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Particle Physics in the LHC Era

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Abstract

During the past 100 years experimental particle physicists have collected an impressive amount of data. Theorists have also come to understand this data extremely well. It was in the first half of the 20th century the efforts of the early pioneers of quantum mechanics laid the ground work for this understanding: quantum field theory. Through the tireless efforts of researchers during the later half of the 20th century many ideas came together to form what we now call the Standard Model (SM) of particle physics. Finally, it was through the ideas of the renormalization group and effective field theory that the understanding of how the SM fits into a larger framework of particle physics was crystallized.

In the past four years the Large Hadron Collider (LHC) has made more precise measurements than ever before. Currently the SM of particle physics is known to have excellent agreement with these measurements. As a result of this agreement with data, the SM continues to play such a central role in modern particle physics that many other theories are simply known as 'Beyond the Standard Model' (BSM) as we know any new models will simply be an extension of the SM.

Despite agreement with experiment, the SM does suffer from several shortcomings that raise deeper questions. In this dissertation we study models that address the two of the outstanding theoretical problems of the SM - the Strong CP Problem and the fine tuning of the Higgs mass. We study models that solve or ameliorate these problems, and their implications for collider physics and astrophysics.

Particle Physics in the LHC Era

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To my parents and grandparents,
who wanted to see me get an education.
I did the most badass thing I could find.

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Chapter 1

Introduction

In physics we are trying to answer a deceptively simple question, 'what are the fundamental laws of nature?' The deception is twofold. On one level, it is deceptive in that this appears to be a simple-minded question that does not do justice to the wide range of phenomena it is seeking to inquire about. On a deeper level, it is deceptive in that it turns out we actually *can* describe nature in an extremely concise framework. Over 300 hundred years ago Newton began this consolidation when he unified two previously distinct arenas, the heavens and the earth, with his law of universal gravitation. With this conceptual breakthrough Newton swept in a new era of science in which new phenomena are not just named and catalogued, but progressively incorporated into more and more concise models. No one could have guessed how far it can go and that now much of nature can be summarized on one side of a cocktail napkin.

Given the plethora of phenomena we are trying to describe, it is unclear where to even start building the next generation of particle physics models. An analogy due to Feynman [1] will help elucidate an algorithm. Trying to figure out the laws of nature is like not knowing the rules of chess and trying to figure them out only by watching a game unfold. We might watch in bewilderment of the players as everything seems random and chaotic at first. It seems like anything is possible and players make up the rules as they go along. Slowly we might be able to start to surmise that the game is constrained in some fashion, and that not just anything is possible. At some point we will want to go beyond just noticing that stuff is happening, and when something seems like it happens with reasonable frequency, we posit it as a rule. In other words, we stick our neck out and make a prediction. Sometimes these predictions withstand the test of time, other times they are overturned, and we are unsettled when something we were fairly certain of turns out to be wrong. For example, we might try to deduce how a particular piece is allowed to move, then we could propose this as a 'rule' and see if future moves are consistent with this. Over time, if more moves are shown to be consistent with the proposed rule, we feel more confident in it.

On occasion our confidence can be completely dashed just when we think we have everything figured out. Such would be the case if one witnessed castling, in which a rook and a king are simultaneously moved. First of all, the move is rare so we will have to collect a large amount of data to ever see it once. Furthermore, there are a strict set of circumstances that govern castling, so we will have to collect an extremely large amount of data to understand the circumstances under which it is a legal move. One might be tempted to abandon everything when confronted with such confidence shattering discoveries.

Alas, the analogy is a rough one, and it does not pay to press it too far. It turns out that the situation in physics is both better and worse than the chess game. It is worse in that we do not know all the outcomes yet and there is always the chance that some particle may decay in a way that we have not yet seen. We could continue to enumerate all the possible ways that we have it worse off, however it is not all bad, and there is one way we are much better off than the case of the chess game. It turns out there is quantitative meaning to being partially correct in the answer to our original question when it comes to physics. We do not need to know the most concise set of rules and only those. We can get an 'effective' set of rules and work with these. These rules will not be as concise and beautiful as the 'master' rules, but they do just fine from an experimental perspective in that we can make a finite set of measurements and predict things beyond what we had to measure in the first place.

The reason that the effective rules are good enough is decoupling. Decoupling is the statement that physics at different length scales do not affect one another. As we gradually uncover new layers of physics, we can analyze them in an orderly fashion and then systematically incorporate them. Decoupling is the reason we do not need a detailed understanding of planetary geography and composition to understand how the sun and planets interact with one another. It is also the reason why we can treat protons and neutrons as point particles when we study chemistry.

In particle physics, the formal process of analyzing relevant length scales and neglecting irrelevant scales is known as effective field theory (EFT)[2]. As we resolve smaller and smaller sizes, and equivalently larger and larger energy scales, we resolve new structure. In particle physics, this means we need to incorporate new degrees of freedom (DOF), that is new particles, in order to properly account for new phenomena. We will see that even though we start with a low energy effective theory, there are always clues in the low energy theory as to the correct high energy theory, and we can use these to bootstrap our way up to better models.

In this dissertation, decoupling enables us to implement a ground up approach by starting with the bare minimum of a model. We will then implement a utilitarian approach of pushing this model to its breaking point, and then incorporating the necessary new physics in order to go beyond this threshold. We will begin developing the tools by first building up to the Standard Model (SM) and using this as a springboard into beyond the Standard Model (BSM). During this exploration we will weave in the development of one of our most powerful tools in physics - dimensional analysis. In section 2.1 we outline the basic machinery of quantum field theory, in particular EFT. Beginning in 2.2 we implement a ground up approach using the minimal EFT we need in order to have a working description of particle physics. By deliberately analyzing one energy scale at a time, using minimal amount of inputs and some basic reasoning, we are able to build up to the SM. Using the lessons and tools we learned in exploring the SM, we then move on to use these to explore BSM in section 2.3. In section 2.4 we touch on one of the more radical proposals of the past 20 years of physics- the AdS/CFT correspondence. In chapters 3 and 4 we explore two specific models of BSM physics that address the limitations of the SM.

Appendix 1.A List of Conventions

Throughout this dissertation, we have adopted the following conventions:

- Greek indices $\mu, \nu, \lambda, ...$ label the components of four-vectors and take values $0, 1, 2, 3$.

- Repeated indices are summed over.

- The metric has a $(+, -, -, -)$ signature.

- We work in units such that $\hbar = c = 1$. This results in units where

  $[\text{Mass}] = [\text{Energy}] = [\text{Momentum}] = [\text{Length}^{-1}] = [\text{Time}^{-1}]$.

- The σ^i are the Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- We further define $\sigma^\mu \equiv (\mathbb{1}, \sigma)$ and $\bar{\sigma}^\mu \equiv (\mathbb{1}, -\sigma)$.

- The Dirac matrices are

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

- We employ the Feynman slash notation $\not{p} \equiv \gamma^\mu p_\mu$.

- We can use the Dirac matrices as a basis for 4×4 matrices Γ_A which consist of:

$$\begin{array}{cc} 1 & \text{one of these} \\ \gamma^\mu & \text{four of these} \\ \gamma^{\mu\nu} = \gamma^{[\mu}\gamma^{\nu]} \equiv -i\sigma^{\mu\nu} & \text{six of these} \\ \gamma^{\mu\nu\rho} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho]} & \text{four of these} \\ \gamma^{\mu\nu\rho\sigma} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]} & \text{one of these} \end{array}$$

Appendix 1.B List of Abbreviations

Throughout this dissertation, we have used the following abbreviations:

- BSM: Beyond the Standard Model
- DOF: Degrees of Freedom
- EFT: Effective Field Theory
- GR: General Relativity
- NDA: Naive Dimensional Analysis
- QED: Quantum Electrodynamics
- QCD: Quantum Chromodynamics
- RG: Renormalization Group
- RS: Randall Sundrum
- SM: Standard Model
- SUSY: Supersymmetry
- vev: vacuum expectation value

Chapter 2

Effective Field Theory and the Standard Model

2.1 A Crash Course in Model Building

2.1.1 The Path Integral

We began with a rather general philosophical inquiry of the fundamental laws of nature. In particle physics our basic probe of nature is the particle collider. From a collider perspective we ask a much more utilitarian question, 'What is the minimal amount of measurements that we need to make before we can predict all the other possible outcomes in our particle collider?' More precisely, we are interested in the *probability* of an outcome in our particle collider.

From a practical standpoint we first calculate the amplitude for a set of asymptotically free particles in the infinite past with momenta $\{\mathbf{k}_i\}$ to evolve in time (scatter) by the S-matrix, and then find the overlap with some final state of free particles in the infinite future with momenta $\{\mathbf{p}_j\}$ which we will denote as

$$\langle f|i\rangle = \langle \mathbf{p}_1 ... \mathbf{p}_n|S|\mathbf{k}_1 ... \mathbf{k}_m\rangle. \tag{2.1}$$

The amplitude is related to the probability of this outcome by

$$P(i \to f) \propto |\langle f|i\rangle|^2. \tag{2.2}$$

The matrix elements can be written as the Fourier transform of $n+m$ point correlation functions of the fields $\phi(x_i)$

$$\langle \mathbf{p}_1 ... \mathbf{p}_n | S | \mathbf{k}_1 ... \mathbf{k}_m \rangle \propto \prod_{i=1}^{n} \int d^4 x_i \, e^{i p_i \cdot x_i} \prod_{j=1}^{m} \int d^4 y_j \, e^{-i k_j \cdot y_j} \langle T[\phi(x_1) .... \phi(x_n) \phi_(y_1) .... \phi(y_m)] \rangle \tag{2.3}$$

where T is the time ordering operator. And the correlation functions can be in turn calculated by taking functional derivatives with respect to the sources J_i

$$\langle T[\phi_1 .... \phi_n] \rangle = \frac{\int \mathcal{D}\phi_i \; \phi_1 .... \phi_n \; e^{iS}}{\int \mathcal{D}\phi_i \; e^{iS}} = \left(-i \frac{\delta}{\delta J_1}\right) ... \left(-i \frac{\delta}{\delta J_n}\right) \log Z[J_i] \tag{2.4}$$

where we have used the condensed notation $\phi_i \equiv \phi(x_i)$. $Z[J_i]$ is the vacuum-to-vacuum transition in the presence of sources J_i

$$Z[J] = \int \mathcal{D}\phi_i \; e^{iS} \tag{2.5}$$

where S is the action $S = \int d^d x \, (\mathcal{L}(\phi_i) + \phi_i J_i)$, $\mathcal{L}$ is the Lagrangian, ϕ_i are fields (bosonic or fermionic), and the integration is over field configurations. Equation 2.5 is also known as the path integral.

2.1.2 The Lagrangian

With the machinery we have outlined above, we have further reduced the problem outlined at the beginning of the dissertation of ‘What are the fundamental laws of nature?’ to that of

$$\mathcal{L} = ?^1 \tag{2.6}$$

This may not seem like progress since we have just replaced one question with another one, but actually we have managed to strip away a huge amount of bookkeeping with some known formalism in order to reduce our original question with the much simpler question. The Lagrangian is a highly constrained object, and using these

[1]Recent developments indicate that perhaps not all of physics can be phrased in this way, or perhaps this is not the most fundamental way to phrase things. See [3] for an example. From the bottom-up approach of defining a low energy effective field theory we will be following, this is a good starting point.

constraints we can outline a concise algorithm for model building. In fact the action, $S = \int d^4x\ \mathcal{L}$, is such a constrained object it is totally boring and completely trivial, it is a singlet (does not transform) under all symmetries. The trick will be to specify these symmetries and then write the Lagrangian in terms of the objects (the fields) that do transform under these symmetries. Thus the first step towards model building is to specify the symmetries and their associated groups, and find representations of the groups.

A large portion of constraints results just from enumerating the representations under the Lorentz group SO$(3, 1)$. A standard method of dealing with any group more complicated than SU(2) is to label its representations in terms of its SU(2) subgroups. In this case representations of the Lorentz group are labeled by representations under two SU(2) subgroups which we label as two half integers (j_1, j_2). [2] The most frequently encountered fields and their corresponding representations are:

Common name	Also known as[3]	(j_1, j_2)	Index
scalar	'spin 0'	$(0, 0)$	none
left handed spinor	'spin 1/2'	$(\frac{1}{2}, 0)$	latin $(a, b, ...)$
right handed spinor	'spin 1/2'	$(0, \frac{1}{2})$	dotted latin $(\dot{a}, \dot{b}, ...)$
Dirac spinor	'spin 1/2'	$(0, \frac{1}{2}) \oplus (\frac{1}{2}, 0)$	latin $\oplus$ dotted latin
vector	'spin 1'	$(1, 1)$	greek $(\mu, \nu, ...)$

An immediate question that arises is, 'What about the other representations beyond spin one?' It turns out that while the mathematical objects, the representations, are constructed easily enough, physics poses a severe constraint on the fields in a Lagrangian. Any massless fields with spin greater than or equal to one have unphysical degrees of freedom that must cancel out of calculations precisely.[4]

[2]Technically we find representations of the universal covering group of the Lorentz group, SL$(2, \mathbb{C})$, which is isomorphic to the complexification of SU(2) $\otimes$ SU(2). See [27] for a concise review.

[3]These names are informal at best and belie the subtle nature of the DOF associated with the fields. In particular, we follow the time honored tradition of confusing left-handed, right-handed, and Dirac spinors with the single name 'spin 1/2'.

[4]We will consider massive gauge fields and their relation to massless gauge fields in section 2.2.2.

For example, take a spin one particle. The expectation value of a spin one creation and annihilation operator is proportional to the metric tensor by Lorentz invariance

$$\langle 0|a_\mu(\mathbf{k})a_\nu^\dagger(\mathbf{k})|0\rangle \sim \eta_{\mu\nu}. \tag{2.7}$$

If we set $\mu = \nu$ (no sum) this is the norm of a single particle state,

$$|a_\nu^\dagger(\mathbf{k})|0\rangle|^2 \sim \eta_{\nu\nu}. \tag{2.8}$$

But we can see that since that $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ some of the norms will be positive and others negative. If we are to successfully give a theory a probabilistic interpretation, we can not have states of negative norm as they have no physical meaning.

This issue is resolved by always coupling a field that has unphysical polarizations to a conserved current so that the unphysical degrees of freedom always cancel, such that they will never be observed, and measurable probabilities are always positive definite. For spin one particles like the photon, we couple it to the electromagnetic four-current J^μ_{em}, which is conserved, $\partial_\mu J^\mu_{em} = 0$ (which is just a Lorentz invariant way of saying that electric charge is conserved).

From a practical model building standpoint, it usually ends up being easier to simply enforce that our Lagrangian obeys a (space-time dependent) 'gauge symmetry' in which we 'gauge' a (space-time independent) global symmetry.[5] This is accomplished by replacing any derivatives in a Lagrangian with gauge-covariant derivatives

$$\partial_\mu \rightarrow \partial_\mu - igA_\mu \tag{2.9}$$

where g is the gauge coupling (charge), and A_μ is referred to as a 'gauge field', which lives in the adjoint representation of the gauge group.

In principle, we could have gone on to include spin 3/2. The conserved current that these fields will be coupled to is the super-current of supersymmetry (SUSY). This makes adding spin 3/2 particles to the SM a highly nontrivial extension. If

[5] One must always ensure that the global symmetry is a genuine symmetry, that is, that the globally symmetry is still present once quantum corrections are accounted for.

SUSY is indeed a symmetry of nature, it is at least broken at high energies giving these spin 3/2 particles and the SM super-partners a mass outside our current reach [19].

Likewise we could have also added spin two particles. Now the only conserved current around in this case is the second rank stress energy tensor, $T^{\mu\nu}$. But we already know what this model is, a theory of massless spin 2 gauge bosons *is* a theory of gravity [5]. As we will see in section 2.3.1 the effects of gravity are well beyond the reach of current colliders.

The process of finding conserved currents of successively higher rank to couple gauge fields to terminates at this point. Although we can easily find the representations of spin greater than two, there is no known way of consistently coupling fields that live in these representations to matter fields so as to eliminate unphysical DOF, since there are no remaining nontrivial conserved currents of rank greater than two.

One final clarification is worth making. Global symmetries of a phenomenological model are either ultimately broken by small corrections, or in the case they are exact, they are likely the remnant of a gauge symmetry. An example of the former case would be baryon number conservation. We know the global symmetry associated with baryon number conservation to be conserved to an excellent degree, however, it will likely end up being violated by some very small corrections. An example of the latter is the case with Lorentz symmetry being the global remnant of the exact local symmetry of General Relativity.

With the fields in hand, we combine these with the objects ∂_μ and the matrices $\sigma^\mu_{a\dot{a}}$, ϵ_{ab} and $\epsilon_{\dot{a}\dot{b}}$ to create the singlets. For example,

$$\epsilon_{ab}\psi^a\psi^b,\ \sigma^\mu_{a\dot{a}}\psi^a\partial_\mu\bar{\psi}^{\dot{a}},\ \partial_\mu\phi\partial^\mu\phi,\ \sigma^\mu_{a\dot{a}}\psi^a A_\mu\bar{\psi}^{\dot{a}},\dots \tag{2.10}$$

Any singlet is a candidate term in the Lagrangian. Finally, to construct a general Lagrangian we take all possible singlets with arbitrary coefficients.

2.1.3 Beginner Dimensional Analysis

At this point we can write down a formal expression for a desired quantity. For example, the amplitude for a $\phi\phi \to \phi\phi$ scattering

$$\mathcal{M}_{\phi\phi\to\phi\phi} = \langle T[\phi_1\phi_2\phi_3\phi_4]\rangle = \frac{\int \mathcal{D}\phi\ \phi_1\phi_2\phi_3\phi_4\ e^{i\int d^4x\mathcal{L}}}{\int \mathcal{D}\phi\ e^{i\int d^4x\mathcal{L}}} \tag{2.11}$$

in a scalar field theory with the Lagrangian

$$\begin{aligned}\mathcal{L} = \ & m^2\phi^2 + a_4\phi^4 + \frac{a_6}{\Lambda^2}\phi^6 + ... \\ & + \partial_\mu\phi\partial^\mu\phi + \frac{b_2}{\Lambda^2}\phi^2\partial_\mu\phi\partial^\mu\phi + \frac{b_4}{\Lambda^4}\phi^4\partial_\mu\phi\partial^\mu\phi \\ & + \frac{c_0}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \frac{c_2}{\Lambda^6}\phi^2(\partial_\mu\phi\partial^\mu\phi)^2 + \frac{c_4}{\Lambda^8}\phi^4(\partial_\mu\phi\partial^\mu\phi)^2 \\ & + ...\end{aligned} \tag{2.12}$$

where we have assumed ϕ is a pseudo-scalar for simplicity (which forbids odd terms in the Lagrangian) and added all possible terms consistent with Lorentz symmetry for a scalar field. A dimensionful scale Λ has been extracted so all the coefficients $\{a_i, b_i, ...\}$ are dimensionless. The coefficient of $\partial_\mu\phi\partial^\mu\phi$ is normalized to one. We have identified the coefficient of ϕ^2 as the mass since the simple Lagrangian $\mathcal{L} = \partial_\mu\phi\partial^\mu\phi - m^2\phi^2$ corresponds to the classical equation of motion $(\partial^2 - m^2)\phi = 0$, which means plane wave solutions have the dispersion relation $E^2 = p^2 + m^2$, which is the dispersion relation for a relativistic particle of mass m. We have further identified the dimension of all the other operators as follows. Since [S]= $[\int\ d^4x\mathcal{L}] = 1$, $[\mathcal{L}] = \text{mass}^4$, thus $[\partial_\mu\phi\partial^\mu\phi] = \text{mass}^4$ and since $[\partial_\mu] = \text{mass}$, then $[\phi] = \text{mass}$.

Unfortunately there is little hope of performing the integrals in equation 2.11 for this Lagrangian. Moreover there is a more devious aspect to the Lagrangian 2.12 - it has infinitely many parameters, hence it has no predictive power. Given this, we will have to make some utilitarian assumptions in order to move forward.

Note that if we assume all operators beyond the quadratic portion of the Lagrangian are small, and expand in these, we will be left with Gaussian integrals to perform, the solutions to which are well known. For example, naively expanding in the coefficient $a_8\frac{1}{\Lambda^4}$ of the operator ϕ^8 will contribute to lowest order a term of the form $a_8 \frac{E^4}{\Lambda^4}$ to a scattering amplitude, where E is the energy of the particles. That is, our expansion is retroactively justified if we assume that $E \ll \Lambda$. Additionally we need to assume a dimensionless coefficient like a_4 is small since there will be no energy suppression for operators of this form.

In terms of our Lagrangian 2.12 this means if we restrict ourselves to low energies, we really only need to keep the minimal number of operators so that our model is nontrivial. In this case that means we will keep the operators with coefficient a_2 and a_4.[6] With this assumption our Lagrangian takes a much more simple form,

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 - a_4 \phi^4. \tag{2.13}$$

From here on out we will supplement our prescription for constructing quantum field theories with the assumption we are working at sufficiently low energy to neglect higher dimensional operators, and only keep the minimal number of nontrivial operators to describe scattering. There still remains the concern of what happens at energies on the order of the scale Λ, but we will postpone this issue until we have a concrete model and Λ in hand to analyze.

2.2 The Standard Model from the ground up in 3 segues

Now that we have some model building principles in place, we can start building realistic models. We follow the steps outlined in the previous section, which we now formalize:

1. List the fields (particles) we want to describe, and their representation under the Lorentz group and any gauge group.

2. Write down all possible terms that are singlets under these symmetries.

[6] In other words, if for some reason a_4=0, we would then need to keep the a_6 term around to describe nontrivial scattering processes.

3. Assume the low energy and keep only the minimum number of nontrivial operators in order to describe scattering processes.

4. The sum of these operators with coefficients is the Lagrangian.

2.2.1 A sub-Standard Model Part I

With this algorithm we can write down a model of the stuff we know, that is, the particles that we have direct evidence for. First of all, we see ourselves - we are made of matter particles - electrons, protons and neutrons. We also know that light (photons) interacts with any particles carrying electric charge.

We also assume we know that a proton, neutron and an electron interact in processes which a neutron will decay into a proton and an electron (beta decay). If we want to model this interaction, there is no Lorentz invariant term that we can write down that involves just 3 fermions (there will always be a un-contracted Lorentz index). To solve this problem we will follow Fermi [6] and take the first in a series of radical steps of postulating new particles in order to patch up our model. In this case we will postulate a new neutral fermion, the neutrino, so that charge is still conserved in the interaction and we have something to contract the remaining Lorentz index with.

Following the algorithm outlined at the beginning of this section yields:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_i \left(i\bar{\psi}_i \not{\partial}\psi_i - m_i\bar{\psi}_i\psi_i\right) + eA^\mu J_\mu \\ &+ \frac{1}{\Lambda}\sum_i c_i F_{\mu\nu}\bar{\psi}_i\sigma^{\mu\nu}\psi_i + \frac{1}{\Lambda^2}\sum_{i_j} c_{iAB}\ \bar{\psi}_{i_1}\Gamma_A\psi_{i_2}\bar{\psi}_{i_3}\Gamma_B\psi_{i_4} \qquad (2.14)\end{aligned}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $J_\mu = \sum_i q_i\bar{\psi}_i\gamma_\mu\psi_i$, i is a sum over fermions, q_i are the electric charges of those fermions in units of the electron charge, the ψ_is are Dirac spinors,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{a\dot{a}} \\ \bar{\sigma}^\mu_{a\dot{a}} & 0 \end{pmatrix}, \qquad (2.15)$$

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and the Γ_A are a basis for 4×4 matrices constructed out of the γ^μs.

The sum over all fermions in the four-fermion operator is a sum over all fermions that conserve charge. Terms of the form $F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi$ are informally known as 'Pauli' terms.

It turns out experimentally that only a much smaller subset of the coefficients c_i and c_{iAB} are nonzero to a high degree of precision. That is, in order to account for experimental data, we only need

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_i \left(i\bar{\psi}_i\partial\!\!\!/\psi_i - m_i\bar{\psi}_i\psi_i\right) + eA_\mu J^\mu_{em} \\ + \frac{c_p}{\Lambda}F_{\mu\nu}\bar{p}\sigma^{\mu\nu}p + \frac{c_n}{\Lambda}F_{\mu\nu}\bar{n}\,\sigma^{\mu\nu}n + 2^{3/2}G_F(J^{+\mu}J^-_\mu + J^\mu_z J_{z\mu})\end{aligned} \tag{2.16}$$

where

$$\begin{aligned}J^{+\mu} &= c_B\bar{n}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - g_A\gamma^5\right)p + \bar{e}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - \gamma^5\right)\nu \\ J^{-\mu} &= c_B\bar{p}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - g_A\gamma^5\right)n + \bar{\nu}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - \gamma^5\right)e \\ J_z{}^\mu &= J^\mu_3 - s^2_w J^\mu_{em} \\ J^\mu_3 &= \bar{p}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - \gamma^5\right)p - \bar{n}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - \gamma^5\right)n + \bar{\nu}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - \gamma^5\right)\nu - \bar{e}\gamma^\mu\frac{1}{2}\left(\mathbb{1} - \gamma^5\right)e \\ J^\mu_{em} &= \bar{p}\gamma^\mu p - \bar{e}\gamma^\mu e\end{aligned} \tag{2.17}$$

and where e is the magnitude of the electron charge, $c_{\{B,p,n\}}$, s_w are arbitrary coefficients, and

$$\gamma^5 = \begin{pmatrix} -\mathbb{1}_{2\times 2} & 0 \\ 0 & \mathbb{1}_{2\times 2}\end{pmatrix} \tag{2.18}$$

so that $P_{L,R} = \frac{1}{2}(\mathbb{1} \pm \gamma^5)$ project out the left and right handed states of the Dirac fermion.

First of all, 2.16 is a *significantly* simpler result than we started with in equation 2.14. Secondly, it appears as though the proton and neutron (nucleons) are affected by physics that the electron and neutrino (leptons) are not as there are Pauli terms only for the nucleons. Given this, let us concentrate on the leptons for the time being and come back to the nucleons later.

With a concrete model in place, we can get back to the question we have postponed so far. We had to assume that any higher dimensional operators were suppressed by mass scales much greater than the energy of particles we are scattering in order to make progress. If now we push our model up to an energy $E \sim 1/\sqrt{G_F}$ terms of the form $E\sqrt{G_F}$ in our scattering amplitude, that we previously assumed were small, will no longer be small. Perturbation theory will break down and the model will no longer be predictive.[7] Experimentally we have

$$G_F = 1.2 \times 10^{-5}\text{GeV}^{-2} \tag{2.19}$$

and so this scale places a threshold on the utility of this model

$$G_F^{-\frac{1}{2}} \approx 300 \text{ GeV}. \tag{2.20}$$

Before going any further it is worth noting the fact that perturbation theory breaking down is a *good thing*. There is an automatic governor built in where the theory is telling us exactly where it fails. There is no need to speculate as to which regime it holds.

Now how can we go on to improve on this theory in order to go beyond this energy scale? We need to identify the new physics, namely the new DOF that need to come in near $E \sim \sqrt{G_F}$. Concentrating on the leptons, the interactions have dramatically simplified in 2 ways. First, there is a single scale $G_F^{-1/2}$ associated with all the 4 fermion operators. This implies that the same physics is at work in all the four fermion interactions. Moreover, the interactions take a very simple form (suggested by the notation in 2.16) of a current-current interaction. This interaction is similar to what we get when a photon mediates a force between 2 electromagnetic currents. What we need is a photon-like particle that is negligible at low energies, that is, we need a vector boson with mass.

The correct solution ends up being a set of 3 massive vector bosons - the $W^{\pm}$ and

[7] Technically something much worse is going on than perturbation theory failing. In fact, unitarity bounds are being violated at the scale $E \sim \sqrt{G_F}$ which is a fancy way of saying that the total probability of anything happening is greater than 100 % which implies the theory is fundamentally sick.

the Z. Taking these into account we write our improved model as

$$\mathcal{L} = i\bar{E}\partial\!\!\!/ E - m_E\bar{E}E + i\bar{\nu}\partial\!\!\!/\nu + eA_\mu J^\mu_{em} + gW^+_\mu J^\mu_- + gW^-_\mu J^\mu_+ + \frac{g_Z}{\sqrt{2}}Z_\mu J^\mu_Z$$
$$- \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} + m_W^2 W^+_\mu W^{-\mu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \text{ [8]} \qquad (2.21)$$

where the new parameters g, m_w and m_z related to the old G_F, by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{g_Z^2}{16m_Z^2}. \qquad (2.22)$$

If we treat the W and Z as very heavy, they essentially act as auxiliary fields (i.e. constraints) that should be integrated out. Integrating them out returns us back to our original Lagrangian 2.16. With this model we have to assume the dimensionless couplings e, g and g_z are small.

