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# Interface Motion in Random Media at Finite Temperature

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We have studied numerically the dynamics of a driven elastic interface in a random medium, focusing on the thermal rounding of the depinning transition and on the behavior in the $T = 0$ pinned phase. Thermal effects are quantitatively more important than expected from simple dimensional estimates. For sufficient low temperature the creep velocity at a driving force equal to the $T = 0$ depinning force exhibits a power-law dependence on $T$, in agreement with earlier theoretical and numerical predictions for CDW's. We have also examined the dynamics in the $T = 0$ pinned phase resulting from slowly increasing the driving force towards threshold. The distribution of avalanche sizes $S_{\|}$ decays as $S_{\|}^{-1-\kappa}$, with $\kappa = 0.05 \pm 0.05$, in agreement with recent theoretical predictions.

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## 1. Introduction

The study of the dynamics of driven interfaces in random materials is relevant to a wide class of physical problems, from fluid invasion in porous media [1-3] to the motion of domain walls in random magnets [4,5]. The interface dynamics in these systems is controlled by the competition between an external field which exerts a driving force per unit area $F$ on the interface and the pinning by impurities which impedes the motion. The interface is often modeled as an elastic medium that is distorted by disorder, but cannot break. The dynamics of such elastic interfaces driven through quenched disorder in the absence of thermal fluctuations has been studied extensively both analytically and numerically [1-10]. At zero temperature there is a sharp transition from a sliding state above a critical driving force $F_T$ to a pinned state below $F_T$. The transition has been described as a critical phenomenon in terms of scaling laws and critical exponents [9,10]. The critical exponents depend on the dimensionality of the interface and of the embedding space, as well as on the geometry of the quenched disorder. Thermal fluctuations are expected to round the transition. Closely related systems that exhibit the same type of nonlinear collective dynamics and have been studied very extensively are charge density waves (CDW's) in low dimensional conductors [6-11].

The dynamics of weakly pinned flux lines in type-II superconductors is another problem in this general class [15]. A transport current density $\mathbf{J}$ flowing through a type-II superconductor in the mixed state in the plane normal to the external field exerts a Lorentz force per unit volume $F \sim J$ on the vortex lines, which then moves across the current causing electric fields proportional to the vortex velocity and hence resistance. Impurities and other defects in the material act to pin the vortices and impede their motion. Point-like material impurities, such as $O_2$ vacancies, yield a quenched random pinning potential with short-range correlations.

At low temperatures and fields, when the characteristic pinning energy barriers for a single vortex line exceed the energy associated with intervortex interactions, dissipation is controlled by single vortex dynamics. The problem can then be modeled as that of a one-dimensional elastic interface (string) driven through a random medium. It is important to distinguish between two types of short-range disorder that are relevant in different physical systems. These are usually referred to as random-field (or random-force) disorder and random-bond (or random-potential) disorder. The disorder is of the random-field type for domain walls in disordered ferromagnets [16] or fluid interfaces in porous media [17]. In

this case the pinning energy is the sum of the contributions from all the impurities in the area or volume spanned by the invading fluid during its motion. In contrast, the disorder that controls the dynamics of domain boundaries in ferromagnets with random exchange interactions [18] or flux lines in type-II superconductors is of the random bond or potential type. In this case the pinning energy is determined by the instantaneous position of the interface and it arises only from the impurities in its vicinity. The *static* behavior, i.e., at finite temperature and zero driving force, has been studied extensively and is known to depend strongly on the type of disorder [16,18]. In contrast, Narayan and Fisher recently argued that the critical behavior of *driven* interfaces at zero temperature is essentially independent of the type of disorder [10]. Numerical work has not yet addressed this point conclusively.

In this paper we present results of numerical simulations of the dynamics of an elastic string in quenched disorder of the random bond type. Our earlier studies of this model at zero temperature have focused on the critical properties at the depinning transition and the dynamics in the sliding state [19,8]. We found that at $T = 0$ there is a sharp transition at a threshold force $F_T$ from a state where the string is pinned for $F < F_T$ to a sliding state for $F > F_T$. The mean velocity vanishes as the threshold is approached from the sliding state as $v \sim f^{\beta}$ with $f = |F - F_T|/F_T$ and $\beta = 0.24 \pm 0.1$ [20]. The data can also be fit by a form $v \sim 1/ln|f|$. The spatial range of velocity correlations in the sliding state is determined by a correlation length $\xi$ that diverges at threshold as $\xi \sim f^{-\nu}$, with $\nu = 1.05 \pm 0.1$. Due to the effect of random forces the driven interface profile is rough and can be characterized by a roughness exponent $\zeta$, defined by

$$< \overline{\Big[u(z,t) - u(0,t)\Big]^2} >\sim |z|^{2\zeta}, \tag{1.1}$$

where $u(z,t)$ is the instantaneous position of a point on the string. Our numerical work at $T = 0$ yielded $\zeta = 0.97 \pm 0.05$. Numerical studies of related lattice models have yielded $\zeta = 1.25 \pm 0.01$ [21]. The continuous elastic model of interface dynamics was studied recently by Narayan and Fisher [10] at $T = 0$ via a systematic expansion in $\epsilon = 5 - d$, where $d$ is the dimensionality of the embedding space and 5 the upper critical dimensionality. For the case $d = 2$ of interest here they found $\nu = \zeta = 1$ exactly to all order in $\epsilon$ and $\beta = 1 - 2\epsilon/9 + \mathcal{O}(\epsilon^2) = 1/3 + \mathcal{O}(\epsilon^2)$. These results are consistent with those obtained from our numerical work.

