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Liberating the Inflaton from Primordial Spectrum Constraints

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I discuss a mechanism that renders the spectral index of the primordial spectrum and the inflationary stage independent of each other. If a scalar field acquires an appropriate time-dependent mass, it is possible to generate an adiabatic, Gaussian scale invariant spectrum of density perturbations during any stage of inflation. As an illustration, I present a simple model where the time-dependent mass arises from the coupling of the inflaton to a second scalar. The mechanism I propose might help to implement a successful inflationary scenario in particle physics theories that do not yield slow-roll potentials.

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I. INTRODUCTION

Observations impose significant constraints on eventually successful inflationary models. Current experimental results are consistent with a nearly scale invariant spectrum of Gaussian, adiabatic perturbations [1]. There is a wide class of inflationary models that yield such a spectrum [2]. In essentially all these models, the spectrum is nearly scale invariant because the universe expansion closely resembles a de Sitter stage [3]. In many cases however, particularly when trying to embed inflation within particle physics theories, it turns out that it is difficult to obtain quasi de Sitter inflation, either because potentials are too steep [4] or because the slow-roll regime does not overlap with the regime where the theory is under control [5].

Unlike the slope of the primordial spectrum, its amplitude does not necessarily depend on the inflationary epoch itself. If primordial perturbations originate from the decay of a "curvaton" field [6], or from the fluctuating couplings "constants" of the inflaton [7, 8], the final amplitude of the spectrum turns out not to be directly related to inflation. Nevertheless, these scenarios still had to contain a stage of de Sitter inflation, in order for the perturbations in the curvaton or the coupling constants of the inflaton to be to scale invariant.

In this paper, I propose a mechanism that additionally decouples the spectral index from the physics of the inflaton. In this scenario, fluctuations are imprinted on a "test" scalar field whose mass changes with time. The time-varying mass reproduces the effects of gravity during a de Sitter stage, even though the universe is not expanding exponentially fast. At the end of inflation the fluctuations imprinted in the test field are transferred to the decay products of the inflaton by the mechanism proposed by Dvali, Gruzinov and Zaldarriaga [7]. In that way, the liberated inflaton does not have to satisfy constraints from the amplitude and slope of the primordial spectrum.

The paper is organized I follows. In Section II, I describe how a scalar with a time-varying mass can lead to a scale invariant spectrum of primordial density perturbations. In Section III, I present an example where the changing mass is due to the coupling to the scalar that drives inflation. In Section IV, I try to extend the mechanism to a non-inflating universe, and in Section V I draw the conclusions.

II. TIME-VARYING MASS

Consider a test scalar field φ with mass m in an expanding, flat, linearly perturbed Friedmann-Robertson-Walker universe,

$$ds^2 = a^2(\eta)\left[(1+2\Phi)d\eta^2 - (1-2\Phi)d\vec{x}^2\right]. \qquad (1)$$

Here, a is the scale factor and Φ the gravitational potential (in longitudinal gauge). Neglecting for the moment metric perturbations, the scalar field equation of motion is

$$v_k'' + \left(k^2 + m^2 a^2 - \frac{a''}{a}\right) v_k = 0, \qquad (2)$$

where $v = a\,\varphi$, a prime denotes a derivative with respect to conformal time η, and the subindex k denotes the k Fourier component. Because Eq. (2) is linear in v, the equation of motion for φ and its perturbations $\delta\varphi$ agree.

For simplicity, let me momentarily consider a power-law inflating universe,

$$a \propto |\eta|^{\frac{\beta}{1-\beta}}. \qquad (3)$$

In cosmic time, the last equation corresponds to $a \propto t^\beta$. Hence, the universe inflates for $\beta > 1$, and in that case conformal time η runs from $-\infty$ to 0. Suppose now that the squared mass of the field is proportional to the squared Hubble parameter,

$$m^2 = c \cdot H^2, \qquad (4)$$

where c is a (dimensionless) constant coefficient. Such a relation simply arises for instance if the scalar is non-minimally coupled to gravity,

$$L_\varphi = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{c\cdot\beta}{12(2\beta-1)}\,R\,\varphi^2, \qquad (5)$$

or, as I discuss in Section III, it can arise from the coupling of φ to a second scalar field χ,

$$L_\varphi = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2(\chi)\varphi^2. \qquad (6)$$

Also, it has been observed that supersymmetry breaking in the early universe induces scalar field masses of the order of the Hubble parameter along flat directions [9]. For our present purposes though, it will suffice to treat Eq. (4) as a phenomenological relation.