2.2.2 A sub-Standard Model Part II

Taken at face value, it would appear that the Lagrangian we have constructed in equation 2.21 exceeds our expectations. Not only have we accommodated the known interactions (the four-fermion interactions) in a way that holds to higher energies than the model we started with, this model does not contain any operators of inverse mass dimension at all. That means that this model appears to hold up to arbitrarily high energies since there are no terms at risk for diverging in an amplitude.

It turns out that this conclusion ends up being too naive. In order to see this, we should think more carefully about taking the high energy limit of our Lagrangian. Let us consider the simpler model below for now of a single massive vector field X^μ,

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m^2 X_\mu X^\mu \qquad (2.23)$$

where $X^{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$. We know intuitively the mass term will become irrelevant at high energies, since if we expand in the mass operator it will contribute terms of

[8] If you are worried that the above Lagrangian violates gauge invariance then you are getting ahead of the story. Keeping in mind the goal of writing down a succession of theories that hold at higher and higher energies we are doing just fine. Gauge invariance is a redundancy in our coordinates anyway, not a physical symmetry of the system.

the form $\left(\frac{m}{E}\right)^n$ to an amplitude. So at high energies we expect

$$\mathcal{L} \approx -\frac{1}{4} X_{\mu\nu} X^{\mu\nu}. \tag{2.24}$$

This is fine from the naive dimensional analysis of looking at mass dimension of operators, but we have unwittingly done something very wrong.

For the sake of simplicity, let us consider the particles on on-shell for the time being. We start with four independent DOF $X^\mu = (X^0, X^1, X^1, X^3)$. The equation of motion for the massive field X^μ is

$$\partial_\nu \partial^\nu X_\mu - \partial^\mu \partial^\nu X_\nu + m^2 X_\mu = 0. \tag{2.25}$$

Acting on the above equation with ∂^ρ and setting $\mu = \rho$ we get:

$$m^2 \partial_\mu X^\mu = 0. \tag{2.26}$$

Hence the original equation 2.25 is equivalent to the two equations

$$\partial_\nu \partial^\nu X_\mu + m^2 X_\mu = 0, \tag{2.27}$$

$$m^2 \partial_\mu X^\mu = 0. \tag{2.28}$$

Thus we have one constraint on four DOF, knocking it down to only three DOF on-shell for a massive vector boson.

For $m = 0$, quite a different situation arises. Now we notice from the outset the EOM is invariant under $X^\mu \to X^\mu + \partial^\mu \alpha(x)$. The freedom in this redefinition reflects a redundancy in our field variables X^μ. Moreover, this is a redundancy we are free to do without if we wish. As an example we are free to make a redefinition such that $X^0 = 0$ and $\partial^i X^i = 0$ demonstrating that there are only two DOF in actuality.

If massless vector bosons have two DOF, and massive vector bosons have three DOF, this presents an unanticipated issue for the massless limit since DOF are physical and can be measured. For example, in thermal equilibrium each DOF contributes $kT/2$ to the energy of a system so there is a distinct difference between a system with two or three DOF. That is, there is a discontinuous difference between the Lagrangians 2.23 and 2.24, and something unanticipated is happening in the naively continuous limit $m \to 0$.

To see what is happening we can perform a change of variables in order to rearrange the DOF more appropriately in the massless limit. Once again beginning with the massive vector boson Lagrangian,

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m^2 X_\mu X^\mu, \tag{2.29}$$

we make the field redefinition $X_\mu \to X_\mu + \frac{1}{m}\partial_\mu \pi$ and separate out the spin zero DOF which we denote π, giving us

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \partial_\mu \pi \partial^\mu \pi + 2m\partial_\mu \pi X^\mu + m^2 X_\mu X^\mu. \tag{2.30}$$

Now the X_μ only has two DOF on-shell and we can safely take the $m \to 0$ limit. In this limit the coefficient of the mixing term goes to zero and we get

$$\mathcal{L} \approx -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \partial_\mu \pi \partial^\mu \pi. \tag{2.31}$$

Hence at high energies the two spin one DOF and the one spin zero DOF decouple into three free massless DOF, and the massless limit makes sense. For future reference, we could have also written Lagrangian 2.30 as a 'linear sigma model'

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + v^2|(\partial_\mu - igX_\mu)e^{i\frac{\pi}{v}}|^2 \tag{2.32}$$

where $m = gv$.

The non-abelian version of this story is dramatically different. Now we start with a multiplet V_μ^a where $a = 1, 2, 3$. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^a V^{a\mu\nu} + \frac{1}{2}m^2 V_\mu^a V^{a\mu}. \tag{2.33}$$

We now have three vector bosons, each with three DOF, and each vector boson has one spin zero DOF. Similar field redefinitions and manipulations to expose the three spin zero components π^a yield the 'nonlinear sigma model'

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^a V^{a\mu\nu} + v^2 \mathrm{Tr}\; |(\partial_\mu - igV_\mu^a t^a)e^{i\frac{\pi^a t^a}{v}}|^2 \tag{2.34}$$

where $t^a = \frac{\sigma^a}{2}$ and σ^a are the Pauli matrices. Again this seems innocent enough, but there is a disaster lurking in the above Lagrangian. Expanding the above in powers of the spin zero fields π^a we get successively higher powers of the inverse mass scale v,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{6v^2}\left(\pi^a\pi^a\partial_\mu\pi^b\partial^\mu\pi^b - \pi^a\pi^b\partial_\mu\pi^a\partial^\mu\pi^b\right) + \mathcal{O}\left(\frac{1}{v^4}\right). \tag{2.35}$$

Our naive dimensional analysis has unwittingly led us astray. The massless, high energy limit exists, but it comes with a price - once again we are confronted with a model that contains operators with coefficients of inverse mass dimension, and so the model will fail above a particular cutoff.

In practical calculations the new model of 2.21 holds up to $\sim$ 1000 GeV[7] so we have bought about a factor of three over the old model 2.16. So we have made progress, but the situation is not as good as we had led ourselves to believe.

This means we need to repeat the earlier process and add new DOF in order to hopefully render this model consistent to higher energies. One reasonable guess is to note the following, if the scale m is generated spontaneously by the vacuum expectation value (vev) of a scalar field, we could write the 2.34 as

$$\mathcal{L} = -\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} + \mathrm{Tr}\;|(\partial_\mu - igV^a_\mu t^a)\langle\phi\rangle e^{i\frac{\theta^a t^a}{v}}|^2, \tag{2.36}$$

where $\langle\phi\rangle$ is the vet of a scalar field. We could go further and rewrite ϕ as a two component spinor as $\phi \to (0,\phi)^T$ so that we can also rewrite $e^{i\frac{\theta^a t^a}{m}}\phi \to H$ as a general two component scalar: $H \equiv (\phi_1 + i\phi_2, \phi_3 + i\phi_4)^T$ and we have

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} + |(\partial_\mu - igV^a_\mu t^a)H|^2 + \frac{\lambda}{4!}\left(|H|^2 - v^2\right)^2 \\ &= -\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} + (D_\mu H)^\dagger D^\mu H + \frac{\lambda}{4!}\left(|H|^2 - v^2\right)^2.\end{aligned} \tag{2.37}$$

where we have added a potential for the scalar to ensure it gets a vev v. Now this model definitely has a well defined massless limit for the V^a_μ since there is no vector boson mass to even worry about anymore. Moreover we still have only operators with at most dimensionless coefficients.[9]

[9]We will address how the Higgs couples to fermions when we discuss the SM in full.

One might argue that we have pulled a fast one - we have just rewritten the Lagrangian such that it is 'primed' and ready to give the V_μ^a their masses. However, the crucial point is that we can write the model so there are no operators of dimension higher than four.

So far this was just a toy model. The scalar particle we would add in order to patch up the model 2.21 is known as the Higgs. Keep in mind this is not proof of what nature chooses, it is just a guess for a model that holds up to higher energies. It turns out that this guess is a good one and as of March 2013, we have seen a boson that is consistent with the SM Higgs [8].

2.2.3 A sub-Standard Model Part III

We need to tidy up some loose ends. First of all, we neglected the proton and neutron in section 2.2.1 since we were not sure how to handle the Pauli terms for them at that point. These terms contribute to the magnetic moment of the particles, which experimentally deviate substantially from the values we would expect from a point particle [19].[10]This implies that the proton and neutron are not fundamental particles, and in fact have substructure. While we are putting everything on the table, we totally ignored one aspect entirely in section 2.2.1 when we failed to mention the pions for the sake of the story.

The complete low energy model involving the nucleons and pions is the Chiral Lagrangian

$$\begin{aligned}\mathcal{L}_{chiral} = &-\frac{1}{4}f_\pi^2 \mathrm{Tr}\ D^\mu U^\dagger D_\mu U + v^3 \mathrm{Tr}\ \left(MU + M^\dagger U^\dagger\right) + i\bar{N}\not{\partial}N \\ &- m_N \bar{N}\left(U^\dagger P_L + UP_R\right)N - \frac{1}{2}(g_a - 1)i\bar{N}\gamma^\mu(UD_\mu U^\dagger P_L + U^\dagger D_\mu U P_R)N + \dots\end{aligned} \tag{2.38}$$

where $U(x) = \exp[\frac{2i\pi^a(x)T^a}{f_\pi}]$, $\pi^a(x)$ is the pion field, N is the nucleon field (a doublet consisting of the proton and neutron), f_π is the pion decay constant, M is the quark mass matrix, v^3 is the value of the quark condensate, m_N is the nucleon mass, g_A

[10]Being neutral, naively the neutron should not have a magnetic moment at all!

is the axial vector coupling, D_μ is the covariant derivative and the dots represent subleading terms in the quark masses [9].

For all its complexity, the important point to notice is this is just another nonlinear sigma model. We know the story already - a nonlinear sigma model will break down, in this case at the scale f_π. Our first guess might be to add a scalar condensate as we did in section 2.2.2. Although this might formally fix the model to hold to higher energies, it is not going to yield a theory in which the pions and nucleons reveal their composite DOF; it would still just be a model of pions and nucleons and the new scalar DOF.

What we need is a model where the elementary ultraviolet DOF are locked in bound states at low energies. However, these bound states must be very different than say, the bound state of a electron and proton in a hydrogen nucleus, as no one has ever actually seen the constituent DOF of a nucleon. That is, we do not just want something that is attractive at low energies, we want something that is 'confining'.

Roughly speaking, we need an interaction associated with a scale Λ that gives contributions to an amplitude that go like negative powers of the energy (contributions like $\left(\frac{\Lambda}{E}\right)^n$) so when $E \to 0$ this becomes the dominating term. Formally speaking, we would say we need an operator in our Lagrangian that is relevant at low energies that creates a confining potential between fermions, and irrelevant at high energies where it reveals the composite DOF. One candidate interaction is $m\bar{\psi}\psi$, however this interaction will not do the trick, since a Lagrangian consisting of this term and a kinetic term is just a theory of free fermions. Any operator with more fermions, like the four fermion operator we have already explored, will be irrelevant at low energies and lead to a weak interaction at large distances.

Let us start from scratch then. We need something that not only has good high energy behavior, but in some sense has worse low energy behavior. In other words, we want an interaction that totally blows up at low energies. In order to do this, we need to develop our arguments a little bit more then we have up until this point.

2.2.4 Intermediate Dimensional Analysis

Previously we used dimensional analysis to probe the structure of the theory to see how it scaled with energy, and in particular, to find out when the model fails. Let us formalize the naive dimensional analysis arguments we have been making so far.

Let us analyze one of the simplest possible models we can, a free, real, scalar field with mass m. The starting point is the Lagrangian

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi - m^2\phi^2. \tag{2.39}$$

At high energies we know we can ignore the mass so that we just have

$$\mathcal{L}_{E\gg m} \approx \partial_\mu\phi\partial^\mu\phi \tag{2.40}$$

and at very low energies the fluctuations in the ϕ field freeze out completely (that is, ϕ should be integrated out) and we are left with

$$\mathcal{L}_{E\ll m} \approx 0\ . \tag{2.41}$$

In order to interpolate between these three cases properly, we should remember the basic tenant of physics, that one should only perturb in dimensionless parameters. That is, if we really want to keep track of how the mass affects our model, we should define a new parameter to keep track of

$$\nu \equiv \frac{m}{E} \tag{2.42}$$

where E is the energy scale at which we work. It makes sense to define this parameter because when we say we work with 'small mass' we really mean small mass with respect to energy and when we perform a perturbative expansion, ν is the parameter in which we will be expanding.

We want to track how a parameter for perturbation theory changes with energy, so we define the 'beta function'

$$\beta_\nu \equiv \frac{\partial\nu}{\partial\log E} = E\frac{\partial\frac{m}{E}}{\partial E} = E(-1)\frac{m}{E^2} = (-1)\frac{m}{E} = -(1)\nu.^{11} \tag{2.43}$$

[11] If you find the $\log E$ offensive dimensionally, just think of it as shorthand for $E\frac{\partial}{\partial E}$.

The 1 reflects the fact that mass term has mass dimension 1 and the $-$ reflects the fact that the importance of ν as a parameter decreases as we increase energy. That is, the mass is relevant at low energies and irrelevant at high energies. Thus equation 2.43 formalizes the dimensional analysis arguments we have been making up until this point. It seems silly that we write down such a simple relation since it captures something we already knew before, however the power of the beta function is that we can calculate quantum corrections to it, and thus capture non trivial behavior in the quantum domain that is not so clear or intuitive.

Up until now, we have only been taking the *naive* or *classical* scaling dimension into account. If we take quantum corrections into account, the beta function will look like

$$\beta_\lambda = \frac{\partial \nu}{\partial \log E} = -(1+\gamma_\nu)\nu. \tag{2.44}$$

where we have defined the 'anomalous dimension', γ_ν. The 'anomalous' does not refer to a deviation from the correct behavior, but rather it reflects a deviation from the naive classical behavior. Now by including quantum corrections we can more completely investigate how a model behaves throughout a range of energies. This is the same thing we have been doing up until now, but we have been analyzing models 'classically'.

As an example we can calculate the beta function for charge of the electron, e, in QED, where

$$\mathcal{L}_{QED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi. \tag{2.45}$$

Classically e is dimensionless (in 'natural' units) and we have

$$\beta = \frac{\partial e}{\partial \log E} = 0. \tag{2.46}$$

This is referred to as a 'marginal' coupling as it does not scale with energy at all. We can compute quantum corrections to this and obtain

$$\beta = \frac{\partial e}{\partial \log E} = +\hbar\frac{e^3}{12\pi^2} + \mathcal{O}(\hbar^2).\ [12] \tag{2.47}$$

[12]Some comments are in order. First of all, because we can always simultaneously rescale fields

This is a highly nontrivial result. The + sign implies that quantum mechanical corrections alter the naive scaling dimension of the electric charge, making it grow at high energies and nudging it from a marginal coupling to a relevant coupling. At high energies the electric charge will continue to grow and QED will become strongly coupled. This result is almost what we want, we just want a coupling between fermions that becomes relevant, and then continually grows until the model becomes strongly coupled at low energies. That is, we want a minus sign instead of a plus sign in equation 2.47.

Let us push on and consider a SU(N) non-abelian gauge theory with n_f fermions transforming under the representation r

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \bar{\psi}_i(\not{D} - m_i)\psi_i. \tag{2.48}$$

The one-loop quantum corrected beta function for this model is

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(r)\right) \tag{2.49}$$

where $C_2(G)$ and $C(r)$ are invariants of the SU(N) group. For a particular choice of fermions the quantity inside the round brackets will be positive and the beta function will be negative. A negative beta function means the coupling in a non abelian gauge theory is pushed from being marginal to relevant at low energies. It fact, it keeps growing without bound at low energies. This sort of behavior is exactly what we need if we want a model of strongly coupled fermions at low energies.[13]

This argument is rough so far and we can do a bit better. We would like to see what happens when we take two fermions charged under a non-abelian gauge group

and coupling constants, the beta function will in general depend on the normalization we choose for our fields. For further discussion see [10]. Furthermore, the presence of the $\hbar$s seem to invalidate the argument that we should only work with dimensionless quantities. At any rate, expansions in $\hbar$ should always be taken with a grain of salt as their presence here is only to roughly illustrate the departure from the lowest order results. The $\hbar$s in this equation is only to formally illustrate that the correction to the beta function is quantum in its origin. All other equations have $\hbar = 1$.

[13] "I admit this is blatant hand-waving. However this is not some new hand-waving...but the same old hand-waiving that accompanies any discussion of the large-scale behavior of non-abelian gauge field theories." - Sidney Coleman [11]

and try to separate them. This corresponds to taking the expectation value of the Wilson loop,

$$W_C = \mathrm{Tr}\left(\mathcal{P}\ \exp\ i \oint_C A_\mu dx^\mu\right), \tag{2.50}$$

where $\mathcal{P}$ is the path ordering operator, and C is a closed curve. $\oint_C A_\mu dx^\mu$ corresponds to moving a charge along the closed curve C in the presence of a gauge field A_μ. For simplicity we choose a closed path of size R in the space direction for a duration time T which has area $A = RT$ where $T \gg R$ so that the perimeter $P \approx T$. The expectation value of the Wilson loop should yield the effective potential between the particles,

$$e^{-V(R)_{eff}T} = \langle 0|W_C|0\rangle = \frac{\int \mathcal{D}A^\mu\ W_C\ e^{iS}}{\int \mathcal{D}A^\mu\ e^{iS}}. \tag{2.51}$$

As a warm up let us perform the calculation for QED. The result is

$$e^{-V(R)_{eff}T} = \langle 0|W_C|0\rangle \sim e^{-\left(-\frac{\alpha}{R}\right)T}. \tag{2.52}$$

Solving for the effective potential we obtain

$$V(R)_{eff} = -\frac{\alpha}{R} + c \tag{2.53}$$

where c is a constant. This is exactly what we expect for a theory like QED - a Coulomb potential.

Let us continue with the non-abelian calculation for which we obtain

$$e^{-V(R)_{eff}T} = \langle 0|W_C|0\rangle \sim e^{-\tau A} \tag{2.54}$$

where τ is the 'string tension' $\sim \frac{\log g^2}{a^2}$, g is the gauge coupling, and a is a lattice cutoff. Hence,

$$e^{-V(R)_{eff}T} \sim e^{-\tau RT} \tag{2.55}$$

thus,

$$V(R)_{eff} = \tau R + c \tag{2.56}$$

where c is a constant. Thus, for a non-abelian gauge theory, charges experience a linear potential. This gives credibility to the notion that particles charged in a non-abelian gauge theory will remain confined in bound states at low energies (large distances). The particular gauge group nature chooses is SU(3) and the model is known as Quantum Chromodynamics (QCD). This by no means constitutes a proof that QCD confines and yields the pions and nucleons of the Chiral Lagrangian at low energies. In fact, no proof exists [12], but detailed numerical studies back up our naive arguments [13].

2.2.5 The Standard Model

All the pieces are now in place. Putting them all together, in compact notation, the SM is

$$\begin{aligned}
\mathcal{L}_{SM} = &-\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}A^{a\mu\nu}A^a_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} \\
&+ i\bar{Q}_{Li}\not{D}Q_{Li} + i\bar{d}_{Ri}\not{D}d_{Ri} + i\bar{u}_{Ri}\not{D}u_{Ri} - (\lambda_d^{ij}\bar{Q}^i_L \cdot H d^j_R + \lambda_u^{ij}\epsilon^{ab}\bar{Q}^i_{La}H^\dagger_b u^j_R + \text{h.c.}) \\
&+ i\bar{E}_{Li}\not{D}E_{Li} + i\bar{e}_{Ri}\not{D}e_{Ri} + i\bar{\nu}_{Li}\not{D}\nu_{Li} - (\lambda_l^{ij}\bar{E}^i_L \cdot H e^j_R + \text{h.c.}) \\
&+ D_\mu H^\dagger D^\mu H + \mu^2 H^\dagger H - \lambda(H^\dagger H)^2
\end{aligned} \tag{2.57}$$

where λ_{ij} are 3×3 matrices and the $\cdot$ in the Yukawa term is an SU(2) contraction. The gauge group of the model is $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The covariant derivative is $D_\mu = \partial_\mu - igA^a_\mu\tau^a - iYg'B_\mu - ig_sG^a_\mu t^a$ where $\tau^a = \frac{1}{2}\sigma^a$, where σ^a are the Pauli matrices, and t^a are the Gell-Mann matrices. The SM is anomaly free - that is, the global symmetries that have been 'gauged' really are honest symmetries, which we need in order to cancel the unphysical polarizations we have mentioned. All quarks (leptons) are charged (neutral) under $\mathrm{SU}(3)_c$. The left handed fermions organize themselves into $\mathrm{SU}(2)_L$ doublets

$$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \text{and} \qquad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \tag{2.58}$$

and then every left handed fermion gets a right handed partner that is an $\mathrm{SU}(2)_L$ singlet.

The lone scalar of the SM, the Higgs, is also an SU(2) doublet, is charged under the $\mathrm{U}(1)_Y$, and is a $\mathrm{SU}(3)_c$ singlet. A critical component of the SM is that the minimum of the Higgs potential is not at zero so that the Higgs gets a vev

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{2.59}$$

where $v = 246$ GeV.

This vev 'breaks' electroweak symmetry, dramatically altering the features of the model and yielding the world we see. The Lagrangian is much uglier after electroweak symmetry breaking so we will just summarize the qualitative features in the broken phase. The $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is broken to its U(1) subgroup of electromagnetism by the vev of the Higgs. Three of the Higgs DOF are eaten to make the $W^{\pm}$ and the Z massive. This makes the forces associated with these gauge bosons short ranged. The fourth DOF from the Higgs is the particle we typically refer to as *the Higgs*. Finally, $\mathrm{SU}(3)_c$ remains unbroken and it confines by the process described in section 2.2.4.

Each fermion, except for the neutrinos, get a mass through the Higgs, albeit in a slightly different mechanism than the gauge bosons. For example, looking just at the left and right handed electron's coupling to the Higgs in the unbroken phase,

$$\mathcal{L} = -\lambda \bar{E}_L \cdot H e_R + h.c. \tag{2.60}$$

which after the Higgs gets a vev looks like

$$\mathcal{L} = -\lambda \frac{1}{\sqrt{2}} \bar{e}_L e_R + h.c. \tag{2.61}$$

which is typically written as a Dirac spinor mass term

$$\mathcal{L} = -m_e(\bar{e}_L e_R + h.c.) = -m_e \bar{\psi}_e \psi_e. \tag{2.62}$$

So the fermions do not get mass by eating any DOF from the Higgs, they get an effective mass as a Dirac electron swaps right and left handed helicity components as it bounces off a Higgs condensate.

It is nice to come full circle to one of the magical simplifications we noticed early on in 2.2.3 when we noted that the Pauli terms for the leptons were vanishingly small. The reason is the Pauli terms were not there for the leptons in the first place, as they broke $\mathrm{SU(2)_L}$. The operator that would give a Pauli term would have been

$$\frac{1}{\Lambda^2} B_{\mu\nu} H \cdot \bar{E}_L \sigma^{\mu\nu} e_R + h.c. \tag{2.63}$$

which after the Higgs gets a vev, contains a term

$$\frac{v}{\Lambda}\frac{1}{\Lambda}F_{\mu\nu}\bar{e}\sigma^{\mu\nu}e. \tag{2.64}$$

This has an additional power of suppression of v/Λ instead of what we naively wrote down at first which was

$$\frac{1}{\Lambda}F_{\mu\nu}\bar{e}\sigma^{\mu\nu}e. \tag{2.65}$$

2.3 Beyond the Standard Model

Many tools are on the table and now we can explore how we can use them to go beyond the SM.

2.3.1 Cracks in the Standard Model

It is important to note that there are no more surprises lurking like there was in section 2.2.2. There really are only dimension four operators in the Lagrangian above. But the lack of higher dimensional operators in the SM is just a feature of the model, not necessarily a feature of nature. It could easily turn out that we have been neglecting some other operators that we should take into account once we probe high enough energy. Let us try to extend this algorithm of adding higher dimensional operators, finding when perturbation theory fails, and then adding the necessary DOF in order to patch up the model.

We know for sure there is an outstanding issue looming over us, and that is gravity. The Einstein Hilbert action for General Relativity (GR) is

$$S = \int \sqrt{g}\, d^4x \left(\frac{1}{16\pi G}R + \mathcal{L}_M\right) \tag{2.66}$$

where $g^{\mu\nu}$ is the metric tensor, $g = \det g^{\mu\nu}$, R is the Ricci Scalar and $\mathcal{L}$ is the matter content (which is just the SM Lagragian with $\eta_{\mu\nu} \to g_{\mu\nu}$ for the time being). Note that Newtons constant G has dimensions of mass^{-2} so it is standard to define the Planck mass $M_P \equiv 1/\sqrt{16\pi G}$. Expanding out in terms of fluctuations about a flat space background metric $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$ and canonically normalizing the $h_{\mu\nu}$ field we obtain

$$S = \int d^4x \, \left((\partial h)^2 + \frac{1}{M_P} h(\partial h)^2 + \frac{1}{M_P^2} h^2 (\partial h)^2 + \dots \right) \tag{2.67}$$

where we have just written the operators schematically and neglected careful index contractions. We can see that there are a whole slew of higher dimensional operators suppressed by inverse powers of the Planck mass signaling the breakdown of perturbation for $E \sim M_P$. While this problem is present, it is not of immediate concern since the Planck mass $M_P \sim G^{-1/2} \sim 10^{15}$ TeV and this will have negligible impact at our colliders operating at $\mathcal{O}(\text{TeV})$. Thus, while gravity is a long way off from being a concern for collider phenomenology, it is always there reminding us that at the very least the SM is not the last word. This is a nice verification of why effective field theory works so well - we simply do not have to bother with gravity at the relatively meager energy scales of our colliders.

What we really want is higher dimensional operators that have effects at energy scales we can actually probe. For example, two candidate operators that we could write down and investigate are

$$\mathcal{O}_S = \frac{1}{\Lambda^2} |H^\dagger D_\mu H|^2, \tag{2.68}$$

and

$$\mathcal{O}_T = \frac{1}{\Lambda^2} H^\dagger \sigma^i H A^i_{\mu\nu} B^{\mu\nu} \tag{2.69}$$

where H is the Higgs field, σ^i are the Pauli matrices, $B_{\mu\nu}$ is the U$(1)_Y$ field strength, $A^i_{\mu\nu}$ is the SU$(2)_L$ field strength, and Λ is the unknown scale of new physics. Then we can have the full Lagrangian

$$\mathcal{L}_{tot} = \mathcal{L}_{SM} + a_S \mathcal{O}_S + a_T \mathcal{O}_T. \tag{2.70}$$

It is standard to define

$$S \equiv \frac{4 \sin\theta_w \cos\theta_w v^2}{\alpha} a_S \quad \text{and} \quad T \equiv -\frac{v^2}{2\alpha} a_T \tag{2.71}$$

where v is the Higgs vev, θ_w is the weak mixing angle, and α is the fine structure constant. This definition of S and T is so that new physics at the weak scale would have $\mathcal{O}(1)$ contributions to S and T. The plot S and T in figure 2.1 shows that these parameters are extremely small, and are consistent with $S = T = 0$. The fact that S and T are so small suggests that the scale of new physics contributing to these operators is well above the TeV scale.

