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Predictions and Observations in Theories with Varying Couplings

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Abstract

We consider a toy universe containing conventional matter and an additional real scalar field, and discuss how the requirements of gauge and diffeomorphism invariance essentially single out a particular set of theories which might describe such a world at low energies. In these theories, fermion masses and g-factors, as well as the electromagnetic coupling turn to be scalar field dependent; fermion charges and the gravitational coupling might be assumed to be constant. We then proceed to study the impact of a time variation of the scalar field on measurements of atomic spectra at high redshifts. Light propagation is not affected by a sufficiently slow change of the fine structure constant, but changes of the latter as well as variations of fermion masses and g-factors do affect the observed atomic spectra. Finally, we prove the independence of these predictions on the chosen conformal frame, in a further attempt to address differing views about the subject expressed in the literature.

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I. INTRODUCTION AND SUMMARY

Recent analysis of distant quasar spectra seem to imply that the constants of nature are changing with time [1, 2]. Many different specific models have been proposed to explain such eventual changes, and most of them involve a scalar field governing the value of the constants of nature [3, 4, 5]. On completely different measurements rests the solid evidence that the universe contains a component presently driving accelerated cosmic expansion [6]. Yet again, time-evolving scalar fields have been widely considered as an alternative to explain the origin of late time cosmic acceleration [7]. Scalar fields play indeed a prominent role in modern cosmology. Their importance arises from their simplicity, their power to explain many seemingly unrelated different problems and their ubiquitous appearance in theories of fundamental particle interactions. However, although substantial progress has been achieved in the past, it is fair to say that the latter are far from being able to make concrete (realistic) low-energy predictions. Given the possibility that the universe contains a scalar field in addition to conventional matter, and given the to some extent lack of theoretical guidance, it is important to ascertain what is a generic consequence of the existence of such a scalar field and what is not.

In Section II of this paper, we assume that in addition to ordinary matter there exists a single real scalar field potentially relevant at low energies, and construct the "most general" low-energy theory that one might expect from basic symmetry principles like diffeomorphism and gauge invariance. The resulting effective action has the form one would expect from general relativity and electromagnetism with the exception that all "constants" of nature (gravitational and electromagnetic couplings, magnetic moments and fermion masses as well) can depend on the value of the scalar field. Theories of this form have been long advocated as a theoretical framework for phenomenological studies of gravity [8]. Our construction is useful not only because it delimits up to what extent they are general, but also because it shows the precise nature of the assumptions made during the derivation of the effective action, and what could be different if any of the assumptions were relaxed.

The effective action found in Section II can be considerably shortened. In Section III we simplify its form by "renaming" fields and couplings. In particular, some of of the above mentioned varying quantities can be assumed to be constant without loss of generality. We argue that in the present context the most natural choice is to fix the gravitational constant.

In order to do so, one needs to perform a "conformal transformation", which is nothing else than a particular field redefinition that involves the metric field. Whereas it is commonly accepted that field redefinitions do not affect the "meaningful" predictions of any theory, i.e. the predictions which can be verified in an experiment, the special role played by the metric in general relativity has caused a long debate about the physical equivalence of two actions that differ by a conformal transformation [9]. Although our purpose is not to provide a general proof of such equivalence, we have considered worthwhile to show in Appendix A with an explicit example that the predictions made by two conformally related actions are indeed the same.

Once the effective action has been cast in its simplest form, the next goal of the paper is to state some of the predictions that the theory makes. A priori, the presence of the additional scalar field could be responsible for strong violations of the equivalence principle [10], and therefore, the different field-dependent quantities in our theory have to obey strong experimental constraints [8, 11]. We shall simply assume that changes in the scalar field might induce variations in the couplings and parameters of the effective action, without making further suppositions about the nature of these changes. In order to subsequently study the phenomenological implications of these changes, it is then important to cast any prediction in an "experimentally" meaningful way. Most of the experimental evidence about time variation of the constants of nature [1, 2] and cosmic acceleration [6] stems from measurements of photon spectra emitted by distant objects. In fact, if the constants of nature are allowed to vary in time, it is natural to expect a larger departure from their present values the further one looks back in the past. Hence, in Section IV we focus on light emission by atoms and on the propagation of photons in an expanding universe. It turns out that if the change of the electromagnetic coupling is "slow", light propagation is not affected by changes in any of the other varying parameters. However, because in our theory frequencies in atomic transitions depend on fermion masses, the electromagnetic coupling and the magnetic moment of the electron, a careful analysis shows that ratios of frequencies of atomic spectral lines also depend on all those parameters. Therefore, measurements of deviations from the expected values could a priori be due not only to changes in the electromagnetic coupling, but also to variations of some of the remaining "constants" of nature. This fact might have implications in the analysis of experimental data suggesting a time-varying fine structure constant [1].

II. THE LOW ENERGY EFFECTIVE ACTION

According to the present paradigm in physics, at sufficiently low energies nature can be described by an effective action which accounts for its low energy degrees of freedom, and is invariant under its symmetries. The low energy effective action consists of an expansion in the derivatives of the fields (particles) observed at low energies. The higher the amount of field derivatives, the more the effects of the corresponding terms are expected to be suppressed in the predictions of the theory.