We have now extended our work in two ways. First we have considered the string dynamics in the presence of both quenched disorder and thermal fluctuations. Secondly,

we have investigated the properties of the zero temperature pinned phase below threshold. As expected, thermal fluctuations round the depinning transition and replace it by a smooth crossover. For temperatures well below a characteristic depinning temperature $T_{dp} \sim (\Delta K R_p^3)^{1/3}$, where $\Delta$ is the strength of the quenched disorder (of range $R_p$) and $K$ the string elastic constant, the temperature dependence of the mean velocity at the zero temperature threshold $F_T$ scales with temperature as $v(F_T, T) \sim T^{\beta/\tau}$, where $\beta$ is the $T = 0$ depinning exponent and $\tau$ is a nonuniversal exponent, with $\tau = \frac{3}{2}$ for continuous pinning potentials (see Figure 1). This scaling form was predicted some time ago [14] by D. S. Fisher through a mean field theory for the Fukuyama-Lee-Rice model of CDW's. It was also recently verified numerically for both continuous and discrete CDW models in two and three dimensions [22]. It is not *a priori* obvious that interfaces with short-range pinning potentials should behave like models of CDW's, where the pinning potential is periodic - and therefore correlated - along the direction of motion. In fact the critical exponents describing the $T = 0$ depinning transition are different in these two classes of models. On the other hand, our results indicate that in both cases the dynamics at finite temperatures occurs via thermally induced hopping over small energy barriers and is insensitive to the details of the pinning potential, provided the time scale of the thermal hop is sufficiently short. The mechanism for destabilization of the soft modes of the system is the same for models with short-range pinning potentials and CDW's and the mean field argument proposed by Fisher applies in both cases. The details of the pinning potential only enter through the $T = 0$ depinning exponent, $\beta$. The string dynamics for $T > T_{dp}$ is qualitatively different and the scaling just described fails. This difference is clarified by the high force perturbation theory discussed in Section 3. Numerical studies of interface dynamics in $1 + 1$ dimensions at finite temperature were carried out recently by Kaper et al. [23]. Our work on thermal effects complements this earlier studies by focusing on the scaling of velocity with temperature for $T < T_{dp}$, that was not discussed in [23], and on the qualitative difference in the string dynamics at low ($T < T_{dp}$) and high ($T > T_{dp}$) temperatures.

We have also investigated the properties of the zero temperature pinned phase below threshold. The response of the string when the driving force is increased towards threshold from below is dominated by localized forward jumps or "avalanches". The number distribution of avalanche sizes $S_\parallel$ in the string direction, $D(S_\parallel; f)$, was conjectured to

obey a scaling form near threshold [10], characterized by a diverging correlation length $\xi_- \sim (F_T - F)^{-\nu_-}$,

$$D(S_\parallel; f) = \frac{1}{S_\parallel^{1+\kappa}} \hat{D}(S_\parallel/\xi_-), \tag{1.2}$$

where $D(S_\parallel; f) dS_\parallel df$ is the number of avalanches of diameter between $S_\parallel$ and $S_\parallel + dS_\parallel$ that occur when the reduced force is changed from $f$ to $f + df$. Also, $\hat{D}(x)$ is a scaling function that decays rapidly for $x >> 1$. Using an $\epsilon$ expansion in $5-\epsilon$ dimensions, Narayan and Fisher predicted $\kappa = 0$ and $\nu_- = \nu = 1$, with $\nu$ the correlation length exponent when the transition is approached from above threshold [10].

We have studied the scaling of avalanches numerically by evaluating the distribution $\mathcal{D}(S_\parallel)$ of avalanche sizes $S_\parallel$ obtained in response to a small increase of the driving force for all forces below $F_T$,

$$\mathcal{D}(S_\parallel) = \int_{-1}^{0} df D(S_\parallel; f) \sim \frac{1}{S_\parallel^{1+\kappa'}}, \tag{1.3}$$

and we find $\kappa' = 1.0 \pm 0.2$. The exponents $\kappa$ and $\kappa'$ are related by

$$\kappa' = \kappa + \frac{1}{\nu_-}. \tag{1.4}$$

We find $\kappa' = 1.0 \pm 0.2$ and $\kappa = 0.05 \pm 0.05$, corresponding to $\nu_- = 1.13 \pm 0.30$, consistent with the theoretical values. The value $\kappa = 0$ indicates that avalanches of large size are likely and the description of the interface or as an elastic string breaks down. This is also consistent with our result that the roughening exponent $\zeta$ defined as $A \sim S_\parallel^{1+\zeta}$, where $A$ is the area of an avalanche, is $\zeta = 1.0 \pm 0.05$. As discussed by Coppersmith [24] in the context of CDW, large strain may develop in the $d = 2$ case of interest here. In a real system the string will relax these strains by pinching off a vortex loop around an impurity, a mechanism that is excluded in our model. Overhangs and loops could modify qualitatively both the dynamics in the pinned region and the $T = 0$ depinning transition. Further studied are needed to address their role.

The remainder of the paper is organized as follows. After briefly describing the model in section 2, we discuss simple dimensional estimates and the results of the high field perturbation theory in section 3. The numerical result at finite temperature are presented in section 4, while the scaling of avalanches in the $T = 0$ pinned state is discussed in section 5.