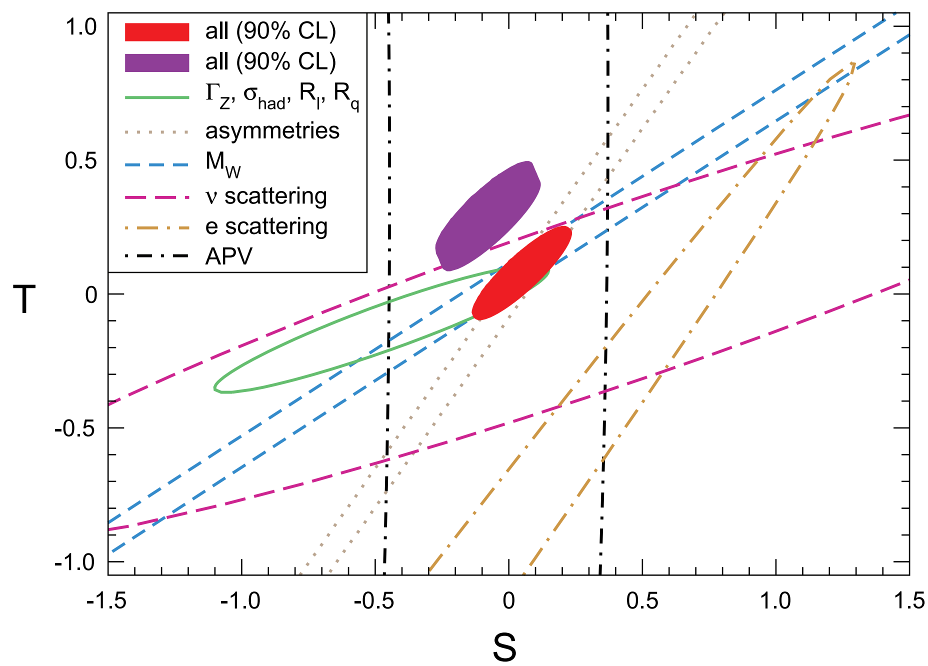


Figure 2.1: 1 σ constraints (39.35%) on S and T from the various inputs combined with M_Z. S and T represent the contributions of new physics only. The contours assume 115.5 GeV $< M_H <$ 127 GeV except for the larger (violet) one, for which the data is for 600 GeV $< M_H <$ 1 TeV. The relevant portion of the data for this discussion is the red ellipse centered near $S = T = 0$ [19].

At this point, we could enumerate all other higher dimensional operators (see [14] for a complete list), but the story is the same. There does not seem to be new physics beyond the SM that we can detect with any significance. On one hand it is wonderful that we have a model that holds well to $\mathcal{O}$(TeV) and likely well beyond this, but we expect some small corrections in the form of higher dimensional operators which will eventually invalidate the model much above this scale. This makes the SM unreasonably effective.

We could continue to enumerate all the possible motivations for new physics, but before moving on there is one thing worth mentioning. Getting back to our original question, we want concise, beautiful laws of physics. The SM has lots of parameters, and lacks a deep explanation of the mass hierarchies that appear and so is not beautiful by this criteria. We would much rather have a more concise description of nature, and for this reason alone it is worth pursuing BSM physics even though the SM works very well in the meantime. That is, from an experimental point of view, the SM is quite satisfactory but it is our desire and intuition for something simpler that makes us look for a more fundamental model.

2.3.2 Advanced Dimensional Analysis

Given that our naive dimensional analysis is not yielding any indications for new physics, let us again turn to the (quantum-corrected) beta functions to see if there are any clues for new physics. For starters, let us return to one of our earlier calculations for the QED beta function,

$$\beta = \frac{\partial e}{\partial \log E} = \frac{e^3}{12\pi^2}. \tag{2.72}$$

Recall that the + sign in the beta function means that the electron's charge grows with energy.

We can go further than just noting the qualitative features of this equation and solve it as a differential equation to obtain

$$e^2(\mu) = \frac{e^2(M)}{1 - \frac{e^2(M)}{6\pi^2}\log\left(\frac{\mu}{M}\right)}. \tag{2.73}$$

The meaning of this solution is that we can relate the electric charge at one energy μ to the electric charge at another energy M. Since the charge grows with energy this means at some point perturbation theory will break down, namely when e is $\mathcal{O}(1)$. If we take $\mu < M$ and $e(\mu)$ to be the charge we see at typical energies, then M will be the scale at which perturbation theory breaks down. Solving for M we obtain

$$M = \mu e^{6\pi^2\left(\frac{1}{e(\mu)^2} - \frac{1}{e(M)^2}\right)}. \tag{2.74}$$

For $e(M) = 1$, $e(\mu)^2 = 4\pi\alpha \approx \frac{4\pi}{137}$, and $\mu = 1\ GeV$ we obtain

$$M \approx e^{60\pi^2} GeV \sim 10^{255}\ GeV. \tag{2.75}$$

This result means that quantum corrections do indeed push the electron charge outside the perturbative regime and perturbation theory will fail at this high scale.[14] New physics will have to come in to render the theory perturbative, as it did with the W and Z, and the Higgs, but the consequences of this new physics are far removed as this is many orders of magnitude larger than the Planck scale we already said we would not worry about. While this example did not yield any consequences at terrestrial energies, the result was intriguing. Using this new technology we can probe the consistency of the SM up to higher energies in other ways.

Let us consider another simple model

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m_\psi\bar{\psi}\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}{m_\phi}^2\phi^2 - \frac{\lambda}{4!}\phi^4 + ig\phi\bar{\psi}\gamma^5\psi.\ ^{15} \tag{2.76}$$

Performing a one-loop calculation we obtain the one-loop corrected beta functions and anomalous dimensions,

$$\beta_g = \frac{\partial g}{\partial \log E} = \frac{5g^3}{16\pi^2} \tag{2.77}$$

$$\beta_\lambda = \frac{\partial \lambda}{\partial \log E} = \frac{1}{16\pi^2}\left(3\lambda^2 + 8\lambda g^2 - 48g^4\right) \tag{2.78}$$

$$\gamma_{m_\psi} = \frac{\partial m_\psi}{\partial \log E} = \frac{g^2}{16\pi^2} \tag{2.79}$$

$$\gamma_{m_\phi} = \frac{\partial m_\phi}{\partial \log E} = \frac{g^2}{8\pi^2}\left(1 - 2\frac{m_\psi^2}{m_\phi^2}\right) + \frac{\lambda}{32\pi^2} \tag{2.80}$$

For g we can solve and obtain

$$g(E)^2 = \frac{g(M)^2}{1 + \frac{5g(M)^2}{8\pi^2}\log\left(\frac{M}{E}\right)}, \tag{2.81}$$

and for the fermion mass

$$m_\psi(E)^2 = \frac{m_\psi(\mu)^2}{\left[1 + \frac{5g(M)^2}{8\pi^2}\log\left(\frac{M}{E}\right)\right]^{5/2}}. \tag{2.82}$$

[14]Given that we assumed perturbation theory in order to perform this calculation in the first place casts doubt on this precise energy, but the conclusion is the same - at some energy we will still lose perturbative control.

[15]In the Yukawa coupling there is a factor of γ^5 since we have made ϕ a pseudo-scalar. It is not critical to the discussion that ϕ is a pseudo scalar, but it makes the calculation much easier since we do not have to introduce counter terms for odd powers of ϕ.

Both these equations have a similar form to that of the electron charge in QED in that the coupling constants grow at high energies, but very slowly.

The γ_{m_ϕ} equation has a much more complicated expression. Since g and the fermion mass run slowly we can neglect their energy dependance in the γ_{m_ϕ} equation and assume them to be constant. Additionally, assuming λ is small we neglect it, and solve the γ_{m_ϕ} equation for the physical Higgs mass

$$m_\phi(E)^2 = m_\phi(M)^2 + \frac{g^2}{4\pi^2} \log \frac{M}{E} \left(2m_\psi^2 - m_\phi(M)^2\right). \tag{2.83}$$

This equation has a subtly different form than the other solutions to the renormalization group equations. For very large fermion mass m_ψ we have the simple relation

$$m_\phi(E)^2 \sim m_\psi^2. \tag{2.84}$$

This equation reflects the fact that the scalar mass is getting dragged all the way up to the scale of the heavy fermion. That is, if the fermion is heavy, it is unavoidable that the scalar is too. Taken at face value, this is not a concern. It is actually a good thing, it is a prediction and the model is telling us that scalars are naturally heavy. [16]

2.3.3 Fine-Tuning of the Higgs Mass

The case we just considered in equation 2.83 is a simple model. The SM is much more complicated than this and the RG equations are hideously coupled. Nonetheless, it turns out this feature persists and in general, scalars are naturally as heavy as the heaviest mass scale around. Being a scalar, the Higgs displays this feature - no matter what we set the Higgs mass to at a high scale, as we run it down to low energies, quantum corrections will drag it up to the highest mass scale around. A priori this is not a huge concern as experiment tells us that the Higgs is around 126 GeV, which is

[16] Fermions do not suffer this fine-tuning as can be seen for example in 2.82 - if we set the boundary condition $m_\psi(M) = 0$ then no fermion mass is generated perturbatively. However fermion masses can be generated when chiral symmetry is broken spontaneously by non-perturbative effects such as in QCD or as we do in chapter 4.

not too far from the heaviest mass scale in the SM, the top quark mass at 173 GeV. So there is not a huge fine-tuning problem if we consider the SM in isolation.

But recall what we explored in section 2.3.1- there is always gravity lurking so naively the Higgs mass should be $\mathcal{O}(M_p)$.[17] Furthermore we expect there to be new physics, and thus new mass scales that arise between the Top mass and the Planck mass which only compound the problem. The only way the SM plus gravity would produce such a light Higgs is if there was an incredibly delicate cancelation of the mass terms on the right hand side of 2.83. This delicate cancelation is known as a fine-tuning. The conclusion is that, by any reasonable metric, the Higgs mass is incredibly fine-tuned. This fine-tuning of the Higgs mass in particular is also known as the hierarchy problem.

Given the staggering degree of fine-tuning of the Higgs mass, we should explore options that assuage this tuning at least to some degree. In general, the Higgs mass equation will be much more complicated, with many mass scales M_i coming in at every order of perturbation theory in the dimensionless couplings g_i which we express as

$$m_\phi(E)^2 = m_\phi(M)^2 - \frac{g_1^2}{4\pi^2}\log\frac{M}{E}\left(m_\phi(M)^2 \pm M_1^2 \pm M_2^2 + \ldots\right) \pm g_2^2 + \ldots \tag{2.85}$$

where the $+(-)$ are for boson (fermion) masses.

First of all, if the masses M_i were all the same for some reason then a cancelation would be believable. From a physics perspective, that means the masses must be related in some fashion, that is, there must be some sort of symmetry amongst them. The oldest idea along these lines is supersymmetry (SUSY). Since fermion's masses and boson's masses appear with opposite signs in the equation 2.85 for the Higgs mass, the natural thing to do is have a symmetry that relates bosons and fermions - this is SUSY. Unfortunately if SUSY is a symmetry of nature, it is broken at a scale higher than the weak scale [19] leaving at least some fine-tuning remaining for the Higgs. While SUSY has many other effects and properties that make it interesting to

[17]Since all the interactions in the Einstein Hilbert Action equation 2.66 have inverse powers of the Planck mass technically this will never happen in perturbation theory, so it is really non-perturbative contributions to the Higgs mass that concern us when it comes to gravity.

study on its own, and moreover it could still be a symmetry of nature at very high energies, it may not be the solution to the hierarchy problem.

We can also take note of the fact that there is one place where we have seen naturally light scalars already - the pions of the Chiral Lagrangian in 2.2.3. Because the pions are composite DOF, they are not sensitive to energy scales above the scale at which their constituents confine. By making the Higgs a composite DOF it would only be sensitive to physics up to the scale that the given model breaks down at since the Higgs is no longer an appropriate DOF above this scale. Models of this type have long been of interest, in particular, models of top condensation [76]. We explore a toy model for a composite Higgs in chapter 4.

A final word on fine-tuning is worth reflection. Fine-tuning is a guide we use to build new models of physics. Being natural is nothing nature has to adhere to or respect. That is, the hierarchy problem *does not* have to be solved. This is in stark contrast to the position we found ourselves in at the outset of sections 2.2.1, 2.2.2, and 2.2.3. In those cases, perturbation theory was breaking down and we had no choice, if we wanted a model that had predictive power above a certain energy scale we were forced into considering new physics beyond the model in consideration. Being fine-tuned on the other hand is something that nature may or may not address in time. Only in retrospect will we be able to tell what amount of fine-tuning nature has chosen to be acceptable.[18] If it turns out that nature is indeed not fine-tuned, it will be a tremendous advance purely based on our intuition of how nature should work.

2.3.4 The Strong CP Problem

We now turn our attention to a fine-tuning problem of a different sort in the SM, the Strong CP problem. When we discussed the perturbative expansion in section 2.1.3

[18]Note that the level at which people will accept fine-tuning is a function of time. In the 80s and 90s when the minimal supersymmetric SM (MSSM) was fashionable, people did not think there would be much fine-tuning at all. Nowadays it is considered acceptable to have some degree of fine-tuning. See [17] and [18] as examples of models with some degree of fine-tuning.

we were not very precise about what we were expanding around, now we need to be a bit more precise. In this section we follow chapters 93 and 94 of [9] closely.

As a toy model, let us consider the one dimensional Hamiltonian

$$H = \frac{p^2}{2m} + \frac{\lambda}{4!}(x^2 - v^2)^2. \tag{2.86}$$

To find the ground state, we extremize the potential and find two stable solutions at $x = \pm v$. Classically the particle will be confined to one of the minima and it will undergo simple harmonic motion about the minimum it chooses. Quantum mechanically the situation is more subtle because there will be tunneling between the two minima so one can not simply isolate a particle to one or the other. The true vacuum is going to be some linear combination of the classical minima $|0_{true}\rangle = \alpha|x = +v\rangle + \beta|x = -v\rangle$.

A similar situation arises in a SU(N) non-abelian gauge theory where we have the action

$$S = -\int d^4x \; \frac{1}{2g^2}\mathrm{Tr}\;[F^{\mu\nu}F_{\mu\nu}]. \tag{2.87}$$

The ground states of this model are given by solutions to the equation $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu = 0$. Solutions to this are a 'pure gauge' and are of the form $A_\mu(x) = iU(x)\partial_\mu U(x)^\dagger$. The solutions are classified by the winding number which is written as a surface integral over the field configuration

$$\begin{aligned} n &= \frac{1}{24\pi^2}\int dS_\mu \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\;[(U\partial_\nu U^\dagger)(U\partial_\rho U^\dagger)(U\partial_\sigma U^\dagger)] \\ &= \frac{i}{24\pi^2}\int dS_\mu \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\;[A_\nu A_\rho A_\sigma] \\ &= \frac{1}{16\pi^2}\int d^4x \; \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\;[F_{\mu\nu}F_{\rho\sigma}]. \end{aligned} \tag{2.88}$$

When we quantize this theory there will be tunneling between the vacua labeled by the integers n. The true vacuum will be linear combination of the vacua $|n\rangle$, denoted the 'theta' vacuum

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}|n\rangle. \tag{2.89}$$

The vacuum-to-vacuum transition for the theta vacuum is given by

$$\mathcal{Z}_\theta = \sum_{n=-\infty}^{\infty} e^{-in\theta} \int \mathcal{D}A^\mu \; e^{-\int d^4x \; \frac{1}{4g^2}\mathrm{Tr}\;[F^{\mu\nu}F_{\mu\nu}]}$$
$$= \int \mathcal{D}A^\mu e^{i\int d^4x \; \left(\frac{1}{2g^2}\mathrm{Tr}\;[F^{\mu\nu}F_{\mu\nu}] - \frac{\theta}{16\pi^2}\mathrm{Tr}\;[F^{\mu\nu}\tilde{F}_{\mu\nu}]\right)}. \tag{2.90}$$

Strikingly, a similar contribution to the path integral comes from a completely different source. There is nothing stopping us from considering the more general fermion mass term

$$\mathcal{L}_{mass} = -m\bar{\psi}e^{i2\alpha\gamma^5}\psi. \tag{2.91}$$

It is convenient to perform a chiral rotation to study the effects of the parameter α,

$$\psi \to e^{-i\alpha\gamma^5}\psi,$$
$$\bar{\psi} \to e^{-i\alpha\gamma^5}\bar{\psi}. \tag{2.92}$$

For fermions charged under a non-abelian gauge group this change of variables induces a Jacobian from the fermion integration measure [20]

$$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \mathcal{D}\psi\mathcal{D}\bar{\psi} \; e^{-i\int d^4x \; \frac{\alpha}{8\pi^2}\mathrm{Tr}\;[F^{\mu\nu}F_{\mu\nu}]} \tag{2.93}$$

which has the effect on the action

$$S \to S - \int d^4x \; \frac{\alpha}{8\pi^2}\mathrm{Tr}\;[F^{\mu\nu}F_{\mu\nu}]. \tag{2.94}$$

An analogous situation arises in QCD where the θ_{QCD} angle combines with the argument of the determinant of the quark mass matrix to create an additional term in the SM Lagrangian,

$$\mathcal{L}_{CP} = -\frac{n_f g^2\theta}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\; F_{\mu\nu}F^{\rho\sigma} \tag{2.95}$$

where $\theta = \theta_{QCD} - \arg(|M_q|)$ and n_f is the number of quarks. We did not include this term in the SM originally because it is a total derivative,

$$\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\; F_{\mu\nu}F^{\rho\sigma} = \partial_\mu J^\mu = \partial_\mu 2\epsilon^{\mu\nu\sigma\rho}\mathrm{Tr}\;(A_\nu F_{\sigma\rho} + \frac{i2}{3}gA_\nu A_\sigma A_\rho), \tag{2.96}$$

which will not contribute anything to an amplitude in a perturbative expansion in QCD.

However, this is not the end of the story as we know QCD is not a convenient way to analyze low energy physics as we explored in section 2.2.3. Instead we should be analyzing effects of the theta term on the low energy DOF, the nucleons and pions of the Chiral Lagrangian. Since QCD depends on the combination $\theta = \theta_{QCD} - \arg(|M_q|)$ this means in the Chiral Lagrangian wherever we see $\arg(|M_q|)$ we should take $\arg(|M_q|) \to -(\theta_{QCD} - \arg(|M_q|))$, or simply $\arg(|M_q|) \to -\theta$.

Taking our generalized fermion mass matrix 2.91 into account in the Chiral Lagrangian 2.38 and expanding in small θ yields an additional interaction term

$$\mathcal{L}_\theta = -\frac{\theta c_+ \tilde{m}}{f_\pi} \pi^a \bar{\mathcal{N}} \sigma^a \mathcal{N} + \dots \tag{2.97}$$

where $c_+ = 1.7$ and $\tilde{m} = \frac{m_u m_d}{m_u + m_d}$. This term yields an electric dipole moment of the neutron

$$d_n = 3.2 \times 10^{-16}\ \theta\ e\,\mathrm{cm}. \tag{2.98}$$

The limit on the dipole moment is $|d_n| < 6.3 \times 10^{-26}\ e\,\mathrm{cm}$ which puts a bound on theta, $|\theta| \leq 2 \times 10^{-10}$. If θ were just any old parameter its incredibly small value would be strange in its own right. However, the problem is confounded by the fact that θ is the *difference* between two parameters, θ_{QCD} and θ_q, the physical origin of which are totally unrelated. Left alone, this implies an implausible cancelation and thus a huge fine-tuning. The θ term in 2.95 violates CP, hence this fine-tuning, and associated lack of CP violation, is known as the Strong CP problem. We can either accept this tuning, or try to understand if there is some sort of physics that drives θ so close to zero.

One possible resolution to this problem is based on the fact that the QCD vacuum energy $E(\theta)$ is minimized for $\theta = 0 \pmod{2\pi}$ [21]. We can see this as follows. Starting from the Euclidean path integral for QCD (with just massive quarks charged under QCD) in a volume V

$$e^{-VE(\theta)} = \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q}\ \exp\left(-\int d^4x \mathcal{L}\right) \tag{2.99}$$

where

$$\mathcal{L} = -\frac{1}{4g^2} Tr(G_{\mu\nu}G_{\mu\nu}) + \bar{q}_i(\gamma^\mu D_\mu + m_i)q_i + \frac{i\theta}{32\pi^2} Tr(G_{\mu\nu}\tilde{G}_{\mu\nu}). \tag{2.100}$$

If we integrate out the quarks we obtain

$$\begin{aligned} e^{-VE(\theta)} &= \int \mathcal{D}A \det(\gamma^\mu D_\mu + m_i)\mathcal{D}q\mathcal{D}\bar{q}\times \\ &\times \exp \int d^4x \left(\frac{1}{4g^2} Tr(G_{\mu\nu}G_{\mu\nu}) - \frac{i\theta}{32\pi^2} Tr(G_{\mu\nu}\tilde{G}_{\mu\nu}) \right). \end{aligned} \tag{2.101}$$

In pure QCD the quarks have vector-like couplings. Thus for each eigenvalue λ of the operator $\gamma^\mu \mathcal{D}_\mu$ there is another eigenvalue of opposite sign. Thus

$$\det(\gamma^\mu D_\mu + m_i) = \prod_\lambda (i\lambda + M) = \prod_{\lambda>0} (i\lambda + M)(-i\lambda + M) = \prod_{\lambda>0} (\lambda^2 + M^2)^2 > 0 \tag{2.102}$$

and so $\det(\gamma^\mu \mathcal{D}_\mu + m_i)$ is positive and real. Hence if θ was zero, the integrand consisting of the gluon action and the fermion determinant would be real and positive. If we include the θ term (with its i coefficient), this contributes a phase which will only *reduce* the value of the path integral, and thus increases the value of $E(\theta)$. Thus $E(\theta)$ is minimized at $\theta = 0$. This further implies that if θ was a dynamical field, it would be driven to a vev of zero in order to minimize energy. If theta is a dynamical field there will be a new particle, the axion, associated with fluctuations in the field. In chapter 3 we explore a model that provides an axion candidate.

2.4 Way Beyond the Standard Model

2.4.1 Randall-Sundrum Space

Thus far when confronted with a discrepancy between experiment and theory we have resorted to the algorithm of introducing new DOF in order to render the theory viable. A priori the hierarchy problem does not necessitate new DOF like the previous problems we encountered, that is, there is no discrepancy between theory and experiment. Given that we do not necessarily need to add any new particle content to our model to rectify the SM, let us consider a novel departure from the methods outlined so far in this dissertation that was first proposed in [22]. In this section and next, we follow [23] closely.

We work on the background metric

$$ds^2 = g_{MN}dx^M dx^N = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \tag{2.103}$$

where $\eta_{\mu\nu}$ is the flat 4 dimensional space Minkowski metric. The y coordinate is truncated on the interval: $0 < y < \pi R$. We postulate the action

$$\begin{aligned} S_{Higgs} &= \int d^5x\ \sqrt{g}(\nabla_\mu\phi\nabla^\mu\phi - M_5^2\phi^2 - \lambda\phi^4)\delta(y-\pi R) \\ &= \int d^4x\ (e^{-2\pi kR}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - M_5^2 e^{-4\pi kR}\phi^2 - \lambda e^{-4\pi kR}\phi^4) \end{aligned} \tag{2.104}$$

where $g = \det g_{MN}$ and M_5 is the 5d scalar mass that is naively susceptible to the large quantum corrections we have discussed. In order to analyze the 4d effective model we canonically normalize the scalar field with the rescaling $\phi \to e^{\pi kR}\phi$ which results in the 4d effective action,

$$S_{eff} = \int d^4x\ (\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (M_5 e^{-\pi kR})^2\phi^2 + \lambda\phi^4). \tag{2.105}$$

The 4d effective scalar mass is identified as $m_4 = M_5 e^{-\pi kR}$.

Assuming that $M_5 \approx k \approx M_p$ and that we want $m_4 \sim$ TeV, this means $\pi kR \approx 35$ or $R = \frac{35}{\pi k} \approx 10M_p^{-1}$. Hence a mild hierarchy between the k and Planck mass results in an exponentially large hierarchy between the scalar mass and the Planck mass. All mass scales at $y = \pi R$ are similarly warped, making an effective cutoff on the brane at $y = \pi R$ a TeV. In particular if we localize the Higgs field to the 'TeV' brane its mass will be similarly protected from large corrections.[19]

This model is known as the Randall-Sundrum (RS) model and in fact, this geometry could be used to explain any hierarchy. Generally speaking, we can 'geometrize' our model building by mapping an energy scale into the geometry of the extra dimension. Given the success we have had so far, let us consider a larger framework that builds on what we have uncovered.

[19]Before going any further, we should temper the excitement that we have potentially solved the hierarchy problem once and for all. RS space does not necessarily solve the hierarchy problem, but recasts it in a geometric fashion so that we might be able to think about it in a different way.

2.4.2 AdS/CFT Correspondance

We now consider an even more radical proposition than we did in the last section, first considered by Maldacena in [24]. The conjecture is summarized compactly as

$$\begin{array}{ccc} \text{Type IIB String Theory} & \text{DUAL} & \\ \text{on } \mathrm{AdS}_5 \times \mathrm{S}^5 & \Longleftrightarrow & \mathcal{N} = 4 \text{ SUSY Yang Mills.} \end{array}$$

On the left hand side we have a ten dimensional manifold: a five dimensional anti de Sitter space (the maximally symmetric Lorentzian manifold with constant negative scalar curvature) and five other dimensions compactified into a five sphere S^5. On the right hand side is a four dimensional supersymmetric gauge theory with $\mathcal{N} = 4$ supercharges. By 'dual' we mean that either side is an equivalent description of the same physics.

The meaning of the correspondence is elucidated by the relationship between the symmetries and parameters on either side of the correspondence. The symmetries are related as follows. The isometry group of the five dimensional AdS is equivalent to the group SO(2,4), the conformal group in four dimensions. The isometry group of the compactified space S_5 is SO(6), which is isomorphic to the group SU(4), the R-symmetry group of the SUSY gauge theory. The parameters of this correspondence are related by

$$\frac{R_{ADS}^4}{l_s^4} = 4\pi g_{YM}^2 N \tag{2.106}$$

where the AdS_5 curvature is $R_{ADS} = \frac{1}{k}$, l_s is the string length and g_{YM} is the Yang-Mills coupling.

In terms of concrete calculations, we are interested in the correspondence between generating functionals

$$Z[\phi_0] = \int \mathcal{D}\phi_{CFT}\ e^{iS_{CFT}[\phi_{CFT}]+i\int d^4x\ \phi_0\mathcal{O}} = \int \mathcal{D}\phi e^{iS_{bulk}[\phi]} \equiv e^{iS_{eff}[\phi_0]} \tag{2.107}$$

where the ϕ_0 are the AdS fields evaluated on the AdS boundary at $z = -\infty$, where we denote $\phi_0(x^\mu) \equiv \Phi(x^\mu, z)|_{\text{AdS Boundary}}$. This means we can calculate the correlation functions of the CFT treating the AdS fields as source terms for the CFT operators,

$$\langle T[\mathcal{O}_1 \dots \mathcal{O}_n]\rangle = \left(-i\frac{\delta}{\delta\phi_{0,1}}\right) \dots \left(-i\frac{\delta}{\delta\phi_{0,n}}\right) \log Z[\phi_0]. \tag{2.108}$$

If we are to make contact with anything that resembles the SM, we first need to analyze the limit where we can neglect the full stringy-ness of the model. In order to do this, we will need to work in the regime of small l_s. In particular we will take $\frac{R^4_{ADS}}{l_s^4} \gg 1$ and hence $4\pi g^2_{YM} N \gg 1$.[20] This means we are probing a strongly coupled theory on the right hand side of the correspondence. Alternatively we could have taken the weak coupling limit on the right hand side of the correspondence in order to probe the strong coupling regime on the left hand side. Thus, this correspondence enables us to probe and model strongly coupled physics that is otherwise outside our perturbative description.

Even after taking the $\frac{R^4_{ADS}}{l_s^4} \gg 1$ limit, we are a long way from the SM. We would like to deform this elegant, constrained, framework into something we can use to model BSM physics. We want to expand the model from the last section where we modeled a naturally light scalar to include features like ultraviolet and infrared cutoffs and symmetry breaking.

First let us go to a different coordinate system for the AdS space by defining $z = e^{ky}/k$ so that the metric is

$$ds^2 = \left(\frac{1}{kz}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \tag{2.109}$$

where $1/k < z < e^{\pi k R}/k$.[21] Now the scaling invariance of the AdS space is transparent under $z \to \alpha z$ and $x^\mu \to \alpha x^\mu$. On the CFT side rescaling x^μ means changing the energy scale, that is, zooming in ($x^\mu \to \alpha x^\mu$ for $\alpha < 1$) in position space is akin to probing higher energies. This suggests the coordinate z on the AdS side is like an energy scale of the CFT. This implies manipulating localized fields throughout the AdS space is akin to modeling the energy scales of the CFT.

[20] Technically we also need the string coupling $g_s \to 0$ so that the masses of non-perturbative string states $m_s \sim \frac{1}{g_s}$ become negligible. Since $g_s \sim \frac{1}{N}$ we must additionally require that we work in the large N limit.

[21] A common criticism that arises at this point is that in these coordinates we seem to have reintroduced the hierarchy problem between the Tev and Planck branes. In practice what really matters is that we can stabilize this geometry in a natural way. A standard phenomenological solution to this is the Goldberger-Wise mechanism [25].

