quark mass ratio A_3/A_1 and the precise mass of the very broad $\Pi(1300)$ resonance. Varying these within the allowable ranges gives rise to the allowed range of predictions displayed in Figs. (2) and (3). One can see that raising m[$\Pi(1300)$] and/or lowering A_3/A_1 yields better agreement for the predicted semi-leptonic decay width of the $f_0(980)$. Clearly, the simple model here provides reasonable estimates for the semileptonic decay widths of the $D_s^+(1968)$.

IV. SUMMARY AND DISCUSSION

We saw that the partial widths for semi-leptonic decays of the $D_s^+(1968)$ into isoscalar scalar singlets and pseudoscalar singlets plus leptons could be well estimated in a simple model where the hadronic current was taken to be the Noether current associated with a minimal linear sigma model.

The agreement between experiment and theory was better for the decays into the η and η' than for the decay into the $f_0(980)$. The former involve the hadronic vector current, which is "protected" according to the conserved vector current hypothesis, while the latter involves the "unprotected" axial vector current.

Clearly it would be interesting to try this technique for other semi-leptonic decays of charmed mesons and also for bottom mesons. We considered the case when the charged lepton was e^+ rather than the cases of μ^+ or τ^+. In those two cases an additional form factor as in the calculation of the $K\ell 3$ decay discussed in Appendix A should be taken into account.

Information about the scalars, involving however more work for disentangling the effects of the strong interaction, can also be obtained from the non-leptonic decay modes of the charm and bottom mesons. A treatment of $D_S^+ \to$

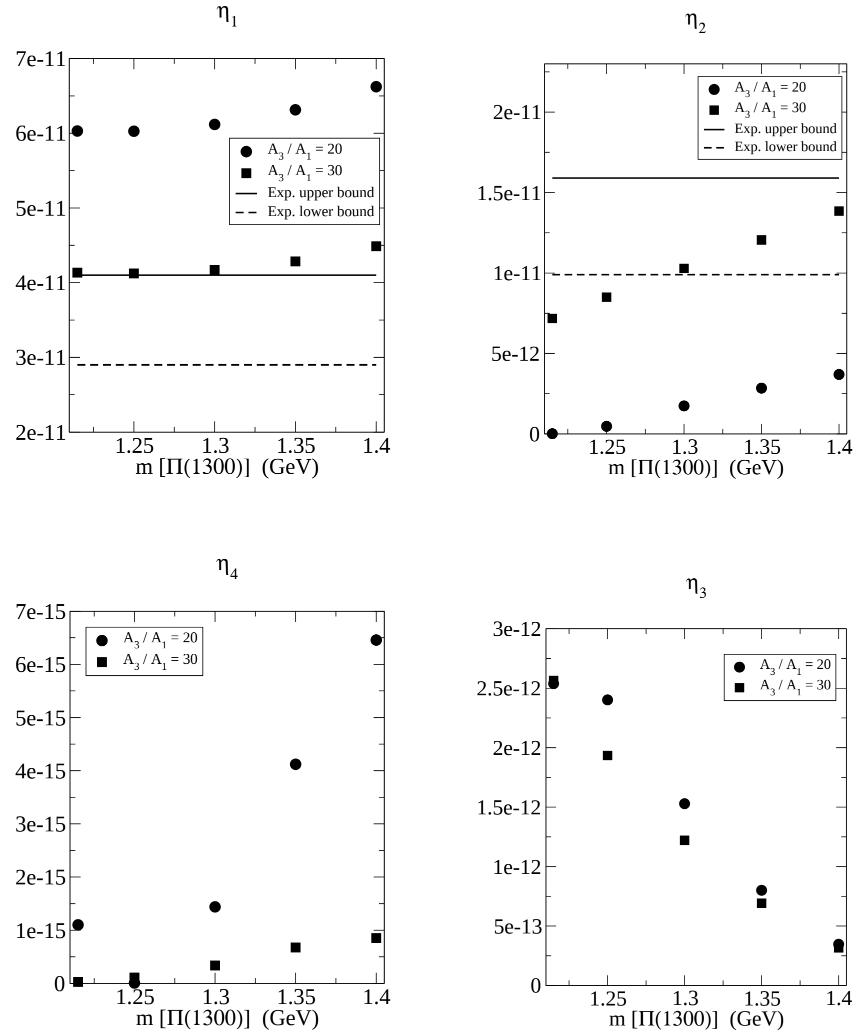

FIG. 2: Starting from the upper left and proceeding clockwise: The dependences of the pseudoscalar partial widths on the current quark mass ratio A_3/A_1 and on the value of the $\Pi(1300)$ mass.

$f_0(980) + \pi^+$ has already been carried out [42]. The study of the decay like $B_c^+ \to scalar + e^+ + \nu_e$ might be useful for learning about mixing between a $c\bar{c}$ scalar and the lighter three flavor scalars.

A straightforward, but not necessarily short, improvement of this calculation would be to include both vector and axial vector mesons in the starting Lagrangian from which the currents are calculated.

