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Feng, Qu and Horrace, William C., "Estimating Technical Efficiency in Micro Panels" (2012). *Economics - Faculty Scholarship*. 13.
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Economics Letters 117 (2012) 730–733

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Estimating technical efficiency in micro panels

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ARTICLE INFO

Article history:
Received 31 January 2012
Received in revised form
11 August 2012
Accepted 16 August 2012
Available online 21 August 2012

JEL classification:
C15
C23
D24

Keywords:
Max operator
Stochastic frontier model
Bootstrap
Micro panel data

ABSTRACT

Bootstrap confidence intervals on fixed-effects efficiency estimates in micro panels exhibit low coverage probabilities. We propose an alternative efficiency measure involving the mean of the firm effects. With the same estimated efficiency ranks as the traditional measure, its corresponding bootstrap confidence intervals have better coverage probabilities.

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1. Introduction

In the panel stochastic frontier literature, fixed-effects estimation (Schmidt and Sickles, 1984) has the advantage of not requiring distributional assumptions on the error components. Consequently, the model is often employed in empirical research (e.g., Kumbhakar, 1987 and Bauer and Hancock, 1993). Confidence intervals for technical efficiency may be based on asymptotic normality, when the sample size is large (Horrace and Schmidt, 1996). However, in micro panels with small time dimensions, the preferred method of constructing confidence intervals without distributional assumptions is to perform the bootstrap. Simar (1992), Simar and Wilson (1998), Hall et al. (1993), and Kim et al. (2007) investigate confidence interval coverage rates for various bootstrap techniques based on the fixed-effects estimation, and find that the coverage is generally poor when the time dimension of the data is small. As discussed in Kim et al. (2007), the coverage erosion is due to the bias induced by the estimated maximum of the firm effects involved in the traditional efficiency estimate (Schmidt and Sickles, 1984). The received literature encourages techniques that improve coverage rates by accounting for this bias. However, the improved coverage of bootstrap confidence intervals is still not satisfactory in micro panels with small time dimensions and large numbers of firms. Even worse, when "ties" among firms occur (i.e., several firms are equally best) no bootstrap confidence intervals are reliable (Kim et al., 2007). Recently, Satchachai and Schmidt (2010) show that removing the "max" operator bias is possible using the generalized panel jackknife, but the resulting estimate has a large variance.

We approach this inference problem from a different perspective. The traditional efficiency measure defined in Schmidt and Sickles (1984) shifts the firm-specific effects by their maximum. We propose an alternative measure which shifts the firm effects by their mean. This efficiency measure yields the same efficiency rank of firms as the traditional measure. However, unlike the estimated maximum of firm effects, the estimated mean is unbiased, so the corresponding bootstrap confidence intervals for the new measures have better coverage probabilities. Thus, when the inference based on the traditional efficiency measure is inaccurate in micro panels or in the case that the identity of the best firm is uncertain, the proposed measure can be used as an alternative and reliable way to estimate firms' technical efficiencies. It should be made clear at the outset that the proposed measure is not a substitute for the traditional measure; it is an additional metric that should be a part of the empiricist's toolbox.

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doi:10.1016/j.econlet.2012.08.031

2. Fixed-effects efficiency measures

In a stochastic frontier model for panel data,

$$y_{it} = \alpha + x_{it}'\beta - u_i + v_{it}, \quad i = 1, \ldots, N, \; t = 1, \ldots, T. \tag{1}$$