This indicates we can go further and break the full conformal invariance of the CFT by simply truncating the AdS space. A brane at small z which truncates the extra dimension corresponds to an ultraviolet (high energy) cutoff for the CFT. The dual of a brane at large z has a more subtle interpretation on the CFT side of the correspondence. We can note that a brane at large z sets the scale for a tower of massive states on the AdS side. Hence, the appearance of such a massive spectrum on the CFT side means conformal invariance has been (spontaneously) broken and so the brane at large z provides the dual of this spontaneous breaking [26].

This implies we were doing something very different than we thought we were when we were manipulating the Higgs action on seemingly arbitrary background metric. In reality we were probing a strongly coupled theory with composite DOF. So in some sense this is nothing entirely new, but a new way of probing strongly coupled field theories.

So far this is just a sketch of the correspondence. A full summary is well outside the scope of this dissertation, but a tidy dictionary of the AdS/CFT correspondence relevant to model builders is nicely summarized in table 2.1 [27].

A host of questions arise for this new framework. In particular the gravitational sector on the AdS side will be dramatically different than four dimensional GR and there will be TeV gravitational fluctuations associated with the distance between the branes in the z coordinate. In chapter 3 we explore these gravitational excitations and other light DOF arising from warped extra dimensions.

Bulk of AdS	$\Longleftrightarrow$	CFT
Coordinate z along AdS	$\Longleftrightarrow$	Energy scale in CFT
UV brane	$\Longleftrightarrow$	CFT cutoff
IR brane	$\Longleftrightarrow$	CFT's Conformal symmetry spontaneously broken
KK modes localized to IR brane	$\Longleftrightarrow$	Composites of CFT
KK modes localized to UV brane	$\Longleftrightarrow$	Elementary fields coupled to CFT
Gauge fields in bulk	$\Longleftrightarrow$	global symmetry of CFT
Bulk gauge symmetry broken on UV brane	$\Longleftrightarrow$	Global symmetry not gauged
Bulk gauge symmetry unbroken on UV brane	$\Longleftrightarrow$	Global symmetry weakly gauged
Higgs on IR brane	$\Longleftrightarrow$	CFT becoming strong produces composite Higgs
Bulk gauge symmetry broken on IR brane by BCs	$\Longleftrightarrow$	Strong dynamics that breaks CFT also breaks gauge symmetry

Table 2.1: AdS/CFT Dictionary

Chapter 3

Revealing Randall-Sundrum Hidden Valleys and a Warped Solution to the Strong CP Problem

Progress in particle physics is typically imagined as a slow march towards higher energy, with increasingly advanced colliders and the discovery of particles with ever larger mass, as outlined in sections 2.2.1 and 2.2.2. Hidden Valley models are a departure from this paradigm and its associated collider signatures. In these models there is an energy barrier, that once overcome, will give way to a new spectrum of light states in colliders that were previously inaccessible. Their phenomenology is unique and unlike standard BSM scenarios. Moreover, they have unanticipated collider signatures that experimentalists might not otherwise be looking for. Standard methods of data analysis such as looking for missing transverse energy are not effective in searching for Hidden Valleys. Instead, we need to search specifically for the novel collider signatures of these models in order to see signals of Hidden Valleys.

One class of light particles are of considerable interest in particular - axions. This is due to the fact that the axion is a candidate solution to the strong CP problem, as was reviewed in section 2.3.4. In standard axion models the SM is enlarged by an additional global chiral U(1) symmetry, the Peccei-Quinn (PQ) symmetry. This symmetry is then spontaneously broken resulting in a Goldstone boson, the axion.

Explicit breaking from QCD instanton effects then gives the axion a potential such that it relaxes so as to eliminate the CP violating phase in the QCD lagrangian. In the early minimal axion models the scale of symmetry breaking was the electroweak scale which is in severe disagreement with astrophysical and cosmological data. However, the axion concept itself still appears an attractive solution to the strong CP problem due to its simplicity, and remains an active research area.

Both of these concepts are naturally incorporated in the context of compact extra dimensions, where light degrees of freedom can arise from 5D gauge symmetries. The 5D gauge symmetries then manifest themselves in the 4D effective theory as light DOF either as Goldstone bosons or 4D gauge fields. Although standard 4D gravity would result in extremely suppressed effective interactions, compact extra dimensional scenarios lower the scale of gravitational interaction. The TeV gravitational fluctuations will then create a bridge to otherwise hidden light sectors. Models of this form were collectively investigated in [62].

3.1 Introduction

In this chapter we study cases in which there may be additional light fields which reside within the same RS geometry. In principle, such fields may be playing an important role in solving issues within the SM, such as the strong CP problem [34], however we take the approach of studying a generic class of models in which there are new light particles that have greatly suppressed couplings to SM fields. The most likely candidates for such light particles would be Goldstone bosons, whose masses are small in comparison with the weak scale due to a(n approximate) shift symmetry, or new gauge fields, protected by a 4D gauge symmetry. Both classes of particles, Goldstone bosons and 4D gauge fields arise naturally from 5D gauge symmetries as a consequence of the different boundary conditions [35] that one may impose on the 5D gauge transformations (see [36] for reviews and additional references).

A main result of this analysis is the observation that extra-dimensional gravitational excitations [37, 38, 39], whose couplings to such hidden sectors (HS) is independent of gauge-coupling [40], create a bridge between the visible and hidden fields. Randall-Sundrum models are thus a natural setting for Hidden Valley models, in which a new sector is separated from the SM through an "energy-barrier" [41, 42]. In RS scenarios, the role of the energy-barrier is played by the extra-dimensional gravitational excitations of the Randall-Sundrum geometry.

As an explicit example, we construct a novel axion solution to the strong CP problem which is in some senses a revival of the earliest axion models where *electroweak scale* physics produces a Peccei-Quinn (PQ) axion [43, 44]. This 5D axion is hidden by a small extra-dimensional gauge coupling, but has TeV-scale associated Kaluza-Klein excitations, unlike in previous models [45], in which the IR brane is coincident with the scale of PQ symmetry breaking. This model shares some features with composite axion models [46, 47, 48], although the effective compositeness scale in this case is close to the electroweak scale, and is decoupled from the scale associated with the axion coupling constant. The gravity sector can act as a bridge to the axion sector, resulting in a greatly altered collider phenomenology, and necessitating a re-evaluation of the usual astrophysical bounds on such light fields.

In Section 3.2 we describe the basic setup for an RS hidden gauge sector. In section 3.3 we discuss direct couplings of SM fields to the hidden sector. In section 3.4, we calculate the couplings of RS gravitational fluctuations to hidden sector fields, and in section 3.5 we describe a toy model in which the RS hidden sector is responsible for producing an axion which resolves the strong CP problem. In section 3.6 we describe the collider phenomenology of such hidden sectors, while in section 3.7 we discuss astrophysical constraints on light hidden RS Goldstone bosons. In Appendices 3.9 and 3.10, we give Feynman rules for the interactions of hidden sector fields with RS gravity, and describe details concerning gauge fixing.

3.2 Basic Setup

We work in an RS geometry, using the coordinate convention where the metric is conformally flat:

$$ds^2 = \left(\frac{R}{z}\right)^2 \left[\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right] \tag{3.1}$$

Branes at $z = R, R'$ truncate the extra dimension, with $R \sim M_{\rm Pl}^{-1}$, and $R' \sim \text{TeV}^{-1}$. The electroweak hierarchy problem is alleviated as the cutoff scale for radiatively divergent observables in the low energy theory is lowered to near the TeV scale. It is presumed new physics comes in near this scale which softens this dependence on the UV scale. The model is constructed on an S_1/Z_2 orbifold in order to obtain the chiral spectrum required to reproduce the SM. The gauge fields are assumed to propagate in the bulk, and the mechanism of electroweak symmetry breaking is left unspecified, as it is model-dependent. A TeV brane localized Higgs [22], a Higgsless mechanism [28, 29, 30], or a 5D gauge field Higgs [31, 32], or some combination of these ideas could be responsible for the generation of fermion and gauge boson masses.

We gauge a new symmetry (not necessarily abelian) in the bulk of the extra dimension. The 5D Lagrangian for this gauge symmetry is given by:

$$\mathcal{L}_{5\rm D} = -\frac{1}{4}\sqrt{g}\left[g^{MN} g^{RS} B^a_{MR} B^a_{NS}\right] - \frac{1}{2}\sqrt{g}\left(G^a\right)^2 + \sqrt{g} c^a \frac{\delta G^a}{\delta \beta^b} c^b \tag{3.2}$$

The first term is the usual 5D gauge kinetic term, and the second term is a gauge fixing term which removes 5D kinetic mixing between the B_μ and B_5 fields. The last term is a ghost Lagrangian that restores unitarity to the gauge-fixed non-abelian theory. In Appendix 3.10, we provide further discussion of gauge fixing.

To determine the spectrum of the gauge sector, we expand the bulk gauge fields in terms of eigenvalues of the 4D gauge equations of motion: $B_\mu(x, z) = \epsilon_\mu(p) f(z) e^{ip\cdot x}$.The bulk equations of motion for the 4D vector-field wave functions in this geometry are:

$$f'' - \frac{1}{z} f' + M^2 f = 0 \tag{3.3}$$

and the solutions to this eigenvalue problem are

$$f(z) = z\left(A J_1(m_n z) + B Y_1(m_n z)\right). \tag{3.4}$$

The coefficients A,B and eigenvalues, m_n, are found by choosing and imposing boundary conditions and suitably normalizing the 4D effective fields.

We study two scenarios. First we take boundary conditions that produce B_5 zero modes (5D Goldstone bosons) due to breaking the 5D gauge symmetry twice, once on each brane. In the other scenario, we assume that the 4D gauge symmetry is unbroken on both branes, with resulting B_μ zero modes. We also discuss explicit and spontaneous breaking of these symmetries which would lead to Goldstone (gauge) field masses in each of these models, respectively.

3.2.1 Hidden RS Goldstones

To obtain Goldstone bosons from the 5D gauge symmetry, a subgroup of the gauge symmetry must be broken twice, once at the UV brane, $z = R$, and again at the IR brane, $z = R'$. The boundary conditions which achieve this, and which satisfy the 5D action principle, are $B^a_\mu|_{z=R,R'} = 0$. In this section, we assume that the entire gauge group is broken twice in this way, and thus the number of Goldstone bosons is equal to the rank of the original bulk gauge symmetry. We additionally suppress the internal gauge indices, and take the rank of the coset space (the number of Goldstone bosons) to be N.

For the B_5, the equation of motion in the gauge we choose is:

$$\Box B_5 - \partial_z \left[z\partial_z \left(\frac{1}{z} B_5 \right) \right] = 0 \tag{3.5}$$

There is a zero mode solution to this equation where $\Box B_5 = 0$. In this case, the wave function for the B_5 zero-mode is given by $B_5 = B_5(x)\zeta(z)$ with $\frac{R}{z}\zeta(z) = A_0 + B_0 \log z$.

The boundary condition $B_\mu|_{R,R'} = 0$ ensures that there are no necessary boundary gauge fixing terms, and so the boundary conditions for B_5 simply arise from the terms coming from integration by parts of the bulk gauge fixing term. These impose:

$$\partial_z \left(\frac{1}{z} B_5 \right) \Big|_{R,R'} = 0, \tag{3.6}$$

and thus the B_5 zero mode takes the following form:

$$B_5^{(0)}(x,z) = \frac{\sqrt{2} g_{5\mathrm{D}} \sqrt{R}}{\sqrt{R'^2 - R^2}} \frac{z}{R} B_5^{(0)}(x), \tag{3.7}$$

where the overall coefficient ensures that the $B_5^{(0)}$ is canonically normalized in the 4D effective theory.

The residual gauge symmetry, after adding the gauge fixing term specified in Appendix 3.10, is given by:

$$\Box\beta - z\partial_z\left(\frac{1}{z}\partial_z\beta\right) = 0, \tag{3.8}$$

implying that there is a residual subgroup which is global from the perspective of the 4D coordinates: $\beta(z) = \beta_0 + \beta_2 z^2$.

The spectrum of B_μ modes can be found by imposing the boundary conditions on the solutions to the bulk equations of motion, (3.3). The eigenvalue problem is then:

$$\frac{J_1(m_n R')}{Y_1(m_n R')} = \frac{J_1(m_n R)}{Y_1(m_n R)} \tag{3.9}$$

with approximate solutions $m_n R' = 3.83, 7.02, 10.17, 13.32, ...$ The B_5 Goldstone bosons and their associated vector KK-modes are hidden from the standard model in one of two ways. Either the gauge coupling associated with this 5D symmetry is very small, $g_{5\mathrm{D}} \ll \sqrt{R}$, or the SM does not carry quantum numbers under the new symmetry.

The effective scale of symmetry breaking that this Goldstone boson corresponds to is given by (see also [45]), as we will show explicitly in Section 3.3:

$$f_{\mathrm{eff}} = \frac{1}{\sqrt{2}R'}\frac{\sqrt{R}}{g_{5\mathrm{D}}}, \tag{3.10}$$

and we will also see that couplings of this Goldstone boson to other light fields transforming under the 5D gauge symmetry are suppressed by this breaking scale. We note that the scale f_{eff} can be parametrically larger than the IR scale, $1/R'$ if the 5D gauge coupling is chosen such that $g_{5\mathrm{D}} \ll \sqrt{R}$. [1]

[1] Such choices may be in conflict with the conjectured bounds on gauge couplings that arise by considering the spectrum of charged Planck scale black hole remnants [49]. While perfectly sound from an effective field theory point of view, it is likely that a new effective cutoff is introduced which is given approximately by $\Lambda = g_{5\mathrm{D}}\sqrt{R}$, parametrically smaller than the 5D Planck scale. New physics (perhaps stringy in nature) must appear at this scale which drive the gauge coupling to be strong enough to avoid these bounds.

3.2.2 Hidden RS Gauge symmetries

In this section, we briefly analyze the scenario in which the 4D portion of bulk gauge symmetry is completely unbroken, and there are B_μ zero modes in the theory. In this case, the boundary conditions are:

$$\partial_z B_\mu|_{z=R',R} = 0 \tag{3.11}$$

In this scenario, the residual gauge symmetry on the branes corresponds to transformations that are a function only of the 4D coordinates: $\partial_z \beta|_{z=R,R'} = 0$. In this case, there is a subgroup of the residual gauge transformations where the gauge transformation parameter is a function of the 4D coordinates only: $\beta = \beta(x)$. Thus this 5D gauge symmetry has a residual unbroken 4D gauge symmetry corresponding to the B_μ zero-mode. The remaining gauge freedom contains z-dependence, and corresponds to transformations of the tower of B_μ Kaluza-Klein modes.

Using the 5D bulk solution in Eq. (3.4) in coordination with these boundary conditions, the eigenvalue problem is

$$\frac{J_0(m_n R')}{Y_0(m_n R')} = \frac{J_0(m_n R)}{Y_0(m_n R)} \tag{3.12}$$

with approximate solutions $m_n R' = 0, 2.45, 5.56, 8.70, 11.84, ...$ The effective gauge coupling for the zero-mode in terms of the geometrical parameters and the 5D gauge coupling is:

$$g_{\mathrm{4D}} = \frac{g_{5D}}{\sqrt{R \log \frac{R'}{R}}}. \tag{3.13}$$

3.3 SM Couplings to RS Hidden Sectors

Matter fields in the standard model may have couplings to the HS fields which are suppressed by a small extra dimensional gauge coupling. In this section we discuss the nature of these couplings to an unbroken HS gauge symmetry, and to a HS gauge symmetry which is broken to a global subgroup, producing a light 5D Goldstone-boson. We work out the case of a 5D fermion coupled to the HS; couplings to fields with different spin can be derived straightforwardly.

The action for a 5D fermion coupled to a HS $U(1)$ with gauge fields B_M is given by:

$$S = \int d^5x\sqrt{g}\left[\bar{\Psi} i \not{D}\Psi + \frac{c}{R}\bar{\Psi}\Psi\right] \tag{3.14}$$

where D_M is the hermitian gauge covariant derivative

$$D_M = \frac{1}{2}\left(\overrightarrow{\partial}_M - \overleftarrow{\partial}_M\right) - iqB_M, \tag{3.15}$$

and c is the 5D bulk Dirac mass in units of the curvature. The additional terms involving spin connections that can appear in non-trivial geometries vanish with this metric. The 5D Dirac fermion can be expanded in terms of KK-modes:

$$\Psi = \sum_n \begin{pmatrix} g_n(z)\chi_n(x) \\ f_n(z)\bar{\psi}_n(x) \end{pmatrix}. \tag{3.16}$$

The functions $\chi_n(x)$ and $\bar{\psi}_n(x)$ are solutions to the 4D Dirac equation, each with mass m_n, while the wave functions f_n and g_n are solutions to the 5D equations of motion with eigenvalues m_n.

We choose boundary conditions for the 5D fermion such that there is a massless mode (e.g. $(++,--)$ boundary conditions, where $-$ refers to Dirichlet boundary conditions). Depending on the choice of the bulk mass term, c, the zero-mode fermion is either localized on the UV brane ($c < 1/2$), or on the IR brane, $c > 1/2$.

3.3.1 Fermion Couplings to a B_5 zero mode

In the case that the extra dimensional gauge symmetry is broken on both branes, and there is a massless B_5, there is a set of field redefinitions that may be performed that elucidate the Goldstone nature of this field. This is in close analogy with the standard prescription in 4D theories with spontaneous global symmetry breaking, where a field Φ may be redefined as $\Phi \to e^{i\pi/f}\Phi'$, where π are the Goldstone degrees of freedom that couple derivatively, and Φ' contains only the vev f, and the radial fluctuations of the field. Similarly, fermions Ψ which carry charge q under the global symmetry broken by the vev of Φ can be redefined as $\Psi \to e^{iq\pi/v}\Psi'$, where the transformation law for Ψ' is trivial, with the transformation of Ψ being carried by the shift symmetry of the Goldstone boson.

For the fermion field in our discussion, the field redefinition can be taken to be [50]

$$\Psi(z,x) = \exp\left[iq\int_{z_0}^{z} dz' B_5(x,z')\right]\Psi'(z,x). \tag{3.17}$$

The transformation law for Ψ' is then

$$\Psi'(z,x) \to e^{iq\beta(z_0)}\Psi'(z,x), \tag{3.18}$$

independent of z. The constant z_0 is arbitrary, however it can be chosen in a convenient manner that depends on the 5D EWSB model into which this HS is embedded. Under this redefinition, for an abelian HS, the fermion gauge invariant kinetic term is modified in the following way:

$$\bar{\Psi}i\not{D}\Psi \to \bar{\Psi}'i\not{D}_4\Psi' - \bar{\Psi}'i\gamma^5\partial_5\Psi' - q\int_{z_0}^{z} dz'\partial_\mu B_5(z')\bar{\Psi}'\gamma^\mu\Psi'. \tag{3.19}$$

Note that the B_5 now couples derivatively in the 4D coordinates, as expected for a Goldstone boson. In the presence of additional fields, such as Higgs scalars which carry HS quantum numbers, (as was the case in [50]), the most convenient redefinition may be slightly different, and could involve the scalar degrees of freedom.

We can now determine the effective global symmetry breaking scale that produces the B_5 Goldstone boson, and read off its corresponding classically conserved current. From the action after the redefinition, we see that the interactions of the B_5 zero mode with fermions is given by:

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= -q\int dz\sqrt{g}\int_{z}^{R'} dz' A_0\left(\frac{z'}{R}\right)\left(\partial_\mu B_5^{(0)}(x)\right)\bar{\Psi}'e_a^\mu\gamma^a\Psi' \\
&= -q\partial_\mu B_5^{(0)}(x)\int_R^{R'} dz\frac{g_{5D}}{\sqrt{2R}}\left(\frac{R}{z}\right)^4\frac{z^2-z_0^2}{R'}\left(\bar{\Psi}'\gamma^\mu\Psi'\right) \\
&\equiv -q\partial_\mu B_5^{(0)}(x)\sum_{m,n}\left[\frac{1}{f_L^{mn}}\bar{\chi}_m\sigma^\mu\chi_n + \frac{1}{f_R^{mn}}\psi_m\bar{\sigma}^\mu\bar{\psi}_n\right]
\end{aligned} \tag{3.20}$$

where

$$\frac{1}{f_L^{mn}} = \frac{g_{5D}}{\sqrt{2R}}\int_R^{R'} dz\left(\frac{R}{z}\right)^4\frac{z^2-z_0^2}{R'}g_m(z)g_n(z)$$

$$\text{and}\quad \frac{1}{f_R^{mn}} = \frac{g_{5D}}{\sqrt{2R}}\int_R^{R'} dz\left(\frac{R}{z}\right)^4\frac{z^2-z_0^2}{R'}f_m(z)f_n(z). \tag{3.21}$$

The most convenient choice for z_0 is model dependent, depending primarily on additional brane localized sources of explicit breaking of the 5D gauge symmetry. For example, a Dirac-type mass that mixes 2 5D fermions on the IR brane (one producing a LH zero mode, the other a RH zero mode) would transform under the above redefinition as:

$$M\bar{\Psi}_L\Psi_R + \text{h.c.} \to M \exp\left[i(q_R - q_L)\int_{z_0}^{R'} dz' B_5\right]\bar{\Psi}'_L\Psi'_R + \text{h.c.}, \tag{3.22}$$

thus introducing additional interactions of the B_5 zero mode with fermions which are physically equivalent to the types of interactions in Eq. (3.20). Such interactions contribute to the amplitudes in such a way as to give the same effective coupling in any physical process. Choosing $z_0 = R'$ for such a model eliminates this additional contribution to the coupling, such that the entire interaction with fermions can be read from Eq. (3.20).

Let us assume that there is a χ zero mode arising in Ψ', and that there is a bulk Dirac mass term, c, that determines the localization of this zero mode. The zero mode profile is then given by:

$$g_0(z) = \kappa\left(\frac{z}{R}\right)^{2-c}. \tag{3.23}$$

This fermion is localized towards the UV (IR) brane for $c > (<)1/2$. Plugging this wave function into the expressions in Eq. 3.21, we find that the associated breaking scale for left handed zero mode fermions as a function of the c-parameter is given by:

$$f_L^{00} = \begin{cases} \frac{1}{R'}\frac{\sqrt{R}}{\sqrt{2}g_5} & c > 1/2 \quad \text{UV localized} \\ \frac{1}{R'}\frac{\sqrt{R}}{\sqrt{2}g_5}\frac{1}{3/2-c} & c < 1/2 \quad \text{IR localized,} \end{cases} \tag{3.24}$$

roughly confirming the interpretation of the 5D gauge coupling in terms of a symmetry breaking scale, Eq. (3.10).

3.3.2 Gauge Field Couplings to a B_5 zero mode

The redefinition (3.17) may produce a non-trivial Jacobian in the path integral measure, reflecting explicit breaking of the global shift symmetry of the B_5 Goldstone boson through anomalies [51, 20]. Such anomalies result in couplings of the B_5 zero mode to the 5D gauge fields, including SM gluons and photons [50, 45].

In the bulk, the theory is vector-like, and there can be no anomalies, however the boundary conditions are chosen to project out a chirality on the branes to obtain a low energy chiral spectrum. The contributions of a single 5D fermion with a chiral zero mode to the anomaly are evenly distributed on the boundaries of the space, with half of the chiral anomaly localized on the UV brane, and the other half on the IR brane [52, 53]. Under an anomalous 5D gauge transformation, the action shifts by:

$$\delta\mathcal{S} = \int d^4x \int_R^{R'} dz\ \beta\partial_M J^M - \int d^4x\ \beta J^5\big|_R^{R'} \equiv \int d^5x\ \beta\mathcal{A}, \tag{3.25}$$

with J^M given by

$$J^M \equiv \sqrt{g}\bar{\Psi}\gamma^M\Psi, \tag{3.26}$$

and the anomaly, $\mathcal{A}$, is given by:

$$\begin{aligned}\mathcal{A}(x,z) &= \tfrac{1}{2}\left[\delta(z-R)+\delta(z-R')\right]\textstyle\sum_f q^f\left(\frac{q_Y^{f2}}{16\pi^2}F\cdot\tilde{F} + \frac{\mathrm{Tr}\ \tau_a^f\tau_a^f}{16\pi^2}W\cdot\tilde{W} + \frac{\mathrm{Tr}\ t_a^f t_a^f}{16\pi^2}G\cdot\tilde{G}\right)\\ &\equiv \tfrac{1}{2}\left[\delta(z-R)+\delta(z-R')\right]\mathcal{Q}(x,z)\end{aligned} \tag{3.27}$$

Such anomalies are not an indication of a "sick" theory, since the transformation is only anomalous on the boundaries of the extra dimension, where the 5D gauge symmetry is restricted to be global with respect to the 4D coordinates.

The resulting action after the field redefinition (3.17), is augmented by the following term:

$$S_{\text{anomaly}} = -\frac{1}{2}\int d^4x\left[\int_R^{z_0} dz' B_5\mathcal{Q}(R,x) - \int_{z_0}^{R'} dz' B_5\mathcal{Q}(R',x)\right]. \tag{3.28}$$

In terms of the zero mode B_5, which has the profile given above, these interactions are:

$$\mathcal{L}_{\text{anom}} = \frac{1}{2}A_0 B_5^{(0)}(x)\left[\left(z_0^2-R^2\right)\mathcal{Q}(R) - \left(R'^2-z_0^2\right)\mathcal{Q}(R')\right]. \tag{3.29}$$

Defining $\mathcal{Q}^{\pm} \equiv \mathcal{Q}(R') \pm \mathcal{Q}(R)$, we have,

$$\mathcal{L}_{\text{anom}} = \frac{1}{2} A_0 B_5^{(0)}(x) \left[\left(2z_0^2 - R^2 - R'^2 \right) \mathcal{Q}^+ - \left(R'^2 - R^2 \right) \mathcal{Q}^- \right]. \tag{3.30}$$

As mentioned above, the physics is not dependent on the choice of z_0, however there are choices which are more convenient than others. Again, in the presence of a Dirac mass term on the IR brane, a sensible choice is $z_0 = R'$. If another value is chosen, the interactions of the Goldstone boson with fermions arising in equation (3.22) will lead to additional triangle loop diagrams which contribute to the interaction in Eq. (3.30) in such a way as to render the physical result independent of z_0. The anomaly interaction with the choice $z_0 = R'$ is then given by:

$$\mathcal{L}_{\text{anom}} = \frac{1}{2} A_0 \left(R'^2 - R^2 \right) B_5^{(0)}(x) \left[\mathcal{Q}^+ - \mathcal{Q}^- \right]. \tag{3.31}$$

Finally, plugging in the normalization coefficient for the B_5 zero mode, the effective interaction of the B_5 zero mode is given by:

$$\mathcal{L}_{\text{anom}} = \frac{1}{\sqrt{2}} \frac{g_5}{\sqrt{R}} \sqrt{R'^2 - R^2} B_5^{(0)}(x) \left[\mathcal{Q}^+ - \mathcal{Q}^- \right]. \tag{3.32}$$

And the effective suppression scale for the anomalous interactions of the B_5 zero mode with SM gauge bosons is approximately

$$f_{\text{anom}}^{00} = \frac{1}{R'} \frac{\sqrt{R}}{\sqrt{2} g_5}, \tag{3.33}$$

in agreement with the effective Goldstone boson scale arising from the couplings to fermion zero-modes in Section 3.3. There are additional interactions of the B_5 with gauge boson KK-modes when the anomalies $\mathcal{Q}^{\pm}$ are expressed in terms of a KK-mode expansion.

3.4 Couplings to RS Gravity

Unlike the couplings of the Goldstone sector to SM fields, the couplings of the excitations of RS gravity to the gauge fields B_μ (or physical B_5 Goldstone bosons) are independent of the 5D gauge coupling [39, 40]. Thus while the gauge sector may be "hidden" from the SM fields, the couplings of the hidden sector to TeV brane localized gravitational waves are suppressed only by the IR brane local cutoff scale. In this section, we calculate the couplings of RS gravitational excitations (the radion and the first two tensor modes) to the hidden sector gauge fields.