Acknowledgments

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Appendix A: $K\ell3$ decay

As an illustration of Eq. (6) we consider the matrix element, between an initial K^+ state with 4-momentum k and a final π^0 state with four momentum p, of the strangeness changing vector current $V_{\mu 1}^3$,

$$< \pi^0(p)|V_{\mu 1}^3|K^+(k) > \sim f_+(t)(k+p)_\mu + f_-(t)(k-p)_\mu, \tag{A1}$$

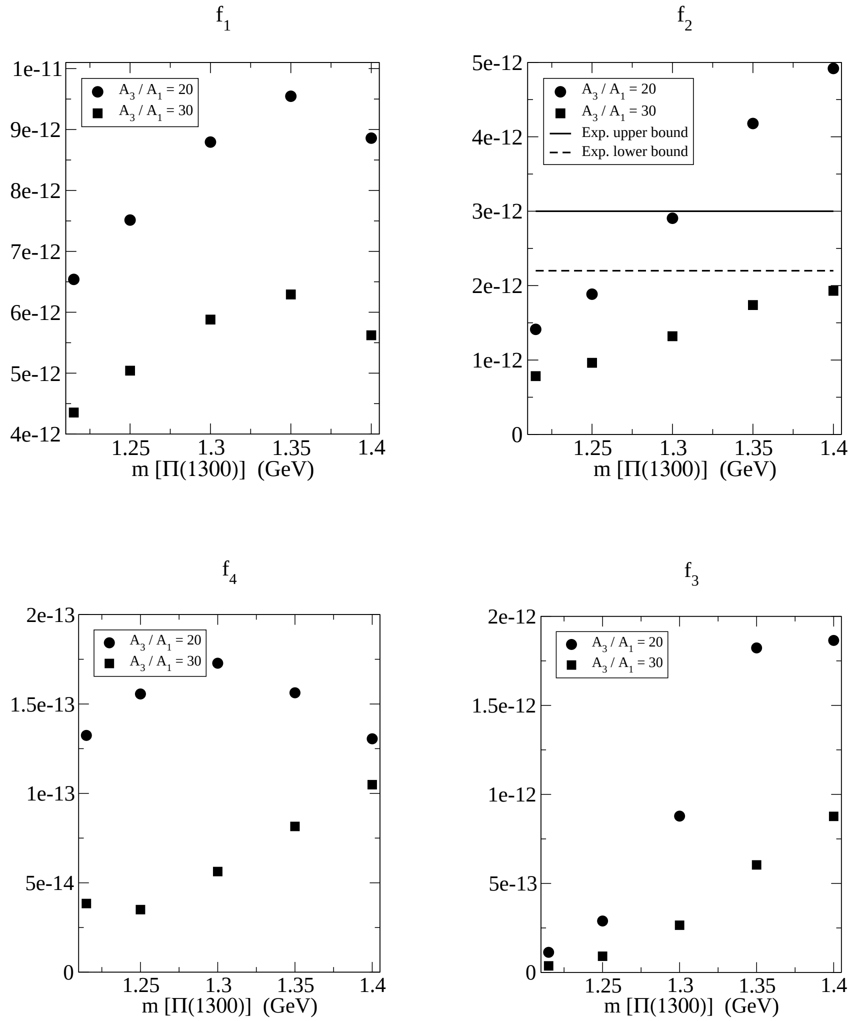

FIG. 3: Starting from the upper left and proceeding clockwise: The dependences of the scalar partial widths on the current quark mass ratio A_3/A_1 and on the value of the $\Pi(1300)$ mass.

where t = -$(k-p)^2$.

The first term of Eq. (6) contributes at tree level to the f_+ form factor while the second term contributes to the f_- form factor. These two contributions are illustrated in Figs. 4a and 4b in which the W boson which is connected to the leptonic current acts at the points X. Here we are evaluating this matrix element in the framework of the plain SU(3) linear sigma model in which, furthermore, the vector and axial vector mesons have not been included.

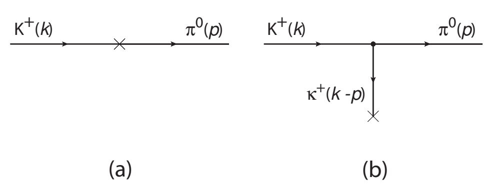

FIG. 4: $K\ell 3$ decay hadronic current.

According to the usual Feynman rules,

$$f_+ \;=\; -\frac{1}{\sqrt{2}},$$

$$f_- \ = \ -\frac{1}{\sqrt{2}}[\frac{\alpha_3-\alpha_1}{\alpha_3+\alpha_1}][\frac{m_\kappa^2-m_\pi^2}{m_\kappa^2-t}], \tag{A2}$$

wherein m_κ denotes the mass of the strange scalar particle contained in this model. Furthermore, the explicit form of the $K\kappa\pi$ coupling constant in the model was used in the expression for f_- [21]. Notice that the first bracket in the equation for f_- evaluates to about 0.16 and that the physical kappa mass is about 800 MeV in the plain SU(3) linear sigma model.

It is interesting that this decay allows one to learn something about the properties of the kappa meson. For this purpose it is necessary to use the process where a final μ^+ is observed rather than a final e^+. That is because the contribution of $f_-(t)$ to the decay width is proportional to the final lepton mass. Of course the effect of the $K^*(892)$, which contributes importantly to the $f_+(t)$ form factor should also be included to get increased accuracy.

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