If a is given by Eq. (3), and m^2 is given by Eq. (4) the equation of motion (2) reads

$$v_k'' + \left(k^2 - \frac{\nu^2 - 1/4}{\eta^2}\right) v_k = 0, \tag{7}$$

where

$$\nu = \frac{\sqrt{(9-4c)\beta^2 - 6\beta + 1}}{2(\beta - 1)}. \tag{8}$$

The solution of Eq. (7) with appropriate "adiabatic vacuum" initial conditions [10] is

$$v_k = \sqrt{\frac{\pi(-\eta)}{2}} H_\nu(-k\eta), \tag{9}$$

where H_ν is the Hankel function of the first kind. Because the fluctuations in φ arise from vacuum fluctuations, $\delta\varphi_k$ is a Gaussian variable.

The power spectrum $\mathcal{P}_\varphi$ is a measure of the mean square fluctuations of φ on comoving lengthscales $1/k$, and it is defined by [11]

$$\mathcal{P}_\varphi = \frac{k^3}{4\pi^2} \frac{|v_k|^2}{a^2}. \tag{10}$$

Cosmologically relevant modes are larger than the Hubble radius at the end of inflation. In this long-wavelength limit, the power spectrum is then

$$\mathcal{P}_\varphi = \frac{2^{2\nu}|\Gamma(\nu)|^2}{8\pi^3} \left(\frac{\beta-1}{\beta}\right)^{2\nu-1} H^2 \cdot \left[\left(\frac{H}{H_*}\right)^{\beta-1} \frac{k}{k_*}\right]^{n_s-1}. \tag{11}$$

Note that the amplitude of the spectrum is time-dependent. The comoving scale k_* is an arbitrary reference scale, and H_* denotes the value of the Hubble constant when that scale crosses the Hubble radius, $a/k_* = H_*^{-1}$. In the following, I denote by k_* the scale that corresponds to our present Hubble radius. The spectral index is

$$n_s - 1 = 3 - 2\nu. \tag{12}$$

A scale invariant Harrison-Zeldovich spectrum, corresponds to $n_s = 1$. Hence, from Eqs. (8) and (12) scale invariance requires

$$c = \frac{3\beta - 2}{\beta^2}. \tag{13}$$

Because cosmic microwave background anisotropies [1] limit the departures from $n_s = 1$ to less than about ten per cent, for given β or order one, c has to agree with Eq. (13) roughly to that accuracy. Therefore, from Eq. (4), the squared mass is positive during power-law inflation, negative (tachyonic) during pole-like inflation and vanishing for de Sitter inflation. Here, I restrict myself to $1 < \beta \leq \infty$. If c is given by Eq. (13) the power spectrum is then

$$\mathcal{P}_\varphi = \left(\frac{\beta-1}{\beta}\right)^2 \frac{H^2}{4\pi^2}. \tag{14}$$

Thus, whereas the fluctuations of φ in de Sitter approach a constant value $H/2\pi$, they decay as $a^{-1/\beta}$ if $\beta \neq \infty$.

The mechanism I have just described successfully generates a scale invariant spectrum of perturbations in φ, which is *not* what is required. Current experiments favor a scale invariant spectrum of Gaussian, adiabatic, *density perturbations.* Hence, perturbations in φ need to be transferred to perturbations in the radiation produced at the end of inflaton. As I mention in detail in next Section, this can be accomplished if the field φ determines the value of the couplings constants of the inflaton to its decay products [7, 8]. Of course, if primordial density perturbations originate form fluctuations in φ, it is important that perturbations due to the inflaton be significantly smaller than the ones due to the spatial variation of φ. The power spectra of scalar metric perturbations $\mathcal{P}_\Phi$ and gravitational waves $\mathcal{P}_h$ seeded during inflation are (see for instance [10])

$$\mathcal{P}_\Phi^{inf} \sim \beta \frac{H_*^2}{M_{Pl}^2} \left(\frac{k}{k_*}\right)^{-\frac{2}{\beta-1}} \sim \beta\, \mathcal{P}_h^{inf}. \tag{15}$$

Because both spectra are red, the highest amplitude in an observable mode is attained for the present horizon k_*. Cosmic microwave measurements have determined that $\mathcal{P}_\Phi \sim 10^{-10}$. Hence, in order for scalar perturbations seeded during inflation not to account for the observed anisotropies, cosmic inflation has to occur at a low energy scale,

$$\beta \frac{H_*^2}{M_{Pl}^2} \ll 10^{-10}. \tag{16}$$

This also implies that gravitational waves have a negligible impact on the cosmic microwave background.