## 2. The Model

The specific model we have studied is an elastic string embedded in two dimensions and on the average aligned with the $z$ direction (in superconductors this is the direction of the applied magnetic field). We describe the interface by its displacement $u(z,t)$ in the direction of the applied force per unit length, $F$. The displacement is assumed to be a single-valued function of $z$ at any time $t$, i.e., we ignore overhangs in the string and vortex loops that could be pinched off from it during the motion. In the absence of driving force the Hamiltonian of the string is the sum of the elastic energy of the string and a pinning potential from random impurities in the medium,

$$H = \int_0^L \Big\{ \frac{K}{2} (\frac{\partial u}{\partial z})^2 + V(u(z,t);z) \Big\}, \tag{2.1}$$

where $L$ is the size of the system in the $z$ direction and $K$ the elastic constant. Assuming purely relaxational dynamics, the equation of motion for the string in the presence of a constant driving force $F$ per unit length in the transverse direction is given by

$$\begin{aligned} \gamma \partial_t u(z,t) =& -\frac{\delta H}{\delta u} + F + \eta(z,t) \\ =& K\partial_z^2 u(z,t) + F_p(u;z) + F + \eta(z,t), \end{aligned} \tag{2.2}$$

where $\gamma$ is a friction coefficient, $F_p(u;z) = -\partial V(u;z)/\partial u$ is the pinning force per unit length, and $\eta(z,t)$ is a Gaussian correlated Langevin force per unit length describing thermal noise, with $< \eta(z,t) >= 0$ and correlations

$$< \eta(z,t)\eta(z',t') >= 2\gamma T \delta(z-z')\delta(t-t'). \tag{2.3}$$

The angular brackets $< \cdots >$ denote the thermal average. In the numerical model the pinning potential is written explicitly in terms of the interaction of the string with $N_p$ short-ranged pinning centers randomly distributed at positions $\vec{R}_i$ in the plane,

$$V(u;z) = \sum_{i=1}^{N_p} U(|\vec{r} - \vec{R}_i|), \tag{2.4}$$

where $\vec{r} = (u(z,t), z)$. The interaction $U(|\vec{r} - \vec{R}_i|)$ of the string with the $i^{th}$ pin is approximated by a potential well centered at the pin location $\vec{R}_i$ of finite range $R_p$ and maximum depth $U_0$. The pins are uniformly distributed with areal density $n_p$. When the mean pin spacing $1/\sqrt{n_p}$ is large compared to the potential range $R_p$, the pinning potential can also

be described as a continuum gaussian random variable, with mean and correlations given by,

$$\overline{V(u;z)V(u';z')} \sim \Delta\delta(u-u')\delta(z-z'), \tag{2.5}$$

where

$$\Delta \approx (U_0 R_p)^2 n_p R_p^2 \{1 + O(n_p R_p^2)\} \tag{2.6}$$

and the overbar denotes an average over disorder realizations.

The overall motion of the string is described by a "center of mass" velocity, defined as

$$v_{cm}(F,t) = \frac{1}{L}\int_0^L dz v(z,t), \tag{2.7}$$

where $v(z,t) = \partial_t u(z,t)$ is the instantaneous velocity of a point on the string. The center of mass velocity is a fluctuating quantity since it depends of both the random positions of pins and the thermal noise. The average or drift velocity of the string is given by

$$v(F) = \overline{< v_{cm}(F,t) >}. \tag{2.8}$$

In the numerical calculation the average over different realizations of disorder (denoted by the overbar) is performed by averaging over time, since as time evolves the string samples different impurity configurations.

## 3. Dimensional estimates and perturbation theory

Considerable insight on the behavior of the driven string can be gained by simple dimensional estimates and by a perturbation theory in the pinning force. Most of the results discussed in this section have been obtained elsewhere for a general d-dimensional interface [25]. It is, however, instructive to summarize them here for the $1+1$ dimensional case of interest.

The relative importance of thermal fluctuations and quenched disorder in governing the *equilibrium* properties of elastic interfaces, in the absence of external drive, has been studied extensively. Thermal fluctuations always dominate at small length scales. In this regime they are responsible for small amplitude vibrations of the interface within a given pinning well and therefore lead to a smoothing of the pinning potential. At large length scales the importance of thermal fluctuations depends on dimensionality. For a one dimensional interface in $d = 2$ the disorder is always dominant at large length scales. This

is because the value of the roughness exponent $\zeta_{eq}$ of an elastic string disordered only by random bond quenched disorder, $\zeta_{eq}(d=2) = 2/3$, is larger than the corresponding value for a thermal string in equilibrium, $\zeta_{th} = 1/2$.

A disordered elastic interface in the absence of thermal fluctuations is characterized by the Larkin-Ovchinnikov (LO) collective pinning length, $L_c$. This is estimated by dimensional analysis by considering the energy fluctuation associated with displacing a segment of string of length $L$ by a transverse distance $R_p$ across a single pinning well,

$$\delta F = K\frac{R_p^2}{L} + (U_0 R_p)\sqrt{n_p R_p L}. \tag{3.1}$$

The *LO* pinning length is obtained by minimizing (3.1), with the result $L_c \approx R_p\big(K/U_0\sqrt{n_p R_p^2}\big)^{1/3}$. When $L_c >> R_p$, the string is pinned collectively by many pins. It is this weak pinning regime that is relevant for flux lines in superconductors. When thermal fluctuations are important the mean thermal displacement of the string exceeds $R_p$ and the string experiences a random potential averaged over the length of its root mean square thermal excursion about equilibrium, $< u_{th}^2 >^{1/2}$. The temperature dependent pinning length is estimated by considering the energy associated with a fluctuation of length $L$ and transverse distance $< u_{th}^2 >^{1/2}$,

$$\delta F = K\frac{< u_{th}^2 >}{L} + U_0 R_p\Big[n_p R_p L\frac{R_p}{< u_{th}^2 >^{1/2}}\Big]^{1/2}, \tag{3.2}$$

where $< u_{th}^2 >$ is determined by requiring $K < u_{th}^2 > /L \sim T$. We find $L_c(T) \approx L_c(T/T_{dp})^5$, where the depinning temperature $T_{dp}$ is defined by $< u_{th}^2 > \approx L_c(T)T/K = R_p^2$ , which yields $T_{dp} \approx (U_0 R_p)^{2/3}(n_p R_p^3 K)^{1/3}$. The temperature-dependent pinning length is defined by interpolating between the $T = 0$ LO length and the finite temperature result,

$$L_c(T) = L_c[1 + (T/T_{dp})^5]. \tag{3.3}$$

A more rigorous derivation of these results can be found in [25].

