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DIFFUSION AND FRICTION-GARCH

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Abstract

In this paper we propose a friction model with a Bernoulli jump diffusion and a friction with GARCH to examine the exchange rates movements in Taiwan. The proposed models resolves the estimation problem associated with the stepwise movements of observed exchange rates. The specification maintains the desirable economic properties associated with movements in exchange rate returns and is empirically tractable. The AIC apparently favors the model based on Friction-GARCH model.

1. Introduction

The specification of a statistical distribution that accurately models the behavior of exchange rates continues to be a salient issue in financial economics (e.g., Baillie and McMahon 1989). With the introduction of arithmetic and geometric Brownian motion models, much attention has recently focused on the Poisson mixture of distributions (e.g., Jorion 1989; Kao and Wu 1990) and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process (e.g., Engle 1982; Bollerslev 1986), as appropriate specifications of exchange rates. Consistent with empirical evidence, these models yield lepthkuetic distributions.

For the Poisson mixture, one decomposes the total change in exchange rates into normal and abnormal components. The normal component is modeled as a lognormal diffusion process. The abnormal change is due to the receipt of any information that causes a more than marginal change in the exchange rates and is modeled as a Poisson process. The discontinuities in the sample path of exchange rates have important implications for the pricing of currency options. For example, Bodurtha and Courtadon (1987) report that the existence of jumps in exchange rates leads to significant deviations between the Black and Scholes (1973), and Merton (1976) option valuation models. Thus the jumps may be operationally significant for the currency option market. On the other hand, since asset price and foreign exchange markets show characteristics of time varying volatility, GARCH models of foreign exchange were modeled by Giovannini and Jorion (1987), McCurdy and Morgan (1987), and Engle and Bollerslev (1986), among others.

However, the currency system in Taiwan has some unique features and the movements of exchange rates are characterized by step functions over time. Therefore, the exchange rate's

behavior is not captured by the lognormal diffusion process, Poisson mixture process or GARCH models.

In this paper, we propose a friction model with a Bernoulli jump-diffusion and a friction model with a GARCH process to examine the exchange rates' movements in Taiwan. The proposed models resolve the estimation problem associated with the stepwise movements of observed exchange rates. This paper also attempts to estimate empirically currency bands or government limits on currency-rate trading ranges. One of the purposes is to establish some facts about exchange rate behavior in Taiwan and perhaps give some impetus to further refinements of target zone models.

The paper is organized as follows. Section 2 describes some empirical facts about Taiwanese intervention policy. Section 3 presents the friction model with a Bernoulli jump diffusion. Section 4 analyzes the friction model with a GARCH process. Section 5 discusses the data and empirical results. Conclusion is given in section 6. The maximum likelihood estimation for these two models is derived in Appendices A and B.

2. Description of the Taiwanese Intervention Policy

A brief review of the foreign exchange market in Taiwan suggests that Krugman's (1991) target zone model may not directly apply to Taiwan. Exchange rate bands in Taiwan have never been explicitly set in terms of the exchange rate itself, although there have been some forms of bands in terms of the rate of changes. The foreign exchange system in Taiwan was converted from a fixed rate system to a managed floating rate system in February 1979, and an operating rule was promulgated. Under the new rule exchange rates were to be limited to a very small margin on either side of a central rate that was set daily by a group of five banks acting together with the central bank. Instead of setting exchange rate

bands, the day-to-day fluctuation of the central rate was initially limited to 0.5 percent. Subsequently, the central bank withdrew participation from the rate setting and the limit on the day to day fluctuation was increased to 2.25 percent.

