7-18-1994

Twin-Boundary Pinning of Superconducting Vortex Arrays

M. Cristina Marchetti
Syracuse University

Valerii M. Vinokur
Argonne National Laboratory

Twin-Boundary Pinning of Superconducting Vortex Arrays

M. CRISTINA MARCHETTI †
Physics Department
Syracuse University
Syracuse, NY 13244

and

VALERII M. VINOKUR *
Material Science Division
Argonne National Laboratory
Argonne, IL 60439

We discuss the low-temperature dynamics of magnetic flux lines in high-temperature superconductors in the presence of a family of parallel twin planes that contain the c axis. A current applied along the twin planes drives flux motion in the direction transverse to the planes and acts like an electric field applied to *one-dimensional* carriers in disordered semiconductors. As in flux arrays with columnar pins, there is a regime where the dynamics is dominated by superkink excitations that correspond to Mott variable range hopping (VRH) of carriers. In one dimension, however, rare events, such as large regions void of twin planes, can impede VRH and dominate transport in samples that are sufficiently long in the direction of flux motion. In short samples rare regions can be responsible for mesoscospic effects. The phase boundaries separating various transport regimes are discussed. The effect of tilting the applied field out of the twin planes is also considered. In this case the resistivity from flux motion is found to depend strongly on the tilt angle.
5/94

† E-mail: cris@npac.syr.edu
* E-mail:valerii_vinokur@qmgate.anl.gov

1. Introduction

The static and dynamical properties of magnetic flux lines in copper-oxide superconductors are strongly affected by pinning by point, linear and planar disorder [1]. Twin boundaries are an example of planar disorder that is generally present in superconducting $YBa_2Cu_3O_{7-x}$ and La_2CuO_4, where they are needed to accomodate the strains produced by a crystallografic tetragonal-to-orthorhombic transition. Twins most often occurs in two orthogonal families of lamellae forming a mosaic [2]. It is also possible to prepare samples that contain a single family of parallel twin planes [3,4]. Columnar or linear defects can be produced by bombardment of the crystal with energetic heavy ions [5] or are embodied in forests of screw dislocations parallel to the crystal growth direction [6]. Both twins [7,3,4] and columnar defects [8] constitute examples of macroscopic correlated disorder that can be responsible for a sharp decrease in the resistivity for specific field orientations.

Extensive investigations of twin-boundary pinning have been carried out by Kwok and coworkers[4]. These authors studied a variety of YBCO single crystal samples containing single families of parallel twins lying in planes spanned by the c axis, with spacings ranging from microns down to several hundred Angstroms. As indicated by earlier decoration experiments [9], the order parameter is suppressed at a twin boundary at low temperatures and the twin attracts or pins the vortices. Transport experiments show clear evidence of strong twin-boundary pinning even in the flux liquid phase [7]. The pinning is most effective when the external field is applied along the c axis and the driving current lies in the ab plane and is parallel to the plane of the twins, resulting in a Lorentz force normal to the twin planes (Fig. 1a). In this case twin planes act like strong pinning centers and the linear resistivity as a function of temperature drops sharply at a characteristic temperature that marks the onset of twin-boundary pinning [7]. In samples with only a few widely spaced twin planes this drop in the resistivity is followed by an abrupt shoulder at a lower temperature, corresponding to the first order freezing transition into an Abrikosov lattice. The abrupt shoulder is not observed in heavily twinned samples where the freezing transition is apparently suppressed by disorder. Additional experimental evidence for twin-boundary pinning comes from the observation of a sharp downward dip in the resistivity as a function of angle as the external field is rotated out of the twin plane through the c direction [4] (see Fig. 1b). This strong angular dependence is a clear signature of anisotropic pinning by twin planes since point disorder in the form of oxygen vacancies, while certainly present in all these samples, can only yield a weak dependence of the resistivity on the tilting angle.

Another experimental probe of flux-line dynamics that has been recently used to study the effect of correlated disorder is real-time imaging of flux profiles [10,11]. Imaging experiments of the field penetrating into single crystals with families of twins lying in planes spanned by the c axis (for fields directed along the c axis) show strong pinning by twin planes for flux motion in the direction normal to the planes, confirming the strong twin-boundary pinning seen by transport measurements for Lorentz forces applied normal to the twin planes. Imaging of flux motion along the twin planes have, however, given contradictory results. Durán and coworkers [10] argued that flux penetrates more easily in the twin regions than in the channels between the twin planes, in apparent contrast with the observation of a lowered flux-flow resistivity by twin-boundary pinning by the Argonne group for the parallel geometry, where the Lorentz force is applied along the plane of the twins [12]. A more recent imaging experiment by Vlasko-Vlasov and collaborators [11] shows, however, that the twin planes act as planar pinning barriers for all directions of flux flow, giving rise to guided vortex motion. In this paper we are only concerned with transport with flux motion in the direction transverse to the twin planes. For this geometry all experimental probes of flux dynamics confirm that twin planes provide attractive potential wells for the vortices. A detailed theoretical understanding of the imaging experiments and their relationship to transport transport in the parallel geometry remains, however, an open question.

The static and dynamical properties of flux line assemblies in the presence of a random array of *columnar pins* have been studied in detail by mapping the physics of magnetic flux lines onto the problem of localization of quantum mechanical bosons in *two dimensions* [13]. This mapping exploits methods developed to understand the behavior of He^4 films on disordered substrates [14] and to decribe electronic transport in disordered superconductors [15]. At low temperatures there is a "Bose glass" phase, with flux lines localized on columnar pins, separated by a phase transition from an entangled flux liquid of delocalized lines. In the Bose glass phase the linear resistivity vanishes and the current-voltage characteristics are nonlinear. Transport in this regime closely resembles the variable-range hopping (VRH) of electrons in disordered semiconductors in two dimensions [15].

In this paper we employ similar methods to study flux-line dynamics in the presence of a *single family of parallel twin boundaries* lying in planes containing the c axis. For $\vec{H} \parallel \hat{c}$, the flux lines are localized by the pinning potential in the direction normal to the planes. At low temperatures, when the average vortex spacing $a_0 \approx (\phi_0/B)$ exceeds the average distance d between twin planes, all flux lines are localized on the twins, progressively "filling" the planar pins as the field is increased. We only consider vortex arrays that are sufficiently dilute ($B << B_f \approx \phi_0/d^2$) that transport is controlled by single-vortex creep. This regime is experimentally relevant and has in fact been probed in flux arrays with columnar defects [16]. Flux motion in this regime is dominated by thermally activated jumps of the vortices over the relevant pinning energy barriers $\mathcal{U}(L,J)$, yielding a resistivity $\rho = \mathcal{E}/J$, given by [1],

$$\rho(T) \approx \rho_0 e^{-\mathcal{U}(L,J)/T}, \tag{1.1}$$

where ρ_0 is a characteristic flux-flow resistivity. Here we focus on flux motion transverse to the twin planes at low fields and temperatures, which resembles the hopping of electrons in *one-dimensional* disordered superconductors. The energy barriers $\mathcal{U}$ corresponding to the various low-lying excitations that can contribute to transport are evaluated and are summarized in Table 1. Typical phase diagrams in the (J, L) plane displaying the regions where the different transport mechanisms dominate are shown in Fig. 2. There is a characteristic current scale $J_L \sim 1/L$, where L is the sample thickness in the field direction, that separates the regions of linear and nonlinear response. As $L \to \infty$ the response is always nonlinear. For large enough current the dominant excitations are the half-loop configurations shown in Fig. 3a, of transverse width smaller than the average separation d between twin planes. For currents below J_1 the width of a typical half-loop excitation exceeds the mean distance between pins. In this case standard VRH arguments [15,13], where an electron hops larger and larger spatial distances to find "good" traps of low energy (Fig. 3c), suggest a nonlinear current-voltage characteristic $V \sim \exp[-(E_k/T)(J_0/J)^{1/2}]$ in the localized phase, where E_k and J_0 are characteristic energy and current scales given below. The phase diagrams shown in Fig. 2 are qualitatively similar to the phase diagrams one would obtain for vortex arrays in the presence of columnar pins. In that case one finds a nonlinear current-voltage characteristic at low currents typical of VRH in two dimensions, with $V \sim \exp[-(E_k/T)(J_0/J)^{1/3}]$.

The main difference between flux motion in the presence of columnar defects and flux motion transverse to an array of parallel twin planes is that in the latter case the low temperature transport is *one-dimensional*. In one dimension VRH can be impeded by the presence of large rare regions void of energetically favorable pins [17,18]. These rare regions free of localized states are exponentially rare, but have a very large resistance and can suppress VRH or even dominate transport (Fig. 4) in one dimension since the vortices cannot get around them. For samples that are sufficiently long in the direction of flux-line motion so that they contain a large number of such rare regions, this mechanism yields a nonlinear current-voltage characteristic of the form $V \sim (T/E_k)(J/J_0)^{1/2} \exp[-(E_k/T)^2(J_0/J)]$. Shorter samples will typically contain only a few rare regions and these will determine the sample's resistance. In sufficiently short samples there will be a spread of values of the resistance between different samples resulting in reproducible sample-to-sample resistance fluctuations.

