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M. Cristina Marchetti
Syracuse University

Valerii M. Vinokur
Argonne National Laboratory

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Low-Temperature Vortex Dynamics in Twinned Superconductors

M.Cristina Marchetti
Physics Department, Syracuse University, Syracuse, NY 13244

Valerii M. Vinokur
Argonne National Laboratory, Materials Science Division, Argonne, IL 60439

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We discuss the low-temperature dynamics of magnetic flux lines in samples with a family of parallel twin planes. A current applied along the twin planes drives flux motion in the direction transverse to the planes and acts like an electric field applied to *one-dimensional* carriers in disordered semiconductors. As in flux arrays with columnar pins, there is a regime where the dynamics is dominated by superkink excitations that correspond to Mott variable range hopping (VRH) of carriers. In one dimension, however, rare events, such as large regions void of twin planes, can impede VRH and dominate transport in samples that are sufficiently long in the direction of flux motion. In short samples rare regions can be responsible for mesoscopic effects.

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The static and dynamic properties of magnetic flux lines in copper-oxide superconductors are strongly affected by pinning by point, linear and planar disorder [1]. *Linear* correlated disorder in the form of columnar defects produced by bombardment of the crystal with energetic heavy ions has been shown to greatly enhance pinning in both yttrium and thallium-based compounds [2]. Twin boundaries are an example of *planar* disorder that is ubiquitous in superconducting $YBa_2Cu_3O_{7-x}$ and La_2CuO_4. Early decoration experiments indicated that the superconducting order parameter is suppressed at a twin boundary and the twin attracts the vortices [3]. Extensive investigations of twin-boundary pinning have been carried out by Kwok and coworkers [4,5]. These authors studied a variety of YBCO single crystal samples containing single families of parallel twins lying in planes spanned by the c axis, with spacings ranging from microns down to several hundred Angstroms. Transport experiments [4] show clear evidence of strong twin-boundary pinning even in the flux liquid phase for external fields along the c axis and driving currents in the ab plane and parallel to the plane of the twins (resulting in a Lorentz force normal to the twin planes). For this geometry the linear resistivity drops sharply at a characteristic temperature where twin-boundary pinning sets in [4]. In addition, there is a sharp downward dip in the resistivity as a function of angle as the external field is rotated through the $\hat{c}$ direction [5].

The static and dynamic properties of flux-line assemblies in the presence of a random array of *columnar pins* have been studied by mapping the physics of magnetic flux lines onto the problem of localization of quantum mechanical bosons in *two dimensions* [6]. At low temperatures there is a "Bose glass" phase, with flux lines localized on columnar pins, separated by a phase transition from an entangled flux liquid of delocalized lines. Transport in the Bose glass phase closely resembles the variable-range hopping (VRH) of electrons in disordered semiconductors in two dimensions [7].

In this paper we study flux-line dynamics in the presence of a *single family of parallel twin boundaries* lying in planes containing the c axis, for $\vec{H} \parallel \hat{c}$. We find that due to the quasi-one-dimensional nature of vortex transport a new regime can arise at low currents where flux-line dynamics is dominated by rare events, such as large regions voids of twin planes, that can be responsible for new mesoscopic phenomena.

At low temperatures, when the average vortex spacing $a_0 \approx (\phi_0/B)$ exceeds the average distance d between twin planes, all flux lines are localized by the pinning potential in the direction normal to the twins, progressively "filling" the planar pins as the field is increased. We only consider fields below $B_f \approx \phi_0/d^2$, where the flux lines fill the twins, and neglect any additional weak point disorder in the sample. We focus on flux motion transverse to the twin planes, which resembles the hopping of electrons in *one-dimensional* disordered superconductors. The current density in the usual hopping conductivity problem corresponds to the vortex velocity (i.e., voltage) and the electrical conductivity maps onto the resistivity from vortex motion. Transport in this low temperature and field regime is dominated by single-vortex dynamics. Flux motion is described in terms of thermally activated jumps of the vortices over the relevant pinning energy barriers $\mathcal{U}(L,J)$, yielding a resistivity $\rho = \mathcal{E}/J$, [1]

$$\rho(T) \approx \rho_0 e^{-\mathcal{U}/T}, \tag{1}$$

where ρ_0 is a characteristic flux-flow resistivity. In the following we determine the barrier heights $\mathcal{U}(L,J)$ (Table 1) corresponding to various transport regimes and the boundaries between the different regimes in the (L,J) plane.