Starting in 1984, the year the U.S. Congress enacted the Trade and Tariff Act, a fast accumulating trade surplus and rising trade disputes with the United States prompted Taiwan to undertake a series of trade liberalization (Tsao 1992). Nevertheless, in accordance with the Omnibus Trade and Competitiveness Act of 1988, the U.S. Treasury Department submitted its trade report on October 1988 to Congress and accused Taiwan of exchange rate manipulations and capital flow controls. At the Treasury Department's request and to avoid trade retaliation, on April 3, 1989, Taiwan's central bank reduced interventionist activities and introduced a new foreign exchange system. The exchange rate for all NT dollar-U.S. dollar transactions to $30,000 and greater was then freely determined without any band or limitation. The exchange rate for small retail transactions under $30,000 are determined by rotating groups on nine foreign exchange banks based on prevailing free market rates. Banks can set their own rates based on this "reference rate." More volatile trading rates could be result in a wilder band for the "reference rate."

The control over capital flows was also relaxed at the same time. In 1988, an individual could remit within a year as much as $5 million out of the country without approval from the central bank and could only remit $50,000 capital into Taiwan. The motive of imposing restrictions on capital inflows was to prevent new Taiwan dollar from appreciating. The ceiling of capital inflow was raised to $1 million at the end of 1989. In 1990, Taiwan virtually became a well-liberalized financial market and distanced itself from the history. Figures 1 and 2 present the movements of the NT$/US$ rates from October 3, 1980 to December 30, 1991.

3.   A Friction Model with a Bernoulli Jump Diffusion

This section presents the stochastic processes under investigation as well as the maximum

likelihood estimation (MLE) procedure. We define x_t as the logarithm of the price relatives

$\log(p_t / p_{t-1})$, where p is the dollar price of the exchange rates, i.e., NT\$/US\$ rates. The assumption

that prices follow the diffusion process $dp(t) / p(t) = \alpha dt + \sigma dz$ implies that $x_t \sim N(\mu, \sigma^2)$ with

$\mu \equiv \alpha - \sigma^2 / 2$, where z is the standardized Wiener process. Discontinuities can be modeled by the

mixed jump-diffusion process $dp(t) / p(t) = \alpha dz + dq$ in which the Poisson process q is characterized

by a mean number of jumps occurring per unit time, λ, as well as a jump size Y, which is assumed

independently lognormally distributed, i.e., $\log Y \sim N(\lambda, \delta^2)$. This results in the following daily exchange

rates return, x_t, whose density is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} \phi(x; \mu + n\gamma, \ \sigma^2 + n\delta^2) \tag{1}$$

where $\phi(\bullet)$ is the normal density.

Ball and Torous (1983) suggest a Bernoulli jump-diffusion process to approximate the Poisson

jump-diffusion process. The distinguishing characteristic of the Bernoulli jump diffusion process is that

over a fixed period either no information impacts upon the exchange rates or, at most, one significant

information arrival occurs. If jumps in exchange rates correspond to the arrival of abnormal information,

by very definition the number of such information arrivals should not be very large. For practical

considerations, if t corresponds to one trading day, then on average there is no more than one abnormal

information arrival is to be expected. Furthermore, if returns were computed for finer time intervals, the

Bernoulli model would converge to the Poisson model. Then the daily exchange rates return can be modeled as a Bernoulli mixture of normal densities:

$$f(x) = (1-\lambda)\phi(x;\mu,\sigma^2) + \lambda\phi(x;\mu+\gamma,\sigma^2+\delta^2). \tag{2}$$

Yet, as indicated earlier, one of the major difficulties in Taiwan's exchange rates is those observed exchange rates usually do not change from one day to another. When there is no change in exchange rates, no information will be conveyed to the market. If each day is treated equally, as in most previous exchange rates' studies, then estimation and test will be biased. The economic interpretation of these biases is simple. Since the band restriction is part of the information set for the rational agents involved. The agents can exploit the information to predict future monetary interventions. For example, when the returns of exchange rates are too close to the upper limit, it can be expected to move downward, and the agents know how much room (the lower limit) there is for the downward movement. Therefore, the band restriction on the returns has to be imposed in the empirical estimation and testing. This may explain the lack of empirical studies obtained from exchange rates in Taiwan.

