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High School Teachers' Use of Graphing Calculators When Teaching Linear and Quadratic Functions: Professed Beliefs and Observed Practice

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ABSTRACT

This study was designed to explore secondary mathematics teachers' beliefs about graphing calculators, their practices with the graphing calculators when teaching linear and quadratic functions, and the relationship between the teachers' beliefs and their practices. The study was conducted in two phases. In the first phase, 81 teachers responded to a questionnaire about their beliefs regarding the use of graphing calculators in the teaching and learning of linear and quadratic functions. Six of these teachers then participated in the second phase involving task-based interviews and classroom observations.

A major finding from the survey was a possible link between teachers' frequency of calculator use and their views regarding sequencing of function representations. I found that low frequency users held the view that algebraic symbols should always precede tables while high frequency users did not hold a similar view. Teachers in this study were also split on which type of tasks students should be allowed to use graphing calculators. Some teachers stated that they would encourage their students to use the graphing calculator when the students felt it was appropriate regardless of the task while others stated that they would always want their students to learn to solve each type of problem with paper and pencil before they could use a calculator.

Findings from the interviews and classroom observations highlighted some differences among the moderate and high frequency users in terms of how they guide their classes – teacher direction and student exploration – and the level of direction they provide to their students when working with graphing calculators. In terms of classroom dynamics, I found that classes taught by high frequency users seemed to involve more

student exploration than those taught by moderate frequency users. I also found that when lessons were characterized by teacher direction, the graphing calculator was used as a computational tool and when lessons were characterized by student exploration, the graphing calculator was used as a visualizing tool and checking tool.

HIGH SCHOOL TEACHERS' USE OF GRAPHING CALCULATORS WHEN TEACHING LINEAR AND QUADRATIC FUNCTIONS: PROFESSED BELIEFS AND OBSERVED PRACTICE

By

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DISSERTATION

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CHAPTER 1: INTRODUCTION

The concept of function has been widely recognized as being foundational to school mathematics and mathematics in general (Romberg, Carpenter & Fennema, 1993). This is because the concept cuts across virtually all areas of mathematics and has a robust ability to provide meaningful representations of complex situations in the real world and in the world of mathematics (Wilson & Krapfl, 1994). However, research has shown that students of all ages have difficulty mastering the topic using traditional instructional approaches. Mathematics educators are concerned with the difficulties that students have in shifting among representations of a function. Research has shown that graphing calculators can improve students' conceptual understanding of functions by allowing students to explore the various representations of a function (Penglase & Arnold, 1996). This study extends the research base by considering how teachers' beliefs about graphing calculators may influence their use of multiple representations. The study also examines how the nature of classroom dynamics – teacher directed lessons versus lessons involving student exploration – influences the role of the graphing calculator and subsequently the exploration of multiple representations. In this introduction, I discuss the aims of this research, the rationale for the study, and the theoretical framework that influenced the study.

Aims of Research

The purpose of this study was three-fold: (a) to investigate secondary mathematics teachers' professed beliefs about graphing calculators, (b) to investigate how these teachers use graphing calculators to teach the concept of function, and (c) to investigate the extent to which the professed beliefs explain the teachers' use of graphing calculators. The study was guided by the following inquiry questions:

1) What are secondary mathematics teachers' professed beliefs about using graphing calculators in the teaching and learning of linear and quadratic functions with respect to the following areas:
    a) Influence on use and exploration of various representations of functions?
    b) Teacher direction versus student exploration?
2) How do secondary school mathematics teachers use graphing calculators when teaching linear and quadratic functions?
    a) What function representational choices do secondary mathematics teachers make when using graphing calculators?
    b) How specific are the teachers' directions to students about how the calculators may be used?
3) What is the relationship between the teachers' professed beliefs about graphing calculators and observed practice?
    a) What is the nature of the similarities and/or differences between reported and observed calculator usage trends?
    b) To what extent do professed beliefs about graphing calculators explain observed usage trends?

Rationale for this Study

The National Council of Teachers of Mathematics [NCTM] (1989, 2000) advocates a curriculum based on multiple representations, arguing that by encouraging students to incorporate many different types of representations into their sense-making, the students will become more capable of solving mathematical problems and understanding underlying concepts. Research has shown that teaching and learning approaches that emphasize problem solving and exploration, where students actively construct and negotiate meaning for the mathematics they encounter, are more compatible with the use of graphing calculators (Harvey, Waits, & Demana, 1995). Some studies have shown that some teachers hold negative beliefs about the use of graphing calculators (Fleener, 1995a). For the purpose of this study, I used the concept of belief to characterize a teacher's idiosyncratic unity of thought and convictions about objects, people, events and their characteristic relationships that affect his/her planning and decision making (Nespor, 1987). Teachers' beliefs about the use of graphing calculators refer to their conceptions of the process of integrating the graphing calculator into problem solving, what behaviors and mental activities are involved on the part of the learner, and what constitutes appropriate and prototypical learning activities. Negative beliefs about graphing calculators could therefore impede the teachers' use of this technology.

Research has shown that beliefs teachers hold regarding the teaching and learning of mathematics have significant influence on their instructional practices (Cohen, 1990; Thompson, 1992). In some cases, teachers' professed beliefs about mathematics teaching and learning are consistent with what is found in observation of classroom practice (Thompson, 1985). In other cases, however, there are inconsistencies between professed beliefs and classroom practice (Cohen, 1990; Thompson, 1984). More recent research has shown that beliefs can dictate how teachers perceive and interpret classroom interactions and influence the construction of goals as the teachers respond to those interactions (Aguirre & Speer, 2000). It is possible therefore for a teacher to begin teaching with a set of goals but change those goals depending on how s/he perceives classroom interactions.

To date, there have been numerous studies on the use of graphing calculators in the mathematics classroom. Findings have been mixed, with some suggesting that graphing calculators can cause changes in teaching styles (e.g., Farrel, 1996) and others showing that graphing calculators do not have any direct impact on teaching styles (Simmt, 1997; Tharp, Fitzsimmons, & Ayers, 1997). Some studies have shown that graphing calculators cause a shift away from teacher-centered instruction type of teaching to more student-centered instruction (e.g., Simonsen & Dick, 1997). Yet other studies have shown that some teachers are still not sure how to use the calculators in their classrooms to their full potential (Milou, 1999; Simmt, 1997). Such teachers face not only the uncertainty of how to best use the graphing calculator, but they are also sometimes faced with classroom situations that are unfamiliar to them.

Theoretical Framework

For this research study, I drew on Vygotsky's (1978) sociocultural theory of learning. According to Vygotsky, education is both a theory of development and a process of enculturation whereby mediated activity helps shape higher human mental functions. The mediator may be a sign system (e.g., language, tabular or graphical representation of a pattern) or a technological tool (e.g., computer, graphic calculator). Vygotsky contends, "if one changes the tools of thinking available to a child, his mind will have a radically different structure" (p. 126). In this study, I took the position that the graphing calculator is an instrument of access to the knowledge, activities and practices of a social group that is the mathematics classroom (Meira, 1998). In this case, using the calculator can be seen as an external activity (using graphs, tables, and numbers to manipulate mathematical concepts), which is then transformed into an internal activity (gaining an understanding of the mathematical concepts) (Berger, 1998).

Sociocultural theory influenced my study in the sense that I treated the graphing calculator as a tool that is available for teachers to use in mediating their teaching activities and classroom interactions (teacher-student as well as student-student); while at the same time I considered teaching strategies and instructional tasks as tools designed by teachers to mediate calculator usage as an activity. During data collection, I looked for teacher-planned whole-class activities as well as small-group activities, paying attention to the types of interaction that took place in the classrooms.

In addition to Vygotsky's sociocultural theory, I also drew on the work of Goos, Galbraith, Renshaw and Geiger (2003), which provides metaphors for studying the interaction between calculator and user. In particular, I drew on three of these metaphors, namely, "technology as servant, technology as partner, and technology as extension of self" (p. 78). With regard to "technology as servant" (p. 78), Goos et al. (2003) contend that technology used this way can be counterproductive and may lead to misconceptions. According to Goos and her colleagues, it is not worthwhile to use technology as "a supplementary tool that amplifies cognitive processes without using it in creative ways to change the nature of activities" (p. 78). They cite using the overhead projection panel as an electronic chalkboard to provide a medium for demonstrating calculator operations to the class as an example of inappropriate use of technology. Goos et al. contend that using technology in this manner only helps reinforce the teacher's preferred teaching methods and this may not be beneficial to students. They suggest that teachers should use the graphing calculator in conjunction with other material resources in ways that further enhance the calculator's capacity for linking multiple representations of concepts.

With regard to the metaphor of technology as partner, Goos et al. (2003) refer to this level of using a graphing calculator as the "cognitive re-organization effects" (p. 79). According to Goos and colleagues, this is characterized by using technology to explore new tasks or new approaches to existing tasks and to mediate mathematical discussion in the classroom between students and teacher or between small groups of students. They suggest, "instead of functioning as a transmitter of teacher input, the overhead projection panel can become a medium for students to present and examine alternative mathematical conjectures" (p. 79). Finally, the metaphor of "technology as extension of self" asserts that a teacher who attains this level would write unit plans that support integrating technology into the teaching program. That is, the teacher would incorporate technological expertise as a natural part of his or her mathematical and/or pedagogical repertoire.

In Chapter 2, I review the literature related to the problem and purpose of this study. I analyze the body of research in terms of the role of multiple representations in the learning and teaching of functions, teachers' attitudes towards and beliefs about the use of graphing calculators, and teachers' knowledge about the use of graphing calculators.

In Chapter 3, I present the design of the study and methods employed in data collection. In short, I conducted the study in two phases; the first phase consisting of a survey which was completed by 81 secondary school teachers. In the second phase a small subset of these teachers (six) participated in task-based interviews. Additionally, I observed each teacher three times during the course of the study. I audiotaped the interviews, transcribed them and coded them for analysis.

The results of the analysis are presented in Chapters 4 and 5. In Chapter 4, I present the results obtained from the first phase of the study. These results concern all the 81 teachers and mainly address Research Question 1. In Chapter 5, I present a combination of results from both the first phase and the second phase. First, I present the results obtained from the interviews and classroom observation thereby addressing Research Question 2. Then I present the results of the analysis of the survey responses of only the six teachers who participated in the second phase against their responses on the interview tasks and actions they took in their classrooms. This addresses Research Question 3. A summary and discussion of all the results is included in Chapter 6.

CHAPTER 2: LITERATURE REVIEW

In this chapter, I present a review of research studies relevant to the problem and purpose of this study. This review is divided into three parts: (a) the role of multiple representations in understanding of functions, (b) teachers' attitudes towards and beliefs about the use of graphing calculators, and (c) teachers' knowledge about the use of graphing calculators.

The concept of function is of fundamental importance in the learning of mathematics. Critical to understanding this concept is the ability for one to move or transfer from one representation of a function to another. As mentioned in Chapter 1, there have been many studies addressing the central role that functions play in the study of mathematics with increasing recognition that being able to move or transfer between their representations plays a key role in student understanding (Knuth, 2000). Likewise, there is a rich body of literature on research studies involving use of graphing calculators in mathematics classrooms.

The Role of Multiple Representations in Understanding of Functions

In this section, I present an analysis of the roles that multiple representations play in the teaching of and learning about functions. Representing functions in multiple ways is critical to student understanding of the function concept and therefore their success in mathematics. This perspective is reflected in the *Principles and Standards for School Mathematics* (NCTM, 2000), which recommends that instructional programs enable students to select, apply, and transfer among mathematical representations to solve problems.

Research on multiple representations indicates that multiple representations cater for a wide range of students with different learning styles and hence promote conditions for effective learning and leads students into deeper understanding of the subject as each representation emphasizes different aspects of the same concept (Berthold, Eysink & Renkl, 2009). By using different representations and transferring between them, learners are not limited by their strengths and weaknesses in understanding of one particular representation (Ainsworth 2006; Ainsworth, Bibby & Wood, 2002). Furthermore, it is expected that if learners are provided with a rich source of various representations from one domain, they build references across these representations (Ainsworth, 2006).

Research has also shown that many students are unable to transfer between representations of functions when solving problems. Learners experience difficulties particularly when relating the multiple representations to each other. Often times they only concentrate on one type of representation or fail to link different representations to each other. As a result, the expected positive effects that were intended by multiple representations do not occur (Ainsworth et al. 1998; Knuth, 2000; Moschkovich, Schoenfeld & Arcavi, 1993). Thus, while multiple representations offer unique possibilities of fostering understanding, these positive effects often do not occur. Some studies have further indicated that instructional factors could be possible sources of these difficulties. For instance, in a study aimed at helping students gain experience in creating and coordinating multiple representations of functions, Brenner et al. (1997) reported qualitative differences in the learning outcomes produced by two different instructional treatments. The study involved 128 seventh and eighth grade pre-algebra students from six classes in three junior high schools where the students were learning about linear functions over a 20-day instructional period. Three of the classes (72 students) formed the experimental group while the other three classes (56 students) made up the control group. All of the students were in their first year of pre-algebra and were using the same textbook, which emphasized learning to solve equations and using equations to solve word problems. At the time of the study, all of the students had finished an introductory chapter on algebra that covered translating expressions and sentences into variable expressions and equations, solving one-variable equations, and solving one-step word problems. The classes were taught by three teachers (two sections – one experimental and one control – for each teacher) who had between four and six years of teaching experience. The experimental group used an instructional unit that emphasized using multiple representations where the learning was anchored in a meaningful thematic context (choosing the best pizza provider for the school cafeteria) with students solving problems in cooperative groups. The control classes, on the other hand, were taught with traditional direct instruction methods from the textbook. Brenner and colleagues found that the experimental group had a better understanding of functional relationships and were better at problem representation tasks such as translating word problems into tables and graphs than the control group.

According to Ainsworth (1999), multiple representations serve three main functions in learning situations, namely (a) to complement each other, (b) to constrain possible interpretations or misinterpretations in each other, and (c) to encourage learners to construct a deeper understanding of a situation. Zazkis and Liljedahl (2004) have described two roles of representations in mathematics, namely as tools for manipulation and communication and as tools for conceptual understanding. The first role of multiple representations as described by Zazkis and Liljedahl (2004) is consonant with the first two roles of multiple representations as suggested by Ainsworth (1999), while the second role of multiple representations described by Zazkis and Liljedahl (2004) is the same as the third role suggested by Ainsworth (1999). Because of this overlap I will only discuss the roles outlined by Ainsworth (1999) in the following sections.

Complementary Roles

Different representations support different computational processes and possess different inferential power. By using different representations in complementary roles, learners are likely to communicate more information or display more processes than if they were to use only a single representation. For example, an equation of the form $y = f(x)$ will show how the output, *y* is obtained from the input, *x*, while a table will clearly show ordered pairs and empty cells (where applicable) and support accurate read-off thereby highlighting patterns and regularities across sets of values. However, these two representations do not readily reveal trends, as would be the case if an informationally-equivalent graph were used. Zazkis and Lijedahl (2004) contend that representations not only help an individual (learner) get his or her ideas across, but also facilitate communication between the individual and other individuals. They caution, however, that a representation can only come to life when the learner maps the symbols to the mathematical notion in a bi-directional fashion whereby the learner is able to communicate ideas efficiently while at the same time recognizing and interpreting what ideas are being communicated.

Constraining Roles

Ainsworth (1999) argues that learners can develop better understandings of problem situations by using one representation to constrain (or focus) their interpretation of a second representation. This, Ainsworth (1999) suggests, can be achieved by either "employing a familiar representation to support the representation of a less familiar or more abstract one, or by exploiting inherent properties of one representation to constrain interpretation of a second" (p. 139). For example, students may overgeneralize the meaning of absolute value functions and have a misconception that these functions must take only positive values and hence have misinterpretations as to the graphs of such functions. Using graphs of absolute value functions reflected over the *x*-axis or shifted downward can help to constrain the students' conceptions of the graphical representations of absolute value functions. Hence, when multiple representations are used for constraining, the purpose is not necessarily to provide new information but "to support a learner's reasoning about a less familiar one. It is the learner's familiarity with the constraining representation, or its ease of interpretation, that is essential to its function" (Ainsworth, 1999, p.139).

Borba and Confrey (1996) presented a detailed case study of a 16-year-old student, Ron, working on transformations of functions in a computer based multi-representational environment. This study was intended to investigate vertical and horizontal translations, reflections around vertical and horizontal lines, and vertical and horizontal stretches of functions. Borba and Confrey (1996) present Ron's attempts to interpret the horizontal translation of a parabola as "problematic" (p. 325) and then follow up by showing how he learned to coordinate visual actions with changes in other representations. Ron’s misinterpretation of a horizontal transformation using the icons and rescale facilities of the Function Probe™ (FP) software was constrained by his use of the algebraic techniques. Ron had rapidly used the icons and the rescale facilities of FP to find that a horizontal translation of the graph of the equation $y = x^2 + 3x + 5$ by five units to the right would imply a change in "c" from 5 to 15. Since he saw the graph move five units to the right as he moved the mouse to the right, he thought this implied that the variable terms in the equation would also increase by five, stating that the equation would become $y = (x + 5)^2 + 3(x + 5) + 5$. After checking with paper and pencil and finding that his new equation led to a “c” of 45 instead of 15, Ron used both the graphical and equation displays to reconcile the discrepancy in his result. The researchers concluded that “visual reasoning, seeing graphical transformations as movements *on or ‘of’* the plane, is a potentially powerful form of cognition, and one which requires that students be provided with adequate time, opportunities and resources to make constructions, investigations, conjectures and modifications” (p. 326). The researchers also contended that students can develop effective strategies of inquiry when presented with environments that support the use of multiple representations.

Conceptual Understanding

Multiple representations help learners develop deeper understanding of concepts by promoting abstraction, encouraging generalization, and exposing relations between the representations. Zazkis and Liljedahl (2004) concur with Ainsworth (1999) that when a representation is used as a tool for thinking and gaining insights, students’ understanding is connected to their ability to apply various representations and to choose those representations that are appropriate to particular problem situations.

Kaput (1989) suggested that the cognitive linking of representations creates a whole that is more than the sum of its parts and that it enables us to see complex ideas in a new way and apply them more effectively. Ainsworth claims that construction of deeper understanding occurs through abstraction, generalization (or extension) and relations. With regard to abstraction, exposure of multiple representations leads the learner to construct references across the representations. This knowledge is then assumed to allow the learner to find out the underlying structure of the concept under investigation. Generalization refers to a learner's extension of his/her knowledge without fundamentally changing the nature of that knowledge. For example, one may know how to interpret increasing or decreasing functions on the basis of their algebraic representations. He/she may later extend this knowledge to the interpretations of such representations as the increasing (or decreasing) graphs or tables of values. Finally, construction of deeper understanding can also occur through teaching the relations among different representations. The pedagogical concern here is not so much with teaching each representation but rather with teaching to translate between two or more representations which are introduced simultaneously. Mathematics teachers should therefore strive to guide their students to communicate better using different representations, resolve differences across representations, and see connections between various representations. This is not always the case though, as several studies have indicated.

Cunningham (2005) surveyed algebra teachers regarding their use of problems requiring transfer between algebraic, numeric and graphic representations. Participants were 28 algebra teachers who, at the time of the study, taught grades eight through ten and had teaching experiences ranging from three to 41 years. Cunningham (2005) reported that the teachers used problems requiring transfer between graphic and numeric representations the least while using those requiring transfer from algebraic to graphic the most – almost by a two to one ratio. He reported further that problems requiring transfer from algebraic to numeric were the second least utilized by his respondents. Cunningham stated that the teachers devoted a smaller number of class periods to problems requiring transfer to numeric representations and used them less frequently on assessments.

The NCTM (2000) suggests that digital technologies provide visual models or representations that many students are unable to generate through their independent efforts. Ruthven, Deaney and Hennessy (2009) and Zbiek, Heid, Blume and Dick (2007) suggested that technology can potentially underline the important qualities of individual representations, making it easier for students to interconnect them and hence achieve a robust understanding. Teachers are perhaps one of the most important factors that make a difference in successful use of multiple representations in technology rich-environments. Ruthven et al. (2009) observed two teachers as part of a larger study aimed at investigating how mathematics teachers in British secondary schools conceived the incorporation of computer-based tools and resources into their classroom lessons. The researchers made records of each lesson, incorporating a transcript of the main episodes, integrated with further observational material including copies of other resources used and records of the graphs displayed. They also conducted post-lesson interviews with the teachers after each observed session – they observed two lessons for each teacher – asking teachers about their thoughts, first while preparing the lesson (what they wanted pupils to learn; how they expected use of the technology to help pupil learning), then looking back on the lesson (how well pupils learned; how well the technology helped pupil learning; important things that they were giving attention to and doing). The researchers reported that the teachers noted the theme of focusing on overarching issues and accentuating important features, adding that the teachers talked of how use of graphing software helped students to "get to grips with, get an idea of or see straight away the effect of altering a coefficient in the equation on the properties of its graph" (p. 290). They further reported that the teachers highlighted particular software devices which facilitated apprehension of equation/graph matching, comparison of gradients and examination of limiting trends.

The NCTM (2000) states that "effective use of technology in the mathematics classroom depends on the teacher . . . The teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways" (p. 25–26). Hence teachers' knowledge about the representations, how they use them for teaching, and how they make use of technology in addressing the multiple representations are all important issues to be considered while teaching with multiple representations through technology.

A number of studies suggest that teachers' beliefs and views influence their practices in the classroom (Ball, Lubienski & Mewborn, 2001; NCTM, 1991; Stipek, Givvin, Salmon & MacGyvers, 2001; Thompson, 1984, 1992). Furthermore, teachers typically control whether or not technologies – including graphing calculators – are among the instructional materials used to enhance student mathematical understanding.

Teachers' Attitudes Towards and Beliefs about Use of Graphing Calculators

In this section, I present an analysis of studies that investigated teachers' attitudes and beliefs with regard to use of graphing calculators in mathematics classrooms. The methods used in these studies range from survey questionnaires to case studies, interviews, and classroom observations. Findings from the studies have not all been uniformly positive. Even though there are substantial enablers to the use of graphing calculators in mathematics classrooms, there are some barriers as well.

Many mathematics educators and organizations believe that mathematics curricula should shift their emphasis from computation to problem solving and conceptual understanding (Simonsen & Dick, 1997). Research indicates that calculator use does not undermine computational ability (Ellington, 2003, 2006; Hollar & Norwood, 1999), while it improves problem solving and conceptual understanding (Dunham & Dick, 1994; Ellington, 2003, 2006; Hennessy, Fung & Scanlon, 2001). Studies have also shown that students instructed with graphing calculators demonstrate improved understanding of functions and graphing (Hollar & Norwood, 1999), greater ability to connect multiple representations of algebraic concepts (Graham & Thomas, 2000), and increased understanding of a dual approach to problem-solving, using both symbolic and graphical solution methods (Harskamp, Suhre & Van Streun, 2000). Additionally, students instructed with technology demonstrate less compartmentalization of mathematical concepts and techniques compared to those receiving more traditional instruction. Jones (2000) argued that when students work with graphing calculators, they have the potential to form an intelligent partnership, as the graphing calculator can undertake significant cognitive processing on behalf of the user. Graphing calculators can therefore be useful tools for both concept development and problem solving.

In a study that investigated the perceptions of three pre-service teachers regarding the use of graphing calculators as instructional tools, Fine and Fleener (1994) reported that the pre-service teachers' view of mathematics as a body of rules to memorize and skills to perfect seemed to prevent them from perceiving calculators as anything other than computational, time saving tools. The participants in this study were volunteers from a clinical testing class at a regional state university who were completing their coursework within a few weeks and would then proceed to a student teaching experience. Data consisted of open-ended individual interviews conducted both before and after the onset of the student teaching experience. The researchers attributed this rather negative attitude to the fact that the participants had few opportunities in pre-college mathematics classes, teacher training courses, or student teaching experiences to learn how calculators may be used in instruction. These results may reflect the time of the study early 1990s and it is possible that the situation is different today.

In another study, Fleener (1995a) surveyed 94 middle and secondary mathematics teachers' attitudes towards and use of graphing calculators after the completion of four two-hour workshops on graphing calculator instruction. Participants were volunteers attending calculator workshop sessions at an annual meeting of the state affiliate of the National Council of Teachers of Mathematics. Findings indicated that the teachers who participated in this study had shared beliefs about the motivational effects of graphing calculators but their beliefs were divergent with regard to cognitive benefits of calculator use. Fleener (1995b) reported that there was consensus among the teachers (i.e., 70% or more) that all students should learn to use calculators, that using calculators makes students better problem solvers, and that calculators are motivational. She further identified two factors as important for deciding issues related to calculator use in the mathematics classroom, namely (a) experience with calculators for instructional purposes, and (b) beliefs about whether students should master the concept or the procedures before they use calculators.

Teachers who felt that students must master concepts and procedures prior to calculator use generally agreed that calculator use would cause a decline in basic arithmetic facts and consequently cause students to lose basic computational skills. In contrast, teachers who felt that students do not have to master procedures before they can use calculators viewed calculators as an alternative to paper and pencil computation. In another study, Fleener (1995a) extended the use of her survey instrument by deleting 12 items from experiential and affective categories and adding 15 new items which focused on beliefs about how calculators can be used, and the consequences of calculator use. She surveyed 233 classroom teachers (elementary (11%), intermediate (49%), and high school (40%)) and 78 pre-service teachers. In order to gain an understanding of the relationship between pre-service and practicing teachers' responses to the survey items and mastery levels, pre-service/classroom teacher groups were separated by responses to the question on mastery. Results suggested that philosophical orientation pertaining to calculator use is a function of both experience and attitudes related to the conceptual mastery issue discussed earlier. Fleener (1995a) concluded that change efforts must address both experience with and mastery orientation towards calculator use in the classroom.

