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We use a hydrodynamics theory to discuss the response of a viscous flux-line liquid to an ac perturbation applied at the surface of the sample. The theory incorporates viscoelastic effects and describes the crossover between liquid-like and solid-like response of the vortex array as the frequency of the perturbation increases. A large viscosity from flux-line interactions and entanglement leads to viscous screening of surface fields. As a result, two frequency-dependent length scales are needed to describe the penetration of an ac field. For large viscosities the imaginary part of the ac permeability can exihibit, in addition to the well-know peak associated with flux diffusion across the sample, a new low-frequency peak corresponding to the transition from solid-like to liquid-like behavior.

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1. Introduction

The study of the response of high-temperature (HTC) superconductors in the mixed state to alternating magnetic fields and transport currents provides direct information on the dynamics of magnetic flux lines in these materials [1-4]. Such ac measurements can be performed by superimposing a small alternating magnetic field to the constant applied field, **H**. Alternatively, one can measure the change in resonance frequency of the superconducting crystal undergoing mechanical vibrations [5,6]. Due to the pinning of the flux lines to the crystal lattice, the periodic tilting of the superconductor is equivalent to the application of an ac field normal to **H**.

AC techniques are also used to determine the irreversibility line, $T_{irr}(H)$, in disordered HTC superconductors [7]. In magnetization measurements this is defined as the locus in the (H, T) phase diagram where the magnetization for field-cooled samples differs from the result obtained in samples cooled initially at zero field. This criterion is experimentally ambiguous since it depends on the rate at which data are taken. For this reason ac techniques, such as ac permeability, ac transport measurements, ac field penetration, mechanical oscillator [5,6], and others are often preferable. When cooling through the irreversibility line during an ac experiment the dissipation goes through a maximum. Experimentally the location of this maximum is very close to $T_{irr}(H)$ and is used as an alternative definition of T_{irr}. The location of $T_{irr}(H)$ has great technological as well as intellectual interset since it generally coincides with the line below which the resistivity becomes unmeasurably small.

Much theoretical work has been done to investigate the high-frequency electrodynamic response of a vortex solid pinned by weak impurity disorder [8,11]. When the perturbing field is step-like in time, the perturbation initially interacts with the flux line only at the surface of the sample. The field, transport current or mechanical tilt generate a surface current that flows in a layer of thickness of the order of the static penetration length. This surface current exerts a force on the vortices, deforming the vortex array. Flux-line interactions cause the surface deformation to propagate in the interior of the sample, while impurity pinning and friction tend to impede its propagation. Using standard descriptions of flux-line dynamics, one finds that for a periodic perturbation modulated at frequency ω, the penetration of the applied field is governed by a single complex penetration length $\lambda_{ac}(\omega)$ [1,8,9]. This length is closely related to the skin depth in normal metals and determines directly the surface impedance and the ac permeability of the material. The peak in the ac dissipation occurs when $\lambda_{ac}(\omega)$ is of the order of the sample size and is conventionally interpreted as associated with the thermally activated depinning of the flux lines [9,10]. More recently the peak in the ac dissipation in disordered superconductors has been interpreted as the signature of a continuous transition from a high temperature flux-line liquid to a low temperature vortex glass [12]. Good fits of ac measurements in single crystals and in thin films of YBCO have been obtained with the scaling relations of the phenomenlogical vortex glass model [2,3]. In contrast the dissipation peak from mechanical oscillator data in clean samples has been interpreted in the context of a melting of the flux-line crystal to a flux-line liquid [5,6].

On the other hand, in a large part of the region of the (H, T) phase diagram probed by both dc and ac transport experiments the flux-line array is in a liquid state. In this case it clearly no longer makes sense to invoke the traditional description of flux-line dynamics [13] in terms of the collective motion of crystalline flux bundles. The question then arises of how to incorporate collective effects in the description of flux-line dynamics in the liquid phase.

A popular phenomenological approach for describing the dynamics of flux-line liquids is the " thermally activated flux-flow " (TAFF) idea [9,11,14], which essentially assumes an ideal gas of disconnected flux elements moving in a tilted washboard potential. This method generalizes the usual Bardeen-Stephen (BS) flux flow model [15] to incorporate thermally-activated depinning of the flux lines. At temperatures higher than a typical pinning energy it reduces to the BS model. The TAFF phenomenology has been used to describe both dc [16] and ac response [9]. This picture takes, however, little account of intervortex interactions in the flux liquid and is not very useful for describing a system undergoing a phase transition, where collective effects can lead to very large or diverging correlation lengths near the transition.

As discussed elsewhere, a useful framework for the description of the long wavelength dynamical response of a flux-line liquid is hydrodynamics [17]. In this context intervortex interactions are naturally incorporated in a viscosity that increases and goes to infinity at the phase transition from a vortex liquid to a vortex crystal or glass. In fact the behavior of the viscosity at the transition serves as a signature of the order and nature of the transition itself. The hydrodynamic model naturally incorporates the intrinsic nonlocality of the electrodynamic response of a viscous flux liquid. Marchetti and Nelson used hydrodynamics to describe the response to dc perturbations [18,17]. They showed that a large viscosity, as one can have as the phase transition is approached from the liquid side, allows the effect of large scale spatial inhomogeneities, such as macroscopic twin boundaries, to propagate large distances into the flux liquid and slows down considerably the motion of the flux lines. More recently Huse and Majumdar [19] used the hydrodynamic model to describe multiterminal dc transport measurements in the vortex liquid regime of the cuprate superconductors. By calculating the dc response of a viscous flux liquid for realistic geometries including the effect of the sample boundaries, they demonstrated that the nonlocal resistivity arising from a finite flux liquid viscosity can explain the voltage patterns seen in experiments [20].

In this paper we use the hydrodynamic model to describe the response of a viscous flux-line liquid to an ac perturbation applied at the surface of the sample. A large viscosity can impede the penetration of a surface field, since the viscous flux liquid cannot quickly adjust to the external field change. This is reflected in a nonlocal relationship between the vortex liquid flow velocity, which determines the electric field from flux flow, and the applied fields and currents. The nonlocality arises from viscous forces, which are proportional to the second spatial derivatives of the vortex flow velocity, and incorporates the force that remote fluid elements exerts on each other via intervortex interactions and entanglement. As a consequence of this nonlocality, the amplitude of the penetrating ac field is given by the linear superposition of two exponentially decaying contributions with different frequency-dependent penetration lengths, λ_1 and λ_2. Recently Sonin et al. [21] showed that the penetration an ac surface field that generates a periodic tilt of the flux lines is also governed by two characteristic length scales. In this case the new penetration length is associated with an elastic tilt mode of the vortex array that can impede surface penetration and was neglected in previous theoretical studies. In a viscous flux liquid for surface perturbations that compress or shear the vortex array there is a viscous mode that incorporates collective effects from intervortex interactions and entanglement and is responsible for the additional surface screening. This is to be contrasted to conventional TAFF or flux flow models, where the resistivity is local and the penetrating field has the form of a single exponential controlled by the single length $\lambda_{ac}(\omega)$. The largest of the two penetration lengths, $\lambda_1(\omega)$, controls field penetration in the bulk of the material. At low frequency it simply describes Meissner screening. When the fluid viscosity is negligible $\lambda_1(\omega) \approx \lambda_{ac}(\omega)$. The additional penetration length, $\lambda_2(\omega)$, is associated with viscous screening that prevents the magnetic field from building up to its maximum value right at the surface. The maximum of the penetrating field is then found at a distance of the order of $|\lambda_2(\omega)|$ inside the sample. Viscous screening is effective in a thin surface layer of width

$|\lambda_2(\omega)|$ ($|\lambda_2(\omega)| << |\lambda_1(\omega)|$ at all frequencies) and leads to the nonlocality of the electric field from flux motion. In a dense vortex array at low frequency $|\lambda_2(\omega)|$ is of the order of the intervortex spacing.