Considerable insight on the competing roles of quenched disorder and thermal noise can be gained by considering the string dynamics at large velocities, where the disorder can be treated as weak. For $F >> F_T$, the effect of pinning is negligible and the string advances uniformly, with $v \approx v_0 = F/\gamma$. The deviations from this asymptotic behavior can be studied by a perturbation theory in $F/F_p$ that was introduced first by Schmid and Hauger and by Larkin and Ovchinnikov for $T = 0$ and recently extended by Vinokur et al. [26,27]

to incorporate thermal fluctuations. To carry out the perturbation theory it is convenient to consider a frame of reference moving at the mean velocity $v$ of the string. The instantaneous position of a point on the string is then written as $u(z,t) = vt + u_p(z,t) + u_{th}(z,t)$, where $u_p(z,t)$ is the deformation due to quenched disorder, treated as small, and $u_{th}(z,t)$ is the contribution from thermal fluctuations to the displacement from the uniform sliding state. It is defined as the solution of,

$$\partial_t u_{th}(z,t) = K\partial_z^2 u_{th}(z,t) + \ \eta(z,t). \tag{3.4}$$

The mean velocity is then given by

$$\delta v = \frac{1}{\gamma} < \overline{F_p(vt + u_p + u_{th}; z)} >, \tag{3.5}$$

with $\delta v = v - v_0$ and

$$\gamma\partial_t u_p(z,t) = K\partial_z^2 u_p(z,t) + F_p\big(vt + u_p(z,t) + u_{th}(z,t)\big). \tag{3.6}$$

The right hand side of Eq. (3.5) is evaluated in perturbation theory by treating $u_p$ as small. The first order correction to the average string velocity as compared to the asymptotic value $v = v_0$ is given by the self-consistent solution of the equation

$$\frac{\delta v}{v} = \frac{\Delta}{\gamma}\int_{-\infty}^{+\infty}\frac{dk}{2\pi}k^3|p(k)|^2\int_0^\infty dt\frac{\sin kvt}{v}G(0,t)S_s(k,t), \tag{3.7}$$

where $p(k)$ is the single-vortex form factor that provides a short-length scale cutoff at $k \sim 1/R_p$, with $k$ the wavevector in the direction of motion. Also, $G(0,t)$ is the vortex Green's function evaluated at $z = 0$, given by

$$G(0,t) = \Theta(t)\int{}' \frac{dq}{2\pi}\frac{1}{\gamma}e^{-q^2Kt/\gamma}. \tag{3.8}$$

The prime on the integration over the wavevector $-\infty < q < +\infty$ in the $z$ direction denotes a short distance cutoff at $|q| \sim 2\pi/R_p$ which controls the crossover to single particle behavior at short times. Thermal fluctuations are responsible for the appearance in Eq. (3.7) dynamical structure factor $S_s(k,t)$ of a single vortex in the absence of disorder, given by

$$S_s(k,t) =< e^{ik[u_{th}(z,t) - u_{th}(z,0)]} > \approx e^{-k^2<[u_{th}(z,t) - u_{th}(z,0)]^2>/2}. \tag{3.9}$$

In the absence of thermal fluctuations $S_s(k,t) = 1$ and the cutoff $k_0$ on the $k$-integration in Eq. (3.7) is provided by the form factor $p(k)$, i.e. $k_0 \sim 1/R_p$. At finite temperature this cutoff is replaced by $k_0 \sim [R_p^2 + < u_{th}^2(t) > /2]^{-1/2}$.

By using the fluctuation-dissipation theorem the mean square thermal displacement can be expressed in terms of the Green's function given in Eq. (3.8), with the result

$$< u_{th}^2(t) >= \frac{2T}{K} \int^{'} \frac{dq}{2\pi} \frac{1}{q^2} \Big(1 - e^{-Kq^2 t/\gamma}\Big). \tag{3.10}$$

The $q$-integration on the right hand side of Eq. (3.10) is easily carried out, with the result

$$\begin{aligned} < u_{th}^2(t) > &\approx \frac{T}{\pi\sqrt{K\gamma}}\sqrt{t}, \qquad t >> t_0, \\ &\approx \frac{4T}{\gamma R_p} t, \qquad t << t_0, \end{aligned} \tag{3.11}$$

where $t_0 = R_p^2/D_0$, with $D_0 = K/\gamma$ the diffusion constant of a point vortex, is the time scale for diffusion across the range $R_p$ of the pinning potential. For $t << t_0$ different bits of string of length $\leq R_p$ are essentially uncorrelated along the $z$ direction and one recovers single-particle behavior, $< u_{th}^2(t) > \sim t$.

As discussed earlier, at large times or long length scales disorder is always important and the long time divergence of Eq. (3.11) is cut off by disorder. The effect of disorder can be approximately incorporated in Eq. (3.10) by introducing a large distance cutoff at $q \sim 2\pi/L_c(T)$ in the q-integration. Neglecting for simplicity the crossover to single particle behavior, we obtain three distinct regimes

$$\begin{aligned} < u_{th}^2(t) > \ &< R_p^2, \qquad t_0 < t < t_{ph} = t_0 \Big(R_p K/T\Big)^2, \\ < u_{th}^2(t) > \ &\approx \frac{T}{\pi}\sqrt{\frac{t}{K\gamma}}, \qquad t_{ph} < t < t_d = t_{ph}\Big(T/T_{dp}\Big)^{12}, \\ < u_{th}^2(t) > \ &\approx \frac{TL_c(T)}{K}, \qquad t > t_d. \end{aligned} \tag{3.12}$$

We have introduced here two new time scales. The time $t_{ph}$ characterizes the time scale for small phonon-like vibrations of the string within the potential well of a single pin, while $t_d$ is the time scale where disorder becomes dominant. The intermediate regime in Eq. (3.12) only occurs provided $t_{ph} < t_d$ which corresponds to $T > T_{dp}$.

