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New Analytical Results on Anisotropic Membranes

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We report on recent progress in understanding the tubular phase of self-avoiding anisotropic membranes. After an introduction to the problem, we sketch the renormalization group arguments and symmetry considerations that lead us to the most plausible fixed point structure of the model. We then employ an ε-expansion about the upper critical dimension to extrapolate to the physical interesting 3-dimensional case. The results are $\nu = 0.62$ for the Flory exponent and $\zeta = 0.80$ for the roughness exponent. Finally we comment on the importance that numerical tests may have to test these predictions.

1. Introduction

The statistical properties of D-dimensional objects embedded in d-dimensional space have been the subject of intense analytical and numerical work in the last ten years. An introduction to the problem, as well as an update with some recent results has been already presented in M. Bowick's talk [1] and in the plenary talk [2]. These studies are of direct experimental interest for cases such as $(D = 2, d = 3)$ (membranes) or $(D = 1, d = 3)$ (polymers) (see [1]).

In [3] it was shown that anisotropy has a remarkable effect in a model of phantom crystalline surfaces; there is a new phase, the tubular phase, characterized by being flat in one internal direction and crumpled in the other ones. This new phase has been nicely corroborated by numerical simulations [4], see [1] for an update.

While the phantom membrane model is completely understood, both analytically and numerically, the situation for the more physical self-avoiding case has been rather controversial. Once the self-avoidance perturbation is added to the phantom model it was found in [5] that the large distance properties of self-avoiding anisotropic membranes are described by a new Fixed Point (the SAFP), different from the phantom one (the TPFP), but perturbative in $\varepsilon = 11 - d$. The phase diagram implied is shown in fig. 1. In [6] it was argued that the SAFP is infrared unstable, and consequently, the large distance properties of anisotropic self-avoiding membranes were described by a new Fixed Point (BRFP), which has a non-trivial anomalous dimension for the bending rigidity term. The phase diagram implied in [6] is the one depicted in fig. 2.

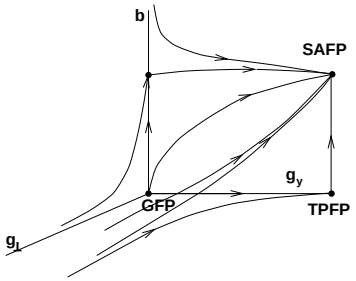

Figure 1. The phase diagram for self-avoiding anisotropic membranes with the Gaussian fixed Point (GFP), the phantom tubular fixed point (TPFP) and the self-avoidance fixed point (SAFP).

In this talk we report on new analytical results [7]. The aim of this work is twofold; first to clarify the phase diagram of the model, and second to compute the critical exponents which provide predictions to be tested numerically or even in actual experiments.

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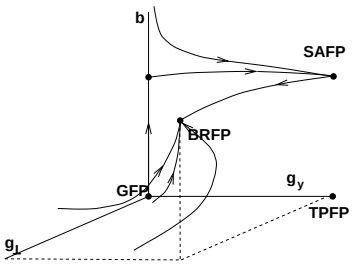

Figure 2. The phase diagram for self-avoiding anisotropic membranes with the Gaussian fixed Point (GFP), the phantom tubular fixed point (TPFP), the self-avoidance fixed point (SAFP) and the bending rigidity fixed point (BRFP).

2. The Phase diagram

The configuration of a membrane is described by giving the position $\vec{r}(\mathbf{x})$, in the d-dimensional embedding space, of a point in the membrane labeled by a D-dimensional internal coordinate $\mathbf{x}$. In the tubular phase, where the membrane is crumpled in $D-1$ dimensions (denoted by $\mathbf{x}_\perp$) and flat in a distinguished direction y, $\vec{r}$ is expanded as

$$\vec{r}(\mathbf{x}) = (\zeta_y y + u(\mathbf{x}), \vec{h}(\mathbf{x})). \tag{1}$$

The relevant degrees of freedom are then in-plane phonons u, and out-of-plane ones $\vec{h}$. Let us remark that those fields have different engineering dimensions, namely, different rescalings in the language of the Wilsonian renormalization group.

The most general free energy for this system was first written in [3]. It is constructed as a derivative expansion in $\vec{r}$ together with the requirements of translational and rotational symmetry. A direct general analysis of this free Energy is a very difficult task, close to impossible if self-avoidance is considered. Our interest, fortunately, is focused on the universal properties of the model, that is, its large distance properties. The challenge becomes to correctly identify the terms that do not affect the universal properties (the irrelevant ones). There is a powerful tool available for that task, the Renormalization Group.