In Section 2 we discuss the basic equations needed to describe the electrodynamic response of a type-II superconductor in the mixed state for the case when the flux array is in a liquid regime. These are the usual coupled Maxwell and London equations and a set of hydrodynamic equations for the viscous flux liquid. In Section 3 we present the solution for an ac field penetrating at the surface of a semiinfinite superconductor. The role of viscosity and its effect on macroscopic properties, such as the material's surface impedance are discussed. The solution for a superconducting slab is given in Appendix A.

2. Flux-Line Hydrodynamics

We consider a uniaxial type-II superconductor in a static field $\mathbf{H}$ applied along the c axis of the material, which is chosen as the z direction, i.e., $\mathbf{H} = H\hat{\mathbf{z}}$. The static field produces an array of flux lines that are on the average aligned with the field direction. The mean spacing a_0 of the flux lines in the xy plane is determined by the average equilibrium induction $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ that penetrates the sample, with $a_0 \simeq 1/\sqrt{n_0}$ and $n_0 = B_0/\phi_0$ the areal density of flux lines ($\phi_0 = hc/2e$ is the flux quantum). For $H >> H_{C1}$ and neglecting demagnetizing effects, $B_0 \simeq H$. For high κ superconductors, such as the HTC materials, there is a large part of the (H, T) phase diagram where $\xi_{ab} << a_0 << \lambda_{ab}$ (i.e., $H_{C1} << B << H_{C2}$), with λ_{ab} and ξ_{ab} the penetration and coherence lengths in the ab plane, respectively. Here we restrict ourselves to this region. In addition, we are interested in properties of the flux array on length scales larger than the mean intervortex spacing where one can describe the system in terms of a few hydrodynamic fields coarse-grained over several flux-line spacings.

The electrodynamics of a type-II superconductor is described by Maxwell's equations for the local fields $\mathbf{b}(\mathbf{r}, t)$ and $\mathbf{e}(\mathbf{r}, t)$,

$$\boldsymbol{\nabla} \times \mathbf{e} + \frac{1}{c}\partial_t \mathbf{b} = 0, \tag{2.1}$$

$$\boldsymbol{\nabla} \times \mathbf{b} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\partial_t \mathbf{e}, \tag{2.2}$$

with $\boldsymbol{\nabla} \cdot \mathbf{e} = 4\pi\rho$ and $\boldsymbol{\nabla} \cdot \mathbf{b} = 0$ [22]. These equations have to be supplemented with a constitutive relation for the current density, $\mathbf{j}$. Following Coffey and Clem [9], we use a two fluid model and write $\mathbf{j} = \mathbf{j}_n + \mathbf{j}_s$. The normal current density $\mathbf{j}_n$ is specified by Ohm's law, $\mathbf{j}_n = \sigma_n \mathbf{e}$, with σ_n the electrical conductivity of the normal metal. The constitutive equation for the supercurrent $\mathbf{j}_s$ is obtained by minimizing the Ginzburg-Landau free energy functional for an anisotropic superconductor in the presence of vortices. As customary, the anisotropy is incorporated in an effective mass tensor. In the London approximation, where the magnitude of the order parameter is assumed constant and only fluctuations in the phase θ of the superconducting order parameter are retained, we obtain

$$\mathbf{\Lambda} \cdot \mathbf{j}_s = -\frac{c}{4\pi}(\mathbf{A} - \phi_0 \boldsymbol{\nabla}\theta), \tag{2.3}$$

where $\mathbf{A}$ is the vector potential, with $\mathbf{b} = \boldsymbol{\nabla} \times \mathbf{A}$, and $\mathbf{\Lambda}$ is a diagonal tensor with components $\Lambda_{xx} = \Lambda_{yy} = \lambda_{ab}^2$ and $\Lambda_{zz} = \lambda_c^2$. Here λ_c is the penetration length along the c direction, with $\lambda_c/\lambda_{ab} = \sqrt{m_c/m_{ab}} = p >> 1$. Taking the curl of Eq. (2.3) and averaging over lengths large compared to the intervortex spacing, one obtains the London equation in the presence of vortices,

$$\boldsymbol{\nabla} \times \mathbf{\Lambda} \cdot \mathbf{j_s} = \frac{c}{4\pi}(-\mathbf{b} + \phi_0 \mathbf{T}). \tag{2.4}$$

Here $\mathbf{T} = \hat{\mathbf{z}}n + \boldsymbol{\tau}$, with $n(\mathbf{r}, t)$ and $\boldsymbol{\tau}(\mathbf{r}, t)$ the coarse-grained hydrodynamic density and tilt fields of the flux array, respectively. The microscopic density and tilt fields are given by

$$n_{mic}(\mathbf{r}, t) = \sum_{i=1}^{N} \delta(\mathbf{r}_\perp - \mathbf{r}_i(z, t)), \tag{2.5}$$

and

$$\boldsymbol{\tau}_{mic}(\mathbf{r}, t) = \sum_{i=1}^{N} \frac{\partial \mathbf{r}_i(z, t)}{\partial z} \delta(\mathbf{r}_\perp - \mathbf{r}_i(z, t)), \tag{2.6}$$

where $\mathbf{r}_i(z)$ is the position of the i-th vortex line in the xy plane as it wanders along the z direction, and $\mathbf{r} = (\mathbf{r}_\perp, z)$. The tilt field describes the local deviation of an element of flux-line liquid from the alignment with the z direction. The hydrodynamic fields are obtained by coarse graining the corresponding microscopic fields over several vortex spacings. The density and tilt fields are not independent, but are related by the constraint that flux-lines cannot start nor stop inside the medium. This requires

$$\partial_z n + \boldsymbol{\nabla}_\perp \cdot \boldsymbol{\tau} = 0. \tag{2.7}$$

Using Maxwell's equations one can eliminate the current from Eq. (2.4) in favor of the fields $\mathbf{b}$ and $\mathbf{e}$. This can be done by multiplying Eq. (2.2) with the tensor $\mathbf{\Lambda}$ from the left and then taking the curl, with the result,

$$\nabla\times\mathbf{\Lambda}\cdot(\nabla\times\mathbf{b})+\mathbf{b}=\phi_0\mathbf{T}-\frac{4\pi\sigma_n}{c^2}(\lambda_{ab}^2\partial_t\mathbf{b}-\lambda_c^2\nabla_\perp\times\hat{\mathbf{z}}e_z)-\frac{1}{c}\partial_t(\frac{\lambda_{ab}^2}{c}\partial_t\mathbf{b}-\lambda_c^2\nabla_\perp\times\hat{\mathbf{z}}e_z). \quad (2.8)$$

The second term on the right hand side of Eq. (2.8) comes from the normal part of the current density. The last term arises from the displacement current and is negligible for all frequency $\omega << \frac{c}{\lambda} \approx 10^{15}$Hz. Since we are interested in experiments carried out at frequencies no higher than microwave, we will neglect this term here, even though it can be easily incorporated in the calculation.