We now proceed to approximately evaluate the correction $\delta v$ given in Eq. (3.7) . The right hand side of Eq. (3.7) can be reduced to a one-dimensional integral that can be

evaluated numerically. It is, however, more instructive to simply carry out a dimensional estimate of the integral. Neglecting for now the short distance cutoff in the $q$-equation in Eq. (3.8) , we obtain $G(0,t) = \Theta(t)\sqrt{\gamma/\pi K t}$. Inserting this on the right hand side of Eq. (3.7) and noting that the main contribution to the time integration comes from $kvt \sim 1$, we obtain

$$\frac{\delta v}{v} = \frac{\Delta}{\sqrt{\pi\gamma K}} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} k^4 \int_0^{1/kv} dt \sqrt{t} e^{-k^2[R_p^2 + < u_{th}^2(t) > /2]}. \tag{3.13}$$

As the driving force $F$ - and therefore the string mean velocity $v$ - increases, the time cutoff $1/kv$ decreases. Inserting on the right hand side of eq. (3.13) the form for $< u_{th}^2(t) >$ from Eq. (3.12) appropriate to each time regime and carrying out the integration, we obtain

$$\begin{aligned}
&\frac{\delta v}{v} \sim \frac{\Delta}{\gamma R_p^3 \sqrt{K\gamma R_p}} \frac{1}{v^{3/2}}, && t_0 < \frac{R_p}{v} < t_{ph}, \\
&\frac{\delta v}{v} \sim \frac{2\Delta}{15} \left( \frac{2^7 K^2 \gamma^2}{\pi T^7} \right)^{1/3} \frac{1}{v^{1/3}}, && t_{ph} < \frac{R_p}{v} < t_d, \\
&\frac{\delta v}{v} \sim \frac{2\Delta}{15} \frac{1}{\sqrt{\pi K \gamma}} \left( \frac{\epsilon}{T L_c(T)} \right)^{7/2} \frac{1}{v^{3/2}}, && t_d < \frac{R_p}{v},
\end{aligned} \tag{3.14}$$

or

$$\begin{aligned}
&1 - \frac{v}{v_0} \sim F^{-3/2}, && \frac{T^2}{K R_p^3} < F < \frac{K}{R_p}, \\
&1 - \frac{v}{v_0} \sim F^{-1/3}, && \frac{T_{dp}^2}{K R_p^3} \left( \frac{T_{dp}}{T} \right)^{10} < F < \frac{T^2}{K R_p^3}, \\
&1 - \frac{v}{v_0} \sim F^{-3/2}, && F < \frac{T_{dp}^2}{K R_p^3} \left( \frac{T_{dp}}{T} \right)^{10}.
\end{aligned} \tag{3.15}$$

For $F > K/R_p$, corresponding to the case where the time cutoff is realized at times shorter than $t_0$, the string is sliding so fast that correlations among different string elements have no time to develop and one recovers single particle behavior, with $\delta v/F \sim F^{-2}$. The results summarized in Eqs. (3.15) show that the shape of the $v$-$F$ curve is qualitatively different at high and low temperatures. The perturbation theory yields $v/F \sim 1 - CF^{-\alpha}$, where C is a constant that depend on temperature and the value of $\alpha$ in the various regimes can be inferred from Eqs. (3.15). The curvature of the $v$-$F$ curve is determined by $\frac{d^2 v}{d^2 F} \sim -\alpha(\alpha - 1) F^{-\alpha - 1}$ and is positive if $\alpha < 1$ and negative if $\alpha > 1$. If $T << T_{dp}$ the intermediate region described by the second of Eqs. (3.15) does not occur and the perturbation theory yields $\alpha = 3/2$ in the entire region where perturbation theory applies,

up to $F \sim K/R_p$. The $v$-$F$ curve has a negative curvature in this regime and thermal effects only affect the coefficient of the correction $\delta v$. When $T > T_{dp}$ there is an intermediate region described by the second of Eqs. (3.15) where $\alpha = 1/3$ and the $v$-$F$ curve has a positive curvature. This behavior is apparent in our data described in the following section(see Fig. 2).

## 4. Numerical Results at Finite Temperature

We have integrated numerically the discretized version of the equation of motion (2.2) for a string composed of discrete elements, each of dimensionless size $R_p$ in the $z$ direction. Here $R_p$ is chosen as the unit of lengths. All forces are measured in units of the string tension $K$. The string elements interact via nearest neighbor elastic forces and are constrained to move only in the direction of the driving force. This model is appropriate for flux lines in layered superconductors, where each flux line can be thought of as a stack of interacting two-dimensional "pancake" vortices residing in the $CuO_2$ planes. Periodic boundary conditions are imposed in the $z$ direction. Each string element is subject to attractive potential wells of finite size $R_p = 1$ and maximum depth $U_0$ in both the $z$ and $u$ directions, centered at the randomly distributed pin locations. The displacements of the discrete string elements are treated as continuous variables and the coupled equations of motion are integrated by a fourth order Runge-Kutta algorithm with a time step much smaller than the typical time to cross a single potential well (typically $\Delta t \sim 0.1$, where time is measured in units of $t_0 = R_p^2/D_0$). As in our $T = 0$ simulations, we have been able to obtain reliable data at very small velocities thanks to our "pinning cells" method. The method consists in dividing the plane in "pinning cells" of dimension $R_p$. At $T = 0$ the string can only move forward and each section of the string only "sees" the disorder in the pinning cell that neighbors it in the forward direction of motion. Thermal fluctuation can also kick the string to move backward, in the direction opposite to that of the driving force. At every iteration each string element needs to know the disorder in the cells that it has left behind, since these may be revisited. We have developed an algorithm to store at each step an adjustable number of "past" pinning cells for each section of the string, in addition to the "future" cell. The list of past cells is updated in parallel at each time step. For driving forces within 10% of the zero temperature threshold, the length of the simulation usually exceeds $10^6$ time steps, while shorter simulation times gave good averages for forces further from threshold. We have investigated systems of size from 256