The technical inefficiency of firm i is characterized by the time-invariant $u_i \geq 0$.[2] The idiosyncratic disturbance v_{it} is assumed i.i.d.$(0, \sigma_v^2)$. Usually, technical efficiency is defined as $r_i = \exp(-u_i)$ under a logarithmic specification of the Cobb–Douglas production function. The slope parameter β can be estimated consistently using fixed-effects estimation. Call this estimate $\widehat{\beta}$. The firm-specific production level (or firm effect) $\alpha_i = \alpha - u_i$ can be estimated by $\widehat{\alpha}_i = \overline{y}_i - \overline{x}_i'\widehat{\beta}$, where $\overline{y}_i$ and $\overline{x}_i$ are within-group averages. Since u_i cannot be identified without additional assumptions, Schmidt and Sickles (1984) suggest the relative efficiency measures,

$$u_i^* = u_i - \min_j u_j = \max_j \alpha_j - \alpha_i, r_i^* = \exp(-u_i^*),$$

with fixed-effects estimates,

$$\widehat{\alpha} = \max_j \widehat{\alpha}_j, \widehat{u}_i^* = \widehat{\alpha} - \widehat{\alpha}_i, \widehat{r}_i^* \equiv \exp(-\widehat{u}_i^*).$$

Per Schmidt and Sickles (1984), $\widehat{\beta}$ converges to β for large N or T, while $\widehat{\alpha}_i$ converges to α_i for large T only. Therefore, in micro panels with small T, asymptotic approximations for confidence intervals on functions of α_i are inappropriate, and a bootstrap method should be employed.

Kim et al. (2007) provide a detailed discussion of methods for bootstrap confidence intervals for technical efficiency $r_i^* = \exp(\alpha_i - \max_j \alpha_j)$, including percentile intervals, bias-corrected with acceleration (BC_a) intervals, bias-corrected (BC) percentile intervals and a parametric method. Overall, BC_a and BC percentile seem preferred considering the coverage and width of the intervals. However, in their simulation study (Kim et al., 2007, Table 3), even the BC_a intervals and BC percentile intervals have low coverage rates in the usual panel case. For example, the coverage rate of the BC_a intervals with nominal 90% significance is only 68.7% when $N = 100$, $T = 10$, $\gamma = 0.1$.[3] The corresponding BC percentile intervals have slightly higher coverage than BC_a intervals, but with bigger width. As we will see, the coverage rate is even lower when $T < 10$ and $N > 100$. The problem arises from the "max" operators in $\widehat{u}_i^*$ and $\widehat{r}_i^*$. Per Kim et al. (2007), the "max" operator induces an upward bias in $\widehat{\alpha}$ and $\widehat{u}_i^*$, and hence a downward bias in $\widehat{r}_i^*$. The "max" operator favors positive noise. When T is large, $\widehat{\alpha}_j \to \alpha_j$, $\max_j \widehat{\alpha}_j \to \max_j \alpha_j$, and the "max" operator bias is mitigated. However, this bias cannot be ignored in micro panels with small T and large N, which is the focus of this paper.

Since the "max" operator bias is the main source of the inaccurate inference for technical efficiency in micro panels, removing the bias could be a solution. A successful attempt of bias correction is made by Satchachai and Schmidt (2010) using the generalized panel jackknife, but the resulting estimate has a large variance. Alternatively, Feng and Horrace (2012) propose two efficiency measures involving $\min_j \alpha_j$, whose corresponding fixed-effects estimates preserve the same ranking as $\widehat{r}_i^*$. Although the bias issue persists for the fixed-effects estimates of their two measures, they show that bias is mitigated in certain situations related to the skew of the efficiency distribution.

This paper approaches this inference problem from a different perspective. Since the sample ranking of estimators $\widehat{r}_i^* = \exp(\hat{\alpha}_i - \max_j \hat{\alpha}_j)$ is the same as the ranking of unbiased estimators $\widehat{\alpha}_i$ with $\widehat{r}_i^*/\widehat{r}_j^* = \exp(\widehat{\alpha}_i - \widehat{\alpha}_j)$, it seems possible to construct an alternative efficiency measure containing the same information as r_i^* but without the "max" operator bias and its related issues. The traditional measure r_i^* is a monotonic transformation of $\alpha_i - \max_j \alpha_j$, a shift of α_i by $\max_j \alpha_j$. Using the maximum can be regarded as a normalization to produce efficiency 1 for the firm with the largest α_i in the sample. Similar in spirit to r_i^*, we propose an efficiency measure,

$$E_i = h\left(\alpha_i - \frac{1}{N}\sum_j \alpha_j\right),$$

where h is a monotonically increasing transformation. E_i has the same efficiency rank of firms as r_i^* in the sample. Unlike r_i^*, our measure E_i shifts α_i by the mean $\frac{1}{N}\sum_j \alpha_j$. $\widehat{E}_i = h(\widehat{\alpha}_i - \frac{1}{N}\sum_j \hat{\alpha}_j)$ is the estimate of E_i.

