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Testing for Shifts in a Time Trend Panel Data Model with Serially Correlated Error Component Disturbances

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Abstract

This paper studies testing of shifts in a time trend panel data model with serially correlated error component disturbances, without any prior knowledge of whether the error term is stationary or nonstationary. This is done in case the shift is known as well as unknown. Following Vogelsang (1997) in the time series literature, we propose a Wald type test statistic that uses a fixed effects feasible generalized least squares (FE-FGLS) estimator derived in Baltagi, et al. (2014). The proposed test has a Chi-square limiting distribution and is valid for both I (0) and I (1) errors. The finite sample size and power of this Wald test is investigated using Monte Carlo simulations.

JEL No. C23, C33

Keywords: Non-Stationary Panels, Time Trends, Serial Correlation, Wald Type Tests

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Testing for Shifts in a Time Trend Panel Data Model with Serially Correlated Error Component Disturbances*

Badi H. Baltagi†, Chihwa Kao‡, Long Liu§

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Abstract

This paper studies testing of shifts in a time trend panel data model with serially correlated error component disturbances, without any prior knowledge of whether the error term is stationary or nonstationary. This is done in case the shift is known as well as unknown. Following Vogelsang (1997) in the time series literature, we propose a Wald type test statistic that uses a fixed effects feasible generalized least squares (FE-FGLS) estimator derived in Baltagi, et al. (2014). The proposed test has a Chi-square limiting distribution and is valid for both $I(0)$ and $I(1)$ errors. The finite sample size and power of this Wald test is investigated using Monte Carlo simulations.

JEL: C23, C3

1 Introduction

Testing for structural change in a time trend model has been an important research topic in the econometrics literature. In a pure time series framework, it has been well studied by Vogelsang (1997), Perron and Zhu (2005), Perron and Yabu (2009a, 2009b), to name a few. Emerson and Kao (2001) extend the Wald test for structural change of Vogelsang (1997) to a panel data setting. They show that the asymptotic distribution of the Wald statistic is different depending on whether the panel data is stationary or nonstationary. Kim (2011) extends the Perron and Zhu (2005) article to large (n, T) panel data with cross-sectional dependence.

*This paper extends the results of Baltagi, Kao and Liu (2014) to allow for possible structural change. Both papers are dedicated in honour of Peter C.B. Phillips's many contributions to econometrics and in particular non-stationary time series analysis and panel data.

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This paper focuses on the testing for structural change in a panel data time trend model with a stationary or nonstationary error term. In the time series literature, various estimation methods have been discussed in Phillips and Lee (1996). In the panel data literature, various estimation methods have been discussed in Baltagi, Kao and Liu (2014), hereafter denoted by BKL. In particular, BKL derive a fixed effects feasible generalized least squares (FE-FGLS) procedure and propose a super-efficient estimate of the autoregressive parameter. Note that BKL did not consider the possibility of a structural change. This is the subject of this paper. Testing for structural change in a panel data model with a stationary or nonstationary regressor and error term has been discussed in Baltagi, Kao and Liu (2017) but not for the time trend panel data model with serial correlation. Also, Baltagi, Kao and Liu (2017) did not discuss the FE-FGLS procedure. Based on the FE-FGLS procedure derived in BKL, this paper proposes a Wald test for structural change for the time trend panel data model with serial correlation that is robust to stationary or nonstationary error terms. We derive the asymptotic distribution of this test and check its finite sample performance using Monte Carlo simulations.

The article is organized as follows. Section 2 introduces the model and the test with an unknown change point. Asymptotic properties of the proposed test are derived. Section 3 discusses some extensions of the model. When additional stationary or nonstationary regressors are included in the regression, we show that the asymptotic distribution of the Wald test stays the same. We also study the case when individual effects are not included in the model. Simulation results are presented in Section 4 and provide the concluding remarks. Mathematical proofs are contained in the supplemental appendix available upon request from the authors. A few words on notation. All limits are taken sequentially as $T \to \infty$ and $n \to \infty$ unless otherwise specified. We use $(n, T) \to \infty$ to denote the sequential limit.[1] Convergence in probability and distribution are denoted as $\xrightarrow{p}$ and $\xrightarrow{d}$, respectively.

[1] The limiting distribution of double indexed integrated processes has been extensively studied by Phillips and Moon (1999, 2000). Under the additional condition, $n/T \to 0$, they show that sequential asymptotic results for their pooled estimators would be equivalent to the joint ones. Our sequential limit results can be extended to joint limit following Phillips and Moon (1999, 2000).

2 The Model and the Tests

Consider the following panel data time trend model with structural change:

$$y_{it} = \delta + \beta t + \gamma DT + \mu_i + v_{it}, \tag{1}$$

for $i = 1, \ldots, n$, $t = 1, \ldots T$, where $DT = 1(t > k)(t - k)$ for a change point $k = [\lambda T]$ for some $\lambda \in (0, 1)$, where $[\cdot]$ denotes the largest integer that is less than or equal to the argument and $1(\cdot)$ is the indicator function. The change point k is unknown. Of course, δ, β and γ are unknown parameters to be estimated. We assume that μ_i are the unobservable individual effects with $\mu_i \sim iid\left(0, \sigma_\mu^2\right)$ and v_{it} are AR(1) stationary disturbance terms with

$$\nu_{it} = \rho \nu_{it-1} + e_{it} \tag{2}$$

with $|\rho| \leq 1$, where e_{it} is a white noise process with variance σ_e^2. The μ_i are assumed to be independent of v_{it} for all i and t. The null hypothesis is $H_0 : \gamma = 0$. Emerson and Kao (2001) derive the Wald statistic based on Ordinary Least Squares (OLS) estimators as

$$\mathrm{W}_{OLS}(k) = \frac{\left(\widehat{\beta}_{1k} - \widehat{\beta}_{2k}\right)^2}{\sigma_v^2 \left[\left(\sum_{i=1}^n \sum_{t=1}^k (t - \bar{t}_{1k})^2\right)^{-1} + \left(\sum_{i=1}^n \sum_{t=k+1}^T (t - \bar{t}_{2k})^2\right)^{-1}\right]},$$

where $\widehat{\beta}_{1k} = \frac{\sum_{i=1}^n \sum_{t=1}^k (t - \bar{t}_{1k}) y_{it}}{\sum_{i=1}^n \sum_{t=1}^k (t - \bar{t}_{1k})^2}$, $\widehat{\beta}_{2k} = \frac{\sum_{i=1}^n \sum_{t=k+1}^T (t - \bar{t}_{2k}) y_{it}}{\sum_{i=1}^n \sum_{t=k+1}^T (t - \bar{t}_{2k})^2}$, $\bar{t}_{1k} = \frac{1}{k}\sum_{t=1}^k t$, and $\bar{t}_{2k} = \frac{1}{T-k}\sum_{t=k+1}^T t$. Emerson and Kao (2001) show that under $H_0 : \gamma = 0$, if $|\rho| < 1$, then

$$\frac{(1-\rho)^2 \sigma_v^2}{3\sigma_\varepsilon^2} \mathrm{W}_{OLS}(k) \xrightarrow{d} \left\{\frac{P(\lambda)(1-\lambda)^3 - \lambda^3 \left[P(1) - P(\lambda) + W(\lambda) - \lambda W(1)\right]}{\left[\lambda^3 (1-\lambda)^3 \left[(1-\lambda)^3 + \lambda^3\right]\right]^{\frac{1}{2}}}\right\}^2,$$

where $P(\lambda) = \lambda W(\lambda) - 2\int_0^\lambda W(s)ds$. However, Emerson and Kao (2001) show that under $H_0 : \gamma = 0$, if $\rho = 1$,

$$\frac{\sigma_v^2}{3T^2\sigma_\varepsilon^2} \mathrm{W}_{OLS}(k) \xrightarrow{d} \left\{\frac{Q(\lambda)(1-\lambda)^3 - \left[Q(1) - Q(\lambda) - \lambda\int_0^1 W(s)ds + \int_0^\lambda W(s)ds\right] r^3}{\left[\lambda^3 (1-\lambda)^3 \left[(1-\lambda)^3 + \lambda^3\right]\right]^{1/2}}\right\}^2,$$

where $Q(\lambda) = 2\int_0^\lambda s\left[W(s) + \widetilde{W}(\pi)\right] ds - \lambda\int_0^\lambda \left[W(s) + \widetilde{W}(\pi)\right] ds$ and $\widetilde{W}(\pi)$ is a different Wiener process from $W(s)$. These imply that, under the null, $\mathrm{W}_{OLS}(k) = O_p(1)$ when v_{it} is $I(0)$ and $\mathrm{W}_{OLS}(k) \to \infty$ when v_{it} is $I(1)$. In view of this and given that the order of integration of v_{it} is not known in practice, it is natural to consider a robust test procedure. In a pure time series model, Perron and Yabu (2009b) suggest a generalized least squares (GLS) procedure using a super-efficient estimate of the autoregressive parameters as a robust test. Following the idea of Perron and Yabu (2009b), we propose a robust test using the FE-FGLS estimator for the panel data model discussed in BKL and modified for the case of structural change in the next section.