To deal with the phenomenon of exchange rate stickiness, we propose a friction model to estimate the Bernoulli mixture in (2). The friction model for exchange rates can be specified as follows:

$$x_t^* \sim f(x_t^*) = (1-\lambda)\phi(x_t^*;\mu,\sigma^2) + \lambda\phi(x_t^*;\mu+\gamma,\sigma^2+\delta^2) \tag{3}$$

and

$$
\begin{aligned}
x_t &= x_t^* - B_1 && \text{if } x_t^* < B_1 \\
&= 0 && \text{if } B_1 \le x_t^* \le B_2 \\
&= x_t^* - B_2 && \text{if } B_2 < x_t^*,
\end{aligned}
$$

where x_t is the observed return, x_t^* is the unobserved desired return of exchange rates which has a Bernoulli mixture of normal densities, $B_1 < 0$, representing a desired decrease in exchange rates, and

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$B_2 < 0$, representing a desired increase in exchange rates. The "desired" return is, of course, related to fundamentals (e.g., monetary variables). Since equation (3) contains a constant term μ, we assume that $B_2 < 0$ for the purpose of identification. The model in (3) is called a model of friction because it implies that returns of exchange rates will remain the same until a change in unobserved returns x_t^* overcome the friction, B_1 or B_2 (see Amemiya 1984, p.28; Maddala 1983:162-164). That is, observed returns do not change for either small negative or positive changes in desired returns x_t^*. As desired returns pass the friction point, x_t either increases or decreases, depending upon the type of stimulus.

The proposed friction model resolves the stickiness problem of observed returns in the parameter estimation; therefore it should provide a more accurate estimate for Taiwan's exchange rate dynamics. The six parameters to be estimated are λ, the intensity of the information arrival; σ^2, the instantaneous variance of the return on the exchange rates; γ, the mean of the logarithm of the jump; δ^2, the variance of the logarithm of the jump; and $\mu \equiv \alpha - \sigma^2 / 2$, where α is the instantaneous expected return. The parameters $\theta = (\mu, \sigma, \delta, \lambda, \gamma, B_1)$ are estimated by maximizing the likelihood function of the parameters given the observed returns x_t:

$$L = \prod_{t=1}^{T} \left[(1-\lambda)\frac{1}{\sigma_1}\phi\left(\frac{x_t + B_1 - \mu}{\sigma_1}\right) + \lambda\frac{1}{\sigma_2}\phi\left(\frac{x_t + B_1 - \mu - \gamma}{\sigma_2}\right) \right]^{d_{t1}}$$

$$\left[(1-\lambda)\left(\Phi\left(\frac{B_2 - \mu}{\sigma_1}\right) - \Phi\left(\frac{B_1 - \mu}{\sigma_1}\right) \right) + \lambda\left(\Phi\left(\frac{B_2 - \mu - \gamma}{\sigma_2}\right) - \Phi\left(\frac{B_1 - \mu - \gamma}{\sigma_2}\right) \right) \right]^{d_{t2}} \qquad (4)$$

$$\left[(1-\lambda)\frac{1}{\sigma_1}\phi\left(\frac{x_t + B_2 - \mu}{\sigma_1}\right) + \lambda\frac{1}{\sigma_2}\phi\left(\frac{x_t + B_2 - \mu - \gamma}{\sigma_2}\right) \right]^{d_{t3}}$$

where $\mu \equiv \alpha - \sigma^2 / 2$, $\sigma_1 = \sigma$ and $\sigma^2 = (\sigma^2 + \delta^2)^{1/2}$. The likelihood function is derived in the Appendix

A. The likelihood function includes the lognormal diffusion process as a special case. Therefore it can

be used in a likelihood ratio test $\Lambda = \dfrac{\sup\limits_{\theta \in \Theta_0} L(\theta, x)}{\sup\limits_{\theta \in \Theta} L(\theta, x)}$, where the likelihood functions have been maximized

over the parameter space $\theta \in \Theta_0 \subset \Theta$ under the null hypothesis $\lambda = 0$, and over the parameter space

$\theta \in \Theta$. Under the null hypothesis, the statistic $-2\log\Lambda$ has a chi-square distribution with degree of

freedom equal to the difference of the number of parameters between the two models.