In an attempt to conceptualize the belief structures of pre-service mathematics teachers with respect to use of technology, Turner and Chauvot (1995) completed a longitudinal study of two teachers. The researchers' concerns included what beliefs were held, how those beliefs were held, and to what extent those beliefs influenced the teachers' use of technology. Participants were followed through four quarters of a secondary mathematics education sequence consisting of two courses in mathematics education, student teaching, and a post-student teaching seminar. Data consisted of an initial survey that involved mathematical tasks and questions about the teaching and learning of mathematics, three interviews during the first quarter, two interviews during the second quarter, one formal observation and interview during student teaching, and three interviews during the post-student teaching seminar. Other data sources included a weekly journal in which the participants were asked to respond to questions related to course activities, and observation of their work on campus as well as their field experiences.

Turner and Chauvot (1995) noted that through experiences in the mathematics education courses the participants were exposed to situations where graphing calculators and computers were used regularly as investigative tools integrated in the teaching and learning of mathematics. Yet analyses of beliefs indicated that one major belief held by both participants was that "their success with technology resulted from the fact that they already knew the mathematics involved in the activities. Thus it was their mathematical knowledge that helped them understand the use of technology; the technology was simply 'an icing on the cake'" (p. 5). This result shows that the pre-service teachers believed that success in the use of technology resulted from a prerequisite knowledge of mathematics; this supports findings reported by Fleener (1995a, 1995b).

In a survey study that sought to investigate teachers' attitudes toward and use of the graphing calculator in the teaching of algebra, Milou (1999) found results similar to those found in studies discussed above. Participants were high school and middle/junior high school teachers from five counties in a northeastern state. Among the major findings, Milou (1999) reported that algebra teachers were still unsure of how to use the graphing calculator in instruction. He noted that teachers were confused whether concepts and/or procedures still needed to be mastered first. Milou (1999) also reported that the cognitive benefits of graphing calculator use were still questioned by many algebra teachers.

More recently, Pierce and Ball (2009) surveyed 92 secondary mathematics teachers in Australia on their attitudes and perceptions regarding use of technology. The researchers found that if teachers did not feel that the school leadership (mathematics coordinator or principal) expected them to use technology, then they were less likely to believe that technology use would motivate students. Pierce and Ball further reported that teachers responded positively to the perception that use of technology can allow students to engage in more real world problems, make mathematics more enjoyable and make students more motivated. Additionally, the researchers noted that there was evidence that those teachers who perceived that students must learn mathematics by hand (pen and paper) first were likely to see teaching students to use technology as an extra, time-consuming task and may see using technology as an addition to the previous curriculum that has already occupied all of the allotted class time. These results suggest that even though the use of graphing calculators and other handheld technologies has become more widespread classrooms across the world, teachers attitudes towards them are in some cases still unchanged.

Teachers' Knowledge about Use of Graphing Calculators

Just as students need to learn how to use graphing calculators to improve their learning ability, teachers need to learn how to use graphing calculators to improve the quality of teaching. In this section, I present research findings on teachers' knowledge, including mathematical knowledge, pedagogical knowledge, and pedagogical content knowledge, about the use of graphing calculators in mathematics classes. Studies in this area have been conducted through experiments, classroom observations, interviews, and case studies. These studies have either examined the effects of different teaching styles on graphing calculator use or the effectiveness of graphing calculators when they are used in conjunction with various teaching approaches. Findings suggest that graphing calculators can significantly change classroom dynamics.

In a survey study that explored calculus instructors' perceived impact of using computers and calculators on specific topics of calculus, student motivation, student learning, and the role of the teacher, Rochowicz (1996) reported that many teachers felt that technology does not replace the teacher but it requires more time and more meaningful and creative teaching from the teacher. Rochowicz (1996) added that most of his respondents felt that the teacher takes on the role of facilitator of learning thereby shifting more responsibility for learning to the students. A major weakness with this kind of study is that what respondents report may not necessarily reflect what they do in classrooms. Perhaps following up the survey with some classroom observations and/or in-depth interviewing could make the results much stronger. Nevertheless, Slavit (1996) and Farrell (1996) seemed to echo Rochowicz's (1996) results. Slavit (1996) conducted a year-long study focusing on the instructional practices of an experienced teacher in one section of an algebra II class and found that when using graphing calculators the teacher allowed students to initiate discussions while he (the teacher) used students' comments to elaborate concepts. Slavit (1996) reported that the teacher in his study "followed the traditional sequencing of topics in most Algebra II courses: a progression from linear, quadratic, polynomial, exponential, to trigonometric functions . . ." (p. 9), but modified "his teaching strategies and the curriculum to accommodate the strategies and topics that arose from the use of graphing calculator" (p. 13). This shows that this teacher was prepared for reactive teaching thereby responding to new types of initiative and opportunities made possible by the graphing calculator.

The data for this study were collected via periodic classroom observations conducted throughout the school year. The study focused on classroom discourse, uses of the graphing calculator, the nature of instructional tasks, and the teacher's questioning patterns. Three units involving linear functions, polynomial functions, and exponential functions were analyzed. Slavit (1996) reported that the teacher used the ZOOM and TRACE features of the graphing calculator extensively both when investigating local aspects of the graph, such as finding zeros and solving for extrema, and when investigating global aspects such as trends. The researcher detailed how the teacher changed the frequencies with which he used the graphing calculator and the kind of tasks in which he encouraged this use. The researcher noted that calculator use increased from about 29% in the linear unit to about 57% in the polynomial unit before dropping to about

27% in the exponential unit. Slavit (1996) explained that by the time of the exponential unit the teacher had begun using the calculator to introduce functional properties graphically and he used it sparingly because he did not want students to rely on it totally. Slavit (1996) further reported that the levels of discourse witnessed in the classroom when the graphing calculator was in use were higher: the teacher posed higher-level questions and the students displayed more active learning behaviors. Slavit (1996) observed that the students controlled the direction of discourse by initiating discussions through “requests for clarification, questions about a problem solving process, factual questions, and conjectures” (p. 10), while the teacher encouraged this by “allowing other students to offer additional comments or corrections” (p. 12) before recapping it himself by restating or extending the students’ comments and posing more questions.

Farrell’s (1996) study focused on six teachers involved in a curriculum development project designed to implement calculators and computers in high school mathematics. Like Slavit (1996), Farrell (1996) reported that there were changes in teacher roles and behaviors when graphing technologies were used in the classroom. The teachers in this study were nearing the end of their first year of teaching pre-calculus using specially designed materials that incorporated graphing technologies, *Calculus and Computers in Pre-calculus (C^2PC)* (Demana & Waits, 1989). Each of the teachers was videotaped for 10 consecutive non-testing sessions, the first six of which were then coded at five-minute segments in terms of teaching activities, student and teacher roles, student learning behaviors, and what type of technology was in use at that time.

Of the total number of five-minute segments coded in Farrell’s (1996) study, it was found that calculators or computers were used 56% of the time. For the time when technology was in use, graphing calculators were used alone 43% of the time, computers were used alone 27% of the time, and both calculators and computers were used simultaneously for 30% of the time. Graphing calculators were therefore the most used technology in these classes. Farrell (1996) found that other than the managerial role, “the roles that teachers exhibited when technology was in use differed from the roles they exhibited when technology was not in use” (p. 42). She argued that while this may sound obvious since we know that new tools are likely to evoke new behaviors, it is worth noting that the behaviors evoked by the graphing technology were not the same for all the teachers on all the days. She observed that while “each teacher maintained an individual style . . . the technology did provide a vehicle for incorporating some new behaviors and roles” (p. 45). She reported similar findings to those of Slavit (1996), stating that “exposition became less prevalent and investigation and group work became more prevalent when technology was in use” (p. 43). However, since her study did not describe the kinds of activities the teachers used in the investigations and group work we cannot tell how rich these discussions were.

Farrell (1996) further reported that all the teachers in the study exhibited the role of manager almost as often when technology was in use (in 96% of the 5-minute segments observed) as when technology was not in use (in 100% of the segments). However, she reported shifts in other roles assumed by the teachers with and without technology: when technology was in use, most of the teachers increased the frequency with which they assumed the roles of consultant and fellow investigator and assumed the roles of explainer and task setter less frequently. Nevertheless, she reported that one teacher was consistently high in displaying the role of explainer whether technology was in use or not, and that two of the teachers assumed the role of task setter more often with technology than without technology.

A study by Simmt (1997) seemed to contradict the findings that graphing calculators can influence (and change) teaching styles and teacher roles. Simmt (1997) interviewed teachers regarding calculator use and noted that teachers generally viewed mathematics as either a collection of skills or as a process of discovery. This study examined how six teachers used graphing calculators in their instruction and how their views of mathematics were manifested in the ways they chose to use the calculators. All the teachers had equal access to the graphing calculators and had the same curriculum requirements. Data collected included classroom observations, interviews, and lesson artifacts including handouts and worksheets.

Simmt (1997) reported that the way the calculators were used varied considerably among the teachers. On transformation of the parabola from the equation $y = a(x - p)^2 + q$, Simmt (1997) reported that four of the six teachers used the graphing calculator as a tool that could enable the students to draw many accurate graphs of quadratic functions and then use them without the teachers' help to discover the roles of the various parameters on transformations. The teachers each designed guided discovery worksheets. These teachers gave their students enough time to work through the activities, a task that involved the students in recording their observations during investigations then making generalizations at the end of the activities. The teachers then summarized or confirmed students' findings. One of the other two teachers, Simmt (1997) noted, used "highly structured and carefully monitored" (p. 276) activities that allowed students only a few minutes to plot a few graphs on their calculators before the teacher called on individual students to suggest the role of each parameter. The teacher then wrote generalizations on the blackboard for the class to take note of. Simmt (1997) reported that the sixth teacher never had students use the graphing calculator at any point in this lesson. He instead demonstrated the roles of the parameters using an overhead model of the graphing calculator.

Simmt (1997) found that both views – that mathematics is either a collection of skills or is a process of discovery – were evident in teachers' "choices of activities for use with the graphing calculator, and the kinds of questions they asked students, and in other interactions with their students" (p. 283). She concluded that the teachers' views about mathematics were not changed, but rather strengthened by using graphing calculators and that the "availability of the graphing calculator simply provided … teachers with an opportunity to further live their philosophies of mathematics education" (p. 286). The last two of the teachers in this study appeared to be explainers. The researcher should have perhaps mentioned what the first four teachers were doing during the time the students were conducting the investigations. This is important for the reader to know given that she mentioned that the teachers would give the students up to two class periods to complete the investigations.

Teachers who are not used to dealing with unfamiliar situations in their classrooms tend to use various ways to avoid such scenarios. When one teacher in Simmt's (1997) study realized that students were having problems fitting their graphs on the calculator screen, he was reported to have "decided to 'fix' his examples so they all fit in the standard viewing window" (p. 279). When this teacher noticed that his students were having problems differentiating a circle from an ellipse – an opportunity he could have used to inspire a discussion that would get the students to discover the differences in the equations, he chose instead to point out the differences himself. Another teacher was reported to have convinced students that algebraic solutions were superior to graphical solutions basing on the fact that the TRACE feature could not yield exact values. Faced with a similar scenario, the teacher in Slavit's (1997) study encouraged his students to use the ZOOM feature in order to get as close as possible to the exact value. These examples illustrate that in Simmt's (1997) study, the teachers' roles as explainers seriously constrained the ways in which they allowed their students to use graphing calculators.

All the teachers in Simmt's (1997) study gave saving time as a reason for choosing to use graphing calculators. Some suggested that the time saved could be used in other areas in the curriculum while others said that they could provide many more examples when using graphing calculators. It is interesting that none of the teachers thought about the type and level of problems that can be investigated by graphing calculators. These teachers also cited student motivation, confidence building, and variation in teaching strategies as other reasons for using graphing calculators. A possible reason for the difficulties faced by the teachers in Simmt's (1997) class in using graphing calculators may be that the teachers lacked deep knowledge about how to use the calculator as an instructional tool.

In a study that could help resolve the contradiction between the findings of Rochowicz (1996), Farrell (1996), and Slavit (1996), on one hand, and those of Simmt (1997) on the other, Tharp, Fitzsimmons, and Ayers, (1997) used both survey data and qualitative data to examine the perceptions of teachers as the teachers engaged in initial instruction using graphing calculators. Participants in this study were also taking part in a four-month technology outreach interactive television course (telecourse) so the survey was administered before and after the telecourse. Making sense of the qualitative data involved analyzing participants' journals in which the participants kept records of among other things, instructional activities they used, students' reactions to these activities, and reflections.

The pre- and post- telecourse responses to the survey revealed that after completion of the program more teachers felt that graphing calculators could help them solve problems they could not otherwise solve and that using graphing calculators in mathematics classes could help emphasize the experimental nature of the subject. On the other hand, the researchers reported that fewer teachers felt that they lacked confidence and skill with the graphing calculators or that their students lacked the ability to work with calculators as complex as graphing calculators. There was, however, a significant positive correlation between the teachers' views of mathematics and the teachers' views on the use of graphing calculators in classrooms.

Tharp and colleagues observed that "teachers who hold a more rule-based view of mathematics are more likely also to hold the view that calculators do not enhance instruction and may even hinder it, while those with a less rule-based view of mathematics are more likely to view calculators as an integral part of mathematics instruction" (p. 558). The qualitative data were coded in relation to the dimensions of conceptual versus procedural instruction, lecturer versus facilitator, observation of affective student reactions versus conceptual student understanding or thinking, and reaction to the use of journal writing as a tool for reflecting on their teaching. The journals were then matched with the first questionnaire which was related to teacher beliefs about mathematics learning and their attitudes and perceptions about calculator use. The researchers reported that there were significant differences between rule-based teachers and non-rule-based teachers in their perceptions of students' reaction to graphing calculator use. Rule-based teachers tended to write in their journals comments that only related to emotional state of their students rather than conceptual understanding.

The researchers also reported slight but not significant differences in teachers' role taking: Rule-based teachers were more likely to assume the role of lecturer while non-rule-based teachers were more likely to assume that of facilitator. These findings may suggest that most of the teachers in Farrell's (1996) study and the one in Slavit's (1996) study were non-rule-based hence their flexibility and adaptability when using graphing calculators. On the other hand, the majority of the teachers in Simmt's (1997) study may have been rule-based, hence structuring their teaching activities more or less like lectures.

In a study that investigated whether respondents had changed in their ways of teaching as a result of using graphing calculators, Simonsen and Dick (1997) found most of the teachers reported that their teaching had become less teacher centered. A majority of these teachers also reported that they were using more open-ended questions, that calculators had fostered discovery approach in their classes, and that they were employing more cooperative learning. However, like any other survey and clinical interview type of study it is difficult to corroborate such self-reported claims unless one goes to the classrooms to observe the teaching and learning activities taking place there.

Later studies showed mixed findings about the roles of teachers in technology-enriched classrooms. Fernandez (2000), and Doerr and Zangor (2000) confirmed Farrell's (1996) and Slavit's (1996) findings that teachers assume the roles of fellow investigator, facilitator, and consultant when they teach with graphing calculators while Goos, Galbraith, Renshaw, and Geiger (2000) reported mixed results, as did Tharp et al. (1997).

In a study focusing on one teacher teaching a week–long unit in five high school mathematics classes (two geometry and three algebra II) towards the end of the academic school year, Fernandez (2000) reported that the teacher displayed the roles of "resource person/ facilitator, fellow investigator, consultant, and technology assistant" (p. 799). Fernandez observed that the teacher facilitated both small group and whole class discussions around students' findings and explorations as well as providing more scenarios for the students to explore. She further stated that the teacher encouraged students to pose their own problems and explore them, while he was ready to answer their questions relating to the situations they were exploring and the graphs they were generating.

A major weakness in Fernandez's study appears to be in the methodology employed. The study was conducted over only one week during which time a unit designed jointly by the researcher and the teacher was used in the classes. The researcher did not indicate whether she collected any data outside that week (i.e., when regular units were taught and without the graphing tools used with the investigation reported in the study). Yet she reported that classroom interactions between the students and teacher during the investigation were different from the typical traditional teacher-directed interactions that characterize these classes – a claim that could be interpreted to mean her study was a comparative one.

Through a case study involving one teacher teaching two pre-calculus classes using a curriculum based on modeling problems in a technology rich environment, Doerr and Zangor (2000) reported that the teacher used the graphing calculator in a flexible way as a result of her confidence in her knowledge of the calculator’s capabilities. The researchers did not state explicitly what the teacher did during group discussions but they did state that “she actively encouraged the students to take over the use of the overhead projection unit during (whole) class discussions” (p. 149). Contrary to what Simmt (1996) reported about some of the teachers in her study, Doerr and Zangor (2000) observed that the teacher in their study was confident with her knowledge about the calculator’s capabilities and its potential uses for student learning to an extent that she gave the students freedom to use the tool as they wished. They further noted that the teacher was willing to take students’ suggestions and examples and explore them with the overhead projection unit.

Doerr and Zangor (2000) also reported that the teacher in their study did not discourage any students from using the calculator even in ways that were seen by other students as not being very useful. Instead, the researchers noted that the teacher required the students to interpret the results they obtained with their calculators in relation to the problem situation. This way the students could then judge for themselves when their approach was not giving them meaningful results and hence abandon that approach. Such a technique by the teacher shows that she did not want to explain everything to her students, therefore displaying facilitative and fellow investigator behaviors consistent with the teachers in the studies by Farrell (1996), Slavit (1996) and others mentioned earlier. Cavanagh and Mitchelmore (2003) reported similar findings as Doerr and Zangor (2000), noting that the "teachers' confidence in their own understanding of the calculator's operation is crucial in determining how effectively they will use the technology in the classroom" (p. 16). Cavanagh and Mitchelmore (2003) observed that teachers who felt uncomfortable using the graphing calculator designed highly structured teacher-centered lessons.

In a three-year, longitudinal study that examined teacher-student, student-student, and technology-humans interactions Goos et al. (2000) found that teachers' roles when technology was in use differed according to the teachers' level of expertise with the technology. This study involved four teachers of which Goos et al. (2000) reported that one admitted to having very little expertise with graphing calculators but made up for this by inviting a student "expert" to take over demonstrations involving the calculator. This teacher, the researchers observed, maintained control and made sure that his agenda was achieved. He did not allow students to use the calculators to explore mathematical ideas outside the current topic. Although Goos et al. (2000) credited this teacher for being on a path "towards greater student participation" (p. 308), I find his style to be consistent with the teacher in Simmt's (1996) study who gave students a few examples to plot on the calculators, gave them a few minutes to explore effects of parameters on transformations and wrapped up by asking students to suggest generalizations. Another teacher who had limited "but growing competence with calculators" (p. 10) allowed students to explore and conjecture in small groups and then share their results with the rest of the class on the overhead unit. The other two teachers in Goos et al.'s (2000) study had high levels of expertise with graphing calculators and provided challenging tasks for their students to explore.

Studies by Harskamp, Suhre, and Van-Streun (1998, 2000) reported findings about a shift in the strategies of the teaching of functions when a graphing calculator was used. Harskamp et al.'s (2000) study was quasi-experimental and comprised of three categories of grade 10 classes in a functions and calculus course. Three of these classes (experimental group 1) used graphing calculators throughout the year, five classes (experimental group 2) used the calculators to cover only one topic for about two months, and four classes (control group) never used graphing calculators. Although the same textbook was used in all the classes, it was supplemented to integrate graphing calculators in the subject matter for the experimental groups. Harskamp et al. (2000) reported that the teachers in both of the experimental groups explained problems using tables and graphs equally often and more often than their counterparts in the control group, adding that the teachers in the control group rarely illustrated what the graph of a function looked like; hence their students relied on the examples in the textbook. The researchers also reported that the majority of the teachers in the experimental groups self-reported to have changed in their styles of teaching: The teachers claimed to have embraced the use of more graphs than before the introduction of graphing calculators.

While the findings by Harskamp et al. (2000) suggest that introducing graphing calculators in the experimental groups may have inspired the teachers in these classes to become more tabular and graphical oriented, one can argue, and genuinely so, that this apparent change was in fact a consequence of the changes in the textbook. For one, the researchers did not follow up on experimental group two after the classes in this group completed the first topic on which they were using graphing calculators. Thus, as pointed out by the researchers themselves, there were no data to determine whether the teachers in this group continued to use tables and graphs after the first topic as often as they did when they taught with graphing calculators. The researchers also noted that the teachers in the experimental groups strictly followed the textbook and never used the calculators on exercises other than those marked in the book as suitable for calculator use. They reported that "about thirty to forty percent of the exercises" in the textbook for these classes were "to be solved with the graphics calculator" (Harskamp et al., 1998, p. 24) and five out of the eight teachers in these groups spent most of the time on instruction with the calculator. The textbook may have driven these teachers to use the calculator, and hence the tables and graphs.

In a recent study, Lee and McDougall (2010) found that factors such as teachers' personal experiences and teaching practices, together with the level of proficiency of the students with the technology, influence how the graphing calculators are used in the mathematics classroom. The study involved three teachers (Victoria, Dawn and Clare) who each viewed mathematics as a dynamic field, emphasized understanding concepts as opposed to mechanical procedures, and preferred the construction and understanding of the concept over the memorization of procedures. All three teachers were willing to use graphing calculators in the mathematics classroom, and all three had a similar goal of attempting to use the graphing calculators to eliminate mechanical processing time and enhance their students' ability to construct their own learning. Data included a self-assessment on beliefs about mathematics, classroom observations, a survey on technology use and interviews. Each teacher was observed at least four times but no more than 13 times and each observation lasted between 45 and 75 minutes depending on the length of the period being observed. The self-assessment comprised of 24 questions divided into three groups that looked at three different conceptions of mathematical knowledge, namely, the status of mathematical knowledge, doing mathematics and learning mathematics. The responses of the three teacher participants were categorized and a score was given to each of the three teacher participants for each of the three conceptions.

Questions on *The Status of Mathematical Knowledge* were meant to determine whether the teacher conceived of the state or nature of mathematics to be dynamic or static. The higher a teacher scored, the more dynamic that teacher believed mathematics to be. The questions categorized as *Doing Mathematics* were meant to determine whether the teacher believed mathematics is about understanding and making sense of the concepts as opposed to being about knowing the correct procedure to use to reach the desired results. The higher the score calculated for each teacher, the more emphasis she believed should be placed on understanding the concept underlying the question, rather than on the mechanics of the question. Finally, questions in the *Learning Mathematics* group were meant to determine whether the teacher believed mathematics is learned through construction and understanding or through a process of memorization. The higher the score reached by a teacher, the more emphasis she believed should be placed on constructing learning rather than on memorization. Victoria scored the highest on all the three categories – 5, 4.88 and 5.38, respectively out of a maximum of 6 in each category. Dawn scored the second highest in two of the categories – status of mathematical knowledge (4.38) and doing mathematics (4.50), while Clare second highest in the category of learning mathematics (4.88) and had the lowest scores on the other two categories.

The researchers noted that from the scores, Victoria reported the strongest belief in each of the three conceptions, Dawn reported fairly strong views on the three conceptions, and Clare had less strong views on the first two conceptions than the other two teachers although she reported fairly strong views on the third conception. Comparing these findings with the observations of the teachers when graphing calculators were present in their classrooms, the researchers reported that Victoria's students were proficient with the graphing calculators and so she never had to give them explicit button pressing instructions on how to perform an operation using their graphing calculators. Instead, Victoria was able to use the graphing calculators to focus her students' attention on the mathematical concepts and assist them in constructing their own understanding of mathematics. Dawn's students used the graphing calculators much less frequently and were much less proficient in their use. Lee and McDougall (2010) pointed out that although Dawn had to spend a significant amount of time guiding her students to use more appropriate domain and range values, like Victoria, she used the graphing calculators to encourage her students to construct their own knowledge and understanding of mathematics. The researchers further reported that Clare used the graphing calculators in her classroom much more frequently than Dawn, although still much less often than Victoria and she spent a lot of time reminding her students of the key strokes required to perform specific operations on their graphing calculators. Only after she had taught and reviewed the keystrokes could she proceed with exploring the mathematical concepts with her students. These findings by Lee and McDougall (2010) suggest that teachers who are more professionally and personally experienced with the use of technology are in a better position to welcome the use of technology in their classroom. Teachers who are proficient in using the graphing calculators can in turn teach their students to effectively and efficiently use their graphing calculators.

Even though many studies have been done regarding graphing calculators, there is still need for more studies to be done in this area. For example, we need to know teachers' views regarding the effects of graphing calculators in exploring various representations of functions. We also need to know the views of teachers regarding the extent to which they would let their students explore with the graphing calculators. Since it is true that in general what one reports about himself or herself may not always be consistent with his or her practice, a good study on these issues is one that is designed to investigate the consistency or variance between what teachers report and what they do in their classrooms. My study is designed to address this. As mentioned earlier, the purpose of this study is to investigate (a) secondary mathematics teachers' professed beliefs about graphing calculators, (b) how these teachers use graphing calculators when teaching linear and quadratic functions, and (c) the interaction between the teachers' professed beliefs about graphing calculators and their use of the calculators.

Summary

Studies summarized here revealed that there are disagreements in terms of teachers' attitudes and beliefs towards graphing calculators. Some studies pointed out that there is a link between teachers' philosophical orientation and attitudes and beliefs about graphing calculator use and called for continued investigation of this issue. It should be noted that none of the studies investigated circumstances under which teachers' beliefs and attitudes towards graphing calculator shift either towards acceptance or towards rejection. Since most of the studies addressing the issue of attitudes and beliefs were conducted via surveys it will be helpful for further studies in this area to combine this method with qualitative methods such as interviewing and classroom observations. My study was designed to incorporate these methods of investigation.

Most of the studies reviewed suggest that in the presence of graphing calculators, teacher roles tend, in general, to shift to fellow investigators, facilitator, or consultant, while teaching strategies tend to involve higher level questioning, more in-depth problem solving, and the classroom discourse grows richer. It cannot be overemphasized that some teaching styles are more compatible with the use of graphing calculators than others. Those teaching styles that use more open-ended questioning and involve engaging students in discovery activities seem to be more compatible with graphing calculator use than those styles that are teacher centered. The studies also reveal that with the presence of graphing calculators there is an increase in cooperative learning where students not only take more responsibility for their own learning but also work together with their peers and learn from each other as well.