In general, E_i can be arbitrarily restricted to $(0, 1)$, so that efficiency can be expressed as a percentage.[4] There are an unlimited number of functions one might use. For example, we consider the function form of standard normal cumulative distribution function Φ,

$$E_i = \Phi\left(\alpha_i - \frac{1}{N}\sum_j \alpha_j\right), \tag{2}$$

with estimate

$$\widehat{E}_i = \Phi\left(\widehat{\alpha}_i - \frac{1}{N}\sum_j \widehat{\alpha}_j\right). \tag{3}$$

The advantage of using a symmetric and smooth cumulative distribution function is that $E_i = 0.5$ is a meaningful reference point corresponding to a firm with the mean value of α_i. Alternative distributions (e.g., Cauchy, student, logistic) can be used for this purpose.

As discussed above, the estimated maximum, $\max_j \hat{\alpha}_j$, is biased. This is the main source of the coverage erosion of r_i^*'s bootstrap confidence intervals. By contrast, the estimated mean $\frac{1}{N}\sum_j \hat{\alpha}_j$ is unbiased as the estimated firm effects $\widehat{\alpha}_i$ are unbiased. Since $\widehat{\alpha}_i - \frac{1}{N}\sum_j \hat{\alpha}_j$ is unbiased, the percentile bootstrap confidence intervals for $\alpha_i - \frac{1}{N}\sum_j \alpha_j$ have no coverage erosion and thus have high coverage even in micro panels. According to Efron and Tibshirani (1993), the percentile bootstrap confidence intervals are *transformation respecting*, implying that the percentile intervals for E_i will have the same coverage rate as those for $\alpha_i - \frac{1}{N}\sum_j \alpha_j$. Therefore, we can always construct percentile intervals for E_i by transforming the intervals for $\alpha_i - \frac{1}{N}\sum_j \alpha_j$. Hence, simple bootstrap confidence intervals like the percentile bootstrap are enough for accurate inference. For empiricists, inference on E_i will be considerably easier to implement than inference on r_i^*. Moreover, the bootstrap confidence intervals for E_i are more reliable than those for the traditional measure r_i^*, no matter how many firms there are, and whether or not the identity of the best firm is in serious doubt.

$E_i = 0.5$ for the firm with mean value of α_i in (2), so we call the firm with $E_i = 0.5$ the *mean firm*. The efficiencies of all firms

2 The relationship between inefficiency and unobserved heterogeneity is not considered here. For a detailed discussion of this issue, see Greene (2004, 2005).

3 γ is defined as the variance ratio $\mathrm{Var}(u)/(\sigma_v^2 + \mathrm{Var}(u))$. A small γ means that the main source of variation of the composite error $-u_i + v_i$ comes from the noise v_i.

4 The usual measure of technical efficiency, $r_i = \exp(-u_i)$, implies that the production function is Cobb–Douglas. However, if the true data-generating process is not Cobb–Douglas, then $r_i = \exp(-u_i)$ is not technical efficiency, *per se*, but merely an *indicator* of technical efficiency. The proposed measure is simply another such indicator.

are distributed around the mean firm. The efficiency measures are above 0.5 for the firms with $\alpha_i > \frac{1}{N}\sum_j \alpha_j$ and below 0.5 for $\alpha_i < \frac{1}{N}\sum_j \alpha_j$. Rather than using the maximum as the reference point, the measure E_i uses the mean firm as the reference point, providing an interesting view for the firms' efficiencies. The reference point $\widehat{E}_i = 0.5$ divides firms into two groupings: "efficient" firms with $\widehat{E}_i$ above 0.5 and "inefficient" firms with $\widehat{E}_i$ below 0.5. If the price of products is related to the cost of the mean firm in a relatively new industry, then the mean firm may have zero profit. Accordingly, the "efficient" firms may have positive profit, while the "inefficient" firms may earn negative profits and be inclined to leave the market.