Our starting point is the model-free energy discussed in [6] for flux lines defined by their trajectories $\{\mathbf{r}_i(z)\}$ as they traverse a sample of thickness L with a family of parallel twin boundaries parallel to the zx plane,

$$F_{(N)} = \sum_{i=1}^{N} F_i + \frac{1}{2}\sum_{i\neq j}\int_0^L V(|\mathbf{r}_i(z) - \mathbf{r}_j(z)|)dz, \qquad (2)$$

where $V(r)$ is the intervortex interaction [8] and F_i is the single-flux line free energy,

$$F_i = \int_0^L dz\left[\frac{\tilde{\epsilon}_1}{2}\Big(\frac{d\mathbf{r}_i(z)}{dz}\Big)^2 + V_D(y_i(z))\right]. \qquad (3)$$

Here $\tilde{\epsilon}_1$ is the tilt modulus ($\tilde{\epsilon}_1 = (M_\perp/M_z)\epsilon_1 < \epsilon_1$, with ϵ_1 the line tension) and $V_D(y)$ represents a z-and x-independent pinning potential [9]. At low temperatures we model $V_D(y)$ as an array of identical one-dimensional square potential wells of depth U_0, width $2b_0$ and average spacing $d >> b_0$, passing completely through the sample in the x and z directions and centered at uniformly distributed random positions along the y axis.

As discussed in [6] and [10], many relevant results regarding the statistical mechanics of flux lines can be obtained from elementary quantum mechanics by mapping the vortex trajectories onto the imaginary time path integrals of *two-dimensional* particles in a static random potential $V_D(y)$. In this mapping k_BT plays the role of $\hbar$, $\tilde{\epsilon}_1$ that of the mass of the fictitious quantum particle, L^{-1} that of the particle's temperature. If we assume that each flux line spends most of its time near the attractive twin planes, the dynamics of flux lines driven by a Lorentz force normal to the twin planes can be described by a tight-binding model for *one-dimensional* bosons, where the lattice sites are the positions of the twin planes. Flux lines can "tunnel" between twins i and j separated by a distance d_{ij} at a rate given by the tunneling matrix element $t_{ij} \sim 2U(T)e^{-E_{ij}/T}$, where $E_{ij} = \sqrt{2\tilde{\epsilon}_1 U(T)}d_{ij}$ is the energy of a "kink" configuration connecting the two pins and $U(T)$ is the effective binding free energy per unit length of a flux line trapped near a twin plane. At low temperature $U(T) \approx U_0$. Thermal fluctuations renormalize U_0 above a crossover temperature, as discussed in [10]. The modeling of the dynamics of *two-dimensional* bosons in terms of a tight binding model in *one dimension* can be understood as a result of vortex-vortex interactions which confine each flux line within a "cage" of radius $\sim a_0$ formed by its neighbors. Each line can be viewed as moving within a one-dimensional "channel" of width $\sim a_0$. Another effect of interactions is the energy cost for an additional flux line to be placed on an already filled twin boundary. This is incorporated into an energy cost V_0 for double occupancy of a site of the one-dimensional tight-binding lattice, which is estimated as $V_0 \approx 4\epsilon_0 d^2/a_0^2$ for $T < T^* d/b_0[\ln(a_0/2d)]^{-1}$ and $d << a_0 << \lambda_{ab}$.

TABLE I. Energy barriers determining the various contributions to the resistivity of Eq. 1, with $\alpha = g(\mu)dU(T)$.

Linear	Nonlinear
$\mathcal{U}_{rf} = UL = E_k(L/w_k)$	$\mathcal{U}_{hl} = E_k(J_1/J)$
$\mathcal{U}_{nnh} = E_k$	
$\mathcal{U}_{Mott} = E_k(L/\alpha w_k)^{1/2}$	$\mathcal{U}_{VRH} = E_k(J_1/\alpha J)^{1/2}$