Although for simplicity I have focused on power-law inflation, this scenario can be easily generalized to any epoch of inflation. Suppose that an arbitrary $a(t)$ is given. All we need is that in the given FRW spacetime there is no particle horizon. Then, the integral

$$\eta_e - \eta \equiv \int_t^{t_e} \frac{d\tilde{t}}{a(\tilde{t})} \tag{17}$$

diverges as t approaches cosmic time origin t_i. Therefore, conformal time runs from $-\infty$ to η_e, where η_e is an arbitrary end time which I shall identify with the end of inflation. In such a spacetime there is an apparent "event horizon", i.e. light emitted at time t can reach an observer before time t_e only if it is emitted within a physical distance

$$d_E = a(\eta)(\eta_e - \eta). \tag{18}$$

Note that this apparent event horizon is not in general a real event horizon, since the upper limit in the integral (17) is kept fixed and finite. The apparent event horizon is not the Hubble radius H^{-1} either, though in many cases they are roughly the same.

Suppose now that the mass of a scalar field is given by

$$m^2 = \frac{a''}{a^3} - \frac{2}{d_E^2}. \tag{19}$$

Because there is an arbitrary freedom in the coupling of φ to a second scalar field, such an evolution of the squared mass can be always achieved. Substituting Eq. (19) into Eq. (2) I get

$$v_k'' + \left(k^2 - \frac{2}{(\eta_e - \eta)^2}\right) v_k = 0. \tag{20}$$

A conformal time shift in Eq. (7) leads to Eq. (20). Hence, the solutions of the latter equation are given by Eq. (9), with $-\eta$ replaced by $\eta_e - \eta$. At early times, $\eta_e - \eta$ approaches infinity, i.e. all modes are initially within the horizon, $a/k \ll d_E$. At late times, $\eta_e - \eta$ approaches zero and all the modes are super-horizon sized, $a/k \gg d_E$. In this long-wavelength limit, the spectral index is still given by Eq. (12), where, from Eqs. (20) and (7), $\nu = 3/2$. Again, this corresponds to a scale invariant spectrum.

III. A CONCRETE EXAMPLE

It might seem that the procedure described above is extremely fine-tuned since, according to Eq. (19), the time evolution of a squared mass has to accurately reflect the a-priori independent expansion history. However, in the presence of (non slow-roll) inflationary attractors, it turns out that the required values of the squared mass come about surprisingly naturally.

Consider for instance two coupled scalar fields in the presence of Einstein gravity,

$$\int d^4x\sqrt{-g}\Bigg[-\frac{M_{Pl}^2}{16\pi}R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \left(1 + \frac{4\pi\,\beta\, c}{3\beta - 1}\cdot\frac{\varphi^2}{M_{Pl}^2}\right)V(\chi) + L_m\Bigg]. \tag{21}$$

Here, $M_{Pl} = G^{-1/2} \approx 10^{19}\,\text{GeV}$ is the Planck mass, and

$$V(\chi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{\beta}}\frac{\chi}{M_{Pl}}\right). \tag{22}$$

In our example, the field φ is assumed to remain at $\varphi = 0$. Hence, it is sufficient to consider a quadratic φ^2 term in the action (21). The inclusion of higher even powers of φ in the action will not change our results[1]. Exponential potentials and couplings naturally appear in string theory (from the dilaton), and/or in theories with extra dimensions (from the radion). L_m stands for additional matter terms only relevant during reheating; I shall write them down below.