to 16384. The computations were performed on the Connection Machine CM-2 and CM-5. In our dimensionless units the parameters of the model are the dimensionless pinning force $\tilde{F}_p = U_0/K$, the dimensionless areal density of pins, $\rho = n_p R_p^2$, and the dimensionless driving force $\tilde{F} = FR_p/K$. In these units the temperature is measured in units of $KR_p$ and the depinning temperature is given by $\tilde{T}_{dp} = T_{dp}/K = \tilde{F}_P^{2/3}\rho^{1/3}$ and the LO collective pinning length is $\tilde{L}_c = \tilde{F}_p^{-1/3}\rho^{-1/6}$, yielding a dimensionless estimate for threshold force $\tilde{F}_T = \rho\tilde{F}_p$ and $\tilde{F}_T = \rho^{2/3}\tilde{F}_P^{4/3}$ for strong and weak pinning, respectively. We will always refer to the dimensionless quantities below and drop the tilde to simplify the notation.

Our simulations were carried out for $F_p = 1$ and $\rho = 0.1$, yielding $L_c = 1.5$ and $T_{dp} = 0.46$. The dimensional estimate for the threshold force gives $F_T = 0.1$, which is consistent with our result $F_T = 0.2435 \pm 0.005$ obtained from the simulations. The mean velocity of the string is shown in Fig. 2 for various temperatures and system sizes. For very large driving forces the effect of pinning is negligible and the string advances uniformly, with $v \approx F$. For $T \neq 0$ there is no sharp transition and the velocity at low driving forces is small but finite. The velocity versus driving force ($v$-$F$) curve always exhibits a tail with positive curvature below the $T = 0$ threshold. At higher driving forces the curvature of the $v$-$F$ curve changes sign and eventually approaches the asymptotic limit $v \sim F$. At $T = 1.0 \times 10^{-2}$, well below our dimensional estimate of $T_{dp} \approx 0.46$, thermal effects have already washed out completely the depinning transition. Thermal effects are quantitatively more pronounced than expected from dimensional estimates.

We distinguish three regimes characterizing the string response at finite temperature. At high driving forces the deviation from the asymptotic behavior $v \sim F$ are well described by the perturbation theory discussed in section 3. If $T << T_{dp}$ our simulation agree with $\delta v/F \sim F^{-3/2}$(with a crossover to $\delta v/F \sim F^{-2}$ at very large driving force) and the $v$-$F$ curve has a negative curvature in this regime. At higher temperature the $v$-$F$ curve shows an intermediate region with positive curvature where $\delta v/F \sim F^{-1/3}$. The qualitative difference in the shape of the $v$-$F$ curve at high and low temperature is consistent with the results of the perturbation theory discussed at the end of Section 3. The value of $T$ above which thermal effects change qualitatively the response is, however, much lower than the value obtained for $T_{dp}$ from dimensional estimates. Finally, at very high forces, $1 - v/F \sim F^{-2}$. This crossover is controlled by the range of the pinning potential correlations in the $z$ direction.

For driving forces within a few percent of the zero temperature threshold and sufficiently low temperature, the mean velocity exhibits the scaling behavior proposed by

Fisher in the context of CDW's. He argued that thermally-induced "hops" over pinning energy barriers are analogue to jumps resulting from ramping up the force at $T = 0$. The velocity evaluated at the $T = 0$ threshold is found to scale with temperature as $v(F = F_T, T) \sim T^{\beta/\tau}$, with $\beta = 0.24$ and $\tau = 3/2$, as shown in Fig. 1. In addition in the region of low $T$ and $F$ close to $F_T$, our data can be fit to the scaling form proposed by Fisher

$$v(f, T) \sim T^{\beta/\tau} B(fT^{-1/\tau}), \tag{4.1}$$

as shown in Fig. 3. Here $B(x)$ is a scaling function that behaves as $B(x) \sim x^{\beta}$ for $x \to \infty$. This scaling fails, however, at higher temperatures.

Finally, at very small driving forces ($F << F_T$) and low temperatures ($T << T_{dp}$) the dynamics occurs via creep over pinning energy barriers and $v \sim e^{-U(F)/T}$. The pinning energy barrier $U(F)$ has been predicted to diverge when $F \to 0$ as $U(F) \sim F^{-\mu}$. The creep exponent $\mu$ has been estimated by dimensional analysis as $\mu = 1/4$. It should, however, be noticed that this result was obtained by assuming that the roughening exponent of the string has the equilibrium value $\zeta_{eq} = 2/3$. Our numerical results in this region are not inconsistent with a small value of $\mu$, as shown in Fig. 4, but are insufficient to either confirm or discard the assumption that the creep dynamics is controlled by the single diverging energy scale $U(F)$ and to determine the creep exponent conclusively. A different approach may be needed to address this point.