Finally, the model presented here is also relevant to the dynamics of Josephson vortices in artificially structured "giant" Josephson junctions. Consider for instance a planar junction of in-plane dimensions large compared to the Josephson penetration length at the contact of two superconductors (the plane of the junction is the xz plane). A magnetic field applied in the plane of the junction (say, in the z direction) penetrates into the junction as a chain of Josephson vortices which lie in the contact plane [19,20]. The intervortex spacing along the x direction is determined by the strength of the applied field. The vortices are localized in the plane of the junction and form therefore a $(1+1)$-dimensional vortex array in this plane. If the junction is not uniform, the vortices are pinned independently at low fields, as indicated by the fact that the critical current does not depend on magnetic field. Additional defects can be artificially introduced in the junction. The junction can be artificially structured by the introduction of an array of defects along the x direction spanning the field axis (z) and the junction thickness (y). At low temperature vortex motion along the junction plane will then occur via thermally activated jumps between these defects and will be described by the one-dimensional tight-binding model introduced below.

In Section 2 we first review the simple model of interacting flux lines in the presence of correlated disorder introduced by Nelson [21] and by Nelson and Vinokur [13]. The analogy with quantum mechanics of *two-dimensional* bosons and the reduction of the low temperature dynamics transverse to an array of parallel twin planes to a tight binding model in *one dimension* are then discussed. In Section 3 we estimate the pinning energy barriers associated with the low-lying excitations from the ground state and the corresponding contributions to the resistivity. The phase boundaries separating the regions of the (L, J) plane where the various contributions dominate are also discussed. A brief summary of these results has been presented elsewhere [22]. In section 4 we consider the case where the external field is tilted at an angle θ away from the c axis and out of the twin planes. The resistivity displays a strong angular dependence with a sharp downward dip at $\theta = 0$. Finally, in Section 5 we discuss the role of rare fluctuations in this one dimensional geometry.

2. Vortex Free Energy and Tight-binding Model

We are interested in transport at low fields and temperatures where each flux line is localized on one or more twin planes. In this regime dominated by single-vortex dynamics a detailed description of transport can be developed. Our starting point is a model-free energy for flux lines in a sample of thickness L in the presence of a family of parallel twin planes [13]. The field is along the c axis, chosen as the z direction, and the flux lines are parametrized by their trajectories $\{\mathbf{r}_i(z)\}$ as they traverse the sample. The twin boundaries are parallel to the zx plane. The model free-energy for a single flux line at $(\mathbf{r}_1(z), z)$ is given by

$$F_1 = \int_0^L dz \Big[\frac{\tilde{\epsilon}_1}{2} \Big| \frac{d\mathbf{r}_1(z)}{dz} \Big|^2 + V_D(y_1(z)) \Big], \tag{2.1}$$

with

$$V_D(y) = \sum_{k=1}^{M} V_1(|y - Y_k|). \tag{2.2}$$

Here V_D is the random potential arising from a set of M x- and z-independent pinning potentials $V_1(|y - Y_k|)$ centered at the locations $\{Y_k\}$ of the twin planes. The first term on the right hand side of Eq. (2.1) is the first term in a small angle expansion of the elastic energy of a nearly straight vortex line, with $\tilde{\epsilon}_1 \approx (M_\perp/M_z)\epsilon_0 \ln(\lambda_{ab}/\xi_{ab})$ the tilt modulus and λ_{ab} and ξ_{ab} the penetration and the coherence lengths in the ab plane, respectively. The effective mass ratio $M_\perp/M_z << 1$ incorporates the material anisotropy and $\epsilon_0 \approx (\phi_0/4\pi\lambda_{ab})^2$ is a characteristic energy scale. For simplicity we model $V_D(y)$ as an array of identical one-dimensional square potential wells of depth U_0, width $2b_0$ and average spacing d, passing completely through the sample in the x and z directions [23]. Assuming the potential wells are centered at uniformly distributed random positions $\{Y_k\}$ and $b_0 << d$, we find $\overline{V_D} \approx U_0\left(\frac{2b_0}{d}\right)$, while the random potential fluctuations $\delta V_D(y) = V_D(y) - \overline{V_D}$ satisfy

$$\overline{\delta V_D(y)\delta V_D(y')} = \Delta\delta(y - y'), \tag{2.3}$$

with $\Delta \approx U_0^2 \frac{(2b_0)^2}{d}[1+\mathcal{O}(2b_0/d)]$. The interaction between vortex lines and a twin boundary has been studied by Geshkenbein in the context of the Ginzburg-Landau theory [24].

In this paper we assume that the twin planes are randomly located along the y direction and that the separations between neighboring twins are Poisson-distributed (see also Section 5 below). This appears to be the case in some of the samples employed by the Argonne group [25]. On the other hand, the twin structures that form naturally in YBCO to accomodate the strains arising from a tetragonal-to-orthorhombic transformation, which take place around $1000K$ as a result of oxygen vacancy ordering, are often quite different [2]. They consists of lamellae or colonies of parallel twins oriented in either the (110) or $(1\bar{1}0)$ directions. Orthogonal twin colonies form a mosaic-type structure, containing colonies of various size. The colony size scales with the square of the average twin spacing d, while the latter remains rather uniform within a given colony[2]. There is a repulsive interaction between the twin planes of a given colony arising from the stress produced by one twin plane in the region of another. This interaction leads to the regular spacing of the twins within a colony which resembles an approximately regular one-dimensional lattice of twin planes. Vortex dynamics in the presence of such a twin structure will not be described by the model presented in this paper. On scales shorter than the typical twin colony size vortices are pinned by a regular array of planar defects, while on scales larger than the colony size the theory developed to describe vortex dynamics in the presence of columnar defects should apply. In contrast, the twin structures observed in the samples used by the Argonne group consist of a single colony of parallel twins with large variations in the twin spacing. A twin structure of this type may arise if the sample is annealed and the twins "fall out of equilibrium" arranging themselves in one-dimensional liquid-like structure within a given colony.

The free energy for an assembly of N flux lines is given by

$$F_{(N)} = \sum_{i=1}^{N} F_i + \frac{1}{2}\sum_{i\neq j}\int_0^L V(|\mathbf{r}_i(z) - \mathbf{r}_j(z)|)dz, \tag{2.4}$$

where $V(r)$ is the pair interaction potential, assumed local in z. It can be shown [13] that both higher order terms in the small angle expansion of the elastic energy and nonlocality in z in the pair interaction are negligible provided $|d\mathbf{r}_i/dz|^2 << M_z/M_\perp$ for the most important vortex configurations. In the following we will simply use the form of the pair interaction for nearly straight flux lines,

$$V(r) \approx 2\epsilon_0\big[K_0(r/\lambda_{ab}) - K_0(r/\xi_{ab})\big], \tag{2.5}$$

where $K_0(x)$ is a modified Bessel function.

At low temperatures, when the average vortex spacing $a_0 \approx (\phi_0/B)$ exceeds the average distance between twin planes, all flux lines are localized on the twins, progressively "filling" the planar pins as the field is increased. Any real sample will, however, also contain point defects, which are known to promote flux-line wandering. Recent analytical and numerical work has shown that in the case of planar pins a single flux line remains localized on the pinning plane even in the presence of additional weak point disorder in the bulk, which is always present in real samples [26]. The stability of the localized Bose glass phase in $1+1$ dimensions - the case relevant to the model considered here - in the presence of point disorder has been studied recently by Hwa et al. [27]. These authors considered a model where flux lines directed along the z direction and confined to the zy plane are pinned by the competing action of randomly distributed linear defects spanning the plane in the z direction and point defects described by a random potential with variance Δ_0. They showed that in $1+1$ dimensions the low temperature phase is of the Bose glass type with flux lines localized on the linear pins when point disorder is weak. The localized phase is marginally unstable to point disorder, but only beyond an astronomically large crossover length scale. Point disorder will in general lower the energy barriers associated with the various low-lying excitations discussed here. It does not, however, have a significant effect on the energy barriers in the rigid flow (or half-loop) regime where $\mathcal{U}_{rf} \sim L$ (or $\mathcal{U}_{hl} \sim 1/J$). This is because the energy gain δF_Δ associated with the pinning of a fluctuation of length L by point defects only grows as $L^{1/2}$ (or as $J^{-1/2}$ in the nonlinear regime), with $\delta F_\Delta \sim (\Delta_0 L)^{1/2}$. Sufficiently strong point disorder can, however, lower considerably the barriers for variable range hopping à la Mott since both the barrier $\mathcal{U}_{Mott}$ and δF_Δ grow like $L^{1/2}$. In this case to assess whether point or correlated disorder dominates one needs to compare quantitatively the relative strengths of these two types of crystal defects. This comparison involves unknown parameters and is beyond the scope of the present paper.