Some of the studies have shown that teachers who have always taught in teacher-centered classrooms are sometimes uncomfortable with the unpredictability that may arise as a result of calculator use, while other teachers are often reluctant to use graphing calculators in creative ways because of their beliefs about what mathematics is and what their role as teachers is. Such teachers tend to confine the graphing calculator to performing computational roles and hence deny their students opportunities to exploit the powerful capabilities of graphing calculators as both instructional and learning tools.

As suggested by Penglase and Arnold (1996), there is still need for further research on the effect of different teaching approaches which incorporate the use of graphing calculators in order to determine pedagogical factors associated with improved understanding of mathematical concepts. Other studies have also suggested the ease with which it is possible to generate and manipulate graphs using graphing calculators allows teachers and students to focus on the interpretative aspects of graphing which cause difficulties without being distracted by the problems of generating the graphs (Wright, 2005). This in turn affords students more opportunities to explore global features of functions. There is therefore need for further research involving the use of graphing calculators to teach the concept of function.

A transition from a traditional mathematics classroom to one where technology is used as an integral part of teaching requires teachers to be prepared to change and to make a commitment to learning to use the technology in an effective manner. Additionally, teachers need to be prepared to face the complexities and challenges of students learning about multiple representations of functions. In this study, I seek to investigate secondary mathematics teachers' professed beliefs about graphing calculators, how the teachers use graphing calculators to teach the concept of function, and the extent to which the professed beliefs explain the teachers' use of graphing calculators.

CHAPTER 3: METHODOLOGY

In this chapter, I discuss the research design, the participants, and the collection and analysis of the data.

Research Design

I used both qualitative and quantitative techniques to address my research questions. The quantitative data came from a survey, while the qualitative data sources consisted of task-based interviews and classroom observations. Research question #1 - What are secondary mathematics teachers' professed beliefs about using graphing calculators in the teaching and learning of linear and quadratic functions? - was addressed by the survey data. Research question #2 - How do secondary school mathematics teachers use graphing calculators when teaching linear and quadratic functions? - was addressed by data from the task-based interviews and classroom observations. Research question #3 - What is the relationship between the teachers' professed beliefs about graphing calculators and observed practice? - was addressed by comparing the findings of the first two questions.

Participants and Data Sources

I conducted the study using a two-phase design. In the first phase, I distributed 110 surveys to high school teachers from New York State; 81 surveys were returned – an approximately 74% return rate. I recruited the participants through various methods; I met some of the teachers during professional development meetings in local schools. I also enlisted the help of heads of the mathematics departments in the schools within the local school district and recruited other teachers through personal contacts with the help of my professors and my former colleagues who were at the time teaching in various high schools.

Of the 81 respondents, 48 (59.3%) were female while 33 (40.7%) were male with teaching experiences ranged from five years to more the thirty years. Fifty-one of the teachers (63%) taught in urban schools while 30 (37%) taught in suburban schools. All of the teachers indicated that they had attended some in-service training workshops on graphing calculators, with 67 (82.7%) of them stating that they had attended such workshops at least three times. Sixty (74.1%) of the teachers stated that they had attended the workshops within the two years prior to the study. All the teachers indicated that calculators in their schools were provided by the schools. Table 1 shows the distribution of respondents by school type, years of teaching experience and the number of workshops on graphing calculator attended.

Table 1

Distribution of Teachers by School Type, Years of Teachers Experience and Number of Workshops on Graphing Calculator Attended

		No. of teachers from urban schools	No. of teachers from suburban schools	Total
Teaching experience	Less than 10 years	10	6	16
	Between 10 and 20 years	33	15	48
	More than 20 years	8	9	17
Graphing calculator workshops attended	One or two	11	3	14
	Between 3 and 6	36	20	56
	More than 6	4	7	11
	Within last 2 years	43	17	60

In New York State, Mathematics I (Math 1) is the typical 9th grade integrated mathematics course. The fundamental purpose of Math 1 is to deepen and extend the understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics II (Math 2) is the typical 10^{th} grade integrated mathematics course. Math 2 is focused on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships.

I used question 10 (If you teach Math 1, how often do you use graphing calculators in your classroom?) from Part I of the survey to place the teachers into three groups: High Frequency Users (nearly every lesson), Moderate Frequency Users (once every two or three lessons), and Low Frequency Users (once every four or five lessons). Thirty-six of the teachers (44.4%) identified themselves as high frequency users, 25 (30.9%) as moderate frequency users and 20 (24.7%) as low frequency users. Table 2 summarizes the distribution of respondents by type of school in each frequency of graphing calculator use group.

Table 2

Summary of Respondents' Frequency of Calculator Use by School Type

Frequency of graphing calculator use	No. of teachers from urban schools	No. of teachers from suburban schools	Total
High	24	12	36
Moderate	15	10	25
Low	12	8	20
Total	51	30	81

I then invited five teachers each from the high frequency and the moderate frequency groups from schools that were within approximately 30 miles from my university to participate in the second phase of the study. I did not include teachers from the low frequency group for this part of the study. I made the decision to not include the low frequency users in the second phase based on the fact that observing the teachers use graphing calculators in their classrooms was a major part of my study yet there was no guarantee that I would have this requirement fulfilled by this group of teachers during the time of the study.

Seven of the invited teachers – three from the high frequency and four from the moderate frequency group – confirmed that they were willing to participate. The schedule for one of the teachers in the moderate frequency group could not fit with mine and since I also intended to have an equal number of participants from each group I chose to proceed with three teachers from each group. I used six letters of the alphabet to represent the names of these teachers as shown in Table 3. The years of teaching experience for these teachers ranged from nine to thirty. Two of these teachers, Mr. L and Ms. T, taught in the same suburban school while the other four came from three schools within an urban school district. Of the four who taught in the urban school district, Ms. K and Ms. S taught in the same school while Ms. M and Ms. R taught in two other schools within the district. Ms. K, Mr. L and Ms. M were the high frequency users while Ms. R, Ms. S and Ms. T were the moderate frequency users. All the six teachers had attended workshops on graphing calculators at least three times and all but Ms. T had attended such workshops within the two years preceding the study.

Among the high frequency users, Ms. K had 22 years of teaching experience. At the time of the study, she was teaching Math 2 and pre-calculus. All three of the lessons that I observed were in the Math 2 class. Her class was comprised of 18 students, seven female and eleven male. Thirteen of the students were African-American, three were Hispanic and two were Caucasian. Mr. L had 15 years of teaching experience; he taught Math 2 and statistics at the time of the study. I observed him in his Math 2 class where he had 17 students; there were nine females and eight males, all of whom were Caucasian. Finally, Ms. M had nine years of teaching experience. She was assigned to teach Math 1 and Math 2 and I observed her teach one lesson in Math 1 and two lessons in Math 2. In Math 1, she had 21 students, 11 females and ten males. Twelve of these students were African-American, five were Caucasian, three were Hispanic and one was Asian. In the Math 2 class she had 19 students, nine females and ten males. The racial composition was 17 African-Americans and two Caucasians.

Ms. R, a moderate frequency user, had 25 years of teaching experience and was assigned Math 1A and Math 2. Math 1A is a mathematics course taken by students who are placed in the lower track and therefore do not take the regular mathematics course (Math 1). I observed Ms. R in the Math 2 class in which she had 15 students. Five of the students were females and ten were males; all the students were African-American. Ms. S had 12 years of teaching experience and was teaching Math 1 and Math 2. I only observed her in the Math 2 class where she had 20 students with females and males in equal numbers. The racial composition was 11 African-Americans, six Caucasians, and three Hispanics. The last moderate frequency user was Ms. T with 30 years of teaching experience. She was assigned Math 2 and pre-calculus. I observed her in her Math 2 class. She had 18 students of whom ten were females and eight were males. Sixteen of the students were Caucasian and the other two were African-American.

Table 3

Teachers Participating in the Second Phase

Frequency of Calculator Use	Teacher	Type of School	Years Teaching
High	Ms. K	Urban	22
	Mr. L	Suburban	15
	Ms. M	Urban	9
Moderate	Ms. R	Urban	25
	Ms. S	Urban	12
	Ms. T	Suburban	30

The second phase was comprised of semi-structured interviews and classroom observations. Data sources included one task-based interview (Goldin, 1999) with each teacher prior to classroom observations and three classroom observations for each teacher with pre-observation (planning) interviews. The pre-observation interviews were short, lasting between ten and fifteen minutes and they were intended to help me know what the teachers were planning on covering in the lesson and what the students would be doing during the lessons. I had intended to follow these observations with more comprehensive post observation (debriefing) interviews, but because of scheduling constraints it was not always possible to do this and so I held short sessions of about five minutes following the observations. During these five minute sessions the teachers generally commented on how they thought the class went. Occasionally, I used email communication as a follow up after an observation.

The Survey Instrument

I developed a survey instrument using items adapted from Fleener (1995). This instrument consisted of two parts, Part I of which had thirteen questions seeking to gather the teachers' background information as well as how often they used graphed calculators, while Part II was comprised of 24 items with Likert-type responses on a five point scale with SA=strongly agree, A=agree, N=neither agree nor disagree, D=disagree, and SD=strongly disagree (for more details see Appendix A).

Most studies cited here that have used survey methodology have used a four-point Likert scale (Fleener, 1995b), but some have also used a five-point scale (Milou, 1999). It appears that researchers who used the four-point scale held the position that teachers' beliefs about use of graphing calculators fall into two mutually exclusive and exhaustive categories, namely MASTERY-YES and MASTERY-NO, depending on whether they prefer that their students master basic concepts first before they can be allowed to use graphing calculators (Fleener, 1995b). I chose to use the five-point scale in order to give teachers who may be uncomfortable with taking sides on some of the items the opportunity to pick the middle ground.

I divided the items in Part II of the survey into three categories, according to the information I intended to get from the responses to the items. The first category consisted of items that either directly or indirectly talked about the role of graphing calculators in the exploration of various representations of functions. These items, shown in Table 4, were meant to address part (a) of the first research question, namely: What are the teachers' beliefs about graphing calculators' influence on use of and exploration of various representations of functions?

Table 4

Survey Items Examining Teachers' Beliefs about the Role of Graphing Calculators in Exploration of Representations

Item Number	Statement
5.	Some problems in a first algebra course are best solved using tables rather than algebraic symbols.
7.	Graphing calculators enable students to solve non-routine problems that would otherwise be inaccessible by algebraic techniques.
8.	Representing a function with a graph helps students who have difficulty using algebraic symbols.
9.	Students should always learn to solve problems using algebraic symbols first before they can use tables.
10.	Graphing calculators help students to recognize connections between graphical, symbolic and numerical representations of functions.
12.	Some problems in a first algebra course are best solved using graphs rather than algebraic symbols.
13.	Graphing calculators support students' learning of linear and quadratic functions by helping them to discuss the various representations of these functions.
14.	When students use graphing calculators on a regular basis, they become better at interpreting tables.
17.	Students should always learn to solve problems using algebraic symbols first before they can use graphs.
20.	When students use graphing calculators on a regular basis, they become better at interpreting graphs.

The second category consisted of items that addressed teachers' beliefs about graphing calculators' influence on teacher directions and student exploration. These items were meant to help me address part (b) of the first research question. The items are shown in Table 5.

Table 5

Survey Items Examining the Teachers' Beliefs about the Role of Graphing Calculators in Influencing Teacher Direction

Item Number	Statement
1.	I encourage my students to use graphing calculators for discovery and/or exploratory activities.
2.	Students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand.
3.	I always give my students specific directions on how they should use the graphing calculator.
6.	Using graphing calculators provides opportunities for students to share ideas.
19.	Students should be free to use the graphing calculator whenever they feel it is appropriate.
22.	Students should be free to explore with the graphing calculator.
23	The teacher should always decide when it is appropriate for students to use graphing calculators.

The third and last category consisted of items that did not fall in either of the first two categories. These items, shown in Table 6, addressed general issues about graphing calculators.

Table 6

Survey Items Examining the Teachers' Beliefs Regarding General issues about Graphing Calculators

Item Number	Statement
4.	Graphing calculators make the study of linear and quadratic functions more accessible to a wider range of students.
11.	Graphing calculators enable students to engage with challenging problems.
15.	Graphing calculators have had almost no impact on how I teach.
16.	Graphing calculators have had almost no impact on what I teach.
18.	I try to take every opportunity to use the graphing calculator when I teach about linear and quadratic functions.
21.	I am a confident user of the graphing calculator.
24.	I have lots of ideas about how I can make use of the graphing calculator in my classroom.

Task-based Interviews

Before I began observing their classes, I conducted semi-structured, task-based interviews (Goldin, 1999) with each teacher. The interviews lasted between forty-five minutes and one hour each. I divided the interview questions and tasks into four major categories, namely planning, sources of teaching tasks, function representations, and issues related to calculator usage. For details about these questions and tasks see Appendix B.

Under the category of planning, I sought to know the key things that teachers consider as they prepared to teach lessons on linear and quadratic functions, particularly when they intended to use graphing calculators in the classrooms. I also sought to know how they perceived the graphing calculators to affect their planning. On sources of tasks, I sought to know where teachers got their teaching activities/tasks and how they used these tasks, that is whether they modified them or not and why or why not. These two categories (planning and sources of tasks) were meant to help me to begin to understand how teachers envisioned a lesson on linear and quadratic functions in which graphing calculators are used and what outcomes they might have expected of their students. This contributed towards addressing research question #2, part (b).

Under the categories of function representations and issues related to calculator usage, I presented the teachers with various tasks and asked them to respond to the tasks as well as speculate on how their students might have responded to those tasks. These categories were meant to help shed light on the teachers' choices of representation in various situations, the kind of partnerships these teachers had developed with graphing calculators, and the kind of expectations the teachers had for their students when using graphing calculators. This was important as the tasks provided a common ground for all the teachers given that no two teachers were teaching the same lesson. This contributed towards addressing research question 2, part (a).

Task-based interviewing (as opposed to open-ended questions) was appropriate in the context of this study because it provided me with the opportunity to structure and focus the mathematical environment for all the teachers that, to some extent, I could control what the teachers did while at the same time allowing the them the flexibility to approach the tasks in different ways. The interviews were intended to focus on the participant as both a doer of and as a teacher of mathematics, thus bringing out problem solving characteristics of the participants that may not have been shown otherwise. This way I was able to glean some aspects of the participants' use of multiple representations before observing their classrooms.

Classroom Observations

Before I went for each classroom observation, I met with the respective teacher ahead of time for a pre-observation interview. These interviews lasted about fifteen and were centered mainly on what the teachers had planned for the upcoming lessons. All the interviews were audiotaped and later transcribed. The interviews were meant to help me touch base with what the teachers would be doing as well as help me focus my observations. While in the classroom I took field notes and recorded various teacher actions with the help of the electronic classroom observation toolkit (eCOVE). Operating under the sociocultural framework influenced the way I collected qualitative data and what kind of data I collected. For example, I was studying the classroom dynamics in terms of teacher-student interaction, student-student interaction, and human-technology interaction. The NCTM (2000) points out that successful communication in the classroom requires the negotiation of meanings and depends on all members of the class expressing genuine respect and support for one another's ideas. I paid special attention to how the teachers facilitated the interaction between students and calculators. In this regard, I was examining the kind of instructions the teachers gave to their students, the actions the students took and the questions they asked their teachers as well as their peers, and how the teachers responded to the students' questions. I was also looking for how teachers organized their classes, small group or whole class, and how this classroom organization was reflected in the ways that graphing calculators were used.

After each classroom observation, I met with the respective teacher immediately for a short, post-observation interview. These interviews mainly centered on what transpired during the lessons. These interviews were audiotaped and transcribed. The interviews were meant to help me obtain the teachers' comments on the lessons as well as clarify or elaborate any questions that I may have had as a result of the observations.

Data Analysis

The quantitative data were analyzed as follows: I used descriptive statistics to obtain frequencies and percentages for the demographic information. Additionally, I assigned numerical ratings of 1 to 5 to the Likert Scale questions as follows: I assigned a 5 to the "strongly agree" response and a 1 to the "strongly disagree" response for positive statements. I then scored the negative statements in reverse order, thus a 5 was assigned to the "strongly disagree" response and a 1 to the "strongly agree" response. I first considered the teachers' responses to the items on the survey without focusing on the teachers' frequency of calculator use groups. I then identified items for which there was an association between the teachers' frequency of calculator use and their responses to the items. In order to achieve this, I ran Chi-Square tests (at 0.05 significance level) for all the items to determine whether there were any associations between responses to the items and the teachers' frequencies of calculator use. Since the Chi-Square test does not tell us exactly which cells cause these associations, if any, for each of the items that had statistically significant values of P, I ran post hoc tests for standardized residuals using the critical value of 1.96.

Qualitative data for addressing research question #2 came from the task-based interviews and classroom. In particular, the data was comprised of the approaches the teachers used when attempting the tasks (including verbal comments), the representations they used on these tasks, and their comments about how they perceived their students might have approached the tasks. As for the classroom observations, the data included actions the teachers made that may have led to calculators being used (such as when, where, for what, etc., type of issues), and were mainly descriptive. These data were meant to help illuminate the findings obtained from the quantitative data. In addressing question #3, I was comparing the graphing calculator usage patterns against the teachers' professed beliefs from the survey.

I employed a grounded theory (Strauss & Corbin, 1998) approach for data analysis. Grounded theory enabled me to identify themes as they emerged from the dominant statements made and the actions taken by the teachers. Working with socio-cultural theory allowed me to view the graphing calculator as a tool that mediated teacher actions as well as student actions. After refining the themes I tied them back to the metaphors for describing the interaction between calculator and user.

I first carried out a microanalysis of the data from the task-based interviews for all the teachers, thereby developing general theme statements from the interviews based on dominant phrases in their responses to items under the categories of planning and sources of tasks and also on the actions they took while attending to items under the categories of function representations and issues related to calculator usage. I then analyzed the data from classroom observations against the statements generated above. I identified episodes from the classes that could support these statements and/or sometimes challenge them. I finally refined the statements by modifying, merging, and/or omitting some into major themes. I also noted the amount of time dedicated to graphing calculator use during each lesson as well as the types of function representation that the teachers specified in their tasks. Additionally, I identified the patterns of shifts between representations used by the teachers. Below is a list of general statements that I generated in the first step of analysis from the interview data:

- Consider students' prior knowledge and experiences
- Use students' own words
- Use examples from real life situations
- Ask students to share their work/solutions with whole class
- Do not specifically plan for calculators
- Consider their use of various representations as balanced
- Shift from equation to table then to graph
- Shift from equation to graph then to table
- Shift from table to equation then to graph
- Shift from table to graph then to equation
- Use ZOOM menu to adjust graph on calculator
- Use WINDOW menu to adjust graph on calculator

In this initial step, I chose not to distinguish between teachers because my aim at this point was not to compare the teachers, but rather to expose in general terms how their use of graphing calculators affected their choices of teaching strategies and instructional tasks and vice versa. I placed statements that appeared to be opposites (e.g., shift from equation to table then to graph vs. shift from equation to graph then to table) in succession not because I wanted to amplify the oppositeness but simply because I wanted to make it easier for the reader to see them as different statements.

In the second step, I analyzed the data from classroom observations against the statements generated above. I tried to identify incidents/episodes from the lessons that could support these statements or sometimes challenge them. I then refined the statements (by modifying, merging, and/or omitting some) into major themes, namely (a) patterns of representations, (b) lessons characterized by teacher direction, (c) lessons characterized by student exploration, and (d) the roles for which the graphing calculator was used.

Table 7 shows the types of representations and the teacher actions that I used to identify the representations.

Table 7

Types of Representations Specified in Tasks Used by Teachers

Representation specified in task	**Teacher actions**
Specifying use of graphs	Using statements like "display the graph(s)", "graph the equation(s)", "find out what will happen to the graph …" Including graphs in student worksheets
Specifying use of tables	Using statements like "make a table of values", "display the table", etc. Including tables in student worksheets
Specifying use of equations	Using statements like "write an equation to model the situation"; "solve the system of equations", "find the equation", etc. Including equations in student worksheets
Not specifying use of any particular representation	Using word problems in which no mention of the words graph, equation, or table is made

Apart from establishing frequency of graphing calculator use in order to assign the participants into groups, the quantitative data and the qualitative data were analyzed independently (addressing questions #1 and #2). I then pulled together the two sets of data after completion of coding and analysis (addressing question #3). Specifically, I analyzed the responses on the survey of the six teachers who participated in the second phase alongside their responses on the interview tasks and the actions they took during the lessons that I observed.

Addressing the Questions

I now discuss the research questions and describe how I used the data to address each one of them.

1. What are secondary mathematics teachers' beliefs about use of graphing calculators in the teaching and learning of linear and quadratic functions?

To address this research question, I sought to determine whether there was consensus or lack thereof within the various frequency of use groups for each of the two areas using corresponding items on the survey as shown in the data collection section. To achieve this, I ran Chi-Squared Goodness-of-Fit Tests (Agresti, 1996) for each item with the frequency of use as rows and the five responses as columns in the contingency tables.

2. How do secondary school mathematics teachers use graphing calculators when teaching linear and quadratic functions?
    c) What function representational choices do secondary mathematics teachers make when using graphing calculators?
    d) How specific are the teachers' directions to students about how the calculators may be used?

I addressed research question 2 by looking at the interview data and the data from classroom observations. From the interview data, I was able to identify patterns of function representation that teachers used while attempting the tasks I asked them to solve. From the classroom observations, I identified the patterns of representation, the amount of class time for which graphing calculators were used, as well as the nature of classroom dynamics. Additionally, I identified the roles for which the graphing calculators were used.

3. What is the relationship between the teachers’ professed beliefs about graphing calculators and observed practice?
    a) What is the nature of similarities and/or differences between reported and observed calculator usage trends?
    b) To what extent do professed beliefs about graphing calculators explain observed usage?

I addressed research question 3 by looking at how the teachers who participated in the second phase of the study responded to the survey and comparing these responses to the teachers’ responses to interview tasks as well as the actions they took while solving the tasks and in their classrooms. I looked for consistencies and inconsistencies between the responses on the survey and on the interview tasks together with the teachers’ instructional choices. In this way I was able to reconcile the results for research questions 1 and 2 for the six teachers in the second phase.

CHAPTER 4: RESULTS FROM SURVEY DATA

In this chapter, I present an analysis of the results of the survey data, structured according to the three categories of items discussed in Chapter Three. The three categories of items are (1) items dealing with teachers' beliefs about the role of graphing calculators in the exploration of multiple representations, (2) items dealing with teachers' beliefs about the role of graphing calculators in influencing teacher direction, and (3) items examining the teachers' beliefs regarding general issues about graphing calculators. For each category, I will first present an analysis of all the 81 teachers' responses to the items without focusing on the teachers' frequency of calculator use groups. I will then identify and analyze items for which there is an association between the teachers' frequency of calculator use and their responses to the items.

Beliefs about Graphing Calculators' Influence on Use of and Exploration of Various Representations of Functions

Survey items in the category of teachers' beliefs about graphing calculators' influence on the use of and exploration of various representations of functions included 5, 7, 8, 9, 10, 12, 13, 14, 17 and 20 (see Table 1), and were aimed at providing me with information about what teachers perceive to be the influence of graphing calculators on students' understanding of multiple representations. Table 8 gives the frequency distribution of responses to each of the items in this category, while Figure 1 shows the corresponding histograms for these responses.

Table 8

Frequency Distribution by Response to Item

Item Number		SA	A	N	D	SD
5	Count	23	32	17	9	0
	Percentage	28.4	39.5	21.0	11.1	0
7	Count	37	37	4	3	0
	Percentage	45.7	45.7	4.9	3.7	0
8	Count	33	41	7	0	0
	Percentage	40.7	50.6	8.6	0	0
9	Count	13	17	10	33	8
	Percentage	16.0	21.0	12.3	40.7	9.9
10	Count	53	25	1	2	0
	Percentage	65.4	30.9	1.2	2.5	0
12	Count	24	35	14	7	1
	Percentage	29.6	43.2	17.3	8.6	1.2
13	Count	34	41	6	0	0
	Percentage	42.0	50.6	7.4	0	0
14	Count	19	29	22	7	4
	Percentage	23.5	35.8	27.2	8.6	4.9
17	Count	9	24	9	30	9
	Percentage	11.1	29.6	11.1	37.0	11.1
20	Count	29	30	16	5	1
	Percentage	35.8	37.0	19.8	6.2	1.2

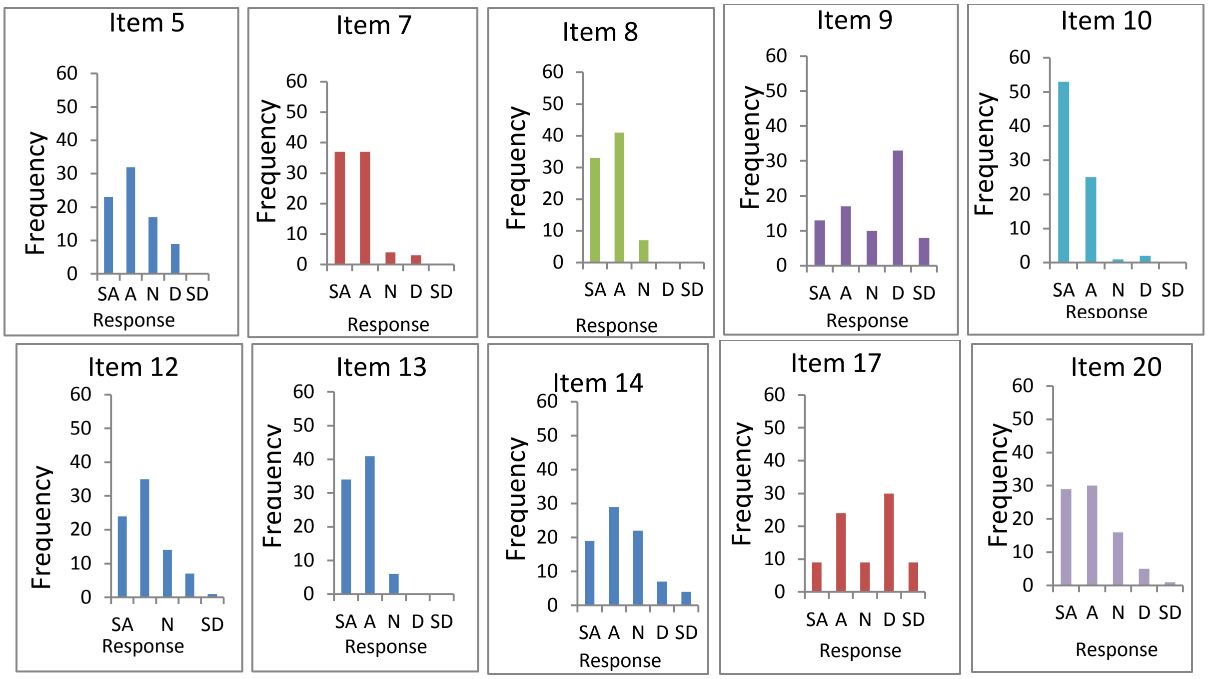


Figure 1. Distribution of teacher responses on each item in category.