As mentioned in the introduction, recent experiments suggest that there is an additional component in the universe which seems to mediate non-electromagnetic interactions. The question we want to answer is: Assuming that this component is a real scalar field, what is the most general effective action we might expect to describe our world? For our purposes, at the energies we are considering, the world consists of electrons, neutrons and protons which are subject to electromagnetic and gravitational interactions. They are respectively described by spin-1/2 fields ψ_f (where f runs through e for electrons, p for protons and n for neutrons), by a massless spin-1 field A_μ (the photon) and a massless, spin-2 field $g_{\mu\nu} = g_{\nu\mu}$ (the graviton). The extra ingredient we want to consider is a scalar field ϕ which might also mediate an additional form of "gravitational" interaction.

The low energy effective action has to be invariant under the symmetries one wants to impose on the system. In our case, these symmetries are intimately related to the field content of the theory. In fact, it seems that the only way of consistently describing the electromagnetic field A_μ is by incorporating gauge invariance into the theory, and analogously, the only way of consistently describing a massless spin-2 field is by incorporating diffeomorphism invariance [12]. By definition, under $U(1)$ gauge transformations the fields transform according to

$$\phi \rightarrow \phi \tag{1a}$$

$$\psi_f \rightarrow \exp\left(i q_f \epsilon\right) \psi_f \tag{1b}$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \epsilon \tag{1c}$$

$$g_{\mu\nu} \rightarrow g_{\mu\nu}, \tag{1d}$$

where ϵ is the gauge parameter and the charges q_f are constants. Note that a *real* scalar field cannot be electrically charged. Under diffeomorphisms $x^\mu \rightarrow \tilde{x}^\mu = \tilde{x}^\mu(x^\nu)$ the fields transform according to

$$\phi \;\to\; \phi \tag{2a}$$

$$\psi_f \;\to\; \psi_f \tag{2b}$$

$$A_\mu \;\to\; \frac{\partial x^\alpha}{\partial \widetilde{x}^\mu} A_\alpha \tag{2c}$$

$$g_{\mu\nu} \;\to\; \frac{\partial x^\alpha}{\partial \widetilde{x}^\mu}\frac{\partial x^\beta}{\partial \widetilde{x}^\nu} g_{\alpha\beta}. \tag{2d}$$

Fermions are diffeomorphism scalars. Indeed, a different approach is needed in order to couple fermions to gravity [13]. For that purpose, one introduces a set of four orthonormal vectors $e^\mu{}_a$ at each point of the spacetime manifold, $e^\mu{}_a e^\nu{}_b\, g_{\mu\nu} = \eta_{ab}$. The vierbein $e^\mu{}_a$ is only determined by the metric up to local Lorentz transformations. In order to avoid the appearance of new degrees of the freedom associated with the vierbein, Lorentz transformations are postulated to be a local symmetry of the theory. Under (local) Lorentz transformations $\Lambda^a{}_b(x)$, the different fields transform according to

$$\phi \;\to\; \phi \tag{3a}$$

$$\psi_f \;\to\; U(\Lambda)\psi_f \tag{3b}$$

$$A_\mu \;\to\; A_\mu \tag{3c}$$

$$e^\mu{}_a \;\to\; \Lambda_a{}^b e^\mu{}_b \tag{3d}$$

$$g_{\mu\nu} \;\to\; g_{\mu\nu}, \tag{3e}$$

where U is the Dirac spinor representation of the Lorentz group. Note that we postulate invariance under arbitrary Lorentz transformations, including "time reversal" $\mathcal{T}$: $e^\mu{}_0 \to -e^\mu{}_0$ and "space inversion" $\mathcal{P}$: $e^\mu{}_i \to -e^\mu{}_i$.

The transformation properties of the different fields essentially determine the precise nature of the particles we are describing, allowing us to distinguish between "fermions", "photons" and "gravitons". The next step is to construct a (local) action functional of the fields ϕ, A_μ $g_{\mu\nu}$ (and e^μ_a) which is invariant under the transformations (1), (2) and (3). At low energies, for slowly varying fields, terms in the Lagrangian with the least possible number of derivatives give the largest contributions. Thus, we shall organize the Lagrangian as an expansion in the total number of derivatives. The least possible number of derivatives is two for a dynamical boson $\sim (\partial\phi)^2$, and one for a dynamical fermion $\sim \overline{\psi}\partial\psi$. We want to ascribe the same weight to the two kinetic terms, and this can be formally accomplished by assigning half an "effective derivative" to fermion fields, as in supersymmetric effective theories [14]. Our theory is certainly not supersymmetric, but since the previous argument just relies on dimensional reasons this procedure should still be a consistent way of organizing the long-wavelength expansion. Hence, we shall include in our effective Lagrangian terms with only two "effective derivatives", i.e. terms where the number of real derivatives plus one half the number of fermion fields is less or equal two. Furthermore, for the purposes of studying atomic spectra it will suffice to consider fermion bilinears.