The problem of one flux line localized near a single twin plane has been studied by Nelson by exploiting the mapping of the statistical mechanics of magnetic flux lines onto the quantum mechanics of two-dimensional bosons [21]. At zero temperature the twin provides a binding energy U_0 per unit length for trapping the flux line. In the presence of thermal fluctuations U_0 is replaced by a smaller binding free energy per unit length $U(T)$, to account for the entropy lost by confining the flux line near the twin plane. In infinitely thick samples ($L \to \infty$) this binding free energy is determined by the zero-point energy of a fictitious two-dimensional quantum mechanical particle confined to a one-dimensional potential well, or $U(T) = -E_0(T)$, where $E_0(T)$ is the ground state eigenvalue of a two-dimensional "Schrödinger" equation,

$$\Big[-\frac{T^2}{2\tilde{\epsilon}_1}\nabla_\perp^2 + V_1(y)\Big]\psi_0(x,y) = E_0\psi_0(x,y), \tag{2.6}$$

and the twin plane is centered at $y = 0$, i.e., $V_1(y) = -U_0$ for $|y| \leq b_0$, $V_1(y) = 0$ for $|y| > b_0$. In the quantum mechanical analogy T plays the role of Planck's constant $\hbar$, $\tilde{\epsilon}_1$ that of the mass m of the fictitious particle and L^{-1} that of the particle's temperature. The x and y degrees of freedom are decoupled in Eq. (2.6) and the ground state wavefunction is the product of a free-particle wavefunction in the x direction and the ground state wavefunction $\phi_0(y)$ of a one-dimensional particle in a well [28], $\psi_0(x, y) = \frac{1}{\sqrt{D}} e^{iq_x x}\phi_0(y)$ [21,13]. Here D is the system size in the x direction and q_x the wavevector. The corresponding ground state energy is $E_0(T) = \frac{T^2 q_x^2}{2\tilde{\epsilon}_1} + E_{0w}$. The first term is the free particle contribution describing the energy cost associated with localizing a flux line within a distance $\sim 2\pi/q_x$ and $E_{0w} < 0$ is the ground state energy of a one-dimensional particle in a well [28]. If no other flux lines are present and the sample is infinite in the x direction, we can take q_x=0 and $U(T) = -E_{0w} = U_0 f(T/T^*)$ [21]. Here $T^* = \sqrt{2U_0\tilde{\epsilon}_1} b_0$ is a crossover temperature above which thermal fluctuations delocalize the flux line, and $f(x) \approx 1 - \frac{\pi^2}{4}x^2$ for $x << 1$ and $f(x) \approx 1/x^2$ for $x >> 1$. The probability of finding a point on the vortex at a transverse displacement $\mathbf{r}_\perp$ is proportional to $|\psi_0(\mathbf{r}_\perp)|^2$ and depends only on the transverse displacement y relative to the center of the twin plane. This corresponds to the fact that the flux line is "free" and therefore completely delocalized in the direction parallel to the twin plane (x), while it is localized by the pinning potential in the direction transverse to the twin plane (y). The corresponding transverse localization length $l_y(T)$ can be defined as

$$[2l_y(T)]^2 = \int_{-\infty}^{+\infty} dy y^2 |\phi_0(y)|^2, \tag{2.7}$$

where the wavefunction is assumed to be normalized. As shown in [21], one finds $l_y(T) \approx b_0[1 + \mathcal{O}(T/T^*)]$, for $T << T^*$. At high temperature thermal wandering is important and the localization length can become larger than b_0, with $l_y(T) \approx \frac{T}{\sqrt{2\tilde{\epsilon}_1 U(T)}} \sim b_0(T/T^*)^2$, for $T >> T^*$.

When many vortices are present, the repulsive intervortex interaction tends to confine each flux line to a "cage" provided by the surrounding vortices in a triangular lattice [29]. For fields below the "filling" field $B_f \approx \phi_0/d^2$, defined as the field where the flux lines fill the twin planes, forming a triangular lattice of spacing $\sqrt{3}d/2$ [30], the additional confining potential provided by the repulsive interaction does not change qualitatively the fluxon states in the direction transverse to the twins, since in this direction flux lines are also localized by the pinning potential. Interactions with neighbors do, however, change qualitatively the behavior along the x direction. Following Ref. [29], a simple description of the role of interactions can be obtained by considering a representative fluxon localized near a single twin plane centered at $y = 0$ and subject to the additional confining potential provided by the surrounding vortices. If the position of all the other vortices is assumed to be fixed, the confining potential can be approximated by a one-body effective potential $V_{eff}(\mathbf{r}_1(z))$,

$$V_{eff}(\mathbf{r}_1(z)) = \frac{1}{N}\sum_{j\neq 1} V(|\mathbf{r}_1(z) - \mathbf{r}_j^0|), \tag{2.8}$$

where $V(r)$ is the pair potential given in Eq. (2.5) and $\mathbf{r}_1(z)$ denotes the position of the representative fluxon. The sum is over all the other vortices that are fixed at their equilibrium positions $\mathbf{r}_j^0$, corresponding to the sites of the triangular lattice. The free energy of the representative fluxon is then given by,

$$F_1^{eff} = \int_0^L dz \left[\frac{\tilde{\epsilon}_1}{2}\left|\frac{d\mathbf{r}_1(z)}{dz}\right|^2 + V_{eff}(\mathbf{r}_1(z)) + V_1(|y_1(z)|)\right], \tag{2.9}$$

where $V_1(|y|)$ is the single-twin pinning potential discussed earlier. If we expand the effective potential $V_{eff}(\mathbf{r}_1)$ about its minimum at $\mathbf{r}_1 = 0$, we find

$$V_{eff}(\mathbf{r}_1) \approx V_{eff}(0) + \frac{1}{2}Cr_1^2, \tag{2.10}$$

where, neglecting logarithmic corrections and constants of order unity,

$$C \approx \frac{2\epsilon_0}{a_0^2}, \tag{2.11}$$

for $\lambda_{ab} >> a_0$, where the pair interaction $V(r)$ is logarithmic ($K_0(x) \approx -\ln x$, for $x << 1$), and

$$C \approx \frac{2\epsilon_0}{\lambda_{ab}^2}\sqrt{\frac{\pi\lambda_{ab}}{2a_0}}e^{-a_0/\lambda_{ab}}, \tag{2.12}$$

for $\lambda_{ab} << a_0$, where the pair interaction decreases exponentially with distance. Again, following Ref. [29], in the limit $L \to \infty$ the partition function of this representative fluxon is written in terms of the ground state eigenfunction and eigenvalue of the "Hamiltonian" operator of a fictitious quantum mechanical particle. Dropping the constant term in Eq. (2.10), the corresponding "Schrödinger" equation is given by

$$\Big[-\frac{T^2}{2\tilde{\epsilon}_1}\nabla_1^2 + \frac{1}{2}Cr_1^2 + V_1(|y_1|)\Big]\Psi_0(\mathbf{r}_1) = E_0\Psi_0(\mathbf{r}_1). \tag{2.13}$$

The x and y degrees of freedom are decoupled and the "Schrödinger" equation (2.13) can be separated into two one dimensional equations (to simplify the notation, we drop the subscript 1 on the location of the representative fluxon),

$$\Big[-\frac{T^2}{2\tilde{\epsilon}_1}\frac{d^2}{dx^2}+\frac{1}{2}Cx^2\Big]g_x(x)=E_xg_x(x), \tag{2.14}$$

and

$$\Big[-\frac{T^2}{2\tilde{\epsilon}_1}\frac{d^2}{dy^2}+\frac{1}{2}Cy^2+V_1(|y|)\Big]g_y(y)=E_yg_y(y), \tag{2.15}$$

with $\Psi_0(\mathbf{r})=g_x(x)g_y(y)$ and $E_0=E_x+E_y$. In the x direction the vortex line is described by the ground state of a one-dimensional harmonic oscillator of frequency $\omega_0=\sqrt{C/\tilde{\epsilon}_1}\approx(1/a_0)\sqrt{2\epsilon_0/\tilde{\epsilon}_1}$. The ground state energy is $E_x=T\omega_0$ and the corresponding eigenfunction is

$$f(x)=\frac{1}{(\sqrt{2\pi}x^*)^{1/2}}e^{-(x/2x^*)^2}, \tag{2.16}$$

where $x^*=(T^2/2\tilde{\epsilon}_1C)^{1/4}$ is the characteristic length scale for vortex fluctuations along the x direction. In the absence of the twin plane, the ground state in the y direction is also that of a harmonic oscillator of frequency ω_0 and the vortex is confined by interactions within a region of radius $r^*=\sqrt{x^{*2}+y^{*2}}$, with $y^*=x^*$, centered at its equilibrium position, $\mathbf{r}=0$. The presence of the twin boundary modifies the potential in the y direction, leading to an additional square well near the center of the harmonic potential, as in Eq. (2.15). The range of the wavefunction $g(y)$ controls the localization length $l_\perp$ in the direction transverse to the twin plane. This is determined by the interplay of the length scale $y^*(T)$ for harmonic fluctuations and the localization length $l_y(T)$ defined in Eq. (2.7) associated with the pinning potential. These two length scales are sketched in Fig. 5 as functions of temperature. For $T<<T^*$, $l_y\approx b_0$. If the temperature is so low that $y^*<l_y\approx b_0$, the range of the wavefunction $g(y)$ is controlled by interactions and $l_\perp\approx y^*$. The characteristic temperature T_{x1} where $y^*=b_0$ is given by $T_{x1}=(b_0/a_0)\sqrt{2\epsilon_0/U_0}T^*<<T^*$, as shown in Fig. 5. For $T>>T^*$, $l_y(T)\approx b_0(T/T^*)^2$ grows more quickly than y^* with temperature. There is therefore a second crossover temperature T_{x2}, as shown schematically in Fig. 5. For $T>T_{x2}$, $l_y>y^*$ and the vortex line is confined only by the harmonic well from intervortex interactions. A lower bound for T_{x2} can be obtained from $y^*(T_{x2})\approx l_y\approx b_0(T_{x2}/T^*)^2$, with the result $T_{x2}=[(a_0/b_0)\sqrt{U_0/2\epsilon_0}]^{1/3}T^*>T^*$. For the parameters of interest here T_{x2} is somewhat smaller than the clean lattice melting temperature T_m, defined by $y^*(T_m)\approx c_La_0$, with $c_L\approx 0.15-0.3$ the Lindeman constant, and is comparable to the isolated vortex depinning temperature, defined by $l_y(T_{dp}) \approx d$. In this paper we only consider the situation where all vortices are pinned in the ground state and $T << \min(T_{dp}, T_m)$. We therefore restrict ourselves to $T < T_{x2}$ and we then find $l_\perp \approx y^*$ for $T < T_{x1}$ and $l_\perp \approx l_y$ for $T_{x1} < T < T_{x2}$. In short, the ground state of a single vortex confined by the pinning potential of a twin plane along the y direction and by the isotropic "cage" provided by the repulsive interaction with the other vortices is localized in all directions, with localization lengths $l_\perp \approx l_y$ in the direction transverse to the twin (for $T_{x1} < T < T_{x2}$) and $l_\| \approx x^*$ in the direction parallel to the twin plane. The total binding free energy renormalized by interactions is of order $U_R(T) \approx U(T) - T\omega_0$. For $T << T_{x2}$ the harmonic oscillator zero point energy is always negligible compared to the pinning energy $U(T)$ and $U_R(T) \approx U(T)$.