2.1 The FE-FGLS Estimator

Rewrite equation (1) in matrix notation as

$$y = \delta \boldsymbol{\iota}_{nT} + Z'\Psi + u \tag{3}$$

where $u = Z_\mu \mu + \nu$, μ is an $n \times 1$ vector of μ_i, $\boldsymbol{\nu}$ is an $nT \times 1$ vector of ν_{it}, $Z_\mu = I_n \otimes \boldsymbol{\iota}_T$, where I_n is an identity matrix of dimension n, $\boldsymbol{\iota}_T$ is a vector of ones of dimension T and $\otimes$ denotes Kronecker product. y is an $nT \times 1$ vector of y_{it}, $Z = \boldsymbol{\iota}_n \otimes Z_i$ where $\boldsymbol{\iota}_n$ is a vector of ones of dimension n and $Z_i = (x_i, DT_i)$ where x_i is a $T \times 1$ vector of $(1, 2, \ldots, T)$ and DT_i is a $T \times 1$ vector of $(0, \ldots, 0, 1, \ldots, T-k)$. $\boldsymbol{\iota}_{nT}$ is a vector of ones of dimension nT. $\Psi = (\beta, \gamma)'$. In order to save space, the FE-GLS estimator of Ψ, $\hat{\Psi}_{FE-GLS}$, is given in Equation (A5) in the supplemental Appendix. Define $R = (0, 1)$, the null hypothesis H_0 can be rewritten into $R\Psi = 0$. Therefore, the Wald statistic of the FE-GLS estimator is

$$\mathrm{W}_{FE-GLS}(k) = \left[R\left(\hat{\Psi}_{FE-GLS} - \Psi\right)\right]' \left[RVar\left(\hat{\Psi}_{FE-GLS}\right)R'\right]^{-1} \left[R\left(\hat{\Psi}_{FE-GLS} - \Psi\right)\right]. \tag{4}$$

The Wald statistic based on the FE-GLS depends on ρ. A popular estimator of ρ suggested in Baltagi and Li (1991) is given by

$$\tilde{\rho} = \frac{\sum_{i=1}^{n}\sum_{t=2}^{T} \hat{\nu}_{it}\hat{\nu}_{i,t-1}}{\sum_{i=1}^{n}\sum_{t=2}^{T} \hat{\nu}_{i,t-1}^2}, \tag{5}$$

where $\hat{\nu}_{it}$ is the within residual in (3). For a panel trend model, BKL show that $\sqrt{nT}\left(\tilde{\rho} - \rho + \frac{1+\rho}{T}\right) \xrightarrow{d} N\left(0, 1-\rho^2\right)$ if $|\rho| < 1$, and $\sqrt{n}T\left(\tilde{\rho} - 1 + \frac{3}{T}\right) \xrightarrow{d} N\left(0, \frac{51}{5}\right)$ if $\rho = 1$ as $(n, T) \to \infty$. When $|\rho| < 1$, a bias-corrected estimator of ρ is $\tilde{\rho} + \frac{1+\tilde{\rho}}{T}$. When $\rho = 1$, a bias-corrected estimator of ρ is $\tilde{\rho} + \frac{3}{T}$. Therefore, we use the bias-corrected estimator $\hat{\rho}$ suggested by BKL as follows:

$$\hat{\rho} = \begin{cases} \tilde{\rho} + \frac{1+\hat{\rho}}{T} & \text{if } 1 - \tilde{\rho} > \frac{3}{T} \\ 1 & \text{if } 1 - \tilde{\rho} \leq \frac{3}{T} \end{cases}. \tag{6}$$

Since change point k is unknown, following Vogelsang (1997), we consider three statistics: $sup\mathrm{W}_{FE-GLS}(k)$, $Mean\mathrm{W}_{FE-GLS}(k)$, and $Exp\mathrm{W}_{FE-GLS}(k)$, where

$$sup\mathrm{W}_{FE-GLS}(k) = \underset{[T\lambda^*]\leq k\leq T-[T\lambda^*]}{sup} \mathrm{W}_{FE-GLS}(k),$$

$$Mean\mathrm{W}_{FE-GLS}(k) = \frac{1}{T}\sum_{k=[T\lambda^*]}^{T-[T\lambda^*]} \mathrm{W}_{FE-GLS}(k),$$

$$Exp\mathrm{W}_{FE-GLS}(k) = \log\left(\frac{1}{T}\sum_{k=[T\lambda^*]}^{T-[T\lambda^*]} \exp\left(\frac{1}{2}\mathrm{W}_{FE-GLS}(k)\right)\right),$$

and λ^* is the fraction of trimming. In a pure time series, the trimming parameter λ^* is chosen to be 0.01 or 0.15 so that there are enough observations before and after the break dates. In a panel data, the break date k could be chosen between 2 and $T-1$. The asymptotic properties are summarized in the following Theorem:

Theorem 1 *For $\hat{\rho}$, under the null hypothesis H_0, we have $\mathrm{W}_{FE-FGLS}(k) \xrightarrow{d} G(\lambda) \sim \chi^2(1)$ for both $|\rho| < 1$ and $\rho = 1$ as $(n,T) \to \infty$, where*

$$G(\lambda) = \begin{cases} L_0^2(\lambda) & \text{if } |\rho| < 1, \\ L_1^2(\lambda) & \text{if } \rho = 1, \end{cases}$$

where $L_0(\lambda) = \frac{\sqrt{3}}{\lambda^{3/2}(1-\lambda)^{3/2}}\left[H^0(\lambda) - (1+2\lambda)(1-\lambda)^2 H^0(0)\right]$ with $H^0(\lambda) = \int_\lambda^1 (r-\lambda)\,dW - \frac{(1-\lambda)^2}{2}W(1)$ and $H^0(0) = \int r dW - \frac{1}{2}W(1)$. $L_1(\lambda) = \frac{1}{\lambda^{1/2}(1-\lambda)^{1/2}}[\lambda W(1) - W(\lambda)]$. Furthermore,

$$sup\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \underset{\lambda^*\leq\lambda\leq 1-\lambda^*}{sup} G(\lambda),$$

$$Mean\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \int_{\lambda^*}^{1-\lambda^*} G(\lambda)d\lambda,$$

$$Exp\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \log\left(\int_{\lambda^*}^{1-\lambda^*} \exp\left(\frac{1}{2}G(\lambda)\right)d\lambda\right).$$

Theorem 1 implies that $\mathrm{W}_{FE-GLS}(k)$ converges to a $\chi^2(1)$ for both $|\rho| < 1$ and $\rho = 1$.

2.2 Local Asymptotic Power

In order to compare the power of the statistics, the following local alternatives were used. When $|\rho| < 1$, the local alternatives are given by $H_1 : \gamma = n^{-1/2}T^{-3/2}\gamma_0$, where γ_0 is a nonzero constant. When $\rho = 1$, the local alternatives are given by $H_1 : \gamma = n^{-1/2}T^{-1/2}\gamma_0$. The local asymptotic power properties are given in the following Theorem.