a. Zero derivatives Gauge, Lorentz and diffeomorphism symmetries only allow the terms

$$c(\phi) \tag{4a}$$

$$\overline{\psi}_f m_f(\phi)\psi_f, \tag{4b}$$

where c and M_f are arbitrary functions of the scalar field ϕ. A term $A_\mu A^\mu$ is not gauge invariant, and Fermi terms $(\overline{\psi}\psi)(\overline{\psi}\psi)$ are left out because they are not fermion bilinears. The Dirac adjoint is given by $\overline{\psi} = \psi^+\gamma^0$, where γ^a is a set of Dirac matrices, $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$. Observe that the only 5 possible independent fermion bilinears are $\overline{\psi}M\psi$, where M is proportional to 1, γ^5, γ^a, $\gamma^a\gamma^5$ and $[\gamma^a, \gamma^b]$. Because electrons, protons and neurons have different charges, gauge invariance forces fermion bilinears to contain only one type of fermion.

b. One derivative Possibly the only way to insure the invariance of our theory is to consider covariant derivatives. Up to multiplications with terms in eqs. (4), the allowed invariant combinations are

$$i\,\overline{\psi}_f \not{D}_f\, \psi_f \tag{5a}$$

$$i\,\overline{\psi}_f \Gamma^\mu \psi_f \partial_\mu\phi \tag{5b}$$

$$i\,\overline{\psi}_f [\Gamma^\mu, \Gamma^\nu]\psi_f F_{\mu\nu}, \tag{5c}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell tensor and $\Gamma^\mu = e^\mu{}_a \gamma^a$. The covariant derivative of a fermion is

$$\not{D}_f = \Gamma^\mu \left(\partial_\mu + \frac{1}{2}\omega_{\mu ab}\Sigma^{ab} - iq_f A_\mu\right), \tag{6}$$

where $\omega_{\mu ab}$ is the (minimal) spin connection and $\Sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$ are the generators of the Dirac representation of the Lorentz group [13]. Without loss of generality we can assume that the covariant derivative of the metric vanishes. Otherwise, it had to be expressible as a properly transforming combination of the fields in the action [for instance $\nabla_\mu\, g_{\nu\rho} = g_{\nu\rho}\partial_\mu\phi + \cdots$]. Invariant terms involving $\nabla_\mu\, g_{\nu\rho}$ [for instance $g^{\nu\rho}\partial^\mu\phi\, \nabla_\mu\, g_{\nu\rho}$] could then be cast as couplings that do not involve derivatives of the metric [in our example $4\,\partial_\mu\phi\partial^\mu\phi + \cdots$]. For analogous reasons we also set the torsion of derivative operators to zero. A non vanishing torsion would manifest itself in the form of specific couplings between the different fields, and because these have to be invariant under the considered symmetries, we will consider them anyway. One can also define the dual of the Maxwell tensor,

$$ {}^*F_{\mu\nu} \equiv \det(e_\alpha{}^a)\, \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, $$

which is a tensor under diffeomorphisms, but changes sign under $\mathcal{P}$ and $\mathcal{T}$. A term proportional to $\overline{\psi}[\Gamma^\mu, \Gamma^\nu]\psi\, {}^*F_{\mu\nu}$ is excluded because it violates the latter symmetries. A contribution proportional to $\nabla_\mu(\overline{\psi}_f \Gamma^\mu \psi_f)$, which vanishes on-shell if fermion flavor is conserved, can be traded for a term proportional to (5b) after an integration by parts.

c. Two derivatives Up to a multiplication by a scalar function the only allowed scalar combinations of two field derivatives are

$$ R \tag{7a} $$

$$ F_{\mu\nu}F^{\mu\nu} \tag{7b} $$

$$ \partial_\mu\phi\partial^\mu\phi, \tag{7c} $$

where R is the scalar curvature. Again, $F_{\mu\nu}{}^*F^{\mu\nu}$ is excluded by $\mathcal{T}$ or $\mathcal{P}$ invariance. The expression $\nabla^\mu\partial_\mu\phi$ can be turned into (7c) [up to a ϕ-dependent coefficient] by an integration by parts. Terms of the form $\overline{\psi}_f\Gamma^\mu\psi_f F_\mu{}^\nu\partial_\nu\phi$ or $\overline{\psi}_f\Gamma^\mu\gamma^5\psi_f\, {}^*F_\mu{}^\nu\partial_\nu\phi$ are expected from a connection with torsion; they are not included because they contain three effective derivatives.