Suppose that initially the field φ sits at the origin $\varphi = 0$. Because the squared mass of the field is positive, $\varphi = 0$ is a stable solution of the background equations. If $\varphi = 0$, the field χ effectively evolves in the single field potential (22). These potentials are known to have power-law inflationary attractors [12], along which the expansion of the universe is given by Eq. (3) and

$$\frac{V(\chi)}{M_{Pl}^2} = \frac{3\beta - 1}{8\pi\beta}H^2. \tag{23}$$

The coefficient in front of φ^2 in the action (21) implies that the field φ has a χ-dependent mass

$$m_\varphi^2(\chi) = \frac{8\pi\,\beta\, c}{3\beta - 1}\frac{V(\chi)}{M_{Pl}^2}. \tag{24}$$

Thus, from Eqs. (24) and (23), along the inflationary attractor the squared mass of φ is precisely given by Eq. (4).

The linearized equations for the perturbations are [13]

$$\begin{aligned}
\delta\varphi'' &+ 2\mathcal{H}\delta\varphi' + k^2\delta\varphi - 4\varphi'\,\Phi' + 2\, m_\varphi^2\, a^2\,\varphi\,\Phi + \\
&+ m_\varphi^2\, a^2\,\delta\varphi + \frac{dm_\varphi^2}{d\chi}\, a^2\,\varphi\,\delta\chi = 0, \qquad (25)\\
\delta\chi'' &+ 2\mathcal{H}\delta\chi' + k^2\delta\chi - 4\chi'\Phi' + 2\frac{dV}{d\chi}a^2\,\Phi + \\
&+ \frac{dm_\varphi^2}{d\chi}\, a^2\,\varphi^2\,\Phi + \frac{d^2V}{d\chi^2}\, a^2\,\delta\chi + \\
&+ \frac{1}{2}\frac{d^2m_\varphi^2}{d\chi^2}\, a^2\,\varphi^2\,\delta\chi + \frac{dm_\varphi^2}{d\chi}\, a^2\,\varphi\,\delta\,\varphi = 0, \qquad (26)\\
\Phi' &+ \mathcal{H}\Phi = \frac{4\pi}{M_{Pl}^2}\left(\varphi'\delta\varphi + \chi'\delta\chi'\right), \qquad (27)
\end{aligned}$$

where $\mathcal{H} \equiv a'/a$. Consequently, to linear order around the solution $\varphi = \varphi' = 0$, perturbations in φ do not couple to inflaton or metric perturbations and vice versa. In particular, substituting $\delta\varphi = v/a$ into Eq. (25) yields Eq. (2). In summary, our model satisfies the assumptions made in Section II.

If the inflaton potential is globally given by Eq. (22) inflation never ends. As in conventional power-law inflationary models, I shall assume that $V(\chi)$ develops a minimum around $\chi = \chi_{end}$. Hence, when χ reaches the vicinity of χ_{end}, inflation ends and χ starts oscillating, thus reheating the universe. During the oscillating phase,

[1] As long as the coefficients of those terms are not too large.

the universe evolves as if dominated by dust, and the time average of $V(\chi)$ is

$$\frac{\langle V(\chi)\rangle}{M_{Pl}^2} = \frac{3}{16\pi}\langle H^2\rangle. \tag{28}$$

During that time the squared mass of the field is still proportional to the squared Hubble parameter and the shape of the spectrum for modes larger than the Hubble radius remains unaltered. I shall assume that the amplitude of the $\delta\varphi$ perturbations hardly changes during a short, almost instantaneous stage of reheating, though parametric resonance might under certain circumstances significantly boost the initial amplitude [14].

Reheating occurs due the coupling of the inflaton χ to additional matter fields. In order to transfer the scale invariant spectrum of perturbations imprinted in φ into the decay products of the inflaton, I assume, following [7], that the action (21) contains terms of the form

$$L_m = -\lambda_0 \left(1 + f\frac{\varphi}{M_{Pl}}\right) \chi\bar{\psi}\psi. \tag{29}$$

Here, the fermion ψ generically stands for the decay products of the inflaton, and λ_0 and f are two dimensionless coupling constants. The energy density at the end of reheating is [15]

$$\rho_R \sim T_R^4 \sim \lambda_0^4 \left(1 + f\frac{\varphi}{M_{Pl}}\right)^4 m_\chi^2 M_{Pl}^2 \tag{30}$$

where T_R is the reheating temperature and m_χ is the inflaton mass during the oscillating stage. Therefore, because during inflation $\varphi = 0$, fluctuations in φ produce fluctuations in the energy density [7]

$$\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T} \sim f\frac{\delta\varphi}{M_{Pl}}, \tag{31}$$