In the presence of a family of parallel twin planes the flux line can "tunnel" between different localized states [29,13]. In this paper we are interested in studying the response of a flux array pinned by a family of parallel twin planes to a Lorentz force normal to the twin planes for $B << B_f$. In the ground state the flux lines are all localized on the attractive twin planes. The transverse driving force promotes motion of the vortices between different twin planes, corresponding to "tunneling" between different localized states along the y direction, while the repulsive interaction confines the vortices in the direction parallel to the twin planes. At low temperature the flux lines will move along the direction of the driving force (y direction) within one-dimensional channels of width $\sim 2x^*$. Using elementary quantum mechanics it can be shown [21,29] that the rate of tunneling between localized states on different twin planes separated by a distance d_{ij} is $t_{ij} \sim 2U(T)e^{-E_{ij}/T}$, with $E_{ij} = \sqrt{2\tilde{\epsilon}_1 U(T)}d_{ij}$. The energy E_{ij} is the energy of a "kink" configuration shown in Fig. 3b, connecting two pins at a distance d_{ij}.

To study the low-lying excitations from this ground state arising from thermal fluctuations one needs to sum over vortex trajectories by evaluating appropriate path integrals. As discussed in [13] and [21], these configuration sums closely resemble the imaginary time path integral formulation of quantum mechanics of *two-dimensional* particles in a static random potential $V_D(y)$. Many relevant results regarding the statistical mechanics of flux lines can then be obtained from elementary quantum mechanics.

The dynamics of flux lines driven by a Lorentz force transverse to the twin planes can then be described by a tight-binding model for *one-dimensional* bosons [13]. The lattice sites in the model are defined by the M positions $\{Y_i\}$ of the twin planes and the tight binding Hamiltonian governing the dynamics in each one-dimensional channel is given by

$$H = -[\mu + U(T)] \sum_i a_i^\dagger a_i + \sum_{i \neq j} t_{ij}(a_i^\dagger a_j + a_j^\dagger a_i) + \frac{V_0}{2} \sum_i a_i^\dagger a_i (a_i^\dagger a_i - 1). \tag{2.17}$$

Here $\mu \approx \phi_0(H - H_{c1})/4\pi$ is the chemical potential which fixes the flux line density, $a_i^\dagger$ and a_i are boson creation and annihilation operators at site Y_i, t_{ij} is a tunneling matrix element connecting localized states i and j and V_0 represents a typical energy cost for double occupancy of a site of the one-dimensional tight-binding lattice. As flux lines move in the transverse direction along the one-dimensional channels, the repulsive intervortex interaction provides an energy cost for an additional flux line occupying an already filled twin. The corresponding on-site repulsion V_0 can be estimated as

$$\begin{aligned} V_0 &\approx V(b_0) - V(d) \\ &\approx 2\epsilon_0 \big[\ln(\lambda_{ab}/\xi_{ab}) - K_0(d/\lambda_{ab})\big], \end{aligned} \tag{2.18}$$

where $V(r)$ is the pair interaction given in Eq. (2.5) and we assumed $b_0 \approx \xi_{ab}$. If $d >> \lambda_{ab}$, we find $V_0 \approx 2\epsilon_0 \ln(\lambda_{ab}/\xi_{ab})$, while for $d << \lambda_{ab}$, we obtain $V_0 \approx 2\epsilon_0 \ln(d/\xi_{ab})$. This estimate assumes that the various one-dimensional channels are completely decoupled.

The first two terms of the tight-binding Hamiltonian determine a noninteracting density of states $g(\epsilon)$ (here ϵ is an energy per unit length and $g(\epsilon)$ has units of 1/energy), such that $\mathcal{N}(\epsilon) = \int_{-\infty}^{\epsilon} g(\epsilon')d\epsilon'$ is the number of localized states per unit length with energy less than ϵ. Note that $g(\epsilon)$ is normalized so that $\mathcal{N}(+\infty) = 1/d$. Even if the pinning sites are all identical in size and well depth, dispersion of energy levels arises because vortices can tunnel between nearby twin planes. The width γ of the impurity band should then be of order $\gamma \approx t(d)$, where $t(d)$ is the tunneling matrix element evaluated at a typical twin spacing d. Interactions will further broaden the band and one can estimate,

$$\gamma \approx \max\{t(d), V_0\}. \tag{2.19}$$

This bandwidth is practically always dominated by V_0. In particular for the case $d << \lambda_{ab}$ one finds $\gamma \approx V_0 \approx 2\epsilon_0 \ln(d/\xi_{ab})$ for all temperatures $T < T^* d/b_0$. If the localized states are filled up to a chemical potential μ such that about half of the twins are occupied by at least one vortex, we can approximate the density of states $g(\mu)$ corresponding to the most weakly bound flux lines with energy $\epsilon \sim \mu$ as $g(\mu) \approx 1/d\gamma$, i.e.,

$$dg(\mu) \approx \min\{1/t(d), 1/V_0\}. \tag{2.20}$$

From our discussion of the bandwidth γ we find that the second term generally dominates for all temperatures of interest. Then $g(\mu)$ is approximately temperature independent, with $dg(\mu) \approx 1/V_0$.

3. Vortex Dynamics at Low Temperatures

We consider vortex transport in the presence of a driving current $\mathbf{J} \perp \mathbf{H}$ parallel to the twin planes, i.e., $\mathbf{J} = -J\hat{\mathbf{x}}$. The applied current exerts a Lorentz force per unit length on the vortices (see Fig. 1a),

$$\mathbf{f}_L = \frac{\phi_0}{c}\hat{\mathbf{z}} \times \mathbf{J} = \hat{\mathbf{y}} f_L, \tag{3.1}$$

with $f_L = \phi_0 J/c$, and drives the vortices to move in the direction transverse to the twin planes, leading to an additional term,

$$\delta F_1 = -f_L \int_0^L y_1(z) dz, \tag{3.2}$$

in the single-vortex free energy, Eq. (2.1). In the context of the analogy with boson quantum mechanics, this term represents a fictitious "electric field" $\mathbf{E} = \frac{1}{c}\hat{\mathbf{z}} \times \mathbf{J} = \hat{\mathbf{y}} J/c$ acting on particles with "charge" ϕ_0. The correspondence between the problem of carrier dynamics in disordered semiconductors and vortex dynamics in the presence of correlated linear or planar disorder is summarized in Table 2.

Up to numerical constants and logarithmic corrections, the critical current at low temperatures can be obtained by equating the Lorentz force to U_0/b_0, with the result $J_c(0) \approx cU_0/\phi_0 b_0$ [13]. Thermal fluctuations renormalize the critical current and one can estimate $J_c(T) \approx cU(T)/\phi_0 l_\perp(T)$. For $T >> T^*$, $J_c(T) \approx J_c(0)(T^*/T)^4$. The crossover temperature T^* is itself a function of temperature. We can define the temperature T_1 above which the entropy from flux-line wandering is important in renormalizing the binding free energy $U(T)$ by the self-consistency relation $T^*(T_1) = T_1$[21]. At low temperature the interaction between a vortex line and a twin plane is always attractive and $U_0 \approx \alpha_b \epsilon_0 \tau$, where $\alpha_b < 1$ is a dimensionless parameter related to the barrier transparency and $\tau = 1 - T/T_c$[24]. Using a mean field parametrization of the critical fields, we find $T^*(T)/T_c = \sqrt{\frac{\alpha_b \ln\kappa}{4Gi}}\tau$, where $\kappa = \lambda_{ab}/\xi_{ab}$ and $Gi = (\lambda_c^2/2\lambda_{ab}^2)(T_c/H_{c0}^2\xi_{ab0}^3)^2$ is the Ginzburg number, with H_{c0} the thermodynamic critical field at $T = 0$ and $\xi_{ab} = \xi_{ab}^0\tau^{-1/2}$. Using $\kappa \approx 10^2$, $Gi \approx 10^{-2}$ and $\alpha_b \approx 0.1$ [31] for YBCO, we find $T_1 \approx 0.77T_c$.

At low temperatures and fields well below B_f, vortex dynamics is determined by the competition between pinning by the one-dimensional array of twin planes and thermal fluctuations of the vortices. In analogy with the case of columnar pins that was discussed in detail in [13], the boson mapping reduces single vortex dynamics to a problem of hopping conductivity of localized particles in one dimension. The current density in the usual hopping conductivity problem corresponds to the vortex velocity (i.e., voltage) and the electrical conductivity maps onto the resistivity from vortex motion (see Table 2). The low temperature dynamics of vortices driven transverse to an array of parallel twin planes presents the same rich variety of hopping conductivity phenomena that occur in semiconductors, as pointed out by Nelson and Vinokur for the case of columnar pins [13]. What is new here is that vortex dynamics maps onto the problem of hopping conductivity in one dimension. In this reduced dimensionality rare events, such as large regions voids of twin planes, can dominate the transport at low currents leading to new mesoscopic phenomena, as discussed in Section 5.