Table 8 shows that 60% or more of the teachers either agreed or strongly agreed with each of items number 5, 7, 8, 10, 12, 13, 14 and 20. These percentages suggest a general consensus among the teachers on these items. Specifically, the teachers agree that graphing calculators help students to solve non-routine problems that would otherwise be inaccessible by algebraic techniques, recognize connections between graphical, symbolic and numerical representations, discuss the various representations of these functions, and become better at interpreting tables and graphs. The teachers further agree that some problems in a first algebra course are best solved using tables and graphs rather than algebraic symbols.

On the other hand, there is a lack of consensus on items 9 and 17. For item 9, just over 50% of the teachers either disagreed or strongly disagreed while 37% either agreed or strongly agreed. Similarly, for item 17, 48% of the teachers either disagreed or strongly disagreed while 40% either agreed or strongly agreed. This lack of consensus suggests that the teachers are divided on whether students should always learn to solve problems using algebraic symbols first before they can use tables or graphs.

Generally speaking, items 7, 8, 10, 13, 14, and 20 examine the perceived benefits that graphing calculators offer students in terms of using different function representations. Therefore, based on the fact that most teachers either agreed or strongly agreed with the items, I conclude that the teachers recognize graphing calculators as valuable to students in using and exploring various representations of functions. Specifically, I argue that most of the teachers believe that graphing calculators help students to recognize connections between various representations of functions, and provide opportunities for students to discuss these representations. Further, the teachers believe that regular use of graphing calculators helps students become better at interpreting tables and graphs. Item 7 states that graphing calculators enable students to solve non-routine problems. Since the majority of the teachers either agree or strongly agree with this item, I conclude that it is a popular belief among teachers that graphing calculators are beneficial for problem solving in a first algebra course. Because of the relationships between items 5 and 12, on the one hand, and items 9 and 17, on the other hand, that is, items 5 and 12 examining teachers' function representation preferences and items 9 and 17 examining ordering preferences (see Table 9 below), I paired up these items and compared the responses across all of the teachers.

Table 9

Comparison of Items 5 and 12 and Items 9 and 17

	Item number	Statement
Function Representation Preferences	5.	Some problems in a first algebra course are best solved using tables rather than algebraic symbols.
	12.	Some problems in a first algebra course are best solved using graphs rather than algebraic symbols.
Representation Sequencing Preferences	9.	Students should always learn to solve problems using algebraic symbols first before they can use tables.
	17.	Students should always learn to solve problems using algebraic symbols first before they can use graphs.

Items 5 and 12 state that *some* problems in a first algebra course are best solved using tables or graphs (respectively) rather than algebraic symbols, while items 9 and 17 state that students should *always* use algebraic symbols first before they can use tables or graphs. Since the majority of the teachers either agreed or strongly agreed with items 5 and 12, I conclude that the teachers believe that the choice of representation depends on the type of problem to be solved. This indirectly implies that the teachers believe that flexibility in choosing representations is important when solving various problems. Due to the split among the teachers on their responses to items 9 and 17, I cannot speculate on their position regarding algebraic symbols always preceding tables and graphs. Items 9 and 17 are almost direct opposites of items 5 and 12 and one would therefore expect that with the majority of teachers agreeing with items 5 and 12, similar numbers would disagree with items 9 and 17. However, as already stated, this is not the case. This inconsistency may have resulted, in part, from the fact that items 5 and 12 use the phrase "*some* problems…" while items 9 and 17 use the phrase "… students should *always* learn …" This may also result from the fact that items 5 and 12 refer to representational preferences while items 9 and 17 refer to ordering preferences. I will discuss these items in pairs, starting with items 5 and 9 (use of algebraic symbols versus tables) followed by items 12 and 17 (use of algebraic symbols versus graphs).

Item 5 states: *Some problems in a first algebra course are best solved using tables rather than algebraic symbols* while item 9 states: *Students should always learn to solve problems using algebraic symbols first before they can use tables*. Both these items examine the use of symbols versus use of tables in solving problems in a first algebra course. Item 5 suggests that the symbolic approach is not always the best but instead using tables is best for some problems. This makes an argument for the choice of representation in the sense that a teacher who agrees with this item holds the opinion that there are problems for which s/he would recommend that students use the tabular approach. Approximately 69% of the teachers agreed or strongly agreed with this item while only about 11% disagreed with the item, and none strongly disagreed. These percentages suggest that most of the teachers hold the view that algebraic symbols are not always the best solution approach. I observe, however, that 21% of the teachers were neutral, which suggests that a sizable number of teachers are non-committal on this item. Of the 69% of the teachers who agreed or strongly agreed with item 5, 49% said they were high frequency users, 29% medium frequency users, and about 22% said they were low frequency users. Clearly, these percentages decrease as we go down the frequency of use categories and reflect the sample population.

On the other hand, item 9 suggests that learning how to solve problems using algebraic symbols should *always* precede learning how to solve problems using tables. This is clearly an argument about which order the two representations should be learned. But more importantly, this item seems to contradict item 5. It therefore makes sense to think that teachers who agree with item 5 would disagree with item 9. Results, however, paint a different picture with the teachers almost split in their responses to item 9, implying that some of the teachers who agreed or strongly agreed with item 5 also agreed or strongly agreed with item 9. Approximately 51% of the teachers disagreed or strongly disagreed with item 9 while 37% agreed or strongly agreed with this item, another 12% were uncommitted. Of the 51% who disagreed or strongly disagreed, approximately 44% said they were high frequency users, 27% reported to be medium frequency users, and 29% said they were low frequency users. Among the 37% who agreed or strongly agreed with item 9, 40% were high frequency users, 33% medium frequency users, and 27% low frequency users.

Because of the relatively high percentages of teachers from the high frequency group who agreed or strongly agreed with item 5 and the corresponding high percentage that disagreed or strongly disagreed with item 9, it is possible that teachers in this group are more likely to shift between using algebraic symbols and tables depending on the problems they are handling. On the other hand the percentages of teachers from the medium and low frequency groups who agreed or strongly agreed with item 5 and the corresponding percentages of teachers from these groups who disagreed or strongly disagreed with item 9 were relatively low, implying that teachers from these groups are more likely to use algebraic symbols rather than tables when approaching problem solving. Table 10 shows the responses within each frequency of use category for both items 5 and 9.

Table 10

Distribution of Responses to Items 5 and 9 by Teachers' Frequency of Use Groups

Frequency of Calculator Use		Item 5			Item 9		
		A/SA	N	D/SD	A/SA	N	D/SD
High	Count	27	6	3	12	6	18
	% within response	49.1	35.3	33.3	40	60	43.9
Moderate	Count	16	6	3	10	4	11
	% within response	29.1	35.3	33.3	33.3	40	26.8
Low	Count	12	5	3	8	0	12
	% within response	21.8	29.4	33.3	26.7	0	29.3
Total	Count	55	17	9	30	10	41
	% within response	100	100	100	100	100	100

Items 12 and 17 are similar to items 5 and 9 in the sense that they also refer to use of algebraic symbols in a first algebra course. This time, though, the comparison is between algebraic symbols and graphs. Item 12: *Some problems in a first algebra course are best solved using graphs rather than algebraic symbols,* makes an argument for use of graphs rather than algebraic symbols on some problems. This, like item 5, deals with making a choice between two representations. A teacher who agrees with this item holds the opinion that under certain circumstances graphs are preferable over algebraic symbols. Approximately 73% of the teachers agreed or strongly agreed with this item, 17% were neutral and only about 10% disagreed or strongly disagreed with the item. These percentages are consistent with those recorded on item 5. The pattern of responses across frequency of use is similar to that of item 5 as well as to the sample population. Out of the 73% who agreed or strongly agreed with the item, slightly over 47% were high users, 29% were medium users, and 24% were low users, as shown in Table 10. These results, like those for item 5 discussed previously suggest that teachers who identified as high frequency users are more likely to shift between algebraic symbols and graphs as the problems dictate.

Item 17: *Students should always learn to solve problems using algebraic symbols first before they can use graphs*, makes the argument that symbols should always precede graphs. As was the case with items 5 and 9, I expected that most of the teachers who agreed with item 12 would disagree with item 17. However, just as was the case with item 9, the teachers were split down the middle on this item. In particular, approximately 41% of the teachers agreed or strongly agreed with item 17 – a clear indication that some of the teachers who agreed or strongly agreed with item 12 did likewise for item 17, 48% disagreed or strongly disagreed, and 11% were neutral. Of the 41% who agreed or strongly agreed, 39% were high frequency users, 36% medium, and 24% low frequency users. On the other hand, out of the 48% who disagreed or strongly disagreed, 46% were high frequency users, 28% medium frequency users, and 26% low frequency users. The language in the item (i.e., using the phrase *always* rather than *some* as in item 12) may have contributed to the near split in teachers' responses to this item. These percentages are shown in the Table 11.

Table 11

Distribution of Responses to Items 12 and 17 by Teachers' Frequency of Use Groups

Frequency of Calculator Use		Item 12			Item 17		
		A/SA	N	D/SD	A/SA	N	D/SD
High	Count	28	6	2	13	5	18
	% within response	47.5	42.9	25	39.4	55.6	46.2
Moderate	Count	17	6	2	12	2	11
	% within response	28.8	42.9	25	36.4	22.2	28.2
Low	Count	14	2	4	8	2	10
	% within response	23.7	14.3	50	24.2	22.2	25.6
Total	Count	59	14	8	33	9	39
	% within response	100	100	100	100	100	100

The pattern of responses for items 9 and 17 are interesting when compared with those for items 5 and 12. As shown in the preceding discussion, the majority of the teachers agreed or strongly agreed with items 5 and 12 but the teachers were split on items 9 and 17. It is possible that since both 9 and 17 deal with which representation students should learn first, this may have influenced the teachers' responses to these items. These results present interesting scenarios and insights into possible sequencing preferences for teachers. I will discuss this issue further in Chapter Five as I look at the interviews and classroom observation data. I will specifically discuss differences or similarities among the six participants in terms of their choice of representations as well as their sequencing of the chosen representations.

As noted earlier, items 9 and 17 are similar and are opposites of items 5 and 12. While items 5 and 12 suggest that some problems in a first algebra course are best solved by tables (item 5) or graphs (item 12) on the contrary, items 9 and 17 suggest that algebraic solutions should be given priority over tabular or graphical solutions. I pointed out in the discussion above that while I expected most of the teachers who agreed with 5 and 12 to disagree with 9 and 17, the results did not reflect this kind of reasoning. What is not very clear is why there was lack of consensus on items 9 and 17. I suggested earlier that a possible answer to this might lie in the fact that items 5 and 12 use the term “some” while items 9 and 17 use the term “always”, as well as the fact that items 5 and 12 deal with choice of representation while items 9 and 17 deal with sequencing preferences. Because of these differences in responses by the teachers, I ran chi-square tests on these items in pairs of opposites; that is, item 5 by item 9, and item 12 by item 17. Both tests revealed possible associations between responses to the paired items as shown in Table 12.

Table 12

Chi-Square Values for Paired Items

	Chi-Square	*df*	*P*
Item 5 by Item 9	29.64	12	0.003*
Item 12 by Item 17	40.58	16	0.001*

*$P < 0.05$

In order to identify the specific cells that caused the associations, I ran the post hoc tests for standardized residuals using the critical value of 1.96. Results showed that teachers who were neutral on item 5 were for the most part neutral as well on item 9 (standardized residual of 2.7) or strongly agreed with item 9 (standardized residual of 2.0). This result suggests that some of the teachers who could not pick a position one way or another on whether some problems in a first algebra course are best solved using tables rather than algebraic symbols (item 5) wound up agreeing with that students should always learn to solve problems using algebraic symbols first before they can use tables (item 9). This is rather surprising given that the wording in item 9 is stricter than in item 5.

For items 12 and 17, those teachers who strongly disagreed with 12 on the most part strongly agreed with 17 (standardized residual of 2.7). In other words, those teachers who strongly disagreed that some problems in a first algebra course are best solved using graphs rather than algebraic symbols also strongly agreed that students should always learn to solve problems using algebraic symbols first before they can use graphs. Here the results are consistent with what one would expect given that the two items are opposites.

Frequency of calculator use and teachers' responses. In the preceding section, I discussed the overall picture of the teachers' responses to all the items in this category. In this section, I first present the results of Chi Square tests on these items and then give analyses of the items for which the test result revealed statistically significant p values. Chi-Square tests for frequency of calculator use by responses showed that only two items in this category, namely, item 14 and item 17, had statistically significant P values (see Table 13).

Table 13

Chi-Square Values

Item No.	Chi-Square	df	P
5	2.7	6	0.846
7	5.56	6	0.474
8	3.33	4	0.503
9	8.35	8	0.400
10	4.58	6	0.599
12	12.75	8	0.121
13	4.62	4	0.328
14	16.42	8	0.037*
17	17.80	8	0.023*
20	11.01	8	0.201

*$P < 0.05$

As stated in Chapter Three, I ran post hoc tests for the items that had statistically significant p values. In this category, these were items 14 (*When students use graphing calculators on a regular basis, they become better at interpreting tables*) and 17 (*Students should always learn to solve problems using algebraic symbols first before they can use graphs*). Table 14 and Figure 2 show, respectively, the results for the cross tabulation of item 14 by frequency of use groups and histograms for the distribution of responses to this item within each frequency of calculator use group.

Table 14

Frequency of Use by Response to Item 14: When students use graphing calculators on a regular basis, they become better at interpreting tables.

Frequency of Calculator Use		SA	A	N	D	SD
High	Count	9	15	9	3	0
	% within response	47.4	51.7	40.9	42.9	0
Moderate	Count	7	8	9	1	0
	% within response	36.8	27.6	40.9	14.2	0
Low	Count	3	6	4	3	4
	% within response	15.8	20.7	18.2	42.9	100
Total	Count	19	29	22	7	4
	% within response	100	100	100	100	100

Chi-Square = 16.423 and P = 0.037

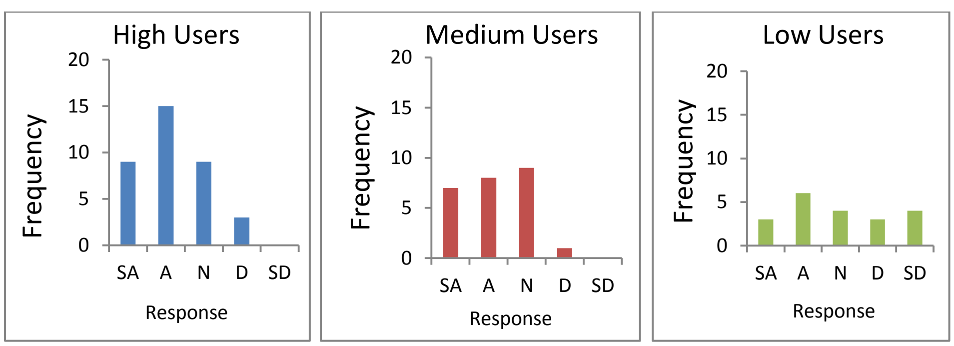


Figure 2. Distribution of responses to item 14 within each frequency of use group.

The histograms show that the pattern of responses within the groups of teachers who identified as high users and medium users are fairly similar. Most of the responses in these groups lie in the Agree or Strongly Agree columns with another sizable number in the Neutral column with only a few in the Disagree column and none in the Strongly Disagree column. However, for those teachers who identified as low users, their responses are spread out almost evenly throughout the five columns. In fact for this item, the cell corresponding to the low frequency users responding with a choice of "Strongly Disagree" returned the largest positive standardized residual (3.0) meaning that this cell was overrepresented. In other words, this suggests that most of the teachers who strongly disagreed with the fact that using graphing calculators on a regular basis helps students become better at interpreting tables identified as low frequency users.

Table 15 and Figure 3 show, respectively, the results for the cross tabulation of item 17 by frequency of use groups and histograms for the distribution of responses to this item within each frequency of calculator use group.

Table 15

Frequency of Use by Response to Item 17: Students should always learn to solve problems using algebraic symbols first before they can use graphs

Frequency of Calculator Use		SA	A	N	D	SD
High	Count	2	11	5	15	3
	% within response	22.2	45.8	55.6	50.0	33.3
Moderate	Count	1	11	2	6	5
	% within response	11.1	45.8	22.2	20.0	55.6
Low	Count	6	2	2	9	1
	% within response	66.7	8.3	22.2	30.0	11.1
Total	Count	9	24	9	30	9
	% within response	100	100	100	100	100

Chi-Square = 17.802 and P = 0.023

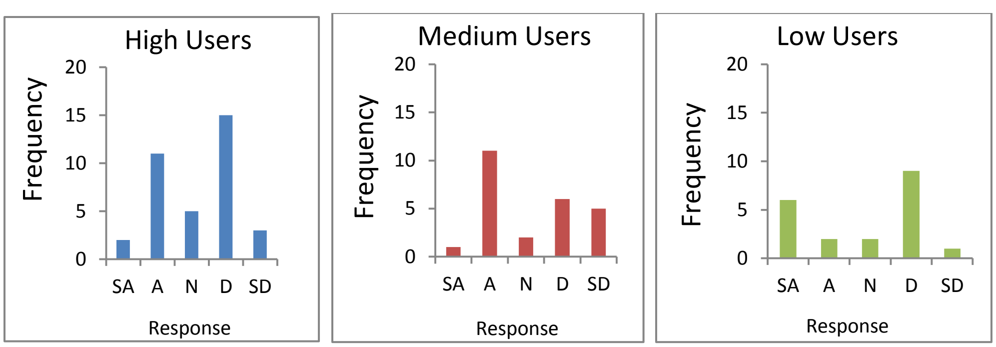


Figure 3. Distribution of responses to item 17 within each frequency of use group.

From the histograms we can see that there is a split among the responses within all the three groups. However, while the pattern of responses within the groups identifying as high users and medium users are very similar, this pattern is slightly different within the group that identified as low users. In this group, the number of teachers responding with Strongly Agree is significantly high relative to the numbers in the other two groups. Indeed, the post hoc tests confirmed this result with a standardized residual of 2.5. This suggests that most teachers who strongly agreed with the statement: Students should always learn to solve problems using algebraic symbols first before they can use graphs, identified as low frequency users.

Beliefs about Graphing Calculators' Influence on Teacher Direction and Student Exploration

Items in this category included 1, 2, 3, 6, 19, 22 and 23 (see Table 16), and were aimed at providing me with information about what teachers perceive to be the influence of graphing calculators on teacher direction versus student exploration. Here, 68% or more of the teachers either agreed or strongly agreed with each of items number 1, 3, 6 and 22 (see Figure 4). This meant that most of the teachers were of the view that they encourage their students to use graphing calculators for discovery and/or exploratory activities, that using graphing calculators provides opportunities for students to share ideas and in fact students should be free to explore with the graphing calculator. Yet most teachers were also of the view that they always gave their students specific directions on how they should use the graphing calculator. There was lack of consensus among the teachers on items 2, 19 and 23 implying that they did not agree on whether or not students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand. Furthermore, the teachers did not agree on whether students should be free to use the graphing calculator whenever they feel it is appropriate or it should always be the teacher's role to make this call.

All the items on which the teachers had consensus examine the issue of teachers giving students freedom to explore with the graphing calculators. It is intriguing though that most of the teachers agreed or strongly agreed with both the positively worded items (1, 6 and 22) as well as the negatively worded item (item 3). The consensus on the positively worded items suggests that the teachers believe in giving their students freedom to explore with graphing calculators; however, agreeing with the negatively worded item would appear to contradict this notion.

Items 2, 19 and 23, on the other hand, examine the role of the teacher in deciding when students should be allowed to use graphing calculators and so the three items are in total contrast to the other 4 items in this category. The split in responses on these items again presents an interesting scenario.

Table 16

Distribution of Responses to Items in the Category of Teacher Direction and Student Exploration

Item Number		SA	A	N	D	SD
1.	Count	45	25	7	0	4
	% within response	55.6	30.9	8.6	0	4.9
2.	Count	17	21	4	25	14
	% within response	21.0	25.9	4.9	30.9	17.3
3.	Count	24	31	16	10	0
	% within response	29.6	38.3	19.8	12.3	0
6.	Count	31	37	12	1	0
	% within response	38.5	45.7	14.8	1.2	0
19.	Count	14	34	7	25	1
	% within response	17.3	42	8.6	30.9	1.2
22.	Count	45	32	4	0	0
	% within response	55.6	39.5	4.9	0	0
23.	Count	8	21	30	20	2
	% within response	9.9	25.9	37.0	24.7	2.5

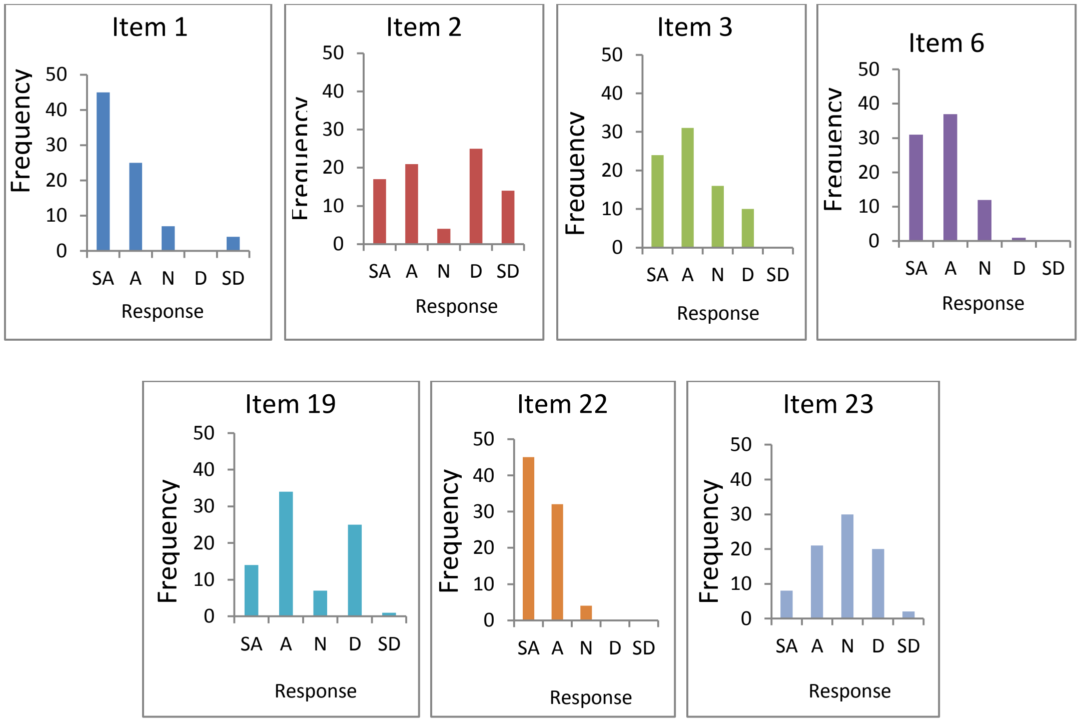


Figure 4. Distribution of responses within each item in category.

The teachers were divided on items 2 and 19, while on item 23 the teachers seemed to cluster around the middle choices. I observe that item 2 deals with sequencing in terms of which representation should be taught first. The pattern of responses to this item is therefore consistent with that of items 9 and 17 which also deal with sequencing. The responses to item 19 are also consistent with those of 9, 17, and 2 since item 19 states: *Students should be free to use the graphing calculator whenever they feel it is appropriate*. Since there were no common positions taken on items that involved sequencing, a common position on item 19 would have been a departure from that trend as it is almost impossible for a teacher to prevent students from using tables or a graph (for example) when the students are working with graphing calculators. Thus, teachers who hold the view that students should always learn how to use algebraic symbols before they can use either a table or a graph (agree with items 9 and 17) are more likely to hold the view that students should not be free to use the graphing calculator whenever they feel it is appropriate (disagree with item 19) and vice versa.

The fact that there was a split among teachers on whether students should be free to use the graphing calculator whenever they feel it is appropriate (item 19) may be an indicator of how the teachers may be split on whether to give students the freedom to explore with the graphing calculators or leave it upon the teacher to determine when it is appropriate for students to use the graphing calculator. However, such a split is not apparent as shown by the responses to item 23; *the teacher should always decide when it is appropriate for students to use graphing calculators*, the opposite of 19. Responses to this item are clustered around the middle, an indication that most of the teachers do not have very strong positions on either side. This then suggests that some of the teachers who agreed or disagreed that with the fact students should be free to use the graphing calculator whenever they feel it is appropriate could not pick a position one way or another on whether the teacher should always make this call.

Frequency of calculator use and teachers' responses. In this category, only items 1 and 2 had significant P values suggesting a possibility of association between the responses and the teacher's frequency of calculator use. I ran post hoc test for these items. Table 17 shows the results of the Chi-Square tests for all the items in this category.