We consider vortex transport in the presence of a driving current $\mathbf{J} \perp \mathbf{H}$ parallel to the twin planes, i.e., $\mathbf{J} = -J\hat{\mathbf{x}}$. The applied current exerts a Lorentz force per unit length on the vortices, $\mathbf{f}_L = \frac{\phi_0}{c}\hat{\mathbf{z}} \times \mathbf{J} = \hat{\mathbf{y}} f_L$ in the direction transverse to the twin planes. In the context of the analogy with boson quantum mechanics, this term represents a fictitious "electric field" $\mathbf{E} = \frac{1}{c}\hat{\mathbf{z}} \times \mathbf{J} = \hat{\mathbf{y}}J/c$ acting on particles with "charge" ϕ_0. At low temperatures the critical current can be obtained by equating the Lorentz force to U_0/b_0, where $b_0 \sim \xi_{ab}$, i.e., $J_c(0) \approx cU_0/\phi_0 b_0$. Renormalization of J_c by thermal fluctuations have been discussed in [6].

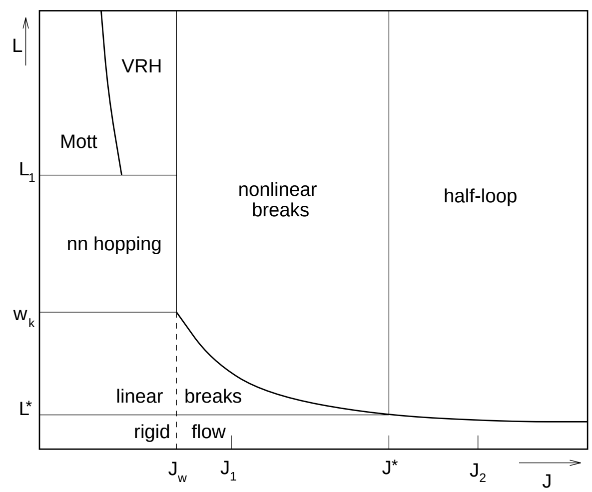

FIG. 1. The (L, J) phase diagram for $\alpha = g(\mu)dU < 1$.

If we neglect rare events, the energy barriers determining the resistivity of Eq. 1 can be estimated as the saddle-point free energy associated with the low-lying excitations from the ground state (where all the lines are localized on twins). The results are summarized in Table 1. The phase diagram in the (L, J) plane is controlled by the parameter $\alpha = g(\mu)dU(T)$, where $g(\mu)$ is the density of states for the most weakly bound flux lines and $\mu \approx \phi_0(H - H_{c1})/4\pi$ is the chemical potential which fixes the flux line density. The low-lying excitations that govern transport at low temperature have been discussed before in the context of flux arrays pinned by columnar defects [6]. The only difference here is in the exponent of the barrier associated with VRH, both in the linear and nonlinear regime. In the presence of linear defects $\mathcal{U}_{VRH} \sim (\tilde{J}/J)^{1/3}$, while for planar defect the corresponding exponent is 1/2, as shown in Table 1.

The boundaries of the (L,J) phase diagrams (Figs. 1 and 2) have not been discussed before in detail. The line $J=J_L(L)$ defines the boundary in the (L,J) plane that separates the regions of linear ($J<J_L(L)$) and nonlinear ($J>J_L(L)$) response, with $J_L=cE_k/(\phi_0 Ld)$ and $E_k=\sqrt{\tilde{\epsilon}_1 U(T)}d$ the energy of a kink connecting neighboring pins separated by d. In the thermodynamic limit $J_L\rightarrow 0$ and the IV characteristic is nonlinear at all currents. In samples of very small thickness L there is a linear resistivity at low currents due to the flow of flux-line segments of length L and typical transverse width $y_{rf}\approx dL/w_k$, with $w_k=E_k/U$ the width of a kink connecting pins separated by d. When $w_k<L<L_1$, where $L_1=E_k/\gamma$ is the length below which dispersion from tunneling and interactions can be neglected, transport occurs via the hopping of vortices between nearest neighbor (nn) pinning sites. Here γ is the width of the impurity band arising from level dispersion from tunneling and intervortex interaction. The region of the phase diagram dominated by nn hopping is only present if $L_1>w_k$, or $\gamma<U$ [11]. When $L>L_1$ the dispersion of energies between different pinning sites makes motion by nearest neighbor hopping energetically unfavorable. Tunneling occurs instead via the formation of "superkinks" that throw a vortex segment onto a spatially remote pin connecting states which optimize the tunneling probability. Tunneling via superkinks is the analogue of Mott variable-range hopping of electrons between localized states in semiconductors. The corresponding energy barriers $\mathcal{U}_{Mott}$ is given in Table 1.