We begin by reviewing the KK-reduction of the 5D metric including linearized fluctuations. The usual Einstein-Hilbert action is given by:

$$S_{\mathrm{EH}} = -\kappa^2 R^3 \int_R^{R'} dz \int d^4x \sqrt{g}\,(\mathcal{R} - \Lambda) \tag{3.34}$$

The distance element on this space, including linearized perturbations which solve the vacuum Einstein equations, is given by:

$$ds^2 = \left(\frac{R}{z}\right)^2 \left[e^{-2F(z,x)}\eta_{\mu\nu}dx^\mu dx^\nu + h_{\mu\nu}dx^\mu dx^\nu - (1 + 2F(z,x))^2\, dz^2\right], \tag{3.35}$$

where $h_{\mu\nu}$ is transverse and traceless, and contains the 4D graviton plus Kaluza-Klein excitations. F is the radion field, expressed after canonical normalization as

$$F(z,x) = \left(\frac{z}{R'}\right)^2 \frac{r(x)}{\kappa\Lambda_r} \tag{3.36}$$

Plugging this radion excitation into the above EH action shows that the normalization factor which sets the scale of the radion coupling to other fields is given by $\Lambda_r = \sqrt{6}/R'$.

The transverse traceless perturbations, $\tilde{h}_{\mu\nu} \equiv (R/z)^2\, h_{\mu\nu}$ satisfy the following bulk equation of motion:

$$\tilde{h}''_{\mu\nu} + \frac{1}{z}\tilde{h}'_{\mu\nu} - \frac{4}{z^2}\tilde{h}_{\mu\nu} - \Box\tilde{h}_{\mu\nu} = 0 \tag{3.37}$$

while the boundary conditions require

$$\left(z^2\tilde{h}_{\mu\nu}\right)'|_{R,R'} = 0. \tag{3.38}$$

After imposing the boundary condition at $z = R$, with the ansatz $\tilde{h}_{\mu\nu} = \sum_n \phi_n(z)\frac{\hat{h}_n(x)_{\mu\nu}}{\kappa\Lambda_n}$, the KK-graviton wave functions are given by:

$$\phi_n(z) = \left(\frac{R}{R'}\right)^2 \left[J_2(m_n z) - \frac{J_1(m_n R)}{Y_1(m_n R)}Y_2(m_n z)\right]. \tag{3.39}$$

Note that we have given the 4D modes $h_n(x)$ mass dimension 1, associating a scale with the couplings of each graviton KK-mode that is calculated by imposing canonical normalization on the 4D modes. The prefactor $(R/R')^2$ is inserted to render the Λ_n's sensitive only to the IR scale (where the lower level KK-gravitons are localized). The scales Λ_n are determined by expanding the EH action to quadratic order in the fluctuations, reading off the coefficient of the kinetic terms and enforcing the low energy theory to reproduce the Fierz-Pauli spin-2 kinetic term. This leads to the following equation for Λ_n:

$$\frac{1}{R^3}\int dz\left(\frac{z}{R}\right)\phi_n^2 = \Lambda_n^2, \tag{3.40}$$

From which we find $\Lambda_1 R' = .285$, $\Lambda_2 R' = .212$.

The final boundary condition at $z = R'$ determines the solutions to the eigenvalue problem for m_n:

$$\frac{J_1(m_n R')}{Y_1(m_n R')} = \frac{J_1(m_n R)}{Y_1(m_n R)} \tag{3.41}$$

This is actually identical in form to the eigenvalue equation for the vector KK-modes of the 5D Goldstone boson in this model, and thus the KK-gravitons have a spectrum identical to the vector KK-modes associated with the Goldstone bosons.

We now calculate the interactions of the radion and KK-gravitons with the light HS fields and the HS KK-modes. The gravitational excitations couple to the matter stress-energy tensor:

$$S_{\text{grav}} = -\frac{1}{2}\int_R^{R'} dz \int d^4x\sqrt{g}(\Delta g)_{MN}T^{MN} \tag{3.42}$$

where the fluctuations including the radion, the graviton, and the KK modes of the graviton are contained in $(\Delta g)_{MN}$. Using Eq. (3.35) for the distance element, one can read off the interactions of the radion with matter:

$$S_{\text{radion}} = -\int_R^{R'} dz d^4x\sqrt{g}F(x,z)\left[\text{Tr } T_{MN} - 3T_{55}g^{55}\right] \tag{3.43}$$

while for the graviton and its KK-modes, we have

$$S_{\text{grav}} = -\frac{1}{2}\int_R^{R'} dz \int d^4x\sqrt{g}\tilde{h}_{\mu\nu}T^{\mu\nu} \tag{3.44}$$

where the Greek indices are limited to the 4D uncompactified directions.

For a gauge theory, the Maxwell stress-energy tensor (before adding gauge fixing terms) is given by:

$$T_{MN} = \frac{1}{4}g_{MN}B_{RS}B^{RS} - B_{MR}B_{NS}g^{RS}. \tag{3.45}$$

Using the ansatz given above for the $\tilde{h}_{\mu\nu}$ fluctuations, interactions of KK-gravitons with the HS are given by:

$$
\begin{aligned}
& -\frac{1}{2\Lambda_n}\int d^4x \hat{h}^n_{\mu\nu}\int_R^{R'} dz\sqrt{g}\phi_n(z)T^{\mu\nu} \\
& = \frac{1}{2\Lambda_n}\int d^4x \hat{h}^n_{\mu\nu}\int_R^{R'} dz\left(\frac{z}{R}\right)\phi_n(z)\left[B_{\rho\kappa}B_{\sigma\lambda}\eta^{\kappa\lambda} - B_{\rho 5}B_{\sigma 5}\right]\eta^{\mu\rho}\eta^{\nu\sigma}, \quad (3.46)
\end{aligned}
$$

Similarly, plugging the normalized radion field into Eq. (3.43), the radion couples in the following way to the HS:

$$
\frac{r(x)}{\Lambda_r}\int_R^{R'} dz\left(\frac{z}{R}\right)\left(\frac{R}{R'}\right)^2\left[\frac{1}{2}B_{\mu\nu}B_{\rho\sigma}\eta^{\mu\rho}\eta^{\nu\sigma} + 2\eta^{\mu\nu}B_{\mu 5}B_{\nu 5}\right]. \tag{3.47}
$$

Using the expressions for the normalized B_5, $B^{(1)}_\mu$, and $\hat{h}^{(n)}_{\mu\nu}$, we find the effective 4D Lagrangian coefficients which are summarized in Appendix A. The couplings are expressed in terms of the normalization factors Λ_n, the hierarchy between the Planck scale and the position of the UV brane, κ, and wave function overlap integrals of the n'th graviton KK-mode with the HS field, parametrized as λ_{nXX}, where X are fields residing in the HS. These coupling constants are robust under variation in the values of R and R', as long as $R' \gg R$.

Note that for a completely brane localized field, X, the coupling ratios $\lambda_{nXX}/\Lambda_n \to \sqrt{2}R'$, bringing our result into agreement with previous publications which have taken the SM fields to be completely localized on the IR brane [39].

The primary process which contributes to production is gluon fusion. The 4D effective Lagrangian for the couplings of the KK-gravitons to gluons are given by (at tree level) [39]:

$$
\mathcal{L}_{\text{glue}} = \hat{h}^{\mu\nu}_{(1)}G_{\mu\rho}G^{\rho}_{\nu}\frac{0.191}{\Lambda_1\log R'/R} + \hat{h}^{\mu\nu}_{(2)}G_{\mu\rho}G^{\rho}_{\nu}\frac{0.028}{\Lambda_2\log R'/R}, \tag{3.48}
$$

and the KK-graviton propagator is given by:

$$
D^{\mu\nu,\rho\sigma}_{(n)} = \left[G^{\mu\rho}_{(n)}G^{\nu\sigma}_{(n)} + G^{\mu\sigma}_{(n)}G^{\nu\rho}_{(n)} - \frac{2}{3}G^{\mu\nu}_{(n)}G^{\rho\sigma}_{(n)}\right]\frac{1}{2\left(k^2 - m_n^2\right)} \qquad G^{\mu\nu}_{(n)} \equiv \eta^{\mu\nu} - \frac{k^\mu k^\nu}{m_n^2}. \tag{3.49}
$$

3.5 A TeV-Scale Axion

In this section, we describe a toy axion model that resolves the strong CP problem and in which a PQ global symmetry is broken at the TeV scale (on the IR brane). We gauge a $U(1)_{\text{PQ}}$ symmetry which is broken by boundary conditions on both branes. The resulting B_5 zero mode plays the role of the axion.

In this model the axion is hidden (and its mass supressed) by taking the 5D gauge coupling to be small. The direct interactions with SM fields are all suppressed by the small extra-dimensional gauge coupling, and with the relation given in Eq. (3.21), we deduce that the effective PQ scale is given by:

$$f_{\text{PQ}} = \frac{1}{R'} \frac{\sqrt{R}}{\sqrt{2} g_5} \tag{3.50}$$

This is the inverse coupling constant that appears in axion interactions that also appear in standard 4D axion models. For instance, the coupling of the axion to photons and gluons from anomalies is given by

$$c_{\text{EM}} \frac{B_5}{f_{\text{PQ}}} F \cdot \tilde{F} + c_{\text{QCD}} \frac{B_5}{f_{\text{PQ}}} G \cdot \tilde{G} \tag{3.51}$$

where F, G and the tildas are the electromagnetic/ gluonic field strengths and their duals. c_{EM} and c_{QCD} are the anomaly coefficients. Below the QCD confinement scale, the second term in Eq. (3.51) leads to an axion mass through instanton effects. This mass is given approximately by [44]

$$m_{B_5}^2 \approx \frac{\Lambda_{\text{QCD}}^4}{f_{\text{PQ}}^2}. \tag{3.52}$$

Standard constraints on f_{PQ} apply, and the allowed ranges of f_{PQ} [54] are roughly 10^9 GeV $< f_{\text{PQ}} < 10^{12}$ GeV, where the lower bound arises from constraints on supernova cooling rates and the upper bound arises from constraints on the relic abundance of coherent axion oscillations (assuming an order one displacement of the axion field from the CP conserving minimum in the early universe).

Charge assignments under the $U(1)_{\text{PQ}}$ symmetry are model dependent. For instance, one could create a hadronic axion model, in which the SM fermions are uncharged, but in which new heavy fermions carrying $SU(3)_C$ charge contribute to the anomaly, and lead to an axion mass. Another option is to model this 5D axion in a manner similar to the DFSZ axion [55] in terms of the charge assignments:

	(H_u)	(H_d)	Q	$\bar{u}$	$\bar{d}$	L	$\bar{e}$
Y	$1/2$	$-1/2$	$1/6$	$-2/3$	$1/3$	$-1/2$	1
PQ	1	1	$-1/2$	$-1/2$	$-1/2$	$-1/2$	$-1/2$

(3.53)

The Higgs fields are placed in parentheses as they are not crucial in extra dimensional theories such as Higgless models of electroweak symmetry breaking. The simplest model in terms of particle content is a Higgsless model augmented by a $U(1)_{\rm PQ}$. The choice of fermion quantum numbers determines the anomaly coefficients $c_{\rm EM}$ and $c_{\rm QCD}$ in Eq. (3.51), and the most convenient fermion redefinition for a Higgsless theory is given in Eq. (3.17), with the choice $z_0 = R'$.

This type of axion model has a strong benefit over previous constructions. This feature concerns explicit global symmetry breaking terms arising from Planck scale physics which must be suppressed in order to preserve the Goldstone nature of the axion [56, 48]. Without some mechanism to forbid or suppress such operators, non-derivative potential terms for the axion arise and displace the axion from the CP conserving minima of the instanton potential. In the extra-dimensional construction, such operators, in the 4D effective theory, take the form:

$$\frac{a}{f}\partial_\mu j^\mu_{\rm PQ} = \frac{a}{f}\left[\frac{g_n}{M^n_{\rm Pl}}\mathcal{O}^{4+n} + c_{\rm QCD} G\cdot\tilde{G}\right]. \tag{3.54}$$

We have also included the term that generates the axion potential from instantons for comparison. To not spoil the strong CP solution, we must have:

$$10^{-10} c_{\rm QCD}\langle G\cdot\tilde{G}\rangle \gtrsim \frac{g_n}{M^n_{\rm Pl}}\langle\mathcal{O}^{n+4}\rangle \tag{3.55}$$

With $c_{\rm QCD}\langle G\cdot\tilde{G}\rangle \sim \Lambda^4_{\rm QCD}$, this becomes

$$g_n \lesssim 10^{-10}\left(\frac{\Lambda_{\rm QCD}}{\mu}\right)^4\left(\frac{M_{\rm Pl}}{\mu}\right)^n \tag{3.56}$$

where μ is the scale associated with fields appearing in the operator $\mathcal{O}^{4+n}$. For dimension 5, 6, 7 operators ($n = 1,\ 2,\ 3$), the scales μ which satisfy this bound (assuming $g_n = 1$) are $\mu \lesssim 4,\ 1 \cdot 10^4,\ 1.4 \cdot 10^6$ GeV. In this extra-dimensional construction, the terms which correspond to spontaneous symmetry breaking reside on the IR brane, and are naturally of order TeV. Thus the scale μ is expected to be of order TeV, and even at dimension 6 such operators are not dangerous, a significant improvement on earlier models, in which μ was tied to the scale $f_{\rm PQ}$ [48, 56].

Irrespective of the gauge coupling, as shown in the previous section, the RS gravity sector bridges between the SM and this axion sector. There are thus operators which are suppressed only by the TeV scale associated with the IR brane which connect the SM with the HS axion and its excitations. In the next sections, we discuss the phenomenology of such hidden sectors, with much of the discussion there being relevant for this axion scenario.

3.6 Collider Phenomenology

Even with greatly suppressed direct couplings, the interactions of the HS with RS-gravity provide a link to SM fields through processes which involve exchange of radions or KK-gravitons. Observation of gravitational resonances have been considered a smoking gun for extra-dimensional models, so it is vitally important to identify how their phenomenology is modified in the presence of these hidden sectors. The most dramatic feature involves decays of the radion and KK-gravitons to HS fields, although direct production of HS fields is also possible.

3.6.1 Radion and KK-graviton decays to 5D Goldstone Bosons

Through the interactions shown in Table 3.1, the radion and the graviton KK-modes can decay to the light B_5 Goldstone bosons. These Goldstone bosons may escape the detector, or decay back to light SM states, depending on the model chosen. In this section, we calculate the partial widths of the radion and KK-gravitons to the light Goldstones.

The radion partial width to Goldstones is given by

$$\Gamma(r \to B_5 B_5) = \frac{1}{32\pi}\frac{m_r^3}{\kappa^2 \Lambda_r^2} \tag{3.57}$$

where $\Lambda_r = \sqrt{6}/R'$, independent of the 5D gauge coupling associated with the HS.

For light radions, where the decay mode W's and Z's is closed, this decay dominates the width, and notably suppresses the $r \to \gamma\gamma$ branching fraction by roughly a factor of 10 for radion masses between 114 GeV and 160 GeV . As $\gamma\gamma$ was the most promising channel in which to search for radions at the LHC [37, 40], this is a significant modification in the phenomenology. The B_5 Goldstone bosons produced in these decays stream through the detector since the Goldstones are only very weakly coupled to SM fields.

The radion may also mix with the Higgs in extra dimensional models that contain a scalar Higgs particle (see e.g. [37]). In this case, the Higgs may have a substantial invisible branching fraction to these Goldstone bosons, even as much as 50 % if the relative splitting of the scalar states is comparable to vR'. The amount of mixing however is very model dependent (there may not even be a scalar Higgs particle in the spectrum), and we leave this area as an avenue for future study.

For heavier radions, where the decays $r \to W^+W^-$ contribute to the width, the branching ratio saturates at a value of roughly 20%.

The KK-graviton partial widths to Goldstones are

$$\Gamma(h_n^{\mu\nu} \to B_5 B_5) = \frac{\lambda_{nB_5B_5}^2}{1920\pi}\frac{m_n^3}{\kappa^2 \Lambda_n^2} \tag{3.58}$$

Where $\lambda_{1B_5B_5} = -.219$, and $\lambda_{1B_5B_5} = .049$, as can be read from Table 3.1. This is in agreement with expectations from the Goldstone theorem that this width should be equal to the width to Z's, the Higgs, and half the width to W's, which have been reported in [39]. The Goldstone equivalence theorem can then be used to obtain the branching fractions of the KK-graviton to light Goldstones,

$$BR(h_{\mu\nu}^1 \to B_5^a B_5^a) = \frac{N}{\frac{\Gamma_{\text{top}}}{\Gamma_Z} + 4 + N} \tag{3.59}$$

where N is the number of 5D Goldstone bosons. We have neglected the contributions of light UV brane localized fermions and to KK-tops to the total width, as these are typically much smaller [39]. The branching fraction to a single $U(1)$ Goldstone boson is typically $\mathcal{O}(10\%)$ for reasonable values of the top-right quark localization parameter, which is the primary variable which determines the ratio $\Gamma_{\text{top}}/\Gamma_Z$.

3.6.2 Radion and KK-graviton decays to hidden 5D gauge fields

The radion width to gauge boson zero modes is given by:

$$\Gamma(r \to B_\mu B_\mu) = \frac{m_r^3}{128\pi\kappa^2\Lambda_r^2 \log^2 R'/R} \tag{3.60}$$

Again, since these light vector modes are assumed to couple only very weakly (or not at all) with SM particles, these particles would manifest as missing energy at colliders. Unlike the Goldstone B_5 HS, these invisible decays only contribute modestly to the total width of the radion, and are of roughly the same size as the branching fraction to $\gamma\gamma$. Thus the radion phenomenology is not greatly altered. The smaller branching fraction relative to the Goldstone HS scenario is due to the extra log suppression in the couplings of the radion to the flat profile of the B_μ zero modes.

The level-1 KK-graviton width to gauge boson zero modes is given by

$$\Gamma(h_{\mu\nu}^{(1)} \to B_\mu B_\mu) = \left(\frac{.191}{\kappa\Lambda_1 \log R'/R}\right)^2 \frac{m_{(1)}^3}{1536\pi} \tag{3.61}$$

and exhibits the same log suppression as the radion decays. Thus the branching fraction to B_μ modes will be very small compared to the fractions to SM massive gauge fields, and similar with the branching fraction to photons (in fact the branching fractions are identical up to loop corrections).

3.6.3 Non-exact shift or gauge symmetries

The symmetries (HS shift/gauge symmetries) can not be exact/unbroken for most choices of the 5D gauge coupling since there are stringent constraints from astrophysics on new massless scalar fields and long range forces. The scalars or vector fields in the HS must have some mass. If the light HS is hidden through a small 5D gauge coupling, and the SM fields have non-vanishing quantum numbers under the gauge symmetry, the Goldstone bosons will decay to SM particles if the HS fields are massive enough. Depending on how small the extra dimensional gauge coupling is, and the masses of the light pseudo-Goldstone fields, their decays may range from prompt to cosmological time scales.

For the lightest range of HS scalar masses, the 5D Goldstone boson may decay to SM fermions. The decay width of a light 5D pseudo-Goldstone boson to SM fermions is given by:

$$\Gamma(B_5 \to \bar{f}f) = \frac{q^2}{4\pi}\left(\frac{m_f}{f_{\rm eff}}\right)^2 m_{B_5} \tag{3.62}$$

The distance traveled by a pseudo-Goldstone boson that couples universally to leptons before decaying to muons (presuming the B_5's have mass less than $2m_\tau$, and assuming a 5D gauge symmetry charge of $q = 1$), is given by:

$$\Delta x = 58\text{cm}\left(\frac{f_{\rm eff}}{10^6\text{GeV}}\right)^2\left(\frac{10\text{GeV}}{m_{B_5}}\right)\sqrt{\left(\frac{E}{m_{B_5}}\right)^2 - 1} \tag{3.63}$$

The pseudo-Goldstone modes may also couple to SM quark fields, in which case there will be displaced hadronic decays.

Thus these RS Hidden sectors are a concrete example of a "Hidden Valley" model [41, 42], in which HS fields may be produced at colliders through on-shell production of RS gravitations resonances which subsequently decay into the HS. The final decay products of the HS fields may be substantially displaced from the production vertex, depending on the choice of the extra dimensional gauge coupling. Searches have been performed at the Tevatron, with null results thus far [57, 58]

In the case of light HS vector fields, the width to fermions is given in the massless fermion limit by:

$$\Gamma(B_\mu \to \bar{f}f) = \frac{g^2 m_{B_\mu}}{4\pi} \tag{3.64}$$

and the decay length in the detector is given by:

$$\Delta x = 20\text{cm}\left(\frac{10^{-8}}{g}\right)^2\left(\frac{10\text{GeV}}{m_{B_\mu}}\right)\sqrt{\left(\frac{E}{m_{B_\mu}}\right)^2 - 1} \tag{3.65}$$

3.6.4 Hidden KK-modes at Colliders

It is also possible that higher level KK-modes of the light HS will be directly produced by collider experiments such as the LHC. The most likely channel for HS KK-mode production in the light Goldstone scenario is a level one KK-mode in association with the light Goldstone boson: $gg \to r(\hat{h}^{(1)}_{\mu\nu}) \to B^{(1)}_\mu B_5$, where the exchanged particle is either a radion or a level one KK-mode graviton. For a HS with a residual gauge symmetry it is $gg \to r(\hat{h}^{(1)}_{\mu\nu}) \to B^{(1)}_\mu B^{(0)}_\mu$.

The production cross sections for these processes are very small for two reasons. Firstly, the KK-modes of the gauge fields are quite massive. The lowest they could be is in the 2 TeV range for a typical Higgsless model. Secondly, the rate is suppressed as RS gravity couplings all come with the normalization factors Λ_r, or Λ_n which are in the TeV range.

For a model with a HS Goldstone boson, taking $R' = (500\text{ GeV})^{-1}$, the LHC cross section at design energy (14 TeV CM energy) is $\sigma(gg \to r \to B^{(1)}_\mu B_5) \approx 1 \cdot 10^{-5}$ pb. For a model with a HS light gauge field, for the same parameters, the cross section is $\sigma(gg \to r \to B^{(1)}_\mu B^{(0)}_\mu) \approx 5 \cdot 10^{-6}$ pb. These are up against the design goals of the LHC, however with high luminosity ($100's$ of fb^{-1}) a few events may be possible. The HS KK-modes dominantly decay via the channel $B^{(1)}_\mu \to B_5 r (B^{(0)}_\mu r)$ for HS Goldstone (gauge) field. If the light HS Goldstone (gauge) field can decay to SM leptons within the detector, there is hope of triggering on and reconstructing even a few such events.

3.7 Astrophysical Constraints on RS Goldstone Bosons

At low energies, the couplings of the hidden sector to RS gravity induce higher dimensional operators involving SM fields that are suppressed only by the TeV scale. In this section, we calculate the effective operators relevant for main sequence star cooling, and supernova energy loss (see [59] for a related study). We take into account the contributions from radion exchange, however we neglect the contributions of KK-gravitons, as these are negligible in comparison. We leave a study of the astrophysical constraints on light RS gauge fields for future work, although we provide expressions

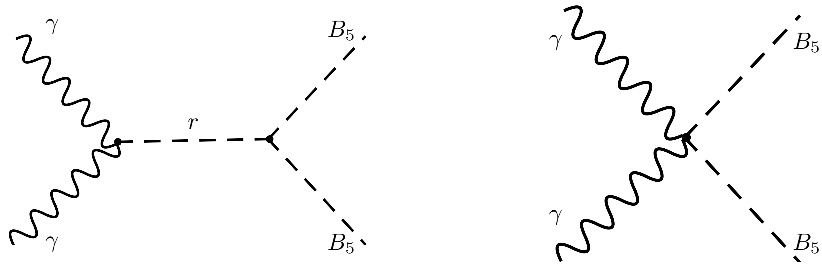


Figure 3.1: The diagram on the left involving the exchange of a RS radion leads to the effective dimension 8 contact operator shown on the right.

for the relevant higher dimensional operators in this section. Existing light scalar search experiments are not sensitive to the operators that arise from integrating out the RS gravitational excitations.

3.7.1 Higher dimensional operators

Diagrams such as the one shown in Figure 3.1 create higher dimensional operators in an effective theory valid at energies below the scale of RS gravitational excitations. In this section, we calculate these higher dimensional operators as functions of the radion mass and the parameters associated with the RS geometry.

Operators for 5D Goldstone Bosons

The coefficients of the irrelevant operators arising from integrating out the radion can be determined by the form of the radion couplings to bulk SM fields [40]. The results are given by:

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{aa\gamma\gamma} &= \frac{\left(\partial_\mu B_5\right)^2 F_{\rho\sigma}^2}{4m_r^2\Lambda_r^2 \log R'/R} \\
\mathcal{L}_{\rm eff}^{B_5B_5gg} &= \frac{\left(\partial_\mu B_5\right)^2 G_{\rho\sigma}^2}{4m_r^2\Lambda_r^2 \log R'/R} \\
\mathcal{L}_{\rm eff}^{aa\bar{f}f} &= \frac{m_f(c_L - c_R)}{m_r^2\Lambda_r^2}\bar{f}f\left(\partial_\mu B_5\right)^2
\end{aligned} \tag{3.66}$$

The last interaction is for fermions which are localized on the UV brane. The coefficients c_L and c_R are the fermion bulk masses which determine the wave-functions of the zero modes. The second interaction, at momentum transfer below the QCD scale, leads to an effective coupling of the Goldstone boson to nucleons:

$$\mathcal{L}_{\text{eff}}^{B_5 B_5 nn} = \frac{(\partial_\mu B_5)^2\, \bar{n}n}{4m_r^2\Lambda_r^2 \log R'/R} m_{n,p} \frac{8\pi}{9\alpha_s}\left[\sum_{q=u,d,s} f_{Tq} - 1\right] \tag{3.67}$$

where $m_{n,p}$ is the neutron/proton mass. The coefficient is obtained by taking the matrix element of the scalar gluon current between nucleons:

$$\bar{n}n\langle n|G_{\rho\sigma}^2|n\rangle \to -\bar{n}n\; m_n \frac{8\pi}{9\alpha_s}\left[\sum_{q=u,d,s} f_{Tq} - 1\right] \tag{3.68}$$

The f_{Tq} coefficients are defined by $\langle n|m_q\bar{q}q|n\rangle \equiv m_n f_{Tq}$.

Operators for unbroken gauge symmetries

Similarly, there are higher dimensional operators involving massless bulk gauge fields, B_μ.

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{BB\gamma\gamma} &= \frac{B_{\mu\nu}^2 F_{\rho\sigma}^2}{16m_r^2\Lambda_r^2 \log^2 R'/R} \\
\mathcal{L}_{\text{eff}}^{BBgg} &= \frac{B_{\mu\nu}^2 G_{\rho\sigma}^2}{16m_r^2\Lambda_r^2 \log^2 R'/R} \\
\mathcal{L}_{\text{eff}}^{BB\bar{f}f} &= \frac{m_f(c_L - c_R)}{m_r^2\Lambda_r^2}\bar{f}fB_{\mu\nu}^2
\end{aligned} \tag{3.69}$$

These are invariant under the 4D gauge symmetry. We leave a full analysis of the effects of these operators for future study.

3.7.2 Main-Sequence Star and Supernova Cooling

In massive astrophysical bodies, processes may occur which produce the light fields within an RS hidden sector. This is the case, for example, with standard axion scenarios, and which leads to significant constraints on the coupling strength of a pseudo-scalar axion to SM fields, f_{PQ}^{-1}. However, our model predicts the existence of new TeV suppressed operators which can contribute to astrophysical pseudoscalar production. If the HS fields are coupled weakly enough, the produced fields will free-stream out of the astrophysical body, and contribute in a straightforward way to its energy loss rate. In main-sequence stars and supernovae, an increased energy-loss rate above that predicted within the SM has not been detected, putting constraints on the higher dimensional operators that arise from integrating out RS gravitational fluctuations.

In this section, we consider only RS hidden sectors containing a light Goldstone boson, not a light gauge field. We leave constraints on HS gauge fields for future study. These constraints are particularly relevant for the RS axion model considered in Section 3.5.

We have calculated the scattering length of a 5D Goldstone boson produced in the core collapse neutron star phase of SN1987a, taking into account only the higher dimensional operators given above. The scattering length is given approximately by

$$fL = 1 \cdot 10^{14}\mathrm{m}\left(\frac{30\ \mathrm{MeV}}{E_a}\right)^4\left(\frac{1/R'}{500\ \mathrm{GeV}}\right)^4\left(\frac{m_{\mathrm{radion}}}{120\ \mathrm{GeV}}\right)^4 \tag{3.70}$$

This scattering length is far larger than the size of the core for reasonable choices of the parameters, and thus any produced Goldstone bosons in the core collapse process are free-streaming [2].