The implementation of the Renormalization Group in this particular problem presents new and interesting features. For example, the rotations of the tubules imply that the free energy should be invariant under

$$\begin{array}{ll} u \to & u\cos\theta + \sin\theta h + (\cos\theta - 1)y \\ h \to & h\cos\theta - \sin\theta u - \sin\theta y \end{array} . \tag{2}$$

This symmetry mixes the $\vec{h}$ and u fields, which, as already pointed out, have different rescalings. It should not come as a surprise then, that the large distance realization of the $O(D-1)$ rotational symmetry is different from Eq. 2, [7]

$$\begin{array}{ll} u \to & u + \vec{A}\vec{h} - \frac{1}{2}\vec{A}^2 y + \mathcal{O}(e^{2(\nu-z)l}) \\ \vec{h} \to & \vec{h} - \vec{A}y + \mathcal{O}(e^{2(\nu-z)l}) \ , \end{array} \tag{3}$$

where $\vec{A}$ is an arbitrary $D-1$ dimensional vector.

The symmetry Eq. 3 provides an important guiding principle to elucidate the phase diagram. A more detailed analysis performed in [7] shows that the most plausible phase diagram when self-avoidance is included is the one in fig. 1. Of course, more complicated situations (reminiscent of the ones depicted in fig. 2 are not completely ruled out, but we do not find enough evidence to sustain them. A further clarification for this debate definitely requires a full treatment of both nonlinear elastic terms and self-avoidance, an open problem so far.

3. Critical exponents

The self-avoidance perturbation is relevant for any embedding dimension $d < d_c^{SA}$, where

$$d_c^{SA}(D) = \frac{6D-1}{5-2D} \ . \tag{4}$$

The fixed point of interest, the SAFP, is perturbative in ε, with $\varepsilon(D,d) = 3D - \frac{1}{2} - (\frac{5}{2} - D)d$.

In [5] a direct ε-expansion was performed at $D = 2$. In that case, $\varepsilon(2,d) = \frac{11-d}{2}$ and the extrapolation to the physical interesting case $d = 3$ was not found to be robust against higher order corrections (recall that $\varepsilon(2,3) = 4$). The values for the critical quantities obtained in [5] show a very large uncertainty. On the other hand, we

have emphasized how important is to get good estimates of the critical indices at the SAFP.

In [7], the one loop critical exponents are computed for arbitrary internal dimension D (For those values of D for which the model is well defined are $3/2 < D < 5/2$). This generalization provides a new extrapolation parameter, and allows for generalized ε expansions [8] that may deliver reliable results at $D = 2$.

The extrapolation techniques we used are rather sophisticated. Essentially they consist in re-expressing the critical exponents in new variables such that the next corrections get minimized. We tested different variables and different corrections to the exponents. We refer the interested reader to the original work [7] for details. We just quote the results for the two main critical exponents coming from the best extrapolations, which are listed in Table 1. The results are compared with the uncontrolled Flory estimate, which is usually a good approximation.

d	ν	ν_{Flory}	ζ	ζ_{Flory}
8	0.333(5)	0.333	0.60	0.600
7	0.374(8)	0.375	0.64	0.643
6	0.42(1)	0.429	0.68	0.692
5	0.47(1)	0.500	0.72	0.750
4	0.54(2)	0.600	0.76	0.818
3	0.62(2)	0.750	0.80	0.900

Table 1
Final results for critical exponents. ν is the Flory exponent and ζ is the roughness exponent. The Flory estimate is quoted as ν_{Flory}.

4. Conclusions and Outlook

In this talk we discussed the phase diagram for anisotropic membranes. We have definite analytical predictions for the critical exponents characterizing the tubular phase of self-avoiding membranes. At this stage, further refinement would necessitate a two-loop calculation for arbitrary D that, as a byproduct, would provide a valuable check for our extrapolation.

There are other aspects which are worth pointing out. As discussed in [1], existing arguments favors the belief that the crumpled phase for self-avoiding isotropic membranes disappears whenever bending rigidity terms are present, although a definite proof is still lacking. One might legitimately ask if the same is true for the tubular phase. Intuitively, one would think that this is not the case, since self-avoidance is less constraining in this case. In fact, the arguments that lead to the considerations above do not apply(and the corresponding molecular dynamics simulations are absent). Our conclusion is that the tubular phase should be observed both in numerical simulations and actual experiments so that our predictions can be tested.

There has been some progress in the numerical analysis of this model as reported in [1]. There is evidence that the tubular phase does survive, but the available data is not still enough to enable reliable estimates for critical exponents. Anyway, we hope that the calculations presented here will inspire further work on the numerical side. Further progress in understanding the self-avoiding tubular phase very much requires the insight of numerical work.

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