4.   A Friction Model with GARCH

This section presents the friction model with a GARCH process. The assumption that prices

follow the diffusion process $dp(t) / p(t) = \alpha dt + \sigma(t)dz$ implies that $x_t \sim N(\mu, \sigma_t^2)$ with

$\mu \equiv \alpha - \sigma_t^2 / 2$, where z is the standardized Wiener process and the conditional variance, σ_t^2, is defined

as

$$\log \sigma_t^2 = \beta_0 + \beta_1 \log x_{t-1}^2 + \beta_2 \log \sigma_{t-1}^2 \tag{5}$$

The conditional variance given by equation (5) is a function of last period's observed return, x_t,

and the conditional variance of returns in period t-1, σ_{t-1}^2. This type of conditional Heteroskedasticity

has some intuitive appeal since it does not depend on some arbitrary exogenous variables. When

$\beta_1 = \beta_2 = 0$, we have conditional homoskedasticity.

Again, the friction model for exchange rates can be specified as follows:

$$x_t^* \sim \phi(x_t^*; \mu, \sigma_t^2)$$

and

$$x_t = x_t^* - B_1 \qquad\qquad \text{if } x_t^* < B_1$$
$$\quad = 0 \qquad\qquad\qquad \text{if } B_1 \le x_t^* \le B_2$$
$$\quad = x_t^* - B_2 \qquad\qquad \text{if } B_2 < x_t^*,$$

The parameters $\theta = (\mu, \beta_0, \beta_1, \beta_2, B_1)$ are estimated by maximizing the likelihood function of the

parameters, given the observed returns x_t:

$$L = \prod_{t=1}^{T} \left[\frac{1}{\sigma_1} \phi\left(\frac{x_t + B_1 - \mu}{\sigma_t} \right) \right]^{d_{t1}} \left[\Phi\left(\frac{B_2 - \mu}{\sigma_t} \right) - \Phi\left(\frac{B_1 - \mu}{\sigma_t} \right) \right]^{d_{t2}} \left[\frac{1}{\sigma_t} \phi\left(\frac{x_t + B_2 - \mu}{\sigma_t} \right) \right]^{d_{t3}} \qquad (6)$$

with

$$\log \sigma_t^2 \equiv \beta_0 + \beta_1 \log x_{t-1}^2 + \beta_2 \log \sigma_{t-1}^2 \qquad\qquad (7)$$

where $\mu \equiv \alpha - \sigma^2 / 2$. The likelihood function is derived in the Appendix B.

5.   Data and Empirical Results

The data consists of daily spot rates of Taiwan's exchange rates (in terms of United States

dollars, i.e., NT\$/US\$) from the *Wall Street Journal* with 2,260 observations spanning the period

October 3, 1980 to December 30, 1991. Table 1 reports the MLE of the parameters of the Friction-

lognormal diffusion, Friction-Bernoulli jump diffusion, and Friction-GARCH process. Asymptotic

standard errors are included in the parentheses.

For the Friction-lognormal process, the mean parameter, μ, the friction value, B_1, and the

standard deviation, σ, are all significantly different from zero at the one percent level. For the Friction-

Bernoulli jump-diffusion process, there are six parameters, $(\mu, \sigma, \delta, \lambda, \gamma, B_1)$ are reported. The estimate

of mean number of jumps, λ, corresponding to the arrivals of abnormal information is also significant at

the one percent level, suggesting the existence of infrequent discrete movements. For the diffusion part,

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the estimates of the mean and standard deviation (μ and σ) are significant at 1 percent level. For the

jump component, the estimate of the standard deviation δ and the mean γ are significantly different

from zero. The log-likelihood functions are reported in the Table 1. Finally, the estimates of B_1 are -

18.533 x 10^3 and -24.101 x 10^3 for Friction-lognormal and Friction-Bernoulli jump, respectively. That

is, after the jump component is factored in, the friction value B_1, decreases about 30 percent. This

indicates that the steps of the observed returns are captured partially by the jump components. Results

suggest that the observed exchange rate returns would not change if their "desired" returns are larger

than -18.533 x 10^3 for Friction-lognormal and -24.101 x 10^3 for Friction-Bernoulli jump.