3. Simulations and empirical example

To better understand the accuracy of inference for the traditional measure $r_i^* = \exp(\alpha_i - \max_j \alpha_j)$ and our proposed measure $E_i = \Phi(\alpha_i - \frac{1}{N}\sum_j \alpha_j)$, we conduct Monte Carlo experiments to compare the coverage rate of bootstrap confidence intervals in micro panels. The data-generating process used here follows the procedure in Kim et al. (2007): $y_{it} = \alpha - u_i + v_{it}$, with $v_{it} \sim \text{i.i.d.}N(0, \sigma_v^2)$ and $u_i = |\mu_i|$, where $\mu_i \sim \text{i.i.d.}N(0, \sigma_\mu^2)$. We set $\alpha = 1$ and define $\gamma = \text{Var}(u)/(\sigma_v^2 + \text{Var}(u))$. Small γ indicates a particularly noisy experiment. A large γ (close to 1) means there is little variation in the idiosyncratic error v_{it}. The Monte Carlo parameters are (γ, N, T). When T is large, the bias issue of traditional efficiency estimate is mitigated. Here, we consider the interesting case when the inference of the traditional measure r_i^* is inaccurate in micro panels with $T = 4, 6, 10$. Experiments are conducted with $N = 20, 50, 100$ and 150, and $\gamma = 0.1, 0.3$ and 0.5, with 1000 bootstrap replications. We repeat the Monte Carlo experiments 1000 times.

Kim et al. (2007) show that the BC_a bootstrap confidence intervals and BC percentile intervals for r_i^* are better than other intervals, so we report these two intervals. The percentile intervals are also presented for comparison. Since $\widehat{\alpha}_i - \frac{1}{N}\sum_j \widehat{\alpha}_j$ is unbiased, only percentile intervals for E_i are presented. Simulated 90% nominal coverage rates for intervals for r_i^* and E_i are in Table 1. The coverage of percentile intervals for r_i^* declines dramatically in N and is, therefore, unreliable for panels with small T and large N. For example, the coverage of the 90% nominal percentile interval (column 4) is 24.6% for $T = 6$, $N = 150$ and $\gamma = 0.5$. As expected, the coverage rates for both BC_a (column 6) and the bias-correct percentile (column 8) intervals improve, but are still far from satisfactory. The coverage rates of BC_a and the BC percentile intervals for r_i^* are 41.0% and 68.5% respectively for $T = 6$, $N = 150$ and $\gamma = 0.5$. Since $\hat{\alpha}_i - \frac{1}{N}\sum_j \hat{\alpha}_j$ is unbiased, the percentile bootstrap confidence intervals for $\alpha_i - \frac{1}{N}\sum_j \alpha_j$ have no coverage erosion. Since E_i is a monotonic function of $\alpha_i - \frac{1}{N}\sum_j \alpha_j$, its percentile bootstrap confidence intervals have the same coverage as those for $\alpha_i - \frac{1}{N}\sum_j \alpha_j$. Unlike the bootstrap confidence intervals for r_i^*, the coverage rates of percentile intervals for E_i (column 9) are effectively constant across N and γ. They are only related to T. The coverage rates of percentile intervals for E_i are about 75%, 81% and 85% for $T = 4$, 6 and 10, respectively. Therefore, based on coverage rates, the confidence intervals for E_i are much more informative and robust than those for r_i^*.