In a generic scattering process within a neutron star, where thermal conditions are semi-degenerate, the energy loss rate per unit volume due to particles which free-stream out of an object is given by

$$Q = \int d\Pi_{\mathrm{PS}} S\left|\mathcal{M}\right|^2 \delta^{(4)}\left(\sum_i p_i - \sum_f p_f\right) E_{\mathrm{stream}} f_1 f_2 \left(1-f_3\right)\left(1-f_4\right), \tag{3.71}$$

where E_{stream} is the energy lost in a single process due to particles streaming out of the object, and the f_i are the thermal occupation functions of the neutrons and protons which scatter to produce the Goldstone bosons:

$$f_i = \frac{1}{e^{(E_i-\mu)/kT}+1} \tag{3.72}$$

[2]There are also other processes due to direct couplings of the Goldstone sector with the SM fields which can lead to re-scattering in a core collapse supernova. However, the relevant range of allowed couplings for very light scalars $f_{\mathrm{eff}}^{-}1$, are small enough to ensure that the light fields are still free streaming.

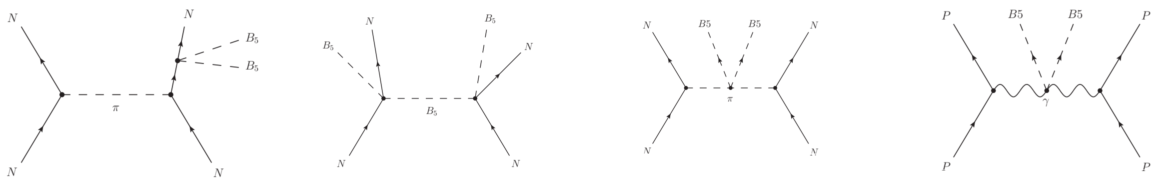


Figure 3.2: These are the new diagrams arising from RS gravitational excitations that contribute to supernova cooling. N is either a neutron or proton, while P is a proton. The higher dimensional operators involving B_5's arise primarily from integrating out the radion.

The phase space integration is over both initial and final state particles, and S contains initial and final state combinatorics for identical particles.

We have estimated the energy loss rate due to nuclear bremsstrahlung in SN1987a. The processes are: $n\,n \to n\,n\,a\,a$, where n is any nucleon, either a proton or neutron. The diagrams which contribute to the matrix element are shown in Figure 3.2. We make a number of approximations in calculating the energy loss rate, but all of these simplifications overestimate the rate, meaning that the actual models are safer than what is reflected in our calculations.

First, we neglect the final state phase space distributions. For Fermi-Dirac statistics, the $1-f_k$ functions vary between 1/2 and 1. We take these to simply be one. We overestimate the energy loss per collision by assuming it is equal to the total initial energy of the system: $E_{\text{stream}} = E_{\text{av}} \equiv E_1 + E_2 - m_{n_1} - m_{n_2}$. This means that the energy lost is purely a function of the initial states in the scattering process, and can be factored out of the final state phase space integration. In reality, the energy lost is generally much less. Once this is done, the phase space integration over final states is the usual one for calculating cross sections:

$$
\begin{aligned}
Q &= \int d\Pi_{\text{PS}} S\, |\mathcal{M}|^2\, \delta^{(4)}\left(\sum_i p_i - \sum_f p_f\right) E_{\text{stream}} f_1 f_2 \left(1-f_3\right)\left(1-f_4\right) \\
&\lesssim \int d\Pi_{\text{is}} E_{\text{av}} f_1 f_2 S \int d\Pi_{\text{fs}}\, |\mathcal{M}|^2\, \delta^{(4)}\left(\sum_i p_i - \sum_f p_f\right) \\
&= \int d\Pi_{\text{is}} E_{\text{av}} f_1 f_2 S \left(2E_1 2E_2 v_{\text{rel}} \sigma\right). \qquad (3.73)
\end{aligned}
$$

We compute the cross sections for the relevant processes using CalcHep [60]. These are relativistically invariant functions of the center of mass energy of the collision, or equivalently the magnitudes of the 3-momenta in the center of mass frame. We numerically interpolate these total cross sections over the relevant range of 3-momenta and perform the above integration numerically.

The final energy loss rates due to the nuclear Bremsstrahlung processes are given by

$$Q_{\rm NN} = 3.9 \cdot 10^{20} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 \left(\frac{36.8}{\log R'/R}\right)^2 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s}$$
$$Q_{\rm PP} = 2.0 \cdot 10^{21} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 \left(\frac{36.8}{\log R'/R}\right)^2 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s}$$
$$Q_{\rm NP} = 3.9 \cdot 10^{20} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 \left(\frac{36.8}{\log R'/R}\right)^2 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s} \tag{3.74}$$

This corresponds to a total luminosity (for a 20 km radius neutron star) of $\mathcal{L}_a = 3 \cdot 10^{40}$erg/s, and temperature $kT = 30$ MeV whereas the luminosity of the neutrino burst phase is estimated to be $\mathcal{L}_\nu \approx 10^{53}$erg/s. Thus, for this choice of parameters, the additional energy loss due to processes involving the couplings of RS gravity to the HS can be neglected. In Figure 3.3, we display the temperature dependence of the total luminosity in Goldstone bosons due to the processes in Figure 3.2.

We have also calculated the energy loss rates in stars due to hidden Goldstone boson production from the processes shown in Figure 3.4. Compton, Primakoff, and Bremsstrahlung diagrams contribute, as well as photon annihilation to Goldstone bosons. The solar energy loss rates, using a temperature $kT = 1.3$ keV, for each process (labelled by the initial states) are given by:

$$Q_{\gamma\gamma} = 6.7 \cdot 10^{-39} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 \left(\frac{36.8}{\log R'/R}\right)^2 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s}$$
$$Q_{e^-\gamma} = 2.1 \cdot 10^{-36} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s}$$

$$Q_{H\gamma} = 6.8 \cdot 10^{-11} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 \left(\frac{36.8}{\log R'/R}\right)^2 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s}$$
$$Q_{He\gamma} = 1.1 \cdot 10^{-10} \left(\frac{100 \text{ GeV}}{m_{\rm radion}}\right)^4 \left(\frac{36.8}{\log R'/R}\right)^2 (R' \; 500 \text{ GeV})^4 \, \text{erg/cm}^3\text{/s} \tag{3.75}$$

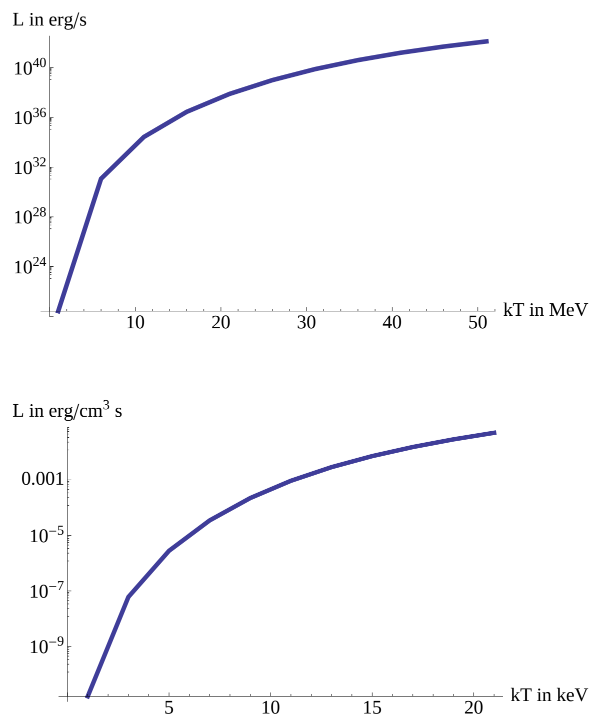


Figure 3.3: In these plots, we show the temperature dependence of the luminosity in hidden sector goldstone bosons in a core collapse supernova (left) and in main sequence stars (right) for a single Goldstone scalar coupled to the SM via RS gravity excitations. The following parameters are used: $m_{\text{radion}} = 100$ GeV, $R' = (500 \text{ GeV})^{-1}$, and $R = 1/M_{\text{Pl}}$.

The Compton process has a different scaling due to the fact that the B_5 couplings to electrons are not dependent on the log of the scale hierarchy. In comparison with usual solar nuclear energy production of a few erg/cm^3/s, these energy loss rates are negligible. In Figure 3.3 we display the temperature dependence of the total energy loss rate, so that the results can be extended to other main-sequence stars. For the higher red-giant core temperatures, the energy loss rate is still small in comparison with nuclear burning rates of about 10^8 erg/cm^{3}s.

3.8 Conclusions

We have examined a class of models embedded in a Randall-Sundrum geometry in which there are new extra dimensional gauge symmetries which contain in their spec-

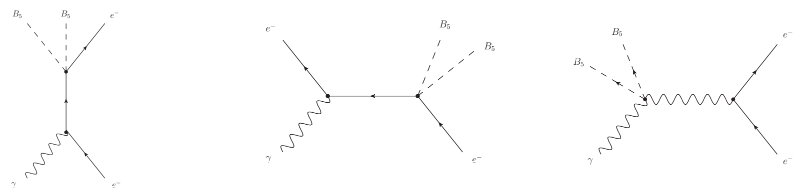


Figure 3.4: These are the new diagrams arising from RS gravitational excitations that contribute to star cooling. In addition to these diagrams, the electron may be replaced by the nuclei of the solar elements. The higher dimensional operators involving B_5's arise primarily from integrating out the radion.

tra either light scalar fields or light gauge fields. These new fields are taken to be hidden from the SM, either through small couplings, or vanishing quantum numbers. Such hidden sectors are still phenomenologically relevant, however, due to sizable couplings to RS gravitational fluctuations which, in turn, couple with similar strength to SM fields. Through these couplings, the collider phenomenology of the radion and KK-gravitons may be drastically modified, and through scalar mixing, Higgs phenomenology may change as well. We also motivate the case for such a hidden sector by describing a simple model which resolves the strong CP problem, and in which a light scalar field arising from an RS gauge symmetry plays the role of an axion. Hidden sectors which contain such light scalar fields contribute new amplitudes relevant for star and supernova cooling. We have calculated constraints arising from these operators, and find them to be well within current bounds.

3.9 Appendix A: Tables of gravitational interactions

In this Appendix, we summarize the interactions of the radion and the gravitational excitations with both broken and unbroken 5D gauge symmetries.

In Table 3.1, we give the interactions of the radion and graviton KK-modes with the massless B_5 and the associated KK-modes in the case where the gauge symmetry is broken twice by boundary conditions. In Table 3.2, we give the couplings of the radion to an unbroken gauge group. Finally, in Table 3.3, we give the couplings of the first two KK-gravitons to an unbroken bulk gauge group.

$rB^{(1)\mu}\partial_\mu B_5$	$1.09\frac{M_1}{\kappa\Lambda_r}$	$\hat{h}^{\mu\nu}_{(1)}B^{(1)}_\mu\partial_\nu B_5$	$-0.134\frac{M_1}{\kappa\Lambda_1}$	$\hat{h}^{\mu\nu}_{(2)}B^{(1)}_\mu\partial_\nu B_5$	$.099\frac{M_1}{\kappa}\Lambda_2$
$rB^{(1)}_\mu B^{(1)\mu}$	$\frac{4}{3}\frac{M_1^2}{2\kappa\Lambda_r}$	$\hat{h}^{\mu\nu}_{(1)}B^{(1)}_\mu B^{(1)}_\nu$	$-.137\frac{M_1^2}{2\kappa\Lambda_1}$	$\hat{h}^{\mu\nu}_{(2)}B^{(1)}_\mu B^{(1)}_\nu$	$.050\frac{M_1^2}{2\kappa\Lambda_2}$
$rB^{(1)}_{\mu\rho}B^{(1)\mu\rho}$	$\frac{1}{3}\frac{1}{2\kappa\Lambda_r}$	$\hat{h}^{\mu\nu}_{(1)}B^{(1)}_{\mu\rho}B^{(1)\rho}_\nu$	$.137\frac{1}{2\kappa\Lambda_1}$	$\hat{h}^{\mu\nu}_{(2)}B^{(1)}_{\mu\rho}B^{(1)\rho}_\nu$	$.053\frac{1}{2\kappa\Lambda_2}$
$r(\partial_\mu B_5)^2$	$2\frac{1}{2\kappa\Lambda_r}$	$\hat{h}^{\mu\nu}_{(1)}\partial_\mu B_5\partial_\nu B_5$	$-.219\frac{1}{2\kappa\Lambda_1}$	$\hat{h}^{\mu\nu}_{(2)}\partial_\mu B_5\partial_\nu B_5$	$.049\frac{1}{2\kappa\Lambda_2}$

Table 3.1: This table contains the Lagrangian coefficients for interactions between RS gravitational excitations and the modes associated with the bulk gauge symmetry that produces light Goldstone modes.

$rB^{(0)}_{\mu\nu}B^{(0)\mu\nu}$	$\frac{1}{4\kappa\Lambda_r\log R'/R}$	$rB^{(0)}_{\mu\nu}B^{(1)\mu\nu}$	$\frac{.483}{\kappa\Lambda_r\sqrt{\log R'/R}}$	$rB^{(0)}_{\mu\nu}B^{(2)\mu\nu}$	$\frac{-.090}{\kappa\Lambda_r\sqrt{\log R'/R}}$
$rB^{(1)}_{\mu\nu}B^{(1)\mu\nu}$	$.556\frac{1}{2\kappa\Lambda_r}$	$rB^{(1)}_\mu B^{(1)\mu}$	$.222\frac{M_1^2}{2\kappa\Lambda_r}$	$rB^{(1)}_{\mu\nu}B^{(2)\mu\nu}$	$-.237\frac{1}{\kappa\Lambda_r}$
$rB^{(1)}_\mu B^{(2)\mu}$	$-.175\frac{M_1M_2}{\kappa\Lambda_r}$	$rB^{(2)}_{\mu\nu}B^{(2)\mu\nu}$	$.377\frac{1}{2\kappa\Lambda_r}$	$rB^{(2)}_\mu B^{(2)\mu}$	$.312\frac{M_2^2}{2\kappa\Lambda_r}$

Table 3.2: This table contains the Lagrangian coefficients for interactions between the radion and the zero and KK-modes of an unbroken RS gauge symmetry.

3.10 Appendix B: Gauge fixing of the Hidden Sector

Since we are including the coupling of gravity to the gauge fields, and we have already chosen a specific gauge in which to express the gravitational fluctuations, we must be sure to respect general covariance in the gauge fixing term we add to restrict the path integral to non-redundant hidden sector gauge field configurations. This is to ensure we do not create spurious interactions which are artifacts of over-constraining the gauge freedom. Note that the general R-ξ gauges often chosen in such models break 5D covariance, even in the bulk, so we must find a new gauge fixing potential. The one we choose is, in the end, equivalent at the quadratic level to the 5D R-ξ gauges [61]

$h^{\mu\nu}_{(1)} B^{(0)}_{\mu\rho} B^{(0)\rho}_{\nu}$	$\frac{.191}{2\kappa\Lambda_1 \log \frac{R'}{R}}$	$h^{\mu\nu}_{(1)} B^{(0)}_{\mu\rho} B^{(1)\rho}_{\nu}$	$\frac{.209}{\kappa\Lambda_1 \sqrt{\log \frac{R'}{R}}}$	$h^{\mu\nu}_{(1)} B^{(0)}_{\mu\rho} B^{(2)\rho}_{\nu}$	$\frac{-.066}{\kappa\Lambda_1 \sqrt{\log \frac{R'}{R}}}$
$h^{\mu\nu}_{(1)} B^{(1)}_{\mu\rho} B^{(1)\rho}_{\nu}$	$\frac{.249}{2\kappa\Lambda_1}$	$h^{\mu\nu}_{(1)} B^{(1)}_{\mu} B^{(1)}_{\nu}$	$\frac{-.082 M_1^2}{2\kappa\Lambda_1}$	$h^{\mu\nu}_{(1)} B^{(1)}_{\mu\rho} B^{(2)\rho}_{\nu}$	$\frac{-.118}{\kappa\Lambda_1}$
$h^{\mu\nu}_{(1)} B^{(1)}_{\mu} B^{(2)}_{\nu}$	$\frac{.086 M_1 M_2}{\kappa\Lambda_1}$	$h^{\mu\nu}_{(1)} B^{(2)}_{\mu\rho} B^{(2)\rho}_{\nu}$	$\frac{.152}{2\kappa\Lambda_1}$	$h^{\mu\nu}_{(1)} B^{(2)}_{\mu} B^{(2)}_{\nu}$	$\frac{-.138 M_2^2}{2\kappa\Lambda_1}$
$h^{\mu\nu}_{(2)} B^{(0)}_{\mu\rho} B^{(0)\rho}_{\nu}$	$\frac{.028}{2\kappa\Lambda_2 \log \frac{R'}{R}}$	$h^{\mu\nu}_{(2)} B^{(0)}_{\mu\rho} B^{(1)\rho}_{\nu}$	$\frac{-.024}{\kappa\Lambda_2 \sqrt{\log \frac{R'}{R}}}$	$h^{\mu\nu}_{(2)} B^{(0)}_{\mu\rho} B^{(2)\rho}_{\nu}$	$\frac{.119}{\kappa\Lambda_2 \sqrt{\log \frac{R'}{R}}}$
$h^{\mu\nu}_{(2)} B^{(1)}_{\mu\rho} B^{(1)\rho}_{\nu}$	$\frac{-.064}{2\kappa\Lambda_2}$	$h^{\mu\nu}_{(2)} B^{(1)}_{\mu} B^{(1)}_{\nu}$	$\frac{-.051 M_1^2}{2\kappa\Lambda_2}$	$h^{\mu\nu}_{(2)} B^{(1)}_{\mu\rho} B^{(2)\rho}_{\nu}$	$\frac{.116}{\kappa\Lambda_2}$
$h^{\mu\nu}_{(2)} B^{(1)}_{\mu} B^{(2)}_{\nu}$	$\frac{.0031 M_1 M_2}{\kappa\Lambda_2}$	$h^{\mu\nu}_{(2)} B^{(2)}_{\mu\rho} B^{(2)\rho}_{\nu}$	$\frac{-.282}{2\kappa\Lambda_2}$	$h^{\mu\nu}_{(2)} B^{(2)}_{\mu} B^{(2)}_{\nu}$	$\frac{-5.9\cdot 10^{-4} M_2^2}{2\kappa\Lambda_2}$

Table 3.3: This table contains the Lagrangian coefficients for interactions between the KK-gravitons and the zero and KK-modes of an unbroken RS gauge symmetry.

with the choice $\xi = 1$, however the non-covariant R-ξ gauge still generates spurious 3-point couplings involving KK-gravitons and the radion.

To begin, we write the gauge kinetic term in an explicitly covariant manner (although as usual the Christoffel symbols cancel by anti-symmetry of the gauge field strength tensor):

$$\begin{aligned} S_{\text{gauge}} &= -\frac{1}{4g_5^2}\int_{\mathcal{M}} dV g^{MN} g^{RS} \left(\nabla_M A_R - \nabla_R A_M\right)\left(\nabla_N A_S - \nabla_S A_N\right) \\ &= \frac{1}{2g_5^2}\int_{\mathcal{M}} dV g^{MN} g^{RS} \left(\nabla_R A_M \nabla_N A_S - \nabla_M A_R \nabla_N A_S\right) \end{aligned} \tag{3.76}$$

where dV is the covariant volume element. We would ideally like to remove the kinetic mixing between the vector fields and the components which are eaten to produce massive 4D vectors in the effective field theory.

A general covariant gauge fixing term which removes the mixing is given by:

$$\mathcal{S}_{GF} = -\frac{1}{2g_5^2}\int_{\mathcal{M}} dV \mathcal{G}(B)^2 = -\frac{1}{2g_5^2}\int_{\mathcal{M}} dV \left(\nabla_M A^M + v_M A^M\right)^2 \tag{3.77}$$

Here, v_M is a vector field whose components we will determine in this section. Expanded in component form, in the absence of gravity fluctuations, this gauge fixing function is:

$$\left(\frac{R}{z}\right)^2 \left[\partial_\mu B_\nu \eta^{\mu\nu} - B_5' + 3\frac{B_5}{z} + \eta^{\mu\nu} v_\mu B_\nu - v_5 B_5\right] \tag{3.78}$$

The residual gauge symmetry with this gauge fixing term obeys the following equation (in the absence of gravity fluctuations):

$$\Box\beta - \beta'' + 3\frac{\beta'}{z} + \eta^{\mu\nu}v_\mu\partial_\nu\beta - v_5\beta' = 0 \tag{3.79}$$

The kinetic mixing term between B_μ and B_5, after summing up the standard kinetic term and the contributions from the gauge fixing term are:

$$\frac{1}{g_{5\mathrm{D}}^2}\left(\frac{R}{z}\right)\left[(\partial_\mu B_\nu\eta^{\mu\nu} + \eta^{\mu\nu}v_\mu B_\nu)\left(B_5' - 3\frac{B_5}{z} + v_5 B_5\right) - B_\mu'\partial_\nu B_5\eta^{\mu\nu}\right] \tag{3.80}$$

Integration by parts of the last term in this expression causes the entire mixing term to vanish if the vector v_M is chosen such that $v_\mu = 0$, and $v_5 = 2/z$.

Note that the gauge fixing function $\mathcal{G}(B)$ is a function of $\partial_\mu B^\mu$ and only the eaten B_5 modes. The variation of the gauge fixing term then, with respect to the metric, is:

$$\frac{\delta}{\delta g^{MN}}\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2g_{5\mathrm{D}}^2}\left(\frac{\delta}{\delta g^{MN}}\sqrt{g}\right)\mathcal{G}(B)^2 + \sqrt{g}\left(\frac{\delta}{\delta g^{MN}}\mathcal{G}(B)\right)\mathcal{G}(B) \tag{3.81}$$

Thus all interactions with gravitational fluctuations involve only the unphysical B_5's, and terms involving $\partial_\mu B^\mu$ which vanish in all matrix elements due to the 4D Ward identities for the HS KK-modes. The interactions listed in Appendix A are thus sufficient to describe the physical couplings of RS gravity to the excitations within the HS.

Chapter 4

A Top Seesaw on a 5D Playground

As we explored in section 2.3.3 the fine-tuning of the Higgs mass is of paramount interest to particle physicists as it represents one of the best indirect probes of BSM physics. Traditionally there are two paths towards eleviating the fine-tuning of the Higgs mass. The first is introducing new symmetries such as SUSY so that a light Higgs is natural. More modest proposals typically concentrate exclusively on the top quark sector of the SM. The focus on the top is because the Higgs couples most strongly to the top than anything else in the SM, hence the top poses the greatest contribution to the quantum corrections to the Higgs mass through its large Yukawa coupling.

Top composite models flip this logic on its head, turning the large coupling between the Higgs and top quarks into a virtue. Roughly speaking, the $\mathcal{O}(1)$ Yukawa coupling of the top quark indicates the possibility of strong coupling between the tops and Higgs sector. Loop corrections show that the situation is actually even more promising than this. The renormalization group flow of the top Yukawa is governed by an infrared fixed point near $\lambda_t \sim 1$ at low energies, implying the Yukawa was in fact much larger at higher energies and the Higgs could be a bound state of top quarks. If the Higgs is indeed a composite particle, then it is immune to mass scales above the scale of compositeness as above that energy it is no longer a DOF.

A toy model of a top composite Higgs in a flat extra dimension was considered in [62]. This work also addressed the more general nature of quantum corrections in a full 5D effective action in a 5D language, instead of quantum corrections in the 4D effective theory.

4.1 Introduction

Models of top-quark condensation [63, 64, 65, 66, 67] are particularly appealing models of electroweak symmetry breaking. These theories are relatively compact and have the feature of automatically generating a large top Yukawa interaction with a composite Higgs field that is a bound state of a top–anti-top pair. While the simplest model is plagued by naturalness issues, subsequent embeddings of top condensation in supersymmetric [68] and strongly coupled models of electroweak symmetry breaking (EWSB) [69, 70, 71] can reproduce the weak scale without excessive fine-tuning. However, a combination of flavor [72, 73, 74, 75] and electroweak precision constraints [77, 78] have consistently put tension on implementations of top condensation within strongly coupled scenarios. For a review with extensive discussion of these issues and a complete citation list, see [76].

Recent focus on extra dimensional models of EWSB, particularly those constructed on geometrically warped backgrounds [22, 79] has shed new light on naturalness issues of the electroweak sector, and how precision tests might be addressed in a weakly coupled framework. In this model, we explore the possibility of embedding top condensation within an extra dimensional setup. Such models with warped geometry are expected to generate natural hierarchies of scales. In this model, we explore the 5D Nambu–Jona-Lasinio (NJL) mechanism [80, 81] in a flat space toy model, with the idea that many of the results will carry over to more realistic extra dimensional scenarios utilizing a warped compactification.

In calculating the 5D effective action for fermion–antifermion bound states, we renormalize a 5D Yukawa theory compactified on an interval. The running is supplemented by an ultraviolet (UV) "composite" boundary condition at a scale Λ_0. At the UV boundary, which we take to be at an energy greater than the compactification scale $1/L$, the theory describes 5D fermions that interact via a four-fermion interaction which arises from unspecified UV dynamics, perhaps from physics above the cutoff due to the strong coupling limit of the extra dimensional model. In deconstruction models [82, 83], where the extra dimension resolves into a product gauge structure at high energies, the four-fermion operator could arise as a result of the (unspecified) dynamics which breaks the product group structure down to the Standard Model (SM) at low energies. The four-fermion operator could also arise due to intrinsically 5D dynamics such as a spontaneously broken 5D gauge theory.

Top condensation has been studied in extra dimensional contexts previously [84, 85, 86, 87, 88], although focus has typically been on the low-energy theory below the scale of compactification. Our analysis includes the effect of 5D running up to the scale associated with the four-fermion interaction, and gives predictions for a Kaluza-Klein (KK) tower of scalar bound states corresponding to a 5D composite field. Of particular interest are the form of and role played by brane localized terms generated by fermion loops. Other top condensation models that simultaneously generate the correct top and W-boson masses generally supplement top condensation with a seesaw mechanism [89, 90] (see chapter 91 of [9] for a general description of the seesaw mechanism). Features of our 5D construction are similar to those found in extra dimensional top see-saw models [91, 92], in which the lightest KK excitations of the fermions play a key role in the formation of the condensate.

We begin with a review of the 4D NJL model, which we then extend to a 5D setup compactified on an interval, or equivalently, an $S_1/\mathbb{Z}_2$ orbifold. Extension of these methods to a compactified model is relatively straightforward, although there are some complications associated with performing quantum corrections in an extra dimensional model, which we discuss. We work in the fermion bubble approximation, valid as long as the scale associated with the four-fermion operator is below the scale at which any additional 5D interactions (i.e. gauge interactions of the SM) become strongly coupled. Section 4.3 contains a study of the relevant fermion loop graphs in 5D flat space. We then calculate the 5D quantum effective action valid at low scales. Solving the scalar equations of motion in this effective theory determines whether or not a chiral symmetry breaking condensate is formed.

We calculate the resulting light fermion and scalar spectrum, requiring a weakly gauged $SU(2)_L \times U(1)_Y$ version of the model to reproduce the observed W-boson mass. We find that the top quark mass and W mass constraints can be simultaneously satisfied by making an appropriate choice of the fermion bulk mass parameters. The lowest lying scalar fluctuation is found to be generically heavy, due primarily to a large effective quartic coupling generated in the model. Lighter values can be generated by going to larger N_c or creating a larger hierarchy between the four-fermion scale and the compactification scale $\Lambda_0 L \gg 1$. The second of these choices is made at the expense of increased fine-tuning of the interaction strength associated with the four-fermion operator and reducing the validity of the fermion bubble approximation.

4.2 Extending the NJL Model to 5D

A toy model for spontaneous breaking of chiral symmetry in four dimensions can be constructed with a low-energy effective theory of massless fermions supplemented with a single chirally symmetric four-fermion contact operator [80, 81]. The Lagrangian for this model, valid at the scale Λ is

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{g^2}{4\Lambda^2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma^5\psi)^2\right] \tag{4.1}$$

where ψ is a 4-component massless Dirac fermion. The Lagrangian is invariant under independent chiral rotations of the left- and right-handed components of ψ.