For $J>J_L(L)$ the resistivity is nonlinear. For $J_1<J<J_c$, with $J_1=cU(T)/\phi_0 d$, flux motion occurs via thermally activated "half-loop" configurations of transverse width $y_{hl}\approx U/f_L$. For $J<J_1$ the size of the transverse displacement of the liberated vortex segment exceeds the average distance d between twin planes and transport occurs via VRH which generalizes the Mott mechanism to the nonlinear case. The characteristic current scales that governs VRH is $J_0=J_1/\alpha$. The VRH contribution to the resistivity dominates that from half loop only if if $\mathcal{U}_{VRH}<\mathcal{U}_{hl}$, or $J<J_2=J_1\alpha$. All current scales in Fig. 1 are much smaller than the pair breaking current $J_{pb}=4c\epsilon_0/(3\sqrt{3}\phi_0\xi_{ab})$. The Mott and the rigid flow regimes are separated by a horizontal line above which $\mathcal{U}_{Mott}<\mathcal{U}_{rf}$. Similarly, the condition $\mathcal{U}_{VRH}=\mathcal{U}_{hl}$ yields the vertical line separating the VRH and half-loop regions.

Transport in flux arrays with columnar pins is described by a phase diagrams similar to that of Fig. 1. The difference for planar disorder is that transport is one dimensional in this case and rare fluctuations in the spatial distribution of twins can impede VRH and dominate flux-line dynamics. The vortex line can encounter a region where no favorable twins are available at the distance of the optimal jump. The vortex will then remain trapped in this region for a long time and the resistivity can be greatly suppressed. Rare fluctuations can also occur in samples with columnar pins, but in that case they will dominate transport only at extremely small fields, when the number of rare regions exceeds the number of vortices.

At a given temperature and for applied currents below J_L, a vortex can jump from one twin plane to another at a distance y if the energy difference per unit length between the initial and final configuration is within a range $\Delta\epsilon\sim E_k y/Ld$. A trap is then a region of configuration space (y,ϵ) void of localized states within a spatial distance y and an energy band $\Delta\epsilon$ around the initial vortex state. A vortex that has entered such a trap or "break" will remain in the trap for a time $t_w\approx t_0\exp(2y/l_\perp)$, where $l_\perp=T/\sqrt{2\tilde{\epsilon}_1 U}$ is the localization length and t_0 a microscopic time scale. The probability of finding such a break is given by a Poisson distribution, $P(y)\approx P_0(y)\exp[-Ag(\mu)y\Delta\epsilon]$, where $P_0(y)$ is the concentration of localized states in the energy band $\Delta\epsilon$, $P_0(y)\approx 2Ag(\mu)\Delta\epsilon$ and $A\sim 1$ a numerical constant. The mean waiting time between jumps is

$$\overline{t_w}\approx\int_0^\infty dyP(y)t_0 e^{2y/l_\perp(T)}. \quad (4)$$

For $L>>(T/E_k)^2\alpha w_k=L^*$, the integral can be evaluated at the saddle point, corresponding to the situation where the mean waiting time is controlled by "optimal breaks" of transverse witdth $y_l^*\approx l_\perp L/L^*$, with the result, $\overline{t_w}\sim t_0\sqrt{L/L^*}e^{L/L^*}$. The optimal breaks correspond to the longest trapping time and are most effective at preventing flux motion. The inverse of the trapping time determines the characteristic rate of jumps, yielding a linear resistivity,

$$\rho_{bl}\approx\rho_0(T/\mathcal{U}_{Mott})e^{-(\mathcal{U}_{Mott}/T)^2}. \quad (5)$$

For currents above J_L the typical energy per unit length available to a flux line for jumping a distance y is $\Delta\epsilon\sim f_L y$. Again, for $J<<(J_1/\alpha)(E_k/T)^2=J^*$ the main contribution to the resistivity is from "optimal traps" of transverse size $y_b^*\approx l_\perp(J^*/J)$, with the result,

$$\rho_b\approx\rho_0(T/\mathcal{U}_{VRH})e^{-(\mathcal{U}_{VRH}/T)^2}. \quad (6)$$

The contribution to the resistivity from tunneling à la Mott (both in the linear and nonlinear regimes) always dominates that from hopping between rare optimal traps if both mechanisms of transport can occur. On the other hand, in one dimension if the sample is wide enough in the direction of flux-line motion to contain optimal traps, tunneling à la Mott simply cannot take place because flux lines cannot get around the traps. These rare traps with large waiting times will then control transport. If W is the sample width in the y direction, the condition for having optimal traps of width $y_{l,b}^*$ is $P(y_{l,b}^*)W>1$.