Theorem 2 *When $|\rho| < 1$, under the local alternative hypothesis $H_1 : \gamma = n^{-1/2}T^{-3/2}\gamma_0$, where γ_0 is a nonzero constant, we have $\mathrm{W}_{FE-FGLS}(k) \xrightarrow{d} H_0(\lambda) \equiv [L_0(\lambda) + M_0(\lambda)]^2$ as $(n, T) \to \infty$, where $L_0(\lambda)$ is defined in Theorem 1 and $M_0(\lambda) = 3^{-1/2}\sigma_e^{-1}(1-\rho)\lambda^{3/2}(1-\lambda)^{3/2}\gamma_0$. Furthermore,*

$$sup\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \sup_{\lambda^* \leq \lambda \leq 1-\lambda^*} H_0(\lambda),$$

$$Mean\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \int_{\lambda^*}^{1-\lambda^*} H_0(\lambda)\, d\lambda,$$

$$Exp\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \log\left(\int_{\lambda^*}^{1-\lambda^*} \exp\left(\frac{1}{2}H_0(\lambda)\right) d\lambda\right).$$

When $\rho = 1$, under the local alternative hypothesis $H_1 : \gamma = n^{-1/2}T^{-1/2}\gamma_0$, where γ_0 is a nonzero constant, we have $\mathrm{W}_{FE-FGLS}(k) \xrightarrow{d} H_1(\lambda) \equiv [L_1(\lambda) + M_1(\lambda)]^2$ as $(n, T) \to \infty$, where $L_1(\lambda)$ is defined in Theorem 1 and $M_1(\lambda) = \sigma_e^{-1}\lambda^{1/2}(1-\lambda)^{1/2}\gamma_0$. Furthermore,

$$sup\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \sup_{\lambda^* \leq \lambda \leq 1-\lambda^*} H_1(\lambda),$$

$$Mean\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \int_{\lambda^*}^{1-\lambda^*} H_1(\lambda)\, d\lambda,$$

$$Exp\mathrm{W}_{FE-GLS}(k) \xrightarrow{d} \log\left(\int_{\lambda^*}^{1-\lambda^*} \exp\left(\frac{1}{2}H_1(\lambda)\right) d\lambda\right).$$

As we can see from Theorem 2, the local asymptotic power depends on σ_e^2, γ_0 and λ. To be specific, both $M_0(\lambda)$ and $M_1(\lambda)$ are functions of $\lambda(1-\lambda)$, which is maximized at $\lambda = 0.5$. If λ is close to 0 or 1, the break date is close to the boundary and is harder to detect. When $|\rho| < 1$, the local asymptotic power also depends on the value of ρ. When ρ increases towards 1, $1-\rho$ decreases and hence the asymptotic power decreases, too. Furthermore, when $|\rho| < 1$, the local alternatives are given by $H_1 : \gamma = n^{-1/2}T^{-3/2}\gamma_0$. Hence large T improves the asymptotic power by a bigger margin than n does. When $\rho = 1$, the local alternatives are given by $H_1 : \gamma = n^{-1/2}T^{-1/2}\gamma_0$. Comparing to the case of $|\rho| < 1$, we know that for the same sample sizes n and T, the asymptotic power is smaller when $\rho = 1$.

3 Further Extensions

3.1 Generalization of the Independent Variables

When other regressors are included in the model, Equation (1) becomes

$$y_{it} = \delta + \beta t + \gamma DT + \theta_1 w_{1,it} + \theta_2 w_{2,it} + \mu_i + v_{it}, \tag{7}$$

where $w_{1,it}$ and $w_{2,it}$ are nonstationary and stationary variables, respectively. Without loss of generality, let us assume $w_{1,it} = w_{1,i,t-1} + \epsilon_{it}$ with $\epsilon_{it} \sim iid\left(0, \sigma_1^2\right)$ and $w_{2,it} \sim iid\left(0, \sigma_2^2\right)$. Rewrite in matrix notation as

$$y = \delta \boldsymbol{\iota}_{nT} + Z'\Psi + W'\Theta + u \tag{8}$$

where $W = (W_1, W_2, \ldots, W_n)'$, $W_i = (W_{i1}, W_{i2}, \ldots, W_{iT})'$, $W_{it} = (w_{1,it}, w_{2,it})$ and $\Theta = (\theta_1, \theta_2)'$. $\boldsymbol{\iota}_{nT}$ is a vector of ones of dimension nT. $\Psi = (\beta, \gamma)'$. The FE-GLS estimators of Ψ and Θ are given by

$$\begin{pmatrix} \hat{\Psi}_{FE-GLS} - \Psi \\ \hat{\Theta}_{FE-GLS} - \Theta \end{pmatrix} = \left[\begin{pmatrix} Z^* \\ W^* \end{pmatrix}' (I_n \otimes E_T^{\alpha}) (Z^*, W^*) \right]^{-1} \begin{pmatrix} Z^* \\ W^* \end{pmatrix}' (I_n \otimes E_T^{\alpha}) v^* \tag{9}$$

$$= \begin{pmatrix} F_{1,Z} & F_{1,ZW} \\ F_{1,WZ} & F_{1,W} \end{pmatrix}^{-1} \begin{pmatrix} F_{2,Z} \\ F_{2,W} \end{pmatrix}, \tag{10}$$

where $F_{1,Z} = Z^{*\prime} (I_n \otimes E_T^{\alpha}) Z^*$, $F_{2,Z} = Z^{*\prime} (I_n \otimes E_T^{\alpha}) v^*$, $F_{1,W} = W^{*\prime} (I_n \otimes E_T^{\alpha}) W^*$, $F_{1,ZW} = Z^{*\prime} (I_n \otimes E_T^{\alpha}) W^*$, $F_{2,W} = W^{*\prime} (I_n \otimes E_T^{\alpha}) y^*$ and $W^* = (I_n \otimes C) W$ with E_T^{α} and C defined in Equations (A4) and (A5) in the supplemental Appendix. Therefore, we have

$$\hat{\Psi}_{FE-GLS} - \Psi = \left(F_{1,Z} - F_{1,ZW} F_{1,W}^{-1} F_{1,ZW} \right)^{-1} \left(F_{2,Z} - F_{1,ZW} F_{1,W}^{-1} F_{2,W} \right). \tag{11}$$

Theorem 3 *When $|\rho| < 1$,*

$$\begin{aligned} n^{1/2} T^{3/2} \left(\hat{\Psi}_{FE-GLS} - \Psi \right) &= \left(n^{-1} T^{-3} F_{1,Z} \right)^{-1} \left(n^{-1/2} T^{-3/2} F_{2,Z} \right) + o_p(1), \\ nT^3 Var\left(\hat{\Psi}_{FE-GLS} \right) &= \left(n^{-1} T^{-3} F_{1,Z} \right)^{-1} + o_p(1). \end{aligned}$$

When $\rho = 1$,

$$\begin{aligned} n^{1/2} T^{1/2} \left(\hat{\Psi}_{FE-GLS} - \Psi \right) &= \left(n^{-1} T^{-1} F_{1,Z} \right)^{-1} \left(n^{-1/2} T^{-1/2} F_{2,Z} \right) + o_p(1), \\ nT Var\left(\hat{\Psi}_{FE-GLS} \right) &= \left(n^{-1} T^{-1} F_{1,Z} \right)^{-1} + o_p(1). \end{aligned}$$

Therefore, the asymptotic distribution $\hat{\Psi}_{FE-GLS}$ is not changed when additional stationary or nonstationary regressors are included in the regression. Hence, for the hypothesis H_0: $R\Psi = 0$, the corresponding Wald test has the same asymptotic properties given in Theorems 1 and 2.

3.2 Generalization of the Error Component

We now consider an extension of the analysis to the case where the error term ν_{it} is allowed to have a more general structure. Following Perron and Yabu (2009a, 2009b), we can modify Equation (2) into

$$\nu_{it} = \rho \nu_{i,t-1} + \epsilon_{it} \tag{12}$$

$$\epsilon_{it} = d(L) e_{it} \tag{13}$$

with $d(L) = \sum_{k=0}^{\infty} d_k L^k$, $\sum_{k=0}^{\infty} k |d_k| < \infty$, $d(1) \neq 0$, and e_{it} is a white noise process with variance σ_e^2. Under these conditions, ν_{it} has an autoregressive representation, say $A(L) v_{it} = e_{it}$, where $A(L) = 1 - \sum_{k=0}^{\infty} a_k L^k$. Let the parameter ρ represent the sum of the AR coefficients. Accordingly, we have the representation

$$\nu_{it} = \rho \nu_{i,t-1} + A^*(L) \Delta v_{i,t-1} + e_{it},$$

where $A^*(L) = \sum_{k=0}^{\infty} a_k^* L^k$ with $a_k^* = -\sum_{j=k+2}^{\infty} a_j$. Based on a truncated autoregression of order K, an estimate $\tilde{\theta} = \left(\tilde{\rho}, \tilde{\zeta}_1, \cdots, \tilde{\zeta}_K\right)'$ can be obtained from the following regression

$$\hat{\nu}_{it} = \rho \hat{\nu}_{i,t-1} + \sum_{k=1}^{K} \zeta_k \Delta \hat{\nu}_{i,t-k} + e_{it},$$

where $\hat{\nu}_{it}$ is the within residual in (3). If $K \to \infty$ and $K^3/T \to 0$ as $T \to \infty$, we have the following theorem:

Theorem 4 *Assume* $(n, T) \to \infty$,

1. *If* $|\rho| < 1$,

$$\sqrt{T}(\tilde{\rho} - \rho) \xrightarrow{p} 0.$$

2. *If* $\rho = 1$,

$$T(\tilde{\rho} - 1) \xrightarrow{p} -\frac{3}{1 - \zeta_1 - \cdots - \zeta_K}.$$

In Theorem 4, the asymptotic bias of $\tilde{\rho}$ depends on $\zeta_1, \ldots, \zeta_K$ when $\rho = 1$. When $\zeta_1 = \ldots = \zeta_K = 0$, the asymptotic bias reduces to -3 as in BKL. Similar to Equation (6), a bias-corrected estimator $\hat{\rho}$ can be obtained replacing the threshold by $3/\left[T\left(1 - \tilde{\zeta}_1 - \cdots - \tilde{\zeta}_K\right)\right]$. Using $\hat{\rho}$, the Wald test statistics can be obtained from Equation (4). To calculate the Wald test statistics, we need an estimate of σ_ϵ^2, i.e., the variance of the error term ϵ_{it}. When ν_{it} is I(0) or I(1), σ_ϵ^2 can be estimated following Equations (12) and (13) in Perron and Yabu (2009b), respectively.

3.3 A Special Case: Without Individual Effects

Let us consider a special case where $\mu_i = 0$ for all i, i.e., there are no individual effects in the panel data model. The variance-covariance matrix of equation (1) is

$$\Phi = E\left(uu'\right) = \sigma_e^2 \left(I_n \otimes \boldsymbol{A}\right).$$

The least squares estimator of the transformed equation yields the GLS estimator:

$$\hat{\Psi}_{GLS} = \left[Z^{*\prime} M_{\boldsymbol{\iota}_{nT}^*} Z^*\right]^{-1} Z^{*\prime} M_{\boldsymbol{\iota}_{nT}^*} y^*, \tag{14}$$

where $M_{\boldsymbol{\iota}_{nT}^*} = I_{nT} - \boldsymbol{\iota}_{nT}^* \left(\boldsymbol{\iota}_{nT}^{*\prime}\boldsymbol{\iota}_{nT}^*\right) \boldsymbol{\iota}_{nT}^{*\prime} = I_{nT} - \bar{J}_n \otimes \bar{J}_T^{\alpha}$ using the fact that $\boldsymbol{\iota}_{nT}^* = (1-\rho)\left(\boldsymbol{\iota}_n \otimes \boldsymbol{\iota}_T^{\alpha}\right)$. It is easy to see that $M_{\boldsymbol{\iota}_{nT}^*} Z^* = \left(I_{nT} - \bar{J}_n \otimes \bar{J}_T^{\alpha}\right)\left(\boldsymbol{\iota}_n \otimes z_i^*\right) = \boldsymbol{\iota}_n \otimes \left(z_i^* - \bar{J}_T^{\alpha} z_i^*\right) = \left(I_n \otimes E_T^{\alpha}\right) z^*$. This proves that $\hat{\Psi}_{GLS}$ and $\hat{\Psi}_{FE-GLS}$ are the same if there are no individual effects in the model. The Wald-statistic based on $\hat{\Psi}_{GLS}$ is in turn the same as the one based on $\hat{\Psi}_{FE-GLS}$ in equation (4). Similar to equation (5) for the general model with individual effects, let

$$\hat{\rho} = \frac{\sum_{i=1}^{n}\sum_{t=2}^{T} \hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{i=1}^{n}\sum_{t=2}^{T} \hat{u}_{i,t-1}^2}, \tag{15}$$

where $\hat{u}_{it}$ denotes the OLS residuals. Define $\hat{u}^*$ as an $nT \times 1$ vector of OLS residuals from the Prais-Winsten transformed regression using $\hat{\rho}$. An estimator of σ_e^2 is $\hat{\sigma}_e^2 = \frac{1}{nT}\hat{u}^{*\prime}\hat{u}^*$. Substituting $\hat{\rho}$ and $\hat{\sigma}_e^2$, the Wald-statistic corresponding to the FGLS estimator can be obtained from equation (4). BKL show that then $\sqrt{nT}\left(\hat{\rho} - \rho\right) \xrightarrow{d} N\left(0, 1-\rho^2\right)$ if $|\rho| < 1$, and $\sqrt{n}T\left(\hat{\rho} - 1\right) \xrightarrow{d} N\left(0, 3\right)$ if $\rho = 1$ as $(n, T) \to \infty$. Therefore, when $\mu_i = 0$ for all i, for $\hat{\rho}$, $\mathrm{W}_{FGLS}(k) \xrightarrow{d} \chi^2(1)$ when $|\rho| < 1$ or $\rho = 1$ as $(n, T) \to \infty$. Hence $\mathrm{W}_{FGLS}(k)$ converges to $\chi^2(1)$ whether the error term is $I(0)$ or $I(1)$ when there are no individual effects in the model. This is an interesting result, i.e., the Wald test based on FGLS effectively bridges the gap between the $I(0)$ and $I(1)$ error terms, if there are no individual effects in the model. This implies that inference on the slope parameter can be performed using the standard normal distribution if there are no individual effects. This is different from the pure time series model discussed in Perron and Yabu (2009b) which requires a super-efficient estimate in order to achieve this goal.

4 Monte Carlo Results

This section reports the results of Monte Carlo experiments designed to investigate the finite sample size and power properties of the proposed test statistic. The model is generated by

$$y_{it} = 5 + 10t + \gamma DT + \mu_i + v_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{16}$$

where $DT = 1(t > k)(t - k)$ with a change point $k = 0.5T$. $\mu_i \overset{iid}{\sim} N(0, 5)$, $\nu_{it} = \rho\nu_{it-1} + e_{it}$ with $v_{i0} = 0$ and $e_{it} \overset{iid}{\sim} N(0, 5)$. γ varies over the range $(0, 0.02, 0.04, 0.06, 0.08, 0.1)$. ρ varies over the range $(0, 0.2, 0.4, 0.6, 0.8, 0.9, 1)$. The sample sizes considered are $n = (50, 200, 500)$ and $T = (20, 50)$. For each experiment, we perform $1,000$ replications. For each replication we estimate the model using: (i) FD: first-difference ignoring serial correlation; (ii) FE: fixed-effects ignoring serial correlation; (iii) FE-GLS: FE-GLS estimator using the true value of ρ; (iv) FE-FGLS$_1$: FE-FGLS estimator using $\tilde{\rho}$ calculated by the method suggested in Baltagi and Li (1991); and (v) FE-FGLS$_2$: FE-FGLS estimator using a bias-corrected estimator $\hat{\rho}$. Tables 1 and 6 report the size and size-adjusted power of the Wald-test for $H_0 : \gamma = 0$ corresponding to each estimator of γ. In Table 1 for example, the size of the Wald-test based on the FD estimator is 0.000 when the true value of $\rho = 0$ and the size of the Wald-test based on the FE estimator is 0.251 when the true value of $\rho = 1$. By comparison, the size of the Wald-test based on the FE-GLS estimator is always close to 0.05. Using $\tilde{\rho}$ in Baltagi and Li (1991), the size of the Wald-test based on the FE-FGLS$_1$ estimator is larger than 0.05 when the true value of ρ is large. For example, when $\rho = 1$, the size of the Wald-test based on the FE-FGLS$_1$ estimator is 0.060 when $(n, T) = (500, 20)$ and 0.087 when $(n, T) = (500, 50)$. Using the bias-corrected estimator $\hat{\rho}$, the size of the Wald-test based on the FE-FGLS$_2$ estimator performs better. For example, when $\rho = 1$, the size of the Wald-test based on the FE-FGLS$_2$ estimator is 0.042 when $(n, T) = (500, 20)$ and 0.053 when $(n, T) = (500, 50)$. This finding is consistent with the asymptotic results in Theorem 1.[2] The size-adjusted power increases as γ increases.