We can construct gauge and coordinate invariant terms in our action functional by combining the previous building blocks into factors with at most two effective derivatives. In addition, in order to define our action as a local field functional, we need a coordinate and gauge invariant "volume element" to integrate those invariant terms. The most general expression of this type is

$$ \sqrt{|\det[v_g(\phi, \partial\phi^2) g_{\mu\nu} + v_F(\phi) F_{\mu\nu} + v_1(\phi)\partial_\mu\phi\partial_\nu\phi + v_2(\phi)\nabla_\mu\partial_\nu\phi]|}\, d^4x, $$

which yields a generalization of the Born-Infeld action [15]. By expanding the square root in powers one recovers to lowest order terms proportional to $F_{\mu\nu}F^{\mu\nu}$, $(\partial\phi)^2$ and $\nabla^\mu\partial_\mu\phi$ plus additional higher order derivatives. Because we have already considered the lowest order ones as part of our building blocks (7), and we keep only up to two derivatives, without loss of generality we can set $v_F = v_1 = v_2 = 0$. For the same reasons, we can assume that v_g only depends on ϕ. The final (unsimplified) action thus reads

$$S = \int d^4x\sqrt{-g}\, v_g^2(\phi)\Bigg\{ -\frac{B_g(\phi)}{16\pi}R - \frac{B_F(\phi)}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{B_\phi(\phi)}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \\ + B_f(\phi)\overline{\psi}_f\left[i\not{D}_f - m_f(\phi)\right]\psi_f - i\overline{\psi}_f\Gamma^\mu\psi_f\partial_\mu r_f(\phi) + i\frac{q_f h_f(\phi)}{16 m_f(\phi)}\overline{\psi}_f[\Gamma^\mu,\Gamma^\nu]\psi_f F_{\mu\nu}\Bigg\}. \quad (8)$$

Note that all the terms containing boson differentiations only involve partial derivatives, and hence, are independent of the way a connection is defined.

In order to delimit the validity of our derivation, let us summarize all the assumptions made:

- Locality
- Diffeomorphism invariance (in 4 spacetime dimensions)
- Invariance under $\mathcal{P}$ or $\mathcal{T}$ and local Lorentz transformations
- $U(1)$ gauge invariance
- Lowest order in derivative expansion (at most two effective derivatives)
- Bilinear in fermion fields

Departures from four dimensional coordinate invariance, as in brane-world models [16] (see however [17]), and certain theories where higher-derivative terms become large [18] are obvious examples that do not fit into our framework. Our derivation is rather to be regarded as a conservative attempt to delimit a basic, but still quite general reasonable set of theories to focus on. In fact, the action (8) is general enough to accommodate scalar-tensor theories of gravity [3], Bekenstein's theory of varying α and its revivals [4, 19], and even the long-wavelength limit of bimetric theories [5]. Finally, let us point out that the arguably only known "ultraviolet-complete" theory of quantum gravity, string theory, is expected to have an action of the form (8) as its low-energy limit [8].

III. SIMPLIFICATIONS

The action (8) can be considerably simplified. First, the overall factor v_g can be absorbed into the remaining ϕ dependent functions B_g, B_F, etc. Next, by performing field redefinitions, some of the B functions can be assumed to be constant. Specifically, by introducing the new scalar field $d\widetilde{\phi} = \sqrt{B(\phi)}\, d\phi$ one can always choose $\widetilde{B_\phi} = \pm 1$, and by defining $\widetilde{\psi}_f = \sqrt{B_f(\phi)}\psi_f$ one can also choose $\widetilde{B_f} = \pm 1$. In addition, by the field redefinition

$$\widetilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \tag{9}$$

where $\Omega^2 = B_g(\phi)$, one can set $\widetilde{B_g}(\phi) = 1$. Under the transformation (9), sometimes called a "conformal transformation", canonically normalized ($B_f = 1$) fermion masses transform according to

$$\widetilde{m_f} = \Omega^{-1} m_f. \tag{10}$$

Thus, instead of requiring $B_g = 1$ we could set one of the masses m_f to one [20]. In such a "conformal frame" the corresponding fermion would be minimally coupled to the "metric" $g_{\mu\nu}$. However, since B_F as well as h_f are invariant under the redefinition (9),

$$\widetilde{B_F} = B_F, \quad \widetilde{h_f} = h_f, \tag{11}$$

and because in general there is no reason to expect the parameters m_f to be proportional to each other, matter would be still coupled to the scalar field. We therefore conclude that in the present framework $B_g = 1$ is the most convenient choice. Consequently, the following action is completely equivalent to (8),

$$S = \int d^4x \sqrt{-g} \Bigg\{ -\frac{R}{16\pi} - \frac{B_F(\phi)}{16\pi} F_{\mu\nu}F^{\mu\nu} \pm \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \pm$$
$$\pm \overline{\psi}_f \left[i\not{D}_f - m_f(\phi)\right]\psi_f - i\overline{\psi}_f\Gamma^\mu\psi_f\partial_\mu r_f(\phi) + i\frac{q_f h_f(\phi)}{16 m_f(\phi)}\overline{\psi}_f F_{\mu\nu}[\Gamma^\mu, \Gamma^\nu]\psi_f \Bigg\}, \tag{12}$$

where the fermionic covariant derivative is given by equation (6). Note that symmetry principles alone do not restrict the signs of the kinetic terms.