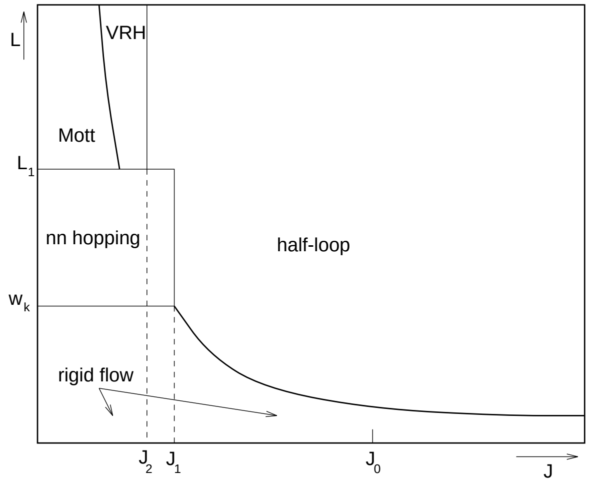

FIG. 2. The (L,J) phase diagram for $\alpha = g(\mu)dU < E_k/T$.

Optimal traps will be present only if $J > J_w = J^*/\ln(2W/l_\perp)$ for $J > J_L$ and if $L < L_w = L^* \ln(2W/l_\perp)$ for $J < J_L$.

A flux line can, however, escape a break by nucleating a half-loop, i.e., by tunneling directly into conduction band, if $y_{hl} > y_b^*$, or $\alpha < E_k/T$. The only effect of rare fluctuations in this case is of suppressing VRH for $J > J_w$, extending to lower currents the region where transport occurs via half-loop nucleation. For instance the right (high current) boundary of the VRH region in the phase diagram of Fig. 1 will be pushed down to J_w if $J_w < J_2$, or $\ln(2W/y_T) > (E_k/\alpha T)^2$. Similar considerations apply to the linear response. Only if $\alpha > E_k/T$, there will be a portion of the (J,L) plane where breaks dominate transport, as shown in Fig. 2. For YBCO, we estimate $E_k \sim 1K\mathring{A}^{-1}d$. Assuming $\alpha \sim U/\gamma$, the condition $\alpha > E_k/T$ can only be satisfied at low fields ($B < 1KG$ for $d \sim 200\mathring{A}$). The sample will contain optimal breaks if $W > 30\mathring{A}\exp(J^*/J)$, with $J^* \sim 4\times 10^5 \mathrm{Amp/cm}^2$ at $80K$.

If the sample is too short to contain optimal breaks, i.e., $WP(y \sim y_{l,b}^*) < 1$, the dynamics is controlled by the trap with the longest waiting time, $t(y_f) \sim \exp(y_f/l_\perp)$, with y_f determined by the condition $WP(y_f) \sim 1$. The corresponding resistivity is $\rho_W \approx \rho_0 e^{-y_f/l_\perp}$. In this case the relevant physical quantity is the logarithm of the resistivity,

$$\begin{aligned}\ln(\rho_W/\rho_0) &= -y_f/l_\perp \\ &\approx -\frac{\mathcal{U}_{VRH}}{T}\Big\{\ln\Big[\frac{2W}{l_\perp}\frac{T}{\mathcal{U}_{VRH}}\Big(\ln(2W/l_\perp)\Big)^{1/2}\Big]\Big\}^{1/2}. \quad (7)\end{aligned}$$

The leading dependence of Eq. 7 on current and temperature is the same as that of the VRH contribution. Equation (7) also contains, however, logarithmic terms that in sufficiently short samples will give a random spread of values of the resistivity from sample to sample. These effects have been discussed for semiconductors [13]. In this case a more relevant physical quantity rather than the resistivity itself is the distribution of the logarithms of the resistivity over different samples. The expression (7) determines the position of the maximum of this distribution.

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