Here we are interested in the low temperature regime where transport is dominated by single-vortex dynamics. In this case the dominant contribution to dissipation can be described in terms of the low-lying excitations from the ground state that correspond to thermally activated jumps of vortex lines over the relevant pinning energy barriers. The resistivity takes the form given in Eq. (1.1). In the following we determine the barrier heights $\mathcal{U}(L, J)$ corresponding to various transport regimes and the boundaries between the various regimes in the (L, J) plane. In samples of finite thickness L in the field direction the typical pinning energy barriers $\mathcal{U}(L)$ grow with L but are independent of current, yielding a linear resistivity. In thick samples there is a nonlinear resistivity associated with barriers $\mathcal{U}(J)$ that grow at low currents. We assume that in the ground state all the flux lines are localized on twin planes. We then study the low lying excitations from the ground state that can be nucleated by a finite temperature T or by a driving current J. The largest contribution to the resistivity from each class of excitations is assumed to be inversely proportional to the shortest time for the nucleation of a given excitation. The latter is determined by the typical energy barrier $\mathcal{U}$ for the formation of the low-lying excitations, which is identified with the saddle point in the single-flux line free energy. The discussion in this section follows closely that of Ref. [13], where the corresponding results for votices pinned by columnar defects were obtained.

Linear response

Consider a fluctuation that extends a length z along the twin and a distance y in the direction of the Lorentz force. The free energy of this fluctuation relative to the case $f_L = 0$ is

$$\delta F(y, z) \approx \tilde{\epsilon}_1 \frac{y^2}{z} + Uz - f_L yz. \tag{3.3}$$

Optimization of Eq. (3.3) with repsect to z for $f_L = 0$ yields the shape of the optimal low temperatute fluctuation,

$$y \sim \sqrt{U/\tilde{\epsilon}_1}\ z. \tag{3.4}$$

At low currents in samples of very small thickness L there is a linear resistivity due to the flow of rigid flux-line segments of length L and typical transverse width $y_{rf} \approx \sqrt{U(T)/\tilde{\epsilon}_1}L$ obtained by letting $z \sim L$ in Eq. (3.3). The corresponding saddle point energy is $\mathcal{U}_{rf}(L) \sim U(T)L$, resulting in a linear "rigid-flow" resistivity,

$$\rho_{rf}(L) \approx \rho_0 e^{-U(T)L/T}. \tag{3.5}$$

At larger currents the contribution from the Lorentz force to the single-line free energy (3.3) becomes comparable to the typical energy barrier $\mathcal{U}_{rf}$. When $f_L y_{rf} L > \mathcal{U}_{rf}$ or $J > J_L = c\sqrt{\tilde{\epsilon}_1 U}/(\phi_0 L)$, the response becomes nonlinear. In the thermodynamic limit $J_L \to 0$ and the IV characteristic is nonlinear at all currents. The characteristic current J_L is also conveniently expressed in terms of the energy of a kink configuration connecting neighboring pins separated by the distance d (see Fig. 3a). The typical thickness w_k of a kink along the z direction is obtained from Eq. (3.4) for $y \sim d$, with the result $w_k = d\sqrt{\tilde{\epsilon}_1/U}$. The kink energy is the corresponding saddle-point free energy, $E_k = w_k U = \sqrt{\tilde{\epsilon}_1 U(T)}d$. The energy barrier associated with the "rigid-flow" resistivity can then be written as $\mathcal{U}_{rf} = E_k(L/w_k)$ and the current scale for nonlinear transport is $J_L = cE_k/(\phi_0 L d)$.

The line $J = J_L(L)$ defines the boundary in the (L, J) plane that separates the regions of linear ($J < J_L(L)$) and nonlinear ($J > J_L(L)$) response (see Fig. 2). The details of the (L, J) phase diagram are controlled by the dimensionless parameter $\alpha = g(\mu)dU(T)$. Typical phase diagrams for $\alpha < E_k/T$ are shown in Fig. 2. The rigid flow mechanism dominates the linear resistivity only in very thin samples. When $w_k < L < L_1$, where $L_1 = E_k/\gamma$ is the length below which dispersion from tunneling and interactions can be neglected, transport occurs via the hopping of vortices between nearest neighbor (nn) pinning sites. This region of the phase diagram is only present if $L_1 > d\sqrt{\tilde{\epsilon}_1/U}$, or $\gamma < U$. For $L > L_1$ dispersion is always important and the relevant excitations are superkinks (Fig. 3c), which correspond to the tunneling of vortices between remote pinning sites analogue to Mott's electronic conductivity in disordered semiconductors. For $J > J_L(L)$ the resistivity is nonlinear. At large currents the typical transverse displacement is smaller than the average spacing d between twin planes and transport is dominated by "half-loop" excitations (Fig. 3a), characterized by an energy barrier that grows linearly as the current decreases, $\mathcal{U}_{hl} \sim 1/J$. Finally, at the smaller current flux motion takes place via VRH, characterized by a diverging energy barrier, $\mathcal{U}_{VRH} \sim 1/J^{1/2}$. We now discuss in more detail the origin of the various contributions summarized in Table 1 and the estimate of the energy barriers.

We first consider the linear portion of the phase diagram ($J < J_L$) in samples of increasing thickness L. When the typical transverse width $y_{rf} \sim \sqrt{U/\tilde{\epsilon}_1)}L$ of a rigidly flowing flux segment becomes comparable to d, transport occurs via nucleation of double kink configurations (Fig. 3b) of energy $\sim 2E_k$. The double kink then separates to $z = \pm\infty$, resulting in the hopping of vortices from one pin to a neighboring one. As discussed in [13], this transport mechanism will dominate only if the sample is so thin that the width γ of the impurity band arising from tunneling and interactions is negligible ($L < L_1 = E_k/\gamma$). In this case flux motion will occur via hopping between nearest neighbor pins, resulting in a linear resistivity $\rho_{nnh} \sim \exp(-aE_k/T)$, with a a numerical constant. In extremely thin samples this transport mechanism will ultimately be suppressed. In fact for $L < w_k$ transport via the flow of rigid flux segments described above is energetically favorable over nn hopping. As a result, a necessary condition for observing a linear nearest neighbor hopping resistivity ρ_{nnh} is $L_1 > w_k$, or $\gamma < U$. If we estimate the density of states as $dg(\mu) \sim 1/\gamma$, the condition $\gamma < U$ requires $\alpha = g(\mu)dU > 1$. The Lorentz force term in Eq. (3.3) will modify the kink energy and thickness. By optimizing Eq. (3.3) with respect to z for $y \sim d$ and $f_L \neq 0$, we find that a finite current increases the thickness of a typical kink, according to $\tilde{w}_k(J) = w_k(1 - J/J_1)^{-1/2}$, where $J_1 = cU/(\phi_0 d)$. This result only applies for $J < J_1$. At higher currents simple nn hopping cannot occur.

In thick samples ($L > L_1$) the dispersion of energies between different pinning sites makes motion by nearest neighbor hopping energetically unfavorable (the energy barrier diverges with the sample thickness L). Tunneling occurs instead via the formation of "superkinks" (Fig. 3c) that throw a vortex segment onto a spatially remote pin connecting states which optimize the tunneling probability. The free energy of a superkink excitation shown in Fig. 3c relative to the case $f_L = 0$ is then [13,27],

$$\delta F_{sk} \approx 2E_k(y/d) + \Delta\epsilon z - -f_L y z. \tag{3.6}$$

We assume all states up to a chemical potential μ are filled. The states available to a weakly bound flux line about to hop a distance y are those within an energy $\Delta\epsilon$ determined by requiring that there is at least one localized state within a region $(y, \Delta\epsilon)$ of configuration space, i.e., $g(\mu)y\Delta\epsilon \simeq 1$. The shape of the most important superkink excitations is obtained by minimizing Eq. (3.6) for $f_L = 0$ and is given by

$$y \sim \sqrt{\frac{dz}{E_k g(\mu)}}. \tag{3.7}$$

In finite thickness samples the saddle point free energy corresponding to superkink fluctuations of width given by Eq. (3.7) for $z \sim L$ yields a linear Mott resistivity, given by

$$\rho_{Mott}(L) \approx \rho_0 e^{-E_k(L/\alpha w_k)^{1/2}}, \tag{3.8}$$

with $\alpha = U(T)g(\mu)d$.

Nonlinear response

In the nonlinear regime ($J > J_L(L)$) the contribution to the free energy from the Lorentz force cannot be neglected when estimating the energy of the dominant excitations. For $J_1 < J < J_c$, with $J_1 = cU(T)/\phi_0 d$, flux motion occurs via thermally activated "half-loop" configurations identical to those discussed in [13] for the case of columnar pins. The length and width of an unbound line segment for the lowest-lying half-loop excitations are obtained by minimizing the free energy (3.3) for $f_L \neq 0$, with the result $z_{hl} \sim (U_0\tilde{\epsilon}_1)^{1/2}/f_L$ and $y_{hl} \sim (U_0/\tilde{\epsilon}_1)^{1/2} z_{hl} \sim U(T)/f_L$, respectively. The saddle point energy of a half-loop excitation is $\mathcal{U}_{hl} \approx \sqrt{\tilde{\epsilon}_1 U^3(T)}/f_L$, yielding a nonlinear resistivity,

$$\rho_{hl} \approx \rho_0 \exp[-(E_k/T)(J_1/J)]. \tag{3.9}$$

In the context of the mapping of flux-line dynamics onto the problem of hopping conductivity, the nucleation of half loops corresponds to tunneling of a carrier from a localized state directly into conduction band, as shown in Fig. 6.