## 5. Avalanches in the $T = 0$ pinned state

In this section we discuss the critical behavior of the interface at zero temperature in the pinned region, as the threshold $F_T$ is approached from below by slowly increasing the driving force. As discussed recently by Narayan and Fisher [10] for the interface problem and in more detail by Narayan and Middleton [28] for CDW's, local instabilities occurs as $F$ is increased towards threshold resulting in "avalanches" of various size. The notion of avalanches was introduced to describe the large response to a local perturbation in models exhibiting self-organized criticality (SOC)[29]. Here we discuss avalanches obtained in response to a small increase of the driving force, which provides a global perturbation that affects equally all the discrete string elements. This perturbation, if sufficiently small, will, however, only trigger instabilities locally. The resulting response consists of discontinuous local jumps of portions of the interface (i.e., avalanches), with a distribution of sizes. The size of an avalanche can be defined in terms of its area (or moment)$A$ or of its diameter

$S_\parallel$ in the direction of interface. We start the system in a pinned configuration at a driving force $F < F_T$. Increasing the reduced force $f = F/F_T - 1$ from f to $f + df$ triggers a forward jump of sections of the string, until a new metastable pinned state is reached. The avalanche is characterized by the total area $A$ swept by the string as a result of the increase in driving force,

$$A = \int_0^{S_\parallel} dz u(z). \tag{5.1}$$

Assuming $u(z) \sim z^\zeta$, with $\zeta$ a roughening exponent, the area and diameter of the avalanche are related by

$$A \sim S_\parallel^{1+\zeta}. \tag{5.2}$$

Figure 5 shows a plot of the area of the avalanches versus their diameter. The straight line has slope 2 and we find $\zeta = 1 \pm 0.05$.

Fisher and Narayan have conjectured that the distribution of avalanche sizes near threshold obeys the scaling form given in Eq. (1.2)[10]. Using Eq.(5.2), one can also immediately obtain the scaling form for the number distribution of avalanche areas, $D_A(A; f)$, given by,

$$D_A(A; f) = \frac{1}{A^{1+\kappa_A}} \hat{D}(A^{1/(1+\zeta)}/\xi_-), \tag{5.3}$$

with

$$\kappa_A = \frac{\kappa}{(1+\zeta)}. \tag{5.4}$$

Rather than considering the distribution of avalanches at a fixed force $f$, we have evaluated numerically the distribution of avalanche areas and diameters integrating over all driving force below $F_T$. The total distribution of avalanche size is given in Eq. (1.3). The corresponding distribution of avalanche areas is given by

$$\mathcal{D}_A(A) = \int_{-1}^{0} df D_A(A; f) \sim \frac{1}{A^{1+\kappa'_A}}. \tag{5.5}$$

The corresponding exponents are related to the exponents defined in (1.2)and (5.3)by

$$\begin{aligned} \kappa' &= \kappa + \frac{1}{\nu_-} \quad , \\ \kappa'_A &= \kappa_A + \frac{1}{\nu_-(1+\zeta)} \quad . \end{aligned} \tag{5.6}$$

To generate avalanches, we start with the string in a metastable pinned configuration well below $F_T$ and study the response to a small increase $\Delta F$ of the force (we have used

$\Delta F = 1 \times 10^{-4}$ in most of our simulations). The increase in force triggers local avalanches which are recorded, until the string reaches a new pinned configurations. The procedure is then repeated by stepping up again the force of an amount $\Delta F$ until $F_T$ is reached. If, however, the string slides as a whole in response to the small perturbation, we discard this event, return to the original pinned configuration and repeat the procedure with a smaller $\Delta F$. These "giant avalanches" are discarded because they are a finite-size effect, characteristic of the response of finite systems above threshold. The distributions are shown in Figs. 6a and 6b for two system sizes. They exhibit a power-law decay with $\kappa' = 1.0 \pm 0.2$ and $\kappa'_A = 0.5 \pm 0.15$.

In order to gain further insight in the "shape" of the avalanches, we define the number distribution $D_u(\Delta u; f)$ of the local displacement advances $\Delta u$ following the ramping of the force. More precisely $D_u(\Delta u; f)d(\Delta u)df$ is the number of displacement advances between $\Delta u$ and $\Delta u + d(\Delta u)$ that occur when the reduced force is increased from $f$ to $f + df$. To develop a scaling ansatz for $D_u(\Delta u; z)$ we proceed as follows. Let $n(\Delta u|A)d(\Delta u)$ be the conditional number of displacements between $\Delta u$ and $\Delta u + d(\Delta u)$ that occurs within a given avalanche of fixed area $A$. We assume that the transverse shape of the avalanche (in the direction of motion) can be characterized by a single length scale $S_\perp$, defined as

$$S_\perp = A/S_\parallel. \tag{5.7}$$

then conjecture a scaling ansatz for $n(\Delta u|A)$ of the form

$$n(\Delta u|A) = \frac{S_\parallel}{S_\perp}\hat{n}(\Delta u/S_\parallel), \tag{5.8}$$

where $\hat{n}(x)$ is a scaling function that depends on the detailed shape of the string and the prefactor in Eq. (5.8) is determined by the normalization condition

$$\int_0^{S_\perp} d(\Delta u) n(\Delta u|A) = S_\parallel. \tag{5.9}$$

The distribution of displacements $D_u(\Delta u; f)$ can then be written as

$$D_u(\Delta u; f) = \int dA D_A(A; f) n(\Delta u|A). \tag{5.10}$$

By inserting Eq. (5.8)in Eq. (5.10)and making use of Eq. (5.2)and (5.7), we obtain

$$D_u(\Delta u; f) \sim \frac{1}{\Delta u^{1+\kappa_u}}, \tag{5.11}$$

with

$$\kappa_u = -\frac{1}{\zeta} + \kappa_A\big(1 + \frac{1}{\zeta}\big). \tag{5.12}$$

Finally, the number distribution of the local displacements $\Delta u$ integrated over all forces up to $F_T$ decays as

$$\mathcal{D}_u(\Delta u) \sim \frac{1}{\Delta u^{1+\kappa'_u}}, \tag{5.13}$$

with

$$\kappa'_u = \kappa_u + \frac{1}{\zeta \nu_-}. \tag{5.14}$$

The distribution $\mathcal{D}_u(\Delta u)$ can be evaluated with excellent statistics and is shown in Fig. 7. We find $\kappa'_u = 0.0 \pm 0.10$.