[2] We thank a referee pointing out that the Wald-test based on true FE-GLS has size distortion when ρ is large. The size performance is actually worse than the Wald-test based on FE-FGLS2. In a pure time series frame work, Theorem 2 of Perron and Yabu (2009b) derived the asymptotic distribution of the Wald test based on GLS estimator in case of local to unity. They find that "a conservative test may be expected for values of α (ρ in our notation) close to 1, relative to the sample size." The FGLS estimator has the same asymptotic distribution as the FD estmiator in the local to unity case. Hence FGLS performs better. Our simulations find similar results in a panel data frame work.

Finally, we examine the performance of the proposed Wald test that is based on the FE-FGLS estimator when the change point is treated as *unknown*. Tables 7–12 report the size and size-adjusted power of the $sup\mathrm{W}_{FE-GLS}$, $Mean\mathrm{W}_{FE-GLS}$ and $Exp\mathrm{W}_{FE-GLS}$ tests. As discussed before, the break date k is chosen between 2 and $T-1$. Following Perron and Yabu (2009b), asymptotic critical values are calculated by simulations using random standard normal variables to approximate the Wiener process. The integrals are approximated by normalized sums with 2,000 steps, and 10,000 replications are used. The Monte Carlo results show that the $Mean\mathrm{W}_{FE-GLS}$ test performs the best. Its size is always close to 0.05 for different values of ρ. When the sample sizes are small, $Exp\mathrm{W}_{FE-GLS}$ has relatively large size distortion. However, when the sample sizes increase to $(n, T) = (500, 50)$, the size of $ExpW_{FE-GLS}$ shown in Table 12 is also close to 0.05. In contrast, Table 12 for example shows that the size of $sup\mathrm{W}_{FE-GLS}$ is still as large as 0.095 when $\rho = 0.9$.[3]

In conclusion, this paper recommends the $Mean\mathrm{W}_{FE-GLS}$ test for testing structural change in a time trend panel data model when the change point is treated as *unknown*. In case, the change point is *known*, we recommend the Wald-test based on the FE-FGLS$_2$ estimator.

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Table 1: Size and Size-Adjusted Power of the Wald Test at a *Known* Date ($n = 50$, $T = 20$)

ρ	γ	FD	FE	FE-GLS	FE-FGLS$_1$	FE-FGLS$_2$
0	0.00	0.000	0.043	0.043	0.042	0.030
0	0.02	0.055	0.082	0.082	0.088	0.084
0	0.04	0.081	0.149	0.149	0.150	0.150
0	0.06	0.117	0.267	0.267	0.272	0.272
0	0.08	0.161	0.405	0.405	0.408	0.407
0	0.10	0.227	0.579	0.579	0.582	0.582
0.2	0.00	0.000	0.099	0.043	0.048	0.031
0.2	0.02	0.044	0.075	0.066	0.069	0.068
0.2	0.04	0.064	0.121	0.122	0.120	0.120
0.2	0.06	0.099	0.191	0.200	0.202	0.198
0.2	0.08	0.138	0.304	0.305	0.312	0.311
0.2	0.10	0.200	0.430	0.425	0.434	0.430
0.4	0.00	0.000	0.167	0.039	0.053	0.033
0.4	0.02	0.049	0.064	0.059	0.061	0.061
0.4	0.04	0.071	0.088	0.097	0.101	0.105
0.4	0.06	0.098	0.150	0.142	0.144	0.144
0.4	0.08	0.131	0.217	0.225	0.220	0.226
0.4	0.10	0.193	0.293	0.304	0.305	0.307
0.6	0.00	0.000	0.248	0.033	0.066	0.031
0.6	0.02	0.044	0.063	0.059	0.058	0.060
0.6	0.04	0.064	0.074	0.076	0.078	0.077
0.6	0.06	0.087	0.114	0.115	0.119	0.117
0.6	0.08	0.112	0.156	0.161	0.165	0.163
0.6	0.10	0.166	0.214	0.224	0.228	0.219
0.8	0.00	0.006	0.317	0.036	0.090	0.047
0.8	0.02	0.058	0.057	0.050	0.062	0.050
0.8	0.04	0.059	0.067	0.063	0.071	0.068
0.8	0.06	0.069	0.084	0.077	0.089	0.082
0.8	0.08	0.090	0.115	0.100	0.118	0.100
0.8	0.10	0.127	0.148	0.132	0.144	0.137
0.9	0.00	0.032	0.334	0.040	0.090	0.057
0.9	0.02	0.056	0.053	0.052	0.052	0.055
0.9	0.04	0.059	0.061	0.058	0.057	0.059
0.9	0.06	0.064	0.072	0.067	0.073	0.072
0.9	0.08	0.085	0.085	0.085	0.092	0.086
0.9	0.10	0.106	0.101	0.113	0.122	0.121
1	0.00	0.043	0.251	0.043	0.067	0.043
1	0.02	0.052	0.050	0.052	0.044	0.052
1	0.04	0.057	0.053	0.057	0.056	0.060
1	0.06	0.065	0.064	0.065	0.061	0.064
1	0.08	0.077	0.078	0.077	0.074	0.078
1	0.10	0.097	0.097	0.097	0.097	0.098

Table 2: Size and Size-Adjusted Power of the Wald Test at a *Known* Date ($n = 50$, $T = 50$)

ρ	γ	FD	FE	FE-GLS	FE-FGLS$_1$	FE-FGLS$_2$
0	0.00	0.000	0.045	0.045	0.044	0.037
0	0.02	0.090	0.392	0.392	0.392	0.390
0	0.04	0.209	0.906	0.906	0.908	0.908
0	0.06	0.468	1.000	1.000	1.000	1.000
0	0.08	0.719	1.000	1.000	1.000	1.000
0	0.10	0.902	1.000	1.000	1.000	1.000
0.2	0.00	0.000	0.116	0.045	0.048	0.043
0.2	0.02	0.087	0.284	0.288	0.289	0.288
0.2	0.04	0.209	0.755	0.763	0.761	0.759
0.2	0.06	0.460	0.986	0.989	0.985	0.985
0.2	0.08	0.709	1.000	1.000	1.000	1.000
0.2	0.10	0.895	1.000	1.000	1.000	1.000
0.4	0.00	0.000	0.194	0.045	0.051	0.041
0.4	0.02	0.085	0.187	0.188	0.173	0.171
0.4	0.04	0.201	0.532	0.544	0.529	0.528
0.4	0.06	0.415	0.873	0.883	0.873	0.878
0.4	0.08	0.671	0.990	0.993	0.989	0.990
0.4	0.10	0.865	1.000	1.000	1.000	1.000
0.6	0.00	0.000	0.296	0.038	0.056	0.036
0.6	0.02	0.081	0.124	0.117	0.118	0.119
0.6	0.04	0.174	0.322	0.335	0.328	0.331
0.6	0.06	0.348	0.581	0.623	0.614	0.612
0.6	0.08	0.563	0.827	0.863	0.855	0.860
0.6	0.10	0.767	0.949	0.965	0.965	0.964
0.8	0.00	0.000	0.455	0.027	0.055	0.033
0.8	0.02	0.081	0.072	0.080	0.081	0.079
0.8	0.04	0.153	0.146	0.173	0.173	0.171
0.8	0.06	0.250	0.267	0.306	0.315	0.304
0.8	0.08	0.406	0.413	0.475	0.471	0.472
0.8	0.10	0.559	0.571	0.656	0.653	0.658
0.9	0.00	0.007	0.510	0.029	0.076	0.044
0.9	0.02	0.066	0.066	0.069	0.066	0.064
0.9	0.04	0.111	0.108	0.099	0.106	0.100
0.9	0.06	0.179	0.159	0.181	0.186	0.190
0.9	0.08	0.267	0.243	0.268	0.264	0.272
0.9	0.10	0.374	0.353	0.383	0.384	0.387
1	0.00	0.048	0.482	0.048	0.077	0.046
1	0.02	0.055	0.061	0.055	0.058	0.056
1	0.04	0.068	0.089	0.068	0.077	0.073
1	0.06	0.103	0.127	0.103	0.108	0.106
1	0.08	0.143	0.171	0.143	0.153	0.146
1	0.10	0.209	0.219	0.209	0.217	0.214

Table 3: Size and Size-Adjusted Power of the Wald Test at a *Known* Date ($n = 200$, $T = 20$)