Table 17

Chi Square Values

Item No	Chi-Square	df	P
1	16.23	6	0.012*
2	19.39	8	0.013*
3	2.01	6	0.911
6	5.80	6	0.447
19	10.61	8	0.225
22	3.89	4	0.421
23	11.68	8	0.166

*P < 0.05

Table 18 and Figure 5 show, respectively, the results for the cross tabulation of item 1 by frequency of use groups and histograms for the distribution of responses to this item within each frequency of calculator use group.

Table 18

Frequency of Use by Response to Item 1

Frequency of Calculator Use		SA	A	N	D	SD
High	Count	19	12	5	0	0
	% within response	42.2	48.0	71.4	0	0
Moderate	Count	14	9	2	0	0
	% within response	31.1	36.0	28.6	0	0
Low	Count	12	4	0	0	4
	% within response	26.7	16.0	0	0	100
Total	Count	45	25	7	0	4
	% within response	100	100	100	100	100

Chi-Square = 16.259, P = 0.012

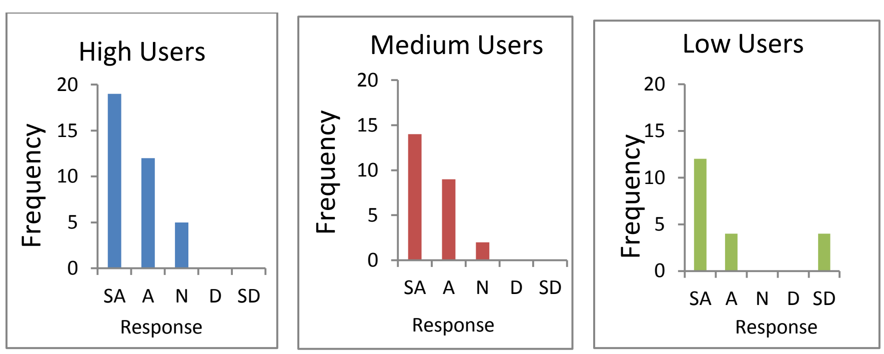


Figure 5. Distribution of responses to item 1 within each group.

Just like for item 14 above, the cell corresponding to the low frequency users responding with a choice of “Strongly Disagree” on item 1 returned the largest positive standardized residual (3.0) meaning that this cell was over represented. In other words, this suggests that most of the teachers who strongly disagreed with the statement: *I encourage my students to use graphing calculators for discovery and/or exploratory activities, also identified as low frequency users*.

Table 19 and Figure 6 show, respectively, the results for the cross tabulation of item 2 by frequency of use groups and histograms for the distribution of responses to this item within each frequency of calculator use group.

Table 19

Frequency of Use by Response to Item 2

Frequency of Calculator Use		SA	A	N	D	SD
High	Count	11	6	1	12	6
	% within response	64.7	28.6	25.0	48.0	42.9
Moderate	Count	1	7	0	10	7
	% within response	4.0	28.0	0	40.0	28.0
Low	Count	5	8	3	3	1
	% within response	29.4	38.1	75.0	12.0	7.1
Total	Count	21.0	25.9	4.9	30.9	17.3
	% within response	100	100	100	100	100

Chi-Square = 19.389, P = 0.013

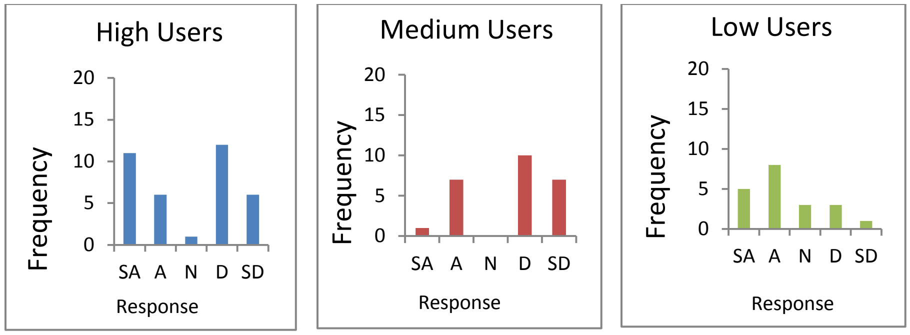


Figure 6. Distribution of responses to item 2 within each group.

On item 2, most of the teachers who selected the "Neutral Choice" identified as low frequency users as well (standardized residual of 2.0). For the histograms we see that there was a clear split among all the teachers across the three groups. There were more teachers taking the neutral position within the low users' group than any other group. It is, therefore, clear that among the high frequency users and the moderate frequency users there were those who held the view that students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand and those who held the view that students could use a graphing calculator to create a graph at any time, even before they have learned to graph by hand.

General Issues Regarding Use of Graphing Calculators

Items in this category included 4, 11, 15, 16, 18, 21, and 24 (see Table 20). These items were aimed at helping me gain insights into the teachers' beliefs on issues related to calculator use but that are not necessarily related to multiple representations or teacher direction. Unlike the previous two categories, in this category, I found that there was consensus (one way or another) on all the items, with most of the teachers agreeing or strongly agreeing with items 4, 11, 18, 21, and 24, which meant agreeing that graphing calculators make the study of linear and quadratic functions more accessible to a wide range of students besides enabling them to engage with challenging problems. The teachers further agreed that they took every opportunity to use the graphing calculator when they taught about linear and quadratic functions, they were confident users of the graphing calculator, and had lots of ideas about how they could make use of the graphing calculator in their classrooms. On the other hand a majority of the teachers disagreed or strongly disagreed with items 15 and 16 (see Figure 7), implying that graphing calculators had had almost no impact on how the teachers taught or what they taught.

Items 4 and 11 address the fact that graphing calculators not only make the study of linear and quadratic functions accessible to a wider variety of students, but they also enable the students to engage with challenging problems. The vast majority of the teachers agreed or strongly agreed with these items (94% for item 4 and 90% for item 11), suggesting that the teachers value graphing calculators as important tools in the study of linear and quadratic functions. Items 18, 21, and 24 address teachers' comfort levels with graphing calculators. Agreeing or strongly agreeing with these items implies that most of the teachers believe they are comfortable with using graphing calculators. Specifically, approximately 73% of the teachers agreed or strongly agreed with item 18 implying that the majority of the teachers believe that they try to take every opportunity to use the graphing calculator when they teach about linear and quadratic functions. With almost 98% of the teachers agreeing or strongly agreeing with item 21, we can conclude that the teachers who participated in the study on the most part believe that they are confident users of the graphing calculator. The percentage of those who agreed or strongly agreed with item 24 is slightly lower than corresponding percentages for items 18 and 21, but at 69% it is still high.

The consensus on items 15 and 16 was that of disagreeing with the items. These items address the issue of whether or not graphing calculators have had any impact on how and what one teaches. Disagreeing with the items imply that the teachers believe that graphing calculators have not only affected what they teach but also how they teach. However, I should point out that while the teachers disagree or strongly disagree almost unanimously with item 15 (93%), the percentage of those disagreeing or strongly disagreeing with item 16 drops to 59%. This drop may be attributed to the fact that what is taught is determined by many other factors, some of which are external to the school.

Table 20

Distribution of Responses to Items in the Category of General Issues Regarding Use of Graphing Calculators

Item Number		SA	A	N	D	SD
4	Count	54	22	3	1	1
	% within response	66.7	27.2	3.7	1.2	1.2
11	Count	44	29	6	1	1
	% within response	54.3	35.8	7.4	1.2	1.2
15	Count	2	4	0	34	41
	% within response	2.5	4.9	0	42.0	50.6
16	Count	6	14	13	25	23
	% within response	7.4	17.3	16.0	30.9	28.4
18	Count	18	41	15	3	4
	% within response	22.2	50.6	18.5	3.7	4.9
21	Count	44	35	0	1	1
	% within response	54.3	43.2	0	1.2	1.2
24	Count	28	28	19	6	0
	% within response	34.6	34.6	23.5	7.4	0

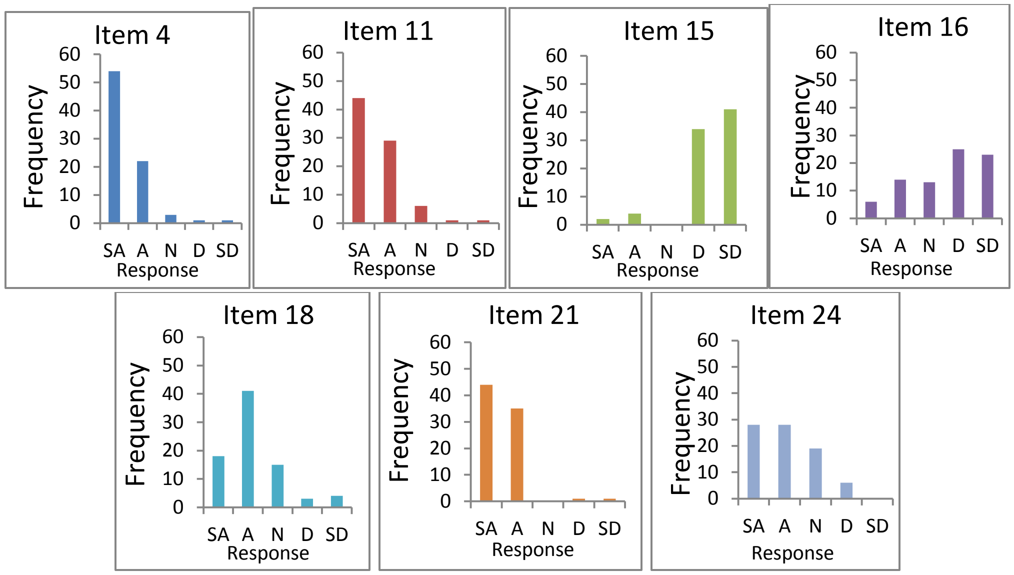


Figure 7. Distribution of responses within each item in category.

Frequency of calculator use and teachers' responses. In this category, only items 15 and 18 had significant P values suggesting a possibility of association between the responses and the teachers' frequency of calculator use. I ran post hoc test for these items. Table 21 shows the results of the Chi-Square tests for all the items in this category.

Table 21

Chi Square Values

Item No	Chi-Square	df	P
4	6.76	8	0.562
11	4.97	8	0.760
15	15.82	6	0.015*
16	10.29	8	0.246
18	21.64	8	0.006*
21	8.24	6	0.221
24	4.03	6	0.672

*P < 0.05

Table 22 and Figure 8 show, respectively, the results for the cross tabulation of item 15 by frequency of use groups and histograms for the distribution of responses to this item within each frequency of calculator use group

Table 22

Frequency of Use by Response to Item 15

Frequency of Calculator Use		SA	A	N	D	SD
High	Count	0	1	0	17	18
	% within response	0	25.0	0	50.0	43.9
Moderate	Count	0	0	0	8	17
	% within response	0	0	0	32.0	68.0
Low	Count	2	3	0	9	6
	% within response	10.0	15.0	0	45.0	30.0
Total	Count	2	4	0	34	41
	% within response	100	100	100	100	100

Chi Square = 15.822, P = 0.015

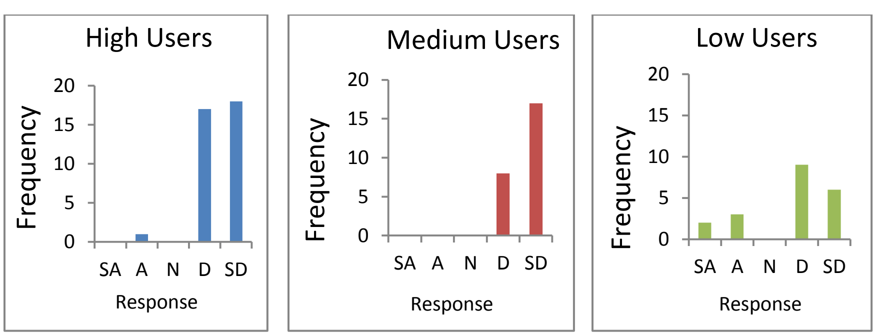


Figure 8. Distrbution of responses to item 15 withn each group.

On item 15, most of the teachers who responded to this item with “Agree” or “Strongly Agree” identified as low frequency users (standardized residuals 2.0 and 2.1, respectively). This was the case across all the three groups. This suggests that most of the teachers who agreed that graphing calculators have had almost no impact on how they taught identified as low users. The high users and moderate users disagreed with this notion almost entirely.

Table 23 and Figure 9 show, respectively, the results for the cross tabulation of item 2 by frequency of use groups and histograms for the distribution of responses to this item within each frequency of calculator use group

Table 23

Frequency of Use by Response to Item 18

Frequency of Calculator Use		SA	A	N	D	SD
High	Count	6	22	7	1	0
	% within response	33.3	53.7	46.7	33.3	0
Medium	Count	8	10	7	0	0
	% within response	44.4	24.4	46.7	0	0
Low	Count	4	9	1	2	4
	% within response	22.2	22.0	6.7	66.7	100
Total	Count	18	41	15	3	4
	% within response	100	100	100	100	100

Chi-Square = 21.639, P = 0.006

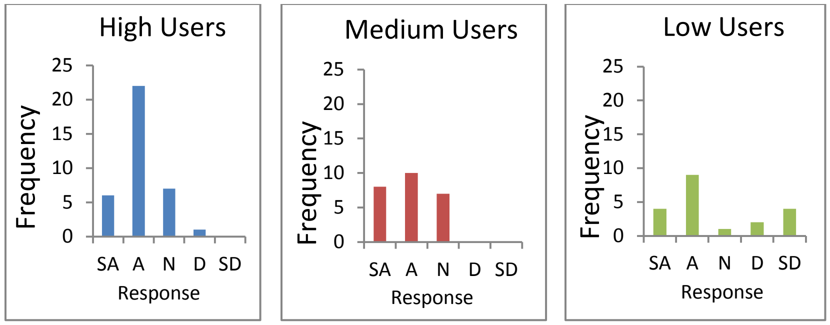


Figure 9. Distribution of responses to item 18 within each group.

Likewise, for item 18 the cell corresponding to the low frequency users responding with a choice of "Strongly Disagree" returned the largest positive standardized residual (3.0) meaning that this cell was over represented. Consequently, we can conclude that most of the teachers who strongly disagreed with the statement: *I try to take every opportunity to use the graphing calculator when I teach about linear and quadratic functions*, identified as low frequency users.

Summary

Data from this survey was aimed at helping me answer research question #1, namely, *What are secondary mathematics teachers' professed beliefs about using graphing calculators in the teaching and learning of linear and quadratic functions with respect to the following areas: (a) Influence on use and exploration of various representations of functions? and (b) Teacher direction versus student exploration?* In order to achieve this, I divided the survey items into three broad categories. The first category was meant to address the teachers' beliefs about the influence of graphing calculators on students' understanding of multiple representations. The second category to explore the teachers' beliefs about the influence of graphing calculators on how much teachers direct their classes versus how much they let students explore on their own. The third category explored the teachers' beliefs on issues related to calculator use but that are not necessarily related to multiple representations or teacher direction.

The results showed that the teachers who participated in this study had consensus (either agreeing/strongly agreeing or disagreeing/strongly disagreeing) on most of the items. Specifically, in the first category items, there was consensus that some problems in a first algebra course are best solved using tables or graphs rather than algebraic symbols (items 5 and 12). The teachers also agreed that representing a function with a graph helps students who have difficulty using algebraic symbols (item 8), and that when students use graphing calculators on a regular basis they become better at interpreting tables and graphs (items 14 and 20). Furthermore, there was consensus that graphing calculators help students to solve non-routine problems that would otherwise be inaccessible by algebraic techniques (item 7), enable students recognize connections between various function representations (item 10), and support students in learning about linear and quadratic functions by enhancing discussions around the various representations (item 13).

However, there was lack of consensus on whether students should always learn to solve problems using algebraic symbols first before they can use tables or graphs (items 9 and 17). Chi Square tests and post hoc tests showed this split in opinion among teachers on whether algebraic symbols should always precede graphs (item 17) cut across all the three frequency of use groups, but when it came to algebraic symbols always preceding tables (item 9) the high frequency users tended to disagree while the low frequency users tended to agree.

In the second category, there was consensus that the teachers encouraged their students to use graphing calculators for discovery and/or exploratory activities (item 1), that students should be free to explore with the graphing calculator (item 22), and that using graphing calculators provides opportunities for students to share ideas (item 6). Consensus was also recorded with respect to the fact that the teachers always give their students specific directions on how they should use the graphing calculator (item 3). But there was lack of consensus on whether students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand (item 2) and whether or not the teacher should always decide when it is appropriate for students to use the graphing calculator (items 19 and 23).

Finally, in the third category, there was consensus on all ideas explored in the items. Specifically, the teachers agreed that graphing calculators make the study of linear and quadratic functions more accessible to a wider range of students and enable them to engage with challenging problems (items 4 and 11), and that the calculators have had almost no impact on how and what the teachers teach (items 15 and 16). Additionally, there was consensus that the teachers were confident users of graphing calculators, had lots of ideas about how to make use of the graphing calculator in the classroom, and they try to take every opportunity to use the graphing calculator when I teaching about linear and quadratic functions (items 18, 21 and 24).

The analysis above indicates that on the most part teachers believe that graphing calculators are valuable for students in the study of linear and quadratic functions. The teachers also generally feel confident about their knowledge of graphing calculators and they believe that they make use of graphing calculators whenever opportunities for doing so are available. This analysis also revealed that teachers do not hold a single position about which particular sequence should be followed when exploring different function representations. The analysis has offered some insights into teachers' views about use of graphing calculators which have opened up possible paths for me to follow as I look at that interview data as well as the classroom observation data. Specifically, I seek to find out the type of representations the six case study teachers use and the pattern for shifting among the representations.

CHAPTER 5: TEACHER PRACTICES

In this chapter, I consider results from the analysis of the data from the interviews and classroom observations guided by my second and third inquiry questions:

2) How do secondary school mathematics teachers use graphing calculators when teaching linear and quadratic functions?
    a) What function representational choices do secondary mathematics teachers make when using graphing calculators?
    b) How specific are the teachers' directions to students about how the calculators may be used?
3) What is the relationship between the teachers' professed beliefs about graphing calculators and observed practice?
    a) What is the nature of similarities and/or differences between reported and observed calculator usage trends?
    b) To what extent do professed beliefs about graphing calculators explain observed usage?

I will first present and discuss the findings that are relevant in addressing my second research question then I will follow this up with the presentation and discussion of the results related to the third research question. I will finally give a summary of all the findings from the two research questions. Recall that my participants were divided into three groups depending on their reported frequency of calculator use, namely, High Frequency Users, Medium Frequency Users, and Low Frequency Users. The results reported in this chapter are for six volunteer teachers from the first two groups (High Frequency Users & Medium Frequency Users). I used six letters of the alphabet to represent the names of these teachers as shown in Table 1. The results are organized into three broad categories as namely, (a) teachers' representational choices when using graphing calculators, (b) teachers' directions on how graphing calculators may be used, and (c) relationship between professed beliefs about graphing calculators and observed use.

Teachers' Representational Choices When Using Graphing Calculators

The ability to identify a function using different representations and flexibility in translating between the representations is powerful in that it can allow students to develop deeper conceptual understanding (Even, 1998). We can therefore judge the meaningfulness of teaching and learning experiences based on how well they prepare learners to develop this ability. For calculators to promote meaningful learning of the concept of function they must be used in ways that promote understanding of relationships between multiple representations. With regard to what function representational choices teachers make when using graphing calculators, I found two major results, namely (1) there was an overwhelming preference for teachers to use graphs and equations with very little use of tables, (2) representational shifts in the classroom were dominated by equation to graph and graph to equation. I will discuss these findings in detail in the next sections.

Overwhelming preference for using graphs and equations with very little use of tables. Analysis of the classroom observation data showed that teachers chose equations and graphs much more frequently than tables. Most instructional tasks made specific reference to either an equation for which a graph would be drawn and various explorations done on it, or a graph on which various explorations would be done. Only a handful of tasks directly specified use of tables. In cases involving word problems, it was common to see equations being generated then graphs drawn with little or no use of the table. During classroom observations, I kept track of the tasks used by the teachers and the function representations that were specified in those tasks. The number of tasks for each teacher over the three lessons I observed ranged from 10 to 14. As discussed in Chapter Three, I classified the function representation initially specified in each task depending on the wording of the task or what was included in students' worksheets (see Table 24).

Graphical approach and algebraic approach were specified more often in the tasks than tabular approach (see Table 24). Overall, out of the 18 lessons I observed (three for each of the six teachers), I identified a total of 75 tasks. Algebraic approaches were specified on 27 tasks (36%), graphical approaches on 23 tasks (31%), and tabular approaches on 13 tasks (17%). These percentages show that even though the teachers stated that they give equal preference to all the function representations that was not the case in these particular observations.

Table 24

Distribution of Function Representations Specified in Instructional Tasks

Frequency of Calculator Use	Teacher	No. of tasks in observed lessons	Initial representation specified in task			
			Graphical	Algebraic	Tabular	Verbal/other
High	Ms. K	10	3 30%	4 40%	2 20%	1 10%
High	Mr. L	14	5 36%	4 29%	3 21%	2 14%
High	Ms. M	13	4 31%	5 38%	2 15%	2 15%
Moderate	Ms. R	12	3 25%	5 42%	2 17%	2 17%
Moderate	Ms. S	14	4 29%	5 36%	2 24%	3 21%
Moderate	Ms. T	12	4 33%	4 33%	2 17%	2 17%
	Total	75	23 31%	27 36%	13 17%	12 16%

From Table 24, we see that there were no major differences between the moderate frequency users and the high frequency users in terms of the representation they initially specified.

Representational shifts in classrooms. Before I discuss the choices made by teachers in their classrooms in terms representational shifts, I will first discuss how the teachers shifted between various representations while responding to tasks number 8 and

9 on the interview (see Appendix B). Task #8, begins with a graph; and task #9 begins with a table. On task #8, all the teachers generally suggested using the fact that the lines were not parallel to determine which pipe pumped the most water in a given time. Ms. R referred to “looking at whether the vertical gap between two graphs is shrinking or expanding as you move from left to right” [Task-Based Interview, Lines 50-52] in order to determine the relative rates of the pipes represented in the graphs. Ms. K and Ms. S used similar arguments as Ms. R only they compared the intercepts. They pointed out the fact that the graph for pump A had the greatest *y*-intercept yet intersected the graphs for pumps B and C meaning that it was falling at a faster rate. When I asked about the graph for pump D, they were both of the view that graph A would intersect it as well. On her part, Ms. T stated that the graph for pump A “appears to make the smallest angle with the vertical axis” [Task-Based Interview, Line 60] and so pump A must be the fastest.

Ms. M and Mr. L made reference to the graphing calculator; Ms. M noted that if the graphs were on a calculator she would have to use the TRACE function to identify two sets of points on each graph then calculate the slopes of the lines and compare them. Mr. L suggested that he could scroll down the tables of values of the graphs and compare the successive differences for each graphs. These solution approaches pointed towards shifts between function representations. While Ms. R’s approach did not make explicit use of tables, it can be inferred that by comparing the “vertical gap” between two graphs from left to right one might as well be comparing successive differences which is the approach taken by Mr. L. Similarly, by identifying points on the graphs and determining the slopes, we can argue that Ms. M was also using the tabular approach. Thus in this case Mr. L, Ms. M and Ms. R were all shifting from graphs to tables while attending to this task**.**

On item #9, which begins with a table and asks for a rule, Ms. M and Ms. K used the successive difference method to identify the function as quadratic then used points from the table to set up two linear equations from which they determined the values of the coefficients of the quadratic equation. Below is part of how Ms. K went about determining the equation:

> We know that the *y*-intercept is six so the equation will look something like this (she writes $y = ax^2 + bx + 6$). Now I can use any two sets of points to write to linear equations which I can then use to solve for *a* and *b*, I'll use this and this (pointing to (-4, 2) and (-2, 0)). The first equation is (writes $2 = 16a - 4b + 6$) and the second is (writes $0 = 4a - 2b + 6$) [Task-based Interview, Lines 44-49].

Ms. K then proceeded to solve the equations by elimination method and found the quadratic equation to be $y = x^2 + 5x + 6$. She pointed out that if she assigned such a problem to her students she would have them check their equations with the graphing calculators. For this reason she entered the data into the calculator as well as input the equation and displayed the graphs on the same set of axes. When I asked her whether she would let her students check the shape of the graph using a calculator before they determine the equation, she said she would not necessarily suggest it but she would be fine if students did that on their own.

Mr. L, Ms. R, Ms. S, and Ms. T used their graphing calculators to first enter the values in their calculators and use the STAT PLOT menu to display the graph from which they determined that it was a quadratic relationship. However, while Ms. R and Ms. T used the same method as Ms. M and Ms. K to determine the equation, Mr. L and Ms. S simply used the quadratic regression method from the CALC sub-menu of the STAT menu on their calculators to determine the coefficients of the quadratic equation. Here again was a case of teachers shifting between representations in different ways even though they were solving the same problem.

I took a cue from these patterns to analyze the data from classroom observations for similar patterns. To achieve this end, I identified the representations mentioned in the tasks used by the teachers and then I kept track of how the teachers shifted from the specified representation to other representations. I was most interested in finding out what the representation following the one that begun the task would be. Since most tasks started with graphical or algebraic representations, I will present the representational shift for only the tasks that started with these representations. In general, there was a bias toward following graphs with equations rather than tables and similarly, following equations with graphs rather than tables. I will discuss sequences in the sections below.

Equation to graph vs. equation to table sequence. Table 25 shows the patterns of representations that emerged when equations were the initially specified representations in the tasks. I consider two patterns, namely, equation to graph and equation to table. The first pattern indicates that when a task starts with an, a teacher shifts from the equation to a graph first. The equation to graph sequence noted here only represents the first change in representation. Depending on the task, sometimes there would be a shift back to equation or, in some cases, a shift to tables. In the second pattern the teacher would shift from the graphical to tabular. It is clear that the equation to graph dominates the other pattern (equation to table) by almost a three to one ratio.