To describe the response of the superconductor to a perturbing field $\delta\mathbf{H}_a$ applied at the surface of the sample, one needs to solve Eq. (2.8) with appropriate boundary conditions on the field. The local field $\mathbf{B}$ is, however, coupled to the vortex distribution through the right hand side of Eq. (2.8) and the vortices in turn move in response to changes in the field, as discussed for instance by Coffey and Clem [9]. Equation (2.8) has to be supplemented with equations describing the vortex dynamics. Here is where our work differs from that of other authors. We assume the flux-line array is in a liquid state and describe its dynamics through the set of hydrodynamic equations for the density and tilt field discussed elsewhere. The hydrodynamic equations consists of a continuity equation for the density,

$$\partial_t n + \nabla_\perp \cdot \mathbf{j}_v = 0, \quad (2.9)$$

with $\mathbf{j}_v = n\mathbf{v}$, and a continuity equation for the tangent field,

$$\partial_t\tau_\alpha + \partial_\beta j^\tau_{\alpha\beta} = \partial_z j_{v\alpha}, \quad (2.10)$$

where $j^\tau_{\alpha\beta}$ are the components of the tangent flux tensor. This is a 2×2 antisymmetric tensor and has therefore only one independent component, $j^\tau_{xy} = \epsilon_{\alpha\beta}j^\tau_{\alpha\beta} = -j^\tau_{yx}$. At equilibrium $n = n_0$ and the tangent field vanishes, $\boldsymbol{\tau}_0 = 0$, since the flux lines are on the average aligned with the applied field. In the following we will only discuss the linear response of the flux liquid to external perturbations of small amplitude [23]. The hydrodynamic equations can then be linearized in the deviations of the hydrodynamic fields from their equilibrium values, $\delta n = n - n_0$ and $\delta\boldsymbol{\tau} = \boldsymbol{\tau}$, with $\mathbf{j}_v \approx n_0\mathbf{v}$,

$$\partial_t\delta n + n_0\nabla_\perp \cdot \mathbf{v} = 0, \quad (2.11)$$

$$\partial_t \tau_\alpha + \partial_\beta j^\tau_{\alpha\beta} = n_0 \partial_z v_\alpha. \tag{2.12}$$

As discussed in [17], these equations have to be closed with constitutive equations for the fluxes. Neglecting the Hall current, and keeping only terms linear in the fluctuations from equilibrium, the constitutive equations for $\mathbf{j}_v$ and j^τ_{xy} are given by,

$$-\gamma \mathbf{v} + \eta_s \nabla_\perp^2 \mathbf{v} + \eta_b \boldsymbol{\nabla}_\perp (\boldsymbol{\nabla}_\perp \cdot \mathbf{v}) + \eta_z \partial_z^2 \mathbf{v} - \frac{1}{c} \mathbf{B}_0 \times \mathbf{j} = 0, \tag{2.13}$$

$$-\gamma_\tau j^\tau_{xy} + \eta_\tau \nabla_\perp^2 j^\tau_{xy} + \eta_{\tau z} \partial_z^2 j^\tau_{xy} + \frac{n_0}{c} \mathbf{B}_0 \cdot \mathbf{j} = 0, \tag{2.14}$$

where $\mathbf{j} = (c/4\pi)\boldsymbol{\nabla} \times \delta\mathbf{B}$, with $\delta\mathbf{B} = \mathbf{B} - \mathbf{B}_0$. The first terms in both Eqs. (2.13) and (2.14) describe the drag on the flux lines arising from interaction with the crystal lattice, with γ and γ_τ friction coefficients per unit volume. The contribution to γ from interaction of the normal core electrons with the underlying crystal lattice can be approximated by the Bardeen-Stephen coefficient $\gamma_{BS}(T, H) = n_0 \pi \hbar^2 \sigma_n / 2e^2 \xi_{ab}^2$[15]. Weak point pinning centers can be approximately incorporated in a renormalized friction by assuming $\gamma \simeq \gamma_{BS} e^{U_p/k_B T}$, where U_p is a typical pinning energy, as done in thermally activated flux flow (TAFF) models [14]. The friction coefficient γ_τ can be related to the relaxation rate of an overdamped helicon and we estimate $\gamma_\tau \approx \gamma$. The next three terms in Eq. (2.13) and the next two terms in Eq. (2.14) describe the viscous drag in the flux-line liquid from intervortex interactions and entanglement. These effects are incorporated in the viscosity coefficients: η_s, η_b and η_τ, denoting the shear and bulk viscosity coefficients of an anisotropic liquid, and the "tilt" viscosities η_z and $\eta_{\tau z}$ associated with velocity gradients in the direction of the applied field. Finally, the last two terms on the left hand side of both Eq. (2.13) and Eq. (2.14) represent reversible forces on an element of flux-line liquid. To linear order in the hydrodynamic densities these can be written as

$$-\frac{1}{c} \mathbf{B}_0 \times \mathbf{j} = -\frac{1}{n_0} \int d\mathbf{r}' \left[\boldsymbol{\nabla}_\perp \hat{c}_L(\mathbf{r} - \mathbf{r}') \delta n(\mathbf{r}') - \partial_z \hat{c}^L_{44}(\mathbf{r} - \mathbf{r}') \boldsymbol{\tau}(\mathbf{r}') \right], \tag{2.15}$$

$$\frac{n_0}{c} \mathbf{B}_0 \cdot \mathbf{j} = \int d\mathbf{r}' \hat{c}^L_{44}(\mathbf{r} - \mathbf{r}') \hat{\mathbf{z}} \cdot [\boldsymbol{\nabla}_\perp{}' \times \boldsymbol{\tau}(\mathbf{r}')]. \tag{2.16}$$

Here $\hat{c}_L(\mathbf{r})$ and $\hat{c}^L_{44}(\mathbf{r})$ are real space elastic constants. Their Fourier transforms, $c_L(\mathbf{q})$ and $c^L_{44}(\mathbf{q})$, are the nonlocal compressional and tilt moduli, respectively [17]. The liquid tilt modulus can differ from that of an elastic flux array because of flux-line cutting. We note that the nonlocality of the elastic constants is automatically incorporated in our treatment.

When discussing the response of a viscous vortex liquid to an ac perturbation one needs to modify Eqs. (2.9) -(2.14) to include viscoelastic effects analogous to those present in entangled polymer melts [24]. These effects will be important in the vortex liquid in the region of temperatures just above the transition to a solid phase because the viscosity gets very large in this region. If velocity gradients are small, viscoelastic effects can be incorporated following the theory of polymer dynamics by modifying Eqs. (2.13) and (2.14) as

$$-\gamma v_\alpha(\mathbf{r},t) + \int_0^t dt' \int d\mathbf{r}' C_{\alpha i\beta j}(\mathbf{r}-\mathbf{r}',t-t')\partial_i'\partial_j' v_\beta(\mathbf{r}',t') - \frac{1}{c}[\mathbf{B}_0 \times \mathbf{j}(\mathbf{r},t)]_\alpha = 0, \quad (2.17)$$

and

$$-\gamma_\tau j_{xy}^\tau(\mathbf{r},t) + \int_0^t dt' \int d\mathbf{r}' C_{ij}^\tau(\mathbf{r}-\mathbf{r}',t-t')\partial_i'\partial_j' j_{xy}^\tau(\mathbf{r}',t') + \frac{n_0}{c}\mathbf{B}_0 \cdot \mathbf{j}(\mathbf{r},t) = 0, \quad (2.18)$$

where latin indices run over the three cartesian coordinates, $i = x, y, z$, while Greek indices only run over the two coordinates normal to the applied field, $\alpha = x, y$. The components of the tensors $C_{\alpha i\beta j}(\mathbf{r},t)$ and $C_{ij}^\tau(\mathbf{r},t)$ are the Green-Kubo correlation functions that determine the viscosity coefficients. It is convenient to introduce the Fourier and Laplace transforms of these correlation functions, defined as

$$\tilde{C}_{\alpha i\beta j}(\mathbf{q},\omega) = \int_0^\infty dt \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \tilde{C}_{\alpha i\beta j}(\mathbf{r},t) \quad (2.19)$$