which cosmic microwave background measurements [1] require to be are around 10^{-5}. Note that the amplitude of the temperature perturbations is fixed by f, whereas the reheating temperature itself is determined by λ_0 and m_χ. For a given reheating temperature, one can compute the number of e-folds of inflation N after the present horizon left the Hubble radius,

$$N \sim \frac{\beta}{\beta - 1}\log\left(\frac{T_0\, T_R}{H_0 M_{Pl}}\right), \tag{32}$$

where, $H_0 \sim 10^{26}\,\mathrm{m}$ and $T_0 \sim 3\,\mathrm{K}$ are respectively the present values of the Hubble constant and the CMB temperature. During this number of e-folds, the seeded modes span a window in k space

$$\frac{k_{end}}{k_*} = \exp\left(\frac{\beta - 1}{\beta}N\right) = \frac{T_0\, T_R}{H_0 M_{Pl}}. \tag{33}$$

Thus, the amount of seeded modes does not agree with the amount of inflation. The cosmologically accessible window k/k_* spans three to four logarithmic decades.

Observations are consistent with adiabatic primordial perturbations, though significant amounts of non-adiabaticity, of the order of several tenths per cent, are still compatible with observations [16]. Because in our scenario reheating is driven by the oscillations of a single component, the χ field, the spectrum of density perturbations produced at the end of reheating is adiabatic [17]. Current observations also restrict the amount of non-Gaussianity of the perturbations. It is typically quantified by means of the relation $\Phi = \Phi_g + f_{NL}(\Phi_g^2 - \langle\Phi_g^2\rangle)$ [18], which expresses metric perturbations in terms of a Gaussian random field Φ_g. The amount of non-Gaussianity produced in our model was carefully estimated in [19], where it was found that f_{NL} is of order one. This is well within the experimental limits $-58 < f_{NL} < 134$ [20], but might be potentially detectable in the future [18].

To conclude this section, let me show that, at least at first sight, this model is phenomenologically viable. Assume $m_\chi \sim 10^2$ TeV and $\lambda_0 \sim 10^{-6}$. From Eq. (30), the reheating temperature is $T_R = 10^3\,\mathrm{TeV}$, well above the nucleosynthesis limit. Substituting into Eq. (33) I find $k_{end}/k_* \sim 10^{16}$, which is much bigger than the required 10^4. Let me set $\beta = 4$. For a vanishing mass ($c = 0$), power-law inflation with such an exponent would yield a spectral index $n_s = 1/3$ [10], very far form the experimentally favored scale invariant spectrum $n_s \approx 1$. With a time-dependent mass and $c = 5/8$, the spectrum is scale invariant. The number of e-folds from the time our present horizon left the Hubble radius till the end of inflation is then $N \sim 49$, and the total number of e-folds of inflation is larger. This suffices to explain the homogeneity and flatness of the universe. Let me choose in addition $V(\chi_{end}) \sim m_\chi^4$. Then, the value of the squared Hubble parameter at crossing was $H_*^2 \sim 10^{-45} M_{Pl}^2$, which satisfies the constraint (16). If in addition we choose $\lambda_0 f/M_{Pl} \sim 10^2/m_\chi$ we find that (31) reproduces the observed value of the primordial spectrum amplitude.

IV. NON-INFLATING UNIVERSE

In conventional inflationary models, primordial perturbations are causally seeded when modes exit the sound horizon (which for a scalar field is of the order of the Hubble radius). In the scenario I have previously described, the Hubble radius and the Compton radius m^{-1} are of the same order, so one could argue that modes are causally seeded because the Compton radius grows slower than the physical wavelength of the perturbations. Similarly, one could envisage a scenario where perturbations are causally seeded in a non-inflating universe because the Compton radius starts large and subsequently does not grow fast enough. Let me illustrate this idea in a concrete setting.

Consider this time an expanding, non-inflating universe. Then, the integral (17) converges as cosmic time approaches the origin t_i. In that case, there is a particle horizon

$$d_P = a(\eta)(\eta - \eta_i) = a \int_{t_i}^{t} \frac{d\tilde{t}}{a(\tilde{t})}. \qquad (34)$$

Suppose that in analogy with Eq. (19), the squared mass of a scalar is given by

$$m^2 = \frac{a''}{a^3} + m_{eff}^2, \qquad (35)$$

where

$$m_{eff}^2 = -\frac{2}{a^2(\eta_e - \eta)^2} \qquad (36)$$

and η_e is a constant with dimensions of length. As in Section III, such a relation can be obtained by coupling φ to a second scalar χ. The rationale of considering this squared mass becomes manifest when writing down the field equation of motion (2), which takes the form of Eq. (20). Thus, the equations of motion of the perturbations both in an inflating as well as in a non-inflating universe are formally the same. As in Section II, this leads to a scale invariant spectrum.