It is important to make a distinction between the parameters and fields that enter the action (12) and the quantities one measures in real experiments. Although we have been talking about the "metric", the "gravitational coupling", fermion "masses" and so on, one should realize that the quantities measured in experiments might be different, even if they are given the same name. For instance, in standard general relativity $g_{\mu\nu}$ determines proper distances and times, and hence, $g_{\mu\nu}$ is generally denoted as the "metric". Because the metric is not invariant under conformal transformations, doubts about the physical equivalence of conformally related actions have been raised in the literature [9]. In our description of gravity however, the "metric" is just an additional field essentially on the same footing as the remaining ones [21], and its precise meaning is determined by its couplings to them. In particular, distance measurements might hinge on other parameters, like fermion masses. We have considered worthwhile to illustrate the issue in Appendix A, where we show the conformal frame independence of the outcome of a redshift measurement.

When we later study the motion of electrons around a nucleus, it is going to be convenient to have a point particle description of the fermion instead of the field description in eq. (12). What matters is how a fermion is accelerated in the presence of the ϕ, A_μ and $g_{\mu\nu}$ fields, so our goal is to compute this acceleration. We shall neglect the effects of the fermion spin, so we shall ignore terms proportional to $[\gamma^a, \gamma^b]$. Then, by varying the action (12) with respect to $\overline{\psi}_f$ one gets the Dirac equation $[i\Gamma^\mu(\partial_\mu - iqA_\mu - i\partial_\mu r) - m]\psi = 0$, where we have dropped the f label. Observe that the function r plays the role of a scalar "electromagnetic potential". The WKB ansatz $\psi = \Psi \exp iS(x^\mu)$, where Ψ is a constant spinor, yields $[\Gamma^\mu(\partial_\mu S - qA_\mu - \partial_\mu r) + m]\Psi = 0$, which implies the on-shell condition

$$g^{\mu\nu}(\partial_\mu S - qA_\mu - \partial_\mu r)(\partial_\nu S - qA_\nu - \partial_\nu r) = m^2. \tag{13}$$

We thus identify the mechanical four momentum of the particle p^μ to be

$$p^\mu \equiv m \cdot u^\mu = \partial_\mu S - qA_\mu - \partial_\mu r, \tag{14}$$

and differentiating the on-shell condition (13) we find then that the force exerted on the particle is given by

$$u^\mu \nabla_\mu (m u_\nu) = qF_{\nu\mu}u^\mu + \partial_\nu m. \tag{15}$$

The first term on the right hand side is the Lorentz force, and the second is the "fifth force" described by Dicke [21]. Because derivatives acting on scalars commute, the particle does not couple to r. The resulting equation of motion can be derived from the action principle

$$S_f = -\int d\lambda \left\{ m_f \sqrt{g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}} + q_f A_\mu \frac{dx^\mu}{d\lambda} \right\}, \tag{16}$$

where m_f and A_μ are evaluated at the position of the particle $x^\mu(\lambda)$. We could have also included a term $\partial_\mu r(dx^\mu/d\lambda)$ in the action (16), which is a total derivative and hence does not alter the equations of motion, in agreement with our previous remark concerning the same fact. Note that the behavior of the fermion mass m_f under conformal transformations (10) can be also derived from eq. (16).

IV. ATOMIC SPECTRA

A significant amount of information about our universe stems from measurements of spectra emitted by distant objects. In this section we consider an ideal measurement whereby an electron in an hydrogen-like atom changes its quantum state and thereby emits a photon of definite frequency. The photon freely propagates to an hypothetical observer who measures its frequency by comparing it to the frequency of photons emitted by a reference "atomic clock" at the observer's site. Hence, in order to predict the result of the measurement we need to know *a*) how the frequency of the emitted photon depends on the parameters of our theory, *b*) how the photon freely propagates in space, and *c*) how the observer compares the frequency of the incoming photon to the one of a photon emitted by the reference atomic clock. In the following we address these points in that order.

a. Emission Consider a hydrogen-like atom consisting of a nucleus of mass m_N and charge $Z \cdot q_p$ surrounded by a single electron. The nucleus itself can be described by the action (16), with $m_f = m_N$ and $q_f = Zq_p$. Because our toy universe does not contain nuclear forces, such an atom could not possibly exist, but this fact is not essential for our purposes. We shall assume that there exist coordinates where the universe looks flat, homogeneous and isotropic [22],

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta)(d\eta^2 - d\vec{x}^2). \tag{17}$$

Therefore, it also follows that ϕ can only depend on time η, and the parameters in our action might hence also be η-dependent. Following the standard procedure to quantize the non-relativistic limit of eq. (16) in the presence of an external electromagnetic field sourced by the nucleus [23] we find that the Hamiltonian of the electron has degenerate eigenvalues given by

$$E^0_{n,l} = -Z^2\alpha^2 \frac{a\,\mu}{2n^2}, \tag{18}$$

where $\mu \equiv m_e m_N/(m_e + m_N)$ is the reduced mass of the electron-nucleus system and

$$\alpha \equiv \frac{q_e^2}{B_F}$$

is the fine structure constant. It is important to notice that this result is valid so long as the scale factor a and the field ϕ can be regarded as constant (we shall later discuss when this assumption is applicable). By expanding the square root in eq. (16) to a higher order in the fermion and nucleus velocities, one gets a relativistic correction [23] to the lowest order result (18),

$$\Delta E^r_{n,l} = -Z^4\alpha^4 \frac{a\,\mu}{2}\left(\frac{\mu^3}{m_e^3} + \frac{\mu^3}{m_N^3}\right)\left(\frac{1}{n^3(l+1/2)} - \frac{3}{4n^4}\right).$$