Table 1 also provides a likelihood ratio test for the presence of jumps in the Taiwan's exchange

rate returns. Under the null hypothesis that exchange rate returns are consistent with a Friction

lognormal diffusion process without the Bernoulli jump structure, -2logΛ is asymptotically distributed χ_3^2

with three degrees of freedom. From Table 1, the -2logΛ of 180.058 amounts to a very strong

rejection of the Friction pure diffusion process. The large -2logΛ indicates that the null hypothesis of no

jumps is rejected for Taiwan's exchange rates.

For the Friction-GARCH process, five parameters $(\mu, \beta_0, \beta_1, \beta_2, B_1)$ are reported. The

estimates of σ, are significant at the one percent level, suggesting the existence of the GARCH effect.

The log-likelihood functions are reported in the Table 1. Finally, the estimate of B_1 is -11.252 x 10^3

for the Friction-GARCH process.

Table 1 also provides a likelihood ratio test for the presence of the GARCH effects in Taiwan's

exchange rates returns. Under the null hypothesis exchange rate returns are consistent with the Friction-

lognormal diffusion process without the GARCH structure, -2logΛ is asymptotically distributed χ_2^2 with

two degrees of freedom. From Table 1, the -2logΛ of 706.906 also amounts to a very strong rejection

of the Friction pure diffusion process. The large -2logΛ indicates that the null hypothesis of no GARCH is rejected for Taiwan's exchange rates.

We have presented estimates from the Friction-Poisson jump and the Friction-GARCH. Which model performs better ? We propose to use Akaike's information criterion (AIC) (see Amemiya 1981:1505-7). The idea is to choose the model for which AIC is smallest. In the case of identical sample information for the models to be compared, Akaike's criterion is given by

$$AIC = -\log L + K$$

where log$L$ is the log likelihood function evaluated at maximum likelihood estimates, and K is the number of estimated parameters. Table 1 presents AIC values for the three models. For Friction-lognormal, $K = 3$, for Friction-jump $K = 6$, and for Friction-GARCH $K = 5$. From the Table 1, the AIC apparently favors the model based on Friction-GARCH model.

6.   Conclusion

The movements of Taiwan's exchange rates are characterized by the stepwise adjustments and clusters of fluctuations. Neither a floating system or EMS can properly depict Taiwan's exchange rate behaviors. The Krugman's types of target zone models are, therefore, not directly applicable.

This paper has put forth a Friction-Bernoulli jump diffusion model and a Friction GARCH model for Taiwan's exchange rates. The proposed models resolve the estimation problem associated with the stepwise movements of observed exchange rates. The specification maintains the desirable economic properties related to the movements in exchange rate returns. The empirical results are strongly supportive of the presence of jumps and GARCH effects. The AIC clearly shows that the Friction-GARCH model outperforms Friction-lognormal and Friction-jump models.

Appendix A
Maximum Likelihood Estimation for a Friction with Jumps

This Appendix summarizes the MLE used in the section 3. The likelihood function in (4) can be derived as follows.

Let $\quad$ $d_{t1} = 1$ if $x^* < B_1$

$\qquad = 0$ otherwise

$\qquad d_{t2} = 1$ if $B_1 \le x^* \le B_2$

$\qquad = 0$ otherwise

$\qquad d_{t3} = 1$ if $B_2 < x^*$

$\qquad = 0$ otherwise.