ρ	γ	FD	FE	FE-GLS	FE-FGLS$_1$	FE-FGLS$_2$
0	0.00	0.000	0.048	0.048	0.047	0.042
0	0.02	0.071	0.119	0.119	0.122	0.119
0	0.04	0.146	0.353	0.353	0.352	0.350
0	0.06	0.291	0.689	0.689	0.688	0.683
0	0.08	0.481	0.909	0.909	0.908	0.904
0	0.10	0.671	0.976	0.976	0.976	0.976
0.2	0.00	0.000	0.096	0.048	0.053	0.042
0.2	0.02	0.070	0.094	0.096	0.095	0.098
0.2	0.04	0.158	0.253	0.256	0.247	0.255
0.2	0.06	0.285	0.531	0.533	0.526	0.526
0.2	0.08	0.469	0.786	0.783	0.775	0.779
0.2	0.10	0.670	0.933	0.936	0.932	0.932
0.4	0.00	0.000	0.162	0.048	0.060	0.042
0.4	0.02	0.083	0.083	0.083	0.083	0.088
0.4	0.04	0.149	0.169	0.174	0.179	0.175
0.4	0.06	0.278	0.332	0.373	0.373	0.379
0.4	0.08	0.441	0.563	0.605	0.606	0.612
0.4	0.10	0.640	0.778	0.795	0.804	0.802
0.6	0.00	0.002	0.236	0.037	0.069	0.039
0.6	0.02	0.068	0.071	0.079	0.076	0.079
0.6	0.04	0.128	0.124	0.136	0.131	0.135
0.6	0.06	0.229	0.222	0.255	0.245	0.259
0.6	0.08	0.350	0.357	0.420	0.414	0.415
0.6	0.10	0.521	0.544	0.594	0.586	0.594
0.8	0.00	0.010	0.308	0.037	0.082	0.051
0.8	0.02	0.062	0.058	0.064	0.058	0.061
0.8	0.04	0.115	0.093	0.114	0.101	0.108
0.8	0.06	0.170	0.146	0.171	0.164	0.165
0.8	0.08	0.266	0.225	0.265	0.259	0.259
0.8	0.10	0.382	0.326	0.389	0.370	0.383
0.9	0.00	0.036	0.337	0.047	0.100	0.062
0.9	0.02	0.060	0.073	0.059	0.057	0.059
0.9	0.04	0.092	0.088	0.090	0.086	0.084
0.9	0.06	0.132	0.131	0.133	0.126	0.129
0.9	0.08	0.212	0.184	0.212	0.202	0.209
0.9	0.10	0.304	0.271	0.312	0.296	0.304
1	0.00	0.060	0.246	0.060	0.084	0.059
1	0.02	0.055	0.059	0.055	0.055	0.055
1	0.04	0.078	0.077	0.078	0.074	0.079
1	0.06	0.113	0.104	0.113	0.115	0.113
1	0.08	0.162	0.157	0.162	0.162	0.162
1	0.10	0.252	0.223	0.252	0.242	0.248

Table 4: Size and Size-Adjusted Power of the Wald Test at a *Known* Date ($n = 200$, $T = 50$)

ρ	γ	FD	FE	FE-GLS	FE-FGLS$_1$	FE-FGLS$_2$
0	0.00	0.000	0.038	0.038	0.040	0.033
0	0.02	0.245	0.917	0.917	0.915	0.914
0	0.04	0.722	1.000	1.000	1.000	1.000
0	0.06	0.962	1.000	1.000	1.000	1.000
0	0.08	0.999	1.000	1.000	1.000	1.000
0	0.10	1.000	1.000	1.000	1.000	1.000
0.2	0.00	0.000	0.100	0.037	0.040	0.035
0.2	0.02	0.240	0.769	0.779	0.775	0.770
0.2	0.04	0.703	1.000	1.000	1.000	1.000
0.2	0.06	0.960	1.000	1.000	1.000	1.000
0.2	0.08	0.999	1.000	1.000	1.000	1.000
0.2	0.10	1.000	1.000	1.000	1.000	1.000
0.4	0.00	0.000	0.182	0.041	0.045	0.037
0.4	0.02	0.226	0.528	0.562	0.564	0.566
0.4	0.04	0.657	0.989	0.991	0.991	0.990
0.4	0.06	0.943	1.000	1.000	1.000	1.000
0.4	0.08	0.997	1.000	1.000	1.000	1.000
0.4	0.10	1.000	1.000	1.000	1.000	1.000
0.6	0.00	0.000	0.295	0.038	0.051	0.037
0.6	0.02	0.186	0.309	0.339	0.339	0.338
0.6	0.04	0.566	0.828	0.852	0.851	0.854
0.6	0.06	0.895	0.994	0.996	0.996	0.996
0.6	0.08	0.988	1.000	1.000	1.000	1.000
0.6	0.10	1.000	1.000	1.000	1.000	1.000
0.8	0.00	0.000	0.437	0.029	0.059	0.031
0.8	0.02	0.124	0.136	0.158	0.159	0.162
0.8	0.04	0.374	0.412	0.483	0.480	0.486
0.8	0.06	0.715	0.725	0.801	0.805	0.807
0.8	0.08	0.917	0.916	0.966	0.963	0.966
0.8	0.10	0.983	0.991	0.998	0.998	0.999
0.9	0.00	0.004	0.534	0.032	0.077	0.042
0.9	0.02	0.097	0.107	0.102	0.108	0.107
0.9	0.04	0.241	0.256	0.259	0.267	0.268
0.9	0.06	0.494	0.447	0.517	0.513	0.517
0.9	0.08	0.727	0.672	0.756	0.749	0.759
0.9	0.10	0.894	0.837	0.903	0.902	0.906
1	0.00	0.048	0.477	0.048	0.081	0.048
1	0.02	0.078	0.058	0.078	0.075	0.077
1	0.04	0.151	0.135	0.151	0.147	0.150
1	0.06	0.264	0.235	0.264	0.260	0.264
1	0.08	0.454	0.376	0.454	0.440	0.451
1	0.10	0.613	0.522	0.613	0.605	0.611

Table 5: Size and Size-Adjusted Power of the Wald Test at a *Known* Date ($n = 500$, $T = 20$)

ρ	γ	FD	FE	FE-GLS	FE-FGLS$_1$	FE-FGLS$_2$
0	0.00	0.000	0.051	0.051	0.051	0.044
0	0.02	0.126	0.240	0.240	0.246	0.241
0	0.04	0.353	0.728	0.728	0.726	0.721
0	0.06	0.658	0.969	0.969	0.969	0.969
0	0.08	0.882	0.999	0.999	0.999	1.000
0	0.10	0.972	1.000	1.000	1.000	1.000
0.2	0.00	0.000	0.105	0.050	0.053	0.047
0.2	0.02	0.132	0.184	0.188	0.194	0.193
0.2	0.04	0.353	0.563	0.570	0.579	0.575
0.2	0.06	0.652	0.898	0.895	0.898	0.896
0.2	0.08	0.876	0.992	0.993	0.994	0.994
0.2	0.10	0.973	1.000	1.000	1.000	1.000
0.4	0.00	0.000	0.180	0.042	0.059	0.040
0.4	0.02	0.118	0.147	0.161	0.152	0.156
0.4	0.04	0.311	0.415	0.434	0.431	0.423
0.4	0.06	0.598	0.736	0.764	0.760	0.758
0.4	0.08	0.832	0.923	0.938	0.935	0.936
0.4	0.10	0.960	0.995	0.994	0.995	0.994
0.6	0.00	0.001	0.259	0.036	0.068	0.035
0.6	0.02	0.099	0.118	0.114	0.121	0.115
0.6	0.04	0.260	0.273	0.312	0.310	0.312
0.6	0.06	0.499	0.512	0.581	0.582	0.589
0.6	0.08	0.729	0.758	0.814	0.815	0.817
0.6	0.10	0.897	0.901	0.939	0.936	0.939
0.8	0.00	0.009	0.336	0.031	0.080	0.043
0.8	0.02	0.092	0.083	0.091	0.096	0.095
0.8	0.04	0.218	0.189	0.216	0.215	0.216
0.8	0.06	0.391	0.353	0.403	0.403	0.404
0.8	0.08	0.589	0.528	0.600	0.606	0.599
0.8	0.10	0.753	0.716	0.769	0.770	0.772
0.9	0.00	0.027	0.356	0.038	0.093	0.056
0.9	0.02	0.078	0.074	0.075	0.078	0.086
0.9	0.04	0.182	0.147	0.174	0.171	0.181
0.9	0.06	0.329	0.293	0.318	0.312	0.323
0.9	0.08	0.496	0.431	0.489	0.483	0.495
0.9	0.10	0.660	0.589	0.656	0.643	0.661
1	0.00	0.044	0.270	0.044	0.060	0.042
1	0.02	0.088	0.093	0.088	0.086	0.088
1	0.04	0.160	0.171	0.160	0.166	0.159
1	0.06	0.271	0.274	0.271	0.283	0.271
1	0.08	0.442	0.422	0.442	0.448	0.440
1	0.10	0.611	0.576	0.611	0.617	0.610