A more accurate treatment of the motion of the electron with the Dirac equation additionally introduces the spin-orbit coupling of the electron magnetic moment to the magnetic field [23],

$$\Delta E^{so}_{n,l} = Z^4\alpha^4 \frac{a\mu}{2}\frac{\mu}{m_e}\frac{1+h_e}{2}\frac{\begin{Bmatrix} l \\ -l-1 \end{Bmatrix}}{n^3 l(l+1/2)(l+1)},$$

where the upper and lower values in the bracket apply for an atom with total angular momentum $j = l \pm \frac{1}{2}$ respectively. In order to compute the frequency of an emitted photon in an electron transition from a state (n_i, j_i, l_i) to a state (n_f, j_f, l_f) one needs to quantize the electromagnetic field too. The photon field (in Coulomb gauge) A_μ is thereby expanded into modes

$$\vec{A} = \sum_k \vec{\mathcal{A}}_k e^{-ik_\mu x^\mu} a_k + h.c$$

where the sum runs over null vectors, $k_\mu k^\mu = 0$. A necessary condition for the emission of a photon of four-momentum k_μ is "energy conservation", $E_i = E_f + k_0$. Therefore, the possible time components of the photon four-momentum are

$$k_0 \approx Z^2\alpha^2 \frac{a\mu}{2}\left\{A - Z^2\alpha^2\left[2B + (C/2 - 3B)\frac{\mu}{m_e} + \frac{C}{2}\frac{\mu}{m_e}h_e\right]\right\}, \tag{19}$$

where

$$A = \frac{1}{n_f^2} - \frac{1}{n_i^2} \tag{20a}$$

$$B = \frac{1}{n_f^3(l_f+1/2)} - \frac{3}{4n_f^4} - \frac{1}{n_i^3(l_i+1/2)} + \frac{3}{4n_i^4} \tag{20b}$$

$$C = \frac{\left\{\begin{matrix} l_f \\ -l_f-1 \end{matrix}\right\}}{n_f^3 l_f(l_f+1/2)(l_f+1)} - \frac{\left\{\begin{matrix} l_i \\ -l_i-1 \end{matrix}\right\}}{n_i^3 l_i(l_i+1/2)(l_i+1)} \tag{20c}$$

only depend on the atomic transition, and where all remaining parameters are to be evaluated at the time of emission. So long as the relative change in those parameters during a time interval $1/k_0$ is negligible small, the assumption that they are constant should be a good approximation. Note that, besides of α and the scale factor a, both the mass ratio μ/m_e and the g-factor of the electron $g_e = 2 + 2h_e$ enter emission frequencies[1].

b. Propagation We assume that once the photon is emitted by an atom, it freely propagate in space until it reaches the observer. Whereas it was necessary to quantize the electromagnetic field in order to compute the possible emission frequencies, a classical consideration suffices to determine its propagation. The Maxwell term in eq. (12) is invariant under conformal transformations, so that the scale factor in the conformally flat metric (17) does not enter the equations of motion with sources set to zero,

$$\partial_\mu \left[B_F \, \eta^{\mu\rho} \eta^{\nu\sigma} F_{\rho\sigma} \right] = 0. \tag{21}$$

Here $\eta_{\mu\nu}$ is the Minkowski metric and $x^0 = \eta$ is still the conformal time in the metric (17). Introducing electric and magnetic fields $E_i = F_{0i}$, $B_i = {}^*F_{0i}$ and defining $\vec{D} \equiv B_F \vec{E}$, $\vec{H} \equiv B_F \vec{B}$ eq. (21) translates into the macroscopic "inhomogeneous" Maxwell equations $\operatorname{div} \vec{D} = 0$, $\operatorname{rot} \vec{H} - \partial \vec{D}/\partial\eta = 0$, whereas by definition $\operatorname{div} \vec{B} = 0$, $\operatorname{rot} \vec{E} + \partial \vec{B}/\partial\eta = 0$. Thus, the problem we are studying is physically equivalent to the study of light propagation in Minkowski space permeated by a medium with time varying permittivity $\epsilon = B_F$ and permeability $\mu = 1/B_F$. The propagation speed of the electromagnetic perturbations, the speed of light, is in our dimensionless units given by $v = (\epsilon\mu)^{-1/2} = 1$, which is constant, regardless of how all the couplings in our theory are evolving.

[1] Perturbative corrections in α also contribute to the "anomalous" magnetic moment of the electron [24]; we absorb them into h_e. Similarly, the mass of the nucleus contains an α-dependent piece due to the electromagnetic interactions between nucleons.