for $Im(\omega) < 0$. There are five independent Green-Kubo correlation functions, defined by

$$\begin{aligned}\tilde{C}_{\alpha i\beta j}(\mathbf{q},\omega) =& \tilde{\eta}_s(\mathbf{q},\omega)\delta_{\alpha\beta}[\delta_{ij} - \delta_{iz}\delta_{jz}] + \tilde{\eta}_z(\mathbf{q},\omega)\delta_{\alpha\beta}\delta_{iz}\delta_{jz} \\ &+ \frac{1}{2}[\tilde{\eta}_l(\mathbf{q},\omega) - \tilde{\eta}_s(\mathbf{q},\omega)][\delta_{\alpha i}\delta_{\beta j} + \delta_{\alpha j}\delta_{\beta i}],\end{aligned} \quad (2.20)$$

and

$$\tilde{C}_{ij}^\tau(\mathbf{q},\omega) = \tilde{\eta}_\tau(\mathbf{q},\omega)\delta_{\alpha\beta}[\delta_{ij} - \delta_{iz}\delta_{jz}] + \tilde{\eta}_{\tau z}(\mathbf{q},\omega)\delta_{\alpha\beta}\delta_{iz}\delta_{jz}, \quad (2.21)$$

where $\tilde{\eta}(\mathbf{q},\omega)$ for $\nu = s, z, l, \tau, \tau z$ are the frequency- and wavevector-dependent viscosity coefficients. These determine the static viscosity coefficients according to,

$$\eta_\nu = \tilde{\eta}(\mathbf{q}=0,\omega=0), \quad (2.22)$$

for $\nu = s, z, l, \tau, \tau z$, where $\eta_l = \eta_s + \eta_b$ is the longitudinal viscosity.

In entangled polymers the relaxation of shear and compressional stresses described by the Green-Kubo correlation functions is nonexponential [25]. This reflects the existence of a wide distribution of relaxation times in the systems. The same feature is expected to occur in strongly interacting flux-line liquids. Here for simplicity we model the time decay of the Green-Kubo correlation functions as exponential according to the phenomenological Maxwell model of viscoelasticity [26]. The wave vector and frequency-dependent viscosities are then given by

$$\tilde{\eta}_\nu(\mathbf{q},\omega) = \frac{\eta_\nu}{1 + i\omega\tau_\nu(\mathbf{q})}. \tag{2.23}$$

The characteristic relaxation times τ_ν are chosen so that the viscoelastic model incorporates the essential feature that the fluid behaves like a viscous liquid on long times scales and as an elastic solid on short time scales. In other words for $\omega >> 1/\tau_\nu$ for all ν, the hydrodynamic equations should reduce to the equations of continuum elasticity for a flux-line array. This requires

$$\tau_s = \eta_s/G, \tag{2.24}$$

$$\tau_l = \eta_l/(c_{11}(\mathbf{q}) + G - c_L(\mathbf{q})), \tag{2.25}$$

$$\tau_z = \eta_z/(c_{44}(\mathbf{q}) - c_{44}^L(\mathbf{q})), \tag{2.26}$$

where c_{11}, G and c_{44} are the compressional, shear and tilt elastic moduli of a flux-line elastic medium. For an Abrikosov flux-line lattice $G \approx c_{66}$ and the wavevector dependence of the shear modulus c_{66} is always negligible. The compressional modulus is given by $c_{11}(\mathbf{q}) = c_L(\mathbf{q}) + c_{66}$ and $c_{11}(\mathbf{q}) \approx c_L(\mathbf{q})$ since $c_{66} << c_L(\mathbf{q})$ at all but the largest wavevector $q \sim k_{BZ}$ with $k_{BZ} = \sqrt{4\pi/a_0}$ the Brillouin Zone boundary, where $c_L(k_{BZ}) \sim c_{66}$. As a consequence we can approximate c_{11} by c_L in eq. (2.25) with the result $\tau_l \sim \eta_l/G$. The wavevector dependence of both the shear and compressional relaxation time is therefore negligible.

3. Surface Impedance and AC Permeability

In this section we discuss the response of a flux-line liquid to a weak ac field $\delta\mathbf{H}_a = \hat{\mathbf{z}}\delta H_a e^{i\omega t}$ applied parallel to the static field $\mathbf{H}$, with $\delta H_a << H$. For clarity we first consider the case of a semiinfinite superconductor occupying the half space $y \geq 0$. The perturbing field is applied at the surface, $y = 0$, and generates a surface current in the $+x$ direction. The surface current in turn exerts a Lorentz force normal to the sample boundary that yields a compression of the flux array. To evaluate the change in induction in the superconductor as a result of this perturbation we need to solve the coupled set of Maxwell and London equations and the hydrodynamic equations of the flux liquid. Since the lines remain on the average aligned with the z direction, the change in induction will be of the form $\delta\mathbf{B} = \hat{\mathbf{z}}\delta B_z(y)e^{i\omega t}$ and $\boldsymbol{\tau} = 0$. The linearized hydrodynamic equations reduce to

$$i\omega\delta n(y) + n_0\frac{dv_y}{dy} = 0, \tag{3.1}$$

$$-\gamma v_y(y) + \tilde{\eta}_l(\omega)\frac{d^2v_y}{dy^2} - \frac{B_0}{c}j_x(y) = 0, \tag{3.2}$$

where $j_x(y) = (c/4\pi)(d\delta B_z/dy)$ is the total current density, including the response of the medium. The field δB includes both the Meissner response to the applied field and the change in induction generated by the motion of the vortices. It is determined by the London equation (2.8). For the simple geometry considered here this becomes,

$$-\frac{d^2\delta B_z}{dy^2} + \Big(\frac{1}{\lambda^2} + \frac{1}{\lambda_{nf}^2}\Big)\delta B_z = \frac{\phi_0}{\lambda^2}\delta n(y), \tag{3.3}$$

where $\lambda_{nf} = \sqrt{c^2/4\pi i\omega\sigma_n}$ is the normal fluid skin depth and λ is the London penetration length in the ab plane. For simplicity we drop in this section the subscript on λ_{ab}, since this is the only penetration length that is relevant for the geometry considered. Equation (3.3) has to be solved with the boundary condition $\delta B_z(y = 0) = \delta H_a$.

When $\tilde{\eta}_l = 0$, corresponding to a simple flux flow model in the absence of pinning, one can eliminate δn between Eqs. (3.3), (3.1) and (3.2) to obtain a second order differential equation for δB_z, which has solution

$$\delta B_z(y) = \delta H_a e^{-y/\lambda_{ac}(\omega)}, \tag{3.4}$$

where

$$\lambda_{ac}(\omega) = \Big[\frac{\lambda^2 + \lambda_f^2(\omega)}{1 + \lambda^2/\lambda_{nf}^2(\omega)}\Big]^{1/2}, \tag{3.5}$$