Table 25

Patterns of representations that begun with Equation approach

Frequency of Calculator Use	Teacher	Tasks with Equation as Initial Representation	Emergent patterns of representations	
			Equations $\rightarrow$ Graphs	Equations $\rightarrow$ Tables
High	Ms. K	4	3 75%	1 25%
High	Mr. L	4	3 75%	1 25%
High	Ms. M	5	4 80%	1 20%
Moderate	Ms. R	5	3 60%	2 40%
Moderate	Ms. S	5	4 80%	1 20%
Moderate	Ms. T	4	3 75%	1 25%
	Total	27	20 74%	7 26%

While exploring the effects of adding a constant to the equation of quadratic function, Ms. R shifted from equation to table in one case and equation to graph in another. To start the lesson, she led her students through a discussion that led to the definition of the vertex of the graph of a quadratic equation in relation to its axis of symmetry and the horizontal line that runs through its minimum or maximum. To help students understand why functions of the form y = C, for any value C are called constant functions, she asked them to graph on their calculators various functions of this type and access their tables of values. A brief discussion led to the conclusion that the values in the output column of any given function would be the same and equal to the value C whenever the equation was of the form y = C. In this exploration, though graphs were displayed before the tables of values were accessed, the graphs were used only to show how they were similar to each other but were never analyzed or discussed beyond this, and so I chose to refer to this as ‘equation to table’ sequence. In the next step, Ms. R asked her students to graph the functions of the form $y = x^2 + C$ for various values of C. She then facilitated a discussion in which it was concluded that the vertex of the graph would always be (0, C) and that the effect to the graph of $y = x^2$ would be a “downward” or “upward” shift of C units. I referred to this as “equation to graph” sequence because the graphs were analyzed beyond the observation that they all had the same shape. Ms. R also showed the class how to find the vertex using the ‘CALC’ menu of the graphing calculator.

Even though the equation graph sequence dominated more than the equation to table, there were some good examples of these less dominant pattern as shown in the following example from Mr. L’s class. In this example the class was working on the task shown below:

> A baseball player throws a ball from the outfield toward home plate. The ball's height above the ground is modeled by the equation y = $-16x^2 + 48x + 6$, where y represents height, in feet, and x represents time, in

seconds. The ball is initially thrown from a height of 6 feet. What is the maximum height that the ball reaches? [Classroom Observation, Lesson 2].

Students worked on this task in their groups. During the wrap up session, Mr. L encouraged students to display both the graph and table on the same screen (see Figure 10) so that they could have a good picture of how the values in the table relate to the various points on the graph.

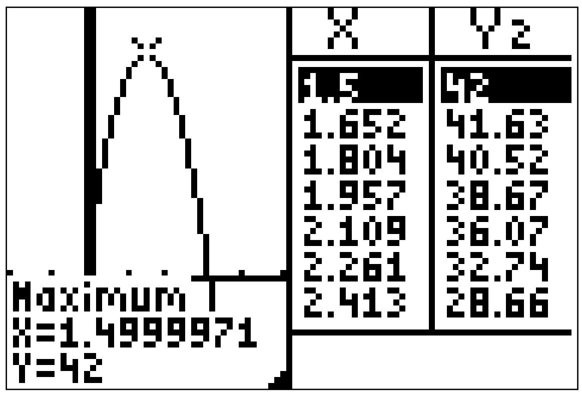


Figure 10. Split screen showing graph and table.

One thing that strikes me most is Mr. L's seeming desire to ensure that his students make the connection between points on the graph and values in the table. He tried to accomplish this by insisting on displaying the graph and table on the same screen as opposed to alternating between graph and table one at a time. This he explained "helps students get a complete visual picture of the function as they can see the cursor on the graph and the corresponding coordinate pair highlighted in the table" [Classroom Observation, Lesson 3]. I should also add that while Mr. L's class used the TI-84 graph calculators the other teachers used the TI-83 plus model. The added capabilities of the TI-

84 model may have contributed to Mr. L being more comfortable in doing certain things that other teachers did not do.

In a lesson on exploring the slope of a line, Ms. T had her students use the table of values (‘equation to table’ sequence) even though she had asked them to graph the functions first. She gave her students a worksheet with this problem [Classroom Observation, Lesson 1]:

Graph the equation $y = 2x$*, display its table of values and follow the instructions.*

a) Find two points on the line. Point #1 (__, __) and Point #2 (__, __)

b) How many spaces up are there between the two y values?

c) How many spaces over are there between the two x values?

d) Divide the y answer by the x answer, what do you get?

In this task, students had to select any two points using values from the tables then calculate the slope and compare the result to the equation. This was a shift from an earlier task in which they had been using the trace function. The opportunity was still there to use the graphs and the trace function but this was not the focus of this particular lesson.

Graph to equation vs. graph to table sequence. Table 26 shows the patterns of representational shifts that emerged when the graphs were the initially specified representations in the tasks. Again, I considered the two patterns, namely, graphs to equations and graphs to tables. As can be seen in Table 26, the graph to equation sequence was used the most, on 70% of all the teachers’ tasks in which graph were the initially specified representations, while the graph to table sequence was only used on 30% of such tasks.

Table 26

Patterns of representations that begun with graphical approach

Frequency of Calculator Use	Teacher	Tasks with Graph as Initial representation	Emergent patterns of representations	
			Graphs → Equations	Graphs → Tables
High	Ms. K	3	2 67%	1 33%
High	Mr. L	5	3 60%	2 40%
High	Ms. M	4	4 100%	0 0%
Moderate	Ms. R	3	2 67%	1 33%
Moderate	Ms. S	4	2 50%	2 50%
Moderate	Ms. T	4	3 75%	1 25%
	Total	23	16 70%	7 30%

One example of this sequence can be illustrated by this episode from Ms. R's class. In episode the class worked on a number of problems involving finding equations of graphs. Ms. R had prepared a worksheet on which she had various graphs with grid marks showing such that the coordinates of points on the graphs could be read easily (see Figure 11) [Classroom Observation, Lesson 2].

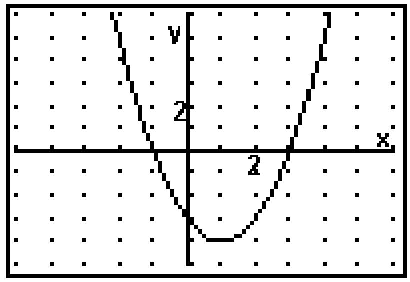


Figure 11. Screen showing a parabola.

Students worked on these graphs by first identifying coordinates (*h*, *k*) of the turning points (minimum or maximum point) then using these coordinates in the equation $y = (x - h)^2 + k$.

As discussed in the previous section Mr. L seemed to prefer combining graphs and tables and investigating them simultaneously. In one episode, his class was investigating graphs of motions previously collected using the Calculator Based Ranger (CBR), Mr. L had the class display both the graphs and their corresponding tables simultaneously. This way, students were able to navigate through the tables of values as well as along the graphs when they used the trace function (see Figure 12) [Classroom Observation, Lesson 3].

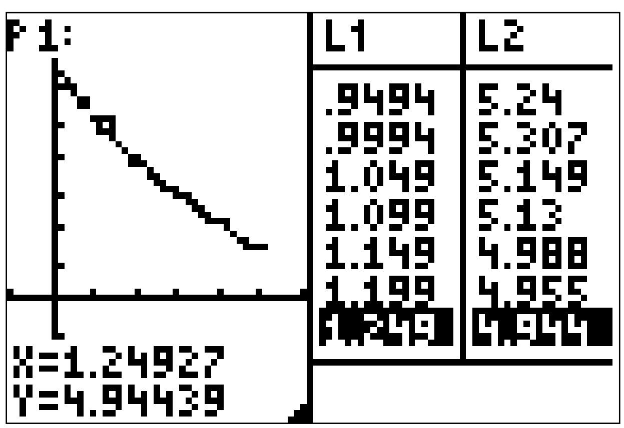


Figure 12. Split screen showing graph and table from CBR data.

Summary. As the foregoing discussion shows, the teachers preferred graphical approaches and algebraic approaches over tabular approaches. The teachers specified algebraic approaches the most (in 36% of all tasks), followed closely by graphical approaches (in 31% of all tasks), then tabular representation approach in 17% of the tasks, and another 16% of the tasks not being specific on any of the three representations. This is contrary to what they stated in the interviews about balancing between representations. It should be noted however, that specifying an initial representation in a task did not restrict the teachers to staying with that representation alone. The discussion above has shown that the teachers shifted from the specified representation to other representations.

Also clear from the discussion above is the fact that when graphical approach was specified in the tasks, the shift to algebraic approach (and vice versa) dominated the shift to tabular approach – about 70% to 30% overall. This goes further to show how much graphical and algebraic representations dominated over tabular representations. A near balance in representations appears to be achieved when the initial specified representation is tabular as the percentages of shifts to graphical or algebraic representations differ only slightly – 54% and 46% respectively. However, this apparent balancing between representations is overshadowed by the fact that there were very few tasks in which tabular representation approaches were initially specified. When the initial representation specified was verbal the teachers switched to algebraic representation more than four times as often as they switched to tabular representation.

Teachers' Directions on How Graphing Calculators May be Used

For part b of my second research question (How specific are the teachers' directions to students about how the graphing calculator s may be used?) I found three major results about classroom dynamics, namely

(1) All teachers in general varied from directing their classes to letting students explore. However, high frequency users involved more student exploration than teacher direction. Also, the nature of classroom dynamics tended to influence the role for which the graphing calculators were used (Doerr & Zangor 2000).

(2) When a lesson was teacher directed, it was characterized by the following teaching strategies: (a) teacher demonstrations to specific calculator functions, (b) teacher making decisions about particular calculator settings, and (c) teacher correcting student errors and confirming similar solutions. During such sessions (teacher directed) the graphing calculator was mainly used as a computational tool.

(3) When a lesson involved student exploration, it was characterized by the following teaching strategies: (a) teacher involving students in decision making regarding calculator use, (b) teacher guiding students in refining their thinking with regard to calculator use, and (c) teacher challenging students to interpret calculator results in the context of the problem situation and communicate this understanding to whole class. During such sessions the graphing calculator was mainly used as a visualization tool, a checking tool, or a data collection and analysis tool (this role was observed in only one teacher's lessons).

Classroom Dynamics and Role of Graphing Calculator

Overall, each teacher's lessons had some blend of teacher directedness and student exploration; however, for some teachers their lessons tended to be more teacher-directed than involving student exploration while the reverse is true for other teachers. Table 27 shows the distribution of the various modes of calculator use by teacher and the percentages of time for which each teacher's lessons were characterized by teacher direction or student exploration.

Table 27

Distribution of class time by teacher directedness and student exploration

Frequency of calculator use	Teacher	Total class time (mins) observed	% of total class time characterized by teacher direction	% of total class time characterized by student exploration	Other
High	Ms. K	128	24%	67%	9%
High	Mr. L	131	21%	65%	14%
High	Ms. M	133	40%	53%	7%
Moderate	Ms. R	130	31%	57%	12%
Moderate	Ms. S	135	48%	44%	8%
Moderate	Ms. T	132	56%	33%	11%

As shown in Table 27, the higher frequency users' classes were in general characterized more by student exploration than teacher directedness, however, the percentage of class time characterized by teacher direction was higher for Ms. M's classes (40%) compared to those of Ms. K (24%) and Mr. L (only 21%). On the other hand, among the moderate users, the results were mixed: Ms. R's classes, for example, were characterized more by student exploration while Ms. T's classes were characterized more by teacher directedness; furthermore, Ms. S's classes while marginally leaning towards teacher directedness, appear to be evenly split – 48% teacher directed to 44% student exploration.

The nature of classroom dynamics tended to influence the role for which the graphing calculators were used. I found four of the five roles of graphing calculator use discussed by Doerr and Zangor (2000), namely, (a) visualization tool, (b) checking tool, (c) data collection and analysis tool, and (d) computational tool. During the times when the lessons learned more toward student explorations the graphing calculator was mainly used as a visualization tool, a checking tool, or a data collection and analysis tool (this role was observed in only one teacher's lessons). During teacher directed sessions the graphing calculator was mainly used as a computational tool and sometimes as a visualization tool. I will discuss these roles of graphing calculator use below.

The role of the graphing calculator as a visualizing tool was the most common, found in 10 of the 18 lessons I observed. This was followed closely by the role of the graphing calculator as a computational tool, in nine of the lessons. The least observed role of graphing calculator use was data collection and analysis, in just one of the 18 lessons. I observed the role of graphing calculator use as a checking tool in five lessons.

Out of the 18 lessons that I observed, I found the role of the graphing calculator as a visualizing tool in at least one of each teacher's lessons, and found the role of the graphing calculator as a computational tool in at least one lesson for all teachers but Ms. K. I did not observe this role of graphing calculator use in any of Ms. K's classes. On the other hand the only time I found the role of the graphing calculator use as a data collection and analysis tool was in Mr. L’s class. I found the role of the graphing calculator use as a checking tool in Ms. K’s, Ms. R’s and Ms. S’s classes. Table 28 summarizes the distribution of these patterns and roles of graphing calculator use in each teacher’s classes. I will discuss these roles in detail in the following sections on teacher direction vs. student exploration.

Table 28

Distribution of patterns and modes of graphing calculator use

Classroom Dynamics	Role of graphing calculator use	Ms. K’s lessons			Mr. L’s lessons			Ms. M’s lessons			Ms. R’s lessons			Ms. S’s lessons			Ms. T’s lessons		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Teacher Directed	Computational tool				x		x	x		x			x	x			x	x	x
Student Exploration	Data collection & Analysis tool					x													
	Visualizing tool	x	x				x		x		x	x			x		x	x	x
	Checking tool	x		x									x		x	x			

Lessons characterized by teacher direction. As stated earlier lessons that were characterized by teacher direction involved (a) teacher demonstrations of specific calculator functions, (b) teacher making decisions about particular calculator settings, and (c) teacher correcting student errors and confirming similar solutions. I will discuss each of these strategies below, but first, Table 29 shows these teaching strategies with the actions taken by the teachers and examples that illustrate how the various actions were taken. In addition, the graphing calculator was mainly used as a computational tool.

Table 29

Teaching strategies employed by teachers in lessons characterized by teacher direction

Teaching Strategies	Teacher actions	Examples
a) Demonstrating the use of specific calculator functions	Providing a sequence of keystrokes while students follow along	Asking students to use self-check points to ensure they are following the right steps.
b) Making decisions about particular calculator settings	Providing specific window settings for the graphing calculator	Giving directions like: I want us to set the y-min at… and the y-max at…
c) Correcting student errors and confirming similar solutions	Providing corrections to student errors without soliciting suggestions from their peers	Using statements such as: your window setting is not correct…, can you put _ for y/x min

Teacher demonstrating specific calculator functions. Teachers tended to direct their classes through demonstrations of specific calculator keystrokes when they introduced particular calculator functions for the first time. For example, Ms. S was just introducing her students to using the graphing calculator when I observed her class the first time. In this lesson, she led her students in finding out how to use various function keys on the graphing calculator. Specifically, she demonstrated to the class how to enter equations into the y = 's editor and use the *trace* function to identify coordinates of points on the graphs. She also demonstrated how to access and read the table of values (Classroom Observation, Ms. S-Lesson I). Similarly, Ms T demonstrated to her class how to find the zeros of a quadratic function using the *trace* menu and then the *calc* menu (Classroom Observation, Ms. T-Lesson I). Ms. T's class was learning about finding roots of a quadratic equation by graphing. Before this they had looked at finding roots by factoring. The lesson started with a review of previously learned material. Ms. T asked the students to find the roots of several quadratic functions by factoring. She started by telling the class that the roots of an equation are also known as zeros because when the equation is graphed the roots correspond to the points where the graph crosses the *x*-axis and so the *y*-values are zeros at such points. Mr. L demonstrated to his class the keystroke sequence for using the linear regression function of the graphing calculator (Classroom Observation, Mr. L-Lesson I).

Teacher making decisions about particular calculator settings. There were times when teachers made decisions about particular calculator settings and had their students follow along. Most of these decisions involved window settings on the calculators. For example, in a lesson on solving systems of linear equations graphically, Ms. M explained to her students how the graphing calculator could be used to eliminate some of the hassles of graphing by hand. She told her students that just as they did with pencil and paper, they needed to solve each equation for "y =" and graph both equations. Then she went on to demonstrate how to solve the system $x – y = 14$ and $2x + 3y = 12$. After she had rewritten the equations as $y = x – 14$ and $y = -2/3x + 6$, Ms. M instructed her students to "enter these equations in the y equals menu and then go to window and put X min equals -5, X max equals 15, Y min equals -10, and Y max equals 10" (Classroom Observation, Lesson 3). She explained that even though they had used negative 10 for X min and positive 10 for X max in the window setting previously, these setting were not "very friendly" for the current problem.

Teacher correcting student errors and confirming similar solutions. Sometimes a teacher would move to correct a student error without having the student or the rest of the class try to figure out the cause of the error. One such example was in Ms R's class in which students were investigating graphs of quadratic equations using the graphing calculator. While investigating the graph of the equation $y = x^2 + 3x - 1$, two students asked Ms. R for help with their graph saying that what they saw in the view screen looked like a straight line. Ms. R went to them and after looking at their graph she quipped "I know what's wrong with your graph, it is the window. See you have *x* and *y* min at negative one so you're only seeing part of the graph. Remember when graphing by hand we have to make sure that we have enough room in all the four quadrants; it's the same with the calculator. The graph needs room to the left of the *y*-axis and below the *x*-axis, you need to change the values for *x* min and *y* min to bigger numbers, try negative ten" (Classroom Observation, Lesson 2). The students then did this and got a better view of the graph. Although Ms. R went to the overhead and talked about this particular incident to the whole class, showing them both the "half graph" and the "full graph" and explaining the differences, she did not check with the rest of the class to find out whether any students could have figured out the error on their own.

I observed some occasions where the teacher confirmed two different solutions as both being correct. In such occasions, the teacher gave the explanation for why both solutions were correct. One such instance was in Mr. L's class on an investigation involving a quadratic function. The problem read:

> *During practice, a softball pitcher throws a ball whose height can be modeled by the equation h = -16t^2 +24t +1, where h = height in feet and t = time in seconds. How long does it take for the ball to reach a height of 6 feet?* [Classroom Observation, Lesson 2].

There seemed to be confusion between two students at one group and when Mr. L inquired about what was happening one student stated that they had two different answers. Mr. L asked for the answers and the students gave them as $x = 0.25$ and 1.25. Mr. L took a quick survey (by show of hands) to find out how many students had found each of the two values for *x* before he explained that both answers were correct and added that "since the ball goes up and then comes back down, it would reach this height twice; first on its way up then on its way down." On this occasion Mr. L did not have the students try to reason among themselves to figure out why the two values were correct before he intervened.

Graphing calculator as a computational tool. I coded the role of the graphing calculator as a computational tool whenever it was used to evaluate numerical expressions and determine numerical solutions. As stated earlier, I observed this role of graphing calculator use in at least one lesson for all the teachers with the exception of Ms. K. In particular, I observed this role of graphing calculator use in all of Ms. T's lessons.

Ms. S directed her class in using the graphing calculator when she was introducing them to using graphing calculators for the first time. For example, she led them through the steps involved in entering an equation in the Y menu and demonstrated how to find the coordinates of a point using the TRACE function. She asked students graph several graphs and record the coordinates of two points on each of the graphs. She walked around and helped students with calculator commands. Similarly, Ms. T directed her class during a lesson on finding the roots of a quadratic equation. Before this lesson the class had covered finding roots by factorization. The lesson begun with Ms. T handing out a worksheet which involved using whole numbers to fill out the four regions formed by an X such that the numbers on the sides multiply to give the number at the top and add up to give the number at the bottom. She then demonstrated to the class, as a reminder, how to fill in two of the X's and asked the class to complete the rest. After the class had completed this exercise and Ms. T had confirmed students' solutions, she told the class that they would apply this to factoring quadratic functions. The lesson proceeded with the students finding the roots of several quadratic functions by factoring using this method. Ms. T then told the class that the roots of an equation are also known as zeros because when the equation is graphed the roots correspond to the points where the graph crosses the *x*-axis and so the y-values are zeros at such points. She had the class find roots of several quadratic equations by factorization and told them that those values they found were the *x* coordinates of the roots and that the y coordinates are zero. Then she led the class into finding the roots of the equation $x^2 - 5x - 24 = 0$ [Classroom Observation, Lesson 3] using the graphing calculator. She demonstrated how to use the zero function of the CALC menu to find the roots.

On his part, Mr. L directed his class in a lesson involving finding the equation of a line given two or more points on the line. The lesson started with a review in which the students solved several problems involving equations of lines. Mr. L then led the class in finding the equations of three lines on the board. They did this by calculating the slopes and then finding the *y* intercepts to express the equations in the form $y = mx + b$. Mr. L then explained that the same equation could be obtained with the graphing calculators without having to calculate the slope first. He used one of the examples they had just done (finding the equation of the line that passes through points (2, 8) and (6, 18)). He demonstrated to the class how to use the linear regression function of the graphing calculator to determine the slope and *y*-intercept. He used the overhead projector for the demonstration and asked students to perform these commands on their calculators after him. He then asked the students to use this method to check the equations of the lines they had found earlier and walked around the class helping students with calculator steps.

As the discussions above show in lessons characterized by teacher direction the teachers were more involving in shaping both the content and focus of discussions in the classrooms. The teachers played bigger roles in decision making while students on the most part followed through these directions. On the other hand in lessons characterized by student exploration the teachers took on a facilitator's role more. I will discuss these types of lessons in the next section.

Lessons characterized by student exploration. When a lesson involved student exploration, it was characterized by (a) teacher involving students in decision making regarding calculator use, (b) teacher guiding students in refining their thinking with regard to calculator use, and (c) teacher challenging students to interpret calculator results in the context of the problem situation and communicate this understanding to whole class. Table 30 shows these teaching strategies with the actions taken by the teachers and examples that illustrate how the various actions were taken. In addition, the graphing calculator was used as a visualizing and checking tool.

Table 30

Teaching strategies employed by teachers in lessons characterized by student exploration

Teaching Strategies	Teacher actions	Examples
Involving students in decision making regarding calculator use	Inviting suggestions from students regarding how calculators may be used in solving problems	Asking questions such as: How can we use the calculator to determine the y-intercept…?
	Letting students work on tasks involving calculators in pairs/small groups without directing them on what steps to take	Giving directions such as: With your partner graph the system of equations on your calculators and determine the point of intersection
Guiding students in refining their thinking with regard to calculator use	Asking questions that help students locate errors in their thinking	Asking questions such as:
		How do you think the 'y-min' and 'y-max' affect the graph?
	Encouraging students to comment on/critique their peers' responses	
Challenging students to interpret calculator results in the context of the problem situation	Encouraging students to check calculator results using other methods	Asking questions such as: How does this solution compare to the one you get using the substitution method?
	Requiring students to state what calculator results say about the problem	What is the real world meaning of the point of intersection?
Encouraging students to communicate their understanding of calculator results	Asking students to restate and/or rephrase their peers' responses	Asking questions such as:
		Who can explain that in a different way?
	Writing conclusions /conjectures in students' vocabulary	Are the two explanations equivalent?
		Did I hear you say …?
	Re-voicing students' responses	If I say …, would I be stating what you meant?

Involving students in decision making regarding calculator use. The students in the classrooms were mainly organized in small groups or in pairs; Mr. L, and Ms. M, preferred having their students work in small groups then share their findings during whole class discussions while Ms. K, Ms. R, Ms. S, and Ms. T preferred having their students work in pairs and occasionally as individuals then share their findings with the rest of the class. All the teachers made efforts in one way or another to have their students provide steps towards solving problems. This would range from asking students to suggest what to do in order to get started with the calculator with respect to given information or asking students to suggest how to modify various calculator settings in order to achieve desired results. For example, a teacher would ask students to suggest window settings and if the graph was not useful, she would ask them to suggest modifications. For example, in one lesson Ms. K's class was exploring transformations of the quadratic functions and they wanted to graph the function $y = x^2 + 9$ [Classroom Observation, Lesson 2]. One student suggested that they use the settings [-10, 10] for both X and Y scales explaining that he chose these settings because the class had used the same settings for some other graph. However, this time the settings did not work since only a tiny piece of the graph could be seen on the screen at the top of the y-axis. Ms. K asked the class to suggest what might be done in order to fix the problem. She asked them to discuss in their pairs and come up with better settings and why those settings worked. By doing this, Ms. K encouraged her students to not only help solve the problem with the window settings for this graph, but she also helped them prepare to deal with similar situations in the future. Teachers demanding that students explain why their new settings worked ensured that students would not simply use random guess and check but that they would try to relate their guesses to the problem situation thereby making more informed guesses.

Typical among these teachers was that often times they would ask students to share their work on the calculator with the whole class using the overhead projection unit. This kind of sharing involved a representative from a given group or pair demonstrating on the overhead unit with his/her group mates chipping in their comments as need arose. All the group members would be responsible for answering questions from their peers in other groups. Although the teachers occasionally interjected, they exercised some level of restraint, stepping in only when they needed to clarify something. For example, in Ms. M's class, classmates challenged one group who volunteered to share their solution to the following problem on the overhead unit for setting up their equations in the opposite way from the rest of the class.

> Edna leaves a trailhead at dawn to hike 12 miles toward a lake, where her friend Maria is camping. At the same time, Maria starts her hike toward the trailhead. Edna is walking uphill so she averages only 1.5 mi/hr, while Maria averages 2.5 mi/hr walking downhill. When and where will they meet?[Classroom Observation, Lesson 3].

While most of the other groups had their system of equations for this problem set up as $y = 1.5x$ for Edna's hike and $y = 12 - 2.5x$ for Maria's hike, this particular group who had volunteered had their equations the other way around. When their colleagues challenged them that this was not the correct system they almost gave in but Ms. M told them to continue with their solution to the end. Some students were surprised that this still led to the same solution so Ms. M pointed out that the only difference in the two systems was the point of reference.