Because Maxwell's equations are linear in the fields and space is homogeneous and isotropic, we can decompose the fields field into plane waves $\propto \exp(i\vec{k}\cdot\vec{x})$. Both sets of equations can then be combined into

$$\left(B_F^{1/2}\vec{E}\right)'' + \left[\vec{k}^2 + \frac{1}{2}\frac{B_F''}{B_F} - \frac{3}{4}\left(\frac{B_F'}{B_F}\right)^2\right]\left(B_F^{1/2}\vec{E}\right) = 0, \tag{22}$$

where a prime means a derivative with respect conformal time η. We assume that changes in B_F are slow enough for a WKB solution to be a good approximation,

$$\vec{E} = \frac{\vec{\mathcal{E}}}{\sqrt{2\Re(\omega_k)}} \exp\left(i\int_{\eta_{em}}^{\eta} \omega_k(\tilde{\eta})d\tilde{\eta}\right). \tag{23}$$

Here, $\vec{\mathcal{E}}$ is a constant transverse polarization vector, $\vec{k}\cdot\vec{\mathcal{E}} = 0$, and $\Re(\omega_k)$ denotes the real part of the "frequency"

$$\omega_k(\eta) = \sqrt{\vec{k}^2 + \frac{1}{2}\frac{B_F''}{B_F} - \frac{3}{4}\left(\frac{B_F'}{B_F}\right)^2} + \frac{i}{2}\frac{B_F'}{B_F}.$$

In order to uniquely solve eq. (22) proper initial conditions are needed. Because the solution (23) should describe light emitted by our atom we require that at the time of emission it behave as $\exp(ik_0\eta)$, where k_0 is any of the frequencies in eq. (19). Under the assumption of slowly varying B_F this condition fixes the length of the wave vector $\vec{k}$, $\omega_{em} \equiv \omega_k(\eta_{em}) \approx k_0(\eta_{em})$. When the photon reaches the observer at time η_{arr}, its frequency is given by $\omega_{arr} \approx \omega_k(\eta_{arr})$. Only if B_F changes either during emission or observation is the frequency line of the emitted light shifted and broadened. For a slowly changing B_F the dominant effect is the broadening, which is of the order

$$\frac{\delta\omega_{em}}{\omega_{em}} \approx \frac{1}{\omega_{em}}\left[\left(\frac{B_F'}{B_F}\right)_{arr} - \left(\frac{B_F'}{B_F}\right)_{em}\right].$$

Hence, if the relative change of B_F during the period of the field oscillations both at the emission and observations is negligible, the effect of varying B_F on the photon propagation is negligible too, regardless of the overall total change of B_F between both times. We shall hence ignore this effect and assume $\omega_{arr} = \omega_{em} = k_0$.

c. Observation When the photon finally reaches the observer at time η_{arr}, he or she can compare its frequency $\omega_{arr} = k_0(\eta_{em})$ with the one of a photon emitted by a reference atomic clock. For simplicity we take the reference clock to be identical to the atom that emitted the photon (same values of Z and m_N). Then, the atomic clock frequencies ω_{clock}

are still given by eq. (19), but the different factors have to be evaluated at the time of observation, $\omega_{clock} = k_0(\eta_{arr})$. The observer determines the ratio of the frequency of the incoming photon to the frequency of the photon emitted by the clock to be

$$\frac{\omega_{arr}}{\omega_{clock}} = \frac{(a\,\mu\,\alpha^2)_{em}}{(a\,\mu\,\alpha^2)_{arr}} \cdot \frac{\left\{A - Z^2\alpha^2\left[2B + (C/2 - 3B)\,\mu/m_e + \frac{C}{2}(\mu/m_e)h_e\right]\right\}_{em}}{\left\{A - Z^2\alpha^2\left[2B + (C/2 - 3B)\,\mu/m_e + \frac{C}{2}(\mu/m_e)h_e\right]\right\}_{arr}}, \tag{24}$$

where subscripts indicate the time where the corresponding expressions should be evaluated (emission or arrival), and the coefficients A, B, C are given by eqs. (20). Expression (24) is the predicted outcome of a frequency measurement.

In general relativity μ and α are constants, and the overall coefficient $a(\eta_{arr})/a(\eta_{em})$ in eq. (24) is the redshift, which we have derived without explicitly assuming that proper times are determined by the metric. In the present context, the redshift z is rather given by

$$1 + z \equiv \frac{a(\eta_{arr})\mu(\eta_{arr})}{a(\eta_{em})\mu(\eta_{em})}.$$

In the absence of fine-structure corrections ($B = C = 0$) variations of α cannot be distinguished from redshifts. However, by considering several transitions (several values of A, B and C), information about the values of $a \cdot \mu$, α, μ/m_e and h_e at different times can be in principle extracted from frequency measurements. At present $\alpha_0 \approx 1/137$, for an hydrogen atom $(\mu/m_e)_0 \approx 1 - 5 \cdot 10^{-4}$, the leading electromagnetic contribution to the g-factor of the electron is $h_e^{em} \approx \alpha/\pi + \mathcal{O}(\alpha^2)$, and the non-electromagnetic contribution is limited by $h_e \lesssim 10^{-10}$ [24]. The important point is that the right hand side of eq. (24) might differ from $(1 + z)^{-1}$ even if α is constant.