with $\lambda_f(\omega) = \sqrt{c_L(0)/i\omega\gamma}$ the ac penetration length of a flux liquid [27]. As in a normal metal, the response of the material to an ac field is entirely characterized by a single length $\lambda_{ac}(\omega)$, that plays the role of the skin penetration depth in metals. The inclusion of the normal current guarantees that $\lambda_{ac}(\omega)$ reduces to the normal metal skin depth at H_{c2} where λ diverges [9]. The frequency dependence of λ_{ac} is controlled by two frequency scales. The first, $\omega_f = c_L(0)/\gamma\lambda^2$ is associated with flux diffusion: ω_f^{-1} represents the time it takes for the field to diffuse a length λ into the sample. The second frequency scale is the frequency governing ac field screening by the normal fluid component, $\omega_{nf} = c^2/(4\pi\sigma_n\lambda^2)$. If the friction coefficient γ is approximated by its Bardeen-Stephen value, we find $\omega_f/\omega_{nf} = B/H_{c2} < 1$. The ratio of these two frequencies is even smaller if γ is renormalized to incorporate the pinning by weak point defects as in TAFF models, since in that case $\omega_f/\omega_{nf} \approx (B/H_{c2})\exp(-U_p/k_BT)$. The frequency dependence of $\lambda_{ac}(\omega)$ is characterized by three regions, as shown in Fig. 1. For $\omega << \omega_f$, $\lambda_{ac}(\omega) \approx \lambda_f(\omega) \sim \omega^{-1/2}$ and the field penetration is controlled by flux flow. For $\omega_f << \omega << \omega_{nf}$, $\lambda_{ac}(\omega) \approx \lambda$ and one has essentially static Meissner screening since the flux lines cannot flow on experimental time scales. Finally, for $\omega >> \omega_{nf}$, $\lambda_{ac}(\omega) \approx \lambda_{nf}(\omega) \sim \omega^{-1/2}$ and the electrodynamic response is controlled by the normal fluid. We remark that the frequency dependence of $\lambda_{ac}(\omega)$ in the region where penetration is controlled by flux flow is the same as in a normal metal. In the flux liquid model considered here, weak point disorder decreases ω_f and widens the region of Meissner screening. As pointed out by Geshkenbeim et al [11]., ω_f is very large (typically $\omega_f \sim 10^{14} Hz$) and in the frequency range where the experiments are usually carried out $(1 - 10^6 Hz)$ one has $\lambda_{ac} \approx \lambda_f$.

The quantity that is usually measured at these frequencies is the surface impedance $Z_s(\omega) = R_s + iX_s$, defined as

$$Z_s(\omega) = \frac{E_x(0)}{\int_0^\infty j_x(y)dy} = \frac{4\pi}{c}\frac{E_x(0)}{\delta B_z(0)}, \tag{3.6}$$

where $E_x(0)$ is the electric field at the surface. The surface resistance, R_s, and the surface reactance, X_s, determine the loss and the resonance frequency, respectively, when the superconductor forms part of a resonant circuit. When $\tilde{\eta}_l = 0$ the surface impedance is given by the familiar expression,

$$Z_s(\omega) = \frac{4\pi i\omega}{c^2}\lambda_{ac}(\omega). \tag{3.7}$$

It is related to the ac resistivity, $\rho_{ac}(\omega) = E_x(y)/j_x(y)$, by $Z_s(\omega) = [4\pi i\omega\rho_{ac}(\omega)/c^2]^{1/2}$. If $|\lambda_f| >> \lambda$, the ac resistivity is independent of frequency and it is simply given by the Bardeen-Stephen dc resistivity, $\rho_{ac} = \rho_f = 4\pi c_L(0)/c^2\gamma$ [27].

Finally, for a superconducting slab of thickness $2W$ occupying the region of space $|y| \leq W$, one can use this simple flux-flow model that neglects intervortex interaction to evaluate the ac permeability, $\mu(\omega)$, defined as

$$\mu(\omega) = \frac{1}{2W\delta H_a}\int_{-W}^{W} dy\delta B_z(y), \tag{3.8}$$

with the result,

$$\mu(\omega) = \frac{\lambda_{ac}}{W} \tanh\Big(\frac{W}{\lambda_{ac}}\Big). \tag{3.9}$$

The ac permeability depends on the sample thickness W through

$$\frac{\lambda_{ac}}{W} = [\frac{\lambda^2/W^2 + \omega_D/i\omega}{1 + i\omega/\omega_{nf}}]^{1/2} \tag{3.10}$$

where $\omega_D = \omega_f \lambda^2/W^2$ is the inverse of the time for flux diffusion across the sample. The normal fluid contribution is negligible provided $\omega << \omega_{nf}$, where $\omega_{nf} >> \omega_D$. In this range of frequencies a crossover in $\mu^{'}$ and a peak in $\mu^{''}$ occur when $|\lambda_{ac}| \sim W$ or $\omega \sim \omega_D$, i.e., the time modulation of the applied field matches the time for flux diffusion across the sample thickness. If $\omega << \omega_D$, $|\lambda_{ac}| >> W$ and the field penetrates completely, yielding $\mu^{'} = 1$. If $\omega >> \omega_D$ the flux liquid cannot diffuse on the time scale over which the perturbation changes and the applied field can only penetrate a surface layer of width λ, yielding $\mu^{'} \approx \lambda/W \tanh(W/\lambda) << 1$. The crossover between these two limits is marked by a maximum in the absorption $\mu^{''}$. Finally, for $\omega \sim \omega_{nf}$ the normal fluid contribution becomes important and leads to a second crossover from $\mu^{'} \approx \lambda/W \tanh(W/\lambda)$ for $\omega_D << \omega << \omega_{nf}$ to $\mu^{'} \sim 0$ for $\omega >> \omega_{nf}$, accompanied by a second peak in $\mu^{''}$. When $\omega >> \omega_{nf}$ the normal quasiparticles cannot follow the changes in the external field which is completely expelled from the sample. If $W >> \lambda$, both $\mu^{'}$ and $\mu^{''}$ are very small at $\omega \sim \omega_{nf}$, where the normal fluid contribution becomes dominant. As a result, the second peak in $\mu^{''}$ is very small, as show in Fig. 2a, where $W/\lambda = 10$. On the other hand, if $W \geq \lambda$ the second peak in $\mu^{''}$ at $\omega \sim \omega_{nf}$ can exceed the peak from flux flow (Fig. 2b). In general, the frequencies ω_{nf} is, however, outside the range of frequency usually probed in experiments.

Recently Coffey and Clem incorporated flux creep and pinning in the model described above by introducing a frequency-dependent single-vortex mobility (the inverse of our friction coefficient) [9]. They found that, while pinning and creep affect the location of the peak in the absorption, no new length scale appears in the electrodynamic response, which is qualitatively similar to that obtained from simple flux-flow models.

In a viscous flux liquid intervortex interactions and entanglement as described by the viscous force in Eq. (3.2) make the electrodynamic response nonlocal. This nonlocality is governed by the new viscous length

$$\delta(\omega) = \sqrt{\tilde{\eta}(\omega)/\gamma} \tag{3.11}$$

that controls the relative importance of the first two terms of the left hand side of Eq. (3.2). To clarify the meaning of this new length scale it is useful to eliminate the flux density and flow velocity from Eq. (3.1)-(3.3) in favor of the electrodynamic fields $\delta B_z(y)$ and $E_x(y) = \frac{B_0}{c} v_x(y) + \frac{4\pi\lambda^2 i\omega}{c} j_s$, with the result,

$$E_x - \delta^2 \frac{d^2 E_x}{dy^2} = \rho_f j_x(y) + \frac{4\pi\lambda^2 i\omega}{c}\big[j_s - \delta^2 \frac{d^2 j_s}{dy^2}\big], \tag{3.12}$$

where $\rho_f = B_0^2/c^2\gamma$ is the flux flow resistivity, and

$$\frac{\lambda^2\delta^2}{1+\lambda^2/\lambda_{nf}^2}\frac{d^4\delta B_z}{dy^4} - (\lambda_{ac}^2 + \delta^2)\frac{d^2\delta B_z}{dy^2} + \delta B_z = 0. \tag{3.13}$$

The second term on the right hand side of (3.12) arises from Meissner screening of the fields over a surface layer of width λ. To understand the physical meaning of the new length scale $\delta(\omega)$ we define a contribution from flux motion , E_x^f, to the electric field by $E_x = E_x^f + \frac{4\pi\lambda^2 i\omega}{c} j_s$. Substituting this into (3.12) , we obtain

$$E_x^f - \delta^2 \frac{d^2 E_x^f}{dy^2} = \rho_f j_x(y). \tag{3.14}$$

When the viscosity is very small the second term on the left hand side of Eq. (3.14) is always negligible and the field from flux motion is simply determined by Ohm's law for a normal metal of resistivity given by the flux flow resistivity, ρ_f. The resistive response of the medium is local and the ac resistivity ρ_{ac} is independent of the frequency, with $\rho_{ac} = \rho_f$. When δ is sufficiently large the resistive response of the medium is nonlocal : the electric field at $\mathbf{r}$ is determined by the current at spatially remote points $\mathbf{r}'$ as a result of the force that remote fluid elements can exert on each other via interactions and entanglement.