A quintessential example of a micro panel in the stochastic frontier literature is the Indonesian rice farm data set with $N = 171$ and $T = 6$. See Horrace and Schmidt (1996, 2000) for a detailed description. Table 2 contains the point estimates and bootstrap intervals for r_i^* and E_i. We report results for the best farm, the 99th, 75th, median, 25th, 1th quantile, and the worst farm in rank order. The first column is the farm number. The third column contains the estimates of the firm effects $\alpha_i = \alpha - u_i$. It is interesting to note that the point estimator $\widehat{E}_i$ (column 5) is similar in magnitude to $\widehat{r}_i^*$ (column 4) at the bottom end of the order statistic. Compare 0.336–0.366 for farm 45 and 0.349–0.379 for farm 117. If one is interested in inference on the least efficient farms in the sample, then $\hat{E}_i$ gives point estimates similar to $\widehat{r}_i^*$ but with higher coverage rates (but for different measures). Fig. 1 contains the graphs of $\widehat{r}_i^*$

Table 1
Coverage of 90% bootstrap confidence intervals for r^* and E.

T	γ	N	r^*			E
			Percentile	BC_a	BC percentile	Percentile
4	0.1	20	37.2	42.7	68.0	74.3
4	0.1	50	21.6	27.1	61.3	74.7
4	0.1	100	13.4	17.6	55.3	74.8
4	0.1	150	9.9	13.4	51.6	74.8
4	0.3	20	47.4	51.4	71.7	74.4
4	0.3	50	29.8	35.0	65.5	74.8
4	0.3	100	20.1	24.9	61.9	75.0
4	0.3	150	15.0	19.2	57.5	74.8
4	0.5	20	55.6	58.0	74.0	75.0
4	0.5	50	37.7	42.0	68.5	74.7
4	0.5	100	25.1	29.6	63.4	74.8
4	0.5	150	19.7	24.1	59.9	74.9
6	0.1	20	44.7	59.9	75.1	80.6
6	0.1	50	26.4	43.6	70.0	80.6
6	0.1	100	16.2	32.0	63.4	80.6
6	0.1	150	11.8	25.8	60.5	80.5
6	0.3	20	55.6	67.0	77.8	81.0
6	0.3	50	36.6	52.6	72.7	80.7
6	0.3	100	24.3	40.9	68.3	80.7
6	0.3	150	18.5	34.6	66.2	80.7
6	0.5	20	62.8	70.8	78.6	80.7
6	0.5	50	45.8	59.6	76.2	80.5
6	0.5	100	30.6	47.5	71.1	80.6
6	0.5	150	24.6	41.0	68.5	80.6
10	0.1	20	52.6	73.3	81.0	84.4
10	0.1	50	31.0	59.5	75.0	84.4
10	0.1	100	19.9	49.2	71.7	84.5
10	0.1	150	14.6	42.6	68.7	84.4
10	0.3	20	64.1	77.8	82.7	84.3
10	0.3	50	44.4	67.8	78.9	84.7
10	0.3	100	29.9	58.1	75.1	84.7
10	0.3	150	23.2	51.8	73.0	84.5
10	0.5	20	72.5	80.8	83.8	84.8
10	0.5	50	53.3	73.1	81.4	84.6
10	0.5	100	38.6	64.5	77.9	84.4
10	0.5	150	30.6	59.1	76.3	84.6

Note: r^*'s BC_a and BC percentile bootstrap confidence intervals are obtained using the indirect method in Kim et al. (2007).

Table 2
Estimates and 90% bootstrap intervals for efficiency in rice farms.

Farm #	Percentile	α_i	r^*	E
164	Best	5.556	1.000	0.668
			[0.912, 1.000]	[0.633, 0.694]
118	99th	5.486	0.932	0.651
			[0.837, 1.000]	[0.618, 0.689]
31	75th	5.072	0.616	0.531
			[0.423, 0.890]	[0.412, 0.623]
15	50th	4.966	0.554	0.498
			[0.448, 0.738]	[0.437, 0.560]
16	25th	4.859	0.498	0.464
			[0.439, 0.608]	[0.410, 0.520]
117	1st	4.586	0.379	0.382
			[0.306, 0.476]	[0.328, 0.436]
45	Worst	4.550	0.366	0.373
			[0.275, 0.531]	[0.303, 0.466]

Note: Bracketed values are 90% bias-corrected percentile intervals for r^*, using the indirect method in Kim et al. (2007), and 90% percentile bootstrap intervals for E. The bootstrap resampling number is 1000.