For $J < J_1$ the size of the transverse displacement of the liberated vortex segment exceeds the average distance d between twin planes and transport occurs via variable range hopping (VRH) which generalizes the Mott mechanism to the nonlinear case. Again a flux line hops to a state within a region $(y, \Delta\epsilon)$ of phase space, with $g(\mu)y\Delta\epsilon \sim 1$. The size of the most important excitations is determined by minimizing Eq. (3.6) with $f_L \neq 0$, with the result $y_{VRH} \sim (g(\mu)f_L)^{-1/2}$ and $z_{VRH} \sim E_k/df_L$. One then obtains a non-Ohmic VRH behavior with

$$\rho_{VRH} \approx \rho_0 e^{-\mathcal{U}_{VRH}(J)/T} = \rho_0 \exp[-(E_k/T)(J_0/J)^{1/2}], \tag{3.10}$$

with $J_0 = J_1/\alpha$. The crossover to the linear Mott resistivity resistivity takes place when $z_{VRH} \sim L$, or $J \sim J_L$, consistent with the result obtained above when discussing half-loop excitations. The VRH contribution to the resistivity dominates that from half loop only if if $\mathcal{U}_{VRH} < \mathcal{U}_{hl}$, or $J < J_2 = J_1\alpha$.

The above results are summarized in Table 1. The corresponding phase diagrams are shown in Figs. 2 for $\alpha < E_k/T$. There are three relevant current scales, $J_0 = J_1/\alpha$, $J_1 = cU/\phi_0 d$ and $J_2 = J_1\alpha$, all much smaller than the pair breaking current $J_{pb} = 4c\epsilon_0/(3\sqrt{3}\phi_0\xi_{ab})$. For $\alpha > 1$ one can have $L_1 > w_k$ and there is a region of the phase diagram where transport occurs via nn hopping (Fig. 2a). For $\alpha < 1$ nn hopping can occur only if the chemical potential μ falls in the tails of the impurity band, so that $g(\mu)d < 1/\gamma$. If μ falls well within the impurity band, so that $g(\mu)d \sim 1/\gamma$, then $\alpha < 1$ requires $\gamma > U$ and nn hopping is always suppressed in this case. The Mott and the rigid flow regimes are separated by a horizontal line above which $\mathcal{U}_{Mott} < \mathcal{U}_{rf}$. Similarly, the condition $\mathcal{U}_{VRH} = \mathcal{U}_{hl}$ yields the vertical line separating the VRH and half loop regions.

Collective effects

At very low currents and in thick samples collective effects are always important and flux motion takes place via the creep of vortex bundles, rather than single vortices. The region where collective effects dominate is shown schematically in Figs. 2 and 4. It corresponds to the upper left portion of the (L, J) phase diagram. As discussed in Ref. [1], the crossover from single vortex creep to creep of vortex bundles occurs when

$$L_z = a_0, \tag{3.11}$$

where L_z is the size of a typical single-vortex fluctuations along the z direction and a_0 the intervortex spacing. The condition (3.11) is simply obtained by equating the tilt energy E_{tilt} of a single disorted vortex to the elastic energy E_{int} of interaction of with its neighbors. The elastic energy associated with displacing a length L_z of vortex line at an average distance a_0 from its neighbors a distance u out of its equilibrium position in the xy plane is $E_{int} \sim c_{66}u^2 L_z$, where $c_{66} \sim \epsilon_0 a_0^2$, and grows with L_z. In contrast, the corresponding single-vortex tilt energy, $E_{tilt} \sim \epsilon_0(u/L_z)^2 L_z$, decreases as L_z increases. Consequently when the longitudinal size of the typical fluctuation is sufficiently large, or $L_z > a_0$, then $E_{int} > E_{tilt}$ and collective effects are important.

In the VRH regime the relevant length scale is the width w_{sk} of a superkink excitation, shown schematically in Fig. 3c, where $w_{sk} \approx \sqrt{\tilde{\epsilon}_1/U} y_{sk}$, with y_{sk} the typical size of a superkink in the direction of flux motion. Collective effects dominate when $w_{sk} \geq a_0$. In the linear regime ($J < J_L$) y_{sk} is given by Eq. (3.7) with $z \sim L$, or $w_{sk} \approx \sqrt{w_k L}$, with $w_k = E_k/U$ the width of a kink (see Fig. 3d). Dissipation is then dominated by creep of vortex bundles for $L \geq L_b = a_0^2/w_k$. In the nonlinear regime ($J < J_L$) the size of the superkinks grows with decreasing current and $w_{sk} \approx w_k(J_0/J)^{1/2}$. Transport is always dominated by collective effects at sufficiently low currents, i.e., for $J \leq J_b = J_0(w_k/a_0)^2$. The crossover from single-vortex creep to creep of vortex bundles is marked by the dashed lines $L = L_b$ and $J = J_b$ in Figs. 2 and 4. For $\alpha > 1$ this crossover takes place well into the VRH region, as shown in Fig. 2a, provided $L_b > L_1$ and $J_b < J_1$, which corresponds to $B < (B_f/\alpha)(U/2\tilde{\epsilon}_1)$ (here and below we assume the chemical potential falls in the middle of the impurity band and $dg(\mu) \sim 1/\gamma$). For $\alpha < 1$ (Fig. 2b) this crossover occurs within the VRH region provided $L_b > w_k/\alpha$ and $J_b < J_2$, or $B < B_f\alpha(U/2\tilde{\epsilon}_1)$

4. Transport in the Presence of Tilt

We now consider another transport geometry investigated in some of the experiments by Kwok et al [4]. Here the external field $\mathbf{H}$ is tilted at an angle θ away from the c axis and out of the twin planes (see Fig. 1b). The transport current is still applied along the twin planes, which contain the c axis, $\mathbf{J} = -\hat{\mathbf{x}}J$, and the resulting Lorentz force, $\mathbf{f}_L = (\phi_0/c)[\hat{\mathbf{z}}\cos\theta + \hat{\mathbf{y}}\sin\theta] \times \mathbf{J}$, has components both normal to the twin planes and along the c axis. Only the y component of the Lorentz force is effective at driving flux motion normal to the twin planes and therefore determines the voltage in the direction of the applied current. The experiments by Kwok et al. [4] have been mostly carried out at high fields for flux arrays in a liquid state, in a regime where intervortex interactions are believed to be important. Here in contrast we neglect intervortex interactions and investigate the dependence of transverse transport on tilt angle in the regime where single-line dynamics dominate. Even though our result are therefore not directly relevant to the experiments by the Argonne group, the strong angular dependence that we predict for the resistivity is qualitatively similar to that reported in the experiments.

The free energy of a fluctuation that extends a length z along the twin and a distance y in the direction of average motion is obtained by adding the tilt energy Eq. (3.3), with the result,

$$\delta F(y, z, \theta) \approx \tilde{\epsilon}_1 \frac{y^2}{z} + Uz - f_L \cos\theta y\, z - \frac{\phi_0}{4\pi} H_\perp y, \tag{4.1}$$

where $H_\perp = H\sin\theta$ is the component of the field along the y direction and $f_L = \phi_0 J/c$, as in the preceeding sections. Optimizing Eq. (4.1) with respect to z for $f_L = 0$, we find that the shape of the optimal low temperature fluctuation is still given by Eq. (3.4) and does not depend on the angle θ. As shown by Hwa et al. [27], the energy $\tilde{E}_k(\theta)$ of a kink fluctuation in the presence of tilt, corresponding to the saddle point of the free energy (4.1) with $f_L = 0$ for $y \sim d$ and $z \sim d\sqrt{U/\tilde{\epsilon}_1}$, is reduced compared to its value for $\theta = 0$, according to,

$$\begin{aligned}\tilde{E}_k(\theta) &= E_k - \frac{\phi_0 H_\perp}{4\pi} d \\ &= E_k\Big(1 - \frac{\sin\theta}{\sin\theta_c}\Big).\end{aligned} \tag{4.2}$$

Here we have introduced a critical angle θ_c defined by $\sin\theta_c = H/H_c$, with $H_c = 4\pi E_k/\phi_0 d$. For $\theta > \theta_c$, the kink energy becomes negative and kinks proliferate, as discussed in [27]. We now consider the angular dependence of transport for $\theta < \theta_c$.

As in the case $\theta = 0$, at high enough currents flux motion will occur via the nucleation of half loops. By identifying the typical energy barrier $\tilde{\mathcal{U}}_{hl}(\theta)$ for a half loop excitation in the presence of tilt with the saddle point energy found by minimizing Eq. (4.1) for $f_L \neq 0$, we obtain,

$$\tilde{\mathcal{U}}_{hl}(\theta) = \frac{\mathcal{U}_{hl}}{\cos\theta}\Big(1 - a\frac{\sin\theta}{\sin\theta_c}\Big), \tag{4.3}$$

with a a numerical constant of order one and $\mathcal{U}_{hl} = E_k(J/J_1)$ the half loop energy barrier for $\theta = 0$. The energy for nucleating a half loop excitation is reduced by the tilt. The angular dependence of the resulting flux-flow resistivity is very strong, since the angle appears in the argument of the exponential. Flux motion will occur via half loop excitation provided the typical transverse size of the half loop does not exceed the average distance between twin planes. This imposes a lower bound on the values of the current where half loop excitation dominates transport, given by $J > J_1(1 + a'\sin\theta/\sin\theta_c)$, with a' a numerical constant of order one. Tilt decreases the range of currents where half-loops dominate.