Using the Maxwell model of viscoelasticity given in Eq. (2.23), the frequency dependent viscous length $\delta(\omega)$ is given by $\delta(\omega) = \delta_0(1 + i\omega\tau_l)^{-1/2}$ where $\delta_0 = \sqrt{\eta_l/\gamma}$ is the static viscous length discussed earlier by Marchetti and Nelson [17]. The static viscosity of a flux-line liquid has been estimated elsewhere employing analogies with the physics of entangled polymer melts[17,28] . Assuming $\gamma \approx \gamma_{BS}$, one finds $\delta_0 \approx a_0 \exp(U_\times/2k_BT)$, where $U_\times(H,T)$ is the typical energy barrier for flux-line cutting, which is expected to vanish at H_{c2}. If the barriers to flux cutting are sufficiently large, at low temperatures the vortex array can get stuck in a polymer glass regime of entangled lines, characterized by infinite viscosity on experimental time scales. A simple estimate of the crossing energy gives $U_\times \approx 2(\sqrt{2}-1)\sqrt{\frac{m_{ab}}{m_c}}a_0\epsilon_0 \ln\kappa$, with $\epsilon_0 = (\phi_0/4\pi\lambda_{ab})^2$ [29]. This can also be written in terms of the clean flux lattice melting temperature $T_m = \alpha_L^2\epsilon_0 a_0\sqrt{\frac{m_{ab}}{m_c}}$, with $\alpha_L \approx 0.15 \sim 0.3$ the Lindemann parameter [29], as $U_\times/T_m = c_\times/\alpha_L^2$, with $c_\times = 2(\sqrt{2}-1)\ln\kappa$. This shows that the crossing barrier associated with entanglement can become very large above T_m and preclude crystallization on experimental time scales. Recent calculations of the energy $U_\times$ by Moore and Wilkin [30] and by Carraro and Fisher [31] confirm these simple estimates. Using $\alpha_L \sim 0.3$ and $\kappa \sim 200$, we find $\delta_0 \sim a_0 e^{50T_m/T}$. It is then clear that the static viscous length δ_0 can become very large in the flux liquid regime and the resulting nonlocality of the dc response can be probed experimentally.

Viscous effects introduce two new frequency scales in the ac response of a flux array. The first is the frequency $\omega_l = 1/\tau_l = G/\eta_l$ describing the relaxation of shear stresses and controlling the frequency dependence of the viscous length $\delta(\omega)$. The second frequency scale is defined by $|\lambda_f(\omega)| \sim \delta_0$, corresponding to $\omega \sim \omega_\eta = c_L(0)/\eta_l$. Both ω_l and ω_η decrease with increasing static viscosity and in flux arrays $\omega_l << \omega_\eta$ since $G << c_L(0)$, as discussed below. The frequency ω_l only enters through the frequency dependence of $\delta(\omega)$ and governs the crossover from liquid-like to solid-like response of the flux array as a function of the frequency ω of the external probe. To understand the crossover it is useful to first neglect the fourth order derivate of the field in (3.13) . The single length scale governing ac field penetration is then $\lambda_{ac}^2+\delta^2 = \lambda^2+\lambda_f^2+\delta^2$. At low frequency ($\omega << \omega_l$), $\delta(\omega) \approx \delta_0$ and $\lambda_{ac}^2 \approx \lambda^2+\lambda_f^2+\delta_0^2$, with $\lambda_f = \sqrt{c_L(0)/i\omega\gamma}$. The model then simply describes penetration via flux diffusion (flux flow) of a vortex liquid of static viscosity η_l. In this case the viscous force provides an additional static damping of the penetration field. This additional damping is, however, negligible if $\omega < \omega_\eta$. If $G << c_L(0)$, then $\omega_l << \omega_\eta$ and the flux flow contribution to the penetration length always dominates the contribution from δ_0 in this low frequency regime. At high frequency ($\omega >> \omega_l$), $\delta(\omega) \sim \sqrt{G/i\omega\gamma}$ and $\lambda_{ac}^2+\delta^2 \sim \lambda^2+\frac{c_L(0)}{i\omega\gamma}+\frac{G}{i\omega\gamma} = \lambda^2+\frac{c_{11}(0)}{i\omega\gamma}$. In this regime field penetration is governed by flux flow of a vortex lattice. The model therefore describes the crossover from liquid-like to solid-like response at the frequency ω_l. This frequency decreases as the static viscosity increases and $\omega_l/\omega_f = (G/c_L)(\lambda^2/\delta_0^2)$. Assuming $G \approx c_{66}$, where c_{66} is the shear modulus of the Abrikosov flux lattice, we find $G/c_L(0) \approx (1-b)^2/4\kappa^2 b^2$, if $b = B/H_{c2} > 0.25$, and $c_{66}/c_L(0) \approx 1/4\kappa^2 b^2$ if $0.3/\kappa^2 < b < 0.25$. [32] Using material parameters of YBCO at $T = 85K$, we estimate $G/c_L(0) \leq 10^{-3}$ [32,33]. As a result, at the frequency $\omega \sim \omega_l$ (when the crossover from liquid-like to solid-like behavior occurs) the conventional flux flow contribution to the penetration length is so large that it always dominates the ac response of the flux array. This result is not unexpected. It simply reflects the fact that the only difference in response between a vortex solid and a vortex liquid arises from the small difference in the compressional moduli. A perturbation that generates a compression of the flux array is therefore not a good probe to distinguish between liquid-like and solid-like response. Viscous effects may be observable only provided $G/c_L \sim 1$, as discussed below.

We now discuss in detail the field penetration for the semiinfinite geometry when $\tilde{\eta}_l \neq 0$. It is clear from (3.13) that an additional boundary condition is needed to solve the equations. The additional boundary condition used here is $\delta n(y=0)=0$, which follows from the analysis of the fields generated by the vortices and their images near the surface[10]. Assuming that all perturbations vanish as $y \to \infty$, we obtain

$$\delta n(y) = \frac{\delta H_a}{\phi_0} \frac{\lambda_f^2}{\lambda_1^2 - \lambda_2^2} \Big[e^{-y/\lambda_1} - e^{-y/\lambda_2} \Big]. \tag{3.15}$$

where $\lambda_1(\omega)$ and $\lambda_2(\omega)$ are two frequency-dependent complex penetration lengths, given by,

$$\lambda_{1,2}^2(\omega) = \frac{1}{2} \Bigg[\lambda_{ac}^2 + \delta^2 \pm \sqrt{(\lambda_{ac}^2 + \delta^2)^2 - \frac{4\lambda^2\delta^2}{1+\lambda^2/\lambda_{nf}^2}} \Bigg]. \tag{3.16}$$