Table 6: Size and Size-Adjusted Power of the Wald Test at a *Known* Date ($n = 500$, $T = 50$)

ρ	γ	FD	FE	FE-GLS	FE-FGLS$_1$	FE-FGLS$_2$
0	0.00	0.000	0.052	0.052	0.051	0.049
0	0.02	0.524	0.999	0.999	0.999	0.999
0	0.04	0.981	1.000	1.000	1.000	1.000
0	0.06	1.000	1.000	1.000	1.000	1.000
0	0.08	1.000	1.000	1.000	1.000	1.000
0	0.10	1.000	1.000	1.000	1.000	1.000
0.2	0.00	0.000	0.093	0.052	0.053	0.047
0.2	0.02	0.518	0.984	0.984	0.986	0.986
0.2	0.04	0.979	1.000	1.000	1.000	1.000
0.2	0.06	1.000	1.000	1.000	1.000	1.000
0.2	0.08	1.000	1.000	1.000	1.000	1.000
0.2	0.10	1.000	1.000	1.000	1.000	1.000
0.4	0.00	0.000	0.183	0.047	0.053	0.046
0.4	0.02	0.472	0.881	0.900	0.897	0.899
0.4	0.04	0.964	1.000	1.000	1.000	1.000
0.4	0.06	1.000	1.000	1.000	1.000	1.000
0.4	0.08	1.000	1.000	1.000	1.000	1.000
0.4	0.10	1.000	1.000	1.000	1.000	1.000
0.6	0.00	0.000	0.291	0.043	0.056	0.044
0.6	0.02	0.356	0.631	0.646	0.647	0.651
0.6	0.04	0.904	0.995	0.996	0.996	0.996
0.6	0.06	0.999	1.000	1.000	1.000	1.000
0.6	0.08	1.000	1.000	1.000	1.000	1.000
0.6	0.10	1.000	1.000	1.000	1.000	1.000
0.8	0.00	0.000	0.450	0.030	0.069	0.037
0.8	0.02	0.234	0.271	0.295	0.302	0.297
0.8	0.04	0.714	0.756	0.833	0.832	0.831
0.8	0.06	0.964	0.976	0.992	0.991	0.991
0.8	0.08	1.000	0.999	1.000	1.000	1.000
0.8	0.10	1.000	1.000	1.000	1.000	1.000
0.9	0.00	0.001	0.523	0.041	0.083	0.054
0.9	0.02	0.140	0.165	0.159	0.159	0.158
0.9	0.04	0.456	0.461	0.513	0.502	0.507
0.9	0.06	0.814	0.781	0.847	0.843	0.845
0.9	0.08	0.962	0.955	0.976	0.976	0.975
0.9	0.10	0.999	0.995	0.999	0.999	0.999
1	0.00	0.053	0.499	0.053	0.087	0.053
1	0.02	0.097	0.102	0.097	0.112	0.100
1	0.04	0.279	0.261	0.279	0.304	0.278
1	0.06	0.547	0.488	0.547	0.566	0.552
1	0.08	0.786	0.718	0.786	0.798	0.786
1	0.10	0.929	0.894	0.929	0.935	0.929

Table 7: Size and Size-Adjusted Power of the Wald Test at an *Unknown* Date ($n = 50$, $T = 20$)

ρ	γ	sup-W	Mean-W	Exp-W
0	0.00	0.044	0.069	0.045
0	0.02	0.054	0.049	0.060
0	0.04	0.080	0.098	0.140
0	0.06	0.129	0.152	0.190
0	0.08	0.226	0.269	0.300
0	0.10	0.329	0.388	0.415
0.2	0.00	0.042	0.051	0.045
0.2	0.02	0.053	0.058	0.050
0.2	0.04	0.066	0.086	0.100
0.2	0.06	0.103	0.128	0.160
0.2	0.08	0.158	0.217	0.215
0.2	0.10	0.237	0.311	0.305
0.4	0.00	0.052	0.050	0.055
0.4	0.02	0.052	0.053	0.025
0.4	0.04	0.056	0.071	0.055
0.4	0.06	0.077	0.094	0.095
0.4	0.08	0.104	0.126	0.135
0.4	0.10	0.147	0.186	0.175
0.6	0.00	0.077	0.047	0.065
0.6	0.02	0.047	0.052	0.045
0.6	0.04	0.047	0.062	0.050
0.6	0.06	0.053	0.070	0.065
0.6	0.08	0.068	0.090	0.100
0.6	0.10	0.082	0.108	0.135
0.8	0.00	0.093	0.043	0.090
0.8	0.02	0.050	0.057	0.050
0.8	0.04	0.051	0.061	0.075
0.8	0.06	0.054	0.068	0.095
0.8	0.08	0.063	0.069	0.100
0.8	0.10	0.065	0.077	0.095
0.9	0.00	0.108	0.032	0.105
0.9	0.02	0.050	0.052	0.055
0.9	0.04	0.055	0.053	0.065
0.9	0.06	0.060	0.054	0.075
0.9	0.08	0.062	0.061	0.080
0.9	0.10	0.066	0.065	0.085
1	0.00	0.017	0.044	0.035
1	0.02	0.052	0.055	0.050
1	0.04	0.055	0.057	0.045
1	0.06	0.060	0.063	0.060
1	0.08	0.068	0.073	0.080
1	0.10	0.076	0.089	0.095

Table 8: Size and Size-Adjusted Power of the Wald Test at an *Unknown* Date ($n = 50$, $T = 50$)

ρ	γ	sup-W	Mean-W	Exp-W
0	0.00	0.110	0.113	0.050
0	0.02	0.039	0.039	0.250
0	0.04	0.025	0.025	0.785
0	0.06	0.023	0.023	0.990
0	0.08	0.015	0.015	1.000
0	0.10	0.006	0.006	1.000
0.2	0.00	0.046	0.043	0.050
0.2	0.02	0.155	0.246	0.180
0.2	0.04	0.551	0.677	0.595
0.2	0.06	0.894	0.954	0.925
0.2	0.08	0.996	1.000	0.995
0.2	0.10	1.000	1.000	1.000
0.4	0.00	0.057	0.041	0.050
0.4	0.02	0.104	0.167	0.095
0.4	0.04	0.328	0.464	0.390
0.4	0.06	0.640	0.749	0.680
0.4	0.08	0.889	0.954	0.925
0.4	0.10	0.989	0.999	0.985
0.6	0.00	0.065	0.041	0.050
0.6	0.02	0.066	0.088	0.065
0.6	0.04	0.133	0.234	0.180
0.6	0.06	0.314	0.448	0.380
0.6	0.08	0.506	0.668	0.600
0.6	0.10	0.716	0.837	0.795
0.8	0.00	0.102	0.038	0.060
0.8	0.02	0.056	0.060	0.040
0.8	0.04	0.074	0.092	0.050
0.8	0.06	0.108	0.148	0.085
0.8	0.08	0.161	0.234	0.145
0.8	0.10	0.247	0.346	0.235
0.9	0.00	0.129	0.031	0.075
0.9	0.02	0.053	0.052	0.045
0.9	0.04	0.054	0.065	0.055
0.9	0.06	0.063	0.087	0.065
0.9	0.08	0.081	0.108	0.080
0.9	0.10	0.110	0.138	0.095
1	0.00	0.023	0.032	0.040
1	0.02	0.051	0.056	0.045
1	0.04	0.053	0.072	0.050
1	0.06	0.063	0.082	0.055
1	0.08	0.078	0.105	0.080
1	0.10	0.091	0.135	0.095

Table 9: Size and Size-Adjusted Power of the Wald Test at an *Unknown* Date ($n = 200$, $T = 20$)