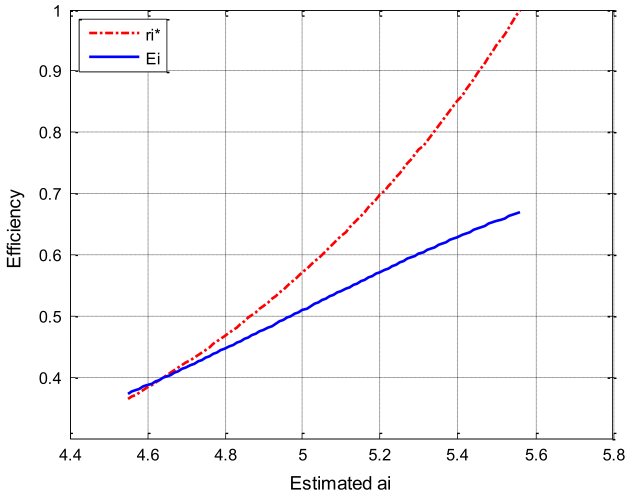

Fig. 1. Comparison of two efficiency estimates in the rice farm example.

(dashed line) and $\hat{E}_i$ (solid line) for various values of $\widehat{\alpha}_i$ over the sample values of $\widehat{\alpha}_i \in [4.550, 5.556]$. Comparing $\widehat{r}_i^*$ and $\hat{E}_i$, we see that over the range of sample values of $\widehat{\alpha}_i$, $\hat{E}_i$ is approximately linear with smaller slope. Also, if we regard efficiency estimate $\hat{E}_i = 0.5$ as an efficiency threshold, the firms above 0.5 efficiency are deemed "efficient", while those below are "inefficient ". In this example, the median firm 15 has efficiency $\hat{E}_i = 0.497$, implying that the efficiency distribution among firms is nearly symmetric, as noted in Feng and Horrace (2012). Table 2 also reports the 90% *BC* percentile interval for r_i^* and 90% percentile interval for E_i in the brackets below their point estimates. Differences in these intervals reflect differences in efficiency measures as illustrated in Fig. 1.[5]

4. Conclusions

Simulated evidence suggests that, in micro panels with small T and large N, development of new efficiency measures may be as important as developing improved bootstrap interval techniques for the traditional measure r_i^* to ensure reliable inference on efficiency. We have investigated one rank-preserving efficiency measure and its fixed-effects estimate that avoids the "max" operator, whose bootstrap intervals are transformation respecting, and that produces good coverage rates. Unlike the traditional measure r_i^*, the new measure, E_i, uses a reference point based on the mean, which can be unbiasedly estimated. Consequently, the new efficiency estimate $\widehat{E}_i$ has no bias issue as does $\widehat{r}_i^*$. Monte Carlo simulations show that the simple percentile bootstrap confidence intervals for E_i exhibit high coverage rates in micro panels. We might conclude that the new efficiency measure E_i (and similar measures) are the only way to ensure accurate inference in micro panels. However, empiricists face a trade-off between the statistical accuracy of inference and the empirical relevance of the estimated measure. It may be the case that E_i is not a reasonable substitute for $\widehat{r}_i^*$ in any particular empirical exercise. As such, E_i should be regarded as a complement to $\widehat{r}_i^*$ and a potentially important addition to the empiricist's toolbox.

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[5] If we construct an efficiency measure without normalization at the mean, e.g., $\Phi(\alpha_i)$, the estimated efficiencies of firms could fall either around 0 or 1. This depends on the magnitude of estimated α_i in a sample. For example, using this empirical sample, all estimated efficiencies based on measure $\Phi(\alpha_i)$ are nearly 1. In this case, without normalization it seems inconvenient to use these efficiencies directly in practice.