Guiding students in refining their thinking with regard to calculator use. Most of teachers used handouts with instructions about tasks to be done in class. However, the teachers did not just plainly accept the answers given by students to these questions. They usually asked probing as well as clarification questions. On many occasions the teachers would ask questions requiring students to compare solutions obtained using different representations and explain the differences if any (e.g., when using the 'TRACE' function and the table of values). Questions like, why is it that when we used the table we got exact values but when we used the trace function we did not? Do you think this is always the case? How can we tell this…?, helped students think beyond tasks at hand.

In most cases the teachers would ask the class whether they understood what their peers (those called upon to respond to questions) had just said, and then if the whole class answered in affirmative, the teachers would pick other students to rephrase the response. If some students said they did not understand what their classmate had said then the teacher would ask either the speaker or one among those who had understood to restate the response. During times when a student would be demonstrating something on the overhead unit and the teachers wanted certain points to be clarified, they would ask the speaker to hold on briefly then pose questions to which either the speaker or any other member of class would respond.

Challenging students to interpret calculator results. The teachers often reminded their students to check their graphical solutions by substituting the values so obtained back into original equations and vice versa. Problems like the hiking example mentioned above had a part that specifically asked for this (*check your solution and explain its real world meaning*). For example, in the problem about the baseball used in Mr. L's class (in the section *Teacher correcting student errors and confirming similar solutions*), after the class had established that the maximum point on the graph had coordinates (1.5, 42) [Classroom Observation, Lesson 2], Mr. L pushed them to explain what this meant in the context of the problem they were solving. Some students responded by saying that y was greatest when x was 1.5 but Mr. L pushed them further asking them to state in a sentence what this meant about the baseball. Finally, with Mr. L providing only some correction on the grammar, one student stated that it meant "the baseball would be at its maximum height above the ground (42 feet) after one and a half seconds from the time it was thrown." Many times students have the misconception that the calculator is never wrong, forgetting that the accuracy of this tool depends on the accuracy of the information it is fed with. Requiring them to check their results is one way of ensuring that they cross-check their entries in case they made a wrong entry. This can also help the students learn to interpret results obtained from the calculator.

I also found that the teachers made conscious efforts to help students learn to communicate. The teachers would do this by asking students to rephrase their colleagues' responses or the teachers themselves would repeat students' responses making only grammatical corrections. For example, a conjecture generated by students about constant functions in Ms. K's class was that "if the equation of a function is given as y = some number, then the y column in the table will always be that number." Another conjecture about translations of the quadratic function was that "adding or subtracting a number from the parent function raises or lowers the graph by that number" (Classroom

Observation, Lesson 3). Both these conjectures were developed and refined by the students with help of the teacher.

Graphing calculator as a visualizing tool. I observed each of the teachers use the graphing calculator as a visualizing tool in at least one of their lessons. Ms. T used the graphing calculator in this manner in one of her classes when she had students investigate the relationship between the factors of quadratic equations and the *x*-intercepts of the resulting graphs. Ms. T's class was learning about finding roots of a quadratic equation by graphing. Before this they had looked at finding roots by factoring. The lesson started with a review of previously learned material. Ms. T asked the students to find the roots of several quadratic functions, and then she asked them to graph these functions on their calculators one by one. After the students graphed each function, Ms. T asked them to use the TRACE function of their calculators to locate the *x*-intercepts then compare these values with roots they had found by factorization. After going through all the functions, students came to a conclusion that the roots were the same as the *x*-intercepts in each case. Ms. T then introduced the term zeros explaining that when the equation is graphed the roots correspond to the points where the graph crosses the *x*-axis and so the *y*-values are zeros at such points.

In a lesson investigating the transformations of quadratic functions, Ms. K's class used the graphing calculator to examine what happens to the graph when various parameters are changed in the equation. One of the investigations involved the effect of adding a constant to the input variable. The equation being investigated was $y = (x-3)^2 + 1$ [Classroom Observation, Lesson 3]. Ms. K wrote the equation on the board and asked the class to display and take a look at what the graph looked like on their graphing calculators. After students had displayed the graphs on their calculators, Ms. K displayed the graph on the projector (the display showed only part of the parabola prompting her to say that she was going to use ZOOM Standard, after which she displayed the graph as shown on the left in Figure 13.

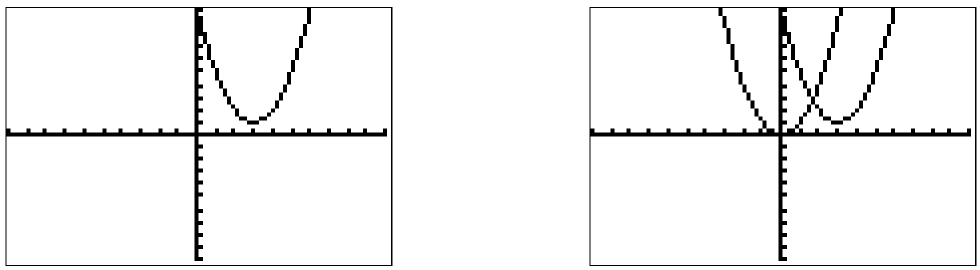


Figure 13. Graphs of the equations $\mathrm{y} = (\mathrm{x} - 3)^2 + 1$ and $\mathrm{y} = \mathrm{x}^2$.

She then immediately said that she was going to include the graph of $y = x^2$ which she did and displayed the graphs shown on the right in Figure 5.4. One student said that his calculator did not display the other graph, to this Ms. K responded that the other graph could not be displayed because the student had not entered the second equation ($y = x^2$). When the student inquired whether he had to enter this other equation Ms. K said no he did not have to, explaining that she used it only the show the original position of the graph.

After a short discussion it was agreed that the function being investigated shifts the graph right three units and up one unit. However, not all students seemed convinced at first as exemplified by one who stated that she thought that for a transformation like the one they had just investigated, the *x*-values do not change only the *y*–values do because *x* is always the same. In response to this, Ms. K went to the board and wrote the equation $y = (x - 3)^2$ and asked the class to think about what *x*-value would yield a *y*-

value of zero. She then used sketches on the board to explain that if we consider the original point as (0, 0) the corresponding point (new point) is (3, 0), implying a shift to the right of 3 units. She further explained that the horizontal shifts are sort of counter-intuitive in that a negative sign in the function means a shift to the right and a positive sign means a shift to the left. This is also an example of the graphing calculator being used as a partner.

Graphing calculator as a checking tool. I coded the role of the graphing calculator as a checking tool whenever it was used verify arguments and test conjectures. For example, in Ms. K's class a discussion about whether adding a constant to the equation $y = x^2$ would only shift the position of the graph up or down, maintaining its shape, led to the exploration and interpretation of different segments of the graphs and corresponding tables of values. One student questioned this argument when she saw that a graph translated upward did not exactly resemble the original graph. The student and her partner had displayed the following graphs for $y = x^2 + 3$ and $y = x^2 + 7$ (they were using a [-10, 10] x [-10, 10]) [Classroom Observation, Lesson 2]. Their argument was that the two graphs would intersect if extended, which should not happen if one graph was obtained by sliding the other one up.

This happened during the second lesson that I observed for Ms. K, the main objective for the lesson was to investigate the effect of adding a constant *k* to the equation of a quadratic function. Before this lesson, the class had previously been introduced to the basic form of the quadratic function ($y = x^2$) so the students knew the definition and shape of its graph (the U-shape) and that such a graph has a minimum point. They also knew that when a negative sign precedes the term x^2, the graph has the shape of an inverted U and consequently has a maximum point. Ms. K told me during the pre-observation interview that she would try to guide students to connect what they were doing in the lesson to something they had learned about linear functions.

Ms. K: I'm hoping that they'll see the similarity between this and what we did with lines when we were covering the linear unit.

Levi: And what was that?

Ms. K: We talked about parallel lines and how they're like images of each other, we discussed the fact that since parallel lines have the same slope you could just slide one line and fit it onto the other. You simply have to slide through the number of units equal to the difference in their *y*-intercepts.

Levi: OK

Ms. K: So I just hope that they'll remember and hopefully they'll get this faster.

Levi: Did you use graphing calculators on that as well?

Ms. K: Oh yeah, yeah, we did. In fact we did lots of graphs and we compared *y*-values for various *x*-values on the graphs, and you know we would talk about how the differences in the *y*-values were the same as the differences in the *y*-intercepts. [Pre-Observation Interview 2, Lines 15 – 30].

In this brief conversation with Ms. K, I got the idea that she would utilize the graphical representation of quadratic functions while using the graphing calculator as a visualizing tool.

Ms. K started the lesson by leading her students through a discussion that led to the definition of the vertex of the graph of a quadratic function. After writing the title

“Quadratic Functions” on the board she wrote the equation $y = x^2$ and then engaged her students in the following discussion:

Ms. K: What do we know about this function? Yes Jon.

Student: It is called a quadratic equation.

Ms. K: Right. What else do we know about it?

Student: The graph has a U-shape.

Ms. K: Great! Now let’s think about the U-shape for a moment (sketches a parabola on the board). Now listen carefully to what I’m going to say as I will be giving you very important information about this graph. This one has a minimum point right here (points to the lowest point on her sketch) and remember if it is the inverted one it will have a maximum point. Anyway, so this graph has a minimum and the *y*-axis runs through that minimum. Now this part of the graph (highlighting the right hand side of her sketched graph) is like a mirror image of the other part (pointing to the left hand side). It’s just like if you look in a mirror you see yourself the other side, okay. So the *y*-axis is like a mirror and we have a special name for it, we call it the axis of symmetry for this graph (writes and underlines the phrase “Axis of Symmetry”). Now, the point where the axis of symmetry meets the minimum point of the graph of a quadratic function has a special name, we call it the vertex (writes and underlines the word “Vertex”). For this graph the vertex is at zero-zero (writes (0, 0)), the *x*-value is zero and the *y*-value is zero as well. Now on your worksheets I want you to look at number one. Alex (calling out one of the students) read for us number one.

Alex: For each of the following equations graph the function on your graphing calculator and find the *x*-value and *y*-value of the vertex.

Problem 1 on the worksheet contained the equations

(a) $y = x^2 + 3$ (b) $y = x^2 + 7$

(c) $y = x^2 - 4$ (d) $y = x^2 - 6$

Ms. K: Thank you Alex. Do we know what we need to do? Is everybody clear about what we have to do?

Class: Yes Miss (in unison)

Ms. K: Okay talk about it with your partner but keep your voices low. [Classroom Observation, Lesson 2, Lines 5-36].

Mrs. K then walked around the classroom as students graphed their functions and recorded the coordinates of the vertices. Students worked on these problems in pairs. After about ten minutes, Ms. K called the class to attention and asked for volunteers to share their results with the rest of the class. She then called on different students to provide the coordinates of the vertices of the various equations. Ms. K then asked the class to think about the relationship in the equations and the coordinates of the vertices. After a brief discussion it was agreed that the coordinates of the vertex of the graph of the equation $y = x^2 + k$ would always be of the form (0, *k*).

Ms. K then started explaining the effect of adding a constant *k* to the quadratic equation $y = x^2$ by comparing to the effect of adding a constant to the linear equation *y* = *x*. She asked the students to recall how adding a constant to the linear equation affected the graph and explained that the same happens when a constant is added to the quadratic equation.

Ms. K: .. so the graph will move up or down depending on whether you add or subtract a constant. For example if we take equations (a) and (b) [referring to the equations in problem 1 of the worksheet] and graph them on the same axes, the graph of equation (b) can be obtained by sliding the graph of (a) up four steps [Classroom Observation, Lesson 2, Lines 41-46].

One pair of students seemed to be having trouble with this comparison though. One of the students in the pair called Ms. K's attention and said that she didn't think the graphs were the same:

Below is an excerpt of the exchange between Ms. K and the students:

Student: Miss, the one on top [graph of equation (b)] looks flatter I don't think the bottom one [graph of equation (a)] can fit on it, but with lines we said one line can fit on the other [referring to sliding parallel lines].

Ms. K: (Looking at the student's calculator) Okay Megan, you say these graphs are not the same?

Megan: Yes miss.

Ms. K: What makes you say they're not the same?

Megan: The lower one is deeper and the upper one looks flatter. I don't think they match.

Ms. K: Hmm. Let me see your window setting. Ok, here we go, I think I know what the problem is! Let me use my TI presenter so that everybody else can see the graphs.

Ms. K then went back to her podium and entered the equations in her calculator, set the WINDOW settings [-10, 10] by [-10, 10] then displayed the graph shown in Figure 14.

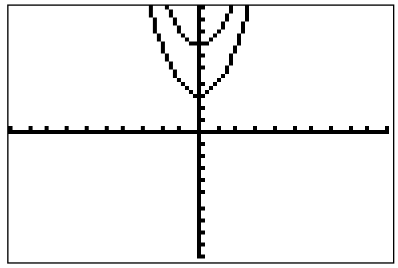


Figure 14. Graphs displayed on standard viewing window.

Ms. K: Okay class Megan says that the lower graph cannot fit onto the one because it is deeper than the upper one. What can we say to her?

There was a brief silence prompting Ms. K to urge the class on.

Ms. K: Think, think. Look at the graphs. What do you see? What do you think?

After a little more silence, one student volunteered an explanation.

Student: I think it's because the upper one only shows a small bit.

Ms. K: Good, good. The upper one shows only a small bit, why is that so?

Student: Cause there's no more space up there for it to extend.

Ms. K: Wonderful! There isn't much space up there. So what should we do? Anybody else wanna try? …Peter.

Peter: Um, I don't know.

Ms. K: Yes Katie.

Katie: I think we can extend the vertical, um, the *y*-axis.

Ms. K: Great suggestion. Let's try to extend the *y*-axis, which means increase the Ymax in the WINDOW setting. So let's try 20.

Ms. K then changed the WINDOW setting accordingly and displayed the graphs shown in Figure 15.

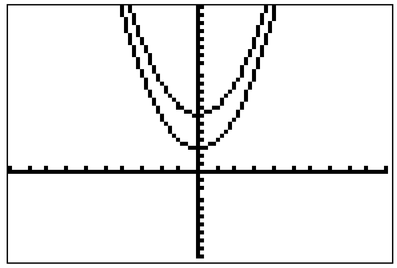


Figure 15. Graphs displayed after y-maximum was changed to 20.

Ms. K: Okay Megan, does this look convincing enough?

Megan: Yes, that's better.

But even as Megan was getting convinced about the likeness of the two graphs, another student was developing fresh doubts.

Student: But Miss, why do the tips look like they're gonna meet. If we get the upper one by sliding the lower one aren't they supposed to remain apart all through? I think these ones will meet if we extend up a little bit more.

Ms. K: Now James says if we keep extending the *y*-axis up the two graphs will meet. Anybody else thinks so? Yes, yes, okay we got a few more so let's try that out and see what happens. How about we try 30 (for Ymax)?

[She changes the settings again and displays the graphs shown in Figure 16]

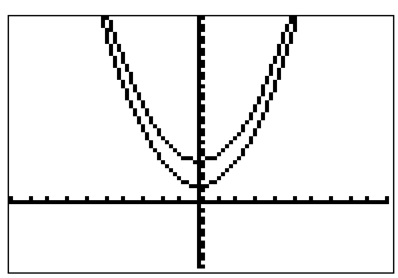


Figure 16. Graphs displayed after y-maximum was changed to 30.

James: See that's what I'm talking about, those meet over there! [Classroom Observation, Lesson 2, Lines 50-91]

At this point Mrs. K decided to display only one half of the graph to show that the two graphs would not meet as predicted by James.

Ms. K: Let's try something different. Let's change the Xmin to zero [Classroom Observation, Lesson 2, Line 93].

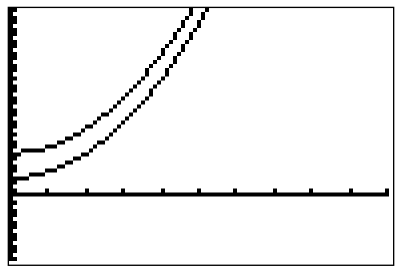


Figure 17. Graphs displayed after x-minimum was changed to 0.

This new screen displaying the graphs shown in Figure 5.8 seemed to convince most of the students. There was then a short discussed before Ms. K wrapped up the lesson.

This discussion in Ms. K's class is one of the examples of how the teachers went about using calculators as checking tools in their classrooms. It is also an example of the graphing calculator being used as a partner as it was used to explore new approaches to an existing task by viewing the graphs in various windows. The graphing calculator was also being used to mediate mathematical discussion in the classroom between students and their teacher. In most cases as illustrated in the discussion above, students would be working on some task and issues relating to graphical displays would arise. It was common for the teachers to use overhead projectors and walk students through a series of displays with input from the students.

Summary

The foregoing discussion dealt with the nature of classroom dynamics during the observed lessons. Two main features characterized portions of lessons, teacher directedness and student exploration. Teacher directed lessons are those in which the teacher took on a more leading role ranging from demonstrating specific calculator functions to making decisions about particular calculator settings, and correcting student errors and confirming similar solutions. On the other hand, lessons characterized by student exploration are those in which the teacher took on a facilitator and fellow investigator role. Teacher actions in such lessons included involving students in decision making regarding calculator use, guiding students in refining their thinking with regard to calculator use, and challenging students to interpret calculator results in the context of the problem situation and communicate this understanding to whole class. I must point out though that each teacher's lessons had a blend of these features, even though one feature might have been predominant in a particular teacher's lessons.

Relationship between Professed Beliefs about Graphing Calculators and Observed Use

For my third research question, I sought to find out the relationship between the teachers' professed beliefs about graphing calculators and observed practice. In particular, I wanted to know (a) the nature of similarities and/or differences between reported and observed calculator usage, and (b) the extent to which professed beliefs about graphing calculators explained observed usage.

Consistency between Professed Beliefs about Graphing Calculators and Observed Usage

I open this section by stating that my interpretation of what the teachers stated to be their position on how they used multiple representations may have been contrary to what they actually meant. For the teachers stated that they believed in balancing among the various representations even though, as I have shown in previous sections, I found their practice to point in a different direction. However, I noticed that whenever the teachers talked about representations in general, they always mentioned graphs and equations but rarely did they mention tables. When asked to comment on the fact that some teachers tend to emphasize certain representations more than others, all the teachers maintained that they balanced the use of various function representations in their own classrooms but stated that they could not speculate whether this was the case with all other teachers. This was the position held by both the high users and the moderate users. All the high users while agreeing that there was a possibility that some teachers might emphasize one representation over others, maintained that this was not the case with them. Given this line of thinking, I am inclined to conclude that when the teachers talked about using various representations they may have been just referring to graphs and equations. The following excerpts from interviews illustrate this point.

Levi: It is alleged that some teachers tend to emphasize some representations more than others. What is your view on this?

Ms. M: It could be true for some teachers even though for me personally, I’d say I balance between various representations, I don’t think one is more important than the other. I use graphs a lot but I also use equations as well [Task-Based Interview, Lines 28-30].

One thing that strikes me here is that Ms. M perhaps unaware of it herself, only mentioned two representations - graphs and equations. Ms. K also mentioned only equations and graphs.

Ms. K: I can't disagree with the statement because I know there's that possibility but I also know that for me that's not the case. I think I emphasize all representations just the same. Sometimes it might be the case that depending on the problem you're working on, you might use say equations a lot more than graphs but I try as much as possible to balance them [Task-Based Interview, Lines 33-37].

Here Ms. K added another dimension - the fact the representation chosen could be task dependent, a fact that was echoed by Mr. L, who stated that

> My view is that all representations are important and I try to show this in my lessons as much as possible. However, as you may know, it is difficult at times to use all representations equally because it also depends on the tasks. There are tasks for which you may find that graphs are most suitable and others where equations are better, and so it actually depends on the tasks [Task-Based Interview, Lines 32-36].

On her part, Ms. T, a moderate user reported that she did not have any particular preferences but usually “goes with what seems appropriate for the task at hand” [Task-Based Interview, Line 28]. Asked to elaborate on what “appropriate for the task” meant, she explained that she considered what would make the concept clearer and students would be comfortable handling. Ms. S responded to this question by echoing Ms. K and Mr. L, stating that the task would definitely determine which representation is best.

A close look at how they explained their position reveals a possibility of preference towards certain representations. What was striking in their responses was that when they mentioned a representation, it was either a graph or an equation, creating the impression that these were the only representations the teachers had in mind. The teachers however spoke positively about using tables while talking about advantages of graphing calculators. Ms. K talked about how she likes to "have students just go to the table and scroll through the values … and in particular it makes it very easy to compare more than one function." Mr. L said he liked the fact that he could have his students instantly switch from a graph to a table and back and thus cover more content in a short time.

Ms. M referred to students' prior knowledge and current level of understanding and whether they have been exposed to similar equations and graphs.

> I have to think about the kind of equations and graphs we shall be dealing with and also think about the level of understanding the students currently have and what prior knowledge they come in with. You know, have they seen similar equations or graphs before or what is new about the current equations and graphs? I just think along those lines. [Task-Based Interview, Lines 5 – 9].

Again Ms. M talks about equations and graphs but does not mention tables. This is consistent with how she spoke about these representations in her earlier responses. Similarly, Mr. L summarized his thinking by stating that he gives consideration to graphing and solving equations and tries to prepare in such a way that his students can have the opportunities to use both representations. It seemed like almost every time the teachers referred to a representation it was either a graph or an equation.

As mentioned earlier, the teachers believed that they did not emphasize one representation more than others in their instruction but some of the teachers also pointed out that which representation they chose depended on the task at hand. In my observations, it emerged that the teachers preferred using equations and graphs more than tables. Several teachers stated that the representations used depended on the tasks being explored; thus, they were well aware that the tasks they chose could determine which representations are used. But most of the time the teachers selected tasks that did not encourage the use of tables. When I analyzed the type of representations specified by the teachers in their tasks I found that overall, there were no major differences between the moderate users and the high users regarding the representation they started with and in general these tended to be equations or graphs. Out of a total of 75 tasks (across all the lessons I observed), equations were the initially specified representations on 27 tasks (36%), graphical approaches on 23 tasks (31%), and tabular approaches on 13 tasks (17%) (see Table 5.2). On individual basis Ms. K, Ms. M, Ms. R, and Ms. S had more tasks that started with the algebraic representation than those that started with the graphical representation while Mr. L had more tasks that started with the graphical representation than those that started with the algebraic representations. Ms. T had the same number of tasks starting with the graphical representation as those starting with the algebraic representation but they were in each case more than those that started with the tabular representation.

Frequency of Calculator Use and Classroom Dynamics

In terms of lessons involving student exploration or teacher direction, there was consistency between what I observed in the classrooms and what the teachers stated in the interviews. While talking about how they plan their lessons, some of the teachers talked about students being able to grasp the idea of function as well as represent a function in different forms, but also implicit in their statements was the degree to which they would let their students explore. For example, for Ms. T the focus is usually on time management, she stated that if she did not give enough thought of what and how she would do with the calculator it could end up being a "chaotic" lesson,

> I have to be cautious of time you know, I don't want us to get caught in the middle of an activity say then we have to put them [graphing calculators] away before we finish what we're doing. I have to make sure we will be able to finish whatever activity I choose or at least reach a point where students can be in a position to say well, this is what we learned from this. It's not possible to achieve this without careful planning. So I would say it [graphing calculator] affects my planning in the sense that I have to budget for time and know when to move to the next step and avoid creating a chaotic situation. [Task-Based Interview, Lines 10 – 17].

Ms. S emphasized class organization:

> I like having kids work in groups of three or sometimes four but with graphing calculators I prefer them to work in pairs so the first thing I have to think about is who is going to work with who. A lot of times I let them

> pick their partners but sometimes this doesn't work well and so I have to pair them up myself. For example if the task is a little challenging I would pair the kids I know have less difficulties with the calculators with those who struggle. [Task-Based Interview, lines 13 – 18].

These views by Ms. T and Ms. S suggest that they would be more likely to direct their classes than leave students to explore. In particular, because of Ms. T's view that she has to manage time properly and make sure she accomplishes what she plans, she's more likely to be more directing in her lessons.

Compare these views with Ms. K's who did not seem to be so concerned about time, as she noted that

> sometimes you just can't be sure what to expect. Students can sometimes be adventurous and I don't think it is bad at all but that means I just have to be more prepared. Sometimes you'll see cases of blank screens, sometimes it is weird graphs and so I find it helpful to just tryout a few things myself beforehand and see what the results might be. And I don't mean to say I can exhaust everything, but very helpful to do this. [Task-Based Interview, lines 11 – 16].

This statement suggests that unlike Ms. T or Ms. S, Ms. R would allow her students to explore more with the calculators. In fact, there appears to be a pattern in the way the teachers responded to the items on the survey that dealt with the issues of student exploration and teacher direction. It appears that in general, the high users either agreed or strongly agreed with items that pointed to student exploration while the reverse is true for moderate users. Table 31 summarizes these responses.

Table 31

Teachers' Responses to Items on Beliefs about Teacher Direction and Student Exploration

Item	High Users			Moderate Users		
	K	L	M	R	S	T
1. I encourage my students to use graphing calculators for discovery and/or exploratory activities.	A	SA	SA	A	A	A
2. Students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand.	D	D	SD	D	A	D
3. I always give my students specific directions on how they should use the graphing calculator.	A	D	A	A	SA	A
6. Using graphing calculators provides opportunities for students to share ideas.	A	SA	N	A	N	A
18. I try to take every opportunity to use the graphing calculator when I teach about linear and quadratic functions	SA	SA	A	SA	A	N
19. Students should be free to use the graphing calculator whenever they feel it is appropriate.	SA	SA	A	A	D	A
22. Students should be free to explore with the graphing calculator.	A	SA	SA	A	A	A
23. The teacher should always decide when it is appropriate for students to use graphing calculators.	D	D	N	D	D	N

This finding was confirmed by my classrooms observations. I found from the classroom observations that there was a connection between the frequency of calculator use and the degree to which a teacher either allowed students to explore or directed the class. Two groups emerged: (1) all the high users and Ms. R from the moderate users mainly allowed students to explore, (2) Ms. S and Ms. T, the other moderate users directed their classes most of the time. Table 32 shows the percentages of class time by teacher directedness and student exploration.