At lower currents transport will take place via VRH. The angular dependence simply replaces the kink energy E_k by the smaller kink energy $\tilde{E}_k(\theta)$ in the presence of tilt given in Eq. (4.2). Carrying then through the standard VRH argument described in the previous section, one obtains a nonlinear angle-dependent resistivity given by Eq. (1.1), with

$$\tilde{\mathcal{U}}_{VRH}(\theta) = E_k(J_1/\alpha J)^{1/2}\frac{1 - \sin\theta/\sin\theta_c}{\sqrt{\cos\theta}}. \tag{4.4}$$

Again the corresponding resistivity is a rapidly varying function of angle, as observed in experiments. On the other hand, a simple estimate using typical parameters for $YBCO$ gives a very small value for the critical angle, $\sin\theta_c \approx 0.1H_{c1}/H$. The transport experiments probe, however, linear transport in the flux-liquid phase, where collective effects in the flux-line dynamics are important. The present dimensional analysis is useful in that it shows that even in the regime of single-line dynamics, the presence of twin planes naturally introduces a very sharp dependence of the resistivity on tilt angle.

5. Rare Fluctuations

The results described in the Section 3 are qualitatively similar to those discussed in [13] for the case of flux arrays in the presence of columnar pins. The most important difference for samples with parallel arrays of twin planes is that due to the one-dimensional nature of vortex transport at low temperature, a new regime can arise at low current, where flux-line dynamics is dominated by rare fluctuations in the spatial distribution of twin planes. The vortex line can encounter a rare region where no favorable twins are available at the distance of the optimal jump. The vortex will then remain trapped in this region for a long time and the resistivity can be greatly suppressed. Rare fluctuations can also occur in samples with columnar pins, but in that case because of the two-dimensional nature of the problem, they will dominate transport and suppress the resistivity only at extremely small fields, when the number of rare regions exceeds the number of vortices.

At a given temperature and for applied currents below J_L, a vortex can jump from one twin plane to another at a distance y only if the energy difference per unit length between the initial and final configuration is within a range $\Delta\epsilon \sim E_k y/Ld$. A trap is then a region of configuration space (y,ϵ) void of localized states within a spatial distance y and an energy band $\Delta\epsilon$ around the initial vortex state. A vortex that has entered such a trap or "break" will remain in the trap for a time $t_w \approx t_0\exp(2y/l_\perp)$, where $l_\perp$ is the transverse localization length and t_0 is a microscopic time scale. The probability of finding such a break is given by a Poisson distribution, $P(y) \approx P_0(y)\exp[-Ag(\mu)y\Delta\epsilon]$, where $P_0(y)$ is the concentration of localized states in the energy band $\Delta\epsilon$, $P_0(y) \approx 2Ag(\mu)\Delta\epsilon$ and $A \sim 1$ is a numerical constant. The mean waiting time between jumps is given by

$$\overline{t_w} \approx \int_0^\infty dy P(y) t_0 e^{2y/l_\perp(T)}. \tag{5.1}$$

For $L >> L^* = \alpha w_k(T/E_k)^2$, the integral can be evaluated at the saddle point, corresponding to the situation where the mean waiting time is controlled by "optimal breaks" of transverse width $y_l^* \approx l_\perp L/L^*$, with the result,

$$\overline{t_w} \sim t_0 \sqrt{L^*/L} e^{L/L^*}. \tag{5.2}$$

The optimal breaks are those that correspond to the longest trapping time and will therefore be most effective at preventing flux motion and dissipation. The inverse of the trapping or waiting time determines the characteristic rate of jumps, i.e., the velocity. The vortex velocity corresponding to the optimal hopping rate of Eq. (5.2) yields a linear resistivity in finite-thickness samples, given by

$$\rho_{bl} \approx \rho_0 \frac{T}{\mathcal{U}_{Mott}} e^{-(\mathcal{U}_{Mott}/T)^2}, \tag{5.3}$$

where $\mathcal{U}_{Mott}$ is given in Table 1 and we have used $L/L^* = (\mathcal{U}_{Mott}/T)^2$.

For currents above J_L, the typical energy per unit length available to a flux line for jumping a distance y is $\Delta\epsilon \sim f_L y$. The corresponding nonlinear contribution to the resistivity from traps of extent $(y, \Delta\epsilon)$ in configuration space is again proportional to the inverse of the average waiting time defined in Eq. (5.1). Again, for $l_\perp^2 g(\mu) f_L << 1$ or $J << J^* = \alpha J_1 (E_k/T)^2$, the integral can be evaluated at the saddle point, corresponding to an optimal break width $y_b^* \approx [g(\mu) l_\perp f_L]^{-1}$, with the result,

$$\rho_b \approx \rho_0 \frac{T}{\mathcal{U}_{VRH}} \exp[-(\mathcal{U}_{VRH}/T)^2]. \tag{5.4}$$

It is clear by comparing Eqs. (3.9) and (3.10) to Eqs. (5.3) and (5.4), respectively, that the contribution to the resistivity from tunneling à la Mott (both in the linear and nonlinear regimes) would always dominate that from hopping between rare optimal traps if both mechanisms of transport can occur. On the other hand, in one dimension if the sample is wide enough in the direction of flux-line motion to contain optimal traps, tunneling à la Mott simply cannot take place because flux lines cannot get around the traps. These rare traps with large waiting times will then control the transport. If W is the sample width in the y direction, the condition for having optimal traps of width $y_{l,b}^*$ is $P(y_{l,b}^*)W > 1$. Optimal traps will therefore be present only if $J > J_w = J^*/\ln(2W/l_\perp)$ for $J > J_L$ and if $L < L_w = L^* \ln(2W/l_\perp)$ for $J < J_L$.

These are, however, only necessary conditions for the sample to contain many optimal breaks. They do not guarantee that these breaks will dominate transport. A flux line can in fact escape a break by nucleating a half-loop excitation or, in the language of semiconductor transport, by tunneling directly from a localized state into conduction band (see Fig. 6). This will occur if the transverse size of a typical half loop exceeds the size of the optimal break, i.e., if $y_{hl} > y_b^*$, or $\alpha < E_k/T$. In the context of the analogy with boson quantum mechanics this condition translates into the requirement that the spatial distance between the occupied localized state and the conduction band edge in the presence of the applied current is shorter than the size of the trap (see Fig. 6). If the flux line can escape the trap by half-loop nucleation, breaks will never dominate transport and their only effect will be that of possibly suppressing VRH in a region of the phase diagram and extending to lower currents the region where transport occurs via half-loop nucleation. This can occur if J_w is smaller than the scale setting the high current boundary of the VRH region, or $J_w < \min(J_1, J_2)$. For instance if $J_w < J_2$, or $\ln(2W/l_\perp) > (E_k/\alpha T)^2$, with $\alpha < 1$, rare fluctuations will modify the phase diagram of Fig. 2b by pushing the high current boundary of the VRH region down to J_w. Similar considerations apply to the linear response. On the other hand, if $y_{hl} < y_b^*$, or $\alpha > E_k/T$, there will be a portion of the (L, J) phase diagram where breaks dominate transport, as shown in Fig. 4.

For YBCO, we estimate $E_k \sim 1K\mathring{A}^{-1}d$. Assuming $\alpha \sim U/\gamma$, the condition $\alpha > E_k/T$ can only be satisfied at low fields ($B < 1KG$ for $d \sim 200\mathring{A}$). The sample will contain optimal breaks if $W > 30\mathring{A}\exp(J^*/J)$, with $J^* \sim 4 \times 10^5 \text{Amp/cm}^2$ at $80K$.

If the sample is too short to contain optimal breaks, i.e., $WP(y \sim y_{l,b}^*) < 1$, the dynamics is controlled by the trap with the longest waiting time, $t(y_f) \sim \exp(y_f/l_\perp)$, with y_f determined by the condition $WP(y_f) \sim 1$. The corresponding resistivity is proportional to this smallest hopping rate,

$$\rho_W \approx \rho_0 e^{-y_f/l_\perp}. \tag{5.5}$$

In this case the relevant physical quantity is the logarithm of the resistivity,

$$\ln(\rho_W/\rho_0) = -y_f/l_\perp \approx -\frac{\mathcal{U}_{VRH}}{T}\Big\{ \ln\Big[\frac{2W}{l_\perp}\frac{T}{\mathcal{U}_{VRH}}\Big(\ln(2W/l_\perp)\Big)^{1/2}\Big]\Big\}^{1/2}. \tag{5.6}$$

The leading dependence of Eq. (5.6) on current and temperature is the same as that of the VRH contribution. Equation (5.6) also contains, however, logarithmic terms that in sufficiently short samples will give a random spread of values of the resistivity from sample to sample. These effects have been discussed for semiconductors [18]. In this case a more relevant physical quantity rather than the resistivity itself is the distribution of the logarithms of the resistivity over different samples. The expression (5.6) determines the position of the maximum of this distribution.

This work was supported by the National Science Foundation through Grants No. DMR91-12330 and DMR92- 17284 and through the U.S. Department of Energy, BES-Material Sciences, under contract No. W-31-109-ENG-38. MCM is grateful to Argonne National Laboratory and to the Institute for Scientific Interchange in Torino, Italy, for hospitality and partial support during the completion of this work. VMV thanks Gianni Blatter and the Swiss National Foundation for supporting his visit at ETH-Zürich were part of this work was carried out. Finally, we both thank David Nelson for many stimulating discussions.