ρ	γ	sup-W	Mean-W	Exp-W
0	0.00	0.036	0.058	0.035
0	0.02	0.091	0.087	0.105
0	0.04	0.234	0.261	0.315
0	0.06	0.469	0.526	0.565
0	0.08	0.730	0.782	0.760
0	0.10	0.907	0.921	0.950
0.2	0.00	0.042	0.055	0.035
0.2	0.02	0.077	0.069	0.090
0.2	0.04	0.158	0.170	0.215
0.2	0.06	0.317	0.373	0.420
0.2	0.08	0.509	0.579	0.615
0.2	0.10	0.728	0.775	0.755
0.4	0.00	0.048	0.051	0.035
0.4	0.02	0.067	0.056	0.070
0.4	0.04	0.107	0.112	0.140
0.4	0.06	0.188	0.211	0.275
0.4	0.08	0.296	0.377	0.405
0.4	0.10	0.451	0.526	0.575
0.6	0.00	0.059	0.048	0.045
0.6	0.02	0.058	0.053	0.065
0.6	0.04	0.078	0.072	0.095
0.6	0.06	0.111	0.113	0.140
0.6	0.08	0.164	0.170	0.210
0.6	0.10	0.235	0.260	0.305
0.8	0.00	0.089	0.041	0.055
0.8	0.02	0.054	0.055	0.055
0.8	0.04	0.060	0.056	0.055
0.8	0.06	0.067	0.068	0.065
0.8	0.08	0.080	0.090	0.090
0.8	0.10	0.098	0.110	0.115
0.9	0.00	0.094	0.039	0.055
0.9	0.02	0.048	0.054	0.050
0.9	0.04	0.054	0.051	0.065
0.9	0.06	0.059	0.057	0.080
0.9	0.08	0.061	0.059	0.080
0.9	0.10	0.073	0.079	0.080
1	0.00	0.022	0.068	0.085
1	0.02	0.057	0.057	0.060
1	0.04	0.066	0.062	0.075
1	0.06	0.083	0.093	0.070
1	0.08	0.107	0.127	0.115
1	0.10	0.151	0.173	0.180

Table 10: Size and Size-Adjusted Power of the Wald Test at an *Unknown* Date ($n = 200$, $T = 50$)

ρ	γ	sup-W	Mean-W	Exp-W
0	0.00	0.082	0.080	0.035
0	0.02	0.396	0.571	0.785
0	0.04	0.998	1.000	1.000
0	0.06	1.000	1.000	1.000
0	0.08	1.000	1.000	1.000
0	0.10	1.000	1.000	1.000
0.2	0.00	0.043	0.034	0.035
0.2	0.02	0.548	0.669	0.530
0.2	0.04	0.997	0.999	1.000
0.2	0.06	1.000	1.000	1.000
0.2	0.08	1.000	1.000	1.000
0.2	0.10	1.000	1.000	1.000
0.4	0.00	0.055	0.035	0.035
0.4	0.02	0.325	0.467	0.360
0.4	0.04	0.899	0.961	0.900
0.4	0.06	1.000	1.000	1.000
0.4	0.08	1.000	1.000	1.000
0.4	0.10	1.000	1.000	1.000
0.6	0.00	0.069	0.034	0.040
0.6	0.02	0.158	0.242	0.190
0.6	0.04	0.541	0.688	0.515
0.6	0.06	0.899	0.964	0.895
0.6	0.08	0.996	0.999	1.000
0.6	0.10	1.000	1.000	1.000
0.8	0.00	0.080	0.030	0.055
0.8	0.02	0.081	0.102	0.095
0.8	0.04	0.190	0.257	0.185
0.8	0.06	0.372	0.480	0.330
0.8	0.08	0.604	0.702	0.520
0.8	0.10	0.802	0.880	0.740
0.9	0.00	0.107	0.025	0.065
0.9	0.02	0.052	0.066	0.055
0.9	0.04	0.085	0.115	0.090
0.9	0.06	0.127	0.191	0.140
0.9	0.08	0.234	0.306	0.205
0.9	0.10	0.347	0.434	0.295
1	0.00	0.023	0.032	0.070
1	0.02	0.059	0.058	0.055
1	0.04	0.073	0.103	0.060
1	0.06	0.123	0.172	0.075
1	0.08	0.213	0.253	0.115
1	0.10	0.292	0.349	0.175

Table 11: Size and Size-Adjusted Power of the Wald Test at an *Unknown* Date ($n = 500$, $T = 20$)

ρ	γ	sup-W	Mean-W	Exp-W
0	0.00	0.036	0.048	0.040
0	0.02	0.139	0.188	0.185
0	0.04	0.504	0.603	0.605
0	0.06	0.873	0.917	0.945
0	0.08	0.985	0.990	0.995
0	0.10	1.000	1.000	1.000
0.2	0.00	0.043	0.048	0.040
0.2	0.02	0.111	0.125	0.145
0.2	0.04	0.318	0.418	0.460
0.2	0.06	0.653	0.749	0.780
0.2	0.08	0.909	0.946	0.960
0.2	0.10	0.986	0.990	0.995
0.4	0.00	0.050	0.042	0.045
0.4	0.02	0.080	0.089	0.105
0.4	0.04	0.189	0.247	0.275
0.4	0.06	0.414	0.504	0.555
0.4	0.08	0.651	0.746	0.775
0.4	0.10	0.873	0.915	0.925
0.6	0.00	0.074	0.041	0.055
0.6	0.02	0.065	0.068	0.065
0.6	0.04	0.120	0.135	0.130
0.6	0.06	0.201	0.260	0.235
0.6	0.08	0.327	0.439	0.415
0.6	0.10	0.508	0.609	0.585
0.8	0.00	0.093	0.037	0.070
0.8	0.02	0.056	0.060	0.055
0.8	0.04	0.075	0.083	0.075
0.8	0.06	0.112	0.115	0.110
0.8	0.08	0.154	0.177	0.155
0.8	0.10	0.213	0.263	0.230
0.9	0.00	0.113	0.032	0.070
0.9	0.02	0.056	0.055	0.050
0.9	0.04	0.074	0.069	0.060
0.9	0.06	0.096	0.091	0.075
0.9	0.08	0.118	0.111	0.110
0.9	0.10	0.154	0.153	0.140
1	0.00	0.022	0.073	0.065
1	0.02	0.066	0.067	0.065
1	0.04	0.107	0.115	0.105
1	0.06	0.173	0.193	0.140
1	0.08	0.263	0.303	0.230
1	0.10	0.388	0.457	0.345

Table 12: Size and Size-Adjusted Power of the Wald Test at an *Unknown* Date ($n = 500$, $T = 50$)

ρ	γ	sup-W	Mean-W	Exp-W
0	0.00	0.052	0.048	0.040
0	0.02	0.992	0.997	1.000
0	0.04	1.000	1.000	1.000
0	0.06	1.000	1.000	1.000
0	0.08	1.000	1.000	1.000
0	0.10	1.000	1.000	1.000
0.2	0.00	0.054	0.047	0.040
0.2	0.02	0.916	0.962	0.965
0.2	0.04	1.000	1.000	1.000
0.2	0.06	1.000	1.000	1.000
0.2	0.08	1.000	1.000	1.000
0.2	0.10	1.000	1.000	1.000
0.4	0.00	0.059	0.047	0.040
0.4	0.02	0.667	0.792	0.805
0.4	0.04	1.000	1.000	1.000
0.4	0.06	1.000	1.000	1.000
0.4	0.08	1.000	1.000	1.000
0.4	0.10	1.000	1.000	1.000
0.6	0.00	0.077	0.046	0.040
0.6	0.02	0.303	0.458	0.500
0.6	0.04	0.909	0.964	0.965
0.6	0.06	1.000	1.000	1.000
0.6	0.08	1.000	1.000	1.000
0.6	0.10	1.000	1.000	1.000
0.8	0.00	0.110	0.045	0.060
0.8	0.02	0.124	0.151	0.155
0.8	0.04	0.345	0.485	0.475
0.8	0.06	0.710	0.813	0.800
0.8	0.08	0.935	0.971	0.975
0.8	0.10	0.997	0.999	1.000
0.9	0.00	0.127	0.035	0.060
0.9	0.02	0.068	0.088	0.095
0.9	0.04	0.135	0.174	0.190
0.9	0.06	0.272	0.367	0.355
0.9	0.08	0.483	0.565	0.565
0.9	0.10	0.689	0.762	0.740
1	0.00	0.034	0.044	0.045
1	0.02	0.057	0.083	0.080
1	0.04	0.131	0.181	0.175
1	0.06	0.269	0.353	0.375
1	0.08	0.463	0.519	0.590
1	0.10	0.638	0.647	0.730