The primed exponents $\kappa', \kappa'_A$ and $\kappa'_u$ governing the scaling of the distribution of avalanches for all forces below $F_T$ are related by the same relations (5.4)and (5.12)that hold among the unprimed exponents,

$$\begin{aligned} \kappa'_A &= \frac{\kappa'}{(1+\zeta)}, \\ \kappa'_u &= \kappa'_A + \frac{1}{\zeta}(\kappa'_A - 1). \end{aligned} \tag{5.15}$$

The exponents obtained from our numerics satisfy well these scaling relations.

The correlation length exponent $\nu_-$ can be inferred from (5.6) or (5.14) if at least one of the distribution at fixed driving force is computed numerically. This is in general more difficult since the distribution is quite sensitive to the value of the force increment used, which needs to be made very small. We have evaluated numerically $D(S_\parallel; f)$ for three different driving force. The distribution for $f = 0.063$ is shown in Fig. 8. We find $\kappa = 0.05 \pm 0.05$. Using the first of Eq. (5.6)and the value $\kappa' = 1.0 \pm 0.2$ quoted earlier, we obtain $\nu_- = 1.13 \pm 0.30$. The result is again consistent with theoretical predictions. Finally, as a consistency check, we can now use $\nu_- = 1.13 \pm 0.30$ in the second of Eqs. (5.6)and Eq. (5.14)to obtain $\kappa_A = 0.07 \pm 0.22$ and $\kappa_u = -0.95 \pm 0.15$. These in turn satisfy the scaling relationships (5.4)and (5.12) .

The distribution of avalanches in discrete interface growth models has been studied numerically by Sneppen [29] and by Sneppen and Jensen [30]. In their model the growing interface is maintained in a "critical state" by a local growth rule similar to that used in invasion percolation models that prevents the formation of overhangs. The spatial and temporal correlations between successive growth events and avalanches in this model

have been studied extensively by Leschhorn and Tang [31]. They find a rather complex behavior where the distribution of growth events shows dynamical scaling only locally. This is because in the Sneppen model the driving force is self-tuned to maintain the interface in a "critical state" at the onset of steady state motion, thereby introducing additional spatial and temporal inhomogeneities in the model.

In contrast, in our continuous model the distributions of various measures of avalanche sizes displays a well defined dynamical scaling characterized by a single correlation length $\xi_- \sim |f|^{-\nu_-}$ as the threshold is approached from below. The correlation length exponent $\nu_-$ equals the correlation length exponent $\nu$ obtained when the transition is approached from above. On the other hand, the values of the scaling exponent of the avalanche size distribution, $\kappa \approx 0$, and of the roughness exponent, $\zeta \approx 1$, indicate that the interface may develop large local gradients, which were explicitly excluded in our model. A physical interface will relax these large strains by pinching off loops around the pinning centers, a mechanism that may modify qualitatively the dynamics in the critical region both below and above threshold. In particular this could reduced considerably the value of the roughness exponent $\zeta$. Further studies of a more general model that incorporates strong elastic nonlinearities and allows for overhangs and loop generation are clearly needed to address these questions.

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## Figure Captions

Fig. 1. Creep velocity as a function of dimensionless temperature $T$ at the $T = 0$ threshold force $F_T$. The sample size is indicated. The line shows the slope $\beta/\tau = 0.16$ for the exponents values discussed in the text.

Fig. 2. Creep velocity as a function of dimensionless driving force at various temperatures. The straight line shows the asymptotic uniform velocity $v \sim F$. The data shown summarize results for three different system size : $L = 1024, 4096, 16384$. The same symbol is used for different system size as no significant size dependence was observed.

Fig. 3. Scaled creep velocity $vT^{-\beta/\tau}$ versus scaled reduced force $fT^{-1/\tau}$ for the three lowest temperatures of Fig. 2, using $\tau = 1.5$ and $\beta = 0.24$.

Fig. 4. Log-log plot of the energy barrier $U(F)$ defined as $U(F) = T\log(F/v)$ versus driving force in the low temperature creep region(from $T = 3.16 \times 10^{-4}$ to $T = 1.00 \times 10^{-2}$). The dashed line has slope $-1/4$.

Fig. 5. The area of avalanches obtained by successively stepping up the driving force by small increments $f \approx 4.0 \times 10^{-4}$ for $-1 \leq f \leq 0$ versus their diameter in the direction of the interface for both $L = 1024$ and $L = 4096$. The straight line has slope 2.

Fig. 6. The distribution of avalanche diameters (a) and areas (b) integrated over all driving forces. The straight line has slope -2 for (a) and $-\frac{3}{2}$ for (b).

Fig. 7. The number distribution of displacement increments $\Delta u$ for all driving forces below threshold for $L = 4096$. The straight line has slope -1.

Fig. 8. The number distribution of avalanche diameters for fixed driving force $f = -0.063$. The dimensionless $\Delta F$ used here is $\Delta F \approx 7.0 \times 10^{-4}$. The straight line has slope $-1.05$.