Then the probability density function (pdf) of observed returns x is

$$f(x_t) = \{P(d_{t1}=1)f(x_t^*|d_{t1}=1)\}^{d_{t1}} \{P(d_{t2}=1)\}^{d_{t2}} \{P(d_{t3}=1)f(x_t^*|d_{t3}=1)\}^{d_{t3}} \tag{A1}$$

Note that

$$P(d_{t1}=1) = P(x^* < B_1) = \int_{-\infty}^{B_1} f(x^*)dx^*$$
$$= (1-\lambda)\Phi(\frac{B_1 - \mu}{\sigma_1}) + \lambda\Phi(\frac{B_1 - \mu - \gamma}{\sigma_2}) \tag{A2}$$

$$f(x_t*|d_{t1}=1) = \frac{1}{P(d_{t1}=1)}[(1-\lambda)\frac{1}{\sigma_1}\phi(\frac{x_t + B_1 - \mu}{\sigma_1}) + \lambda\frac{1}{\sigma_2}\phi(\frac{x_t + B_1 - \mu - \gamma}{\sigma_2})] \tag{A3}$$

$$P(d_{t2}=1) = P(B_1 \le x^* \le B_2) = \int_{B_1}^{B_2} f(x^*)dx^*$$
$$= [(1-\lambda)(\Phi(\frac{B_2 - \mu}{\sigma_1}) - \Phi(\frac{B_1 - \mu}{\sigma_1})) + \lambda(\Phi(\frac{B_2 - \mu - \gamma}{\sigma_2}) - \Phi(\frac{B_1 - \mu - \gamma}{\sigma_2}))] \tag{A4}$$

$$P(d_{t3} = 1) = P(B_2 < x*) = \int_{B_2}^{\infty} f(x*)dx*$$

$$= (1-\lambda)[1 - \Phi(\frac{B_2 - \mu}{\sigma_1})] + \lambda[1 - \Phi(\frac{B_2 - \mu - \gamma}{\sigma_2})]$$

(A5)

$$f(x*|d_{t3} = 1) = \frac{1}{P(d_{t3} = 1)}[(1-\lambda)\frac{1}{\sigma_1}\phi(\frac{x_t + B_2 - \mu}{\sigma_1}) + \lambda\frac{1}{\sigma_2}\phi(\frac{x_t + B_2 - \mu - \gamma}{\sigma_2})]$$

(A6)

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and the cdf for a standard normal random variable, respectively.

Hence, the likelihood function for x_t is

$$L = \prod_{t=1}^{T}[(1-\lambda)\frac{1}{\sigma_1}\phi(\frac{x_t + B_1 - \mu}{\sigma_1}) + \lambda\frac{1}{\sigma_2}\phi(\frac{x_t + B_1 - \mu - \gamma}{\sigma_2})]^{d_{t1}}$$

$$[(1-\lambda)(\Phi(\frac{B_2 - \mu}{\sigma_1}) - \Phi(\frac{B_1 - \mu}{\sigma_1})) + \lambda(\Phi(\frac{B_2 - \mu - \gamma}{\sigma_2}) - \Phi(\frac{B_1 - \mu - \gamma}{\sigma_2}))]^{d_{t2}}$$

$$[(1-\lambda)\frac{1}{\sigma_1}\phi(\frac{x_t + B_2 - \mu}{\sigma_1}) + \lambda\frac{1}{\sigma_2}\phi(\frac{x_t + B_2 - \mu - \gamma}{\sigma_2})]^{d_{t3}}$$

(A7)

The MLE of $\theta = (\mu, \sigma, \delta, \lambda, \gamma, B_1)$ can be obtained by maximizing (A7).

Appendix B
Maximum Likelihood Estimation for a Friction with GARCH

This Appendix summarizes the MLE used in the section 3 The likelihood function in (6) can be derived as follows.

Let $d_{t1} = 1$ if $x^* < B_1$

$\quad\quad = 0$ otherwise

$d_{t2} = 1$ if $B_1 \leq x^* \leq B_2$

$\quad\quad = 0$ otherwise

$d_{t3} = 1$ if $B_2 < x^*$

$\quad\quad = 0$ otherwise.