References

[1] For a review and an extensive list of references, see G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, amd V.M. Vinokur, Rev. Mod. Phys., to appear.

[2] T. Roy and T.E. Mitchell, Phil. Mag. A **63**, 225 (1991); T.E. Mitchell and J.P. Hirth, Acta Metall. Mater. **39**, 1711 (1991).

[3] G.W. Crabtree, W.K. Kwok, U. Welp, J. Downey, S. Flesher, K.G. Vandervoort and J.Z. Lin, Physica C **185-189**, 282 (1991).

[4] W.K. Kwok, S. Flesher, U. Welp, V.M. Vinokur, J. Downey, G.W. Crabtree, and M.M. Miller, Phys. Rev. Lett. **69**, 3370 (1992); and S. Flesher, W.K. Kwok, U. Welp, V.M. Vinokur, M.K. Smith, J. Downey, and G.W. Crabtree, Phys. Rev. **B47**, 14448 (1993).

[5] M. Konczykowski et al., Phys. Rev. B **44**, 7167 (1991).

[6] M. Hawley, I.D. Raistrick, J.G. Beery and R.J. Houlton, Science **251**, 1587 (1991); C. Gerber, D. Anselmetti, J.G. Bednorz, J. Manhort and D.G. Schlom, Nature **350**, 279 (1991).

[7] W.K. Kwok, U. Welp, G.W. Crabtree, K.G. Vandervoort, R. Hulscher and J.Z. Liu, Phys. Rev. Lett. **64**, 966 (1990).

[8] L. Civale, A.D. Marwick, T.K. Worthington, M.A. Kirk, J.R. Thompson, L. Krusin-Elbaum, Y. Sun, J.R. Clem, and F. Holtzberg, Phys. Rev. Lett. **67**, 648 (1991); R.C. Budhani, M. Suenaga and S.H. Liou, Phys. Rev. Lett. **69**, 3816 (1992).

[9] G.J. Dolan, G.V. Chandrashekhar, T.R. Dinger, C. Feild and F. Holtzberg, Phys. Rev. Lett. **62**, 827 (1989).

[10] C. Durán, P.L. Gammel, R. Wolfe, V.J. Fratello, D.J. Bishop, J.P. Rice and D.M. Ginsberg, Nature **357**, 474 (1992).

[11] V.K. Vlasko-Vlasov, L.A. Dorosinskii, A.A. Polyanskii, V.I. Nikitenko, U. Welp, B.W. Veal and G.W. Crabtree, Phys. Rev. Lett. **72**, 3246 (1994).

[12] W.K. Kwok, J. Fendrich, S. Fleshler, U. Welp, J. Downey, G.W. Crabtree, and J. Giapintzakis, in *Proceedings of the XX International Conference on Low Temperature Physics*, August 4-11, 1993.

[13] D.R. Nelson and V.M. Vinokur, Phys. Rev. Lett. **68**, 2398 (1992); D.R. Nelson and V.M. Vinokur, Phys. Rev. B **48**, 13060 (1993).

[14] M.P.A. Fisher, P.B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B **40**, 546 (1989).

[15] B.I. Shklovskii and A.L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlagh, New York, 1984).

[16]

[17] J. Kurkijarvi, Phys. Rev. B **8**, 922 (1973).

[18] M.E. Raikh and I.M. Ruzin, Zh. Eksp. Teor. Fiz. **95**, 1113 (1989) [Sov. Phys. JETP **68**(3), 642 (1989)].

[19] C.S. Owen and D.J. Scalapino, Phys. Rev. **164**, 538 (1967).

[20] V.M. Vinokur and A.E. Koshelev, Sov. Phys. JETP **70**, 547 (1990).

[21] D.R. Nelson, in Proceedings of the Los Alamos Symposium *Phenomenology and Applications of High-Temperature Superconductors*, K.S. Bedell et al., eds. (Addison-Wesley, 1991), and references therein.

[22] M.C. Marchetti and V.M. Vinokur, Phys. Rev. Lett. **72**, 3409 (1994).

[23] G. Blatter, J. Rhyner and V.M. Vinokur, Phys. Rev. B **43**, 7826 (1991).

[24] V.B. Geshkenbein, Sov. Phys. JETP **67**, 2166 (1988).

[25] W.K. Kwok, private communication.

[26] L. Balents and M. Kardar, Europhys. Lett. **23**, 503 (1993).

[27] T. Hwa, D.R. Nelson and V.M. Vinokur, Phys. Rev. B **48**, 1167 (1993).

[28] See, for instance, L.D. Landau and E.M. Lifschitz, *Quantum Mechanics* (Pergamon, New York, 1965), Sec. 45.

[29] D.R. Nelson, in Proceedings of the NATO Advanced Study Institute on *Phase Transitions and Relaxation in Systems with Competing Energy Scales*, T. Riste and D. Sherrington, eds. (Kluwer Academic Pub., The Netherlands, 1993).

[30] We neglect in this estimate the anisotropy of the lattice arising from the pinning of vortices by the twin planes. At low fields the difference between the intervortex spacing a_0 in twin-free regions and the intervortex spacing a_0^{TB} along the twin can be estimated from the magnetic flux decoration images by Dolan et al. [9], with the result $a_0^{TB} \approx 0.6a_0$.

[31] Decoration experiments in twinned samples at low temperatures [9] indicate that $U_0 \sim 0.1\epsilon_0$ [23].

Linear	Nonlinear
$\mathcal{U}_{rf} = UL = E_k(L/w_k)$	$\mathcal{U}_{hl} = E_k(J_1/J)$
$\mathcal{U}_{nnh} = E_k(1 - J/J_1)^{-1/2}$	
$\mathcal{U}_{Mott} = E_k(L/\alpha w_k)^{1/2}$	$\mathcal{U}_{VRH} = E_k(J_1/\alpha J)^{1/2}$

Table 1. Energy barriers determining the various contributions to the resistivity of Eq. (1.1), with $\alpha = U(T)g(\mu)d$.

CARRIERS	VORTICES
m	$\tilde{\epsilon}_1$
$\hbar$	$k_B T$
$\beta\hbar$	L
single impurity level $E_D(E_A)$	$U(T)$
μ	$(\phi_0/4\pi)(H - H_{c1})$
$\vec{E}$	$\frac{1}{c}\hat{z} \times \vec{J}$
carrier velocity $\sim$ current density	vortex velocity $\sim$ voltage
conductivity σ	resistivity ρ
conduction-band transport	flux flow
tunneling from impurity levels to conduction band	half-loop
VRH	superkink

Table 2. Correspondence between carrier dynamics in disordered semiconductors and vortex dynamics in the presence of correlated disorder.

Figure Captions

Fig. 1. Geometry of the transport experiment corresponding to strong pinning by twin boundaries. In (a) the external field is aligned with the c axis and lies in the plane of the twins. In (b) the external field is tilted at an angle θ out of the plane of the twin. In this case only the y component $f_L \cos\theta$ of the Lorentz force is effective at driving flux motion transverse to the twins.

Fig. 2. The (L, J) phase diagram for $\alpha = g(\mu)dU \approx U/\gamma < E_k/T$. The curved phase boundaries between the Mott and VRH regimes and between the rigid flow and half loop regimes are determined by $J_L = cE_k/\phi_0 dL$. For currents below the characteristic current scale J_1 the typical transverse size of a fluctuation in the nonlinear portion of the diagram exceeds the average distance d between pins. The corresponding length scale $w_k = E_k/U$ is the width of a kink connecting pins at the distance d. The crossover from half-loop to VRH is determined by $\min(J_1, J_2)$, with $J_2 = \alpha J_1$. Figure 2a is for $\alpha > 1$, corresponding to $L_1 > w_k$, where L_1 is the sample thickness above which level dispersion is important. In this case flux motion can take place via nn hopping for $w_k < L < L_1$ and $J < J_1$. Figure 2b is for $\alpha < 1$, when $L_1 < w_k$ and nn hopping is suppressed. The typical energy barriers determining the resistivity in the various regimes are given in Table 1. The dashed lines in the upper left corner of the plane delimit the region where collective effects are important (see text).

Fig. 3. Schematic representation of the various low-lying excitations discussed in the text: (a) half-loop excitation, (b) double-kink configuration, with $w_k = d\sqrt{\tilde{\epsilon}_1/U}$, and (c) double-superkink configuration required for VRH.

Fig. 4. The (L, J) phase diagram for $\alpha = g(\mu)dU > E_k/T$. In this case there is a region where the TAFF resistivity is controlled by rare regions, both above and below J_L. The width of this region is controlled by the sample size W in the direction of flux motion.

Fig. 5. The localization lengths in the direction transverse to the twin planes: $l_y(T)$ is determined by the pinning potential of the twin and $y^*(T)$ is determined by intervortex interactions. The various temperature scales are discussed in the text.

Fig. 6. Schematic skecth in configuration space (ϵ, y) illustrating that a half-loop excitation corresponds to tunneling of the fictitious quantum mechanical particle directly into conduction band. The straight line of slope $-f_L/U(T)$ is the conduction band edge in the presence of the fictitious electric field due to the Lorentz force. A carrier occupying a localized state at $y = 0$ near the center of the impurity band, i.e., at an energy $\sim U(T)$ below conduction band edge, is brought directly into conduction band by a hop of transverse size $y_{hl} \approx U(T)/f_L$.