The fields are given by

$$\delta B_z(y) = \frac{\delta H_a}{(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2})} \Bigg[\Big(\frac{1}{\lambda_2^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2} \Big) e^{-y/\lambda_1} - \Big(\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2} \Big) e^{-y/\lambda_2} \Bigg], \tag{3.17}$$

and

$$E_x(y) = \frac{-i\omega}{c} \frac{\delta H_a}{(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2})} \Bigg[(\frac{1}{\lambda_2^2} - \frac{1}{\lambda^2} + \frac{1}{\lambda_{nf}^2}) \lambda_1 e^{-y/\lambda_1} - (\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} + \frac{1}{\lambda_{nf}^2}) \lambda_2 e^{-y/\lambda_2} \Bigg]. \tag{3.18}$$

In a viscous flux liquid the penetration of the ac field is governed by two length scales, $\lambda_1(\omega)$ and $\lambda_2(\omega)$. When $\tilde{\eta}_l = 0$, $\lambda_2(\omega) = 0$ and $\lambda_1(\omega) = \lambda_{ac}(\omega)$. In this case Eq. (3.17) simply reduces to (3.4). The role of the new penetration length, λ_2, can be understood by examining the change $\delta n(y)$ in the vortex density arising from the ac field, given in (3.15) . The density $\delta n(y)$ reaches its maximum near $y_0 \approx [|\lambda_1||\lambda_2|/(|\lambda_1|-|\lambda_2|)] \ln(|\lambda_1|/|\lambda_2|)$. Since $|\lambda_1| >> |\lambda_2|$ for all frequencies of interest here, $y_0 \approx |\lambda_2|$ and $\delta n(y_0) \approx (\delta H_a/\phi_0)\lambda_f^2/\lambda_1^2$. The density grows from zero at $y=0$ to its maximum value at $y_0 \sim |\lambda_2|$ and then decays to zero over a length $|\lambda_1| >> |\lambda_2|$. In other words both the surface currents associated with the Meissner response ($\lambda \neq 0$) and the spatially inhomogeneities in the electric field arising from intervortex interaction ($\tilde{\eta}_l \neq 0$) impede the build up of an appreciable vortex density in a surface layer of width $|\lambda_2|$. When either $\tilde{\eta}_l = 0$ or $\lambda = 0$ the width of this surface layer vanishes. A model that neglects variations on the length scale of order λ (this corresponds to letting $\lambda = 0$ in Eqs. (3.13-16)) assumes that $\phi_0 \delta n(y) = \delta B_z(y)$ and neglects the surface currents responsible for the jump in the tangential component of the average magnetic field at the surface. Similarly, when $\delta = 0$ one neglects spatial inhomogeneities in the electric field from flux motion at the surface and the flux-line density again has an unphysical finite value at the surface, given by $\delta n(0) = (\delta H_a/\phi_0)\lambda_f^2/(\lambda_f^2 + \lambda^2)$. It is also possible to satisify the boundary condition $\delta n(0) = 0$ in a microscopic model that incorporates the interactions of the vortices among themselves and with their images. This boundary condition cannot, however, be satisified in the conventional flux flow model. The hydrodynamic model described here provides a phenomenlogical description of flux dynamics that can satisfy this boundary condition because it incorporates the nonlocalities in the velocity and electric fields due to intervortex interaction.

Due to the nonlocality induced by the viscous screening the electrodynamics response of the system needs to be described in terms of nonlocal response functions. The local real space ac resistivity and ac permeability are defined by

$$E_x(y) = \int dy' \rho_{ac}(y - y', \omega) j_x(y'). \tag{3.19}$$

and

$$\delta B_z(y) = \int dy' \mu_{ac}(y - y', \omega) H_a(y'). \tag{3.20}$$

In an infinite system the above nonlocal relationships in real space simply yield the usual linear relationships between the Fourier components of the fields and currents,

$$E_x(\mathbf{q}, \omega) = \rho_{ac}(\mathbf{q}, \omega) j_x(\mathbf{q}, \omega) \tag{3.21}$$

$$\delta B_z(\mathbf{q}, \omega) = \mu_{ac}(\mathbf{q}, \omega) H_a(\mathbf{q}, \omega) \tag{3.22}$$

Due to this nonlocality the ac resistivity is not simply related to the surface impedance. The voltage drop measured in a transport experiment has to be evaluated for each specific experimental geometry using realistic boundary condition, as discussed in [19]. The effect of the viscosity and the crossover between liquid-like and solid-like behavior should be observable in ac multiterminal experiments of the type described in [20]. Similarily, the nonlocality of the permeability could be probed by measuring local magnetization profiles inside the sample. The net field penetration in a slab of thickness $2W$ is still described by the macroscopic permeability defined in (3.8) , which corresponds to the $\mathbf{q} = 0$ component of $\mu(\mathbf{q}, \omega)$. Its expression is given in Eq. (A.2).

To simplify the discussion of the two length scales $\lambda_1(\omega)$ and $\lambda_2(\omega)$ and of other electrodynamic properties we will drop the normal fluid contribution below. As in the case $\eta_l = 0$, the normal fluid contribution introduces an extra crossover at very high frequency ($\omega \sim \omega_{nf} \sim 10^{15}$Hz) and can be easily distinguished from all the other relaxational modes which occur at frequency scale at least one order of magnitude less than ω_{nf}.

The new length scale λ_2 is in magnitude at most of the order of λ at all frequencies. When the static viscosity is small, i.e. $\delta_0/\lambda << 1$, λ_2 is always negligible compared to both λ_1 and λ, as shown in Figs. 3a and 4a. In this case the field penetration is governed by the single length scale λ_1, with $\lambda_1 \sim \lambda_f$ over the entire frequency range of interest [34]. The ac permeability is well approximated by the familiar formula $\mu(\omega) \approx \lambda_1(\omega)/W \tanh (W/\lambda_1(\omega))$ and is shown in Fig. 3b. Experiments are typically carried out at frequencies $\omega \leq \omega_f$. As discussed earlier, the transition from solid-like to liquid like response takes place at $\omega \sim \omega_l$. Since $\omega_l << \omega_f$ in a viscous liquid due to the small value of the ratio G/c_L, the crossover at $\omega \sim \omega_l$ occurs well into the flux flow regime, in the sense that $|\lambda_f(\omega)| >> \delta_0$ and conventional flux flow dominates the response, as shown in Fig. 3b.

The expression used above for the shear relaxation time τ_l is simply a phenomenlogical estimate. In particular the moduli G and $c_L(0)$ appropriate for an entangled polymer glass are not known. An alternative approach would be to simply consider τ_l or ω_l as a parameter. The effect of viscosity on the ac permeability will then be observable if the G/c_L is not too small, while the viscosity is sufficient large. To illustrate this we show in Fig. 4b-6b the ac permeability for $\omega_l/\omega_f = 0.01$ and $\delta_0/\lambda = 1, 10, 100$. When the viscosity is small ($\delta_0/\lambda = 1$), λ_2 is still negligible at all frequencies and the response is given by conventional flux flow with the maximum at μ'' corresponding to $|\lambda_f| \sim W$ (Fig. 4b). When $\delta_0/\lambda >> 1$, λ_2 can become of order of λ and the width of the surface layer where flux penetration is impeded is no longer negligible. In this regime and for $\omega < \omega_l$ we can approximate $\lambda_1^2 \sim \lambda_{ac}^2 + \delta_0^2$ and $\lambda_2 \sim 2\lambda^2\delta_0^2/(\lambda_{ac}^2 + \delta_0^2)$. There is a new crossover at the characteristic frequency ω_η where $|\lambda_f| \sim \delta_0$, as shown in Figs. 5a and 6a. For $\omega << \omega_\eta$ we find $\lambda_1 \sim \lambda_f$ while $\lambda_2 \sim \delta_0\sqrt{i\omega/\omega_f}$ is negligible. For $\omega >> \omega_\eta$ $\lambda_1 \sim \delta_0$ and $\lambda_2 \sim \lambda$. Correspondingly, a second peak develops in μ'' at $\omega \sim \omega_\eta$ for intermediate viscosity (Fig. 5b). At very large viscosity, the crossover in the ac response occurs at ω_η. To understand this we recall that $|\lambda_2|$ is the distance from the surface over which the vortex density builds up to its maximum value, while $|\lambda_1|$ characterizes the decay of the density from its maximum value to zero within the sample. If $\omega_\eta << \omega_l$ (as it is the case for the parameters of Figs. 6), $|\lambda_1| >> 2W$ at all frequencies ($\lambda_1 \approx \lambda_f \sim 1/\sqrt{\omega}$ for $\omega < \omega_\eta$ and $\lambda_1 \sim \delta_0$ for $\omega_\eta < \omega < \omega_l$) and the ac perturbation simply cannot penetrate into the sample, unless $|\lambda_2|$ is essentially zero, as it is the case for $\omega << \omega_\eta$. The real part of the ac permeability drops therefore from the value corresponding to complete penetration ($\mu^{'} = 1$) to Meissner response ($\mu^{'} = 0.1$) at $\omega = \omega_\eta$, where λ_2 becomes comparable to λ and the viscous screening discussed above becomes appreciable.