Then the probability density function (pdf) of observed returns x is

$$f(x_t) = \{P(d_{t1} = 1)f(x_t^*|d_{t1} = 1)\}^{d_{t1}} \{P(d_{t2} = 1)\}^{d_{t2}} \{P(d_{t3} = 1)f(x_t^*|d_{t3} = 1)\}^{d_{t3}} \quad\quad (B1)$$

Note that

$$P(d_{t1} = 1) = P(x^* < B_1) = \int_{-\infty}^{B_1} f(x^*)dx^* = \Phi(\frac{B_1 - \mu}{\sigma_t}) \quad\quad (B2)$$

$$f(x_t^*|d_{t1} = 1) = \frac{1}{P(d_{t1} = 1)} [\frac{1}{\sigma_t}\phi(\frac{x_t + B_1 - \mu}{\sigma_t})] \quad\quad (B3)$$

$$P(d_{t2} = 1) = P(B_1 \leq x^* \leq B_2) = \int_{B_1}^{B_2} f(x^*)dx^* = \Phi(\frac{B_2 - \mu}{\sigma_t}) - \Phi(\frac{B_1 - \mu}{\sigma_t}) \quad\quad (B4)$$

$$P(d_{t3} = 1) = P(B_2 < x^*) = \int_{B_2}^{\infty} f(x^*)dx^* = 1 - \Phi(\frac{B_2 - \mu}{\sigma_t}) \quad\quad (B5)$$

$$f(x^*|d_{t3}) = \frac{1}{P(d_{t3} = 1)} [\frac{1}{\sigma_t}\phi(\frac{x_t + B_2 - \mu}{\sigma_t})] \quad\quad (B6)$$

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where $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and the cdf for a standard normal random variable, respectively.

Hence, the likelihood function for x_t is

$$L = \prod_{t=1}^{T} [\frac{1}{\sigma_t} \phi(\frac{x_t + B_1 - \mu}{\sigma_t})]^{d_{t1}} [\Phi(\frac{B_2 - \mu}{\sigma_t}) - \Phi(\frac{B_1 - \mu}{\sigma_t})]^{d_{t2}} [\frac{1}{\sigma_t} \phi(\frac{x_t + B_2 - \mu}{\sigma_t})]^{d_{t3}} \qquad \text{(B7)}$$

The MLE of $\theta = (\mu, \beta_0, \beta_1, \beta_2, B_1)$ can be obtained by maximizing (B7).

Table 1. MLE for the Friction Model with Bernoulli Jump Diffusion Process and GARCH for New Taiwan Dollar (NT$)/United States Dollar Rate[a]

Process Parameters					
$\mu \times 10^3$	$\sigma \times 10^3$	$B_1 \times 10^3$	λ	$\gamma \times 10^3$	$\delta \times 10^3$
Friction-Lognormal Diffusion Process					
-10.163*	10.579*	-18.533*			
(0.315)	(0.275)	(0.578)			
Log-likelihood 1628.686	AIC = -1625.686				
Friction-Bernoulli Jump Diffusion Process					
-24.827*	1.169*	-24.101*	0.868*	13.284*	12.529*
(1.574)	(0.315)	(1.660)	(0.040)	(1.063)	(0.701)
Log-likelihood 1772.715					
$-2\log \Lambda$ 180.058	AIC = -1766.715				

$\mu \times 10^3$	$B_1 \times 10^3$	β_0	$\beta_1 \times 10^2$	β_2
Friction-GARCH Process				
-0.687*	-11.252*	-0.360*	1.625*	0.933*
(0.319)	(0.498)	(0.141)	(0.553)	(0.024)
Log-likelihood 1982.139				
$-2\log \Lambda$ 706.906*	AIC=-1977.139			

[a] Asymptotic standard error in Parentheses. The likelihood ratio test, -2logΛ, tests the hypothesis of a pure Friction-diffusion process against a Friction-Bernoulli jump diffusion model, where

$$-2\log\Lambda \sim \chi_3^2 \text{ , with } \Lambda = \frac{\sup_{\theta \in \Theta_0} L(\theta, x)}{\sup_{\theta \in \Theta} L(\theta, x)}.$$

*Significant at the 1 percent level.

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