There is a crucial difference though between an inflating and a non-inflating universe. In an inflating universe, η runs from $-\infty$ to η_e. Therefore, in Eq. (20) the k^2 term dominates at early times (short wavelength regime), and the $1/(\eta_e - \eta)^2$ term dominates at late times (long wavelength regime). This is why modes exit the Hubble radius. In a non-inflating universe though, η runs from 0 to η_e, and hence, modes start in the short-wavelength regime only if η_e is big enough. Of course, the difference between inflating and non-inflating spacetimes rests on the existence or absence of particle and apparent event horizons.

The mode evolution is shown in Figure 1. A given perturbation mode might be initially super-Hubble sized but still within the Compton radius. This requires that the total effective mass of the scalar be small compared to the Hubble radius. Because the coupling to gravity generates a (tachyonic) mass of the order of the Hubble radius, $m^2 \approx -a''/a^3$ (see Eq. (2)), the former requirement forces a cancellation between that term and the one coming from the "true" mass. This is the origin of the first term on the r.h.s of Eq. (35). Note that even though this scenario can seed perturbations on super Hubble scales, it does not violate causality. The reason is that the evolution of perturbations on different lengthscales is dictated by a time-varying mass which is the same in the whole universe. Thus, the ultimate origin of super Hubble correlations is the homogeneity of the universe, which, in the absence of cosmic inflation, we assume rather than explain.

Can this scale invariant spectrum seeded during a non-inflationary stage spectrum account for cosmologically relevant modes? Consider now a stage of power-law expansion, Eq. (3), with $0 < \beta < 1$. In that case, Eq. (36)

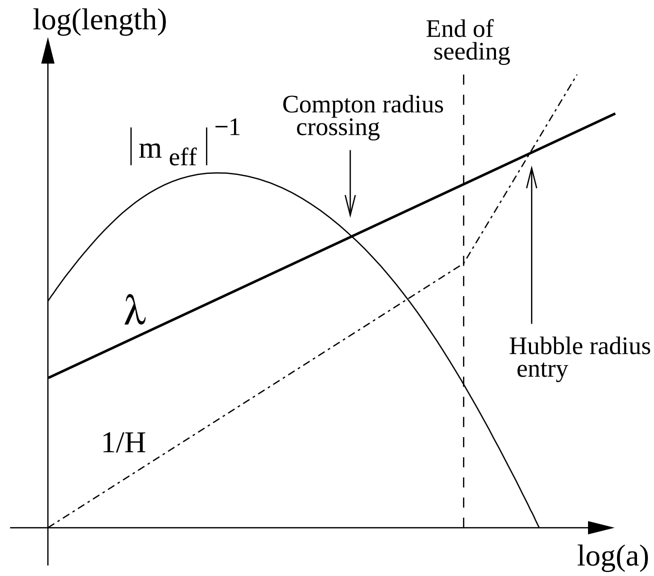

FIG. 1: A schematic plot of the evolution of the different length scales. The physical length of a given mode is shown with a thick line. Initially, the mode is outside the Hubble radius (dot-dashed line) but inside the Compton radius (continuous line). As the mass of the field evolves, the mode exits the Compton radius, freezes and later enters the Hubble radius.

takes the form

$$m_{eff}^2 = -2\,\frac{(1-\beta)^2}{\beta^2}\left[\left(\frac{H}{H_e}\right)^{1-\beta} - 1\right]^{-2} H^2, \qquad (37)$$

where H_e is the value of H when $\eta = \eta_e$ (at that time m_{eff} diverges). Our present comoving horizon is $k_* = a_0 H_0$. Suppose that this mode left the effective Compton radius N e-folds before the end of seeding, which concluded at temperature T_R. The viability of this scenario requires that seeding ends before time η_e. The condition of effective Compton radius crossing, $a/k_* = |m_{eff}|^{-1}$, then translates into

$$\left(\frac{T_R^2}{H_e M_{Pl}}\right)^{1-\beta} - e^{(1-1/\beta)N} \sim \frac{1-\beta}{\beta}\,\frac{T_0^2}{H_0\, M_{Pl}}\,\frac{T_R}{T_0}. \qquad (38)$$