In two component notation, utilizing a complex auxiliary scalar field ϕ, we can re-write this Lagrangian as

$$\mathcal{L} = \bar{\psi}_L i \partial\!\!\!/ \psi_L + \bar{\psi}_R i \partial\!\!\!/ \psi_R + g\phi\bar{\psi}_L\psi_R + \text{h.c.} - \Lambda^2|\phi|^2. \tag{4.2}$$

The field ϕ carries chiral charge such that this Lagrangian has the same symmetry as Eq. (4.1). Running down this theory from the scale Λ to a low scale μ, taking into account only fermion loops, one finds that the scalar field ϕ develops dynamics and a quartic interaction. The fermion loop contribution to the scalar mass^2 is negative, and for sufficiently strong coupling, g, the quantum corrections overcome the positive

$\Lambda^2|\phi|^2$ term. In this case, the scalar field then picks a vacuum expectation value (vev), and breaks the chiral symmetry of the theory.

This mechanism was posited as a method to spontaneously break the electroweak gauge interactions, where the fermion bound state consisted of top/anti-top pairs [67]. A particularly appealing feature of this construction is the presence of a quasi-infrared fixed point in the top Yukawa coupling which renders the top Yukawa relatively insensitive to the compositeness scale [93, 94]. Above this fixed point, the top Yukawa blows up in the UV, and the coupling is in the domain of attraction for this fixed point which resides at a value of $\lambda_t \sim 1$.

We consider a 5D version of the above model, in which there is a four-fermion operator that leads to a composite five dimensional scalar field. This operator must arise from some UV dynamics, as in the case of 4D top condensation models [70]. In this dissertation, we do not specify this dynamics and focus on the mechanics of the renormalization of this theory. A model with better UV behavior is currently under investigation.

The theory at a high scale Λ_0 consists of two 5D Dirac fermions, Ψ_L which contains a left-handed zero mode in the spectrum, and Ψ_R which contains a right handed one. Other assignments are possible, and will have different IR structure, however this theory is the one that most easily generalizes to a standard model-like low-energy spectrum. In addition, the chiral symmetries of this model are identical to those in Eq. (4.2). We write the action for the theory at the scale $\Lambda > 1/L$ as defined on a circle with perimeter $2L$:

$$\begin{aligned} S_{\text{5D NJL}} = \int d^4x \int_{-L}^{L} dz \Big[&\bar{\Psi}_L \left(i\partial\!\!\!/ - M_L(z) \right) \Psi_L \\ &+ \bar{\Psi}_R \left(i\partial\!\!\!/ - M_R(z) \right) \Psi_R + \frac{g^2}{\Lambda_0^3} \bar{\Psi}_L \Psi_R \bar{\Psi}_R \Psi_L \Big]. \end{aligned} \tag{4.3}$$

where $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu + i\gamma^5 \partial_z$ and all fields are assigned periodic boundary conditions. The spectrum of the theory is then reduced by performing the identification $z \leftrightarrow -z$ which restricts the physical region of the space to the interval $z \in [0, L]$. The field solutions that remain can be either odd or even under this identification, although all operators in the Lagrangian must be even. The orbifold assignments that produce the spectrum described above are:

$$\Psi_L(z) = -\gamma^5 \Psi_L(-z), \text{ and } \Psi_R(z) = \gamma^5 \Psi_R(-z). \tag{4.4}$$

In order for the action to be invariant, the fermion mass terms must be odd under the orbifold assignment: $M_{L,R}(z) = -M_{L,R}(-z)$.

While this procedure is equivalent to beginning with an interval and assigning boundary conditions [95, 27], we show in Appendix 4.8 that the orbifold language allows a simple, intuitive explanation for the presence or lack of certain brane localized terms that are induced by quantum corrections.

We assume mass profiles which are constant in the physical region, discontinuously jumping at the orbifold boundaries to satisfy the boundary condition above:

$$M_{L,R}(z) = \begin{cases} +m_{L,R} & z > 0 \\ -m_{L,R} & z < 0. \end{cases} \tag{4.5}$$

The zero modes are then exponentially localized, with profiles given by:

$$\begin{aligned} \Psi_L^0(x; z) &= \sqrt{\frac{m_L}{1 - e^{-2m_L L}}} e^{-m_L |z|} \\ \Psi_R^0(x; z) &= \sqrt{\frac{m_R}{e^{2m_R L} - 1}} e^{m_R |z|}. \end{aligned} \tag{4.6}$$

In the 4D low-energy effective theory and ignoring quantum effects, the zero modes couple via a four-fermion operator that has a form identical to that of Eq. (4.2), with effective four-fermion coupling given by an overlap of the zero mode wave functions:

$$\frac{g_{4D}^2}{\Lambda_{\text{eff}}^2} = \frac{g^2}{\Lambda_0^3} \frac{m_L m_R}{m_L - m_R} \left(\coth m_L L - \coth m_R L \right), \tag{4.7}$$

which is exponentially suppressed in the case that both m_L and m_R are the same sign, and the LH and RH zero modes are localized on opposite boundaries of the physical region. We will show that scalar bound states and chiral symmetry breaking with scales well below the scale $1/L$ can still be obtained, regardless of this suppression. In the KK mode interpretation, these scalars are presumably relativistic deeply bound states of a combination of KK modes. This strongly suggests that a full 5D calculation including all KK modes below the cutoff Λ_0 should be performed in order to properly formulate the low-energy theory.

To analyze the IR behavior of this theory, we write the 5D four-fermion interaction in terms of a complex auxiliary field ϕ. At the scale Λ_0, the theory is then a model of Yukawa interactions in which the scalar field has no dynamics:

$$S_{5\text{D NJL}} = \int d^4x \int_{-L}^{L} dz \bar{\Psi}_L \left(i\partial\!\!\!/ - M_L(z)\right) \Psi_L + \bar{\Psi}_R \left(i\partial\!\!\!/ - M_R(z)\right) \Psi_R$$
$$- \Lambda_0^2 |\phi|^2 + \frac{g}{\sqrt{\Lambda_0}} \phi \bar{\Psi}_L \Psi_R + \text{h.c.} \tag{4.8}$$

Integrating out the field ϕ reduces Eq. (4.8) to Eq. (4.3). The main calculation of this model will be on running this effective Lagrangian down to a low scale $\mu < \frac{1}{L}$, and solving the low-energy equations of motion for the scalar field. We calculate the running in the "fermion bubble" approximation, integrating out only the fermionic contribution to the scalar effective action. This approximation is the analog of re-summing the fermion ladder diagrams in the theory written down in Eq. 4.3.

4.3 Quantum Corrections in 5D

In models with compactified extra dimensions, quantum corrections are complicated by the fact that momenta along the compactified directions are discrete while the 4D momenta span a continuum. In our model, momenta along the compactified coordinate are quantized in units of $n\pi/L$, where L is the size of the physical region. In this section, we compute these quantum corrections for the Yukawa theory in Eq. (4.8).

Quantum effects in extra dimensional models have been studied in some contexts, particularly for the running of gauge couplings [96, 97, 98]. Such calculations are often made simpler due to gauge invariance, which ensures that calculating the running of the coupling of the zero mode gauge field, which has a constant extra dimensional profile, is sufficient to describe all running effects in 5D. Our analysis of a 5D Yukawa theory must be intrinsically five dimensional, taking into account all possible external scalar states, since there is no such underlying symmetry which keeps the lowest lying mode flat.

In determining the quantum effects of the 5D theory, there is the approach of determining the KK spectrum, integrating out the extra dimension, and then truncating the effects of the tower at the desired level of accuracy. It is then a matter of computing usual 4D Feynman diagrams using these few KK modes. This approach, however, obscures 5D translation invariance, and is in fact quite complicated if more than a couple KK modes are included. This is especially the case in this construction, since there are a large number of possible scalar bound states. When the equation of motion is applied on the scalar field ϕ at the scale Λ_0, and the fermions are expanded in terms of their KK towers, we find:

$$\phi = \frac{g}{\Lambda_0^{5/2}} \bar{\Psi}_R \Psi_L = \frac{g}{\Lambda_0^{5/2}} \sum_{m,n} \bar{\psi}_R^m \psi_L^n. \tag{4.9}$$

Quantum effects below the scale Λ_0 mix these fermion bi-linears with each other, and the effective action must then be re-diagonalized. It is much simpler and perhaps more illuminating to instead compute all quantum effects from the 5D viewpoint, and then solve the resulting 5D scalar equation of motion.

The most straightforward method is to compute all quantum corrections in momentum space, where the effects of orbifolding are taken into account in the form of the propagators. Either a hard momentum cutoff or dimensional regularization may then be used to study the divergence structure of the theory. The first of these is most suited to the 5D NJL model, since it explicitly contains information about power law divergences. Dimensional regularization, on the other hand, automatically subtracts these, leaving only poles corresponding to logarithmic divergences. We study both regulators, the former because it applies well to models with an explicit cutoff, and the latter since it is a point of interest to see how the 5D divergence structure, which contains no bulk log divergences, is obtained from the 4D KK tower which contains an infinite number of them.

It is, in principle, possible to use a mixed position-momentum space basis, where the propagators depend on the position in the extra dimensional coordinate, however in this case it is unclear how one would implement a regularization procedure which respects local 5D Lorentz invariance.

4.3.1 Quantum corrections with vanishing fermion bulk masses

In the case that the bulk fermion masses vanish, the fermion propagators are not difficult to compute. The Yukawa theory under consideration is then similar to the one examined in [99], but with slightly different orbifold assignments and field content. In this section we utilize the notation of these authors. In particular, a derivation of the fermion propagators can be found in Section 2 of that publication.

In 5D momentum space, the fermion propagators are given by:

$$S_F^{(L,R)}(p;p_5,p_5') = (2L)\frac{i}{2}\left\{\frac{\delta_{p_5,p_5'}}{\not p + i\gamma^5 p_5} \pm \frac{\delta_{-p_5,p_5'}}{\not p + i\gamma^5 p_5}\gamma^5\right\} \tag{4.10}$$

where the $+$ is for a 5D fermion in which a left-handed zero mode survives the orbifold projection, and the $-$ is for a 5D fermion which contains a right-handed zero mode in the spectrum[1]. The 5D momentum is given by $p_5 = \frac{n\pi}{L}$, where n ranges over all integers. The fermion propagators conserve the magnitude of the 5D momentum, but only up to a sign. The breaking of 5D translation invariance is a manifestation of the reflection conditions at the orbifold fixed points. The remaining conservation of KK number is a tree level symmetry of the theory that is present in the limit of vanishing bulk mass.

We are interested in computing the scalar two- and four-point functions. Since interaction terms in extra dimensional theories are non-renormalizable, higher dimensional operators will be generated as well. For the purposes of illustration in this toy model, we ignore these contributions. One could, in principle, arrange for these terms to be removed via fine-tuning of the coefficients of such operators against the quantum corrections to them. This tuning should then presumably be derived as a natural consequence of some UV complete model.

[1] We have chosen a convention in which the period of the Fourier series appears in the Kronecker-δs of momentum ($2L\ \delta_{p_5,k_5}$), and in sums over unconstrained 5D momenta ($\frac{1}{2L}\sum_{k_5}$). This makes it simpler to compare with the (mostly) standard treatment in non-compact dimensions where the transformation to momentum space comes with a $\frac{1}{2\pi}$ normalization. The dictionary between the compact and non-compact 5D theory consists of replacing sums with integrals, Kronecker-δs with δ-functions, and all factors of $2L$ with 2π.

The scalar two-point function

In the massless fermion bubble approximation, the scalar two-point function at one loop consists of the diagram shown in Figure 4.1. In the compactified 5D theory, this single diagram encapsulates the quantum corrections to the bulk kinetic and mass terms. In addition, it also contains information about brane localized terms which are quadratic in the scalar field. This diagram gives information about how to run the scalar sector of the Yukawa theory from the high scale Λ_0 down to low energies. The value for the diagram is

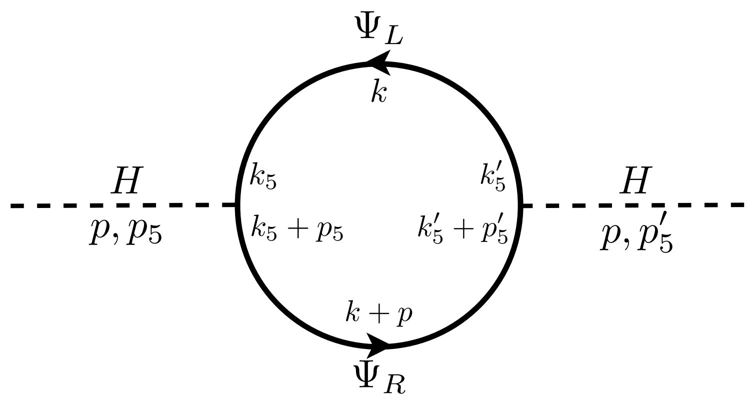


Figure 4.1: The 5D scalar two point function, where the scalar couples to two flavors of 5D Dirac fields, each of which contains either LH and RH zero mode in the KK mode spectrum.

$$-\frac{g^2}{\Lambda_0}\sum_{k_5,k_5'}\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\ \left[\frac{(\not{k}+i\gamma^5k_5)(\delta_{k_5,k_5'}-\gamma^5\delta_{k_5,-k_5'})}{k^2-k_5^2}\right.$$
$$\left.\cdot\frac{(\not{k}+\not{p}+i\gamma^5[k_5'+p_5'])(\delta_{k_5+p_5,k_5'+p_5'}+\gamma^5\delta_{k_5+p_5,-k_5'-p_5'})}{(k+p)^2-(k_5'+p_5')^2}\right]. \tag{4.11}$$

Let us first discuss brane localized divergences of the two-point diagram. In extra dimensional theories, it is now well known that quantum effects generally violate KK-number conservation [99, 100, 101]. The presence of brane localized terms can be identified by divergences which do not conserve 5D momenta. Such divergences signal that a counterterm is necessary, and that the brane term should be included in the tree level action. Expanding the numerator of the diagram and simplifying the

Kronecker-δs, there are in principle terms proportional to δ_{p_5,p'_5}, δ_{-p_5,p'_5}, $\delta_{2k_5,-p_5-p'_5}$, and $\delta_{2k_5,p'_5-p'_5}$. The first two types of terms conserve 5D momentum up to a sign and hence correspond to bulk corrections, while the second two Fourier transform into δ-functions at the brane positions and so correspond to brane localized terms.

Applying the usual Dirac trace identities, the brane localized terms vanish. This is perhaps somewhat surprising at first glance. One might expect that there are brane localized quadratic divergences which renormalize the scalar mass independently on the branes versus in the bulk. One might also expect the generation of brane localized kinetic terms for the scalar field. The reason for the absence of such terms at the one-loop level is that 5D translation invariance is not broken severely enough in this process, as explained in Appendix 4.8. In fact, there are a variety of scenarios in which brane localized terms are not generated at the one-loop level.

Let us now identify the bulk renormalization terms. We expect a cubically divergent mass renormalization, and a linear divergence in the 5D kinetic terms. One of the bulk renormalization terms is proportional to δ_{p_5,p'_5}, the other $\delta_{p_5,-p'_5}$ (effectively reflected and transmitted waves through the orbifold fixed points). From the trace, these have the following momentum structure:

$$\frac{k\cdot(k+p)-k_5(k_5+p'_5)}{(k^2-k_5^2)\left((k+p)^2-(k_5+p'_5)^2\right)} \tag{4.12}$$

The k_5 are quantized on $k_5 = n\pi/L$, with n any integer. This means that the 5D sum cannot be shifted, while the 4D momenta can be redefined in the usual way in order to make the Wick rotated integrand spherically symmetric in Euclidean momentum.

The coefficients of the δ_{p_5,p'_5} and $\delta_{p_5,-p'_5}$ terms are identical. After combining denominators using Feynman parameters, they are given by:

$$-\frac{g^2}{4\Lambda_0}\sum_{k_5}\int\frac{d^dk}{(2\pi)^d}\int_0^1 dx\frac{(l^2-l_5^2)-x(1-x)(p^2-p_5^2)+l_5p_5(2x-1)}{[(l^2-l_5^2)+x(1-x)(p^2-p_5^2)]^2}, \tag{4.13}$$

where $l_5 = k_5 + xp_5$. Unfortunately, one cannot shift the 5D momentum in the sum this way since l_5 is not quantized on the same spectrum as k_5 and the above expression is only a heuristic presentation.

This lack of shift invariance highlights the fact that a naive hard cutoff for the 4D momentum integrals obscures the underlying physics. Such a procedure explicitly violates 5D Lorentz invariance, and will lead to apparent violation of the spacetime symmetries by short-distance interactions. For example, if one performs the sum over *all* unconstrained five-momenta, one obtains an analytic expression as a function of the 4D loop momentum. The remaining integrand can then be performed with a hard cutoff, expanded in small external momenta, and then interpreted as a contribution to the effective action. The resulting expression contains terms proportional to p_μ^2 and p_5^2 with coefficients which differ in general. 5D Lorentz invariance can then be restored by fine-tuning separate counter terms order by order in perturbation theory, but the connection with the original 5D theory defined at the physical scale Λ_0 is then lost. To properly formulate the low energy dynamics, one must choose the regulator more carefully.

We first perform the integration utilizing dimensional regularization. Since there is no explicit cutoff scale, there are no subtleties about the regularization procedure respecting local 5D Lorentz invariance. Performing the 4D momentum integration first, we have

$$\begin{aligned} i\Pi(p^2, p_5) = -i\frac{g^2}{4\Lambda_0}\sum_{k_5}\int_0^1 dx \frac{\Delta^{d/2-2}}{(4\pi)^{d/2}} \times \\ \times \left\{\frac{d}{2}\Delta\Gamma(1-d/2) + \left[x(1-x)p^2 + k_5^2 + p_5 k_5\right]\Gamma(2-d/2)\right\} \end{aligned} \tag{4.14}$$

where Δ is given by:

$$\Delta = -x(1-x)(p^2 - p_5^2) + (k_5 + xp_5)^2. \tag{4.15}$$

Using zeta-function regularization for the remaining sum over 5D internal loop momentum we have

$$\begin{aligned} i\Pi(p^2, p_5^2) = i\Pi(0) - \frac{ig^2}{8\Lambda(4\pi)^{d/2}}\left(\frac{\pi}{L}\right)^{4-d} \times \\ \times \left(2\zeta(4-d) + (\mu_{\mathrm{IR}}L)^{d-4}\right)\Gamma(2-d/2)\left[p^2 + p_5^2\,(2-d)\right]. \end{aligned} \tag{4.16}$$

We have regulated the contribution of the zero mode with an IR cutoff, μ_{IR}. The two point function for vanishing external momentum, $i\Pi(0)$, is given by:

$$i\Pi(0) = -i\frac{g^2}{4\Lambda(4\pi)^{d/2}}\left(\frac{\pi}{L}\right)^{d-2}\zeta(2-d)\Gamma(1-d/2) \tag{4.17}$$

Taking the limit as $d \to 4$, with $\epsilon_{\rm IR} \equiv \mu_{\rm IR} L$, we have the final result:

$$i\Pi(p^2, p_5^2) = \frac{ig^2}{4\Lambda(4\pi)^2}\left[2\left(\frac{\pi}{L}\right)^2 \zeta'(-2) + \log(2\pi\epsilon_{\rm IR})\left(p^2 - 2p_5^2\right)\right] \tag{4.18}$$

Let us point out some aspects of these results: First, all expressions are finite as $d \to 4$. For the field strength term, the pole in the Γ function is canceled by the sum of the zeta function and the contribution of the zero mode. That is, the UV divergences created by the zero mode are canceled by the UV divergences of the tower of KK modes. Second, note that the coefficient of the p^2 and p_5^2 terms differ in the limit $d \to 4$. These finite terms correspond to non-local contributions to violations of 5D translation invariance from the presence of the orbifold fixed points.

The finiteness of the result in this regularization scheme is expected. Since all divergences must be local, the UV structure of the bulk compactified theory should match that of the uncompactified model. All divergences in noncompact odd dimensions are power laws and are automatically subtracted when using dimensional regularization. So both the compact and uncompact models yield finite results for the two-point function in this regularization scheme.

It is possible to utilize a hard cutoff regularization scheme which respects the local spacetime symmetries. This is beneficial, since such a scheme has a better physical interpretation in terms of our physical cutoff, Λ_0. The procedure is described in detail in Appendix 4.7, but in many cases it consists simply of approximating the sum over momenta by an integral, at which point the integrand is manifestly 5D Lorentz invariant, and integration over the interior of a four-sphere in the loop momentum can be performed in the standard way. The substitution required is $\frac{1}{2L}\sum_{k_5} \to \int \frac{dk_5}{2\pi}$. The two-point function in this regularization scheme is then

$$\begin{aligned} i\Pi(p^2; p_5, p_5') = \left(\delta_{p_5,p_5'} + \delta_{p_5,-p_5'}\right)\frac{g^2 L}{2\Lambda_0}\int \frac{d^5k}{(2\pi)^5}\int_0^1 dx \times \\ \times \frac{(l^2 - l_5^2) - x(1-x)(p^2 - p_5^2) + l_5 p_5(2x-1)}{[(l^2 - l_5^2) + x(1-x)(p^2 - p_5^2)]^2}, \end{aligned} \tag{4.19}$$

and we can now shift the full 5D loop momentum in the usual way, and use a 5D hard cutoff Λ. The result, as an expansion in $P^2 = p^2 - p_5^2$, is given by

$$\begin{aligned} i\Pi(p^2; p_5, p_5') &= iL\left(\delta_{p_5,p_5'} + \delta_{p_5,-p_5'}\right)\left[\frac{g^2\Lambda^3}{18\pi^3\Lambda_0} + \frac{g^2\Lambda}{10\pi^3\Lambda_0}P^2\right] \\ &\equiv L\left(\delta_{p_5,p_5'} + \delta_{p_5,-p_5'}\right) i\tilde{\Pi}(P^2). \end{aligned} \tag{4.20}$$

We have kept Λ_0 separate from the regulator cutoff in this expression to highlight the sensitivity to an arbitrary UV scale, although we take them to be equal in our final expression for the effective action. Implicit in Eq. (4.20) is an IR scale, $\mu \ll \Lambda$, which can be put into the effective action with the replacements $\Lambda^n \to \Lambda_0^n - \mu^n$.

The scalar four-point function

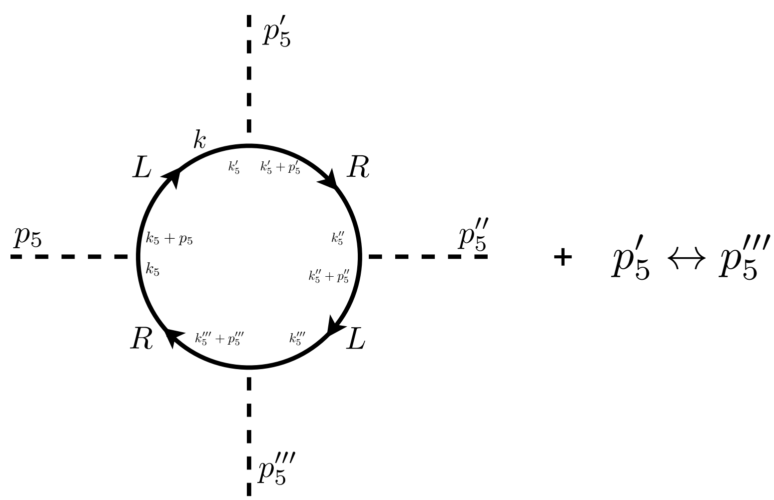


Figure 4.2: The 5D scalar four point function.

The quartic coupling also renormalizes, although we again find that all divergences are confined to the bulk. The relevant Feynman diagrams are shown in Figure 4.2, and evaluate to

$$\begin{aligned} iV_4(0; p_5, p_5', p_5'', p_5''') = -\frac{g^4}{\Lambda_0^2} &\sum_{\substack{k_5,k_5',\\ k_5'',k_5'''}} \int \frac{d^4k}{(2\pi)^4} \times \\ &\times \mathrm{Tr}\ \left[S_F^R(k; k_5, k_5''' + p_5''')S_F^L(k; k_5''', k_5'' + p_5'') \times \right. \\ &\qquad \left. \times\ S_F^R(k; k_5'', k_5' + p_5')S_F^L(k; k_5''', k_5'' + p_5'')\right]. \end{aligned} \tag{4.21}$$

Terms which contribute to bulk running of the quartic arise from an even number of insertions of the 5D momentum conserving Kronecker-δs while terms which contribute to brane running of the quartic involve an odd number of these. The potential brane terms each involve (at leading order in loop momenta) the trace of four identical Dirac matrices, $\not{k}$, with a γ^5, and therefore vanish.

Performing the calculation using dimensional regularization again produces a finite result, with KK modes canceling against the contribution of the zero modes. We only present the result utilizing a 5D Lorentz invariant hard cutoff. We find

$$iV_4(0; p_5, p_5', p_5'', p_5''') = \frac{-ig^4\Lambda}{24\pi^3\Lambda_0^2}(2L)\sum_{\pm}\delta_{0,p_5\pm p_5'\pm p_5''\pm p_5'''}. \tag{4.22}$$

Where the sum is over all 8 permutations of signs in the Kronecker-δ.

To summarize the results of this section, we find that the bulk UV structure of the theory is as expected, where the running is purely power law. We have explicitly shown the cancellation of log divergences in the dimensional regularization scheme for the two-point function.

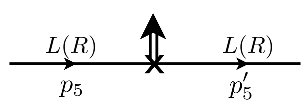
The one-loop brane localized divergence structure is different from naive expectations. Despite the intuition that brane localized terms should be forced by breaking translation invariance via the orbifold identification, they are not generated at one loop. As we discuss in Appendix 4.8, this is due to the interplay of the left- and right-handed components of 5D fermions.

4.3.2 Quantum corrections with fermion bulk masses

The arguments that protect against brane localized terms fail when fermion mass terms are added into the theory. Under the orbifolding procedure, such masses must be odd under the projection since the fermion bilinears $\bar{\Psi}\Psi$ are odd. These masses could arise from a scalar domain wall to which the fermions are coupled via a Yukawa interaction. These domain walls are trapped at the orbifold fixed point by the orbifold quantum numbers of this scalar field and give rise to fermion localization in the extra dimension [103, 104, 105]. Because such fermion masses explicitly break 5D translation invariance at the orbifold fixed points, it is expected that they generate brane localized terms.

In this section, we calculate the quantum corrections in the presence of fermion bulk masses. These mass terms do not conserve even the magnitude of the 5D momenta so that the explicit form of the propagators in momentum space is rather complicated to compute. However, we can accurately capture the divergence structure of the theory by treating the 5D mass term as a perturbation to the massless scenario.

We take the fermion masses to have the profiles given in Eq. (4.5). To obtain the Feynman rule in momentum space, we compute the Fourier series of the fermion mass terms in the action, and read off the interaction vertex. Since the mass term switches sign at the orbifold fixed points, its Fourier series is non-trivial. That is, the mass term acts as a source for 5D momentum which can be injected into a given diagram. The Feynman rule is:



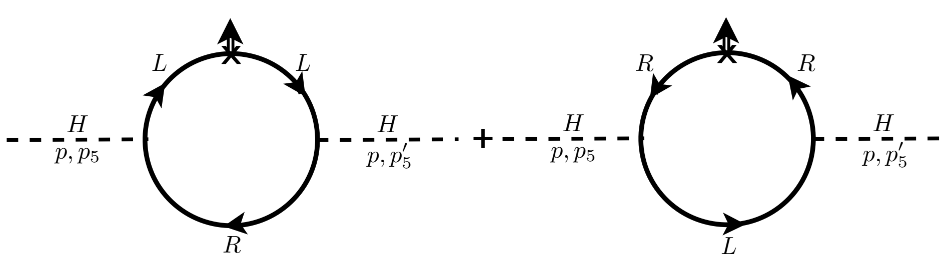
$$= \frac{4m_{L(R)}}{p'_5 - p_5}\, \delta^{\text{odd}}_{p_5,p'_5}\,, \tag{4.23}$$

where

$$\delta^{\text{odd}}_{p_5,p'_5} \equiv \begin{cases} 1 & \text{if } p_5 + p'_5 \text{ is an odd multiple of } \pi/L \\ 0 & \text{if } p_5 + p'_5 \text{ is an even multiple of } \pi/L. \end{cases} \tag{4.24}$$

This is the familiar Fourier transform of the square wave function, with period $2L$.

The corrections to the scalar two-point function arise from two diagrams, one with a mass insertion on the fermion with a LH zero mode, the other with an insertion on the one with a RH zero mode.



$$. \tag{4.25}$$

These contributions to the two-point function are linearly divergent:

$$i\Pi_M(0;p_5,p_5') = i\frac{g^2\Lambda}{3\pi^3\Lambda_0}\left(m_L - m_R\right)\delta^{\text{odd}}_{p_5,p_5'} + \text{finite terms} \tag{4.26}$$

Adding a mass insertion diagram to the four-point function only contributes finite terms.