Table 32

Distribution of class time by teacher directedness and student exploration

Teacher	Total class time (mins) observed	% of total class time characterized by teacher direction	% of total class time characterized by student exploration	Other
Ms. K	128	24%	67%	9%
Mr. L	131	21%	65%	14%
Ms. M	133	40%	53%	7%
Ms. R	130	31%	57%	12%
Ms. S	135	48%	44%	8%
Ms. T	132	56%	33%	11%

While teachers in the high users' category generally tended to allow more student exploration, results were mixed for moderate users with Ms. R's classes involving more student exploration while those of Ms. S and Ms. T tended to be more teacher directed.

The foregoing discussion dealt with the relationship between the teachers' professed beliefs about graphing calculators and observed practice. As illustrated in the discussion, I found lessons taught by the high frequency users tended to involve more student exploration while those taught by two of the three moderate frequency users tended to be more teacher directed.

Summary of Results from Interviews and Classroom Observations

In this section I will present a summary of the results from the interviews and classrooms observations. With regard to my second research question "How do secondary school mathematics teachers use graphing calculators when teaching linear and quadratic functions?" I found that the teachers preferred to use equations and graphs more than tables also that the sequences of representational shifts were dominated by equation to graph and graph to equation. Additionally, I found that all the teachers involved blended between directing their classes and allowing students to explore. Finally I found that when the lessons were characterized by teacher direction, the graphing calculator was used as a computational tool and when the lessons were characterized by student exploration, the graphing calculator was used as a visualizing tool and checking tool.

With regard to my third research question "What is the relationship between the teachers' professed beliefs about graphing calculators and observed practice?" I found that there was consistency between what the teachers said they believed about graphing calculators and what I observed in their classrooms. I also found that the high frequency users seemed to let their students explore more than the medium frequency users.

CHAPTER 6: DISCUSSION AND CONCLUSIONS

The purpose of this study was three-fold: (1) to investigate secondary mathematics teachers' professed beliefs about graphing calculators, (2) to investigate how the teachers use graphing calculators to teach linear and quadratic functions, and (3) to investigate the relationship between the teachers' professed beliefs and the way they use of graphing calculators. This was a mixed methods study in which I used a questionnaire, together with task-based interviews and classroom observations, to collect data in two phases. In the first phase of the study, 81 teachers responded to a survey instrument that I developed using items adapted from Fleener (1995b). The survey was designed to elicit the teachers' beliefs about various aspects related to the use of graphing calculators in the teaching of and learning about linear and quadratic functions, as well as give the teachers an opportunity to report on how often they used graphing calculators in their classrooms.

Based on the teachers' reported frequency of calculator use, the participants were divided into three groups; namely, high frequency users, moderate frequency users, and low frequency users. I then selected six teachers – three from the high frequency users and three from the moderate frequency users – to participate in the second phase of the study, which involved interviews and classroom observations. I designed the interviews to help me begin to understand what the teachers thought about specific ideas related to the use of graphing calculators and multiple representations of functions, while the classroom observations were designed to provide lenses through which I could look into the teachers' practice when they used graphing calculators. I have analyzed the findings from the survey, interviews and the classroom observations to answer the research questions:

4) What are secondary mathematics teachers' beliefs about use of graphing calculators in the teaching and learning of linear and quadratic functions?
5) How do secondary school mathematics teachers use graphing calculators when teaching linear and quadratic functions?
    a) What function representational choices do secondary mathematics teachers make when using graphing calculators?
    b) How specific are the teachers' directions to students about how the calculators may be used?
6) What is the relationship between the teachers' professed beliefs about graphing calculators and observed practice?
    a) What is the nature of similarities and/or differences between reported and observed calculator usage trends?
    b) To what extent do professed beliefs about graphing calculators explain observed usage?

Discussion of Findings

In the following sections, I discuss the major findings organized around (1) secondary mathematics teachers' beliefs about graphing calculators, (2) the teachers' observed practice with graphing calculators, and (3) the relationships between the beliefs and observed practice with regard to use of graphing calculators in the teaching of linear and quadratic functions.

Teacher Beliefs about Graphing Calculators

In this section, I will discuss the findings related to teachers' beliefs about graphing calculators with respect to (a) the effects on students' understanding of multiple representations, (b) the effects on whether classes are teacher directed or involve student exploration, and (c) general issues related to calculator use.

Effects of graphing calculators on students' understanding of multiple representations. One of the major findings of this study concerns areas of agreement among my participants. One area of agreement among the teachers is that some problems in a first algebra course are best solved using tables or graphs rather than algebraic symbols. As I stated in Chapter Two, studies have shown that it is important for teachers to recognize and appreciate the significance of utilizing multiple representations in their instruction (e.g., Even, 1998). It is therefore important for teachers to be aware of what problem types or characteristics are best suited for particular representations and provide guidance for their students. A second area of agreement among my participants is that representing a function with a graph helps students who have difficulty using algebraic symbols. This result supports the finding by Ruthven and Hennessey (2002) who reported that some teachers in their study believed that access to technology actually enables less-able students to participate in exploration.

A third area of agreement is that the teachers perceived that when students use graphing calculators on a regular basis they become better at interpreting tables and graphs. Similar results have been reported in other studies. For example, Tharp, Fitzsimmons, and Brown-Ayers (1997) noted that teachers generally see the graphing calculators as enhancing understanding and promoting exploration. A fourth area of agreement is that graphing calculators help students to solve non-routine problems that would otherwise be inaccessible by algebraic techniques. This finding resonates with survey results reported by Routitsky and Tobin (1998), and also by Tobin, Routitsky and Jones (1999) in which teachers' perceptions of the use of graphics calculators in secondary schools were investigated. The researchers reported that most of the teachers in their study believed that the graphics calculator would improve students' mathematical understanding and make a positive contribution to student learning. A similar study by Hennessy, Ruthven and Brindley (2005) found that teachers were committed to integrating technology into their instruction if they recognized the educational value and believed in the transformative potential of the technology. The teachers in my study seemed to believe in the potential of graphing calculators to enhance learning by students solve non-routine problems.

Other areas of agreement among my participants were that graphing calculators enable students to recognize connections among various function representations, and also that graphing calculators support students in learning about linear and quadratic functions by enhancing discussions around the various representations.

Another finding of this study is that there were some areas of disagreement among the teachers who participated. One notable area of disagreement was on whether students should always learn to solve problems using algebraic symbols first before they can use tables or graphs. This result confirms previous results involving CAS calculators. With regard to whether algebraic symbols should always precede tables, the lack of consensus among teachers appeared to correlate to the frequency of use groups - the high frequency users tended to disagree with the notion that algebraic symbols should always precede tables while the low frequency users tended to agree. This result is a new finding that could be explored more in future studies.

Effects of graphing calculators on whether classes are teacher directed or involve student exploration. Most of the teachers who participated in this study saw themselves as encouraging their students to use graphing calculators for discovery and/or exploratory activities. Furthermore, the teachers agreed that using graphing calculators provides opportunities for students to share ideas. They also stated that they give their students specific directions on how the students should use the graphing calculator. Earlier studies had shown that teachers tend to use technology to foster what they believe (Jost, 1992). Recent studies (Scrimshaw (2004) and Godwin and Sutherland (2004)) have found that technology can support teachers in implementing a student-centered approach to learning. Ruthven, Deaney, and Hennessy (2009) noted that graphing calculators can help build students' confidence in the accuracy of their graphs and enable them to work with less dependence on the teacher. The teachers in my study agreed that students should be free to explore with the graphing calculator, implying that they lean towards having student-centered classrooms.

There was lack of consensus on whether students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand. This may be attributed to individual teacher's preferences with regard to instructional choices. Simmt (1997) found teachers used graphing calculators as an extension of their normal teaching practices. She observed that even though the teachers in her study used similar activities, their differing conceptions of mathematics affected how they followed up those activities with questions and summary notes.

Another area of disagreement among the teachers was on whether or not the teacher should always decide when it is appropriate for students to use the graphing calculator. Research has shown that when using computer algebra systems (CAS), teachers preferred paper and pencil for simple tasks and then technology for more complex tasks (Ball & Stacey, 2005; Herget, Heugl, Kutzler, & Lehmann, 2000; Kendal & Stacey, 2002). In my study (with non-CAS calculators) however, the teachers were split on this issue with some teachers stating that they would encourage their students to use the graphing calculator when the students felt it was appropriate regardless of the task and others stating that they would always want their students to learn to solve each type of problem with paper and pencil before they could use a calculator.

General issues related to graphing calculators. Some studies have shown that access to technology actually enables less-able students to participate in exploration (Ruthven & Hennessey, 2002). I found a similar result in my study with the teachers agreeing that graphing calculators make the study of linear and quadratic functions more accessible to a wider range of students and enable them to engage with challenging problems. They also agreed that the calculators have had almost no impact on how and what the teachers teach. This means that the teachers did not see the graphing calculators as influencing either the content they teach or the methods they used for dissemination. Moreover, the teachers stated that they were confident users of graphing calculators, had lots of ideas about how to make use of the graphing calculator in the classroom, and they try to take every opportunity to use the graphing calculator when teaching about linear and quadratic functions.

Teachers' Observed Practice with Graphing Calculators

In terms of the teachers' practices with graphing calculators, I found that the teachers preferred to use equations and graphs more than tables. Kendal and Stacey (2001) reported that one of the teachers in their study, whom they described as being "content-focused with an emphasis on performance" (p. 155), preferred using the symbolic approaches with CAS while the other teacher, whom they described as "content-focused with an emphasis on conceptual understanding" (p. 155), preferred using both the symbolic and graphical approaches with CAS. In my study, most of the teachers worked with equations and graphs when using graphing calculators more than they worked with tables. Kendal and Stacey (2001) explain that these preferences for particular representations may be as a result of the type of knowledge that the teachers value as most important. I also found that the sequences of representational shifts were dominated by equation to graph and graph to equation.

Another finding in this category was that all the teachers blended their instructional approaches, moving between directing their classes and allowing students to explore. However, all the high frequency users and one moderate user tended to allow their students to explore with the graphing calculators more than they directed their classes. Additionally, I found three ways in which the graphing calculator was used in the classrooms similar to those described by Doerr and Zangor (2000). I found that when the lessons were characterized by teacher direction, the graphing calculator was used as a computational tool. I found that when the lessons were characterized by student exploration, the graphing calculator was used as a visualizing tool and checking tool.

Relationship between Professed Beliefs about Graphing Calculators and Observed Practice

The results showed that there was consistency between what the teachers said they believed about graphing calculators and what I observed in their classrooms. Bartolini (1998) noted that teachers play an important role by constructing meaningful classroom mathematical discussions that foster and support the development of appropriate actions with tools, and are responsible for guiding classroom practice. The teachers involved in the second phase of this study indicated that they used all function representations but only mentioned equations and graphs when they talked about specific representations. This was reflected in their practice in that these were the only representation the teachers employed, for the most part. Noss and Hoyles (1996) observed that there is a mutually constructive relationship between what teachers believe and what they do. There was therefore a consistency in what they considered as balancing between the representations.

Implications

This study provides some interesting features related to teacher beliefs about graphing calculators and how these beliefs may affect the nature of how teachers use graphing calculators. Teachers who believe that students should learn to solve certain problems using paper and pencil approaches first before they can use graphing calculators are more likely to control when and how their students may use calculators. This control may, in turn, lead to more teacher-directed classrooms with less student explorations.

I also found through analyzing the data from this study that when teachers talk about multiple representations in general they may not necessarily be referring to all the representations of functions but rather to a subset. It is therefore important for teacher educators to prepare teachers in such a way that they do not only embrace the use of multiple representations but they can actually do it in practice. Teachers in this study did not use tables much and it could be because they wanted their student to develop good connections between equations and graphs; however, it is worthwhile to note the importance of tables in learning about functions as well.

The findings of this study have shed some light on the relationship between teachers' beliefs about use of graphing calculators and their practices. These findings can benefit various groups involved with teacher preparation in several ways. First, teacher educators can work from these findings towards creating specific content materials for their pre-service teachers that include tasks that would encourage use of tables and foster discussion around meaningful ways of balancing the use of all representations. Professional development providers working with in-service teachers in schools could use similar materials but they may also organize support/discussion groups whereby teachers can meet and share ideas on how to use all the representations, including tables. Curriculum developers who create materials that incorporate technology such as graphing calculators can design specific activities to go along with the graphing calculators. Open-ended activities that involve learners in brainstorming as they go through problem solving processes would be good examples to start with but they need to include inbuilt potential for allowing all representations to be used. The activities need therefore to be well tested to ensure that they can actually offer these opportunities.

Limitations

Like most research studies, this study has some limitations that are worth noting. One such limitation is that even though the first phase of the study had a fairly large number of teachers, the second phase was comprised of only six teachers – three from the high frequency users' group and three from the moderate frequency of use group – therefore limiting the generalizability of the results. Furthermore, I conducted only one task-based interview with each teacher, which means the results about how the teachers responded to various tasks are limited to the episodes of these interviews and to those specific tasks. Another limitation of the study is that there were only three classroom observations for each of the six teachers. The fact that each teacher knew and prepared for when I would be visiting her/his classroom may have had some effect on how the teacher taught and/or used the graphing calculator. Additionally, the differences in the content taught by the teachers at the time of these observations may have had some effect on some of the differences noted in the way the teachers used the graphing calculator. One other limitation is that the teachers gave self reports on their frequency of calculator use and this self reporting may not necessarily have been accurate. Since the selection of teachers who participated in the second phase of the study was based on these self-reported frequencies, there is a possibility that some teachers may have ended up being grouped with those whose frequency of calculator use may not have been the same as their own.

One final limitation of this study is that I did not analyze the survey data until after I had already completed collecting the data in the second phase of the study. Because of this reason, I did not give the teachers opportunities to explain and/or expound on their choices in the survey. Since for each survey item there were only a limited number of choices to pick from, different teachers may have selected the same response to an item but because different reasons. For example, on item #3 which reads: *I* *always give my students specific directions on how they should use the graphing calculator*, depending on how teachers may have interpreted the phrase “specific directions” one teacher may agree while another teacher may disagree with the item but if they are asked to explain their choices it might turn out that they hold the same views.

Questions for Further Study

This study has revealed several areas in which there was lack of consensus among teachers with regard to the best way to use the graphing calculator in the teaching and learning of linear and quadratic functions as well as the teacher’s role in determining how students use the calculator. One such area relates to the issue of which types of tasks on which teachers prefer to have their students use the graphing calculators. Some teachers felt that students should be encouraged to use the graphing calculator at all times regardless of the task while others stated that they would want their students to learn to solve each type of problem with paper and pencil first before they can use the graphing calculator. Teachers in my study were also split along their reported frequency of use groups on whether students should always learn to solve problems using algebraic symbols first before they could use tables. These areas of disagreement among teachers could provide bases for future exploration. One question that I would possibly investigate is “Does using graphing calculators on a regular basis affect a teacher’s choice of sequencing with regard to multiple representations of functions? In what ways is this sequencing affected?”

It might also be helpful to carry out similar studies to this one but carried out over longer periods of time, such as a full semester or a full year, or maybe involving other types of handheld technologies like the TI-Nspire. For such studies, I would recommend having the teachers complete the survey, and then use the analysis from the survey data to generate questions for interviews thereby allowing the teachers to provide more information that would not be supplied through the survey.

Summary

This study was meant to explore secondary mathematics teachers' beliefs about graphing calculators, how the teachers use graphing calculators to teach the concept of function, and the relationship between the teachers' beliefs and their use of graphing calculators. Eighty-one teachers responded to a questionnaire about their beliefs regarding the use of graphing calculators in the teaching and learning of linear and quadratic functions. I then selected six teachers to participate in the second phase of the study, which involved interviews and classroom observations.

The analysis of data revealed that teachers generally believe that graphing calculators are valuable for students in the study of linear and quadratic functions. The teachers also generally feel confident about their knowledge of graphing calculators and they believe that they make use of graphing calculators whenever opportunities for doing so are available. This analysis also showed that teachers have their own preferences in sequencing of representations when exploring different function representations. I further found that these teachers preferred to use equations and graphs more than tables, and that the sequences of representational shifts were dominated by equation to graph and graph to equation. Additionally, I found that all the teachers blended their instructional approaches, moving between directing their classes and allowing students to explore. Finally, I found that when the lessons were characterized by teacher direction, the graphing calculator was used as a computational tool and when the lessons were characterized by student exploration, the graphing calculator was used as a visualizing tool and checking tool.

The data also revealed that there was consistency between what the teachers said they believed about graphing calculators and what was observed in their classrooms. With regard to student exploration, the data showed that the high frequency users appeared to let their students explore more than did the medium frequency users.

A major contribution of this study is that it has highlighted some differences among a particular set of teachers in terms of how they guide their classes – teacher direction and student exploration – and the choices they make in terms of sequencing of function representations. With regard to sequencing of multiple representations, I found that low frequency users held the view that algebraic symbols should always precede tables while high frequency users did not hold a similar view. In terms of classroom dynamics, I found that classes taught by the high frequency users seemed to involve more student exploration that those taught by moderate frequency users.

APPENDIX A: GRAPHING CALCULATOR SURVEY

Part I

1. What is your Name? ____

2. What is the name of your school? ____________________________________

3. How many years have you taught mathematics? __________________________

4. What is your highest qualification to teach mathematics? ___________________

5. At what grade levels have you taught mathematics? (Circle all that apply)

   6th   7th   8th   9th   10th   11th   12th   College

6. What grade(s) are you currently teaching? _____________________________

7. What course(s) are you currently teaching?

   Math 1   Math 2   Pre-Calculus   Calculus   Statistics

   Other (Please Specify) ____

8. What is your gender? _________ Male   _________ Female

9. How old are you? (Circle one)

   20 – 29   30-39   40-49   50-59   60 +

10. If you teach Math 1, 2, how often do you use graphing calculators in your classroom?

    Nearly every lesson __   Once every 2 or 3 lessons ___ Once every 4 or 5 lessons __

11. How do students gain access to graphing calculators?

    School provided ______   Student/Parent purchase _______

    If school provided, can the students take them home?   Yes_____   No ____

12. Have you ever attended in-service training workshops on graphing calculators? (Circle one)

    Never   1 – 2 times   3 – 6 times   More than 6 times

13. Have you attended an in-service training workshop on graphing calculators within the last two years?   Yes _____________   No ______________

Part II

SA=Strongly Agree, A=Agree, N=Neutral, D=disagree, SD=Strongly Disagree

Note: When answering the following, think about 9th and 10th grade levels.

Item Number	Statement	Response				
1.	I encourage my students to use graphing calculators for discovery and/or exploratory activities.	SA	A	N	D	SD
2.	Students should only be allowed to use a graphing calculator to create a graph after they have learned to create the graph by hand.	SA	A	N	D	SD
3.	I always give my students specific directions on how they should use the graphing calculator.	SA	A	N	D	SD
4.	Graphing calculators make the study of linear and quadratic functions more accessible to a wider range of students.	SA	A	N	D	SD
5.	Some problems in a first algebra course are best solved using tables rather than algebraic symbols.	SA	A	N	D	SD
6.	Using graphing calculators provides opportunities for students to share ideas.	SA	A	N	D	SD
7.	Graphing calculators help students to solve non-routine problems that would otherwise be inaccessible by algebraic techniques.	SA	A	N	D	SD
8.	Representing a function with a graph helps students who have difficulty using algebraic symbols.	SA	A	N	D	SD
9.	Students should always learn to solve problems using algebraic symbols first before they can use tables.	SA	A	N	D	SD
10.	Graphing calculators enable students to recognize connections between graphical, symbolic and numerical representations.	SA	A	N	D	SD
11.	Graphing calculators enable students to engage with challenging problems.	SA	A	N	D	SD

12.	Some problems in a first algebra course are best solved using graphs rather than algebraic symbols.	SA	A	N	D	SD
13.	Graphing calculators support students' learning of linear and quadratic functions by helping them to discuss the various representations of these functions.	SA	A	N	D	SD
14.	When students use graphing calculators on a regular basis, they become better at interpreting tables.	SA	A	N	D	SD
15.	Graphing calculators have had almost no impact on <u>how</u> I teach.	SA	A	N	D	SD
16.	Graphing calculators have had almost no impact on <u>what</u> I teach.	SA	A	N	D	SD
17.	Students should always learn to solve problems using algebraic symbols first before they can use graphs.	SA	A	N	D	SD
18.	I try to take every opportunity to use the graphing calculator when I teach about linear and quadratic functions.	SA	A	N	D	SD
19.	Students should be free to use the graphing calculator whenever they feel it is appropriate.	SA	A	N	D	SD
20.	When students use graphing calculators on a regular basis, they become better at interpreting graphs.	SA	A	N	D	SD
21.	I am a confident user of the graphing calculator.	SA	A	N	D	SD
22.	Students should be free to explore with the graphing calculator.	SA	A	N	D	SD
23.	The teacher should always decide when it is appropriate for students to use graphing calculators.	SA	A	N	D	SD
24.	I have lots of ideas about how I can make use of the graphing calculator in my classroom.	SA	A	N	D	SD

APPENDIX B: INTERVIEW TASKS

Planning

1) What things do you consider when planning a lesson on functions?
2) How does using graphing calculators affect your planning?
3) On the survey your response was ____ on the item (a)/(b). Can you comment briefly about your understanding of this item and what your response means?

   (a) I encourage my students to use graphing calculators for discovery and/or exploratory activities

   (b) I always give my students specific directions on how they should use the graphing calculator.

Sources of Tasks

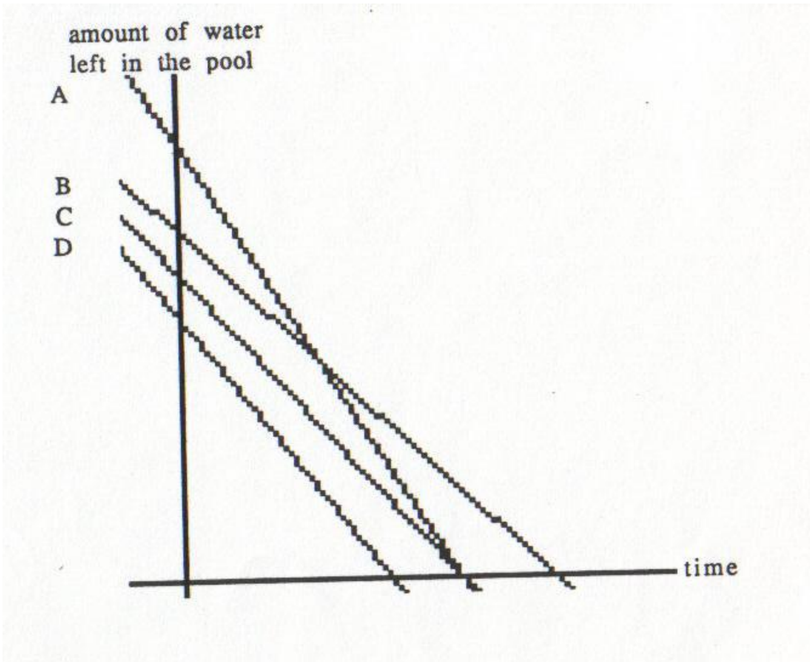
4) Where do you get the activities you use for your lessons?
5) Do you get any activities from other teachers? How about workshops?
6) Do you modify these activities or do you use them in their original form? How/Why?

Function Representations

7) It is alleged that some teachers tend to emphasize some representations more than others. What is your view on this?

8) Students were presented with the following problem:

Here are the graphs of four different pumps emptying four different pools.



Which pump pumps the most water in a given time? Explain your answer.

How would you respond to this problem?

How do you think your students may respond to this problem?

9) The table below shows values of x and corresponding values of y.

x	-10	-8	-6	-4	-2	0	2	4	6	8	10
y	56	30	12	2	0	6	20	42	72	110	156

What type of relationship does this table represent? Justify your answer.

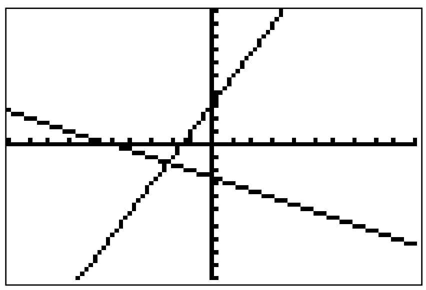
What is the rule defining this relationship? How did you obtain this rule?

Can you think of any other ways students might approach this problem?

Issues Related to Calculator Usage

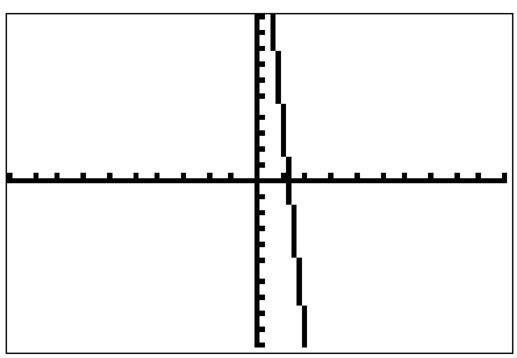
10) What are the advantages and disadvantages of using graphing calculators in your instruction? How do you deal with the disadvantages?

11) A student commented that the lines represented by the equations $y = 2x + 3$ and $y = -0.5x - 2.5$ are perpendicular but their graphs as shown on his calculator screen did not appear at right angles.



How would you help the student understand the graph?

12) A student was asked to find the x-intercepts of the graph of $y = x^2 - 17x + 20$ with the aid of a graphics calculator. The student then produced a graph similar to the following thereby getting only one value for the x-intercept.



What is the problem here? How would you deal with such a case?

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