Let us assume that the seeded ended before nucleosynthesis. Nucleosynthesis occurs around $T \sim 10^{10}\, T_0$. Hence, unless β is very close to one, Eq. (38) implies

$$\left(\frac{T_R^2}{H_e M_{Pl}}\right)^{1-\beta} \geq 10^{-3}\frac{T_R}{T_0} \geq 10^7 \gg 1. \qquad (39)$$

Using the crossing condition for the mode that left the Compton radius at the end of inflation, one can derive the amount of seeded modes,

$$\frac{k_{end}}{k_*} \sim e^N \frac{T_R^2}{H_* M_{Pl}}\,\frac{(H_*/H_e)^{1-\beta} - 1}{(T_R^2/H_e M_{Pl})^{1-\beta} - 1}. \qquad (40)$$

Evaluating the last expression in the limit (39) I find

$$\frac{k_{end}}{k_*} \sim 1, \tag{41}$$

where I have used the fact that energy density decreases during expansion, $H_*^2 \sim \rho_*/M_{Pl}^2 > T_R^4/M_{Pl}^2$. Therefore, this scenario is unable to explain the origin of the cosmologically relevant window $k/k_* \sim 10^3$. There are only two escapes I can think of. The first is to assume that $\beta \approx 1$, but then, the spacetime is already at the verge of inflating. The second is to assume that seeding proceeds during a stage of radiation domination, $\beta = 1/2$. This allows the end of seeding to occur after nucleosynthesis. In that way, the estimate in Eq. (39) can avoided simply by setting $T_R/T_0 \sim 10^3$, which corresponds to the temperature around the time when decoupling occurs. But at that time, perturbations have to be already in place and the universe is rather matter-dominated. It is doubtful that this scenario can work even in such an extreme case.

V. SUMMARY AND CONCLUSIONS

During cosmic expansion, gravity contributes a time-dependent correction to the total effective mass of a minimally coupled scalar field, Eq. (2). If this total effective mass evolves appropriately, a scale invariant spectrum of scalar fluctuations results, Eq. (14). The ultimate origin of the effective time-dependent mass is however not very important. It can arise solely from the expansion of the universe, as in a de Sitter universe or, as I have explored in this paper, it can also arise from the coupling of the scalar to other evolving fields. As a particular example, I have shown that it is possible to seed a scale invariant spectrum of perturbations in a scalar field during any stage of power-law inflation. A concrete model that realizes this setting, Eq. (21), relies on two coupled scalars and looks surprisingly simple. One of the fields is the inflaton, which drives power-law inflation, and the other is a test scalar field sitting at the minimum of its effective potential, upon which a scale invariant spectrum of fluctuations is imprinted.

A scale invariant spectrum of scalar field perturbations does not suffice however to account for the observed spectrum of density perturbations. If there was no way to transfer the field perturbations to energy density perturbations, this scenario would not be realistic. Recently though, a new mechanism has been proposed to transfer scalar field perturbations to density perturbations [7, 8]. If the couplings of the inflaton to its decays products are field dependent, fluctuations in the latter can be converted into fluctuations in matter and radiation. As any model based on the reheating mechanism of [7], our scenario predicts a substantial larger, though observationally consistent, degree of non-Gaussianity in the primordial spectrum. Also, there is no substantial production of gravitational waves because the amplitude of perturbations in the inflaton are assumed to be insufficient to account for the observed temperature anisotropies.

The idea I have discussed can also explain a scale invariant spectrum of density perturbations during a non-inflationary stage of expansion. In this case, modes are seeded when they cross an effective Compton radius which evolves in time. But the generated spectrum cannot encompass a sufficient window of modes around the present horizon. In this case, the origin of large (but not large enough) scale correlations can be traced back to the homogeneity of the universe, which is assumed, rather than explained.

In summary, in the mechanism I have described the primordial spectrum does not depend on the nature of the inflationary stage. As a consequence, the inflaton does not have to account for neither the amplitude nor the spectral index of the primordial spectrum. Hence, "liberated inflation" can be useful in the context of physical theories that do not have slow-roll potentials.

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