V. CONCLUSIONS

We have attempted to construct the most general low-energy action consistent with basic field content and symmetry requirements under the assumption of the existence of a "light" scalar field. Up to redefinitions of fields and couplings, these requirements uniquely determine the form of the effective action. In this framework, it is always possible to choose the gravitational coupling and the fermion charges to be constant. However, fermion masses and g-factors, as well as the electromagnetic coupling strength are generically scalar field dependent, and hence, possibly time-varying.

Because most of the theory parameters and couplings depend on ϕ, the observed frequency of a photon emitted in an atomic transition depends on the values of these parameters both at the time of emission and observation. Concretely, the outcome of such a frequency measurement might be used to determine not only changes in the fine structure constant, but also in the g-factor of the electron and in appropriate mass ratios. Light always propagates along null geodesics, and its frequency is not influenced by changes in the electromagnetic coupling strength as long as these changes are "slow".

We have also addressed the issue about the dependence of our results on the choice of conformal frame. In the context of our toy experiment, frequency measurements are conformal frame independent, as expected.

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APPENDIX A: CONFORMAL FRAME INDEPENDENCE

Our derivation of the "redshift" measured by an observer, eq. (24), has assumed that the action is given by eq. (12). In particular we have worked in the "Einstein" conformal frame, where the function multiplying the scalar curvature is a constant. Our original "Jordan-frame" action (8) actually contained a field-dependent function B_g multiplying R, but we were able to remove it by the conformal transformation (9). A question which has been repeatedly discussed in the literature [9] is whether actions that differ by a conformal transformation are equivalent. If they were not, our simplification might not be justified. Certainly, two such actions are mathematically equivalent, in the sense that solutions to the equations of motion in one frame are mapped by the conformal transformation into solutions of the equations of motion in the conformally related frame[2]. However, this does not automatically imply that they are physically equivalent, in the sense of "experimentally meaningful" predictions being identical. Indeed, because there is no general framework to formulate how "experimentally meaningful" predictions are to be extracted from a a particular set of fields, the issue cannot be addressed in full generality.

[2] We assume that no singularities appear in the transformation [25].

Our goal is to show that in the context of redshift measurements in our toy universe, conformally related actions are indeed equivalent, as might seem to be obvious if one regards the conformal transformation merely as a field-redefinition. For the purpose of illustration, consider an "expanding" universe where the scale factor a in eq. (17) grows with time and fermion masses are constant. In a conformally related metric, eq. (9), the scale factor $\widetilde{a}$ is given by

$$\widetilde{a} = \Omega\, a, \tag{A1}$$

and fermion masses vary according to eq. (10). By an appropriate choice of the arbitrary function Ω, the conformally related universe might be "contracting" (decreasing $\widetilde{a}$). A priori, one expects both expanding and contracting universes to be quite different. Let us nevertheless study how an observer could determine whether a universe expands or contracts. Recall that the first experimental evidence for the expansion of the universe was E. Hubble's measurements of redshifted galaxy spectra. As a matter of fact, our predicted redshift, eq. (24) shows that the observed frequency of the photons is proportional to $1/a$. The conformally related action predicts a frequency which is given simply by replacing the parameters that enter eq. (24) by their analogues in the conformally related frame,

$$\widetilde{\left(\frac{\omega_{arr}}{\omega_{clock}}\right)} = \frac{(\widetilde{a}\,\widetilde{\mu}\,\widetilde{\alpha}^2)_{em}}{(\widetilde{a}\,\widetilde{\mu}\,\widetilde{\alpha}^2)_{arr}} \cdot \frac{\left\{A - Z^2\widetilde{\alpha}^2\left[2B + (\frac{C}{2} - 3B)\widetilde{\mu}/\widetilde{m}_e + \frac{C}{4}(\widetilde{\mu}/\widetilde{m}_e)\widetilde{h}_e\right]\right\}_{em}}{\left\{A - Z^2\widetilde{\alpha}^2\left[2B + (\frac{C}{2} - 3B)\widetilde{\mu}/\widetilde{m}_e + \frac{C}{4}(\widetilde{\mu}/\widetilde{m}_e)\widetilde{h}_e\right]\right\}_{arr}}. \tag{A2}$$

Because of eq. (10) the reduced mass scales as $\tilde{\mu} = \Omega^{-1}\mu$, and therefore, using eqs. (A1), (11) and bearing in mind that $q_e = \widetilde{q}_e$ we find

$$\widetilde{\left(\frac{\omega_{arr}}{\omega_{clock}}\right)} = \left(\frac{\omega_{arr}}{\omega_{clock}}\right). \tag{A3}$$

Both observers measure the same frequency ratios, the two actions are physically equivalent at this level. Nevertheless, the behavior of the scale factor in the two frames appears to be completely different. In fact, as we have seen, even if atomic spectra appear to be redshifted, in some conformal frames the universe might actually be "contracting" [26].

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