Finally, the surface impedance is obtained by inserting (3.18) in (3.6) ,with the result

$$Z_s(\omega) = \frac{-4\pi i\omega}{c^2}\frac{(\lambda_{ac}^2 + \lambda_1\lambda_2)}{(\lambda_1 + \lambda_2)}. \tag{3.23}$$

The surface impedance for a superconducting slab of finite thickness $2W$ in the y direction can be calculated in a similar way. The result is given in Appendix A. Again, when $|\lambda_2| << \lambda << |\lambda_1|$ the field penetration is governed by the longest length scale and Eq. (A.3) can be approximated by

$$\begin{aligned} Z_s(\omega) &\approx \frac{4\pi i\omega}{c^2}\Big[\lambda_1 \tanh(W/\lambda_1) + \frac{\lambda_2^3}{\lambda^2}\Big] \\ &\approx \frac{4\pi i\omega}{c^2}\lambda_1 \tanh(W/\lambda_1). \end{aligned} \tag{3.24}$$

4. Summary

We have studied the linear response of a viscous flux-line liquid to ac perturbations by using a hydrodynamic theory. The flux array is described as a viscoelastic medium that responds elastically to perturbations varying on time scales shorter than the characteristic time ω_l^{-1} for relaxation of shear stresses, while it behaves as a viscous fluid on time scales large compare to ω_l^{-1}. For realistic values of parameters for YBCO, the characteristic frequency ω_l is, however, small compared to the flux flow frequency ω_f. As a result, the crossover from liquid-like to solid-like behavior occurs well into the flux flow regime and it generally cannot be detected in the ac permeability which probes the spatially averaged response of the system. This is because an ac permeability measurement probes the response of the flux array to a compression. Both flux liquid and flux solid have nonvanishing compressional moduli and the values of these moduli in the two regimes are very similar, in virtue of the very small shear modulus of the vortex lattice. A signature of the viscous flux-line liquid that distinguishes it from a flux lattice is the intrinsic nonlocality of the response. This is responsible for the appearance of the second penetration length, λ_2. It should be possible to probe this nonlocality by flux imaging experiments, provided the spatial resolution is sufficiently high to detect variations of length scales of order of $|\lambda_2| \sim \lambda$.

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Appendix A. AC response for slab geometry

In this Appendix we describe the response of a superconducting slab occupying the region $|y| \leq W$ to an ac field $\hat{\mathbf{z}}\delta H_a e^{i\omega t}$ applied at the sample surfaces. The local magnetic induction in the superconductor is given by,

$$\delta B_z(y) = \frac{\delta H_a}{(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2})} \Big[(\frac{1}{\lambda_2^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2}) \frac{\cosh(y/\lambda_1)}{\cosh(W/\lambda_1)} - (\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2}) \frac{\cosh(y/\lambda_2)}{\cosh(W/\lambda_2)} \Big]. \quad \text{(A.1)}$$

The ac permeability defined in (3.8) is

$$\mu(\omega) = \Big(\frac{1}{W}\Big) \frac{1}{\Big(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2}\Big)} \Big[\Big(\frac{1}{\lambda_2^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2}\Big) \lambda_1 \tanh(W/\lambda_1) - \Big(\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2}\Big) \lambda_2 \tanh(W/\lambda_2) \Big]. \quad \text{(A.2)}$$

The surface impedance at one of the boundaries is given by

$$Z_s(\omega, y = \pm W) = \frac{\mp 4\pi i\omega}{c^2} \frac{1}{(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2})} \Big\{ \Big[(\frac{1}{\lambda_2^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2}) \lambda_1 \tanh(W/\lambda_1) \Big] - \Big[(\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2} - \frac{1}{\lambda_{nf}^2}) \lambda_2 \tanh(W/\lambda_2) \Big] \Big\}. \quad \text{(A.3)}$$

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[34] The fact that λ_1 does not vanish as $\omega\to\infty$, but rather approaches λ, is the result of having neglected the normal fluid contribution that becomes dominant at very high frequencies.

Figure Captions

Fig. 1
Modulus of the complex ac penetration length as a function of reduced frequency ω/ω_f for the zero viscosity flux liquid with $\omega_{nf}/\omega_f = 50$. All length scales are measured in units of the London penetration length. Three regimes can be identified (i) conventional flux flow $(\omega << \omega_f, \lambda_{ac} \sim \lambda_f)$, (ii) static Meissner screening $(\omega_f << \omega << \omega_{nf}, \lambda_{ac} \sim \lambda)$ and (iii) normal fluid dominated regime ($\omega >> \omega_{nf}, \lambda_{ac} \sim \lambda_{nf}$).

Fig. 2
Real and imaginary parts of the ac permeability for $\eta = 0$ and $\omega_{nf}/\omega_f = 50$: (a) $W/\lambda = 10.0$ and (b) $W/\lambda = 2$.

Fig. 3
AC penetration lengths (a) and permeability (b) as functions of the reduced frequency ω/ω_f for a viscous flux liquid, with $G/c_L = 10^{-3}$. No appreciable change in the ac response is observable for $\delta_0^2/\lambda^2 = 1, 10, 100$. The normal fluid contribution has been neglected here and in Fig. 4, 5 and 6 below. As a consequence $\mu' \to \frac{\lambda}{W}\tanh(W/\lambda)$ as $\omega \to \infty$.

Fig. 4
AC penetration lengths (a) and real and imaginary parts of the complex ac permeability (b) as functions of the reduced frequency ω/ω_f for $\omega_\eta/\omega_f = 0.01$ and $\delta_0^2/\lambda^2 = 1$. The peak for μ'' at $\omega \sim \omega_f \equiv 1$ corresponds to $|\lambda_f| \sim W$. No effect of the viscosity is observable for $\delta_0/\lambda = 1$.

Fig. 5
Same as Fig. 4 for $\delta_0/\lambda = 10$. The incipient second peak which occurs at $\omega \sim \omega_\eta \sim \omega_f(\lambda^2/\delta_0^2)$ represents the transition from viscous to conventional flux flow. This second peak is not present in Fig. 4b because the viscosity is too low there and the two characteristic frequencies ω_η and ω_f coincide.

Fig. 6
Same as Fig. 4 for $\delta_0/\lambda = 100$. The crossover occurs at $\omega = \omega_0 = \omega_f/(\frac{\delta_0^2}{\lambda^2})$ corresponds to the impediment of the penetration of ac perturbative field due to the viscous screening.