4.4 The quantum effective action

The two- and four-point diagrams we have calculated can now be incorporated into a quantum effective action that is valid at a low scale μ. We can express this action as follows:

$$\begin{aligned} S_{\text{effective}} = \int d^4x \int_{-L}^{L} dz \Big[&\bar{\Psi}_L(i\partial\!\!\!/ - M_L(z))\Psi_L + \bar{\Psi}_R(i\partial\!\!\!/ - M_R(z))\Psi_R \\ &+ \frac{g}{\sqrt{\Lambda_0}} H\bar{\Psi}_L\Psi_R + \text{h.c.} + Z_H \partial_M H \partial^M H^\dagger - \left(\Lambda_0^2 + \delta M^2\right)|H|^2 - \frac{\lambda}{4\Lambda_0}|H|^4 \Big] \\ &- \int d^4x \,\left[m_0^2 \,|H(z=0)|^2 + m_L^2 \,|H(z=L)|^2 \right]. \end{aligned} \tag{4.27}$$

To map between our correlation functions and the terms in this effective action, we first note that each amplitude can be written in terms of projection operators $E_{p_5,p_5'} \equiv L\left(\delta_{p_5,p_5'} + \delta_{p_5,-p_5'}\right)$ acting on "sub-amplitudes." The projection operators are the expression for dynamical external scalar legs when the scalar is even under the orbifolding procedure, $H(z) = H(-z)$. The sub-amplitudes represent Feynman rules arising from bulk and brane localized terms in the effective 5D action.

For the bulk contributions to the two-point function, we have

$$\begin{aligned} i\Pi(p^2;p_5,p_5') &= E_{p_5,p_5'}\, i\tilde{\Pi}(P^2) \\ &= \frac{1}{2L}\sum_{q_5} E_{p_5,q_5}E_{q_5,p_5'}\, i\tilde{\Pi}(Q^2). \end{aligned} \tag{4.28}$$

The contribution arising from the bulk mass insertion diagrams is

$$i\Pi_M(0;p_5,p_5') = i\tilde{\Pi}_M\delta^{\text{odd}}_{p_5,p_5'} = i\tilde{\Pi}_M\left(\frac{1}{2L}\right)^2 \sum_{q_5,q_5'} E_{p_5,q_5}E_{p_5',q_5'}\delta^{\text{odd}}_{q_5,q_5'}. \tag{4.29}$$

We can identify $Z_H \equiv \tilde{\Pi}'(Q^2 = 0)$, and $\delta M^2 \equiv -\tilde{\Pi}(Q^2 = 0)$. The mass insertion diagrams need to be Fourier transformed back into position space. We use the identities

$$\sum_{p_5 \text{ odd}} e^{ip_5 z} = L \sum_N (-1)^N \delta(z - NL)$$
$$\sum_{p_5 \text{ even}} e^{ip_5 z} = L \sum_N \delta(z - NL) \tag{4.30}$$

where the sum over N spans all integers. The Fourier transform thus corresponds to opposite sign δ-functions on the two branes, $\delta^{\text{odd}}_{q_5,q_5'} \to \frac{1}{2}\left[\delta(z) - \delta(z-L)\right]$. The brane localized mass terms are then $m_0^2 = -m_L^2 = -\tilde{\Pi}_M/2$. Finally, the four-point function can be expressed as

$$\begin{aligned} iV_4(0; p_5, p_5', p_5'', p_5''') &= i\frac{\tilde{V}_4}{8} \sum_{\pm} \delta_{0,p_5 \pm p_5' \pm p_5'' \pm p_5'''} \\ &= i\left(\frac{1}{2L}\right)^4 \sum_{q_5,q_5',q_5'',q_5'''} E_{p_5,q_5} E_{p_5',q_5'} E_{p_5'',q_5''} E_{p_5''',q_5'''} \tilde{V}_4 \; \delta_{0,q_5+q_5'+q_5''+q_5'''}. \end{aligned} \tag{4.31}$$

and we make the identification $\tilde{V}_4 = \frac{\lambda}{\Lambda_0}$.

In summary, the effective action can be expressed as a function of the UV parameters as in Eq. (4.27) with coefficients given by

$$\begin{aligned} Z_H &= \frac{N_c g^2}{10\pi^3} \frac{\Lambda}{\Lambda_0} \\ \delta M^2 &= -\frac{N_c g^2}{18\pi^3} \frac{\Lambda^3}{\Lambda_0} \\ \lambda &= \frac{N_c g^4}{3\pi^3} \frac{\Lambda}{\Lambda_0} \\ m_0^2 = -m_L^2 &= \frac{N_c g^2}{6\pi^3} \frac{\Lambda}{\Lambda_0} (m_R - m_L). \end{aligned} \tag{4.32}$$

We now associate the regulator cutoff Λ with the physical scale Λ_0. By defining the coupling constants such that they are dimensionless, with the physical scale explicitly appearing in the interaction terms, the quantum corrections (with the exception of the bulk mass term) are all seen to be independent of the scale Λ_0.

It is interesting that the scalar mass^2 receives brane localized contributions of opposite sign on either brane. This is a severe violation of KK parity. If this parity were preserved, the two brane localized terms are expected to be identical. However, the fermion mass terms explicitly violate KK parity. Quantum effects transmit this breaking of KK parity to the scalar sector in the form of these linear divergences.

These opposite sign, one loop, brane localized terms vanish, however, when the fermion masses are taken to be identical. In this scenario, for positive bulk masses, the LH zero mode is localized on the $z = 0$ brane, whereas the RH zero mode is localized on the $z = L$ brane. If the masses are equal, then the profiles are mirror images of each other, and an "accidental" approximate KK parity is introduced.

We now choose a convenient normalization for the 5D fields. We choose a canonical 5D scalar kinetic term, obtained by redefining $H \to H/\sqrt{Z_H}$,

$$\begin{aligned} S = \int d^4x \int_{-L}^{L} dz \Bigg[&\bar{\Psi}_L \left(i\partial\!\!\!/ - M_L(z) \right) \Psi_L + \bar{\Psi}_R \left(i\partial\!\!\!/ - M_R(z) \right) t_R + \frac{\tilde{g}}{\sqrt{\Lambda_0}} H \bar{\Psi}_L \Psi_R + \text{h.c.} \\ &+ \partial_M H \partial^M H^\dagger - \tilde{m}^2 |H|^2 - \frac{\tilde{\lambda}}{4\Lambda_0} |H|^4 \Bigg] - \int d^4x \left[\tilde{m}_0^2 \left. |H|^2 \right|_{z=0} + \tilde{m}_L^2 \left. |H|^2 \right|_{z=L} \right]. \end{aligned} \tag{4.33}$$

The terms in this 5D effective theory are

$$\begin{aligned} \tilde{g}^2 &= \frac{10\pi^3}{N_c} \\ \tilde{m}^2 &= \left(\frac{10\pi^3}{N_c g^2} - \frac{5}{9} \right) \Lambda_0^2 \\ \tilde{\lambda} &= \frac{100\pi^3}{3N_c} \\ \tilde{m}_0^2 = -\tilde{m}_L^2 &= \frac{5}{3}(m_R - m_L). \end{aligned} \tag{4.34}$$

Above, we have assumed $\Lambda \gg \mu$, where Λ is the scale that our original Lagrangian with the four-fermion operator was defined, and μ is the low scale at which we evaluate our 5D effective action.

There are also finite non-local contributions that arise from quantum corrections. We have neglected these, as they are typically sub-dominant, and do not have an interpretation as terms which are local in the extra dimensional coordinate.

We note that there are no brane localized quadratic divergences at one loop. Such terms might have been expected from considerations of the field content. In the fermion bubble approximation, brane localized terms arise only from diagrams with insertions of the 5D fermion mass, whose profile explicitly violates translation invariance.

In the presence of fermion bulk masses, the conditions under which the chiral symmetry of the low-energy theory is broken are modified. In the absence of the boundary terms, the scalar bound states condense for $g^2 > 18\pi^3/N_c$. However, the brane localized mass terms can drive condensation as well. In the next section we explore the conditions for generation of a chiral symmetry breaking condensate, and the resulting spectrum of the theory.

4.5 Vacuum Solution and Mass Spectrum

We have now shown that the low-energy effective theory is one with an additional 5D composite scalar degree of freedom. The equations of motion and the boundary conditions for this scalar field can be derived from the effective action that we have calculated. These determine the spectrum of the theory.

At the high scale, the 5D scalar Higgs field is equivalent to the fermion bilinear $H(z,x) = \bar{\psi}_L(z,x)\psi_R(z,x)$. With the fermionic orbifold assignments we have made, the orbifold parity transformation of the composite field is

$$H(-z) = \bar{\psi}_L(-z)\psi_R(-z) = (-\bar{\psi}_L(-z)\gamma^5)(-\gamma^5\psi_R(-z)) = \bar{\psi}_L(z)\psi_R(z) = H(z). \tag{4.35}$$

The scalar field is thus orbifold even, which means that when deriving the equation of motion for H, we cannot require that the variation itself vanish on the branes. Rather, the Higgs field is sensitive to the brane localized mass terms.

In this model, chiral symmetry breaking can occur in one of two ways. First, the coupling constant associated with the four-fermion operator may be sufficiently large that the bulk mass term is driven negative, destabilizing the origin as a vacuum solution. The bulk quartic coupling then sets the value for the scalar vacuum expectation value.

The other possibility is that the scalar bulk mass^2 remains positive, but a negative brane localized mass term pushes the field value away from the origin. In this case, it is still the bulk quartic coupling that stabilizes the vacuum field solution away from the origin, since we have shown that no brane localized quartic coupling is induced.

The second solution is more interesting, as it distinguishes the behavior of the compact 5D model from the non-compact one. Unlike the scalar bulk mass, the brane localized terms are sensitive to the values of the fermion bulk mass terms (and thus the relative localization of the fermion zero modes). Whether chiral symmetry breaking occurs in the extra dimensional model is thus a function of the free parameters of the model.

We now consider solutions to the composite scalar equations of motion. In the bulk, the vacuum equation for $\langle H(z,x)\rangle \equiv v(z)/(2\sqrt{L})$ is given by:

$$v''(z) = \tilde{m}^2 v(z) + \frac{\tilde{\lambda}}{8\Lambda_0 L} v^3(z). \tag{4.36}$$

This differential equation can be solved in terms of a Jacobi elliptic function, $\mathrm{sc}(x|m)$. The expression for the vacuum expecation value (vev) is

$$v(z) = \sqrt{\frac{8\Lambda_0 L\kappa_-}{\tilde{\lambda}}}\,\mathrm{sc}\left(|z-z_0|\sqrt{\frac{\kappa_+}{2}}\,\middle|\, 1-\frac{\kappa_-}{\kappa_+}\right), \tag{4.37}$$

where we have introduced the dimensionless quantities $\kappa_\pm = \tilde{m}^2 \pm \sqrt{\tilde{m}^4 - \frac{\tilde{\lambda}\tilde{m}^2 v_0^2}{4\Lambda_0 L}}$. The quantities z_0 and v_0 are determined by imposing the boundary conditions. In order for the low-energy chiral symmetry to be broken, the vacuum energy for the scalar field must be minimized at a non-trivial value for v_0.

The only brane localized terms which survive in the large cutoff limit are scalar mass terms proportional to the difference in bulk fermion masses. These are shown in Eq. (4.34). These mass terms, $\tilde{m}_0^2$ and $\tilde{m}_L^2$, set the boundary conditions for the scalar vev equation:

$$\left.\frac{v'(z)}{v(z)}\right|_{z=0} = \frac{1}{2}\tilde{m}_0^2 \qquad \left.\frac{v'(z)}{v(z)}\right|_{z=L} = -\frac{1}{2}\tilde{m}_L^2. \tag{4.38}$$

We can analytically determine the phase boundary by expanding the solution about small v_0. The result is $v(z) \approx v_0 \sinh(|z-z_0|\tilde{m})$, and the boundary conditions are then:

$$\left.\frac{v'(z)}{v(z)}\right|_{z=0} = \tilde{m}\coth(|z_0|m) = \frac{5}{6}(m_R - m_L)$$
$$\left.\frac{v'(z)}{v(z)}\right|_{z=L} = \tilde{m}\coth(|L - z_0|m) = \frac{5}{6}(m_R - m_L). \tag{4.39}$$

These are satisfied for $z_0 \to -\infty$, and for $\tilde{m} = \frac{5}{6}(m_R - m_L)$. We can express this phase boundary in terms of the original four-Fermi coupling g, which determines $\tilde{m}$ in the low-energy theory. The critical coupling is found to be:

$$g^2_{\text{crit}} = \frac{18\pi^3}{N_c}\left[1 + \frac{5}{4}\frac{(m_R - m_L)^2}{\Lambda_0^2}\right]^{-1}. \tag{4.40}$$

We now scan the parameter space of the model. For these purposes, we presume that the fermions are the 5D analogs of the LH third generation doublet and the RH top quark. In this case, the scalar field then carries the $SU(2)_L \times U(1)_Y$ quantum numbers of a SM Higgs, and when H obtains a vev, the W and Z bosons become massive. We identify the region of parameter space in which we obtain the correct W-boson and top quark masses.

The W mass is well approximated by assuming a flat profile for the lightest W-boson mode, and convoluting the flat profile with the vev^2:

$$m_W^2 = \frac{g_2^2}{4}\left[\left(\frac{1}{2L}\right)\int_{-L}^{L} dz\ v(z)^2\right], \tag{4.41}$$

where g_2 is the $SU(2)_L$ gauge coupling of the SM. The top quark mass is approximated from the Yukawa interaction:

$$m_{\text{top}} = \frac{\tilde{g}}{\sqrt{\Lambda_0 L}}(N_R N_L L)\left[\frac{1}{2L}\int_{-L}^{L} dz v(z) e^{(m_R - m_L)|z|}\right] \tag{4.42}$$

where $N_{R(L)}$ are the normalization factors for the fermion zero mode profiles, $\Psi_L(z) = N_L e^{-m_L|z|}$, and $\Psi_R(z) = N_R e^{m_R|z|}$. Note that the W mass depends only the difference between the fermion bulk mass terms (through the effective Higgs potential), while the top quark mass has a quite different dependence arising from the fermion normalization parameters. The W and top quark masses are thus independently adjustable.

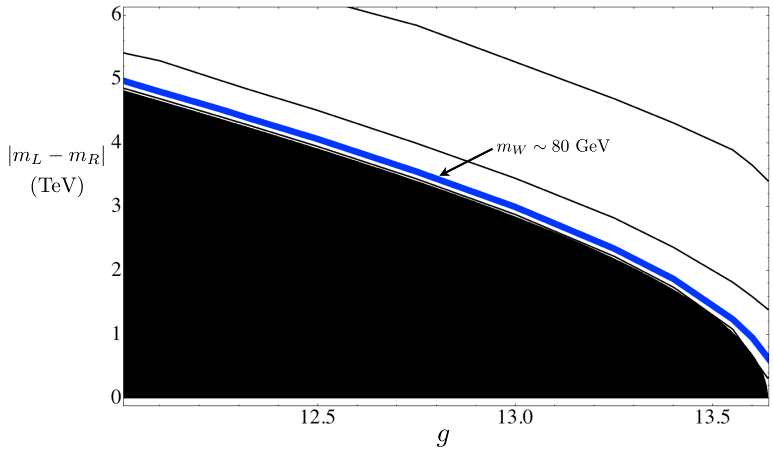


Figure 4.3: The phase boundary of the model is shown, as a function of $|m_L - m_R|$ and the four-fermion coupling g. The size of the extra dimension is $L = 1\ \mathrm{TeV}^{-1}$, and $N_c = 3$. Thin solid black lines indicate contours (moving outwards) of $m_W \sim 40,\ 160,\ 320$ GeV, while the thick blue line corresponds to $m_W \sim 80$ GeV.

The phase boundary is shown in Figure 4.3 along with contours of m_W as a function of the original four-fermion coupling g and the difference between the fermion bulk mass parameters $|m_R - m_L|$. We have set the other free parameters to $N_c = 3$, and $\Lambda_0 L = 10$. In Table 4.1, values of m_L, m_R and L which give the correct top and W mass are shown, along with the associated value for the Higgs mass. Additionally, we quote the value of $g^2/g^2_{\text{crit}} - 1$, a rough measure of the fine-tuning necessary in the four-fermion coupling to achieve the correct W-mass.

We see that for the choice $\Lambda_0 L = 10$, $N_c = 3$, the Higgs is very massive. In fact, it is above the perturbative unitarity bound. This can be alleviated by increasing $\Lambda_0 L$, although the fermion ladder approximation begins to break down as Λ_0 approaches the scale at which the 5D gauge interactions become strong (about $\Lambda L \sim 30$).

4.6 Conclusions

We have considered a compactified 5D version of a Nambu–Jona-Lasinio model. The model is studied by computing quantum corrections to a 5D Yukawa theory in which there are two species of fermions, each with a fermionic zero mode in the spectrum with opposite chiralities. The scalar field is interpreted as a bound state of the two

Table 4.1: Choices of the fermion bulk mass parameters that reproduce the SM values for m_W and m_{top}, and their associated predictions for the Higgs mass. In the third column, we give a rough measure of the fine-tuning necessary to achieve the weak scale from the 5D four-fermion interaction. All dimensionful parameters are given in units of TeV. We have set the other free parameters to $N_c = 3,\ \Lambda_0 L = 10$.

m_L	m_R	L^{-1}	$g^2/g^2_{\text{crit}} - 1$	m_{Higgs}
9.1	18.8	2	0.0035	1.4
9.2	17.1	2	0.0031	1.25
9.45	15.2	2	0.0025	1.1
10.	12.7	2	0.0016	0.85
4.5	9.5	1	0.014	1.4
4.6	8.7	1	0.012	1.3
4.7	7.7	1	0.010	1.1
5.0	6.5	1	0.006	0.85
2.25	5.0	0.5	0.06	1.4
2.3	4.5	0.5	0.053	1.3
2.25	5.4	0.5	0.045	1.1
2.4	3.4	0.5	0.030	0.9

fermion species. The classical 4D effective theory at low energies exhibits a chiral symmetry. Supplementation of the model by a 5D UV composite boundary condition renders the model equivalent at the high scale to one with a 5D bulk four-fermion operator. The quantum corrections to the low-energy Yukawa model are equivalent to a re-summation of fermion bubble diagrams in the fermion four-point function arising from the four-fermion interaction.

Both bulk and brane localized divergences are generated, although the brane localized divergences are softer than might have been expected. An accidental remnant of 5D translation invariance on the parent S_1 space survives, and protects against one-loop quadratically divergent contributions to the scalar mass^2 terms on the branes.

In the presence of fermion bulk mass terms which explicitly violate translation invariance, linear divergences are generated. Under certain conditions, when the four-fermion coupling exceeds a critical value, these brane localized terms destabilize the scalar vacuum, and drive spontaneous chiral symmetry breaking.

If a portion of the chiral symmetry is weakly gauged, it is expected that this symmetry will be spontaneously broken, as in top condensation models. We numerically studied such a model, showing that it is possible to realize simultaneously the correct top quark and W-boson masses. This can be seen as an explicit 5D realization of top seesaw models, a deconstructed version of which was studied in [91, 92]. The Higgs mass is generically quite large in these models due to the large quartic coupling, likely in conflict with perturbative unitarity and/or electroweak precision constraints. A more realistic model implemented in warped space may alleviate both of these tensions.

4.7 Appendix A: 5D Hard Cutoff

There are many ways in which to implement a hard cutoff in 5D theories, although most do not preserve 5D Lorentz invariance. For example, a common procedure is to write 5D propagators in mixed position/momentum space, where the propagators are functions of 4D momenta, and of the extra dimensional coordinate, z. It is not practical however, to implement a short distance cutoff in a manner which respects local 5D Lorentz invariance since the extra dimension has been singled out. Another common approach is to work in a KK-basis, and for each KK mode to integrate over a four-sphere in the 4D momenta. However, the region in full 5D momentum space that is integrated/summed over is not invariant under the 5D Lorentz group.

An ideal regularization procedure respects 5D Lorentz invariance in the UV, with sub-leading terms generated as finite consequences of non-local finite-volume effects. To obtain such a regulator, we recall that the Euler-Maclaurin formula allows one to express a sum over integers in terms of an integral and additional corrections:

$$\sum_{n=-\infty}^{\infty} f(n) = \int_{-\infty}^{\infty} dn f(n) + \lim_{a\to\infty}\left[\frac{f(a)+f(-a)}{2} + \sum_j \frac{B_{2j}}{(2j)!}\left(f^{(2j-1)}(a) - f^{(2j-1)}(-a)\right)\right] \tag{4.43}$$

Where the B coefficients are Bernoulli numbers. 5D loop integrals thus take the form

$$\begin{aligned}\frac{1}{2L}\sum_{k_5}\int \frac{d^4k}{(2\pi)^4} I(k,k_5) = \int \frac{d^5k}{(2\pi)^5} I(k,k_5) \\ + \frac{1}{2L}\lim_{k_5\to\infty}\left[\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left(I(k,k_5)+I(k,-k_5)\right) + \right. \\ \left. \sum_j \frac{B_{2j}}{(2j)!}\int \frac{d^4k}{(2\pi)^4}\left(I^{(2j-1)}(k,k_5) - I^{(2j-1)}(k,-k_5)\right)\right],\end{aligned} \tag{4.44}$$

where the derivatives are with respect to the second argument of the integrand. On the right hand side, to implement a hard cutoff, we wick rotate and then restrict the momentum integration/summation to the interior of a euclidean four-sphere: $K^2 \equiv k_0^2 + k_1^2 + k_2^2 + k_3^2 + k_5^2 \le \Lambda^2$. The final expression for any regulated 5D loop is

$$\begin{aligned}\frac{1}{2L}\sum_{k_5=-\Lambda}^{\Lambda}\int_{K^2\le\Lambda^2} \frac{d^4k_E}{(2\pi)^4} I(k_E,k_5) = \int_{K^2\le\Lambda^2} \frac{d^5k_E}{(2\pi)^5} I(k_E,k_5) + \\ \frac{1}{2L}\lim_{k_5\to\Lambda}\sum_j \frac{B_{2j}}{(2j)!}\frac{\partial^{(2j-1)}}{\partial k_5^{(2j-1)}}\left(\int_{k_E^2\le\Lambda^2-k_5^2} \frac{d^4k_E}{(2\pi)^4}\left(I(k_E,k_5)+I(k_E,-k_5)\right)\right).\end{aligned} \tag{4.45}$$

The contribution from the second term in Eq. (4.44) vanishes, since the region of integration in 4-momentum vanishes as $k_5 \to \Lambda$.

4.8 Appendix B: Brane Localized Terms

In [101], it was stated that brane localized terms are automatically generated in theories with compact extra dimensions. There are, however, many cases in which such terms are not generated at the one-loop level. In this appendix, we discuss these, and provide a symmetry argument for why such terms are protected. For the purposes of this discussion we use the orbifold language, in which the symmetry principle is most clear. The extra dimensional space thus begins as a circle, parametrized by angle θ, and is reduced to an interval by identifying points $\theta \leftrightarrow -\theta$.

The reason why most theories generate brane localized kinetic terms is that the orbifolding procedure explicitly violates 5D translation invariance. In the simplest case, fields can be assigned either even or odd parity under the orbifold identification, a manifestation of this breaking. Quantum effects will then transmit this breaking to other parts of the theory, creating brane localized kinetic terms, mass terms, and interactions.

To see this in action, consider a 5D scalar field with no 5D mass term. The propagator for a scalar which is even or odd under the orbifold assignment is given by [99]:

$$\Delta(p; p_5, p_5') = \frac{i}{2}\frac{1}{p^2 - p_5^2}\left\{\delta_{p_5,p_5'} \pm \delta_{p_5,-p_5'}\right\} \tag{4.46}$$

Now let us add gauge interactions and consider the gauge boson two-point function. There are two diagrams shown in Figure 4.4, although one creates a non-transverse structure which is completely canceled by a portion of the second. This is a consequence of gauge invariance.

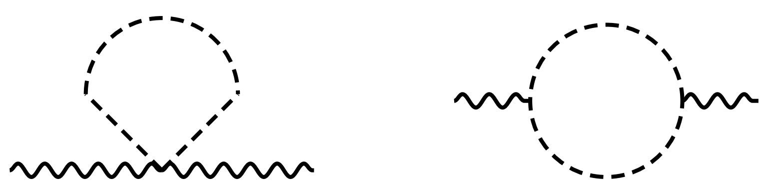


Figure 4.4: Gauge boson two-point diagrams.

The diagram contains the following numerator structure which arises from the two scalar propagators in the loop:

$$\sum_{k_5,k_5'} \frac{1}{D(k_5,k_5')}\left(\delta_{k_5,k_5'} \pm \delta_{k_5,-k_5'}\right)\left(\delta_{p_5+k_5,p_5'+k_5'} \pm \delta_{p_5+k_5,-p_5'-k_5'}\right) \rightarrow$$
$$\sum_{k_5} \frac{1}{D(k_5,k_5')}\left\{\delta_{p_5,p_5'} + \delta_{p_5,-p_5'} \pm \delta_{2k_5,p_5-p_5'} \pm \delta_{2k_5,p_5+p_5'}\right\} \tag{4.47}$$

The last two terms which do not conserve 5D momentum correspond to brane localized terms, and are divergent when the full expression is evaluated. However, note that they come with either positive or negative coefficient depending on whether the scalar has positive or negative orbifold parity. This shows that if a theory is constructed which has two such scalars, with opposite orbifold parity and equal gauge coupling, that the brane localized divergences will cancel. The reason for this is that the enhanced spectrum is identical to that of the theory before the orbifolding has taken place, and therefore has all the field content of the complete circle before orbifolding. 5D translation invariance on the full un-orbifolded circle protects against the generation of brane localized terms.

Now consider a 5D fermion on the same spacetime. If the bulk mass of the fermion vanishes, the fermion propagator (with the Dirac structure made explicit) is given by

$$\Delta(p;p_5,p_5') = \frac{i}{2}\frac{1}{\not{p}+i\gamma^5 p_5}\begin{pmatrix} \mathbf{1}_{2\times 2}\cdot\left(\delta_{p_5,p_5'} \pm \delta_{p_5,-p_5'}\right) & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \mathbf{1}_{2\times 2}\cdot\left(\delta_{p_5,p_5'} \mp \delta_{p_5,-p_5'}\right) \end{pmatrix}. \tag{4.48}$$

The fermion propagator contains two parts, one of which is orbifold even, and the other odd. These correspond to the right- and left-handed components of the 5D Dirac fermion. As with the case of two scalar fields, these degrees of freedom act together in diagrams, and can potentially conspire to make brane localized terms vanish. The question of whether or not brane localized terms are generated thus comes down to the interplay of these two parts of the fermion propagator in particular processes. In the two-point function we calculate for the Yukawa theory, only bulk renormalization takes place, and no brane terms are generated. In contrast, for the case of anomalies, the components of the propagator work together such that only brane localized divergences are generated [99].

Chapter 5

Conclusions

In this dissertation we have reviewed the SM. In the course of doing so we have reviewed EFT, naive dimensional analysis and its advanced cousin, the beta function, in chapter 2. We have also reviewed two of the outstanding theoretical problems of the SM - the fine-tuning of the Higgs mass and the Strong CP problem. In addition, we have discussed one of the modern tools of particle physicists, the AdS/CFT correspondence.

In chapter 3 we analyzed a general class of models that live on a slice of five dimensional AdS background, otherwise known as Randall-Sundrum Space. In particular, we have studied the gravitational fluctuations about the RS background metric. In addition, we studied the generic light DOF these models contain in their four dimensional effective actions. In particular, we analyzed a model that naturally provided an axion candidate for the Strong CP problem. We analyzed the collider and astrophysical phenomenology of these models.

In chapter 4 we analyzed a toy model for composite scalars. We used five dimensional generalization of the NJL model and worked in the fermion bubble approximation to calculate the effective action of the composite scalar. This model had the unique feature that the effective action did not follow the naive power counting rules for brane localized operators that were generated at low energy. Because of the composite nature of the scalar, this has potential implications for studying solutions to the fine-tuning of the Higgs mass. A more realistic model will require extending the work to a Randall Sundrum background in order to make full use of the AdS/CFT correspondence.

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